MCSTHAR++, a statistical hadronization code for MC event generators

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4 July 09

Outline

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1. Introduction
2. The statistical hadronization model
3. My work: `MCSTHAR++`
4. Preliminar results
5. Conclusions

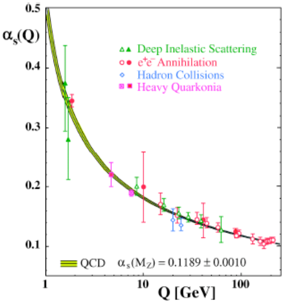
Outline

Part I

Introduction

The strong coupling constant

- The behaviour of the QCD coupling constant is such that at $E \approx 1\ GeV$ perturbative calculations are no more possible



A phenomenological model is needed to describe the hadronization process

References:

S. Bethke,

Prog. Part. Nucl. Phys. **58** (2007) 351

Phenomenological models

- Cluster fragmentation model, implemented in HERWIG and Herwig++ (B.R.Webber, Nucl. Phys. B 238 (1984) 492)
- String fragmentation model, implemented in PYTHIA and PYTHIA8 (B.Andersson, G.Gustafson, G.Ingelman and T.Sjostrand, Phys. Rept. 97 (1983) 31)
- Statistical hadronization model, not yet implemented in any official release of the MC codes

MCSTHAR++

Monte Carlo STatistical Hadron Reaction

Part II

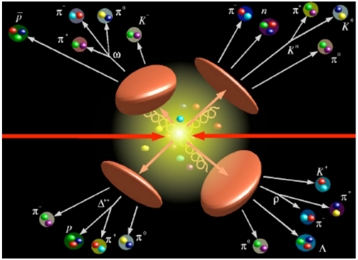
The statistical hadronization model

Statistical hadron production

- In a high-energy collision there is the production of pre-hadronic extended object called clusters or fireballs
- Each of them has well defined physical quantities

$$P, J, I, I_3, Q, \ldots$$

is colour neutral and hadronizes according to a pure statistical law



References:

- R. Hagedorn, Nuovo Cim. Suppl. **3** (1965) 147
- F. Becattini, Z. Phys. C **69** (1996) 485
- F. Becattini, U. W. Heinz, Z. Phys. C **76** (1997) 269.

The microcanonical hypothesis

Microcanonical description

Every localized multi-hadronic state within the cluster compatible with the conservation laws is equally likely

Probability to observe the final state $| f\rangle$

$$p_f \propto \langle f \mid P_i P_V P_i \mid f \rangle$$

- $P_i = P_P P_{Q,S,B}$
- $P_V = \sum_{h_V} \mid h_V \rangle \langle h_V \mid$

Microcanonical partition function

$$\sum_f p_f \propto \sum_f \langle f \mid P_i P_V P_i \mid f \rangle = \Omega$$

- Bose-Einstein and Fermi-Dirac correlations, coming from the finite volume assumption, are included

Interacting hadron gas

- How to take into account the interactions between the confined hadrons?

Gas of hadrons and resonances

Retaining only the resonant part of the interaction, the microcanonical partition function is that of a gas of free hadrons and resonances with distribuited mass

References:

J. Bernstein, R. Dashen, S. Ma, Phys. Rev. **187** (1969) 1

$\Downarrow$

To include the (leading) interactions all the resonances must be included in the hadron samplings with Breit-Wigner distributed mass

Strangeness suppression and free parameters

- To reproduce the observed multiplicities of strange particles a phenomenological parameter γ_s is included in the partition function

Strange particles suppression

$$\langle f \mid P_i P_V P_i \mid f \rangle \Rightarrow {\gamma_s}^{\sum N_s} \langle f \mid P_i P_V P_i \mid f \rangle$$

Free parameters of the model

1. γ_s Strangeness suppression parameter
2. ρ Energy density of the clusters

Part III

My work: MCSTHAR++

Code features

- MCSTHAR++ implements the statistical model in the microcanonical formulation as described in: F. Becattini, L. Ferroni, Eur. Phys. J. C38, 2004.
- The code is written in C++, using an Object Oriented framework, accordingly to the new MC event generators (Herwig++, PYTHIA8, SHERPA, ...)

Why C++?

1. To get an easy implementation in the new MC event generators
2. To get a good flexibility of the hadronization module
3. To speak the same language of the new external libraries of the hep community (HepMC, ROOT, ...)
4. ...and because I like C++!

