The Top Quark Mass

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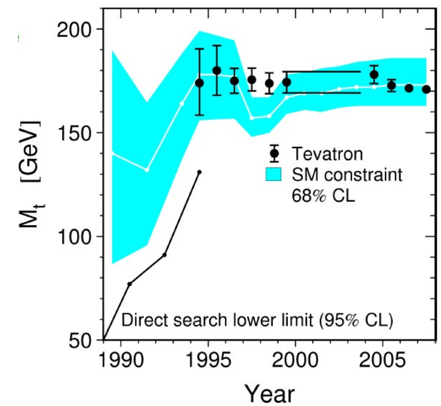
Outline

- Masses and heavy quark mass schemes
  - Pole and short-distance masses
  - Renormalons
- What is measured in top reconstruction
  - Confinement & finite top lifetime
  - Why top reconstruction is conceptually nontrivial.
- What mass is implement in MC's ?
  - A factorization theorem as a guideline
  - My (yet incomplete) answer
- Conclusions



Present top mass: $m_t = 173.1 \pm 1.3 \text{ GeV}$

This talk shall make you aware of the conceptual subtleties that arise with ever decreasing errors and that eventually must be accounted for.

Part I:

Masses and Mass Schemes

Concepts of Mass

Classic Physics: Mass has absolute meaning.

$$F = m_i \left(\frac{d}{dt} v \right)$$ inertial mass

$$F = G \frac{m_{g,1}\, m_{g,2}}{|\vec{r_1} - \vec{r_2}|^2} = g\, m_{g,1}$$ gravitational mass

experimental fact

$$\frac{m_i}{m_g} \sim 10^{-12}$$

Weak (Galilean) equivalence principle: $\frac{m_i}{m_g} = \kappa$ for any object

Special relativity: $p^2 = m^2$ rest mass, mass-shell

Concepts of Mass

Quantum Field Theory:

Particles: Field-valued operators made from creation and annihilation operators

Lagrangian operators constructed using correspondence principle

Classic action: m is the rest mass

No other mass concept exists at the classic level.

$$\mathcal{L}_{\rm QCD} = \mathcal{L}_{\rm classic} + \mathcal{L}_{\rm gauge-fix} + \mathcal{L}_{\rm ghost}$$

$$(p^2 - m^2)\, q(x) = 0$$

$$\mathcal{L}_{\rm classic} = -\frac{1}{4} F^A_{\alpha\beta} F^{\alpha\beta}_A + \sum_{\text{flavors } q} \bar{q}_\alpha (i\not{D} - m_q)_{\alpha\beta}\, q_b$$

$$D^\mu = \partial^\mu + i g T^C A^{\mu C}$$

$$i \frac{p + m}{p^2 - m^2 + i\epsilon}$$

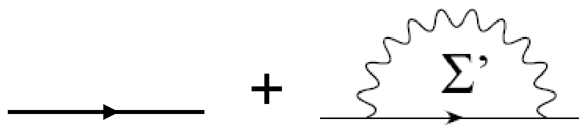
$$-i \frac{\left(g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2}(\xi - 1)\right)}{p^2 + i\epsilon}$$

classic particle poles

Concepts of Mass

Renormalization: UV-divergences in quantum corrections

Fields, couplings, masses in classic action are bare quantities that need to be renormalized to have (any) physical relevance



$$= \not{p} - m^0 + \Sigma(p, m^0)$$

$$m^0 \, \frac{\alpha_s}{\pi} \left[-\frac{1}{\epsilon} + \text{finite stuff} \right]$$

Mass Renormalization Schemes you know:

Pole mass: mass = classic rest mass

$$m^0 = m^{\text{pole}} + \delta m^{\text{pole}} \qquad \delta m^{\text{pole}} = \Sigma(m, m)$$

$\overline{\text{MS}}$ mass: $m^0 = \overline{m}(\mu) - \frac{\alpha_s}{\pi} \frac{1}{\epsilon}$

Concepts of Mass

All mass schemes are related through a perturbative series.

$$m^{\text{schemeA}} - m^{\text{schemeB}} = \#\,\alpha_s + \#\,\alpha_s^2 + \#\,\alpha_s^3 + \ldots$$

Lesson 1: Renormalization schemes are defined by what quantum fluctuations are kept in the dynamical matrix elements and by what quantum fluctuations are absorbed into the couplings and parameters.

Why do we have to care?

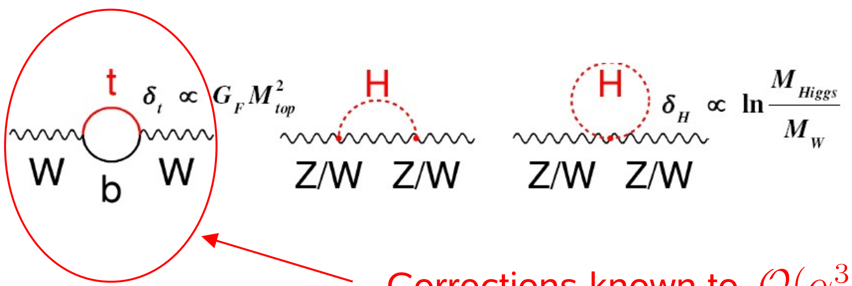
Different mass schemes are useful and appropriate for different applications.

Which is the best mass for a specific application?

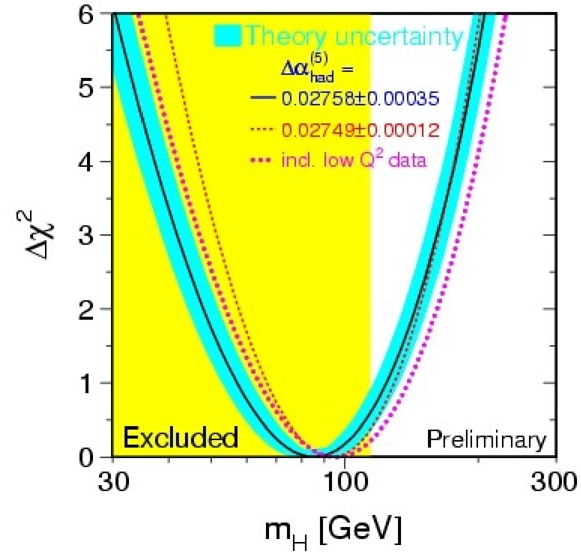
Lessson 2: A good scheme choice is one that gives systematically (not accidentally) good convergence. But there are almost always several alternatives one can use.

