LUND UNIVERSITY

New ideas in hadronization
IPPP, Durham, UK
15–17 April 2009

Old Ideas in Hadronization:

The Lund String

— a string that works —

Torbjörn Sjöstrand

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History
The simple straight string
Colour topologies and the Lund gluon
The perturbative connection: dipoles
Outlook

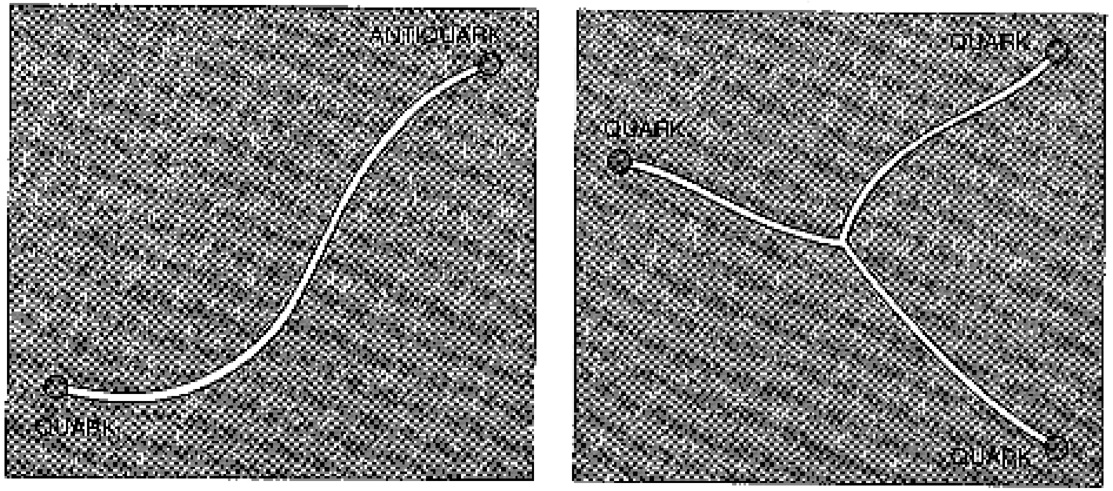
History (subjective)

Scientific American, February 1975

Dual-Resonance Models of Elementary Particles

In this new theoretical approach the strongly interacting particles classified as hadrons are viewed mathematically as massless strings whose ends move with the speed of light in multidimensional space

by John H. Schwarz



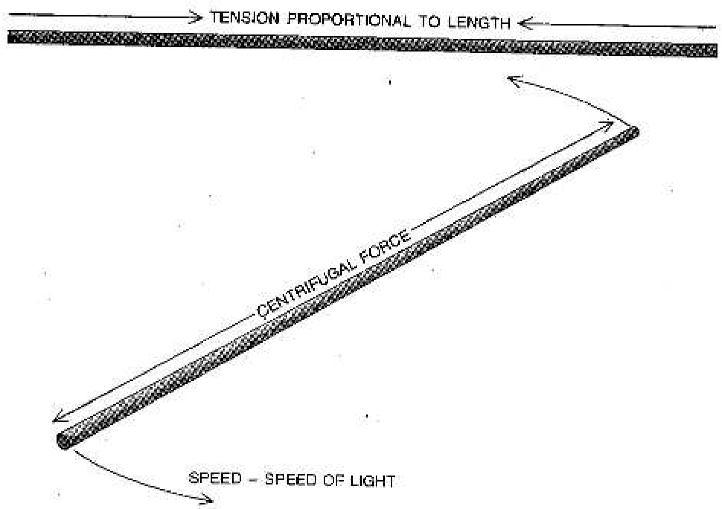
QUARKS CAN BE INCORPORATED into dual-resonance models in several possible ways. For example, if one assumes that quarks are pointlike fundamental particles attached to the ends of light strings, a hadron containing one quark and one antiquark (that is, a meson) would be simple to describe (*left*), whereas a hadron with three quarks (that is, a baryon) would require a more complicated topology (*right*). All the schemes that have been suggested so far for relating quarks to string picture present serious mathematical difficulties.

