

# Introduction to Event Generators

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## Topics of the lectures

- 1 Lecture 1: *The Monte Carlo Principle*
- 2 Lecture 2: *Parton level event generation*
- 3 Lecture 3: *Dressing the Partons*
- 4 Lecture 4: *Modelling beyond Perturbation Theory*

Thanks to

- My fellow MC authors, especially S.Gieseke, K.Hamilton, L.Lonnblad, F.Maltoni, M.Mangano, P.Richardson, M.Seymour, T.Sjostrand, B.Webber.
- the other Sherpas: J.Archibald, T.Gleisberg, S.Höche, S.Schumann, F.Siegert, M.Schönherr, and J.Winter.

## Today's lecture

- Prelude: Orientation
- Stating the problem: Factorial growth
- Efficient matrix element calculation and phase space evaluation at leading order (tree-level)
- Survey of leading order tools
- Next-to leading order

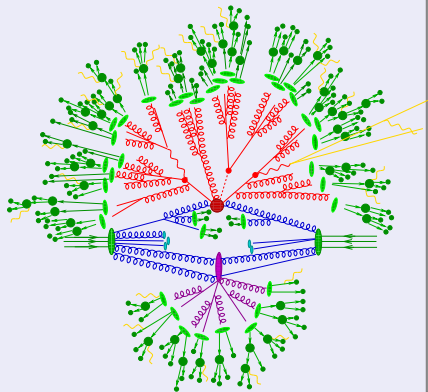
# Prelude: Orientation

## Event generator paradigm

Divide event into stages, separated by different scales.

- **Signal/background:**  
Exact matrix elements.
- **QCD-Bremsstrahlung:**  
Parton showers (also in *initial state*).
- **Multiple interactions:**  
Beyond factorisation: Modelling.
- **Hadronisation:**  
Non-perturbative QCD: Modelling.

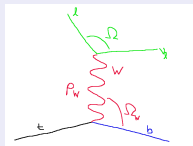
## Sketch of an event



## Simulation of the hard bits (signals & backgrounds)

- Simple example:  $t \rightarrow bW^+ \rightarrow b\bar{l}\nu_l$ :

$$|\mathcal{M}|^2 = \frac{1}{2} \left( \frac{8\pi\alpha}{\sin^2\theta_W} \right)^2 \frac{p_t \cdot p_\nu \ p_b \cdot p_l}{(p_W^2 - M_W^2)^2 + \Gamma_W^2 M_W^2}$$



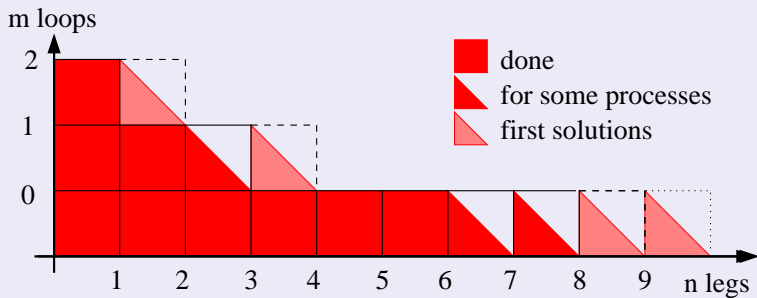
- Phase space integration (5-dim):

$$\Gamma = \frac{1}{2m_t} \frac{1}{128\pi^3} \int dp_W^2 \frac{d^2\Omega_W}{4\pi} \frac{d^2\Omega}{4\pi} \left( 1 - \frac{p_W^2}{m_t^2} \right) |\mathcal{M}|^2$$

- 5 random numbers  $\implies$  four-momenta  $\implies$  “events”.
- Apply **smearing** and/or **arbitrary cuts**.
- Simply **histogram any quantity of interest** - no new calculation for each observable

## Availability of exact calculations (hadron colliders)

- Fixed order matrix elements (“parton level”) are exact to a given perturbative order. (and often quite a pain!)
- Important to understand limitations:  
Only tree-level fully automated, 1-loop-level ongoing.



