MCnet School - Lund July 1st, 2009

Jet Definitions

Matteo Cacciari LPTHE - Paris 6,7 and CNRS

Contains original artwork (as well as many ideas) from Gavin Salam

Outline

Introductory remarks

Jet algorithms, infrared and collinear (un)safety

- Cone-type (progressive removal, split-merge, SISCone)
- Recombination-type (kt, Cambridge/Aachen, anti-kt)
- Third-generation algorithms (C/A-filtering)

Jet definitions

- 🔵 Best R
- Quality measures (mass-peak reconstruction)

Jet areas and subtraction

Summary

Bibliography

Ecs Houches 2007 proceedings, arXiv:0803.0678

Gavin Salam's lectures at CTEQ-MCnet 2008

Gavin Salam, 'Towards Jetography' arXiv:0906.1833

Why jets



A jet is something that happens in high energy events:

a collimated bunch of hadrons flying roughly in the same direction

Note: hundreds of hadrons contain **a lot** of information. More than we can hope to make use of





Often you don't need a fancy algorithm to 'see' the jets

But you do to give them a **precise** and **quantitative** meaning

Matteo Cacciari - LPTHE

Why jets

Jets are usually related to an underlying perturbative dynamics (i.e. quarks and gluons)

00000000000000

Why jets

Jets are usually related to an underlying perturbative dynamics (i.e. quarks and gluons)

The purpose of a 'jet clustering' algorithm is then to reduce the complexity of the final state, simplifying many hadrons to simpler objects that one can hope to calculate

0000000000

Jet algorithm

A **jet algorithm** maps the momenta of the final state particles into the momenta of a certain number of jets:



Most algorithms contain a resolution parameter, **R**, which controls the extension of the jet (more about this later on)

Two main classes of jet algorithms

Sequential recombination algorithms

bottom-up approach: combine particles starting from closest ones How? Choose a distance measure, iterate recombination until few objects left, call them jets

Work because of mapping closeness ⇔ QCD divergence

Examples: Jade, kt, Cambridge/Aachen, anti-kt,

Two main classes of jet algorithms

Sequential recombination algorithms

bottom-up approach: combine particles starting from closest ones How? Choose a distance measure, iterate recombination until few objects left, call them jets

Work because of mapping closeness \Leftrightarrow QCD divergence

Examples: Jade, kt, Cambridge/Aachen, anti-kt,

Cone algorithms

top-down approach: find coarse regions of energy flow.

How? Find stable cones (i.e. their axis coincides with sum of momenta of particles in it)

Work because QCD only modifies energy flow on small scales Examples: JetClu, MidPoint, ATLAS cone, CMS cone,

Cone algorithms

The first rigorous definition of cone jets in QCD is due to Sterman and Weinberg Phys. Rev. Lett. **39**, 1436 (1977) To study jets, we consider the partial cross section $\sigma(E, \theta, \Omega, \varepsilon, \delta)$ for e⁺e⁻ hadron production events, in which all but a fraction $\epsilon \ll 1$ of the total e⁺e⁻ energy E is emitted within some pair of oppositely directed cones of half-angle & << 1, lying within two fixed cones of solid angle Ω (with $\pi\delta^2 \ll \Omega \ll 1$) at an angle θ to the e⁺e⁻ beam line. We expect this to be measur- $\sigma(\mathbf{E},\theta,\Omega,\varepsilon,\delta) = (d\sigma/d\Omega)_{0}\Omega \left[1 - (g_{\mathbf{E}}^{2}/3\pi^{2})\left\{3\ln\delta + 4\ln\delta\ln2\varepsilon + \frac{\pi^{3}}{3} - \frac{5}{2}\right\}\right]$

Good for 2 jets and e⁺e⁻ collisions

In more general cases, where do we place the cones? How many?