Scientific American, November 1976

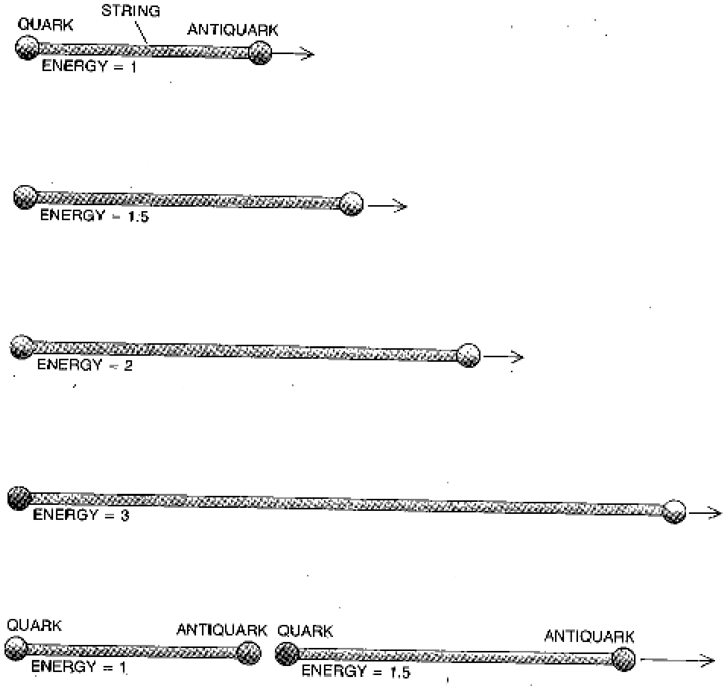
The Confinement of Quarks

How is it that these elementary particles of matter that explain so much about other particles are not seen? It may be that they are held inside other particles by forces inherent in their nature

by Yoichiro Nambu



STRING MODEL of hadron structure leads to another possible explanation of quark confinement. The model assumes that a hadron is made of a massless, one-dimensional string that has as one of its intrinsic properties a constant tension per unit length. Because of its tension the string tends to collapse, but it can be kept in equilibrium by centrifugal force if it is made to spin so that its ends move with exactly the speed of light. These properties of the string imply that its energy is proportional to its length and that its angular momentum is proportional to the square of its energy, a relation that has been experimentally verified for the hadrons.



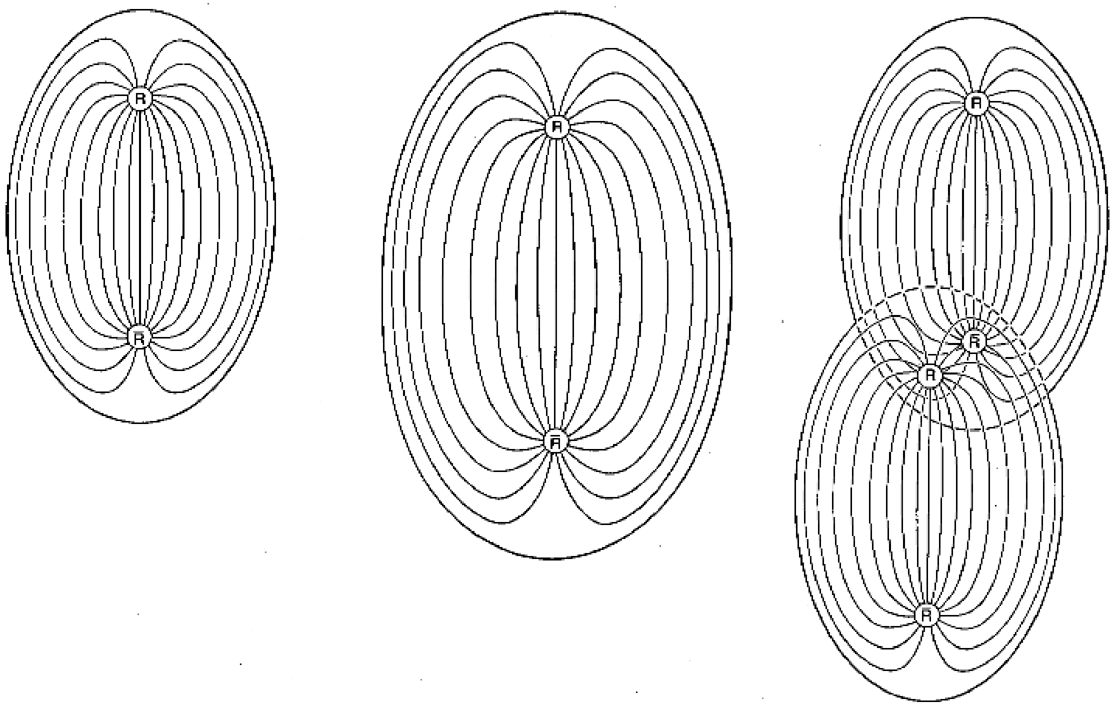
QUARKS WELDED TO STRINGS might be effectively confined. In order to separate the quarks it is necessary to stretch the string, but since the energy of the string is proportional to its length the energy required to pull the quarks apart increases in proportion to the separation. A macroscopic separation could be obtained only at the cost of enormous energy. In fact, isolation of a quark might not be possible at any energy, since as soon as enough energy had been supplied to create a quark and an antiquark the string might snap and these new particles appear at the ends. Thus the result is not the liberation of a quark but the creation of a meson.

