

Matrix Elements and Shower Matching at NLO

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Outline

- NLO + Shower
 - NLO Corrections
 - NLO in MC
 - Going NLO: MC@NLO and POWHEG
 - Alternative proposals
- Results:
 - NLO+S vs. NLO vs. LO SMC's ; NLO+S vs. ME+S
 - POWHEG vs. PYTHIA ME in $2 \rightarrow 1$ processes
 - POWHEG vs. MC@NLO
- Single top in POWHEG: flavour and singularities separation
- Full automation: the POWHEG BOX
- Conclusions

NLO Calculations

SMC with ME-corrections are only leading order accurate. Scale uncertainty

$$\alpha_s^n(2\mu) \approx \alpha_s^n(\mu)(1 - b_0\alpha_s(\mu)\log(4))^n \approx \alpha_s(\mu)(1 - n\alpha_s(\mu))$$

For $\mu = 100 \text{ GeV}$, $\alpha_s = 0.12$;
uncertainty:

$W + 1J$	$W + 2J$	$W + 3J$
$\pm 12\%$	$\pm 24\%$	$\pm 36\%$

To improve on this, need to go to **NLO**

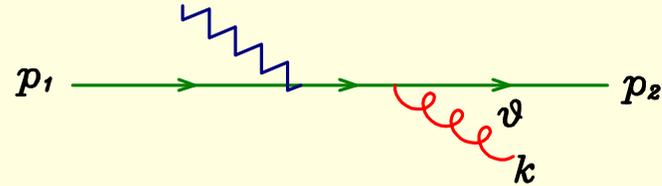
Positive experience with NLO calculations at LEP, HERA, Tevatron
(we TRUST perturbative QCD after LEP!).

Huge NLO effort towards LHC physics

But: NLO results are cumbersome and unfriendly: typically made up of an n -body (Born+Virtual+Soft and Collinear remnants) and $n + 1$ body (real emission) terms, both divergent (finite only when summed up).

Simple example: Z production

“Real” contribution to $q \bar{q} \rightarrow Z + X$:



$$\frac{C_F g_Z^2 g_s^2}{N_c 32\pi^2} \frac{1}{S} [2(1+y^2)\xi^2 + 8(1-\xi)] \left\{ \frac{1}{2} \left(\frac{1}{\xi} \right)_+ \left[\left(\frac{1}{1-y} \right)_+ + \left(\frac{1}{1-y} \right)_- \right] \right\} d\xi dy dY_Z$$

where

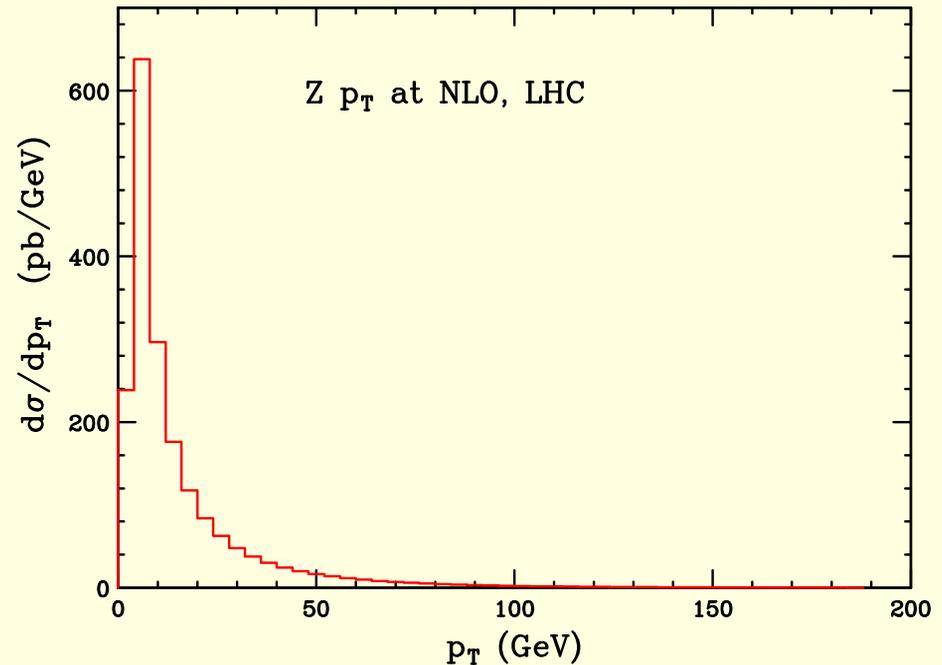
- Y_Z is the Z rapidity
- $y = \cos \theta$, θ being the emission angle of the gluon in the partonic CM
- $\xi = 2k^0/\sqrt{s}$ in the partonic CM ($s = (p_1 + p_2)^2$)

$$\left(\frac{1}{\xi} \right)_+ = \lim_{\epsilon \rightarrow 0} \left[\frac{1}{\xi + \epsilon} - \log \frac{1}{\epsilon} \delta(\xi) \right]; \quad \int_0^1 \left(\frac{1}{\xi} \right)_+ = 0; \quad \int_{-1}^1 \left(\frac{1}{1 \pm y} \right)_+ = 0,$$

Divergent contributions to the cross section for $p_T^Z > 0$ (i.e. $\xi > 0, 1 \pm y > 0$), compensated by negative divergences (i.e. $\delta(\xi), \delta(1 \pm y)$ terms) at $p_T^Z = 0$, that arise from the virtual corrections.

p_T^Z at NLO:

Negative contributions at $p_T^Z = 0$ compensate the diverging real contributions. For small enough histogram bins the first bin will always **turn negative!**



A negative bin means: $\mathcal{O}(\alpha_s)$ corrections larger than Born term:
cannot trust perturbation theory!

One should carefully decide the appropriate bin size around the origin.

For more complex processes this becomes a **requirement on jet parameters.**

So: merging Showers and NLO is **not only** a problem of overcounting (as with LO matrix elements and showers). Some sort of **resummation** of the **diverging virtual corrections** should be carried out, in order to get sensible results in the dangerous regions of collinear and soft emissions.

The key to the solution: the dangerous region is well described by the factorization formula. For example, for $y \rightarrow 1$ our cross section becomes

$$\frac{C_F g_Z^2 g_s^2}{N_c 16\pi^2} \frac{1}{S} \left[\frac{x^2 + 1}{(1-x)_+} \right] \frac{dy}{1-y} d\xi dy dY_W, \quad \text{with } x = 1 - \xi$$

The problem of diverging negative virtual corrections is dealt with and solved in the Shower formalism.

In the following: **assume that the hardest SMC radiation is the first one**, i.e. that the Shower is ordered in relative p_T . We deal later with the subtle issue of the choice of the ordering variable.

Look back at the cross section for the first emission in a Shower Monte Carlo

$$d\sigma = d\Phi_B B(\Phi_B) \left(\underbrace{\Delta_{t_I, t_0}}_{\text{No radiation}} + \sum_{(jk)} \underbrace{\Delta_{t_I, t} \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}}_{\text{radiation}} \right)$$

- t_I is the maximum hardness allowed initially, t_0 is the minimum hardness of emission
- $\Delta_{t_I, t}$ is the no-radiation probability with hardness $> t$

$$\Delta_i(t_I, t) = \exp \left[- \sum_{(jk)} \int_t^{t_I} \frac{dt'}{t'} \int dz \frac{\alpha_s(t')}{2\pi} P_{i,jk}(z) \right]$$

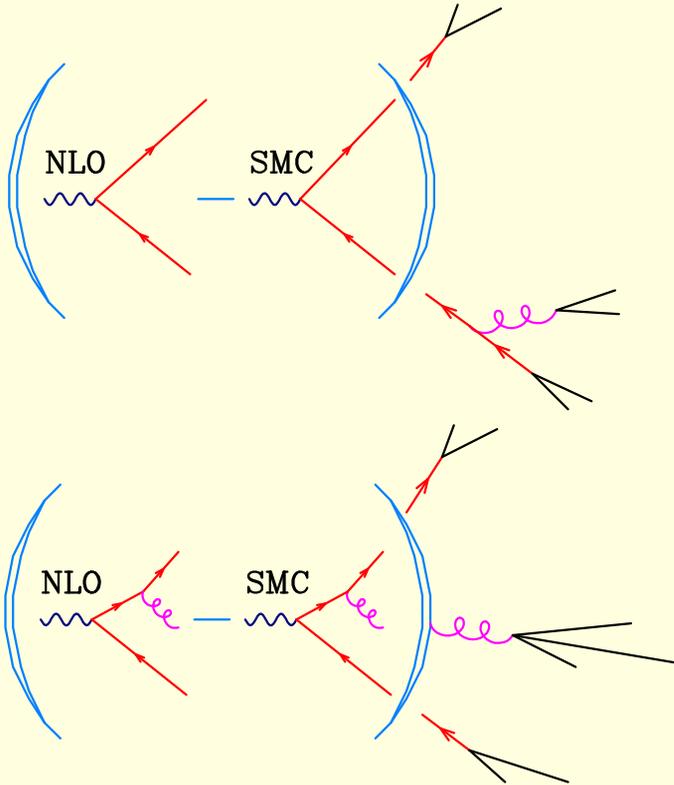
Expand the Shower formula at order $\mathcal{O}(\alpha_s)$:

$$d\sigma = d\Phi_B B(\Phi_B) \left(1 - \underbrace{\sum_{(jk)} \int_{t_0}^{t_I} \frac{dt'}{t'} \int dz \frac{\alpha_s}{2\pi} P_{i,jk}(z)}_{\text{virtual}} + \sum_{(jk)} \underbrace{\frac{\alpha_s}{2\pi} P_{i,jk}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}}_{\text{real}} \right)$$

As in the NLO calculation, we have a **negative divergent contribution** for **no radiation**, and a **positive divergent contribution** for **radiation**.
The divergence cancels for inclusive cross sections.

So: the SMC has his own **approximate** NLO virtual and real terms. To get NLO accuracy these terms should be modified to yield **the exact NLO**.

First solution: MC@NLO (2002, Frixione+Webber)



Add difference between **exact NLO** and **approximate (MC) NLO**.

- Must use MC kinematics
- Difference should be regular (if the MC is OK)
- Difference may be **negative**

Several collider processes already there:
Vector Bosons, Vector Bosons pairs,
Higgs, Single Top (also with W),
Heavy Quarks, Higgs+ W/Z .

How it works (roughly)

The cross section for the hardest event in MC@NLO is

$$d\sigma = \underbrace{\bar{B}^{\text{MC}}(\Phi_B)}_{S \text{ event}} d\Phi_B \left[\underbrace{\Delta_{t_0}^{\text{MC}} + \Delta_t^{\text{MC}} \frac{R^{\text{MC}}(\Phi)}{B(\Phi_B)} d\Phi_r^{\text{MC}}}_{\text{MC shower}} \right] + \left[\underbrace{R(\Phi) - R^{\text{MC}}(\Phi)}_{H \text{ event}} \right] d\Phi$$

$$\bar{B}^{\text{MC}}(\Phi_B) = B(\Phi_B) + \underbrace{\left[\underbrace{V(\Phi_B)}_{\text{infinite}} + \underbrace{\int R^{\text{MC}}(\Phi) d\Phi_r^{\text{MC}}}_{\text{infinite}} \right]}_{\text{finite}}$$

Imagine that soft and collinear singularities in R^{MC} are regulated as in V .

The full phase space Φ is parametrized in terms of the Born phase space Φ_B and the radiation variables of the MC Φ_r^{MC} (typically z, t, ϕ), according to the MC procedure (reshuffling) that yields Φ from Φ_B and Φ_r^{MC} .

B : Born cross section; V : exact virtual cross section.

R^{MC} : radiation cross section in the MC, typically: $R^{\text{MC}} = B \frac{1}{t} \frac{\alpha}{2\pi} P(z)$

R : exact radiation cross section;

We can check that the $\mathcal{O}(\alpha_s)$ expansion of $d\sigma$ coincides with the exact NLO;

$$d\sigma = \bar{B}^{\text{MC}}(\Phi_B) d\Phi_B \left[\Delta_{t_0}^{\text{MC}} + \Delta_t^{\text{MC}} \frac{R^{\text{MC}}(\Phi)}{B(\Phi_B)} d\Phi_r^{\text{MC}} \right] + [R(\Phi) - R^{\text{MC}}(\Phi)] d\Phi$$

$$\bar{B}^{\text{MC}}(\Phi_B) = B(\Phi_B) + \left[V(\Phi_B) + \int R^{\text{MC}}(\Phi) d\Phi_r^{\text{MC}} \right]$$

Expand:

$$d\sigma = \left[B + V + \int R^{\text{MC}} d\Phi_r^{\text{MC}} \right] d\Phi_B \left[1 - \int \frac{R^{\text{MC}}}{B} d\Phi_r^{\text{MC}} + \frac{R^{\text{MC}}}{B} d\Phi_r^{\text{MC}} \right] + [R - R^{\text{MC}}] d\Phi$$

$$= [B + V] d\Phi_B + B d\Phi_B \left[\int \frac{R^{\text{MC}}}{B} d\Phi_r^{\text{MC}} - \int \frac{R^{\text{MC}}}{B} d\Phi_r^{\text{MC}} + \frac{R^{\text{MC}}}{B} d\Phi_r^{\text{MC}} \right] + [R - R^{\text{MC}}] d\Phi$$

$$= [B + V] d\Phi_B + B d\Phi_B \left[\frac{R^{\text{MC}}}{B} d\Phi_r^{\text{MC}} \right] + [R - R^{\text{MC}}] d\Phi = [B + V] d\Phi_B + R d\Phi$$

Recipe for MC@NLO

- Compute totals for S and H events:

$$\sigma_S = \int |\bar{B}^{\text{MC}}(\Phi_B)| d\Phi_B, \quad \sigma_H = \int |R - R^{\text{MC}}| d\Phi$$

- Chose an S or H event with probability proportional to σ_S, σ_H
- For an S event:

- generate Born kinematics with probability

$$|\bar{B}^{\text{MC}}(\Phi_B)| = \left| B(\Phi_B) + \left[V(\Phi_B) + \int R^{\text{MC}}(\Phi) d\Phi_r^{\text{MC}} \right] \right|$$

- Feed the Born kinematics to the MC for subsequent shower with weight ± 1 , same sign as $\bar{B}^{\text{MC}}(\Phi_B)$.

- For an H event:

- generate Radiation kinematics with probability $|R - R^{\text{MC}}|$.
- Feed to the MC (with weight ± 1 , same sign as $R - R^{\text{MC}}$)

Issues:

- Must use of the MC kinematic mapping $(\Phi_B, \Phi_r^{\text{MC}}) \Rightarrow \Phi$.
- $R - R^{\text{MC}}$ must be non singular: the MC must reproduce exactly the soft and collinear singularities of the radiation matrix element. (Many MC **are not** accurate in the soft limit)
- The cancellation of divergences in the expression of \bar{B}^{MC} is taken care of in the framework of the subtraction method (cancellation of divergences under the integral sign) so that the integral in \bar{B}^{MC} becomes in fact convergent.
- Negative weights in the output (not like standard MC's).

an alternative solution: POWHEG

Positive Weight Hardest Emission Generator

Method to generate the hardest emission first, with NLO accuracy, and independently of the SMC (P.N. 2004).

- SMC independent; no need of SMC expert; same calculation can be interfaced to several SMC programs with no extra effort
- SMC inaccuracies in the soft region only affect next-to-hardest emissions; no matching problems
- As the name says, it generates events with positive weight

How it works (roughly)

In words: works like a standard Shower MC for the hardest radiation, with care to maintain higher accuracy.

In a standard MC, the hardest radiation cross section is

$$d\sigma = d\Phi_B B(\Phi_B) \left(\underbrace{\Delta_{t_I, t_0}}_{\text{No radiation}} + \underbrace{\Delta_{t_I, t} \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}}_{\text{radiation}} \right)$$

- t_I is the maximum hardness allowed initially
- $\Delta_{t_I, t}$ in the no-radiation probability with hardness $> t$

SMC algorithm reconstructs from Born kinematics Φ_B and radiation variables t, z, ϕ , the full radiation phase space Φ (momentum reshuffling)

We say that Φ_B is the underlying Born configuration of Φ according to the mapping defined by the MC algorithm

Steps to go NLO:

$$(\Phi_B, t, z, \phi) \Leftrightarrow \Phi \quad \Longrightarrow \quad (\Phi_B, \Phi_r) \Leftrightarrow \Phi, \quad d\Phi = d\Phi_B d\Phi_r$$

$$B(\Phi_B) \quad \Longrightarrow \quad \bar{B}(\Phi_B) = B(\Phi_B) + \underbrace{\left[\overbrace{V(\Phi_B)}^{\text{INFINITE}} + \int \overbrace{R(\Phi_B, \Phi_r)}^{\text{INFINITE}} d\Phi_r \right]}_{\text{FINITE!}}$$

$$\frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi} \quad \Longrightarrow \quad \frac{R(\Phi_B, \Phi_r)}{B(\Phi_B)} d\Phi_{\text{rad}}$$

POWHEG cross section:

$$d\sigma = d\Phi_B \bar{B}(\Phi_B) \left[\Delta_{t_0} + \Delta_t \frac{R(\Phi)}{B(\Phi_B)} d\Phi_r \right], \quad \Delta_t = \exp \left[- \underbrace{\int \theta(t_r - t) \frac{R(\Phi_B, \Phi_r)}{B(\Phi_B)} d\Phi_r}_{\text{FINITE because of } \theta \text{ function}} \right]$$

with $t_r = k_T(\Phi_B, \Phi_r)$, the transverse momentum for the radiation.

In the collinear limit, k_t^2 must be of the order of t .

How does it work: $d\sigma = d\Phi_B \bar{B}(\Phi_B) \left[\Delta_{t_0} + \Delta_t \frac{R(\Phi)}{B(\Phi_B)} d\Phi_r \right],$

For small k_T , the factorization theorem yields

$$\frac{R(\Phi)}{B(\Phi_B)} d\Phi_{\text{rad}} \approx \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}$$

and

$$\bar{B} \approx B \times (1 + \mathcal{O}(\alpha_s))$$

Thus: all features of **SMC**'s are preserved at small k_T .

For large k_T , $\Delta \rightarrow 1$,

$$d\sigma = \bar{B} \times \frac{R}{B} \approx R \times (1 + \mathcal{O}(\alpha_s)),$$

so large k_t accuracy is preserved. Integrating in $d\Phi_r$ at fixed Φ_B

$$\int \delta(\Phi_B - \bar{\Phi}_B) d\sigma = \bar{B}(\bar{\Phi}_B)$$

So NLO accuracy is preserved for inclusive quantities.

Example of mapping $\Phi \Leftrightarrow (\Phi_B, \Phi_r)$: Z pair production

Φ_B variables: choose M_{ZZ} , Y_{ZZ} and θ , where

- M_{ZZ} : invariant mass of the ZZ pair
- Y_{ZZ} : rapidity of ZZ pair
- θ : go in the (longitudinally) boosted frame where $Y_{ZZ} = 0$.
go to the ZZ rest frame with a transverse boost
In this frame θ is the angle of a Z to the longitudinal direction.

Φ_r variables:

- $x = M_{ZZ}/s$, (s is the invariant mass of the incoming parton system)
 $x \rightarrow 1$ is the soft limit
- y : cosine of the angle of the radiated parton to the beam direction
in the partonic CM frame.
- ϕ : radiation azimuth.

Few tricks

Both in MC@NLO and POWHEG, integrals of the form

$$\bar{B}(\Phi_B) = B(\Phi_B) + \underbrace{\left[\overbrace{V(\Phi_B)}^{\text{INFINITE}} + \int \overbrace{R(\Phi_B, \Phi_r) d\Phi_r}^{\text{INFINITE}} \right]}_{\text{FINITE!}}$$

are expressed within the subtraction method as

$$\bar{B}(\Phi_B) = B(\Phi_B) + V_{\text{SV}}(\Phi_B) + \int d\Phi_r [R(\Phi_B, \Phi_r) - C(\Phi_B, \Phi_r)]$$

Needs one Φ_r integrations for each Φ point!. To overcome this, we write

$$\tilde{B}(\Phi_B, \Phi_r) = \frac{B(\Phi_B) + V(\Phi_B)}{\int d\Phi_r} + R(\Phi_B, \Phi_r) - C(\Phi_B, \Phi_r), \quad \bar{B}(\Phi_B) = \int \tilde{B}(\Phi_B, \Phi_r) d\Phi_r.$$

so that

$$\bar{B}(\Phi_B) = \int \tilde{B}(\Phi_B, \Phi_r) d\Phi_r.$$

Use standard procedures (SPRING-BASES, Kawabata; MINT, P.N.) to generate unweighted events for $\tilde{B}(\bar{\Phi}, \Phi_r) d\Phi_r d\bar{\Phi}$,
discard Φ_r (same as integrating over it!).

POWHEG: Interfacing to SMC's

For a pt ordered SMC, nothing else needs to be done.

Use the standard Les Houches Interface for User's Processes (LHI):

put partonic event generated by POWHEG on the LHI;

Run the SMC in the LHI mode.

The LHI provides a facility to pass the pt of the event to the SMC (SCALUP).

As far as the hardest emission is concerned, POWHEG can reach:

- NLO accuracy of (integrated) shape variables
- Collinear, double-log, soft (large N_c) accuracy of the Sudakov FF.
(In fact, corrections that exponentiates are obviously OK)

As far as subsequent (less hard) emissions, the output has the accuracy of the SMC one is using.

Status of POWHEG

Up to now, the following processes have been implemented in POWHEG:

- $hh \rightarrow ZZ$ (Ridolfi, P.N., 2006)
- $e^+e^- \rightarrow \text{hadrons}$, (Latunde-Dada, Gieseke, Webber, 2006),
 $e^+e^- \rightarrow t\bar{t}$, including top decays at NLO (Latunde-Dada, 2008),
- $hh \rightarrow Q\bar{Q}$ (Frixione, Ridolfi, P.N., 2007)
- $hh \rightarrow Z/W$ (Alioli, Oleari, Re, P.N., 2008;)
(Hamilton, Richardson, Tully, 2008;)
- $hh \rightarrow H$ (gluon fusion) (Alioli, Oleari, Re, P.N., 2008; Herwig++)
- $hh \rightarrow H$, $hh \rightarrow HZ/W$ (Hamilton, Richardson, Tully, 2009;)
- $hh \rightarrow t + X$ (single top) **NEW** (Alioli, Oleari, Re, P.N., 2009)
- $hh \rightarrow Z + \text{jet}$, **Very preliminary** (Alioli, Oleari, Re, P.N., 2009)
- The POWHEG BOX, **Very preliminary**, (Alioli, Oleari, Re, P.N., 2009)

In practice

MC@NLO: Code and manuals at

<http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO/>

1 program for all processes

POWHEG: Codes and manuals in

<http://moby.mib.infn.it/~nason/POWHEG>

Examples are provide to link POWHEG to HERWIG or PYTHIA,
or to generate a Les Houches Event File to be fed
later to a SMC for showering.

1 program for each process

In the HERWIG++ code there are few independent implementations of MC@NLO and POWHEG processes

Alternatives

Work in progress in other groups; many attempts to formulate SMC showers that ease the task of producing a MC@NLO type implementation:

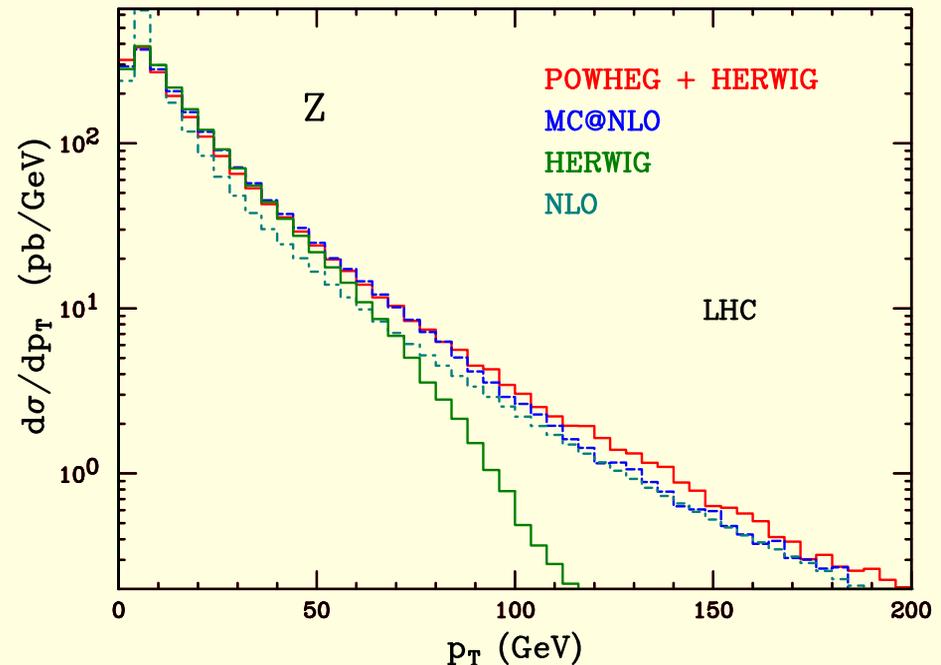
- Shower by **antenna factorization** (Frederix, Giele, Kosower, Skands)
- Shower by Catani-Seymour **dipole factorization** (Schumann)
- Shower with **quantum interference** (Nagy, Soper)
- Shower by **Soft Collinear Effective Theory** (Bauer, Schwartz)

Other approaches in e^+e^-

- Kramer, Mrenna, Soper ($e^+e^- \rightarrow 3$ partons)
- Lavesson and Lönnblad: extension of CKKW-L to NLO completely different from MC@NLO and POWHEG; uses a separation scale, and uses NLO results with jets clustered with the given scale.