Finding cones

Different procedures for placing the cones lead to different cone algorithms

NB: their properties and behaviour can **vastly differ**: there isn't **'a'** cone algorithm, but rather many of them

The main sub-categories of cone algorithms are:

- *** Fixed** cone with **progressive removal** (FC-PR) (PyJet, CellJet, GetJet)
- *** Iterative** cone with **progressive removal** (IC-PR) (CMS iterative cone)
- *** Iterative** cone with **split-merge** (IC-SM) (JetClu, ATLAS cone)
- *** IC-SM** with **mid-points** (IC_{mp}-SM) (CDF MidPoint, D0 Run II)
- ***** IC_{mp} with split-drop (IC_{mp}-SD) (PxCone)
- *** Seedless** cone with **split-merge** (SC-SM) (SISCone)

Probably the simplest cone algorithm



Probably the simplest cone algorithm



Choose hardest particle as seed

Probably the simplest cone algorithm

Choose hardest particle as seed Draw cone around it



Probably the simplest cone algorithm

Choose hardest particle as seed Draw cone around it Call it a jet, remove constituents from set of particles



Probably the simplest cone algorithm

Choose hardest particle as seed Draw cone around it Call it a jet, remove constituents from set of particles

Repeat using hardest particle left



Probably the simplest cone algorithm



Repeat using hardest particle left Etc, etc



2

3

p_t/GeV

60

50

40

30

20

10

0

0

Draw cone

4 _v

Probably the simplest cone algorithm



Repeat using hardest particle left Etc, etc



2

3

p_t/GeV

60

50

40

30

20

10

0

0

Convert into jet

4 _v

Probably the simplest cone algorithm

Choose hardest particle as seed Draw cone around it Call it a jet, remove constituents from set of particles Repeat using hardest particle left

Etc, etc



Probably the simplest cone algorithm



Repeat using hardest particle left Etc, etc



p_t/GeV

60

50

40

30

20

10

Draw cone

4 _v

Probably the simplest cone algorithm



Repeat using hardest particle left Etc, etc

Until no particles left



Seed and cone axis may **not coincide**. Making them do can lead to different jets

Seed and cone axis may **not coincide**. Making them do can lead to different jets

Let us try an **Iterative** Cone with Progressive Removal (IC-PR) (e.g. the CMS Iterative Cone)



Begin with hardest particle as seed

Cluster particles into cone if $\Delta R < R$



Iterate until stable (i.e. axis coincide with sum of momenta) cones found

Eliminate constituents of jet and start over from hardest remaining particle





Choose hardest particle as seed Draw cone around it

So far, identical to FC-PR



Choose hardest particle as seed Draw cone around it Jet axis not centred on seed



Choose hardest particle as seed Draw cone around it Jet axis not centred on seed Redraw cone around new axis



Choose hardest particle as seed Draw cone around it Jet axis not centred on seed Redraw cone around new axis Still, jet axis not centred on seed



Choose hardest particle as seed Draw cone around it Jet axis not centred on seed Redraw cone around new axis Still, jet axis not centred on seed Repeat until it is, finally get to this

This jet differs from the corresponding FC-PR one



IC-PR cone collinear unsafety

A collinear splitting can change the final state



IC-PR cone collinear unsafety

A collinear splitting can change the final state



Splitting the hardest particle **collinearly** has changed the number of final jets

Consequences of collinear unsafety

In QCD perturbation theory, virtual and real configurations can only cancel if they lead to the **same** final state

In this example with IC-PR, we have seen that the final state can differ:



 \Rightarrow no cancellation of divergencies, no convergence of perturbation theory

Jet algorithms using hardest particles as seeds will generally be susceptible to collinear unsafety

Iterative Cone with Split-Merge (IC-SM)

Choosing hardest particles as seed was an issue (collinear unsafety). Let us therefore try taking **all particles**



Use all particles as seed

Cluster particles into cone if $\Delta R < R$

Iterate until stable (i.e. axis coincide with sum of momenta) cones found

Split-merge step (see later on)

Examples of this algorithm are JetClu and the ATLAS Cone

IC-SM

Iterating the cones over all particles as seeds returns 5 stable protojets



The lack of 'progressive removal' means that some protojets can be overlapping (i.e. contain the same particles). Must deal with this: **split-merge**
Split-Merge

'Split-merge' is a further algorithm aimed at disentangling overlapping protojets.