# Parton level simulations

## Stating the problem(s)

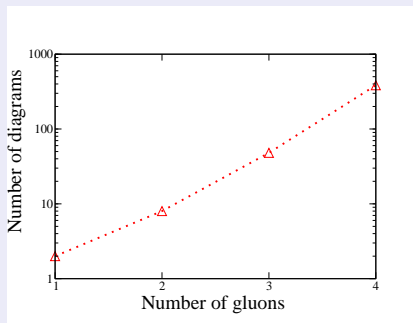
- Multi-particle final states for signals & backgrounds.
- Need to evaluate  $d\sigma_N$ :

$$\int_{\text{cuts}} \left[ \prod_{i=1}^N \frac{d^3 q_i}{(2\pi)^3 2E_i} \right] \delta^4 \left( p_1 + p_2 - \sum_i q_i \right) |\mathcal{M}_{p_1 p_2 \rightarrow N}|^2.$$

- Problem 1: Factorial growth of number of amplitudes.
- Problem 2: Complicated phase-space structure.
- Solutions: **Numerical methods.**

## Example for factorial growth: $e^+e^- \rightarrow q\bar{q} + ng$

n	#diags
0	1
1	2
2	8
3	48
4	384





## Basic ideas of efficient ME calculation

Need to evaluate  $|\mathcal{M}|^2 = \left| \sum_i \mathcal{M}_i \right|^2$

- Obvious: Traditional textbook methods (squaring, completeness relations, traces) fail
  - ⇒ result in proliferation of terms ( $\mathcal{M}_i \mathcal{M}_j^*$ )
  - ⇒ Better: **Amplitudes are complex numbers**,
  - ⇒ **add them before squaring!**
- Remember: spinors, gamma matrices have explicit form could be evaluated numerically (brute force)  
But: Rough method, lack of elegance, CPU-expensive

## Helicity method

- Introduce basic helicity spinors (needs to “gauge”-vectors)
- Write everything as spinor products, e.g.

$$\bar{u}(p_1, h_1)u(p_2, h_2) = \text{complex numbers.}$$

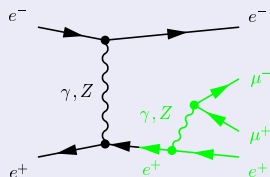
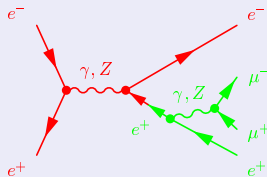
$$\text{Completeness rel'n: } (\not{p} + m) \Rightarrow \frac{1}{2} \sum_h \left[ \left(1 + \frac{m^2}{p^2}\right) \bar{u}(p, h)u(p, h) + \left(1 - \frac{m^2}{p^2}\right) \bar{v}(p, h)v(p, h) \right]$$

- There are other genuine expressions . . .
- Translate Feynman diagrams into “helicity amplitudes”: complex-valued functions of momenta & helicities.
- Spin-correlations etc. nearly come for free.

## Taming the factorial growth

- In the helicity method
  - Reusing pieces: **Calculate only once!**
  - Factoring out: **Reduce number of multiplications!**

Implemented as a-posteriori manipulations of amplitudes.



- Better method: Recursion relations (recycling built in).  
Best candidate so far: Off-shell recursions

(Dyson-Schwinger, Berends-Giele etc.)

## Colour-dressing: Fighting factorial growth in colour

- In principle: sampling over colours improves situation.

(But still, e.g. naively  $\simeq (n-1)!$  permutations/colour-ordering for  $n$  external gluons).

- Improved scheme: colour dressing.

F.Maltoni, K.Paul, T.Stelzer & S.Willenbrock Phys. Rev. **D67** (2003) 014026

- Works very well with Berends-Giele recursions:

C.Duhr, S.Hoche & F.Maltoni, JHEP **0608** (2006) 062

Final State	BG		BCF		CSW	
	CO	CD	CO	CD	CO	CD
2g	0.24	0.28	0.28	0.33	0.31	0.26
3g	0.45	0.48	0.42	0.51	0.57	0.55
4g	1.20	1.04	0.84	1.32	1.63	1.75
5g	3.78	2.69	2.59	7.26	5.95	5.96
6g	14.2	7.19	11.9	59.1	27.8	30.6
7g	58.5	23.7	73.6	646	146	195
8g	276	82.1	597	8690	919	1890
9g	1450	270	5900	127000	6310	29700
10g	7960	864	64000	-	48900	-

Time [s] for the evaluation of  $10^4$  phase space points, sampled over helicities & colour.