Examples: Z production

HERWIG alone fails at large p_T ;
NLO alone fails at small p_T ;
MC@NLO and POWHEG work
in both regions;
Notice:
HERWIG with ME corrections
or any ME program, give the
same NLO shape at large p_T
However: Normalization around
small p_T region is incorrect
(i.e. only LO).



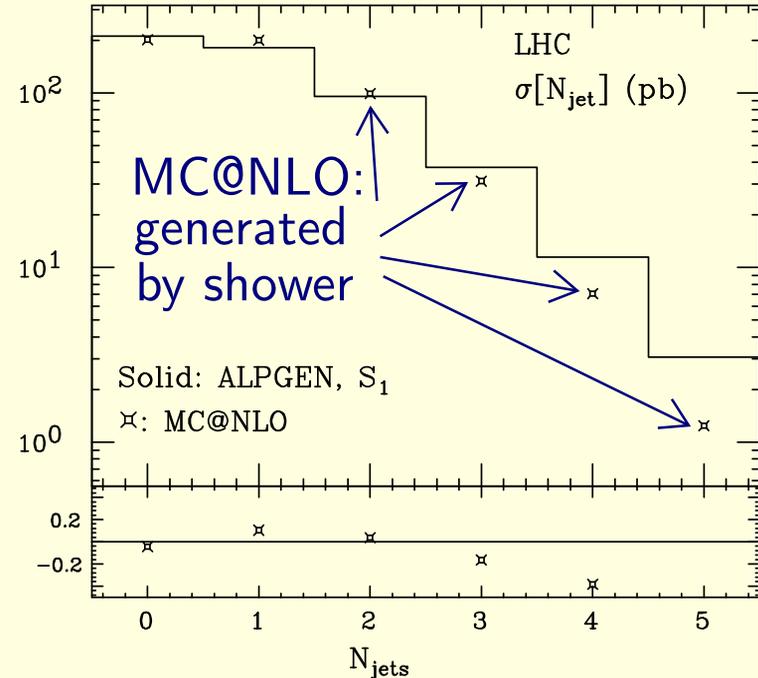
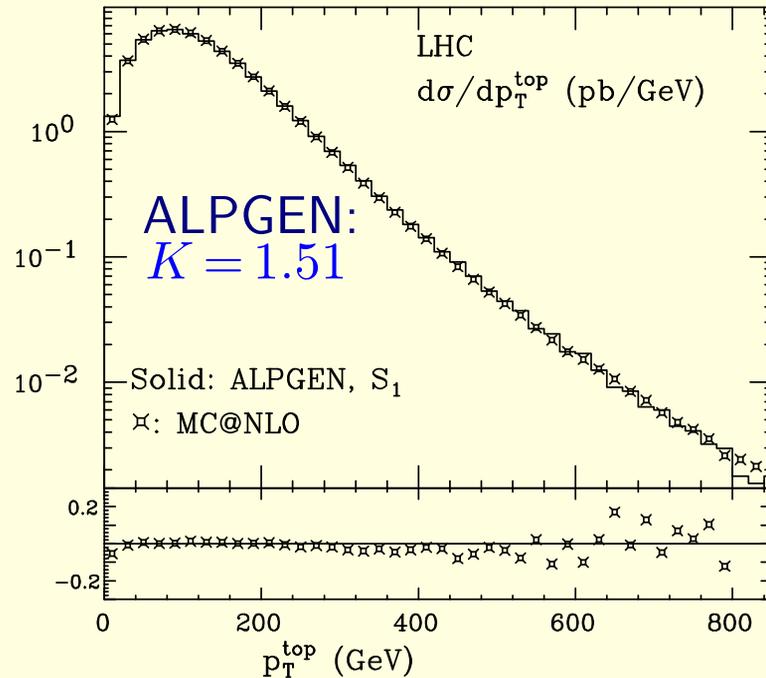
The essence of the improvement with respect to standard shower and ME matched programs is summarized in this plot.

Be careful with the misleading language: Z at LO $\mathcal{O}(1)$, NLO $\mathcal{O}(\alpha_s)$;
At $\mathcal{O}(1)$ there is no Z transverse momentum. Thus, the p_T distribution $p_T > 0$
is of $\mathcal{O}(\alpha_s)$, i.e. has leading order accuracy!

NLO+PS compared with ME programs: ALPGEN and MC@NLO in $t\bar{t}$ production

- expect:
- **Disadvantage:** worse normalization (no NLO)
 - **Advantage:** better high jet multiplicities (exact ME)

(Mangano, Moretti, Piccinini, Treccani, Nov.06)

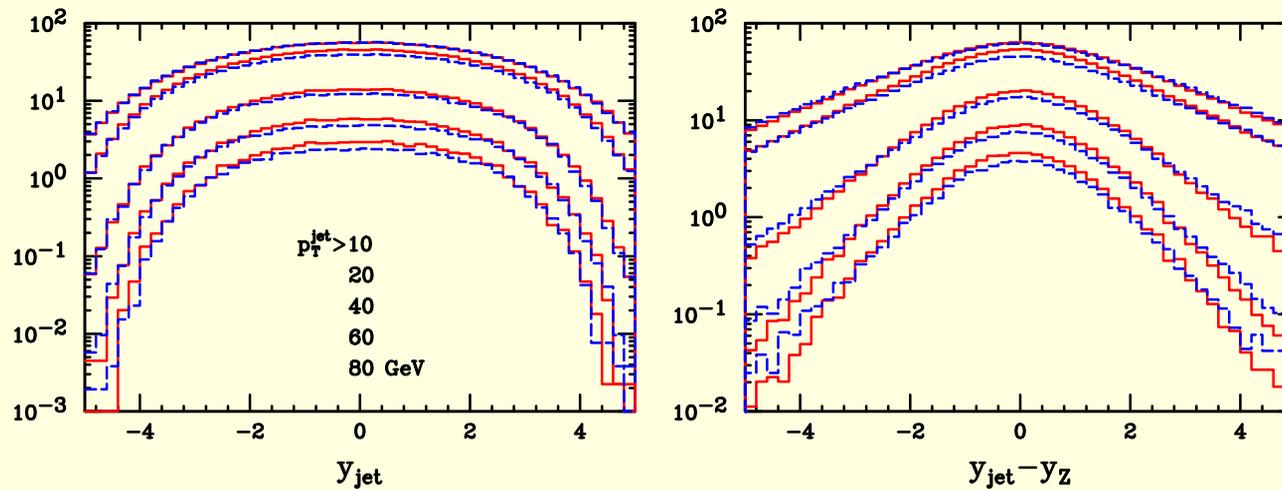


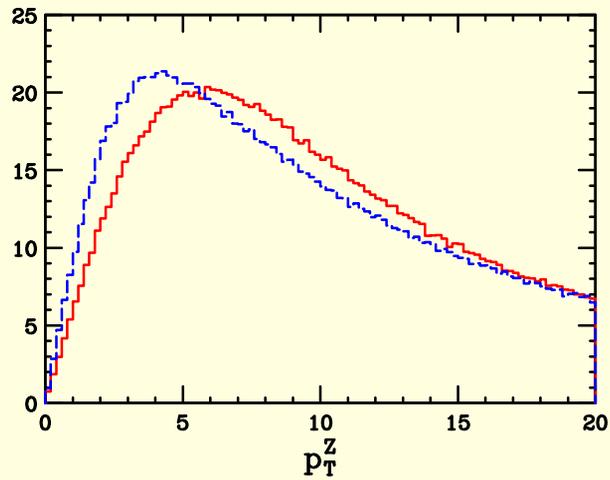
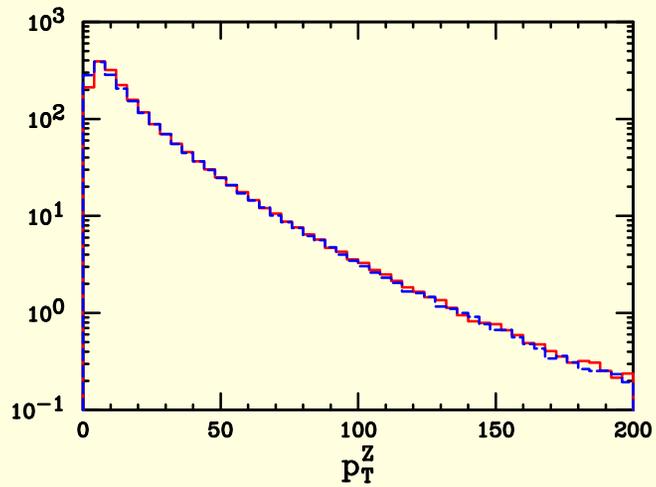
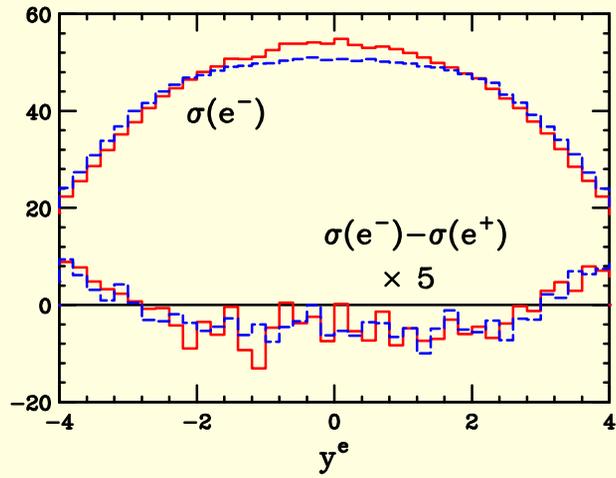
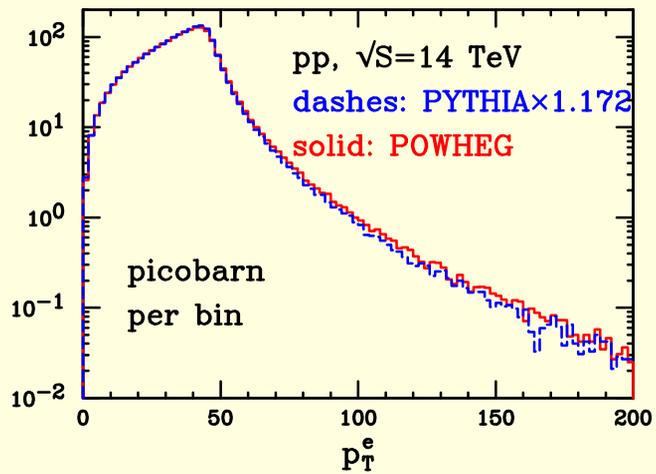
PYTHIA ME vs. POWHEG

For $2 \rightarrow 1$ processes (W/Z and Higgs production), PYTHIA ME corrections are very similar to POWHEG; it implements the formula

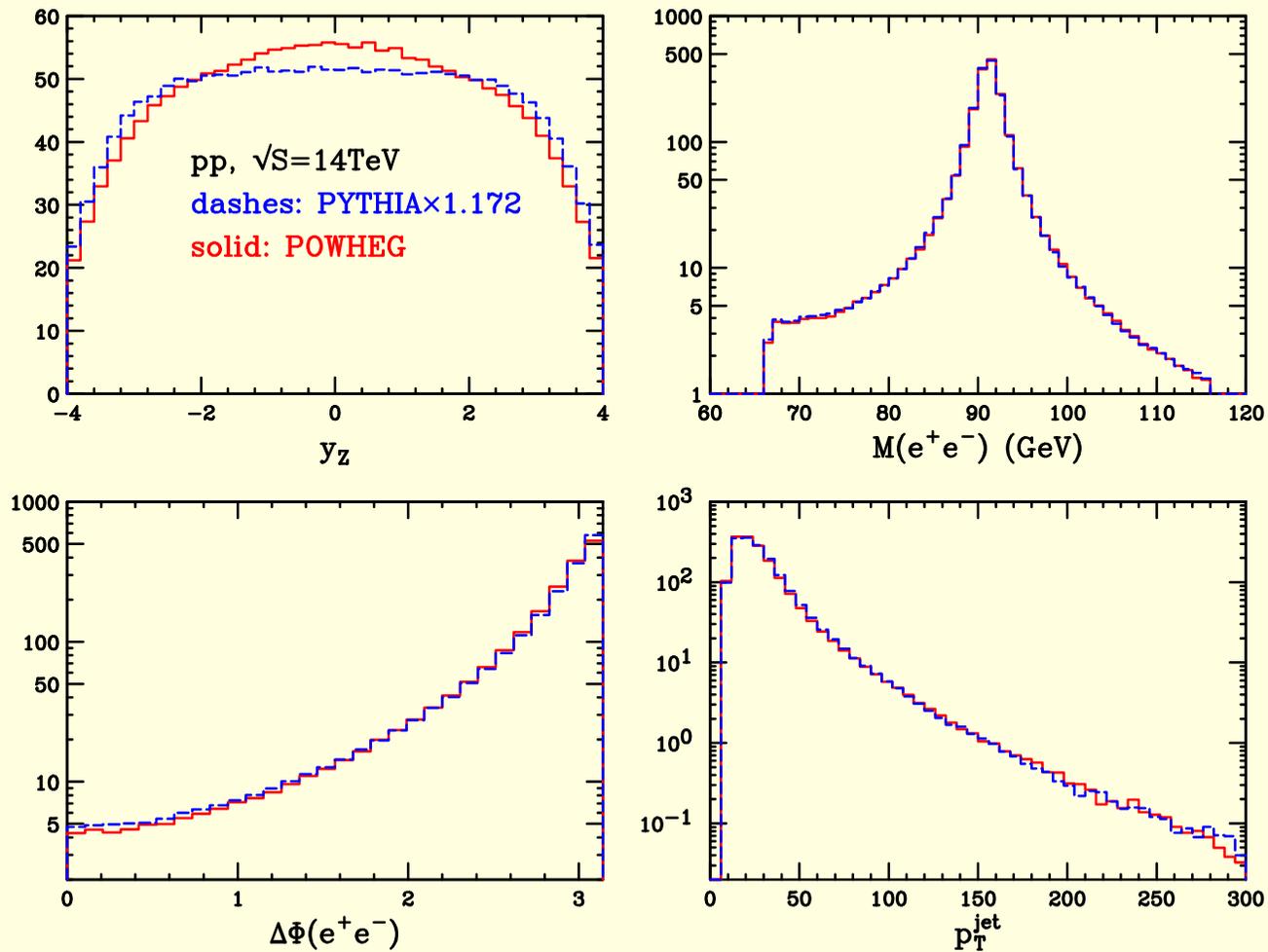
$$d\sigma = d\Phi_B \underbrace{B(\Phi_B)}_{\bar{B} \text{ in POWHEG}} \left[\Delta_{t_0} + \Delta_t \frac{R(\Phi)}{B(\Phi_B)} d\Phi_r \right], \quad \Delta_t = \exp \left[- \int \theta(t_r - t) \frac{R(\Phi_B, \Phi_r)}{B(\Phi_B)} d\Phi_r \right]$$

Dashes: PYTHIA X 1.172, Solid: POWHEG



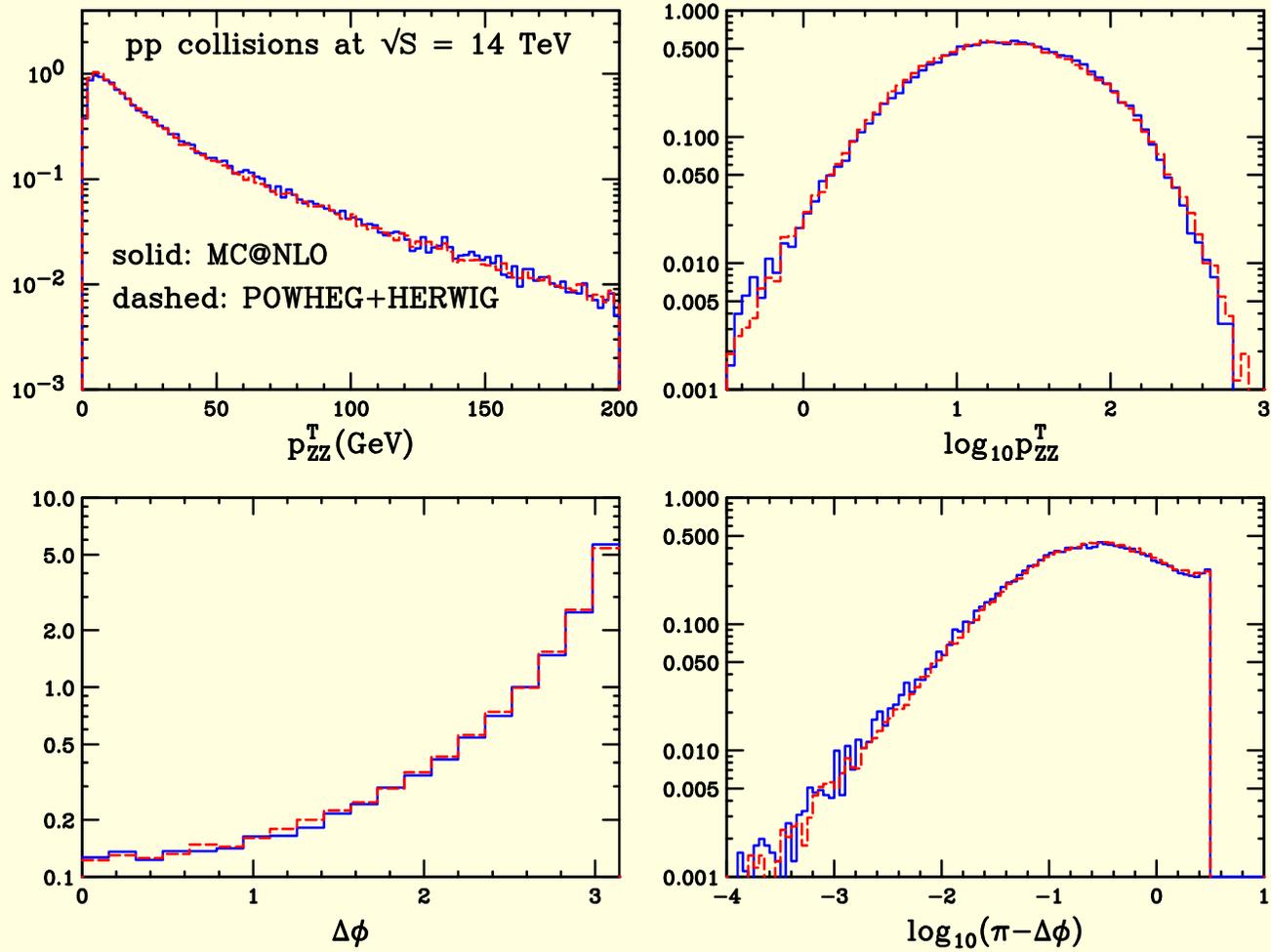


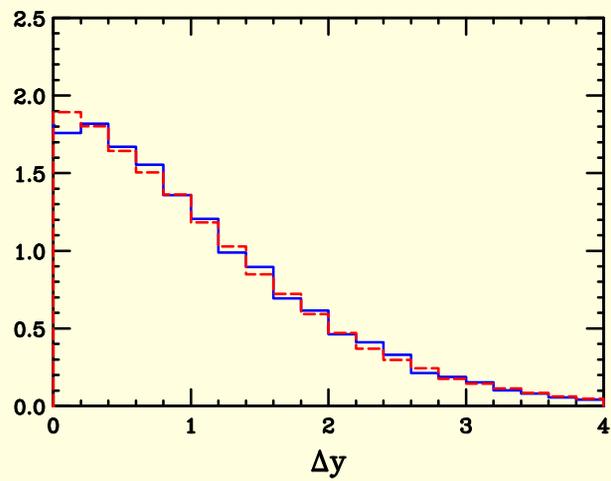
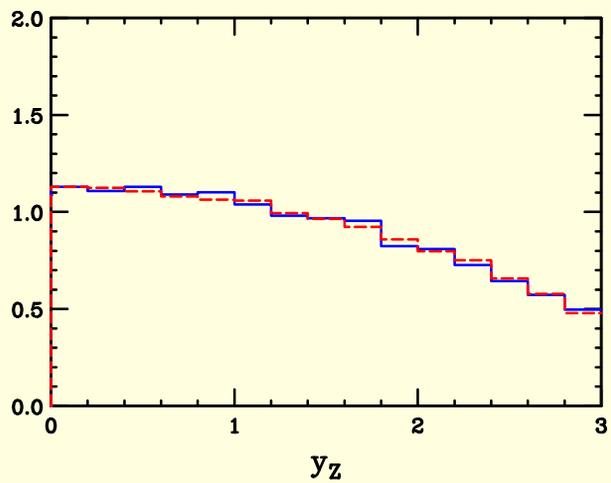
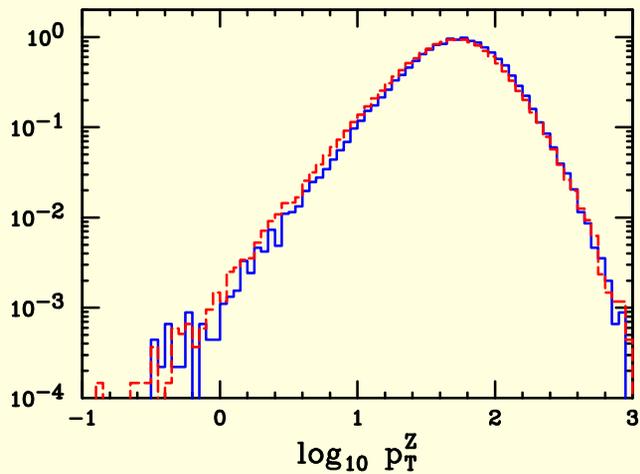
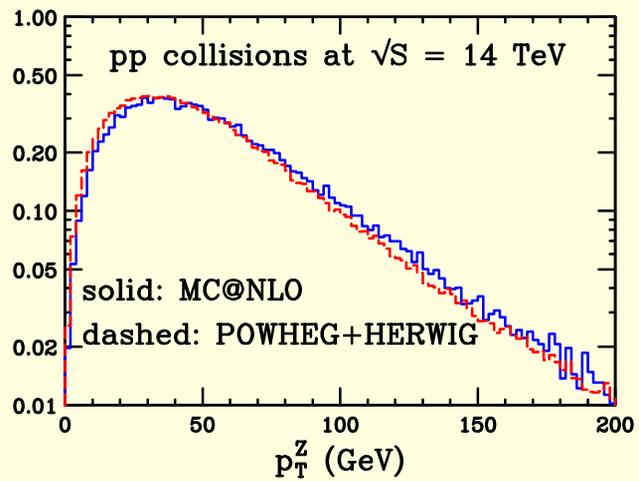
Different shape in y_Z distribution understood as NLO effect