Precise Top Mass Needed!

Fit to electroweak precision observables



Corrections known to $\mathcal{O}(\alpha_s^3)$



$$\sin\theta_W \times \left(1+\delta(m_t, m_H, \ldots)\right)$$

$$= 1 - \frac{M_W^2}{M_Z^2}$$

$$m_t = 173.1 \pm 1.3 \text{ GeV}$$

$$m_H = 90 \pm 24 \text{ GeV}$$

2 GeV change: 15% change in m_H

$$m_H < 163 \text{ GeV} \quad \text{(95\%CL)}$$

Best convergence using the $\overline{\text{MS}}$ top scheme:

$$\overline{m}_t(\overline{m}_t)$$

Precise Top Mass Needed!

Blick in die Zukunft:

Minimales Supersymmetrisches Standard Model

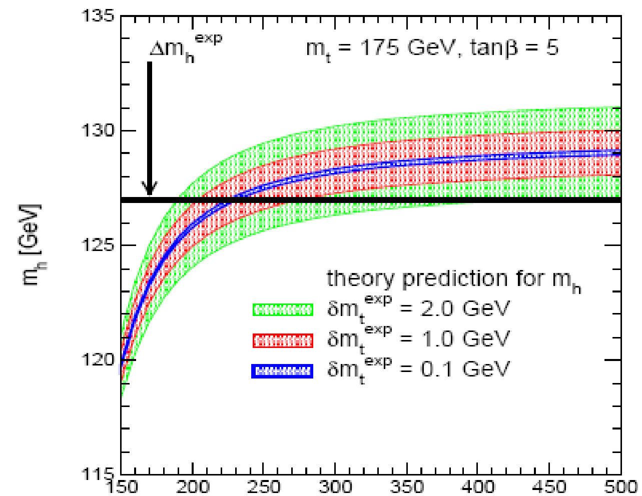
5 Higgs Bosonen:

m_h (skalar, neutral)

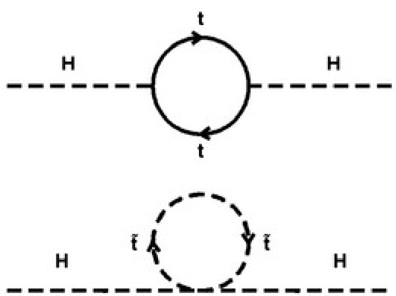
m_H (skalar, neutral)

m_A (speudoskalar, neutral)

$m_H^{\pm}$ (geladen)



Heinemeyer, Kraml, Porod, Weiglein



$$m_h^2 \simeq M_Z^2 + \frac{G_F\, m_t^4}{\pi^2 \sin^2\beta} \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

Corrections known to $\mathcal{O}(\alpha_s^3)$

Best convergence using the $\overline{\text{MS}}$ top scheme:

$$\overline{m}_t(\sqrt{M_{\rm SUSY}\overline{m}_t})$$

Haber, Hempfling, Hoang

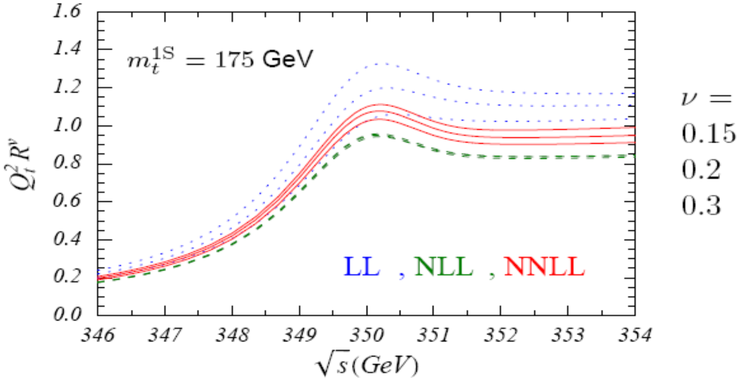
Precise Top Mass Measurable!

Top Pair Threshold Scan at the ILC

$Q \approx 2m_t$

- count number of $t\bar{t}$ events
- color singlet state
- background is non-resonant
- physics well understood (renormalons, summations)

Manohar,Stewart,Teubner,AH



Experimental errors: $\delta m_t^{\rm exp} \sim 50 \text{ MeV}$

Theory errors: $\delta m_t^{\rm th} \sim 100 \text{ MeV}$

Miquel, Martinez; Boogert, Gounaris

Best convergence using the 1S top scheme: $\overline{m}_t^{1S}$

Other good schemes: PS mass, kinetic mass

"Threshold Masses"

Precise Top Mass Measurable!

Top Pair Threshold Scan at the ILC

- count number of $t\bar{t}$ events
- color singlet state
- background is non-resonant
- physics well understood (renormalons, summations)

$Q \approx 2m_t$

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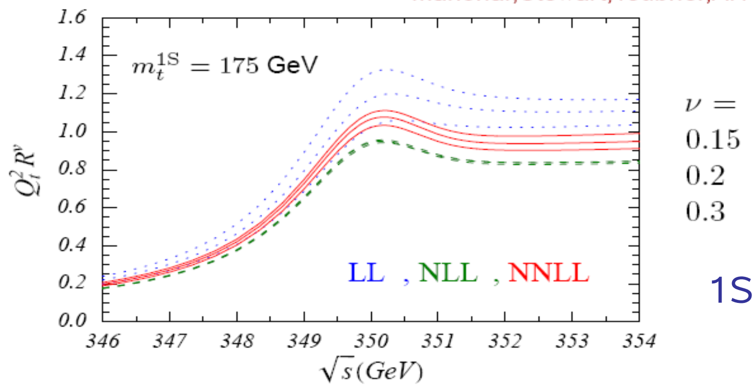
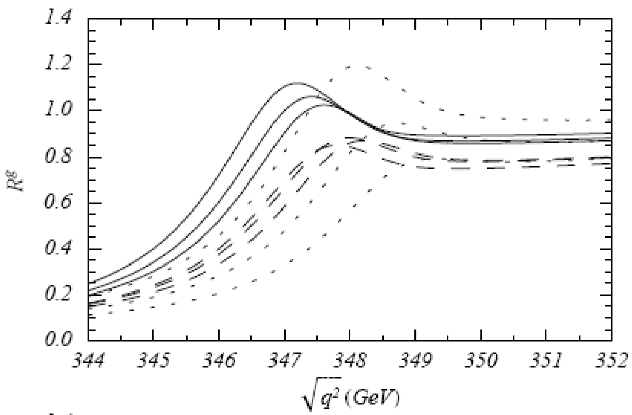
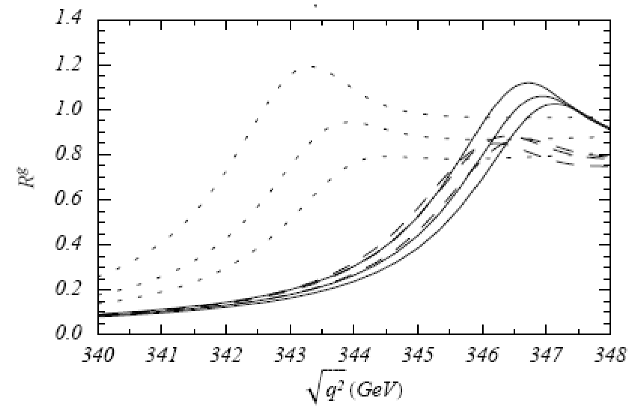