The Tevatron Run II implementation goes like this:



IC-SM infrared unsafety



MCnet School - Lund - July 2009

IC-SM infrared unsafety



MidPoint (IC_{mp}-SM) infrared unsafety

MidPoint fixes the two-particle configuration IR-safety problem by **adding midpoints** to list of seeds.

But this merely shifts the problem to three-particle configurations



The problem is that the stable-cone search procedure used by seeded IC algorithms often cannot find **all** possible stable cones

A long list of cones (all eventually unsafe)

Les Houches 2007 proceedings, arXiv:0803.0678

			1			1		
	S	CDF JetClu		IC_r -SM	IR ₂₊₁	-		
	hm	CDF MidPoint cone		IC_{mp} -SM	IR ₃₊₁	-		
CDF MidPoint searc			chcone	IC _{se,mp} -SM	IR ₂₊₁			
00		D0 Run II cone		IC_{mp} -SM	IR ₃₊₁			
	n, a							
ITA Durgio.		ATLAS Cone		IC-SM	IR_{2+1}			
		PxCone		IC_{mp} -SD	IR ₃₊₁			
	Gen					_		
	5 1 -8	CMS Iterative Cone	;	IC-PR	Coll_{3+1}	_		
		PyCell/CellJet (from Pythia)		FC-PR	$Coll_{3+1}$			
	9	GetJet (from ISAJE	T)	FC-PR	$Coll_{3+1}$			
safety issue								
IC = Iterative Cone			type of	IR : unsafe	e when a soft part	icle is added to		
SD = Split-Drop		n hard particles in a common neighbourhood						
	FC = Fix	FC = Fixed Cone		Coll : unsafe when one of n hard particles in				
	PR = Pr	ogressive Removal	MCrach	a common neighbourhood is split collinearly				
p.	namen v acciari = L		LTIV THET 2CDOOL = 1	1110 - 1111 / 1007		< /		

IRC safety does matter

The best cones seen so far fail at (3+1) partons, others already at (2+1)

	Last r			
	JetClu, ATLAS	MidPoint	CMS it. cone	Known at
	cone [IC-SM]	[IC _{mp} -SM]	[IC-PR]	
Inclusive jets	LO	NLO	NLO	NLO (\rightarrow NNLO)
W/Z + 1 jet	LO	NLO	NLO	NLO
3 jets	none	LO	LO	NLO [nlojet++]
W/Z + 2 jets	none	LO	LO	NLO [MCFM]
$m_{ m jet}$ in $2j+X$	none	none	none	LO

Calculations cost real money: ~ 100 theorists ×15 years ≈100 M€

Using unsafe jet tools essentially renders them useless

IRC safety in real life

Strictly speaking, one needs IRC safety not so much to find jets, but to be able to calculate them in pQCD

If you are not interested in thory/data comparisons, you may think of doing well enough with an IRC-unsafe jet algorithm

However

- Detectors may split/merge collinear particles, and be poorly understood for soft ones
- High luminosity (or heavy ions collisions) add a lot of soft particles to hard event

IRC safety provides resiliency to such effects (plus, at some point in the future you may wish to compare your measurement to a calculation)

Seedless IRC-safe Cone (SC-SM): SISCone

Salam, Soyez, arXiv:0704:0292

Seeds are a problem: they lead to finding only some of the stable cones

Obvious solution:

find ALL stable cones, testing all possible combinations of N particles

Unfortunately, this takes N2^N operations: the age of the universe for only 100 particles

Way out: a geometrical solution \rightarrow SISCone The first (and only?) IRC-safe cone algorithm for hadronic collisions





Key innovation: circular enclosures



In I-dim: slide a 'cone' along the axis Check stability: OK. New protojet



Key innovation: circular enclosures



In I-dim: slide a 'cone' along the axis Check stability: OK. New protojet Move on: first particle on edge: new enclosure

Unstable. Keep going









Key innovation: circular enclosures



Key innovation: circular enclosures



Key innovation: circular enclosures



Key innovation: circular enclosures













Key innovation: circular enclosures



Key innovation: circular enclosures





SISCone v. IC-SM

These are ALL the stable cones



SISCone v. IC-SM



Cones Infrared (un)safety

Q: How often are the hard jets changed by the addition of a soft particle?