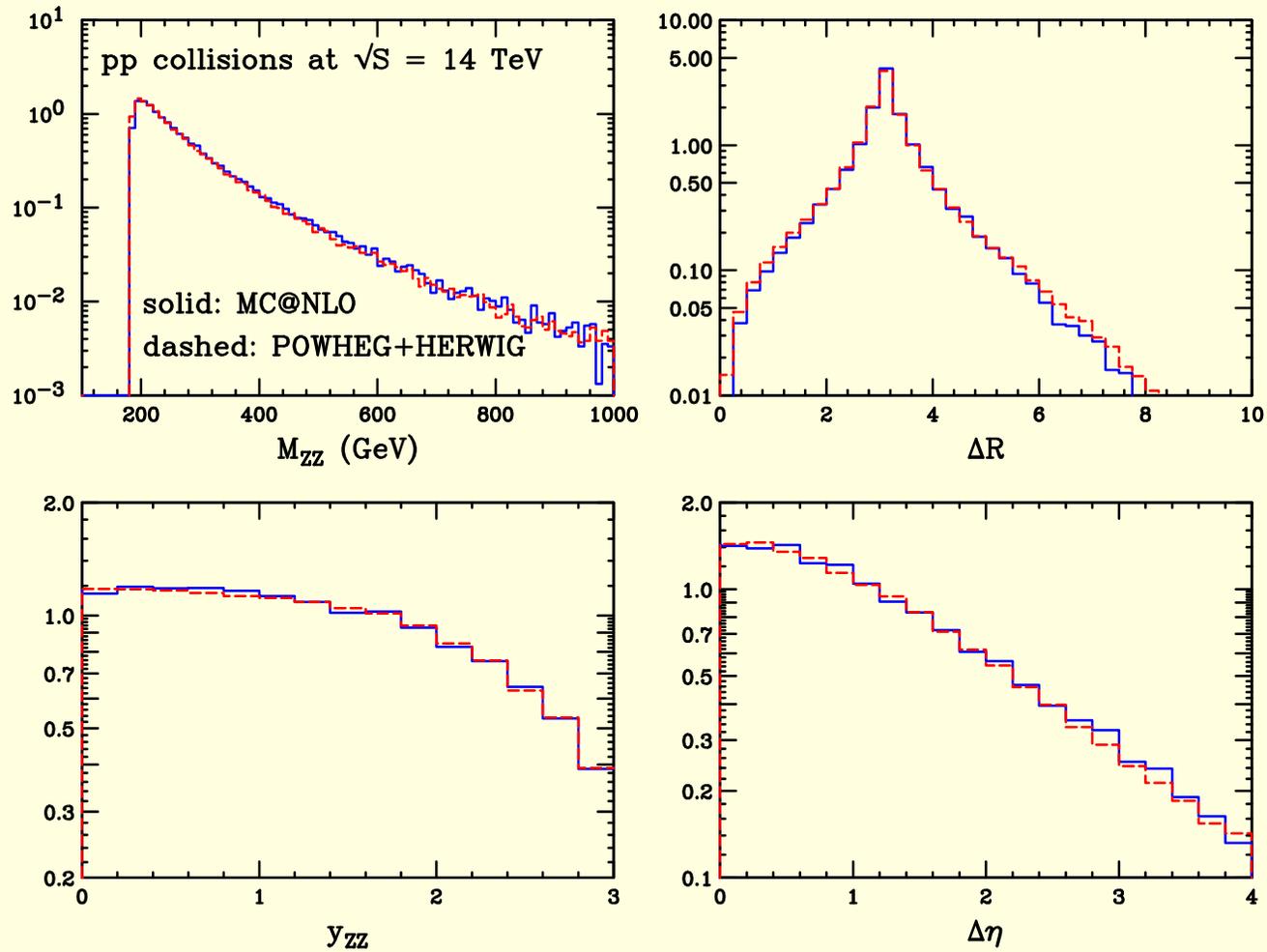


Comparisons of POWHEG+HERWIG vs. MC@NLO

Z pair production

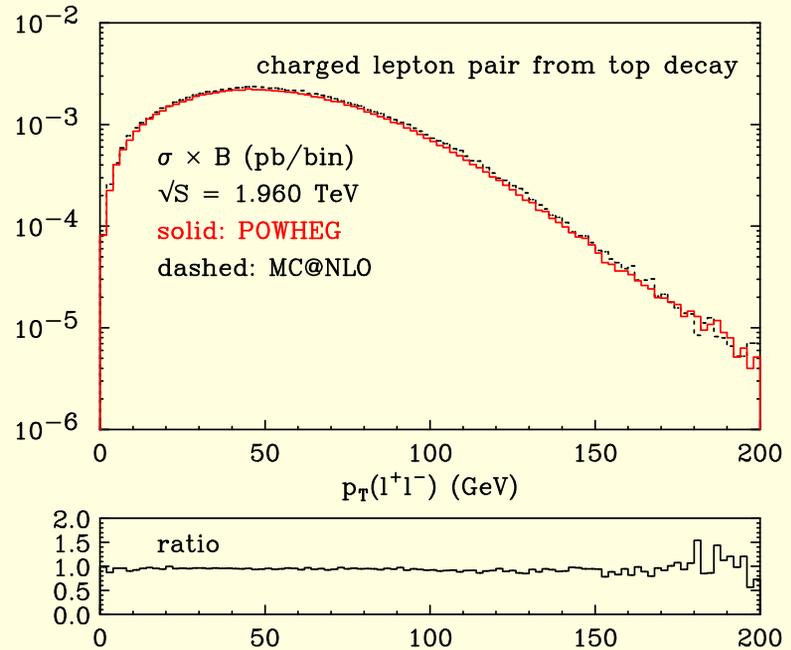
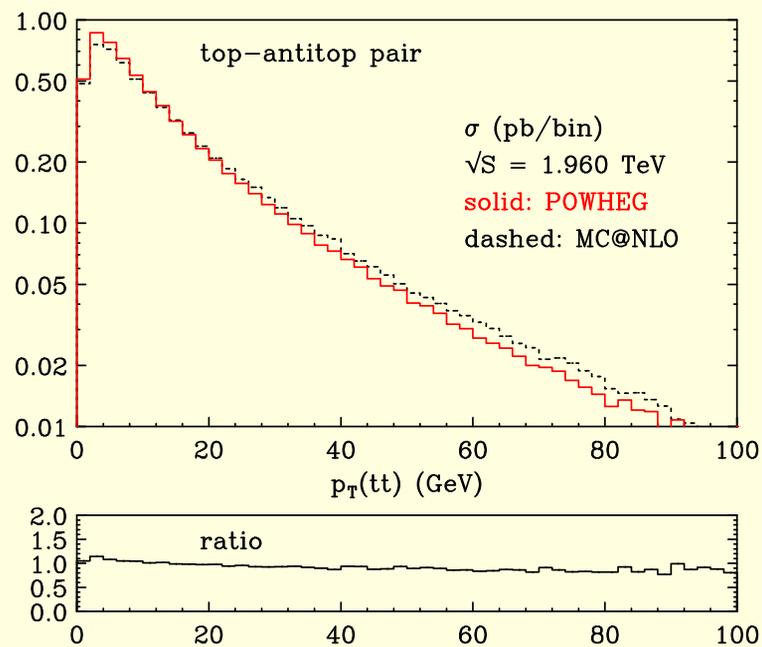






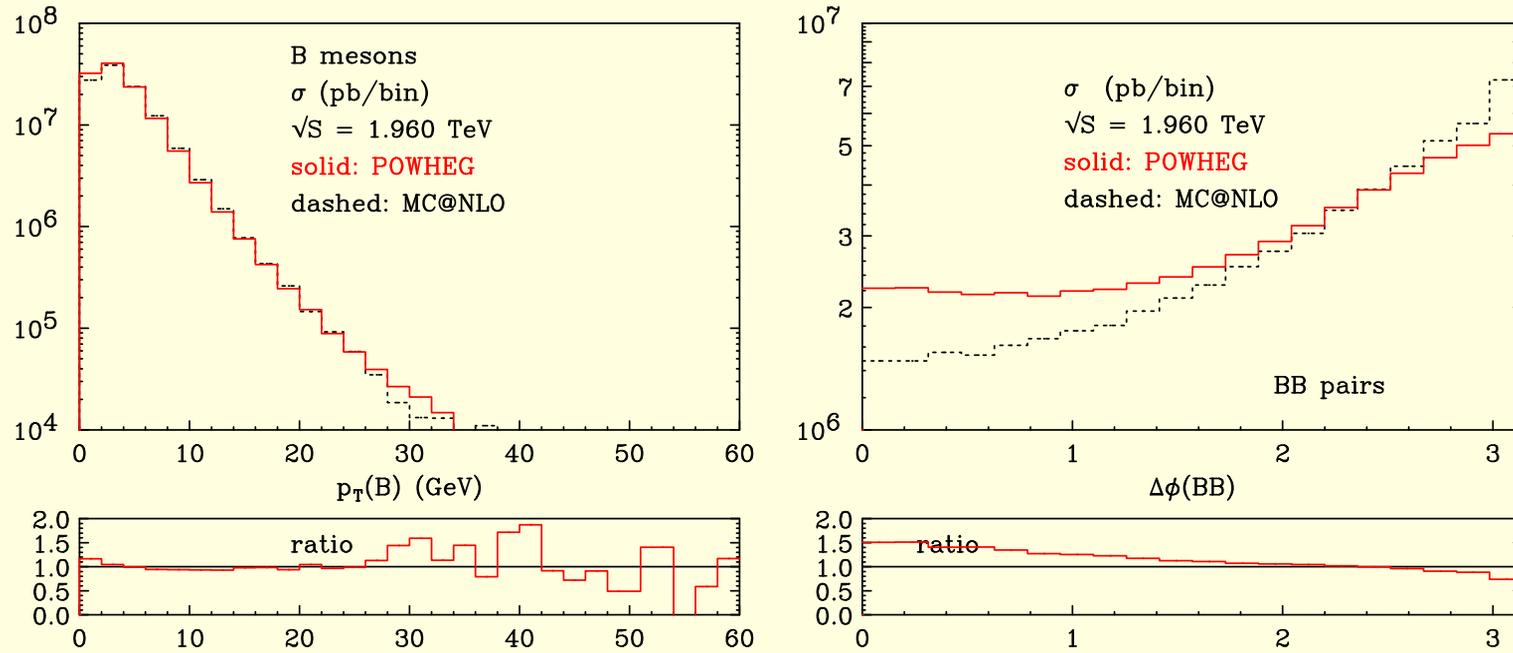
Remarkable agreement for most quantities;

POWHEG and MC@NLO comparison: Top pair production



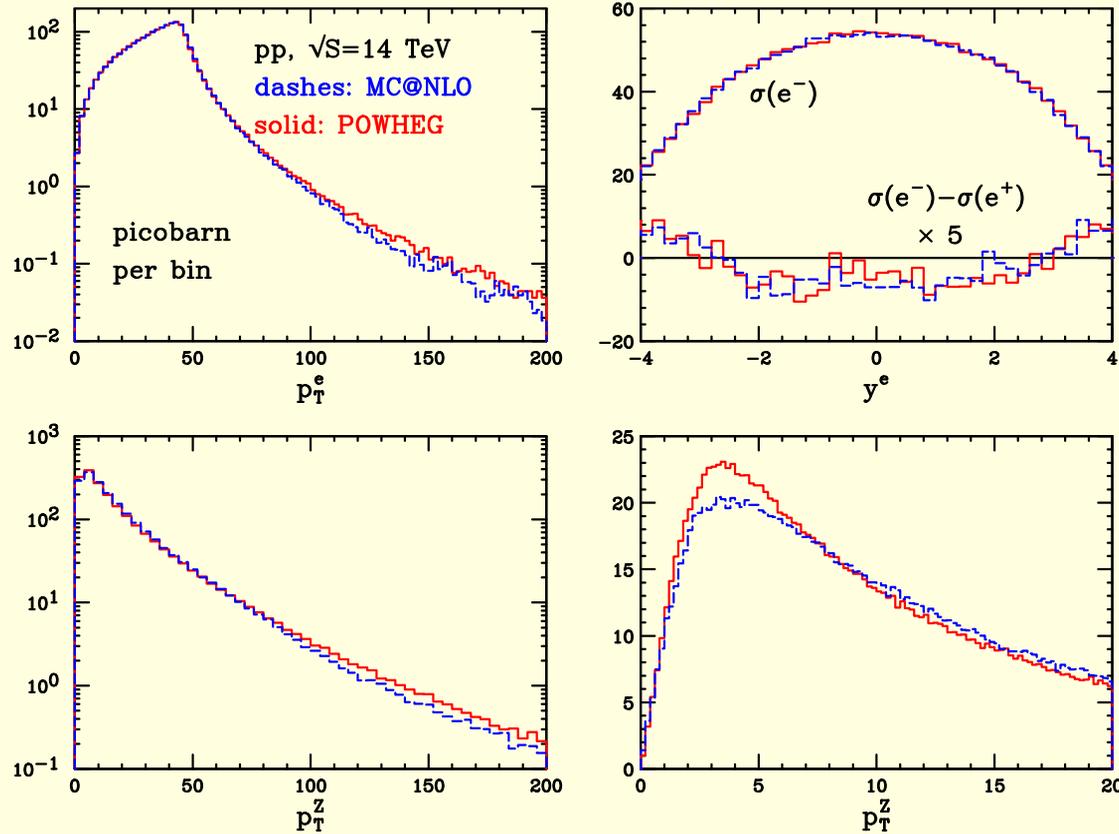
Good agreement for all observable considered
(differences can be ascribed to different treatment of higher order terms)

Bottom pair production



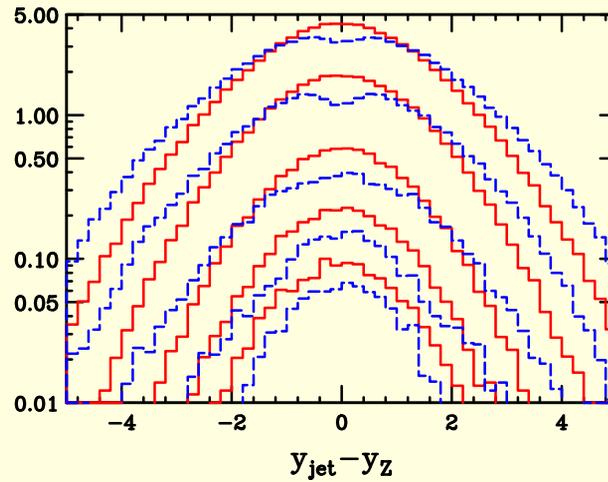
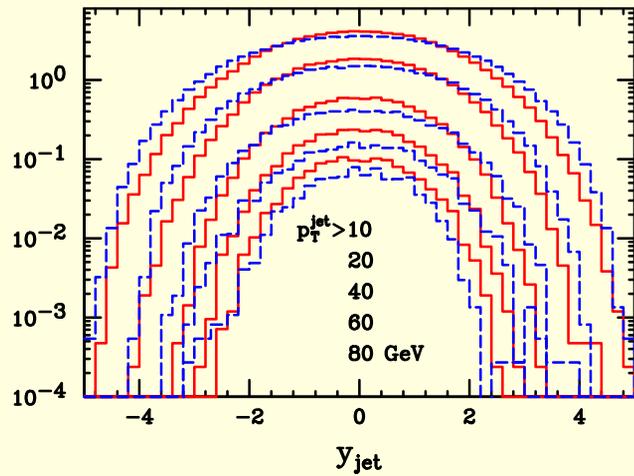
- Very good agreement For large scales (ZZ , $t\bar{t}$ production)
- Differences at small scales ($b\bar{b}$ at the Tevatron)
- POWHEG more reliable in extreme cases like $b\bar{b}$, $c\bar{c}$ at LHC (yields positive results, MC@NLO has problems with negative weights)

Z production: POWHEG+HERWIG vs. MC@NLO

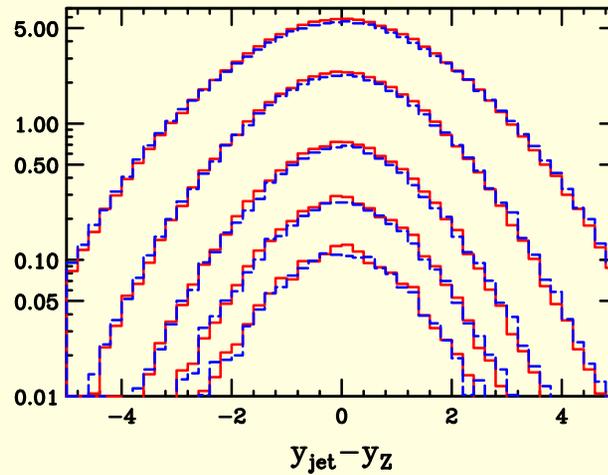
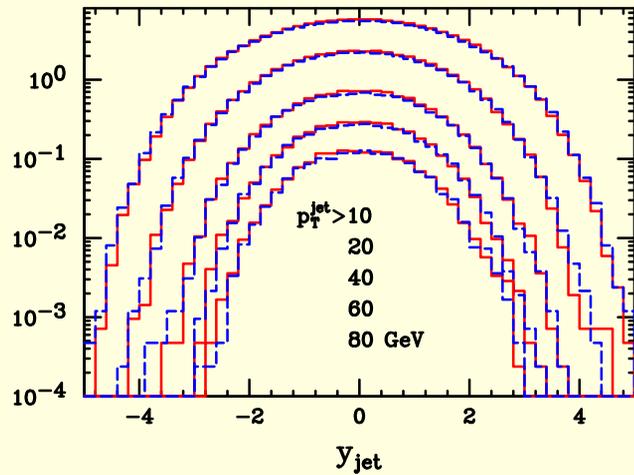


Small differences in high and low p_T region

Z production: rapidity of hardest jet (TEVATRON)

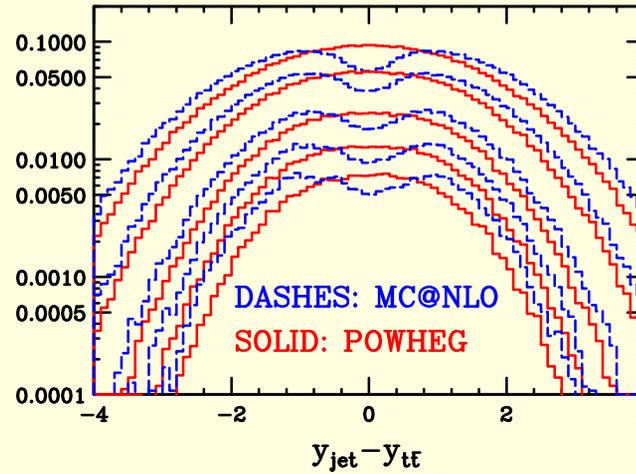
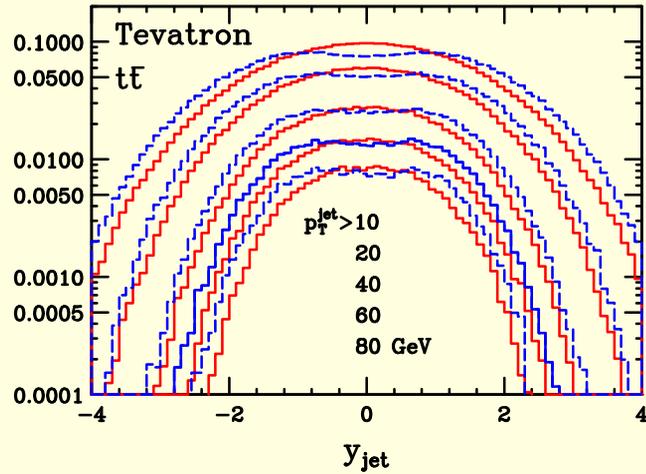


POWHEG+HERWIG
MC@NLO

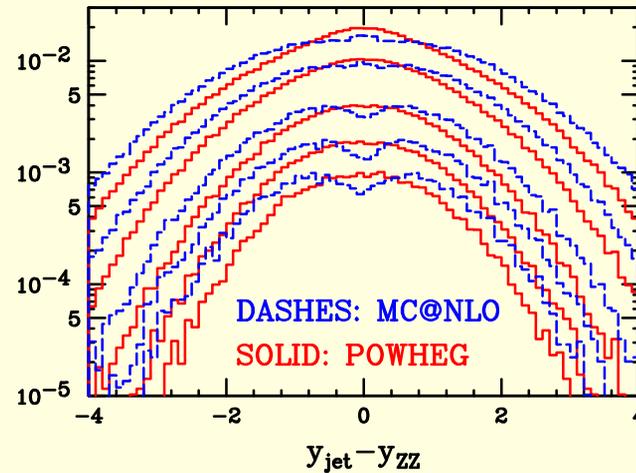
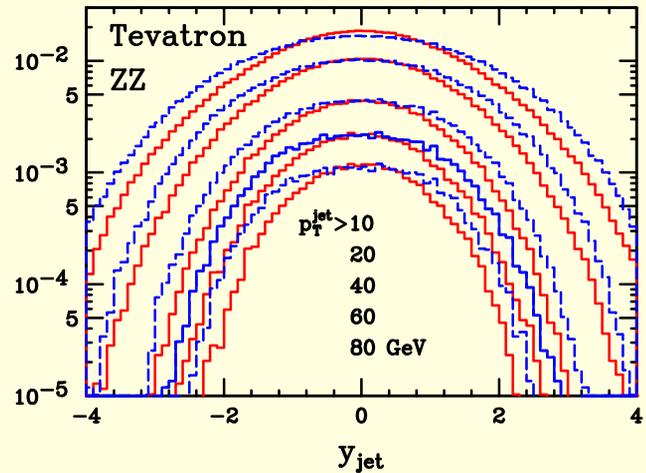


POWHEG+PYTHIA
PYTHIA

Dip in central region in MC@NLO also in $t\bar{t}$ and ZZ

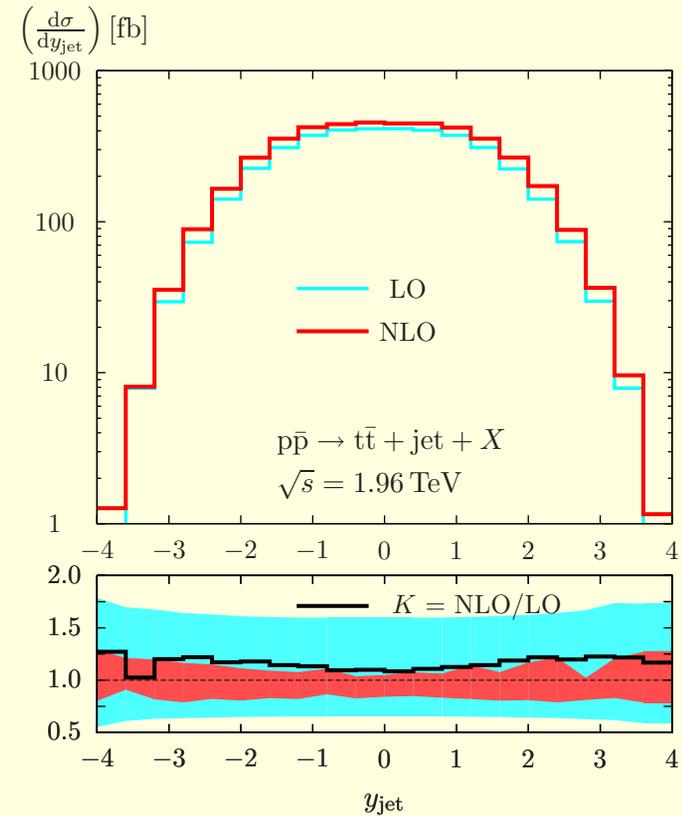
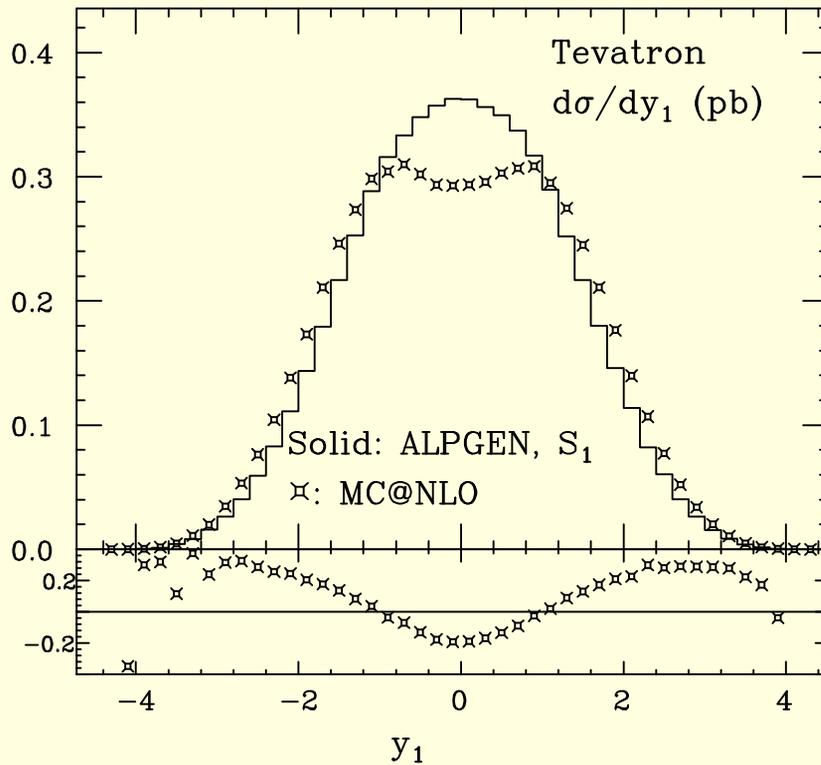