1S scheme



pole mass scheme



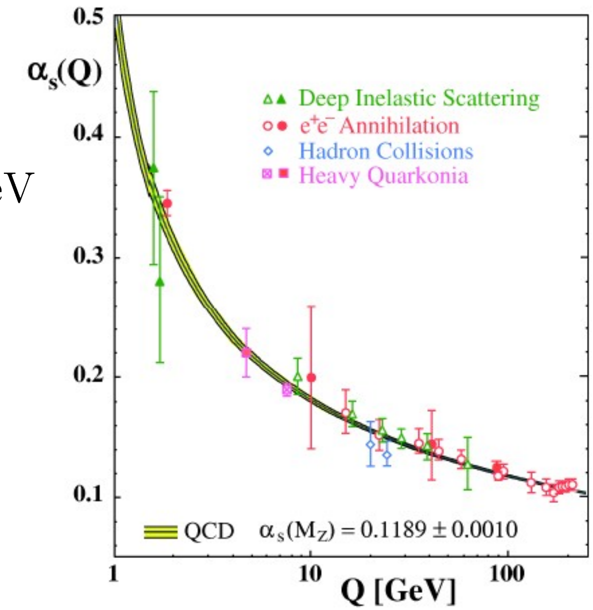
$\overline{\text{MS}}$ scheme

Confinement and Pole Mass

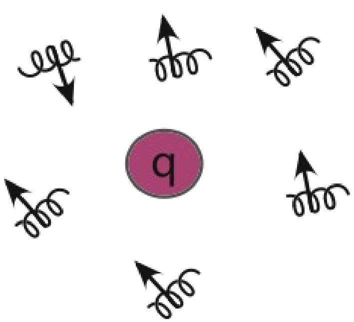
High energies: QCD-effects computable perturbatively:
“Asymtotic Freedom”

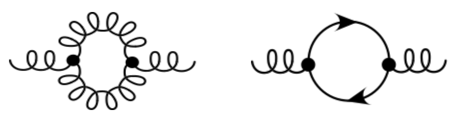
Low energies: QCD-effects nonperturbative **“Confinement”**

$$\Lambda_{\rm QCD} \approx 0.3 \text{ GeV}$$



Concept of free quarks invalid.



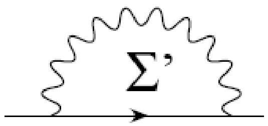


“Anti-Screening”

Lesson 3: Concept of a quark pole (rest) mass invalid a priori.
Can only be used in the context of perturbation theory.

Renormalons

So … do we have to care?



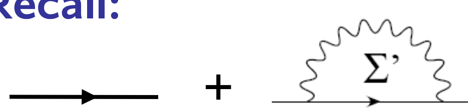
$$\Sigma(m,m) = -\frac{4}{3}\int\frac{d^4q}{(2\pi)^4}\,\alpha_s\,\gamma^\mu\,\frac{q+k+m}{(q+k)^2-m^2}\gamma_\mu\frac{1}{q^2}$$

$$\overset{q\ll m}{=} \frac{2}{3}\int\frac{d^3q}{(2\pi)^3}\frac{\alpha_s(q)}{\vec{q}^{\,2}} = -\frac{1}{2}\int\frac{d^3q}{(2\pi)^3}V(\vec{q}^{\,2})$$

Linear sensitivity to infrared momenta leads to factorially growing coefficients in perturbation theory.

$$\Sigma(m,m) \sim \sum_n \alpha_s^{n+1}(2\beta_0)^n n!$$

Recall:

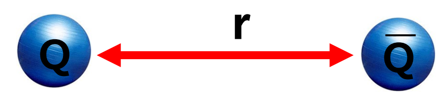


$$= p - m^0 + \Sigma(\not p, m^0)$$
$$\sim p - m^{\rm pole}$$

Isn't this just an argument in favor of the pole mass ?

Renormalons

Static energy of a static heavy quark-antiquark pair



$$E_{\text{static}} = 2m^0 - 2\Sigma(m,m) + V(r)$$

$$\sim 2m^0 - \int \frac{d^3q}{(2\pi)^3}\, V(\vec{q}^{\,2}) + \int \frac{d^3q}{(2\pi)^3}\, V(\vec{q}^{\,2})\, e^{i\vec{q}\vec{r}}$$

Renormalon behavior cancels in the sum of self and interaction energy but UV-divergent.

$$= 2m^{\text{pole}} + V(r)$$

UV-renormalized, but renormalon behavior appears.

$$= 2\left[m^{\text{PS}}(R) - \frac{1}{2}\int_{q<R} \frac{d^3q}{(2\pi)^3}\, V(\vec{q}^{\,2})\right] + V(r)$$

Renormalon behavior cancels in the sum of self and interaction energy and UV-finite.

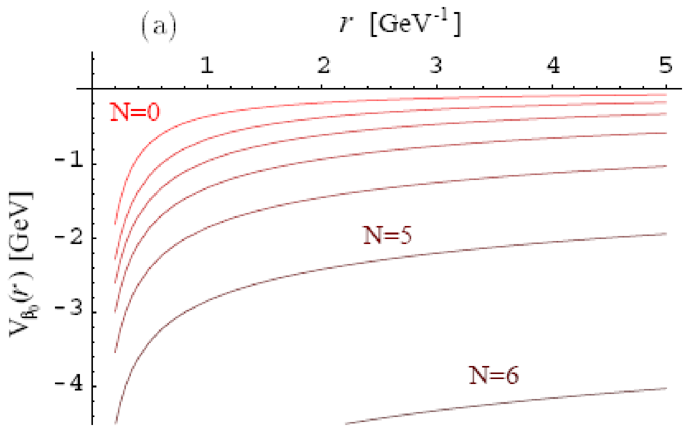
$$= 2m^{\text{PS}}(R) + \left[V(r) - \int_{q<R} \frac{d^3q}{(2\pi)^3}\, V(\vec{q}^{\,2})\right]$$

example of a short-distance mass scheme.