SHM transition probability

Probability to observe the decay $| (N_j) ; p_1, ..., p_N \rangle$

$$p\left((N_j);p_1,...,p_N\right) = \frac{\frac{V^N}{(2\pi)^{3N}} \prod_{i=1}^{N} (2J_i+1)\, \delta^4\left(P_0 - \hat{P}\right) \delta_{\mathbf{Q_0}\hat{\mathbf{Q}}_{\mathbf{f}'}}}{\sum_{(N_j')} \frac{V^{N'}}{(2\pi)^{3N'}} \left(\prod_{i=1}^{N'} \sum_{\sigma_i} \int d^3p_i\right) \delta^4\left(P_0 - \hat{P}\right) \delta_{\mathbf{Q_0}\hat{\mathbf{Q}}_{\mathbf{f}'}}}$$

- The partition function is not computable analytically or by numerical integration for each point of the phase space: the standard unweighted generation is not possible
- Events can be simulated with a weighted generation (importance sampling method) and used...
  1. ...to calculate the mean value of the observabes
  2. ...as input for an unweighting routine

Two projects for MCSTHAR++ (I)

Collaboration of PV (C.B. and F. Piccinini) and FI (F. Becattini)

- Build the hadronization code
- Implement MCSTHAR++ in HERWIG
- Tune the event generator on LEP and Tevatron data, do a tuned comparison with the other models and get some predictions for LHC

Code structure

- MCSTHAR++ works as an external linkable library to the main FORTRAN or C++ code (HERWIG + MCSTHAR++ + HepMC)
- Two external libraries are used: ROOT (random number generators, Lorentz vectors, ...) and HepMC (translation of the event informations from the HEPEVT common to the HepMC record)

Two projects for MCSTHAR++ (II)

Work supported by the MCnet network, in collaboration with the Karlsruhe team of developers of Herwig++ (S. Gieseke)

- Implement MCSTHAR++ in Herwig++
- Tune the event generator on LEP and Tevatron data, do a tuned comparison with the other models and get some predictions for LHC

Code structure

- MCSTHAR++ works as a set of external classes (for the moment using modified ClusterHadronizationHandler and ClusterDecayer classes)
- No external libraries are needed, everything is provided by Herwig++ and ThePEG

Part IV

Preliminar results

Checks: single cluster (I)

- Comparisons on the microcanonical weight (GeV^{-4}) of a single channel:

Channel	MCSTHAR++	BFG	$\Delta\%$
$2\pi^0 + \pi^+ + \pi^-$	$(2.898 \pm 0.001) \times 10^5$	2.91×10^5	0.4%
$2\pi^0 + 2\pi^+ + 2\pi^-$	$(8.921 \pm 0.005) \times 10^5$	8.94×10^5	0.2%
4η	$(6.169 \pm 0.002) \times 10^3$	6.19×10^3	0.3%
6π	$(9.74 \pm 0.02) \times 10^5$	9.7×10^5	-0.4%
6π (QS)	$(1.209 \pm 0.002) \times 10^6$	1.21×10^6	0.08%

- Cluster with mass of $M = 4$ GeV ($M = 5$ GeV in the last two rows), energy density $\rho = 0.4$ GeV/fm^3 and strangeness suppression parameter $\gamma_s = 1$
- Results are at Boltzmann level with exception of the last row
- The column "BFG" is taken from T.Gabriellini Diploma thesis

Checks: single cluster (II)

- Comparisons on the particle multiplicities
- Neutral cluster with energy density $\rho = 0.4$ GeV/fm^3 and strangeness suppression parameter $\gamma_s = 1$
- Results include quantum correlations and interactions

$M = 2$ GeV	MCSTHAR++	BFG
Total	2.47 ± 0.05	2.430 ± 0.002
$\pi^0 * 5$	1.9 ± 0.2	2.10 ± 0.01
$p * 75$	0.6 ± 0.2	0.32 ± 0.01
$\Omega * 2500$	0 ± 0	0 ± 0

$M = 4$ GeV	MCSTHAR++	BFG
Total	4.58 ± 0.05	4.452 ± 0.006
$\pi^0 * 5$	3.3 ± 0.2	3.22 ± 0.03
$p * 75$	2.1 ± 0.7	1.18 ± 0.04
$\Omega * 2500$	0 ± 0	0.02 ± 0.01

Part V

Conclusions

Conclusions

1. Phenomenological models are needed to describe the hadronization process
2. Different models are implemented in the available MC event generators
3. It is worth to have an independent model available for the hadronization:
   - Small number of phenomenological parameters
   - MC generators are tuned on data at energy lower than the one of LHC
   - The availability of independent models gives reliability to the theoretical predictions and their uncertainties
4. MCSTHAR++ is almost ready to be tuned on LEP data with HERWIG and Herwig++

Thank You!