POWHEG+HERWIG
MC@NLO



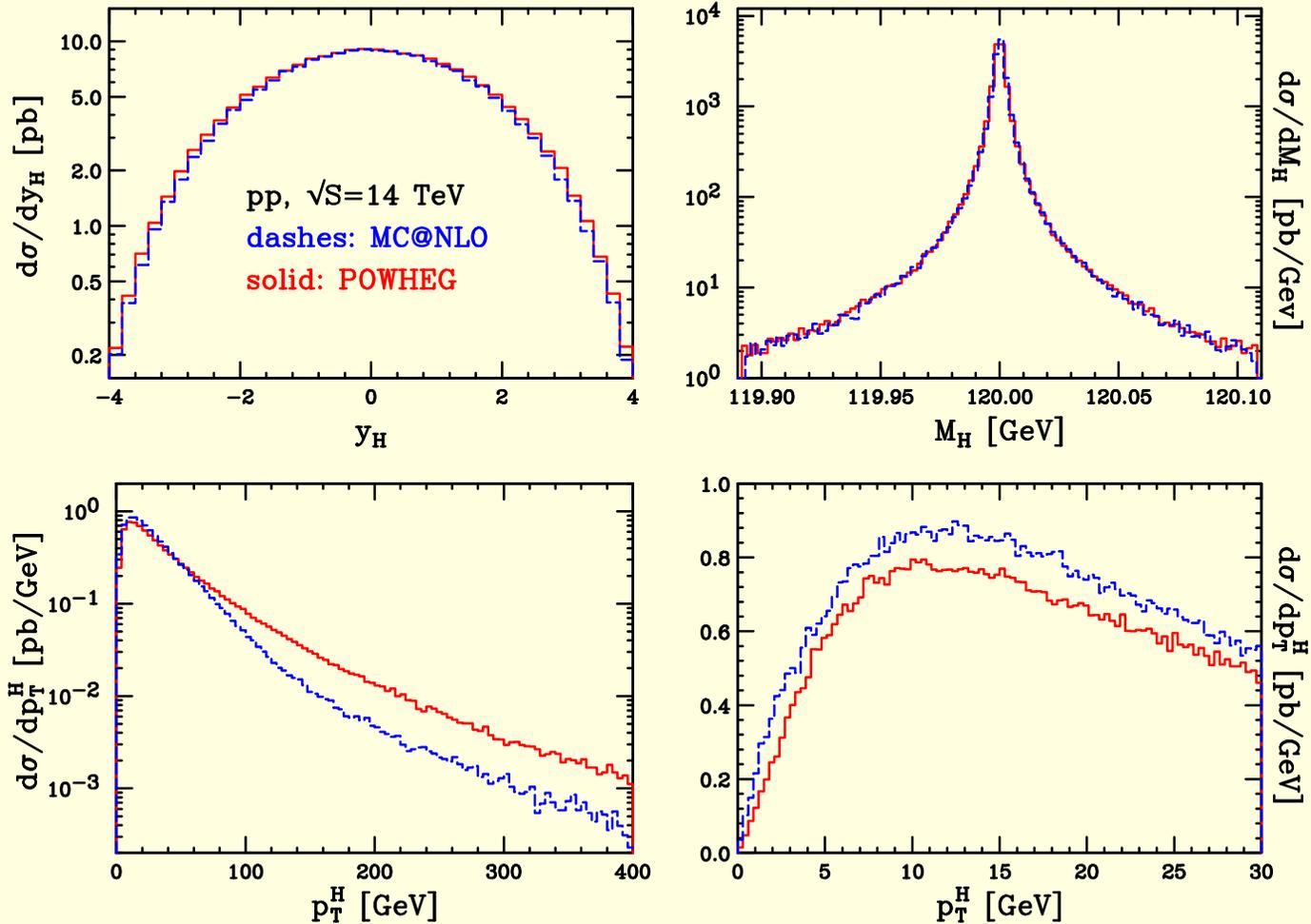
POWHEG+HERWIG
MC@NLO

ALPGEN and $t\bar{t} + \text{jet}$ at NLO vs. MC@NLO

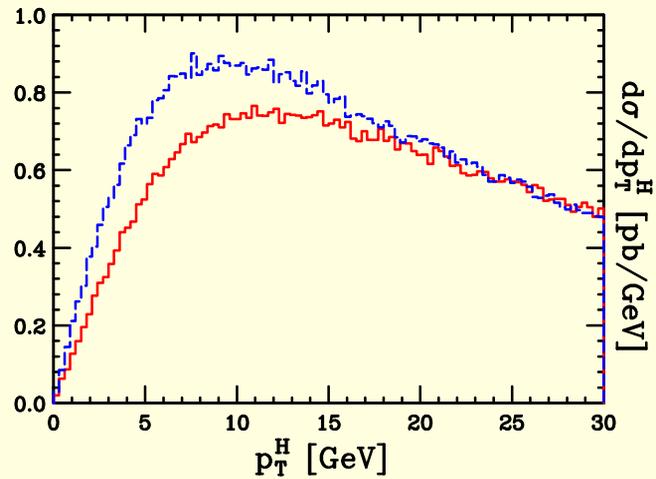
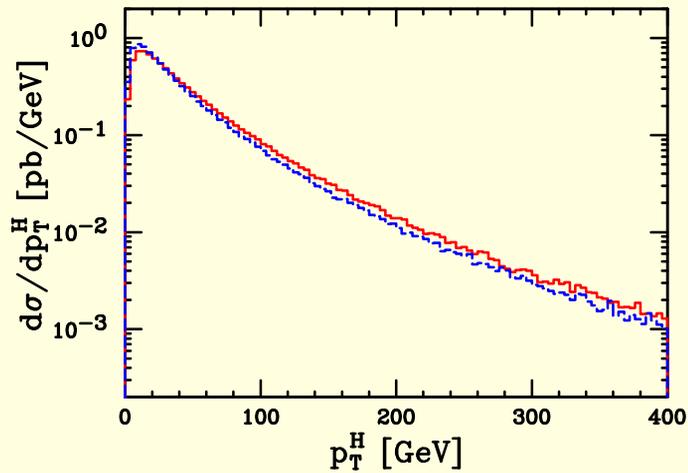
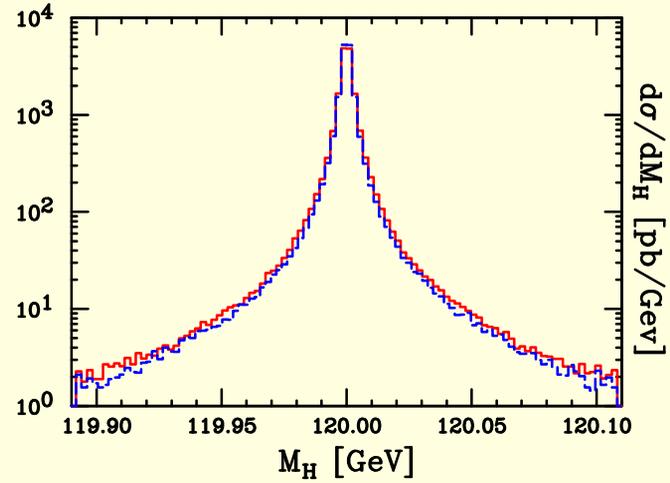
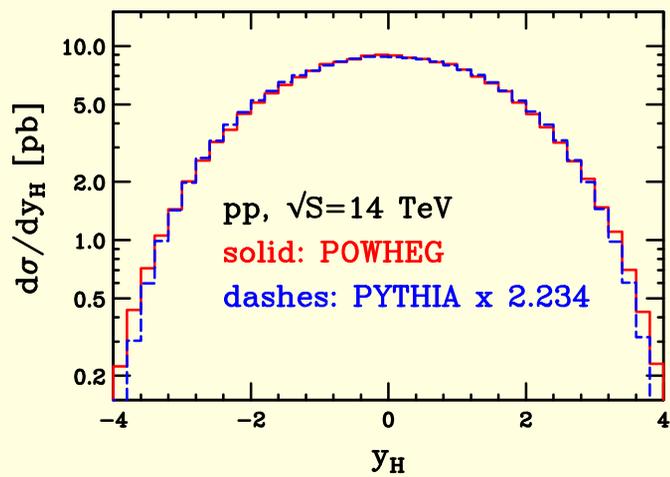


POWHEG distribution as in ALPGEN (Mangano, Moretti, Piccinini, Treccani, Nov.06) and in $t\bar{t} + \text{jet}$ at NLO (Dittmaier, Uwer, Weinzierl) : **no dip present.**

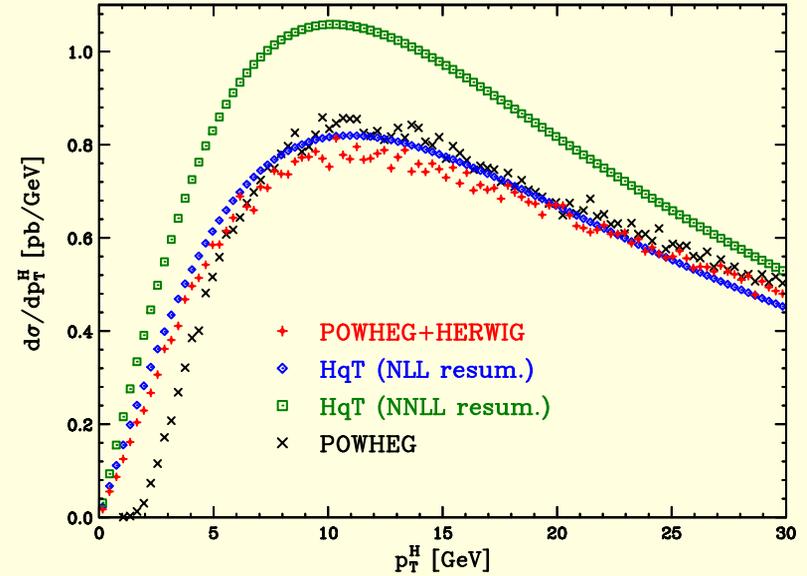
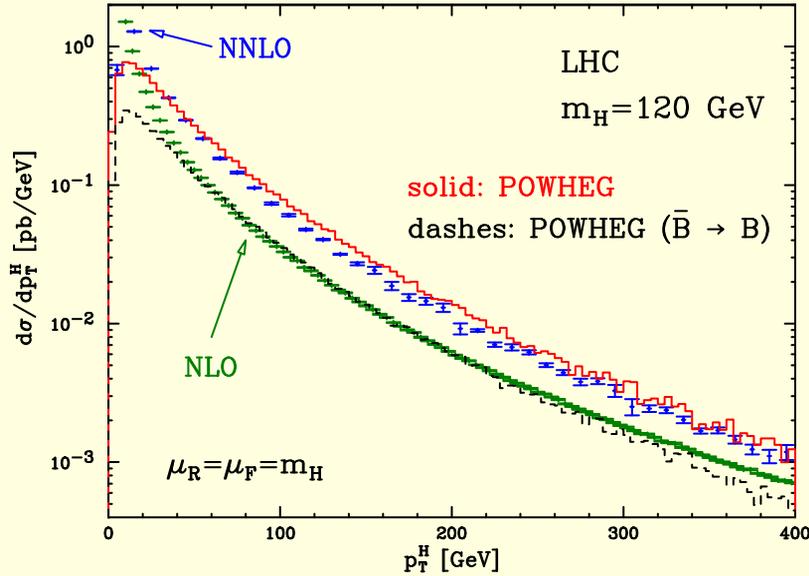
Higgs boson via gluon fusion at LHC



Higgs boson via gluon fusion at LHC



POWHEG vs. NNLO vs. NNLL

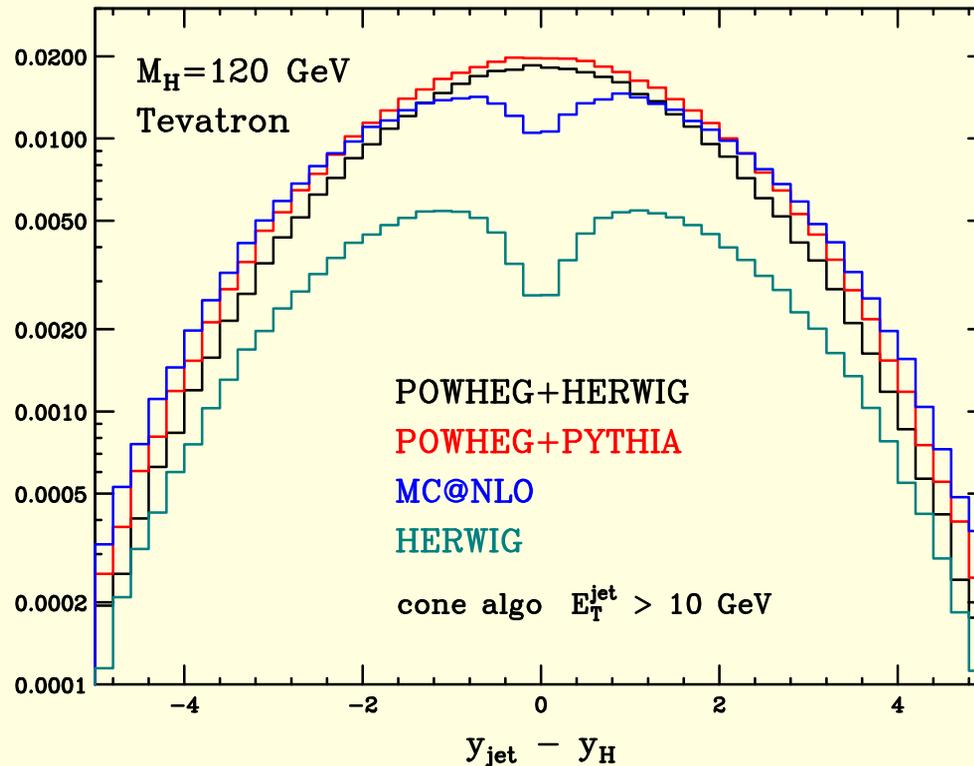


$$d\sigma = \bar{B}(\Phi_B) d\Phi_B \left\{ \Delta(\Phi_B, p_T^{\text{min}}) + \Delta(\Phi_B, p_T) \frac{R(\Phi_B, \Phi_r)}{B(\Phi_B)} d\Phi_r \right\}$$

$$\approx \frac{\bar{B}(\Phi_B)}{B(\Phi_B)} R(\Phi_B, \Phi_r) d\Phi_r = \{1 + \mathcal{O}(\alpha_s)\} R(\Phi) d\Phi$$

Better agreement with NNLO this way.

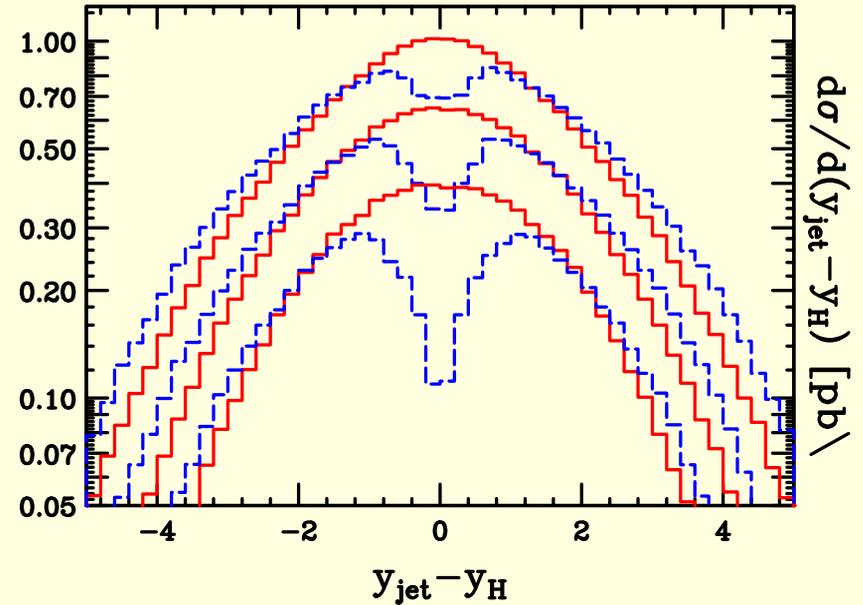
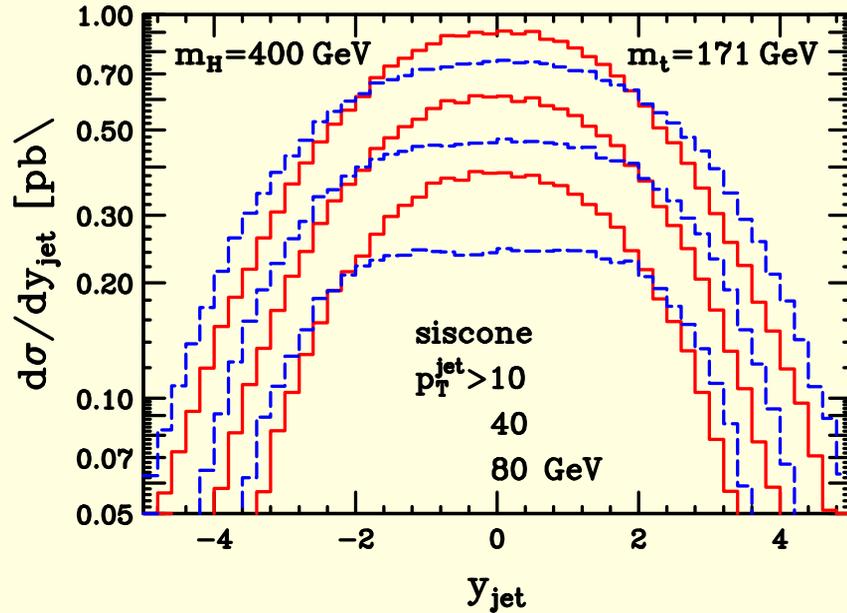
Jet rapidity in h production



Dip in MC@NLO inherited from even deeper dip in HERWIG

(MC@NLO tries to fill dead regions in HERWIG, a mismatch remains).

Gets worse for larger E_T cuts:



Questions:

Why MC@NLO has a dip in the hardest jet rapidity?

Why POWHEG has no dip? Is that because of the hardest p_T spectrum?

Hard p_T spectrum in POWHEG

We understand the cause; we keep it because yields results closer to NNLO;

There is enough flexibility to get rid of it, if one wants!

Go back to the POWHEG cross section:

$$d\sigma = \bar{B}(\Phi_B) \left[\Delta_{t_0} + \Delta_t \frac{R(\Phi)}{B(\Phi_B)} d\Phi_r \right], \quad \Delta_t = \exp \left[- \int \theta(t_r - t) \frac{R(\Phi_B, \Phi_r)}{B(\Phi_B)} d\Phi_r \right]$$

Break $R = R_s + R_f$, with R_f finite in collinear and soft limit, define

$$d\sigma' = \bar{B}^s(\Phi_B) \left[\Delta_{t_0}^s + \Delta_t^s \frac{R_s(\Phi)}{B(\Phi_B)} d\Phi_r \right] + R_f(\Phi) d\Phi$$

with:

$$\Delta_t^s = \exp \left[- \int \theta(t_r - t) \frac{R^s(\Phi_B, \Phi_r)}{B(\Phi_B)} d\Phi_r \right].$$

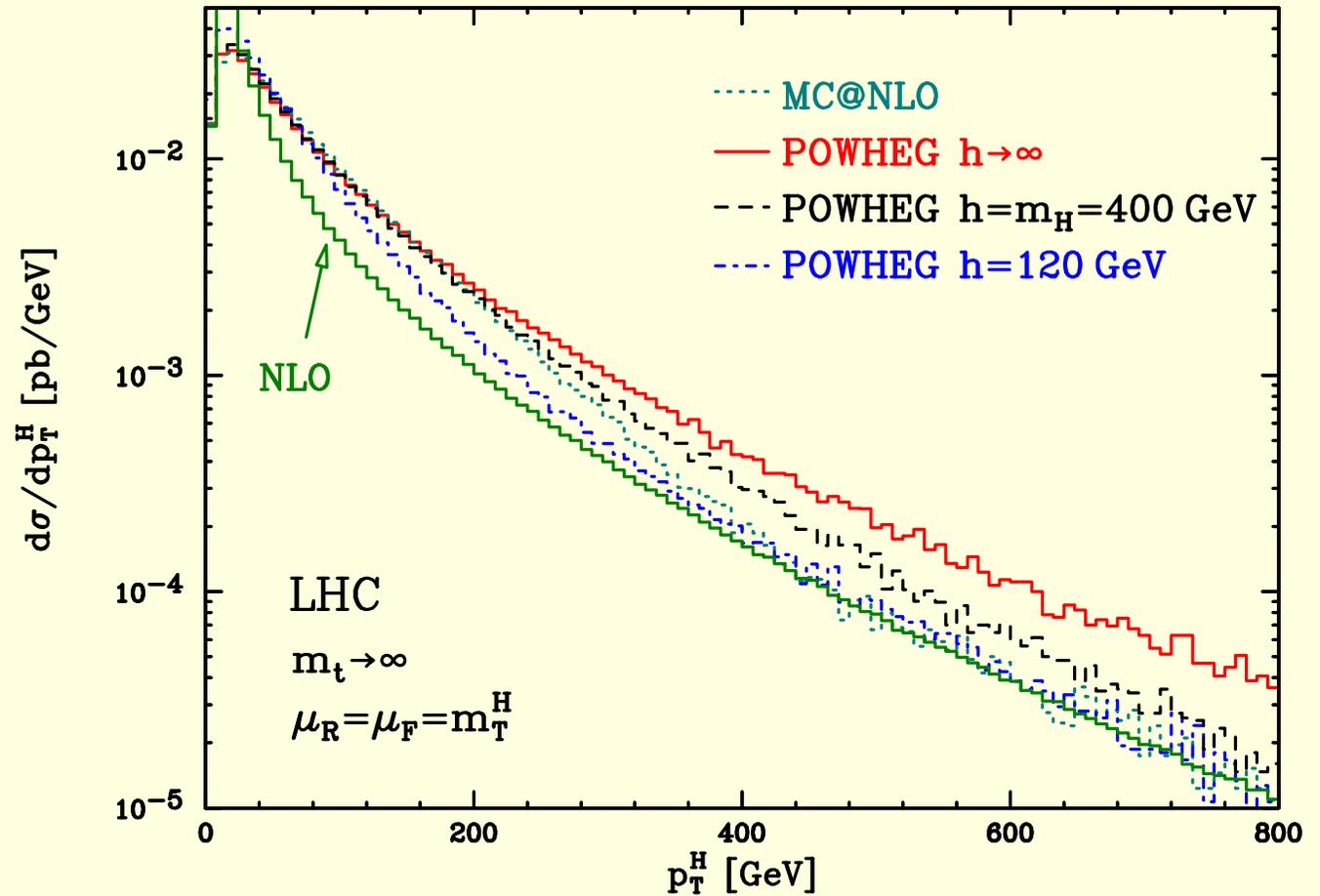
Easy to prove that: $d\sigma'$ is equivalent to $d\sigma$.

In other words, the part of the real cross section that is treated with the Shower technique can be varied.