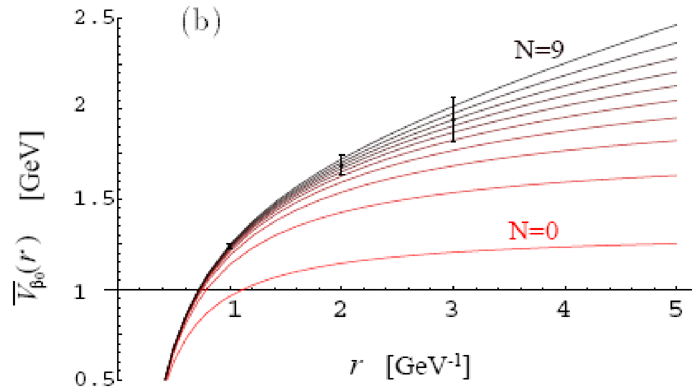
$$R\,(\#\alpha_s + \#\alpha_s^2 + \ldots)$$

Renormalons

pole mass scheme



1S mass scheme



Lessson 4: Pole mass is an conceptually irrelevant concept and leads to artificially large corrections in higher orders. In fields where heavy quark masses need to be known with uncertainties below $\mathcal{O}(1)$ GeV **short-distance** masses must be used. (This is already done in Quarkonium- and B-physics).

Top mass dependent quantities that can be computed with high precision and measured with small errors should expressed in short-distance mass schemes.

Short-Distance Mass Schemes

Short-distance mass schemes:

$$m^{\rm sd}(R) \, = \, m^{\rm pole} \, - \, R\left(a_1\frac{\alpha_s}{4\pi} + a_2\left(\frac{\alpha_s}{4\pi}\right)^2 + \ldots\right)$$

Generic form of a short-distance mass scheme.

$\overline{\rm MS}$ mass: $R = \overline{m}(\mu)\,, \quad a_1 = \frac{16}{3} + 8\ln\frac{\mu}{m}$

Processes where heavy quarks are off-shell and energetic.

Threshold masses (1S, PS, kinetic masses)

$$R \sim m\alpha_s$$

Quarkonium bound states: heavy quarks are close to their mass-shell.

Threshold masses (jet mass)

$$R \sim \Gamma_Q$$

Single quark resonance: heavy quarks are very close to their mass-shell.

The a_i's are chosen such that the renormalon is removed.

The scale R is of order the momentum scale relevant for the problem.

Part II:

Confinement & Finite Top Lifetime

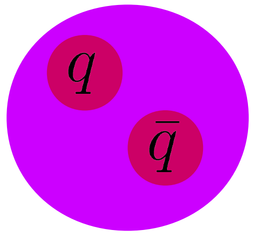
Why Top Mass Recontruction is nontrivial conceptually.

Confinement

Confinement:

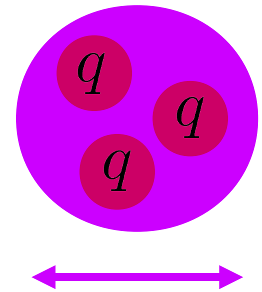
Mesons

$\pi, K, \rho, B, \ldots$



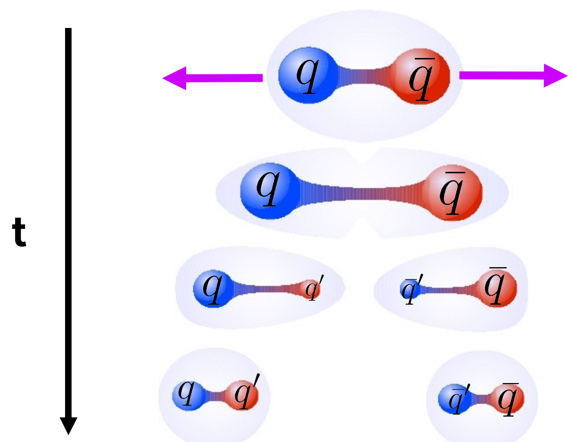
Baryons

$p, n, \Sigma, \Delta, \ldots$



$$r = \Lambda_{\rm QCD}^{-1} \simeq 1 \text{ fm}$$

Stable quarks only appear in the form of color-neutral hadrons.



color-neutral
Meson

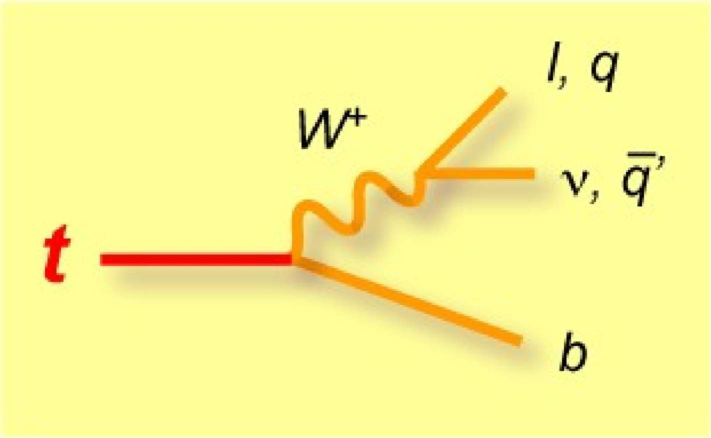
Hadronisation time:

$$\tau_{\rm had} = 10^{-23} \text{ s}$$

Top Decay

Decay of the top quark:

$$\Gamma(t \to bW) \approx 1.5 \text{ GeV}$$



Top quarks cannot ever form hadrons as they decay before that happens.

Color neutralization still relevant for the top quark via its decay products.