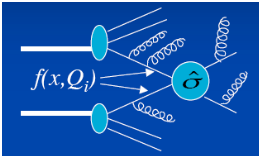
Part VI

Backup slides

High energy process evolution and factorization theorem

- An high-energy event in a Monte Carlo generator is built on the following elements:

1. Parton distribution functions and matrix element
2. Parton shower algorithm
3. Underlying event description
4. Hadronization models, ...



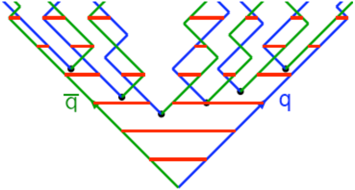
Factorization theorem

$$\frac{d\sigma}{dx} = \sum_{j,k} \int f_j\left(x_1, Q_i\right) f_k\left(x_2, Q_i\right) \frac{d\hat{\sigma}_{jk}\left(Q_i, Q_f\right)}{d\hat{x}} F\left(\hat{x} \to x; Q_i, Q_f\right) \ldots$$

- $F\left(\hat{x} \to x; Q_i, Q_f\right)$ Transition function from the partonic to the observable hadronic states

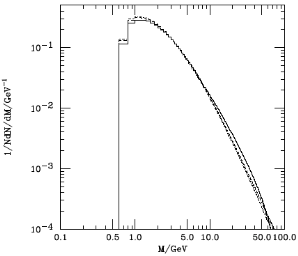
PYTHIA: the string fragmentation model

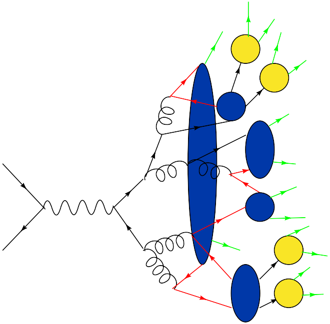
- Implemented in PYTHIA and PYTHIA8
- A colour string is supposed to connect quarks, antiquarks and gluons (linear potential)
- Hadrons come from $q\overline{q}$ pairs produced from the vacuum thanks to the energy coming from string fragmentation
- When the fragmentation process stops we have the hadrons, formed by partons connected by a color string



HERWIG: the cluster model

- Implemented in HERWIG and Herwig++
- Final state quarks and antiquarks are used to build colourless clusters of pre-hadronic matter
- The clusters decay into two hadrons according to spin degeneracy and available phase space (creating $q\bar{q}$ pairs from the vacuum)





HERWIG event weight (I)

Production of the hadrons a and b

$$W\left(a_{(q_1,\bar{q})}, b_{(q,\bar{q}_2)}\right) = P_q w_a s_a w_b s_b p_{a,b}$$

- P_q extraction from the vacuum
- $w_{a,b}$ hadron choice
- $s_{a,b}$ suppression factors
- $p_{a,b} = \frac{1}{2M}\left(\left(M^2 - (m_a + m_b)^2\right)\left(M^2 - (m_a - m_b)^2\right)\right)^{\frac{1}{2}}$
- $w_h = w_{mix}(2J_h + 1)$

References:

- G. Corcella, I.G. Knowles, G. Marchesini, S. Moretti, K. Odagiri, P. Richardson, M.H. Seymour, B.R. Webber, JHEP **0101** (2001) 010
- M. Bähr, S. Gieseke, M.A. Gigg, D. Grellscheid, K. Hamilton, O. Latunde-Dada, S. Plätzer, P. Richardson, M.H. Seymour, A. Sherstnev, B.R. Webber, e-Print: arXiv:0803.0883 [hep-ph]

HERWIG event weight (II)

Event probability

$$P\left(a_{(q_1,\overline{q})}, b_{(q,\overline{q}_2)} \mid q_1\overline{q}_2\right) = P_q \frac{1}{N_{(q_1,\overline{q})}} \frac{1}{N_{(q,\overline{q}_2)}} \frac{w_a}{w_{max(q_1,\overline{q})}} s_a \frac{w_b}{w_{max(q,\overline{q}_2)}} s_b \frac{p_{a,b}}{p_{max}}$$