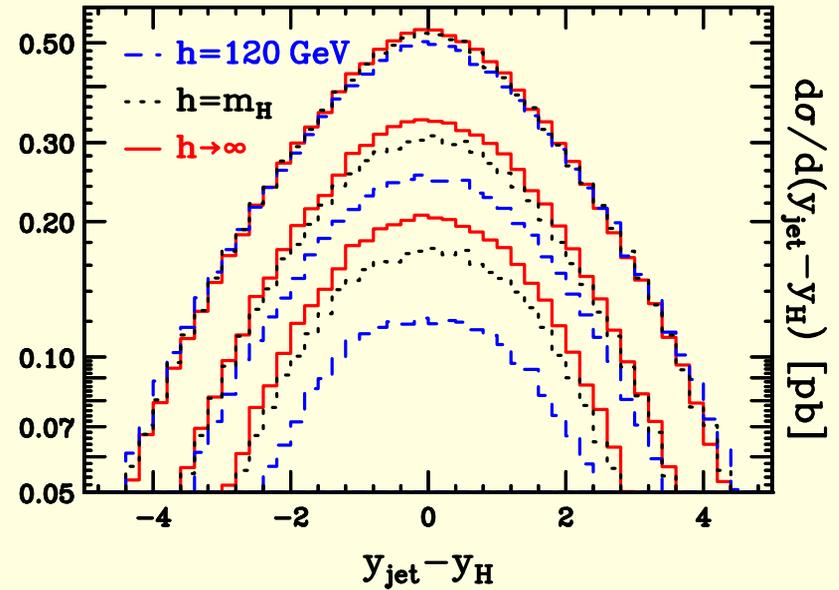
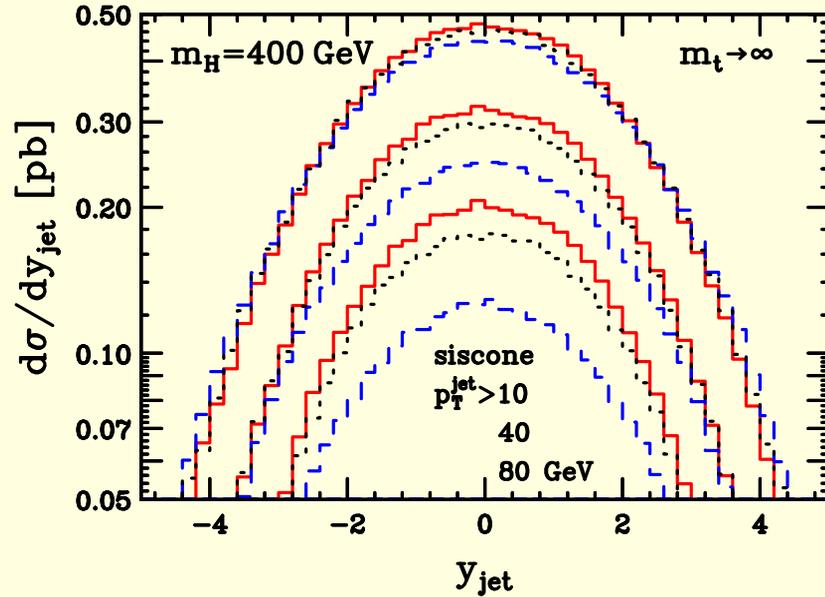
$$R_s = R \frac{h^2}{k_T^2 + h^2}$$

$$R_f = R \frac{k_T^2}{k_T^2 + h^2}$$

Agrees with NLO
at high p_T .



No new features (dips and the like) arise in the other distributions:



So: high k_T cross section and dips are unrelated issues.

Why is there a dip in MC@NLO?

Write the MC@NLO hardest jet cross section in the POWHEG language;
Hardest emission (P.N., 2004) can be written as

$$d\sigma = \underbrace{\bar{B}^{\text{HW}}(\Phi_B) d\Phi_B}_{S \text{ event}} \left[\underbrace{\Delta_{t_0}^{\text{HW}} + \Delta_t^{\text{HW}} \frac{R^{\text{HW}}(\Phi)}{B(\Phi_B)} d\Phi_r^{\text{HW}}}_{\text{HERWIG shower}} \right] + \left[\underbrace{R(\Phi) - R^{\text{HW}}(\Phi)}_{H \text{ event}} \right] d\Phi$$

$$\bar{B}^{\text{HW}}(\Phi_B) = \underbrace{B(\Phi_B)}_{\text{finite}} + \underbrace{\left[\underbrace{V(\Phi_B)}_{\text{infinite}} + \underbrace{\int R^{\text{HW}}(\Phi_B, \Phi_r) d\Phi_r}_{\text{infinite}} \right]}_{\text{finite}}$$

(Imagine that soft and collinear singularities in R^{HW} are regulated as in V !).
 Like POWHEG with $R_s = R^{\text{HW}}$. But now $R_f = R - R^{\text{HW}}$ can be **negative**.
 This formula illustrates why MC@NLO and POWHEG are equivalent at NLO.
 But differences can arise at NNLO ...

For large k_T :

$$\begin{aligned}
 d\sigma &= \left[\frac{\bar{B}^{\text{HW}}(\Phi_B)}{B(\Phi_B)} R^{\text{HW}}(\Phi) + R(\Phi) - R^{\text{HW}}(\Phi) \right] d\Phi_B d\Phi_r^{\text{HW}} \\
 &= \underbrace{R(\Phi)d\Phi}_{\text{no dip}} + \underbrace{\left(\frac{\bar{B}^{\text{HW}}(\Phi_B)}{B(\Phi_B)} - 1 \right)}_{\mathcal{O}(\alpha_s), \text{ but large for Higgs}} \underbrace{R^{\text{HW}}(\Phi)}_{\text{Pure Herwig dip}} d\Phi
 \end{aligned}$$

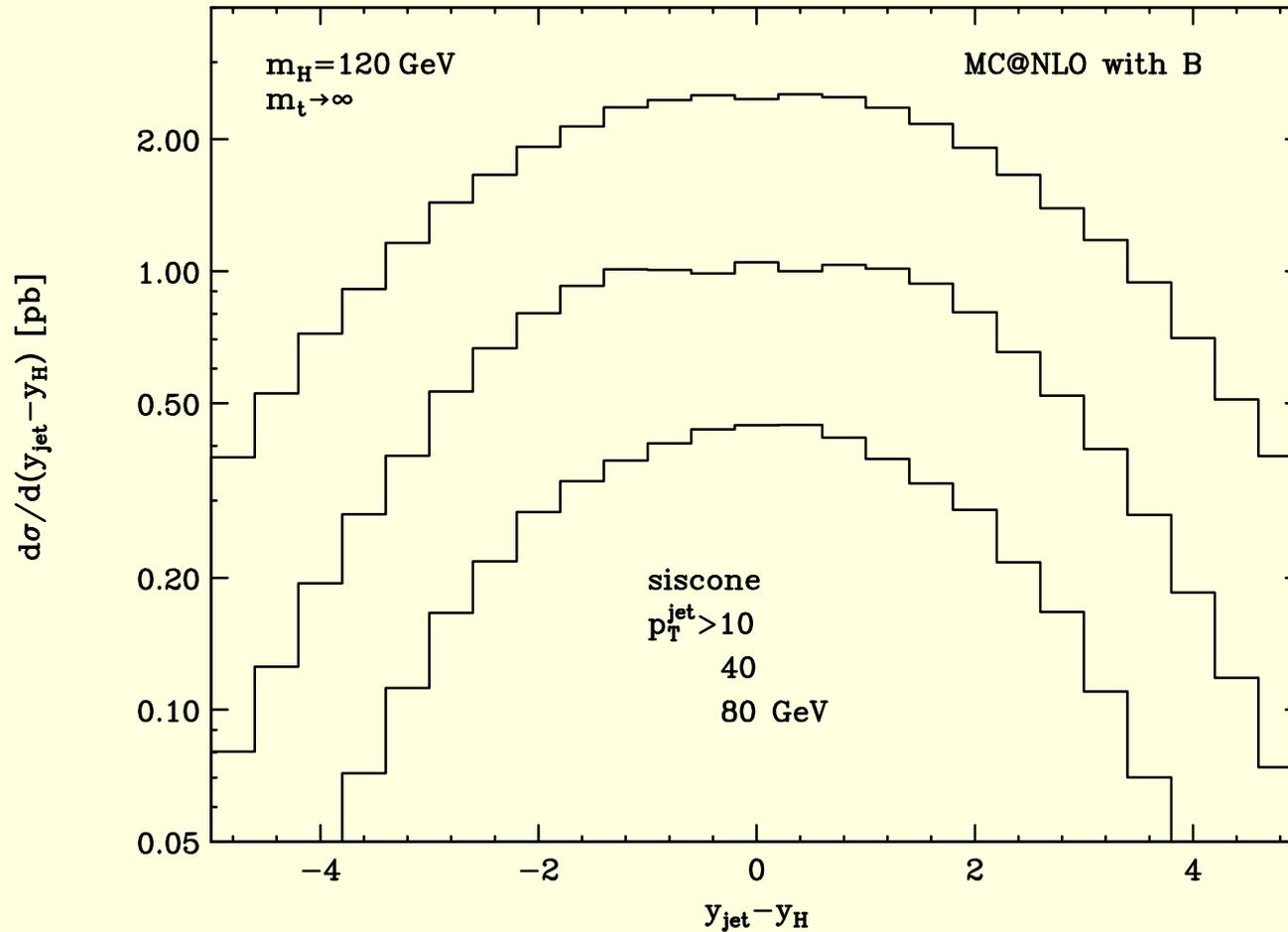
So: a contribution with a dip is added to the exact NLO result;

The contribution is $\mathcal{O}(\alpha_s R)$, i.e. NNLO!

Can we test this hypothesis? Replace $\bar{B}^{\text{HW}}(\Phi_n) \Rightarrow B(\Phi_n)$ in MC@NLO!

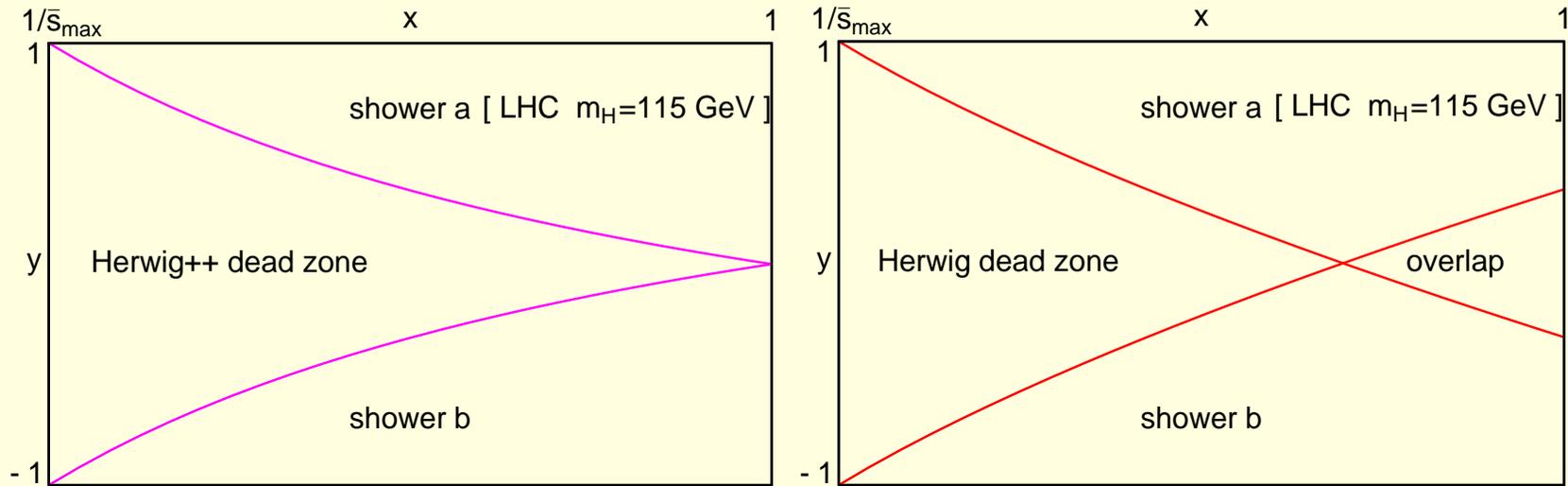
the dip should disappear ...

MC@NLO with B^{HW} replaced by B



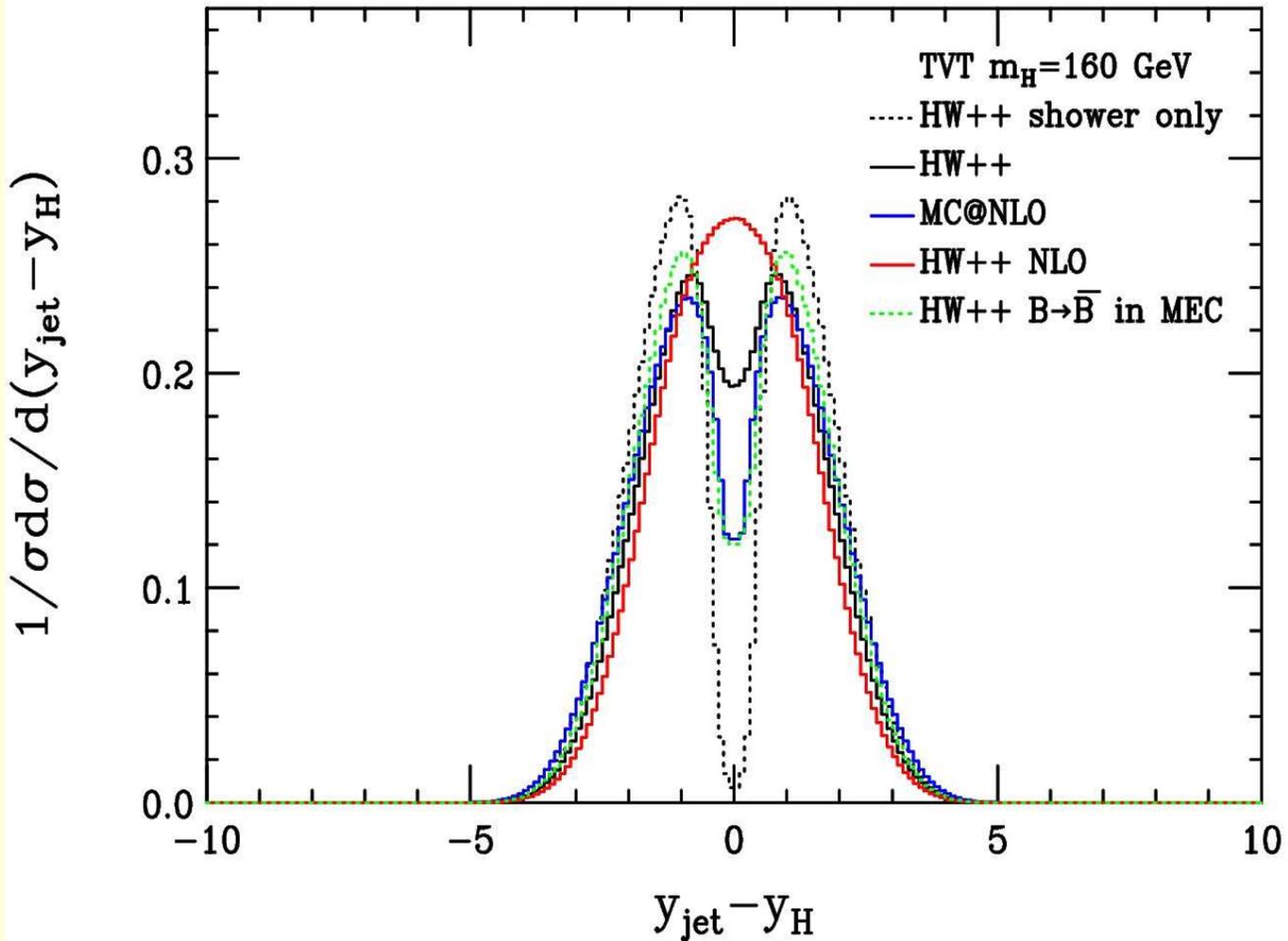
No visible dip is present! (on the right track, more studies needed cd Does...)

Detailed study of the problem also by [Hamilton, Richardson, Tully, 2009](#)



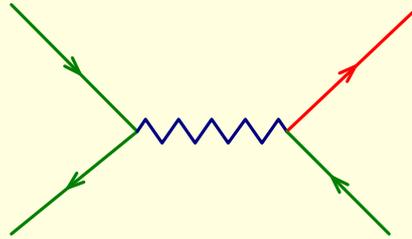
Both HERWIG and HERWIG++ have a dead radiation region corresponding to central rapidity and high energy

Hardest jet rapidity – Higgs rapidity ($p_T > 40$ GeV)

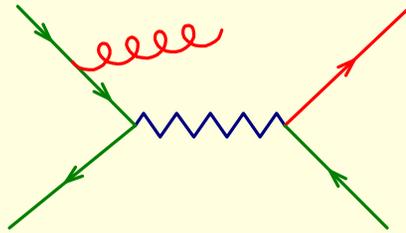


Single Top

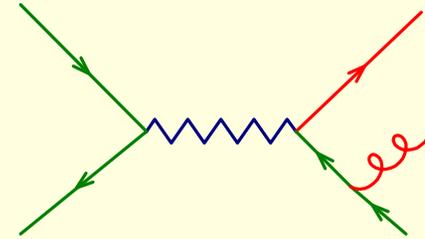
Both initial state and final state radiation is present;



Born



Initial state radiation



final state radiation

The separation of the different singular regions is based upon the general formulation of POWHEG given in [Frixione, Oleari, P.N. 2007](#)

Flavour and singularities separation

There are several allowed flavour structures in the n body process.
A flavour structure is a flavour assignment to the incoming and outgoing partons. B and V contributions are labelled by the flavour structure index f_b .

There are several allowed flavour structures in the $n + 1$ body process.
Thus R is labelled by a flavour structure index f_r .
Each component R_{f_r} has several singularity regions. We thus write

$$R = \sum_{\alpha_r} R^{\alpha_r}$$

where each R^{α_r} has a specific flavour structure, and is singular in only one singular region. In FKS one writes

$$R^{\alpha_r} = R \times S_{\alpha_r}, \quad \sum_{\alpha_r} S_{\alpha_r} = 1$$

The S factors in the FKS formalism are defined as

$$S_i = \frac{1}{Nd_i}, \quad S_{ij} = \frac{1}{Nd_{ij}} h\left(\frac{E_i}{E_i + E_j}\right),$$

where N is define so that $\sum_{\alpha_r} S_{\alpha_r} = 1$,

$$d_i = (\sqrt{s}E_i/2)^a(1 - \cos^2\theta_i)^b, \quad d_{ij} = (E_iE_j)^a(1 - \cos\theta_{ij})^b,$$

$$\lim_{z \rightarrow 0} h(z) = 1, \quad \lim_{z \rightarrow 1} h(z) = 0, \quad h(z) + h(1 - z) = 1.$$

For example:

$$h(z) = \frac{(1 - z)^c}{z^c + (1 - z)^c}$$

So, the S_i factors single out the region where parton i is collinear to either initial state line, or is soft, while S_{ij} single out the region where parton i is collinear to parton j or is soft.

The underlying Born

This is a basic concept in the POWHEG formalism;

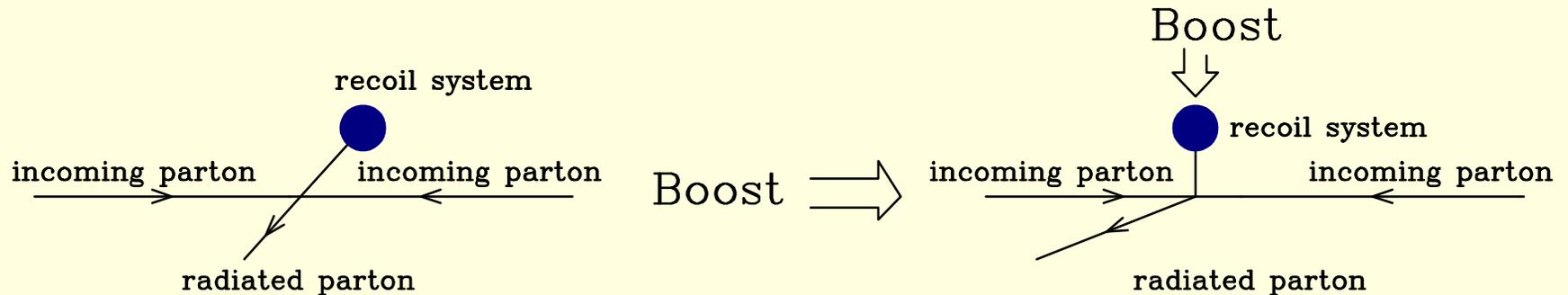
To each region α_r we associate an underlying Born flavour configuration f_b , obtained as follows:

- If the singular region is associated to a parton becoming **soft**, then the parton must be a **gluon**, and it is simply **removed** to get the underlying Born configuration
- If the region is associated to two parton becoming **collinear**, then, in order for the region to be singular, the two partons must come **from the splitting of another parton**. The two partons are removed, and are **replaced by the single parent parton** with the appropriate flavour

Notice that in a shower Monte Carlo one first generates the Born process (i.e. the underlying Born configuration) and then lets one initial or final line undergo collinear splitting. Here we look at each singular region of the real matrix element, and ask from which underlying Born process it could have been produced via a shower.

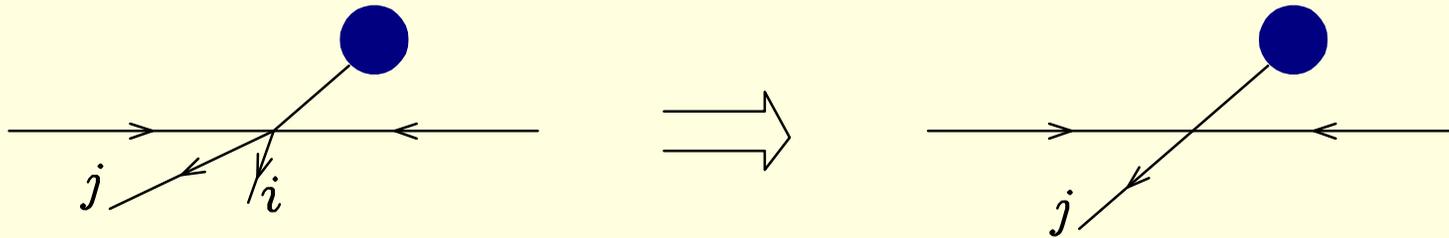
The underlying Born kinematics

To each kinematic configuration for the full radiation phase space Φ , one associates an underlying Born kinematics Φ_B and a set of radiation variables $\Phi_r = (y, z, \phi)$. For initial state radiation Φ_B is obtained by going with a longitudinal boost to the frame where the system recoiling against radiation has zero longitudinal momentum. In this frame one boosts the recoil system in the transverse direction, so that its transverse momentum becomes zero



The radiation variables are $y = \cos \theta$, θ being the angle between the radiated parton and the positive rapidity incoming parton, $\xi = 2E/\sqrt{s}$, where E is the energy of the radiated parton, and ϕ is its azimuth.

For final state radiation, the splitting partons are merged by summing their 3-momenta in the partonic CM frame. The 3-momentum is scaled, and the recoil system is boosted so that momentum and energy are conserved.



The radiation variables are $y = \cos \theta$, θ being the angle between the radiated partons, $\xi = 2E_i/\sqrt{s}$, ϕ is the azimuth of the ij plane relative to $\vec{k}_i + \vec{k}_j$. (This differs from FKS kinematics, where ϕ is relative to \vec{k}_j).