Top lifetime:

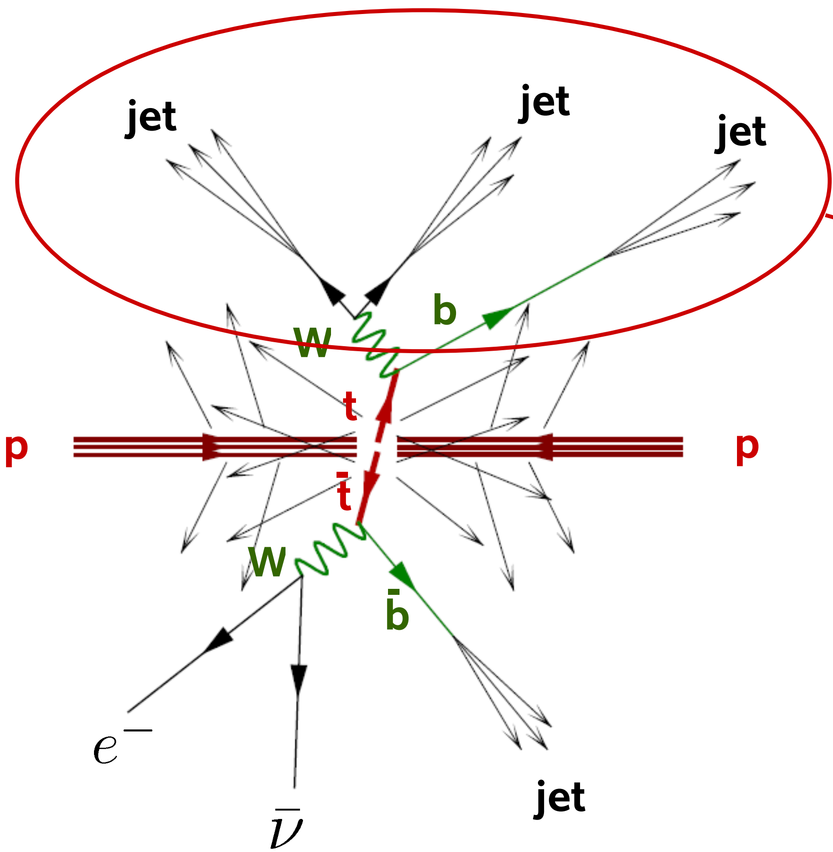
$$\tau_{\text{had}} = 10^{-24} \text{ s}$$

Hadronisation time:

$$\tau_{\text{had}} = 10^{-23} \text{ s}$$

Top Reconstruction

LHC:

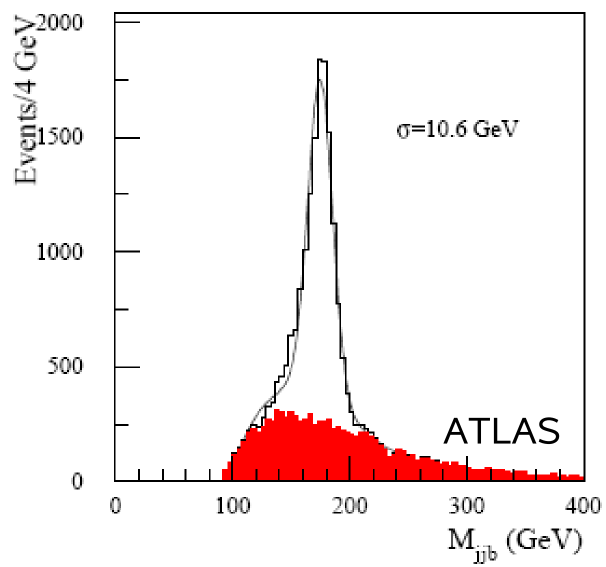


Idea:

Identify top decay products

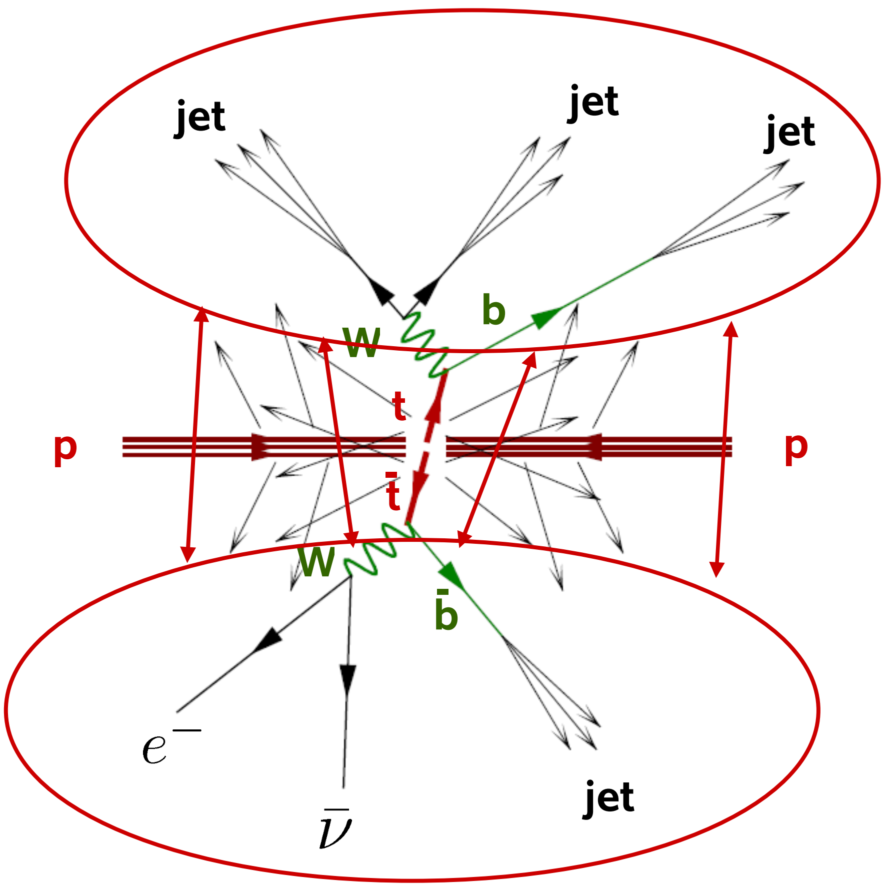
“ $m_{\rm top}^2 = p_t^2 = \left(\sum_i p_i^\mu\right)^2$ ”

Invariant mass distribution



Top Reconstruction

LHC:



Idea:

Identify top decay products

$$\text{“}\; m_{\text{top}}^2 = p_t^2 = \left(\sum_i p_i^\mu\right)^2 \;\text{”}$$

Conceptually this is quite subtle!

The measured quantity does not exist a priori. It is defined only through the experimental prescription.

The idea of an a priori physical object with a well defined mass is not correct.

Details of the color neutralization and hadronization models in MC's affect the simulation of reconstruction and thus the top mass measurement at leading order.