Mixing factor (Mesons with light quarks)

$$\pi^0 = \frac{1}{\sqrt{2}}\left(\mathrm{d}\overline{\mathrm{d}} - \mathrm{u}\overline{\mathrm{u}}\right)$$
$$\eta = \psi_8 cos\theta - \psi_1 sin\theta$$
$$\eta' = \psi_8 sin\theta + \psi_1 cos\theta$$
$$\psi_8 = \frac{1}{\sqrt{6}}\left(\mathrm{d}\overline{\mathrm{d}} + \mathrm{u}\overline{\mathrm{u}} - 2\mathrm{s}\overline{\mathrm{s}}\right)$$
$$\psi_1 = \frac{1}{\sqrt{3}}\left(\mathrm{d}\overline{\mathrm{d}} + \mathrm{u}\overline{\mathrm{u}} + \mathrm{s}\overline{\mathrm{s}}\right)$$

$$\Rightarrow$$

$$w^{\pi^0}_{\mathrm{u}\overline{\mathrm{u}}} = w^{\pi^0}_{\mathrm{d}\overline{\mathrm{d}}} = \frac{1}{2}$$
$$w^{\pi^0}_{\mathrm{s}\overline{\mathrm{s}}} = 0$$
$$w^{\eta}_{\mathrm{u}\overline{\mathrm{u}}} = w^{\pi^0}_{\mathrm{d}\overline{\mathrm{d}}} = \frac{1}{2} cos^2(\theta + \phi)$$
$$\ldots$$
$$\ldots$$

SHM transition probability (I)

Probability to observe the decay $|\left(N_j\right); p_1, ..., p_N\rangle$

$$p\left(\left(N_j\right); p_1, ..., p_N\right) = \frac{\frac{V^N}{(2\pi)^{3N}} \prod_{i=1}^{N} \left(2J_i + 1\right) \delta^4\left(P_0 - \hat{P}\right) \delta_{\mathbf{Q_0}\hat{\mathbf{Q}}_{\mathbf{f}'}}}{\sum_{\left(N'_j\right)} \frac{V^{N'}}{(2\pi)^{3N'}} \left(\prod_{i=1}^{N'} \sum_{\sigma_i} \int d^3 p_i\right) \delta^4\left(P_0 - \hat{P}\right) \delta_{\mathbf{Q_0}\hat{\mathbf{Q}}_{\mathbf{f}'}}}$$

- The partition function is not computable analytically or by numerical integration for each point of the phase space: the standard unweighted generation is not possible
- Events can be simulated with a weighted generation (importance sampling method) and used...
  1. ...to calculate the mean value of the observabes
  2. ...as input for an unweighting routine

SHM transition probability (II)

- The transition probability, including the quantum correlations, is given by

Probability to observe the decay $|\left(N_j\right); p_1, ..., p_N\rangle$

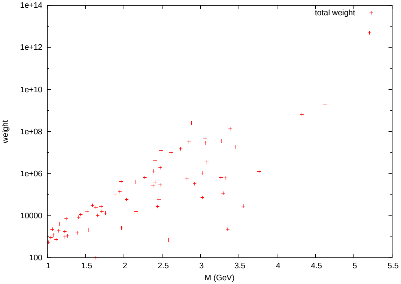
$$\prod_{j=1}^{K}\left[\sum_{r_j}\frac{\chi\left(r_j\right)^{b_j}}{N_j!}\sum_{\sigma_1,\ldots,\sigma_{N_j}}\prod_{i_j=1}^{N_j}\frac{\delta_{\sigma_{i_j}\sigma_{r_j(i_j)}}}{\left(2\pi\right)^3}\int_A d^3x e^{-i\left(\mathbf{p}_{i_j}-\mathbf{p}_{r_j(i_j)}\right)\cdot\mathbf{x}}\right]\times$$

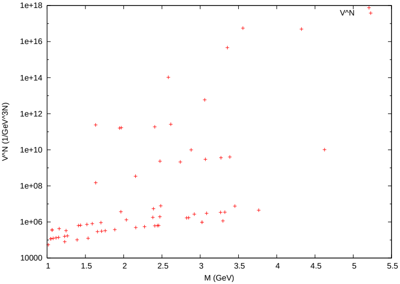
$$\delta^4\left(P_0-\hat{P}\right)\delta_{\mathbf{Q_0}\hat{\mathbf{Q}}_{\mathbf{f}'}}$$