## Efficient phase space integration

(“Amateurs study strategy, professionals study logistics”)

- Democratic, process-blind integration methods:

- Rambo/Mambo: Flat & isotropic

R.Kleiss, W.J.Stirling & S.D.Ellis, *Comput. Phys. Commun.* **40** (1986) 359;

- HAAG/Sarge: Follows QCD antenna pattern

A.van Hameren & C.G.Papadopoulos, *Eur. Phys. J. C* **25** (2002) 563.

- Multi-channeling: Each Feynman diagram related to a phase space mapping (= “channel”), optimise their relative weights.

R.Kleiss & R.Pittau, *Comput. Phys. Commun.* **83** (1994) 141.

- Main problem: practical only up to  $\mathcal{O}(10k)$  channels.
- Some improvement by building phase space mappings recursively: More channels feasible, efficiency drops a bit.

## Monte Carlo integration: Unweighting efficiency

- Want to generate events “as in nature”.
- Basic idea: Use hit-or-miss method;
  - Generate  $\vec{x}$  with integration method,
  - compare actual  $f(\vec{x})$  with maximal value during sampling  $\implies$  “Unweighted events”.
- Comments:
  - unweighting efficiency,  $w_{\text{eff}} = \langle f(\vec{x}_j)/f_{\text{max}} \rangle =$  number of trials for each event.
  - Good measure for integration performance.
  - Expect  $\log_{10} w_{\text{eff}} \approx 3 - 5$  for good integration of multi-particle final states at tree-level.
  - Maybe acceptable to use  $f_{\text{max,eff}} = K f_{\text{max}}$  with  $K < 1$ .  
 Problem: what to do with events where  $f(\vec{x}_j)/f_{\text{max,eff}} > 1$ ?  
 Answer: Add  $\text{int}[f(\vec{x}_j)/f_{\text{max,eff}}] = k$  events and perform hit-or-miss on  $f(\vec{x}_j)/f_{\text{max,eff}} - k$ .

## Best answer at the moment: COMIX (personal bias)

T.Gleisberg & S.Hoeche, JHEP 0812 (2008) 039

- Colour-dressed Berends-Giele amplitudes in the SM.
- Fully recursive phase space generation.
- Example results (cross sections):

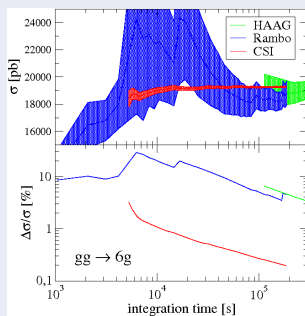
$gg \rightarrow ng$	Cross section [pb]				
n	8	9	10	11	12
$\sqrt{s}$ [GeV]	1500	2000	2500	3500	5000
COMIX	0.755(3)	0.305(2)	0.101(7)	0.057(5)	0.019(2)
Maltoni (2002)	0.70(4)	0.30(2)	0.097(6)		
ALPGEN	0.719(19)				

$\sigma$ [ $\mu\text{b}$ ]	Number of jets						
$b\bar{b}$ + QCD jets	0	1	2	3	4	5	6
COMIX	470.8(5)	8.83(2)	1.826(8)	0.459(2)	0.1500(8)	0.0544(6)	0.023(2)
ALPGEN	470.6(6)	8.83(1)	1.822(9)	0.459(2)	0.150(2)	0.053(1)	0.0215(8)
AMEGIC++	470.3(4)	8.84(2)	1.817(6)				

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T.Gleisberg & S.Hoeche, JHEP 0812 (2008) 039

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- Example results (phase space performance):

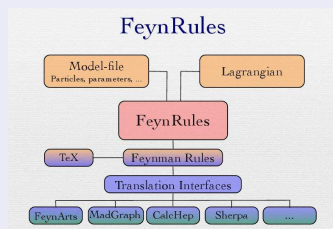




# FEYNRULES: Implementing new models made easy

## Aim

- Portable, transparent & reproducible implementation of (nearly arbitrary) new physics models.
- In most codes: New models given by new particles, their properties & interactions.
- Output to standard ME generators enabled (MADGRAPH, SHERPA, ...)
- Various models already implemented & validated for a list: <http://feynrules.phys.ucl.ac.be>