- Generate event with
 2 < N < 10 hard particles,
 find jets
- Add 1 < N_{soft} < 5 soft particles, find jets again
 A: [repeatedly]
 - If the jets are different, algorithm is IR unsafe.

Unsafety level	failure rate
2 hard + 1 soft	$\sim 50\%$
3 hard + 1 soft	$\sim 15\%$
SISCone	IR safe !

Be careful with split-merge too

	good bad	
[
	JetClu 50.1%	
	SearchCone 48.2%	
	MidPoint 16.4%	
	Midpoint-3 15.6%	
	PxCone 9.3%	
	Seedless [SM-p _t] 1.6%	N
	0.17% Seedless [SM-MIP]	k Soye
	0 (none in 4x10 ⁹) Seedless (SISCone)	lam 8
ا 10	10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1	Sa
	Fraction of hard events failing IR safety test	

Recombination algorithms

(Inclusive version)



Calculate the distances between the particles: d_{ij}

Calculate the beam distances: **d**_{iB}

Combine particles with **smallest distance** or, if d_{iB} is smallest, call it a jet

Find again smallest distance and repeat procedure until no particles are left

IRC safety can usually be seen to be trivially guaranteed

The kt algorithm and its siblings

One can generalise the k_{r} distance measure:

$$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \frac{\Delta y^2 + \Delta \phi^2}{R^2} \qquad d_{iB} = k_{ti}^{2p}$$

 $\mathbf{p} = \mathbf{I} \quad \mathbf{k}_{t}$ algorithm

S. Catani, Y. Dokshitzer, M. Seymour and B. Webber, Nucl. Phys. B406 (1993) 187 S.D. Ellis and D.E. Soper, Phys. Rev. D48 (1993) 3160

The kt algorithm and its siblings

One can generalise the k_{r} distance measure:

$$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \frac{\Delta y^2 + \Delta \phi^2}{R^2}$$
 $d_{iB} = k_{ti}^{2p}$

 $\mathbf{p} = \mathbf{I} \quad \mathbf{k}_{t}$ algorithm

- S. Catani, Y. Dokshitzer, M. Seymour and B. Webber, Nucl. Phys. B406 (1993) 187 S.D. Ellis and D.E. Soper, Phys. Rev. D48 (1993) 3160
- **p = 0** Cambridge/Aachen algorithm ^{Y. Dokshitzer, G. Leder, S. Moretti and B. Webber, JHEP 08 (1997) 001 M. Wobisch and T. Wengler, hep-ph/9907280}

The kt algorithm and its siblings

One can generalise the k_{r} distance measure:

$$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \frac{\Delta y^2 + \Delta \phi^2}{R^2} \qquad d_{iB} = k_{ti}^{2p}$$

p = k algorithm S. Catani, Y. Dokshitzer, M. Seymour and B. Webber, Nucl. Phys. B406 (1993) 187 S.D. Ellis and D.E. Soper, Phys. Rev. D48 (1993) 3160

p = 0 Cambridge/Aachen algorithm ^{Y. Dokshitzer, G. Leder, S. Moretti and B. Webber, JHEP 08 (1997) 001 M. Wobisch and T. Wengler, hep-ph/9907280}

p = -1 anti-k_t algorithm

MC, G. Salam and G. Soyez, arXiv:0802.1189

NB: in anti-kt pairs with a **hard** particle will cluster first: if no other hard particles are close by, the algorithm will give **perfect cones**

Quite ironically, a sequential recombination algorithm is the 'perfect' cone algorithm
	The IRC safe algorithms			
kt	$SR d_{ij} = min(k_{ti}^2, k_{tj}^2) \Delta R_{ij}^2 / R^2 hierarchical in rel P_t$	Catani et al '91 Ellis, Soper '93	NInN	
Cambridge/ Aachen	$SR \\ d_{ij} = \Delta R_{ij}^2 / R^2 hierarchical in angle$	Dokshitzer et al '97 Wengler, Wobish '98	NInN	
anti-k _t	$SR \\ d_{ij} = \min(k_{ti}^{-2}, k_{tj}^{-2}) \Delta R_{ij}^2 / R^2 \\ gives perfectly conical hard jets$	MC, Salam, Soyez '08 (Delsart, Loch)	N ^{3/2}	
SISCone	Seedless iterative cone with split-merge gives 'economical' jets	Salam, Soyez '07	N ² InN	