The \bar{B} function carries a flavour structure index, and is given by

$$\bar{B}^{f_b}(\Phi_B) = [B(\Phi_B) + V(\Phi_B)]_{f_b} + \sum_{\alpha_r \in \{\alpha_r | f_b\}} [d\Phi_r R(\Phi)]_{\alpha_r}$$

The R_{α_r} appearing here have singularities regulated by $+$ prescriptions in the FKS framework.

we have

- $\{\alpha_r | f_b\}$ is the set of all singular regions having the underlying Born configuration with flavour structure f_b .
- $[\dots]_{\alpha_r}$ means that everything inside is relative to the α_r singular term: thus R is R_{α_r} , the parametrization (Φ_B, Φ_r) is the one appropriate to the α_r singular region

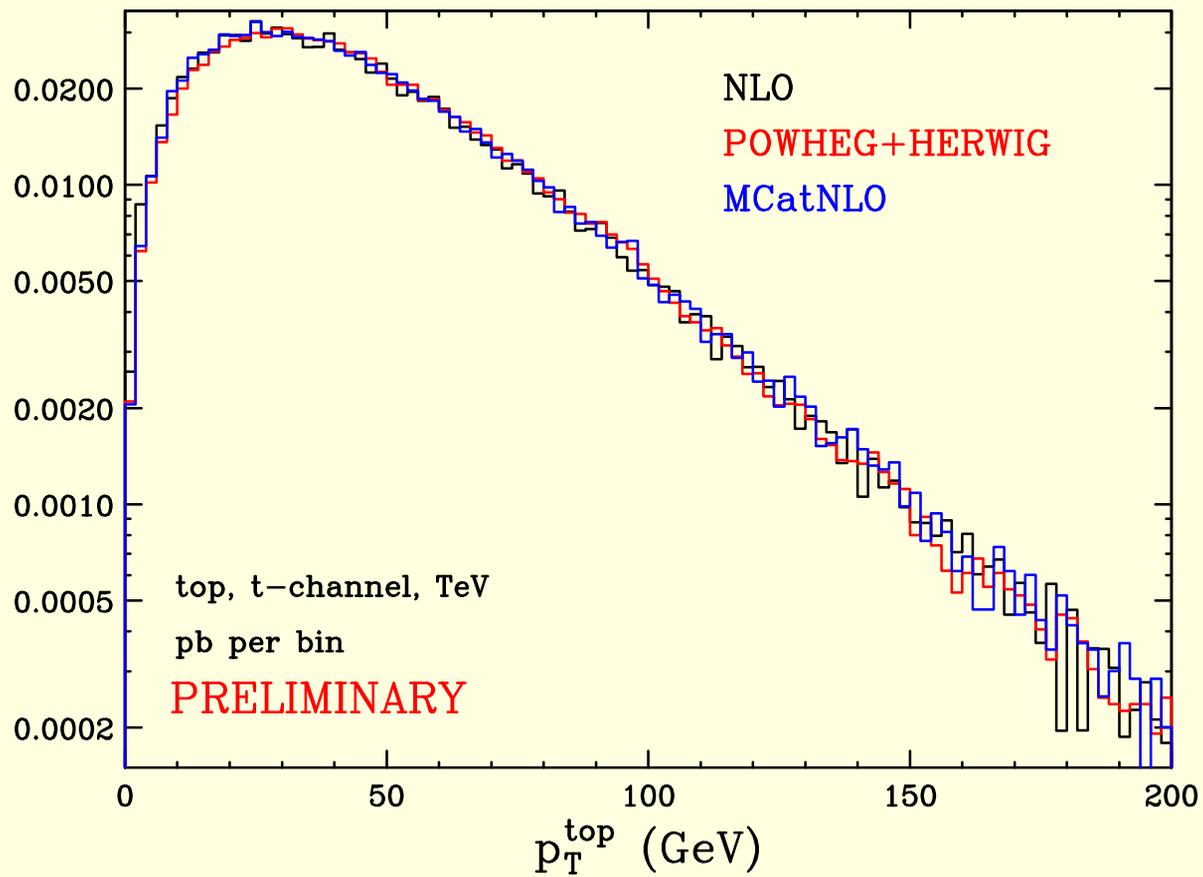
Sudakov FF also carries an f_b index:

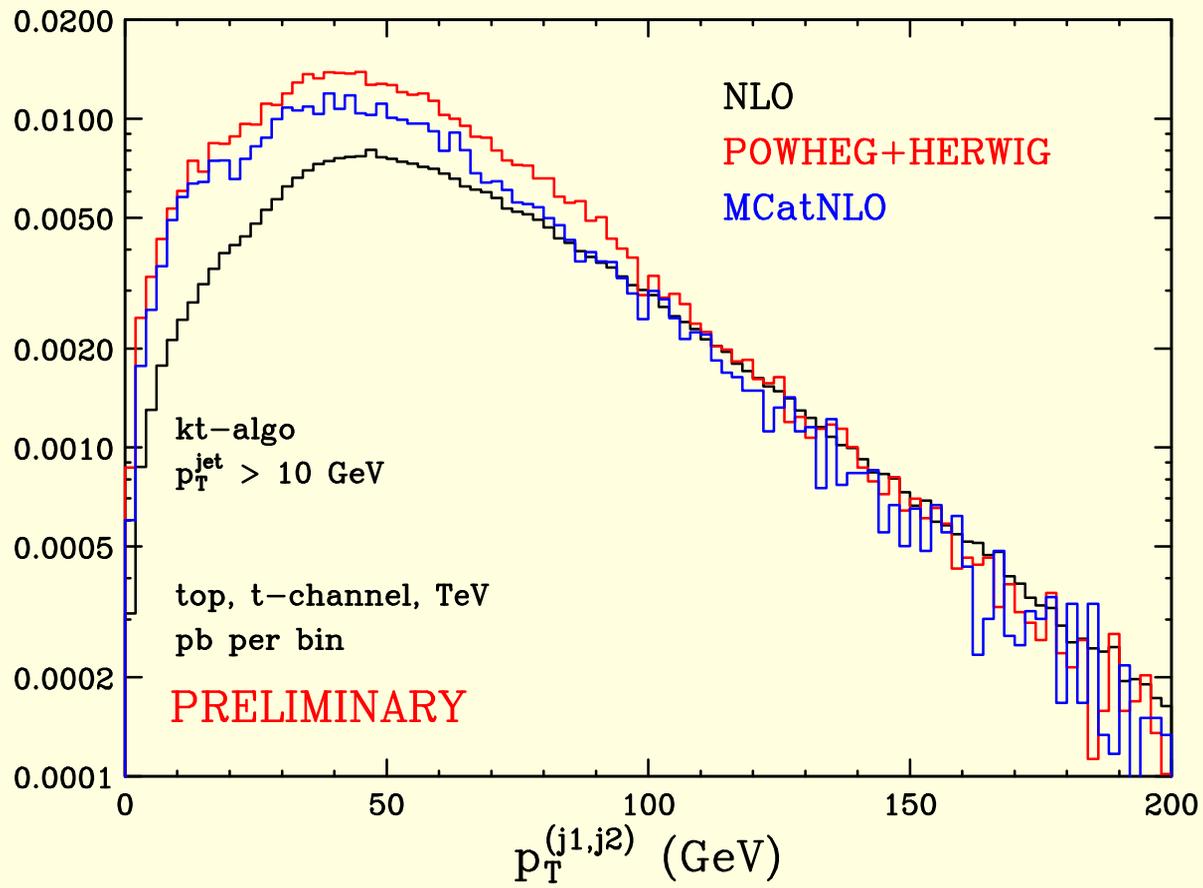
$$\Delta^{f_b}(\Phi_n, \mathbf{pt}) = \exp \left\{ - \sum_{\alpha_r \in \{\alpha_r | f_b\}} \int \frac{[d\Phi_r R(\Phi_n, \Phi_r) \theta(\mathbf{kt} - \mathbf{pt})]_{\alpha_r}}{B^{f_b}(\Phi_n)} \right\}$$

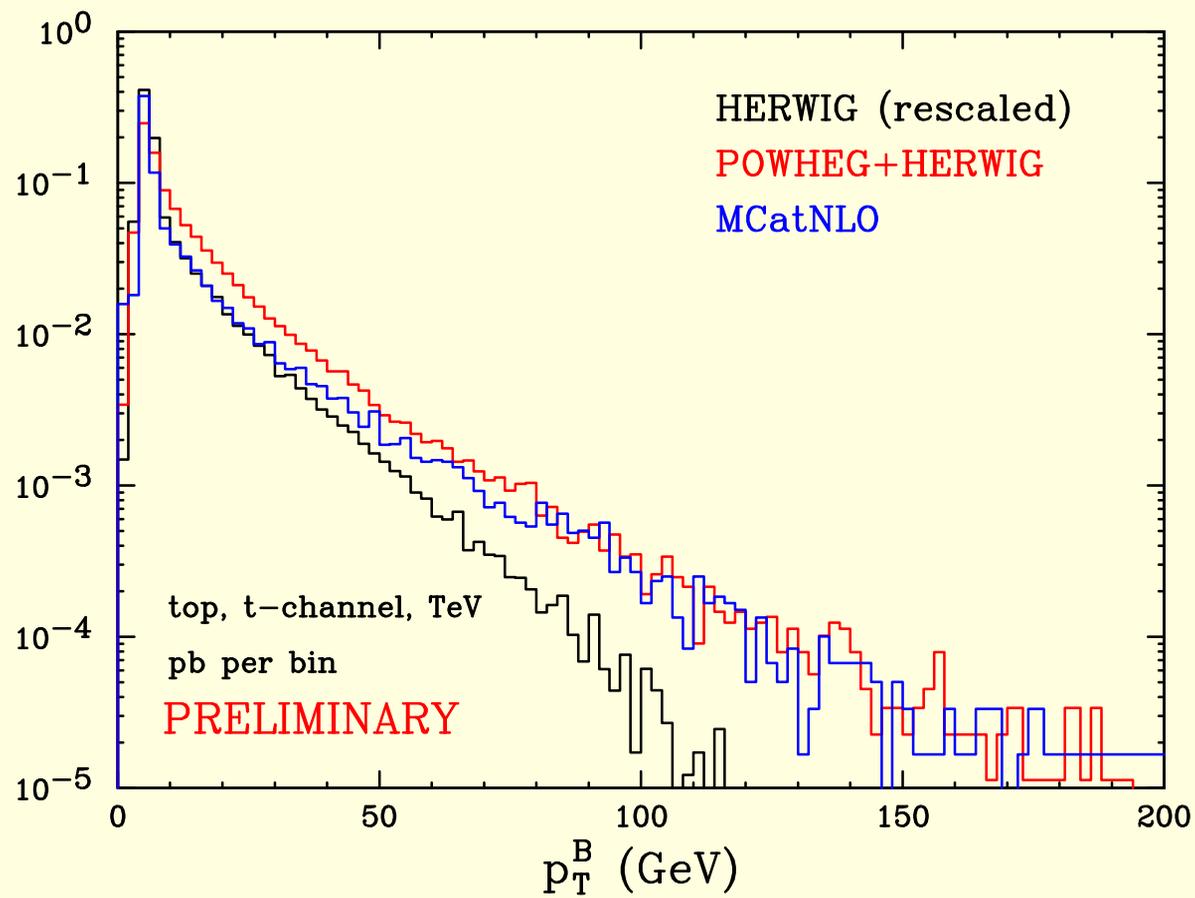
or

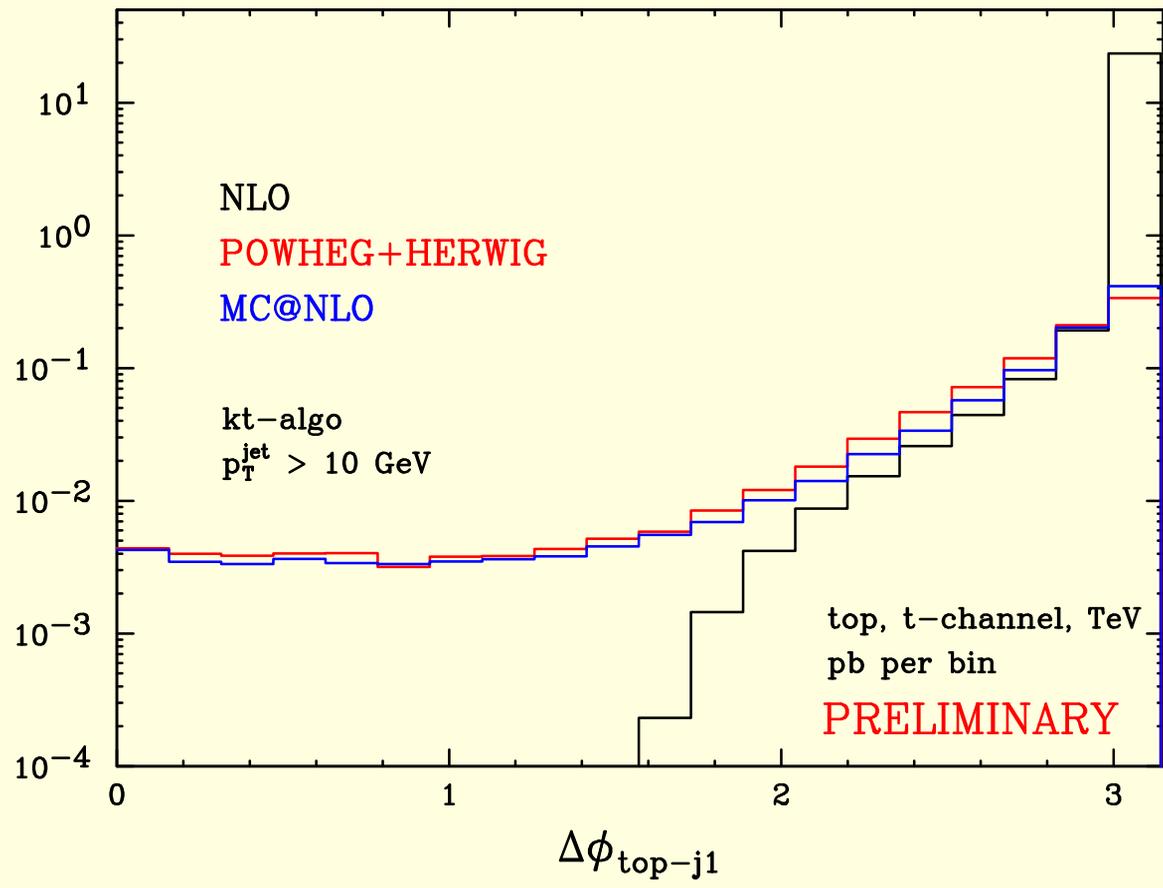
$$\Delta^{f_b}(\Phi_n, \mathbf{pt}) = \prod_{\alpha_r \in \{\alpha_r | f_b\}} \exp \left\{ - \sum \int \frac{[d\Phi_r R(\Phi_n, \Phi_r) \theta(\mathbf{kt} - \mathbf{pt})]_{\alpha_r}}{B^{f_b}(\Phi_n)} \right\}$$

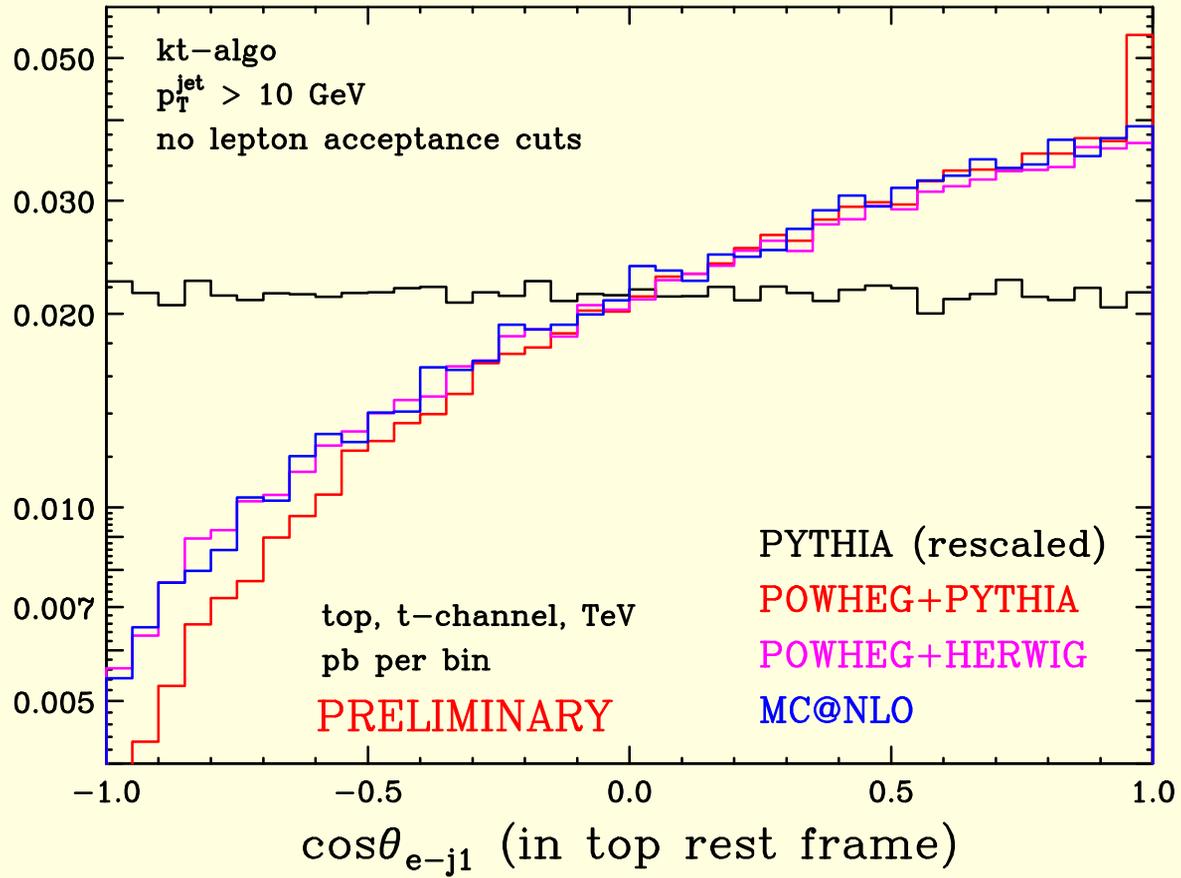
The Sudakov form factor is a product of elementary Sudakov form factors associated with each radiation region. Technically, one generates radiation by generating a k_T with each elementary form factor, and choosing the one with the largest k_T at the end.

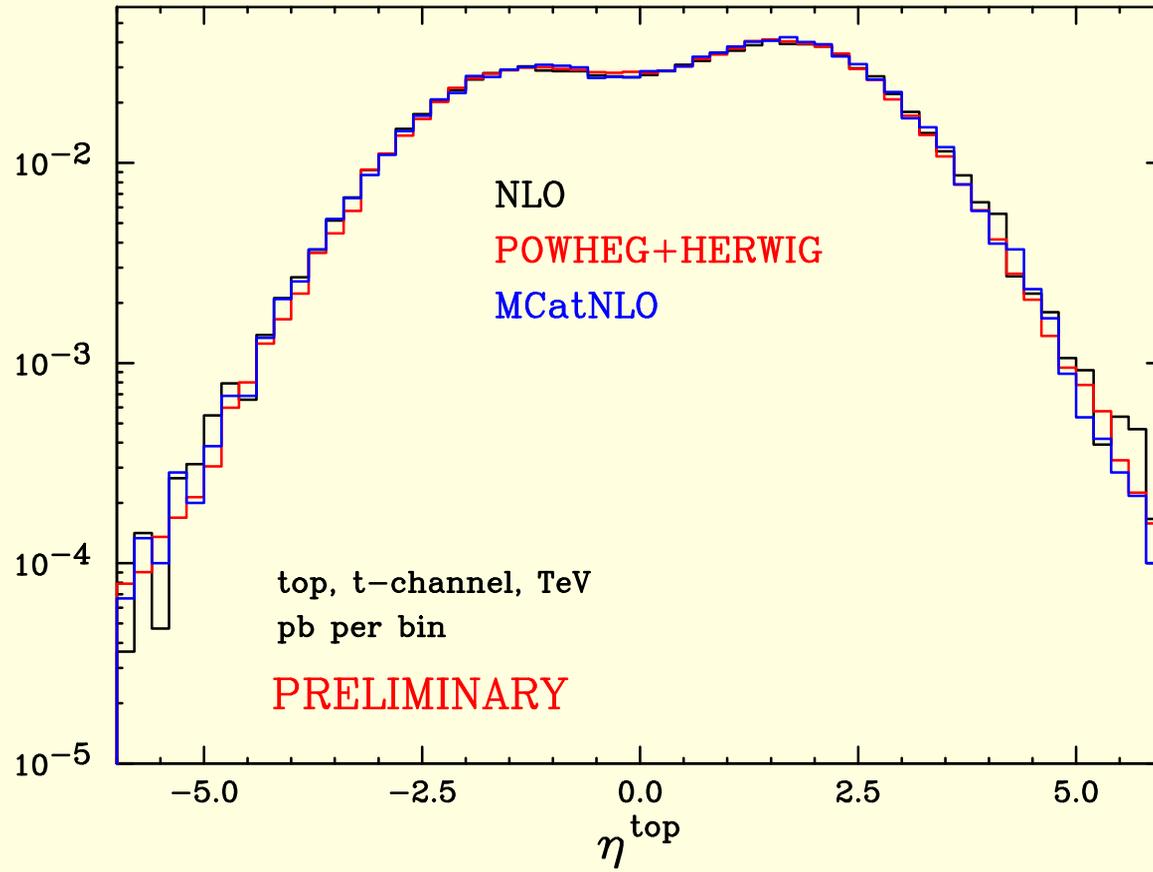


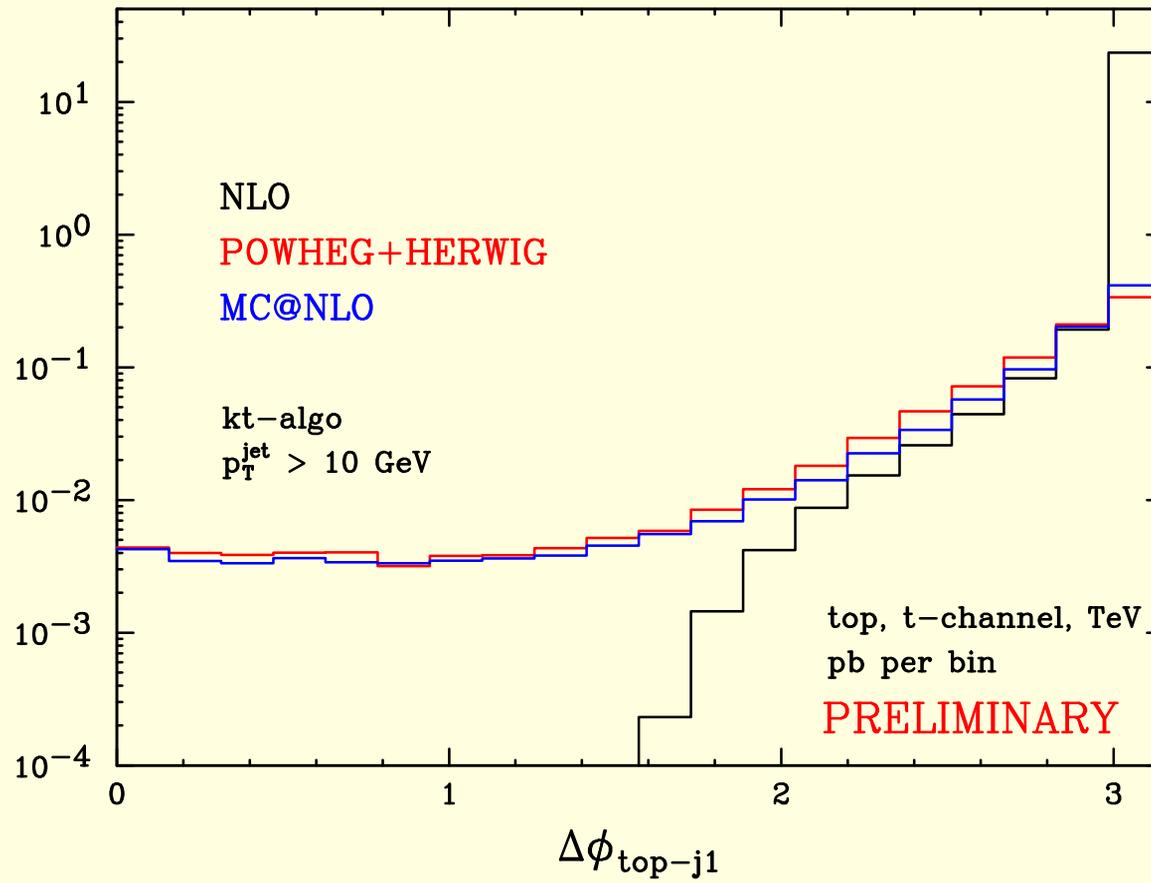












Double logs and angular ordered Showers

We have discussed MC+NLO assuming that the hardest radiation is the first one. This is the case only in dipole shower programs (ARIADNE, newer PYTHIA versions). In virtuality ordered (old PYTHIA) or angular ordered showers the hardest event may not be the first.