# Survey of existing parton-level tools

## Comparison of tree-level tools

	Models	$2 \rightarrow n$	Ampl.	Integ.	public?	lang.
ALPGEN	SM	$n = 8$	rec.	Multi	yes	Fortran
AMEGIC++	SM,MSSM,ADD	$n = 6$	hel.	Multi	yes	C++
COMIX	SM	$n = 8$	rec.	Multi	yes	C++
COMPHEP	SM,MSSM	$n = 4$	trace	1Channel	yes	C
HELAC	SM	$n = 8$	rec.	Multi	yes	Fortran
MADEVENT	SM,MSSM,UED	$n = 6$	hel.	Multi	yes	Fortran
WHIZARD	SM,MSSM,LH	$n = 8$	rec.	Multi	yes	O'Caml

# Limitations of parton level simulation

## Factorial growth

- ... persists due to the number of colour configurations  
(e.g.  $(n - 1)!$  permutations for  $n$  external gluons).
- Solution: Sampling over colours,  
but correlations with phase space  
 $\implies$  Best recipe not (yet) found.
- New scheme for colour: colour dressing

(C.Duhr, S.Hoche and F.Maltoni, JHEP 0608 (2006) 062)

## Efficient phase space integration

- Main problem: Adaptive multi-channel sampling translates “Feynman diagrams” into integration channels  
⇒ hence subject to growth.
- But it is practical only for 1000-10000 channels.
- Therefore: Need better sampling procedures  
⇒ open question with little activity.

(Private suspicion: Lack of glamour)

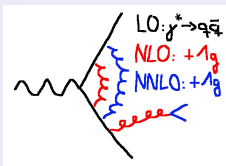
## General

- Fixed order parton level (LO, NLO, ...) implies fixed multiplicity
- No control over potentially large logs  
(appear when two partons come close to each other).
- Parton level is parton level is parton level ...  
**experimental** definitions rely on observable hadrons.

Therefore: **Need hadron level event generators!**

# A short detour to NLO calculations

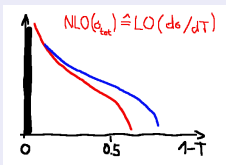
## Nomenclature (example: $\gamma^* \rightarrow \text{hadrons}$ )



- In general:  $N^n\text{LO} \leftrightarrow \mathcal{O}(\alpha_s^n)$
- But: only for inclusive quantities

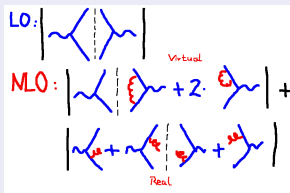
(e.g.: total xsecs like  $\gamma^* \rightarrow \text{hadrons}$ ).

## Counter-example: thrust distribution



- In general, distributions are HO.
- Distinguish real & virtual emissions:  
Real emissions  $\rightarrow$  mainly distributions,  
virtual emissions  $\rightarrow$  mainly normalisation.

## Anatomy: Virtual and real corrections



NLO corrections:  $\mathcal{O}(\alpha_s)$

Virtual corrections = extra loops

Real corrections = extra legs

- UV-divergences in virtual graphs  $\rightarrow$  renormalisation
- But also: IR-divergences in real & virtual contributions  
Must cancel each other, non-trivial to see:  
 $N$  vs.  $N + 1$  particle FS, divergence in PS vs. loop

## MC calculations at NLO QCD

- Calculate two separate, divergent integrals

$$\sigma_{NLO} = \int_{m+1} d\sigma_R + \int_m d\sigma_V$$

- Real emission in  $d\sigma_R$ , virtual loop in  $d\sigma_V$ .
- Divergent structures due to soft/collinear particles.
- Combine before numerical integration to cancel divergences (KLN theorem guarantees cancellation).
- Two solutions: Phase space slicing and subtraction.