- Quantum correlations give corrections of about 20% on the microcanonical weight for the single channel
- They increase (decrease) the weight of the final state configuration with identical bosons (fermions) close to each other in the phase space

MCSTHAR++ phenomenology: event weight (I)

- The importance sampling gives a very large range of value for the microcanonical weights in the full generation





- That is related to the factor V^N in the microcanonical weight and to the large range in the mass distribution of the clusters

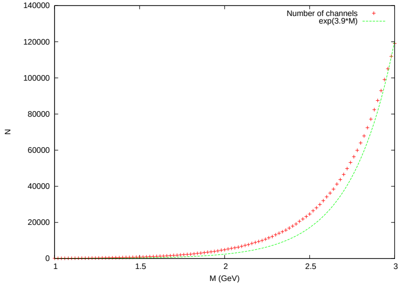
MCSTHAR++ phenomenology: event weight (II)

Some strategies:

1. Calculate the partition function for each cluster coming from Herwig during the simulation (Very slow)
2. Split and merge the clusters to keep the masses in a narrow range (Needs to introduce new phenomenological parameters)
3. Build a set of partition functions to be used during the simulation, for each charge configuration and interpolating in the mass, energy density and γ_s (Approximated solution and the table strongly depends on the particle set)
4. Quit to use the importance sampling and move to Metropolis algorithm (Very slow, again)

MCSTHAR++ phenomenology: event weight (III)

- The number of possible channels for a cluster increases almost exponentially with the mass of the cluster



- Number of channels for a neutral cluster and for a set of 235 hadrons

SHM observable mean values

- The observable mean values are calculated using the Monte Carlo importance sampling method

$$\langle O(f) \rangle = \int O(f) p(f)\, df \approx \sum_{k=1}^{N} O(f_k) \frac{p(f_k)}{\Pi(f_k)}$$

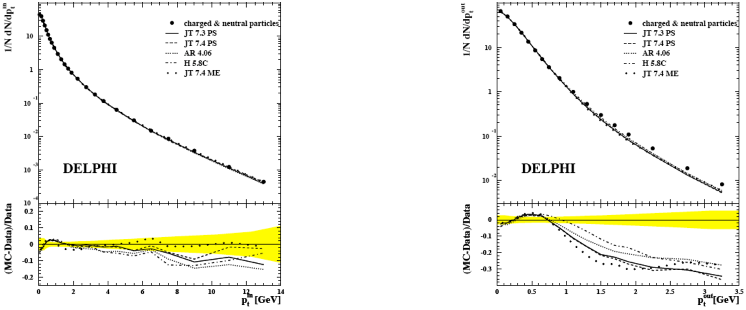
- $p(f_k) = \prod_{j=1}^{N_c} p_j \left(f_k^j \right)$ Total microcanonical weight
- $\Pi(f_k) = \prod_{j=1}^{N_c} \Pi_j \left(f_k^j \right)$ Total sampling weight

Because of the thermodynamical limit grandcanonical probability densities are used as sampling functions

References:

F. Becattini, L. Ferroni,
Eur. Phys. J. C **35** (2004) 243;
38 (2004) 225

Some interesting observables: p_T spectra (String and Cluster models)

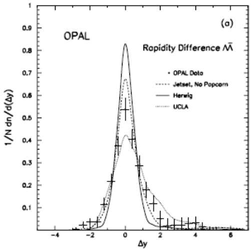


- The transverse directions are defined with respect to the thrust axes:

$$T = \sum_i \frac{\mid \mathbf{p}_i \cdot \mathbf{n}_{th} \mid}{\mid \mathbf{p}_i \mid}$$

- The p_T^{in} distribution is in agreement with data for the two models
- The tail of the p_T^{out} distribution ($p_T^{out} > 1\ GeV$) disagrees with the data for the two model

Some interesting observables: rapidity distribution (String and Cluster models)



References (Numerical results):

- K. Hamacher, M. Weierstall, e-Print: arXiv:9511011 [hep-ex]

- The rapidity is defined as $y = \dfrac{1}{2} \ln \dfrac{E + p_z}{E - p_z}$

Strong coupling constant

- In 1-loop approximation the QCD coupling constant is given by

$$\alpha_s\left(Q^2\right) = \frac{\alpha_s\left(\mu^2\right)}{1 + \alpha_s\left(\mu^2\right)\beta_0 \ln\frac{Q^2}{\mu^2}}$$

- Where $\beta_0 = \frac{33 - 2N_f}{12\pi}$