Part III:

What top mass is contained in MC's ?

MC Top Mass

Isn't it the pole mass?

Recall what we have learned about the pole mass:

In the pole scheme quantum corrections down to zero momentum are kept in the perturbative computations.

In the MC the perturbative contributions in the parton shower are switched off by the shower cutoff.

So, no it is not the pole mass.

For that reason there is also no renormalon problem for the MC mass.

The MC mass is in principle a short-distance mass, but it is difficult to identify with only have leading order showers/matrix elements implemented.

MC and the Top Mass

- MC is a tool designed to describe many **physical** final states, cross sections, etc..

- The concept of mass in the MC depends on the **structure of the perturbative part** and the **interplay of perturbative and nonperturbative part** in the MC.

Parton shower (kinematic expansion)

Perturbative corrections

top mass definition contained here !

scheme-dependent separation **(shower cut)**

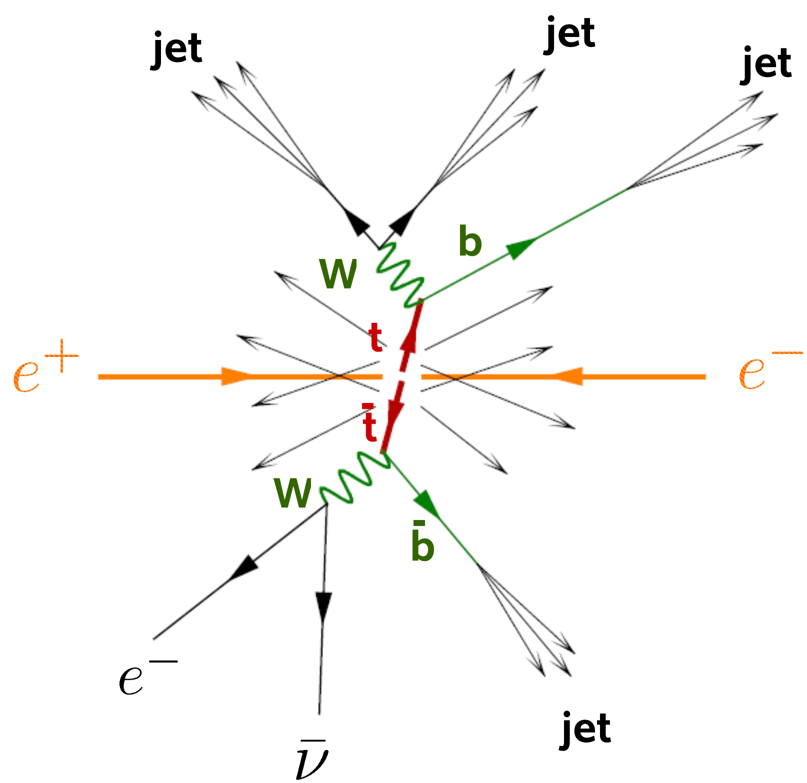
Hadronization Models

affect all top mass dependent observables

Toy Model

Top Invariant Mass Distribution at the ILC:

Fleming, Mantry, Stewart, AH
Stewart, AH
2007 and 2008

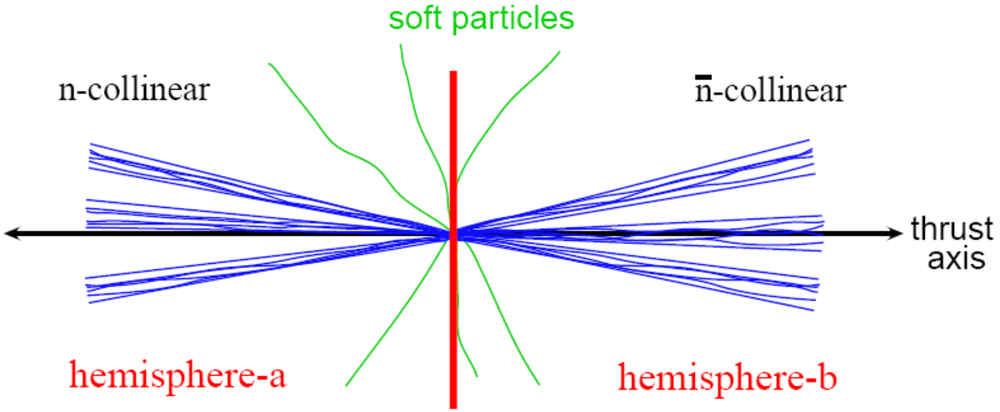


$Q \gg m_t$

QCD-Factorization

Top-Invariante-Massenverteilung:

Definition der Observable

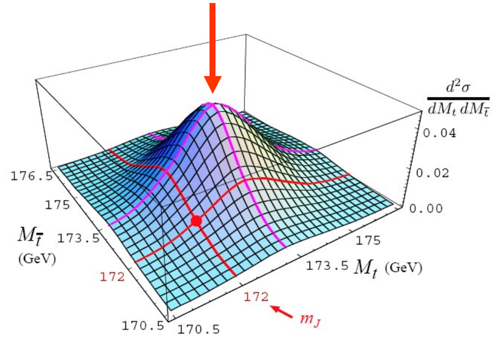


$$M_t^2 = \Big(\sum_{i\,\epsilon\,a} p_i^\mu\Big)^2 \qquad M_{\bar t}^2 = \Big(\sum_{i\,\epsilon\,b} p_i^\mu\Big)^2$$

$$\frac{d^2\sigma}{dM_t\,dM_{\bar t}}$$

Resonance region:

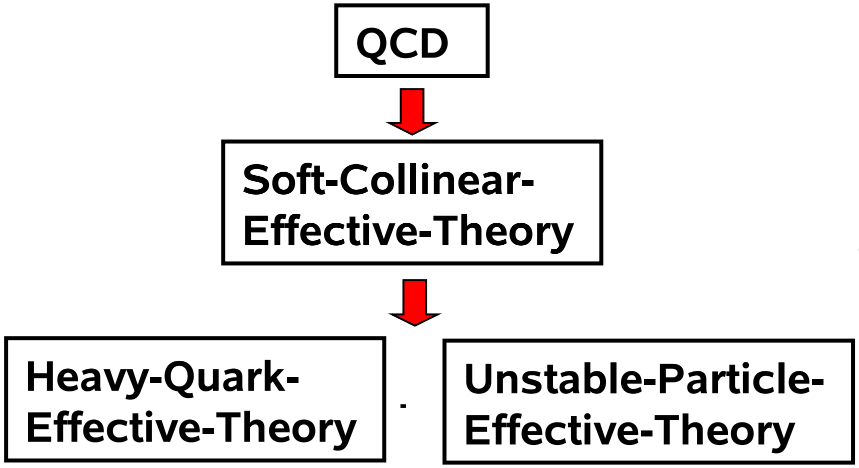
$$M_{t,\bar t} - m_t \sim \Gamma$$

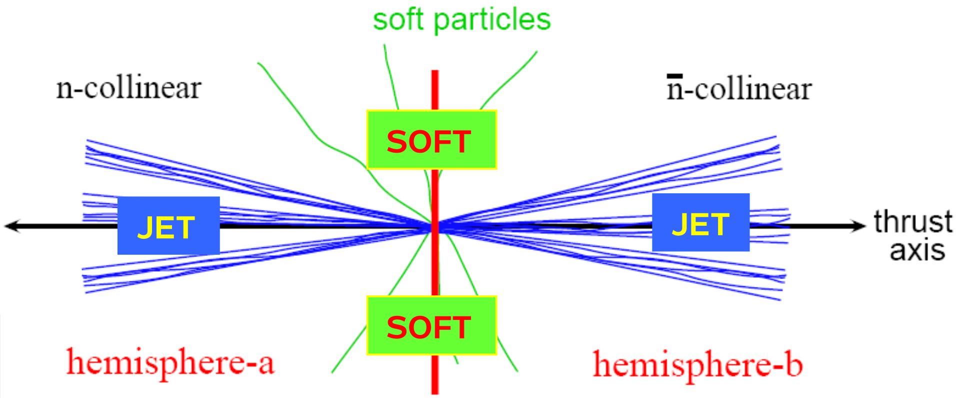


QCD-Faktorisierung

$$Q \gg m_t \gg \Gamma_t > \Lambda_{\rm QCD}$$

Fleming, Mantry, Stewart, AH





Faktorisierungs Formel

$$\left(\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \qquad \hat{s} = \frac{M_t^2 - m_J^2}{m_J}$$

$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^- \; B_+\Big(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\; B_-\Big(\hat{s}_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\; S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

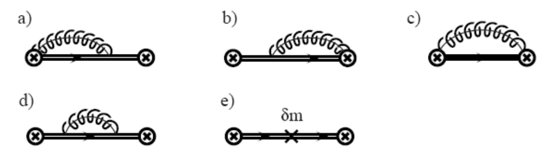
JET JET SOFT

Factorization Theorem

$$\left(\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\; B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

Jet functions: $B_+(2v_+\cdot k) = \frac{-1}{8\pi N_c m}\,{\rm Disc}\int d^4x\, e^{ik\cdot x}\,\langle 0|{\rm T}\{\bar h_{v_+}(0) W_n(0) W_n^\dagger(x) h_{v_+}(x)\}|0\rangle$

- perturbative, mass definition contained here
- depends on m_t, Γ_t
- Breit-Wigner at tree level



$$B_\pm(\hat s, \Gamma_t) = \frac{1}{\pi m_t}\,\frac{\Gamma_t}{\hat s^2 + \Gamma_t^2} \qquad \hat s = \frac{M^2 - m_t^2}{m_t}$$

Soft function: $S_{\rm hemi}(\ell^+, \ell^-, \mu) = \frac{1}{N_c}\sum_{X_s} \delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\langle 0|\overline{Y}_{\bar n}\, Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger(0)|0\rangle$

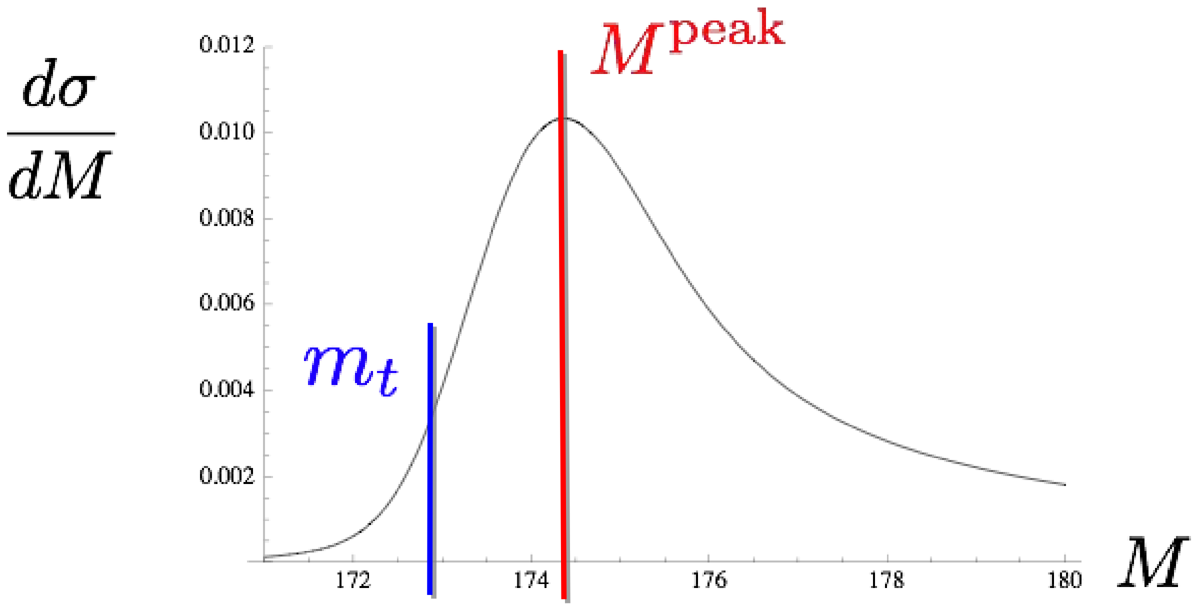
- non-perturbative
- mass+flavor independent
- also governs **massless dijet thrust and jet mass event distributions**

Korshemsky, Sterman, etal.
Bauer, Manohar, Wise, Lee

→ **Short distance top mass can (in principle) be determined to better than $\Lambda_{\rm QCD}$.**