MC@NLO and HERWIG

In this case

$$d\sigma = \underbrace{\bar{B}^{\text{MC}}(\Phi_B)d\Phi_B}_{S \text{ event}} \left[\underbrace{\Delta_{t_0}^{\text{MC}} + \Delta_t^{\text{MC}} \frac{R^{\text{MC}}(\Phi)}{B(\Phi_B)} d\Phi_r^{\text{MC}}}_{\text{MC shower}} \right] + \left[\underbrace{R(\Phi) - R^{\text{MC}}(\Phi)}_{H \text{ event}} \right] d\Phi.$$

The S events should already be treated correctly by the MC; the net effect of the shower development in HERWIG is to generate the hardest radiation according to the above formula (P.N. 2004).

There are however $2 \rightarrow 2$ processes in HERWIG that may need a truncated shower to consistently treat colour connections.

The H event may need truncated shower, as any ME result interfaced to the angular ordered shower; however, being not singular, the region that needs the truncated shower is power suppressed by the phase space.

Interfacing POWHEG with angular ordered SMC's

- Generate event with hardest emission
- Generate all subsequent emissions with a $\langle pt \rangle$ veto equal to the hardest emission $\langle pt \rangle$
- Pair up the partons that are nearest in $\langle pt \rangle$
- Generate an angular ordered shower associated with the paired parton, stopping at the angle of the paired partons:
Truncated shower, (P.N., 2004)
- Generate all subsequent (vetoed) showers

Towards automation: the POWHEG BOX

The MIB (Milano-Bicocca) group (Alioli, Oleari, Re, P.N.) is working on an automatic implementation of POWHEG for generic NLO processes.

This framework is being tested in the process $hh \rightarrow Z + 1\text{jet}$.

The POWHEG BOX

Build a computer code framework, such that, given the Born cross section, the finite part of the virtual corrections, and the real graph cross section, one builds immediately a POWHEG generator. More precisely, the **user** must supply:

- The **Born phase space**
- The **lists of Born and Real** processes (i.e. $u \bar{s} \rightarrow W^+ c \bar{c}$, etc.)
- The **Born squared amplitudes** $\mathcal{B} = |\mathcal{M}|^2$, \mathcal{B}_{ij} , $\mathcal{B}_{j,\mu_j,\mu'_j}$, for all relevant partonic processes; \mathcal{B}_{ij} is the colour ordered Born amplitude squared, $\mathcal{B}_{j,\mu\nu}$ is the spin correlated amplitude, where j runs over all external gluons in the amplitude. All these amplitudes are common ingredient of an NLO calculation.
- The **Real squared amplitude**, for all relevant partonic processes. This may also be obtained by interfacing the program to MADGRAPH.
- The finite part of the **virtual amplitude** contribution, for all relevant partonic processes.

Strategy

Use the FKS framework according to the general formulation of POWHEG given in (Frixione, Oleari, P.N. 2007), hiding all FKS implementation details.

In other words, we use FKS, but the user needs not to understand it.

(Attempts to use the popular Catani-Seymour method have turned out to be too cumbersome).

It includes:

- The phase space for ISR and FSR, according to FNO2006.
- The combinatorics, the calculation of all R_α , the soft and coll. limits
- The calculation of \tilde{B}
- The calculation of the upper bounds for the generation of radiation
- The generation of radiation
- Writing the event to the Les Houches interface

It works! Lots of testing needed now ...

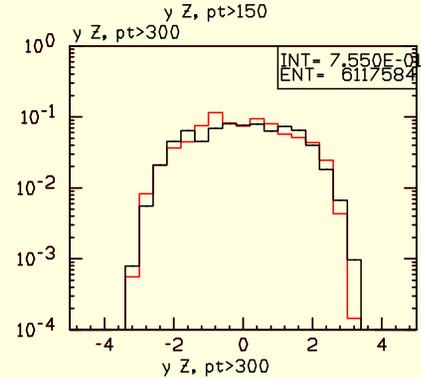
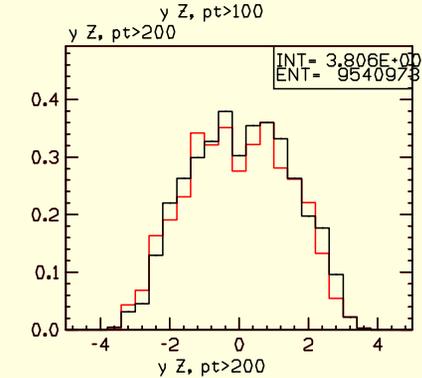
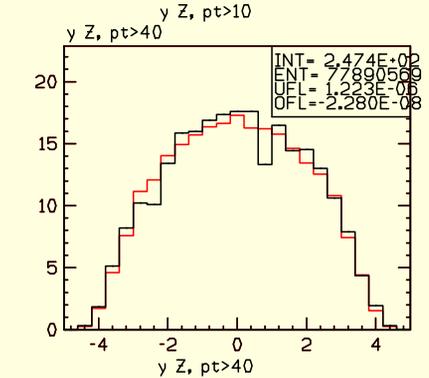
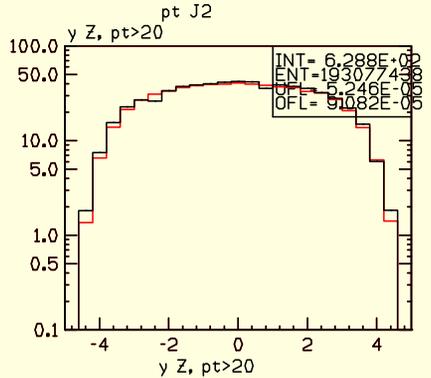
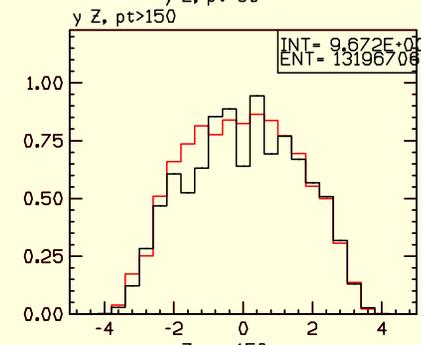
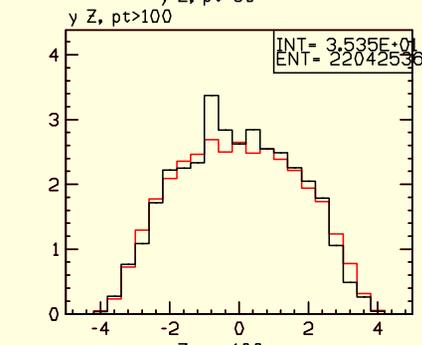
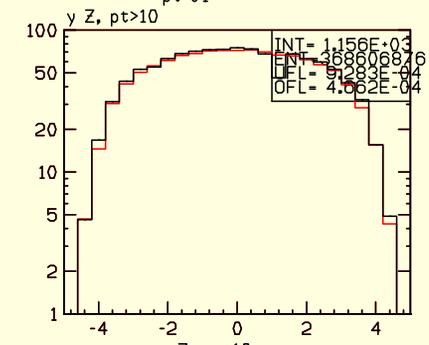
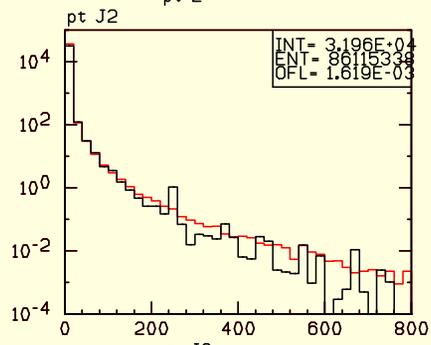
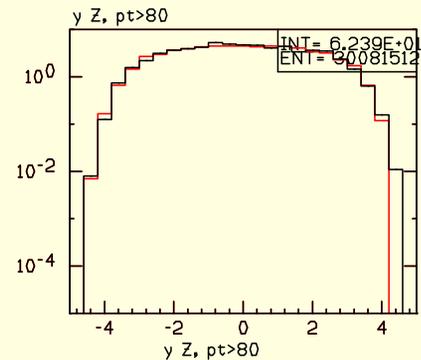
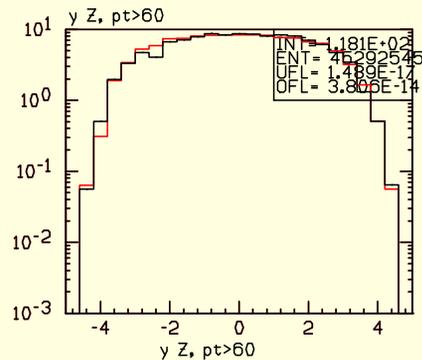
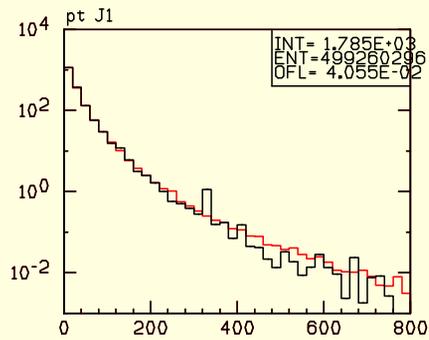
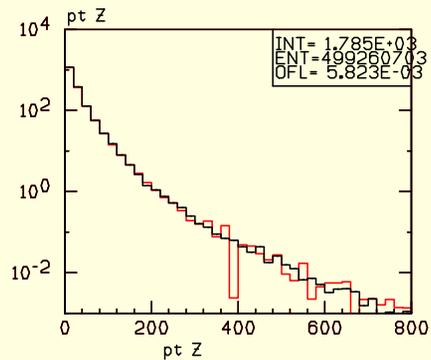
Byproduct: generic NLO implementation using the FKS method

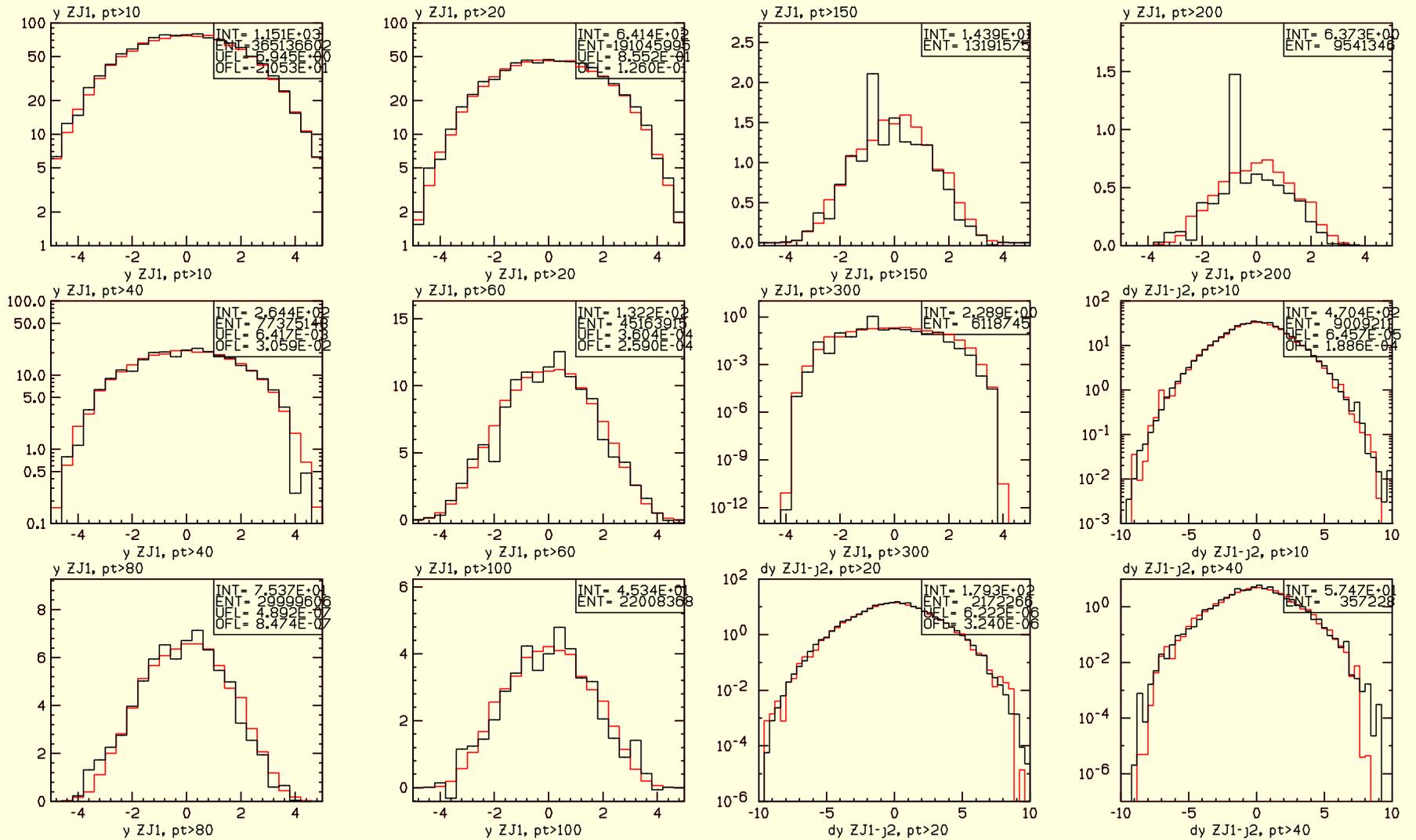
Case study: $Z + \text{jet}$ production

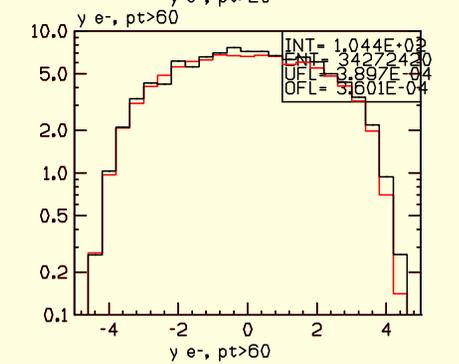
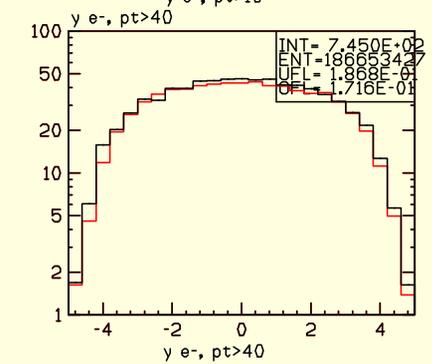
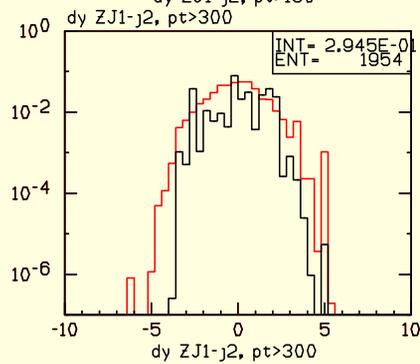
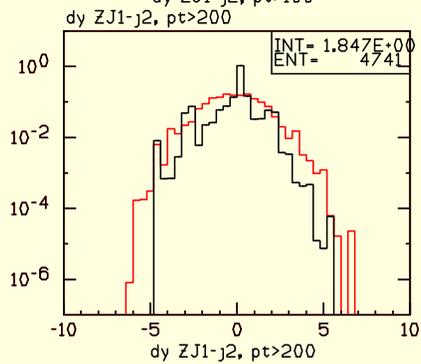
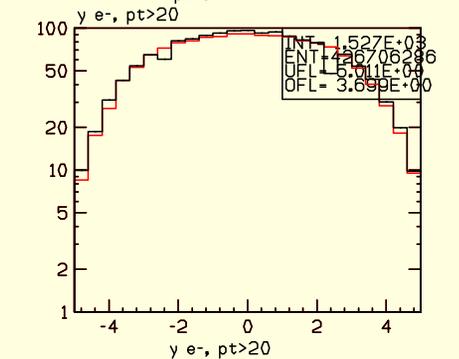
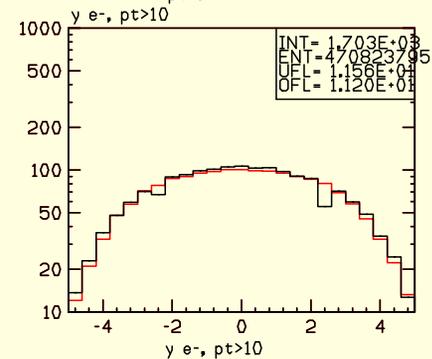
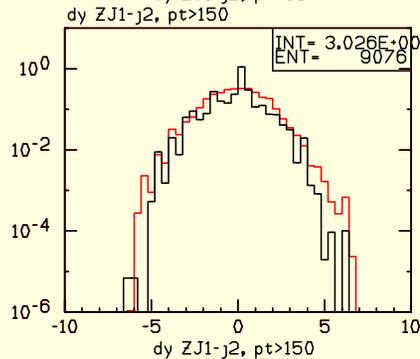
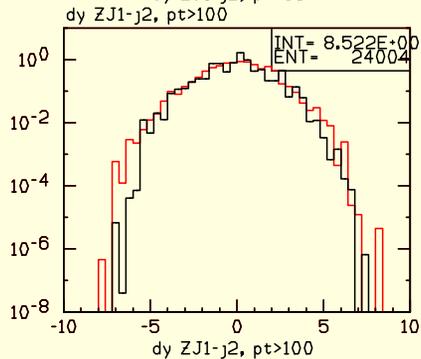
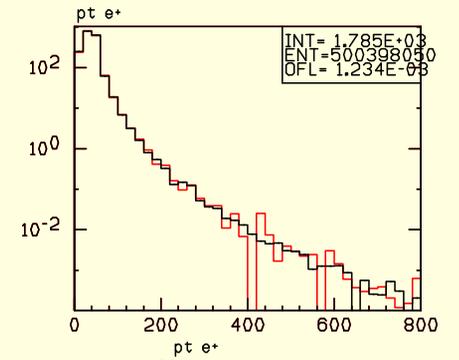
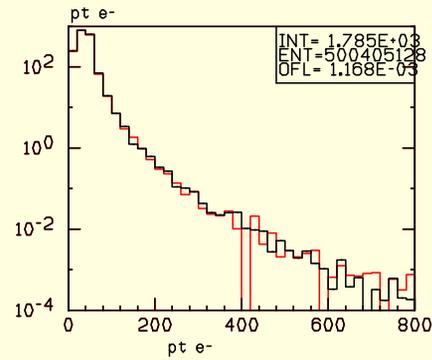
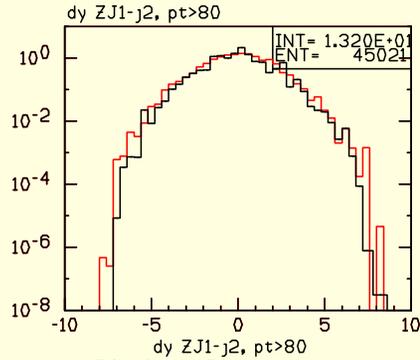
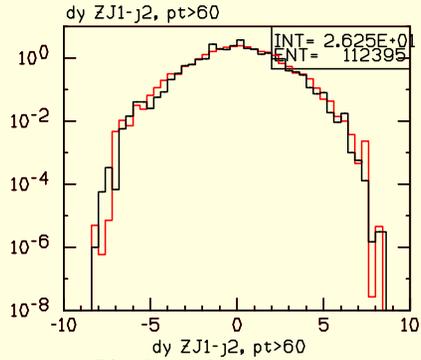
Get virtual matrix elements from MCFM;

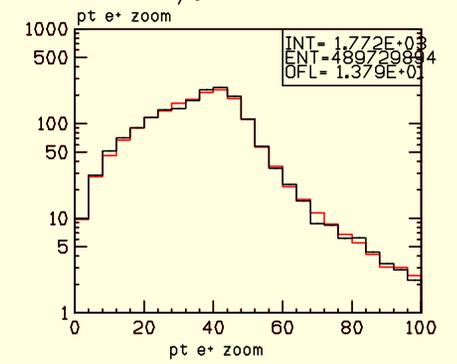
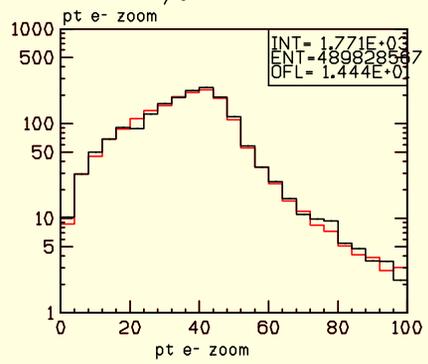
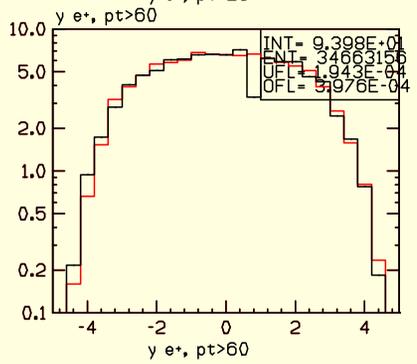
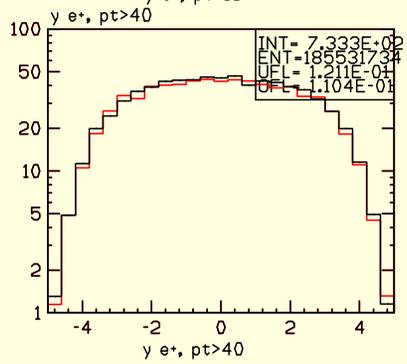
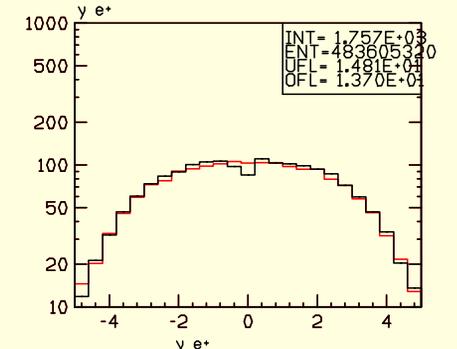
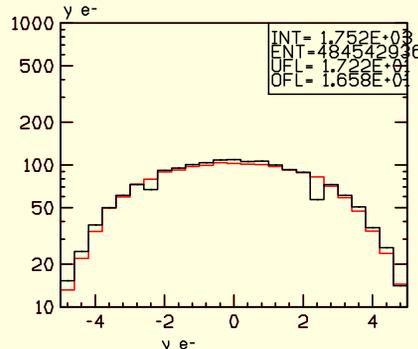
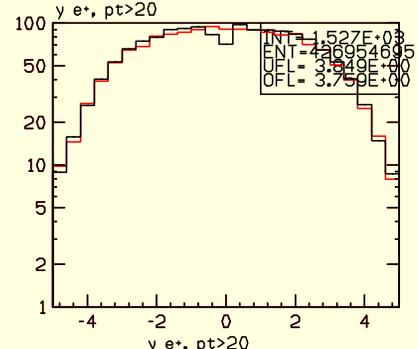
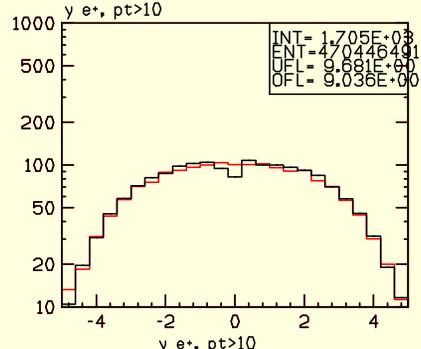
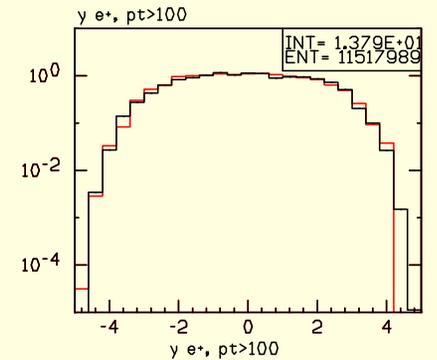
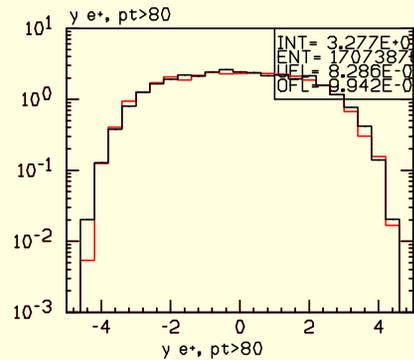
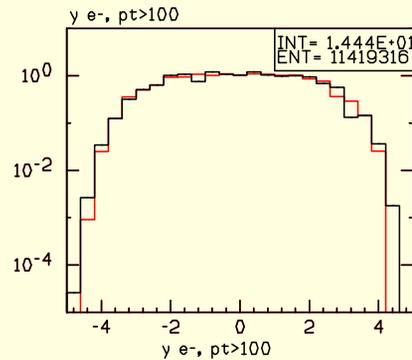
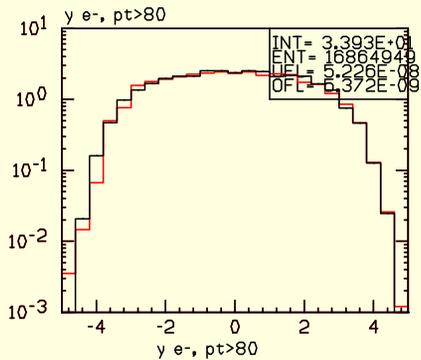
Compare first NLO predictions obtained with MCFM and the POWHEG BOX