	The IRC safe algorithms			
k t	$SR d_{ij} = min(k_{ti}^2, k_{tj}^2) \Delta R_{ij}^2 / R^2 hierarchical in rel P_t$	Catani et al '91 Ellis, Soper '93	NInN	
Cambridge/ Aachen	$SR \\ d_{ij} = \Delta R_{ij}^2 / R^2 \\ hierarchical in angle$	Dokshitzer et al '97 Wengler, Wobish '98	NInN	
anti-k _t	$SR \\ d_{ij} = \min(k_{ti}^{-2}, k_{tj}^{-2}) \Delta R_{ij}^2 / R^2 \\ gives perfectly conical hard jets$	MC, Salam, Soyez '08 (Delsart, Loch)	N ^{3/2}	
SISCone	Seedless iterative cone with split-merge gives 'economical' jets	Salam, Soyez '07	N ² InN	
We call these algs ' second-generation ' ones				

All are available in FastJet, <u>http://fastjet.fr</u>

(As well as many IRC unsafe ones)

Matteo Cacciari - LPTHE







Matteo Cacciari - LPTHE

Replacements

If you care about IRC safety but don't want to stray too far from algorithms used so far, these are possible replacements:



In addition, kt and Cambridge/Aachen can provide further flexibility

Different algorithms spanning a series of <u>different and complementary</u> <u>characteristics</u>: should be enough for most purposes

One should probably try to concentrate on these, both for analytical understanding and practical use in experiments, rather than using IRC unsafe ones

Cambridge/Aachen with filtering

Butterworth, Davison, Rubin, Salam, arXiv:0802.2470

An example of a **third-generation** jet algorithm



Cluster with C/A and a given R

Undo the clustering of each jet down to subjets with radius XfiltR

Retain only the **n**_{filt} hardest subjets

Aim: limit sensitivity to background while retaining bulk of perturbative radiation

Jet definition



Jet definition



+ parameters (usually at least the radius R)

Jet definition



+ parameters (usually at least the radius R)

+ recombination scheme



Les Houches 2007 proceedings, arXiv:0803.0678



Which R to choose?

The value of R matters because it affects, in opposite ways, a number of things:

Small R:Limit underlying event and pileup contaminationBetter resolve many-jets events

Large R: Limit perturbative radiation loss ('out-of-cone') Limit non-perturbative hadronisation effects

The best compromise will in general depend on the specific observable

R-dependent effects

Perturbative radiation: $\Delta p_t \simeq \frac{\alpha_s(C_F, C_A)}{-} p_t \ln R$

Hadronisation:
$$\Delta p_t \simeq \frac{(C_F, C_A)}{R} \times 0.4 \text{ GeV}$$

Analytical estimates, Dasgupta, Magnea, Salam, arXiv:0712.3014

Best R

Minimize $\Sigma(\Delta p_t)^2$



Dasgupta, Magnea, Salam, arXiv:0712.3014

Matteo Cacciari - LPTHE

MCnet School - Lund - July 2009

Flexibility

All IRC safe algorithms are equal, but some are more equal than others

Depending on the analysis you wish to perform, a jet definition may give better results than others

Reconstruction of a di-jet mass peak

To compare different algorithms, define **figures of merit**:

MC, Rojo, Salam, Soyez, arXiv:0810.1304 Les Houches 2007 proceedings, arXiv:0803:0678 $Q_{f=z}^{w}$ 0.02 anti-k. (R=0.4 anti-k. (R=0.7 W reconstruction Q^w_{f=0.18} (GeV) Smallest width of an histogram window containing 0.015 a fraction f=z of the generated objects, i.e. 1/N dN/dm (GeV⁻¹) $\frac{\# \text{ reconstructed massive objects in window of width } w}{\text{Total } \# \text{ generated massive objects}}$ 10.75 GeV 0.01 15.0 GeV 0.005 n 50 60 70 80 90 100 11 reconstructed W mass (GeV)

2. Maximum fraction of events in window of given width: $Q_{w=x\sqrt{M}}^{1/f} \equiv \left(\frac{\text{Max } \# \text{ reco. massive objects in window of width } w = x\sqrt{M}}{\text{Total } \# \text{ generated massive objects}}\right)^{-1}$

Smaller is better







quality.fastjet.fr

Do it yourself



Done

Matteo Cacciari - LPTHE

Jet areas: the link to physics

The definition of **active area** mimics the behaviour of the jet-clustering algorithms in the presence of a **large number of randomly distributed soft particles**

This is like underlying event or pileup!