Numerical Analysis

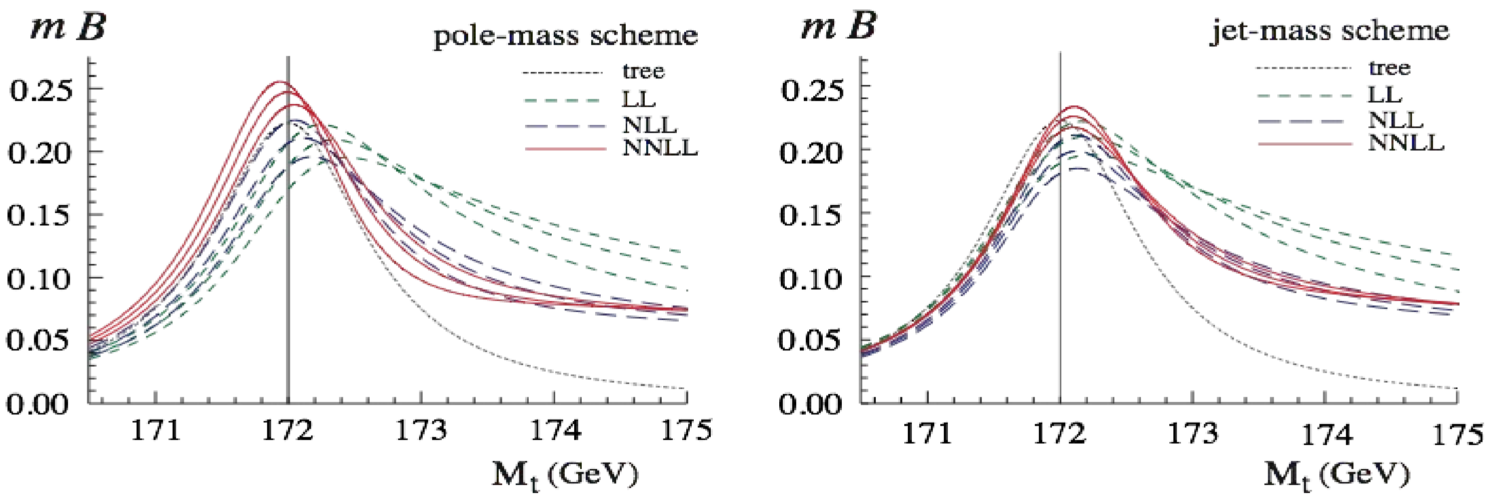
Peak Position und Topquark-Masse:



$$M^{\text{peak}} = m_t + \Gamma_t(\alpha_s + \alpha_s^2 + \ldots) + \frac{Q\,\Omega_1}{m_t} + \mathcal{O}\Big(\frac{m_t \Lambda_{\text{QCD}}}{Q}\Big)$$

Numerical Analysis

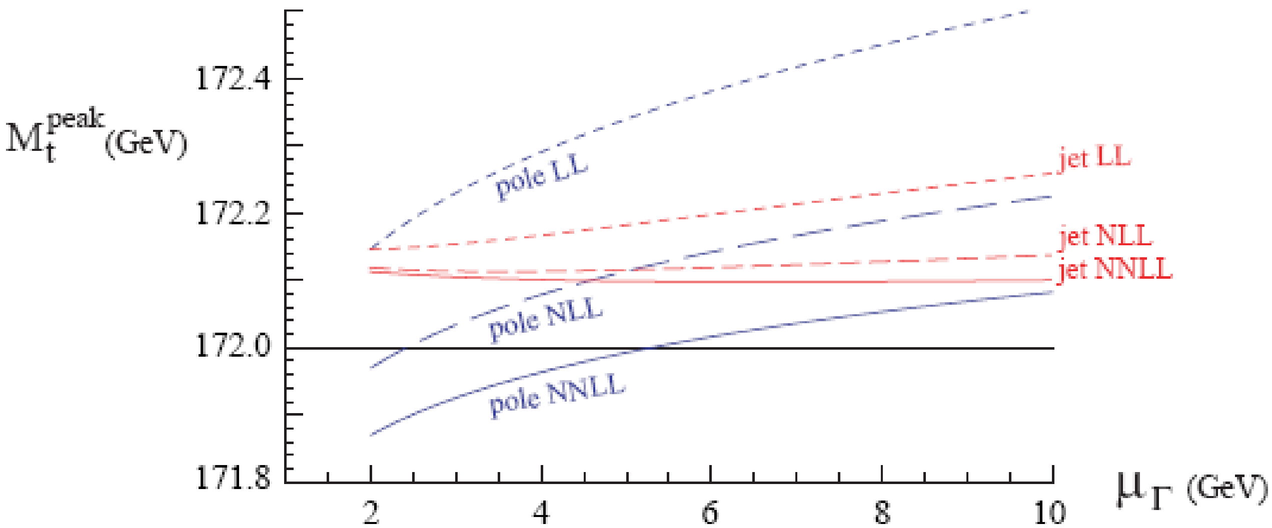
Higher orders and top mass schemes:



$$m_J(R,\mu) \;=\; m_t^{\rm pole} \;-\; R\,\frac{\alpha_s}{\pi}\,C_F e^{\gamma_E}\left[\frac{1}{2}+\ln\frac{\mu}{m}\right]+\ \ldots \qquad\qquad R=\Gamma_t$$

NLL Numerical Analysis

Scale-dependence of peak position



- Jet mass scheme: significanly better perturbative behavior.
- Renormalon problem of pole scheme already evident at NLL.

MC Top Mass

→ Use analogies between MC set up and factorization theorem

Final State Shower

- Start: at transverse momenta of primary partons, evolution to smaller scales.
- Shower cutoff $R_{sc} \sim 1\ \mathrm{GeV}$
- Hadronization models fixed from reference processes

Factorization Theorem

- Renormalization group evolution from transverse momenta of primary partons to scales in matrix elements.
- Subtraction in jet function that defines the mass scheme
- Soft function model extracted from another process with the same soft function

Additional Complications:

Initial state shower, underlying events, combinatorial background, etc

We ignore these issues for now, as they are not included in the factorization theorem yet.

MC Top Mass

Conclusion:

unknown constant of order unity

$$m_t^{\rm MC}(R_{sc}) \,=\, m_t^{\rm pole} \,-\, R_{sc}\, c \left[\frac{\alpha_s}{\pi}\right] \,+\, \mathcal{O}(\alpha_s)$$

correction from 1-loop matrix elements

There is some work to do to complete the relation.

There is some work to improve the MC's.

Conclusions

Be careful thinking about (top) mass definitions. Short-distance masses definition are the better choice if you ask for precision.

The top quark mass is a scheme-dependent parameter. There is no a priori physical quantity associated to the top quark mass.

It seems possible to determine what top mass scheme is in MC's, but at present we do not even know the 1-loop terms.