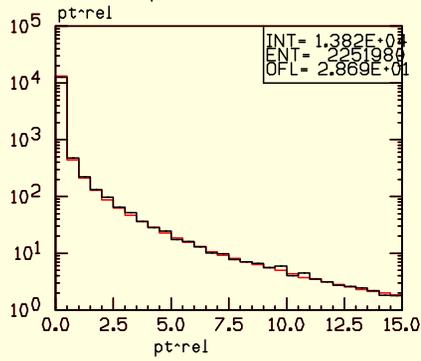
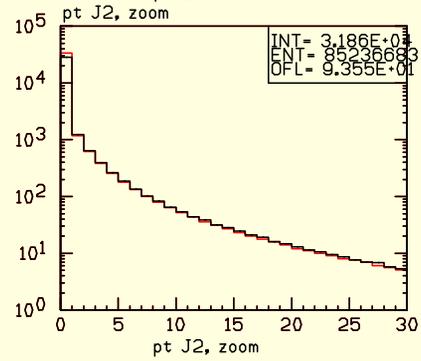
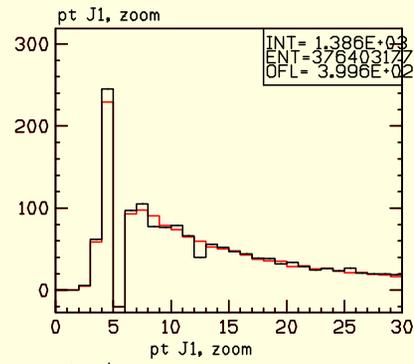
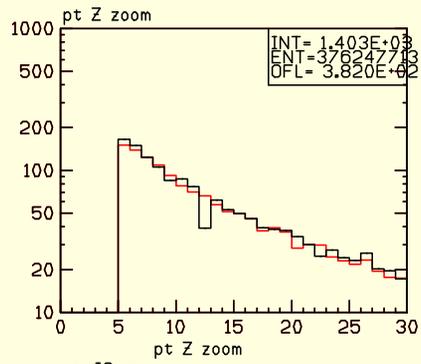
Virtual corrections are the same, but subtraction terms, soft and collinear remnants are all different; non trivial test of setup;





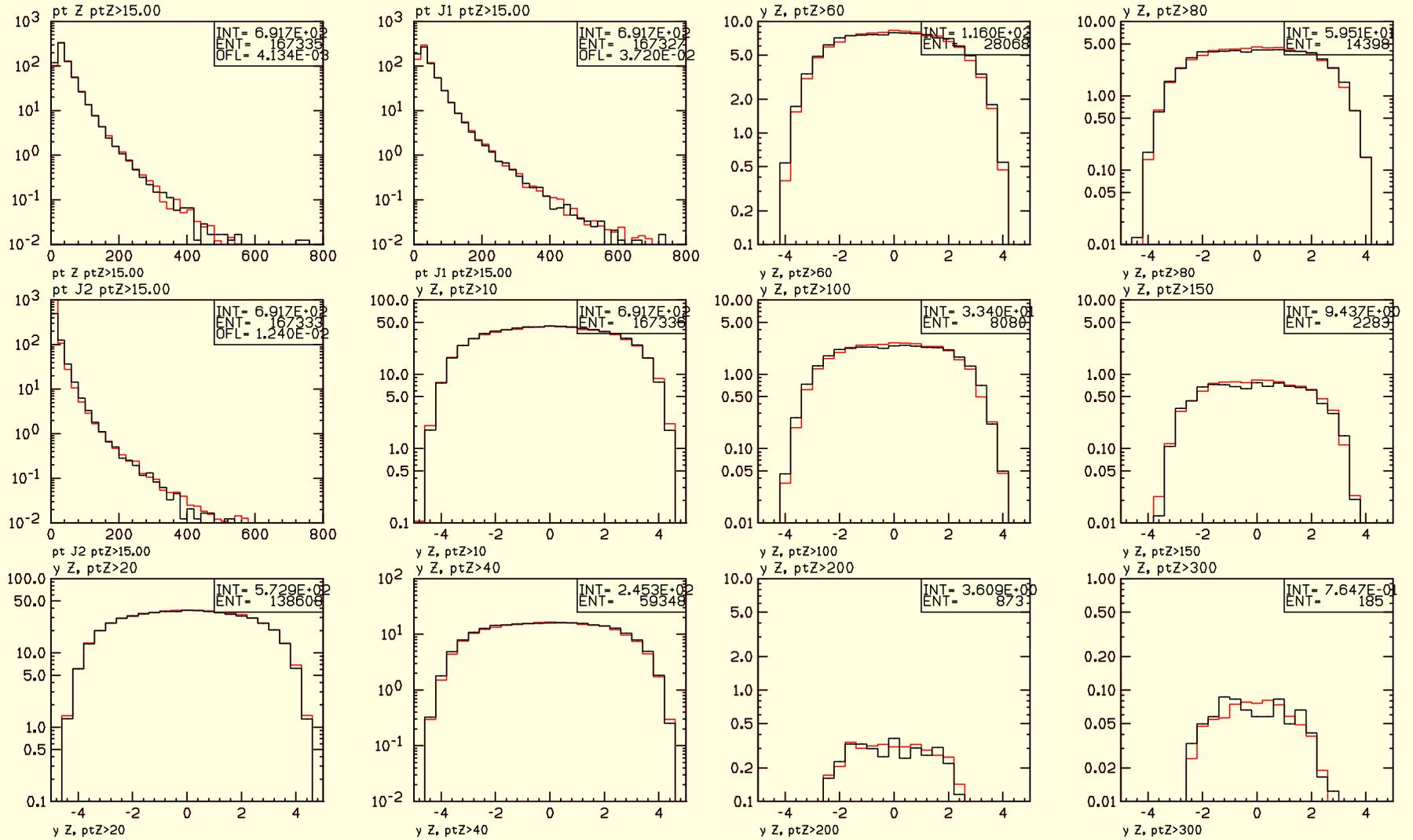


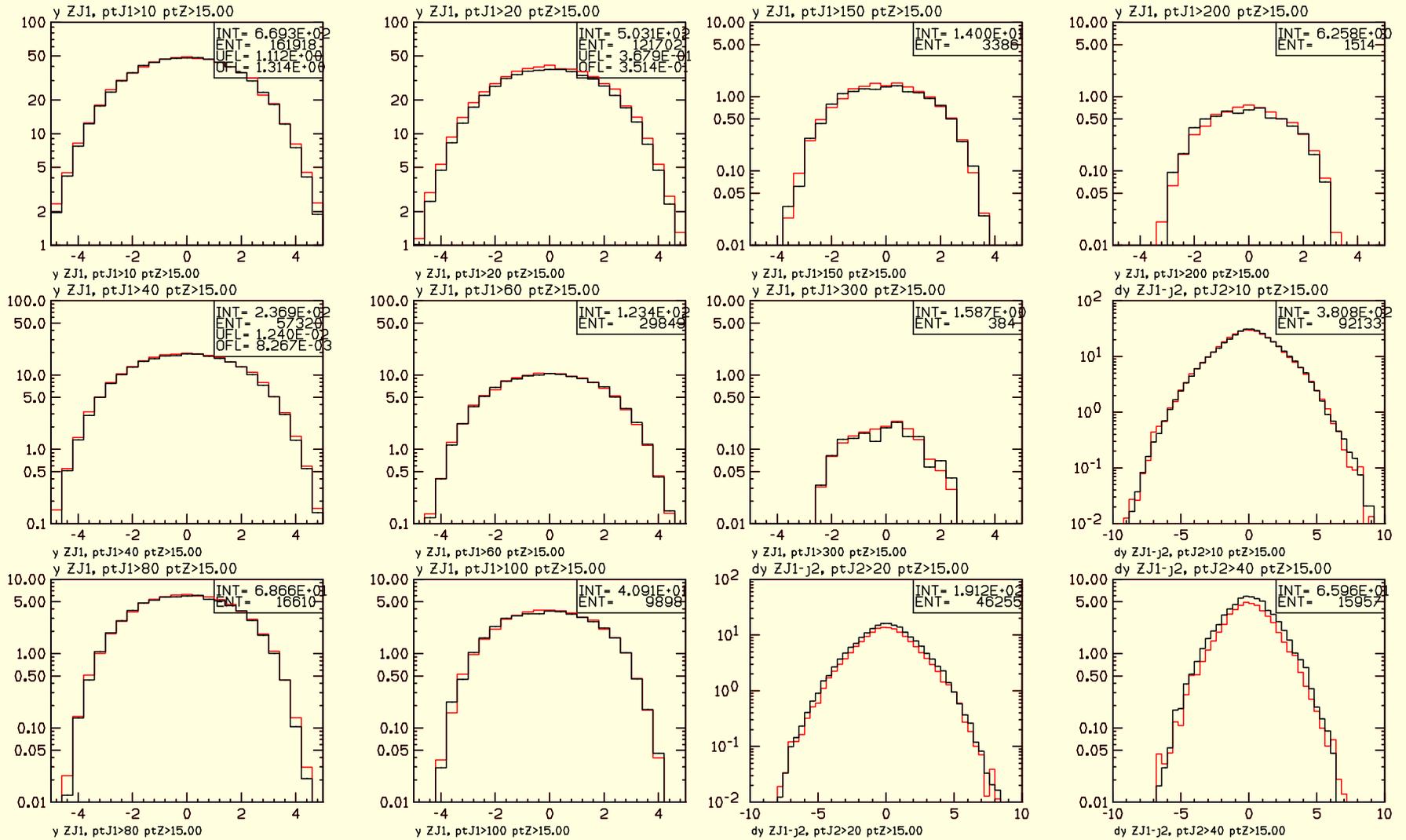


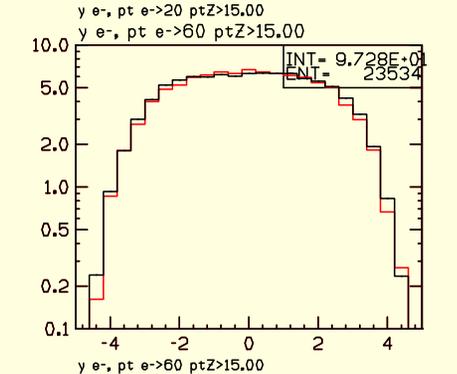
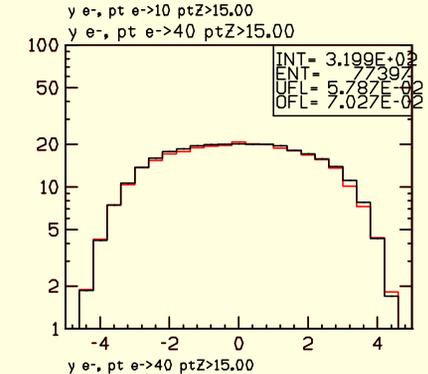
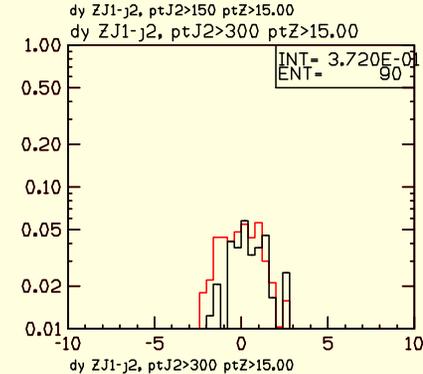
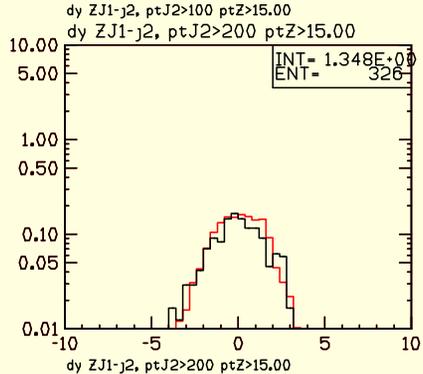
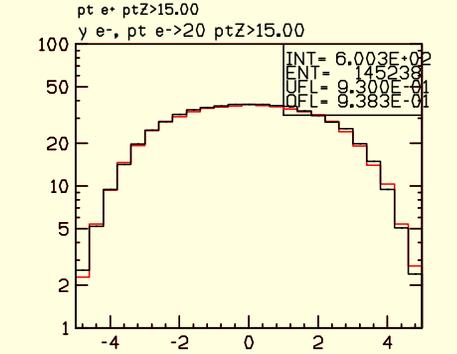
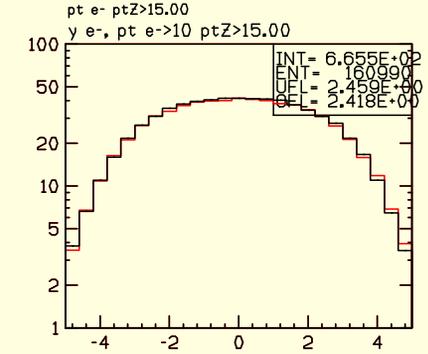
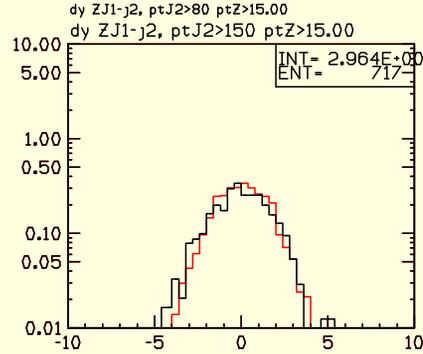
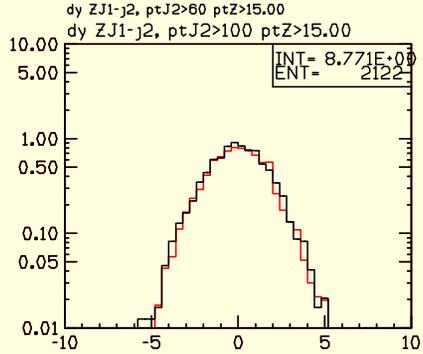
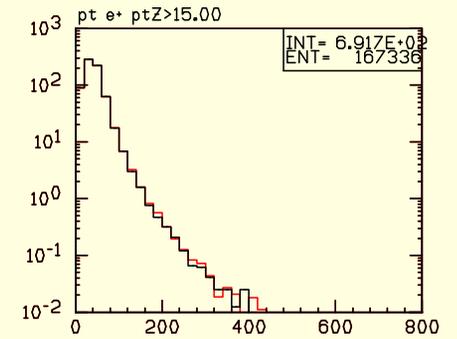
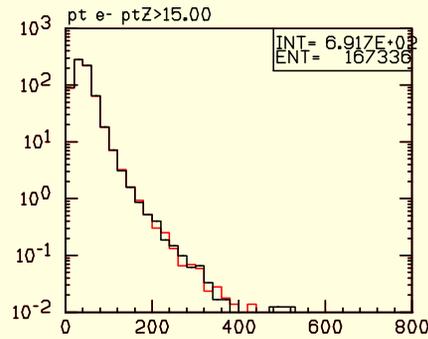
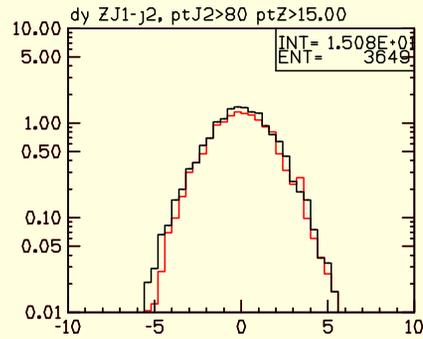
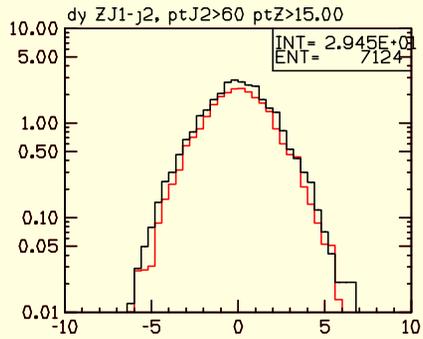


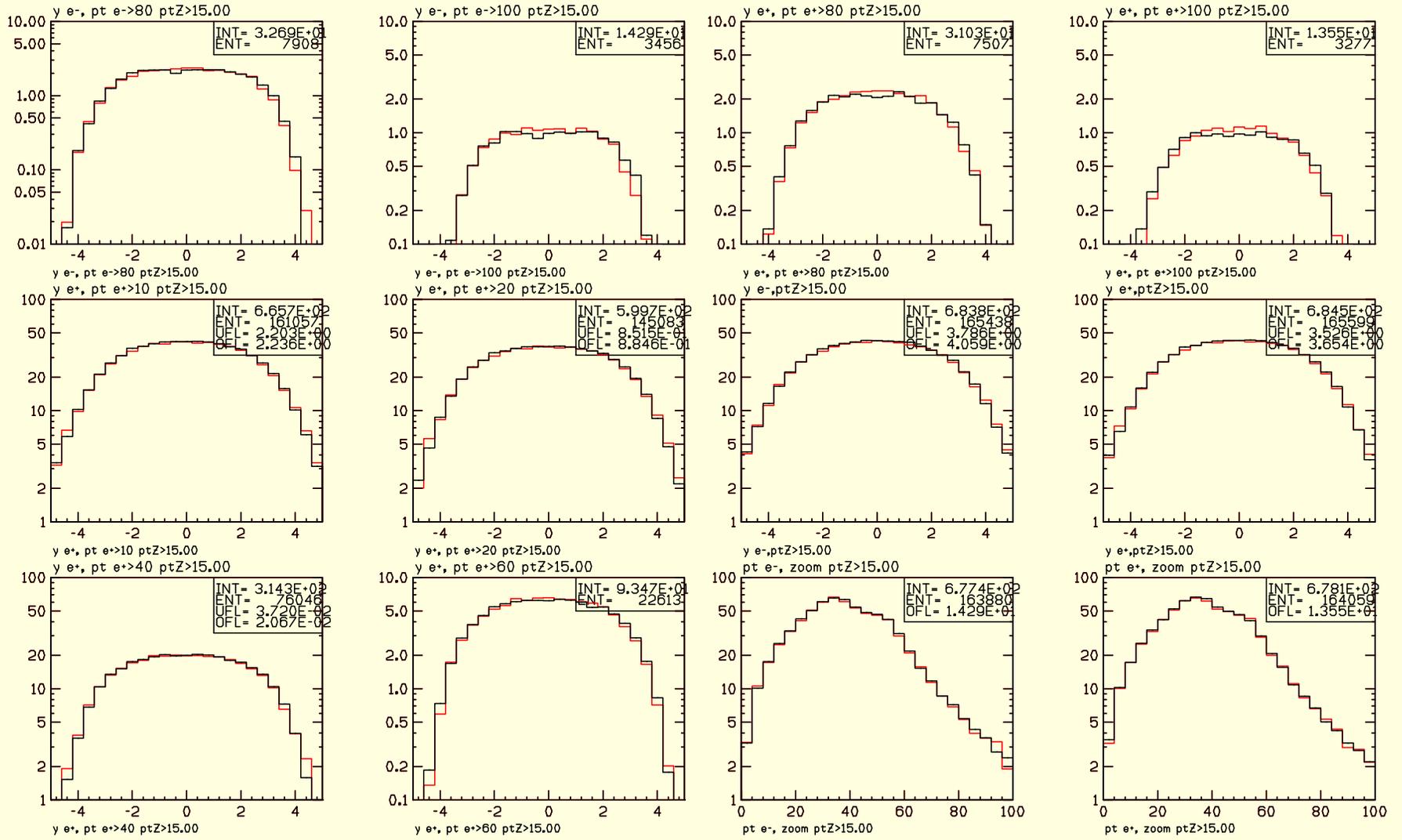
Everything seems to work ...

Now compare POWHEG+HERWIG with NLO (red)

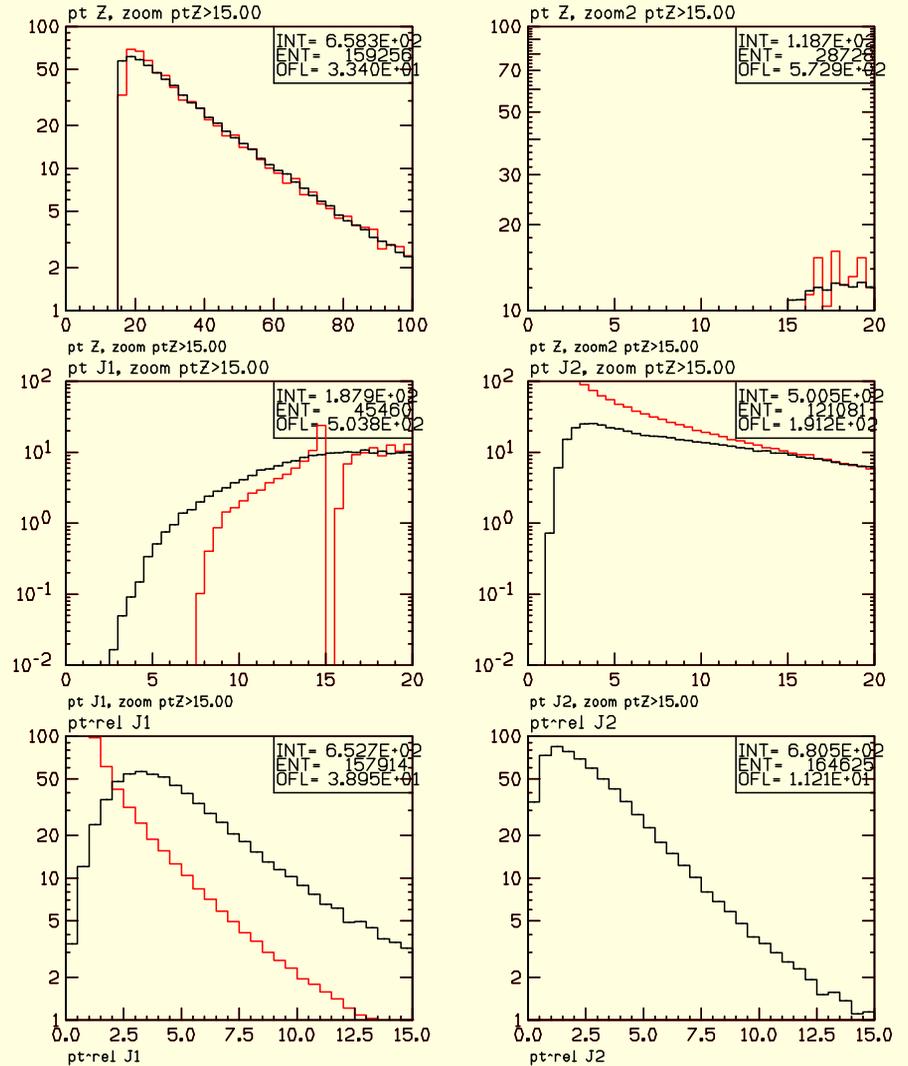








Distributions sensitive to more than two jet show noticeably different.
 All others in agreement with NLO



Conclusions

- NLO accuracy with Shower MC has become a reality in recent years.
- Two methods have provided usable implementations for collider physics MC@NLO and POWHEG.
- Agreement and differences among the two methods are relatively well understood
- A path to full automation of POWHEG implementations of arbitrary NLO calculation is open
- Even so, we are just at the beginning: many interesting problems remain to be addressed, and it is likely that further theoretical progress is still possible in this framework