Tools needed to implement it:

- I. An infrared safe jet algorithm
- 2. A reasonably **fast implementation**

Both are available

Jet active areas



Background subtraction

[MC, Salam, arXiv:0707.1378]

If the area of each jet, and the momentum per unit area of the background are known, one can correct the transverse momentum of the hard jets:

$$p_T^{\text{hard jet, corrected}} = p_T^{\text{hard jet, raw}} - \rho \times \text{Area}_{\text{hard jet}}$$

When ρ is calculated on an event-by-event basis, this procedure will generally improve the resolution of, say, a mass peak

NB. Also be(a)ware of backreaction

(immersing a hard jet in a soft background may cause some particles belonging to the hard event to be lost from (backreaction loss) or added to (backreaction gain) the jet). Small effect for UE, larger for pileup, can be very important for heavy ions. Analytical understanding of this effect available (MC, Salam, Soyez, arXiv:0802:1188)

Matteo Cacciari - LPTHE

Example of pileup subtraction

Let's discover a leptophobic Z' and measure its mass:



Effect of UE subtraction



Effect of UE subtraction: bad jet definitions are improved. Gain in effective luminosity* about 20-30%

(i.e. they were 'bad' due to a large extent to their behaviour with respect to the UE)

* ρ_L = factor of luminosity needed to obtain equivalent signal/background significance



There are many cone-type jet algorithms, but probably a single IRC-safe one

There are many cone-type jet algorithms, but probably a single IRC-safe one

- There are many cone-type jet algorithms, but probably a single IRC-safe one
- * There are also many recombination-type algorithms, usually IRC-safe
 - Their results will normally be 'similar', especially on inclusive quantities. However, if you plan to compare to higher order pQCD you **must** use an IRC-safe algorithm

- There are many cone-type jet algorithms, but probably a single IRC-safe one
- * There are also many recombination-type algorithms, usually IRC-safe
 - Their results will normally be 'similar', especially on inclusive quantities. However, if you plan to compare to higher order pQCD you **must** use an IRC-safe algorithm
- A jet algorithm complemented by its parameters and the recombination scheme is called a **jet definition**

There are many cone-type jet algorithms, but probably a single IRC-safe one

- Their results will normally be 'similar', especially on inclusive quantities. However, if you plan to compare to higher order pQCD you **must** use an IRC-safe algorithm
- A jet algorithm complemented by its parameters and the recombination scheme is called a **jet definition**
 - The proper choice of the parameters of a jet definition can considerably improve the sensitivity of an analysis

There are many cone-type jet algorithms, but probably a single IRC-safe one

- Their results will normally be 'similar', especially on inclusive quantities. However, if you plan to compare to higher order pQCD you **must** use an IRC-safe algorithm
- A jet algorithm complemented by its parameters and the recombination scheme is called a **jet definition**
 - The proper choice of the parameters of a jet definition can considerably improve the sensitivity of an analysis
- * Third-generation algorithms (e.g. filtering) appear promising for analyses where the jet substraction plays a relevant role

There are many cone-type jet algorithms, but probably a single IRC-safe one

- Their results will normally be 'similar', especially on inclusive quantities. However, if you plan to compare to higher order pQCD you **must** use an IRC-safe algorithm
- A jet algorithm complemented by its parameters and the recombination scheme is called a **jet definition**
 - The proper choice of the parameters of a jet definition can considerably improve the sensitivity of an analysis
- Third-generation algorithms (e.g. filtering) appear promising for analyses where the jet substraction plays a relevant role
- Jet areas and subtraction are tools whose full potential has probably not yet been explored and exploited