## Illustrative 1-dim example

- $|\mathcal{M}_{m+1}^R|^2 = \frac{1}{x}R(x)$ , where  $x$ =gluon energy or similar.
- $|\mathcal{M}_m^V|^2 = \frac{1}{\epsilon}V$ , regularised in  $d = 4 - 2\epsilon$  dimensions.
- Cross section in  $d$  dimensions with jet measure  $F^J$ :

$$\sigma = \int_0^1 \frac{dx}{x^{1+\epsilon}} R(x) F_1^J(x) + \frac{1}{\epsilon} V F_0^J$$

- Infrared safety of jet measure:  $F_1^J(0) = F_0^J$   
 $\implies$  "A soft/collinear parton has no effect."  
 (Tricky issue - without it, no reliable NLO calculation!)
- KLN theorem:  $R(0) = V$ .

## Phase space slicing in 1-dim example

W.T.Giele and E.W.N.Glover, Phys. Rev. D 46 (1992) 1980.

- Introduce arbitrary cutoff  $\delta \ll 1$ :

$$\begin{aligned}
 \sigma &= \int_0^\delta \frac{dx}{x^{1+\epsilon}} R(x) F_1^J(x) + \frac{1}{\epsilon} V F_0^J + \int_\delta^1 \frac{dx}{x^{1+\epsilon}} R(x) F_1^J(x) \\
 &\approx \int_0^\delta \frac{dx}{x^{1+\epsilon}} V F_0^J + \frac{1}{\epsilon} V F_0^J + \int_\delta^1 \frac{dx}{x} R(x) F_1^J(x) \\
 &= \log(\delta) V F_0^J + \int_\delta^1 \frac{dx}{x} R(x) F_1^J(x)
 \end{aligned}$$

- Two separate finite integrals - both numerically large  
 $\implies$  error blows up (trial and error for stability)

## Subtraction method in 1-dim example

S.Catani and M.H.Seymour, Nucl. Phys. B **485** (1997) 291

- Rewrite

$$\begin{aligned}\sigma &= \int_0^1 \frac{dx}{x^{1+\epsilon}} R(x) F_1^J(x) - \int_0^1 \frac{dx}{x^{1+\epsilon}} VF_0^J + \int_0^1 \frac{dx}{x^{1+\epsilon}} VF_0^J + \frac{1}{\epsilon} VF_0^J \\ &= \int_0^1 \frac{dx}{x^{1+\epsilon}} \left( R(x) F_1^J(x) - VF_0^J \right) + \mathcal{O}(1) VF_0^J.\end{aligned}$$

- Two separate finite integrals, with no large numbers to be added/subtracted.
- Subtraction terms are universal (analytical bit can be calculated once and for all).

# Parton level tools: Loop level

## Specific solutions

- Currently only process-specific codes/calculations, e.g.:
  - NLOJET++ (jets only),
  - VBFNLO (VBF-type processes),
  - and MCFM (the interesting rest)

Common to both: Reach so far to  $2 \rightarrow 3$  processes.

- Bottleneck so far: virtual contributions

$\implies$  a general solution is in sight

keywords: OPP-method, generalised unitarity

(both calculated  $W + 3$  jets in hadronic collisions)

tiny [R.K.Ellis, K.Melnikov and G.Zanderighi, JHEP 0904 \(2009\) 077](#); [C.F.Berger et al., arXiv:0902.2760 \[hep-ph\]](#)

## Automated real subtraction algorithms

- Remaining major nuisance in NLO calculations: real contributions & subtraction  $\implies$  has been “solved”, i.e. automated.
- In principle: simple (“only” tree-level) & general (process-independent subtraction schemes).
- A problem that begs for automation.
- Status by now:
  - Various implementations documented in different stages, all building on Catani-Seymour subtraction.
  - Only one fully public version (MADDIPOLE), but lacks integrated dipoles.
  - A private version in SHERPA has been used to calculate the  $W + 3$  jet cross section in BLACKHAT.

tiny [C. F. Berger et al., arXiv:0902.2760 \[hep-ph\]](#)

## Summary of lecture 2

- A first level of simulation: parton level.
- Brief review of state-of-the-art there.
- Discussed automated generation of matrix elements and their phase space integration.
- Many tools available for tree-level multi-leg.
- Going to loop-level in an automated way just started now.
- Discussed some intricacies of NLO calculations.