Scientific American, July 1979

The Bag Model of Quark Confinement

Quarks appear to be real, and yet they have not been observed in isolation. One hypothesis for why they have not been is that they are confined in bags analogous to the bubbles in a liquid

by Kenneth A. Johnson



TRANSFORMATION OF A MESON (*left*) into two mesons requires a "polarizing field" acting to separate the color charges of the quark pair in the meson (*middle*). A new quark-antiquark pair appears spontaneously in the polarizing field between the quark and the antiquark of the meson (*right*). The color lines link the quark of the meson with the newly created antiquark and the antiquark of the meson with the newly created quark. Both quark-antiquark pairs are colorless, and so color lines do not join them. Because a constant force does not act between the pairs (as it does between the quark and the antiquark in each pair), they can separate and form two mesons.

X. Artru, G. Mennessier, Nucl. Phys. **B70** (1974) 93

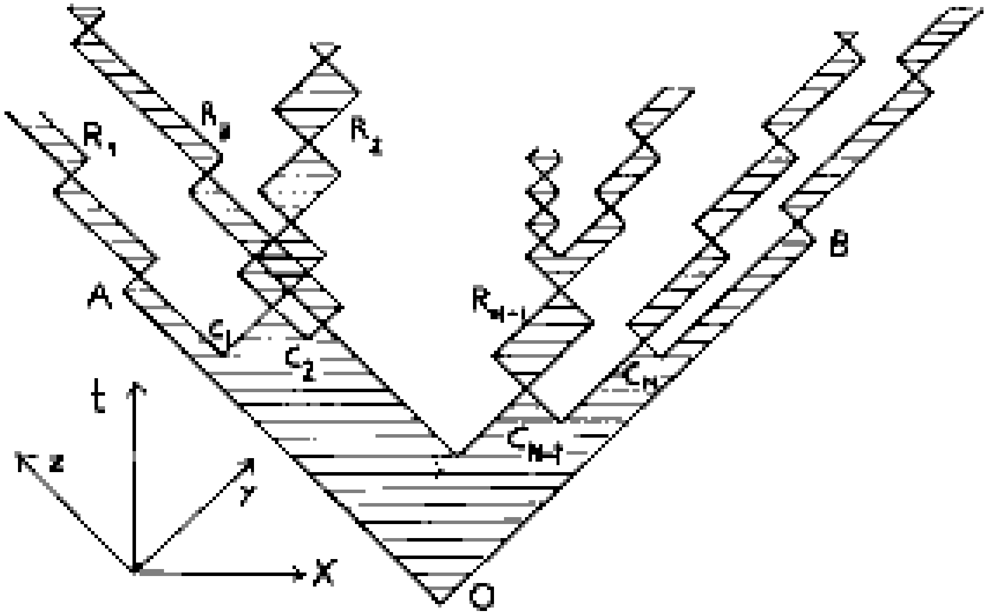


Fig. 10. Decay of a dart in the two-dimensional model, y and z are the new coordinates defined in (5.4). R_i and C_j denote the first generation resonances and cuts respectively. Here the initial quarks are taken away by resonances R_1 and R_{N+1}. There is a rapidity inversion for the resonances R_2 and R_3. The decay of R_{N-1} into two second generation resonances (or particles) is shown.

$$\mathrm{d}\mathcal{P} = b\, \exp(-bA_-)\, \mathrm{d}A$$

A_- string area in backwards lightcone

unphysical mass spectrum; m_{cut} to get $\langle m \rangle \sim$ right

pure $1+1$ dimensions: no $p_\perp$

$\mathrm{u\overline{u}} : \mathrm{d\overline{d}} : \mathrm{s\overline{s}} = 1 : 1 : 1$; only PS

R.D. Field, R.P. Feynman, Nucl. Phys. **B136** (1978) 1

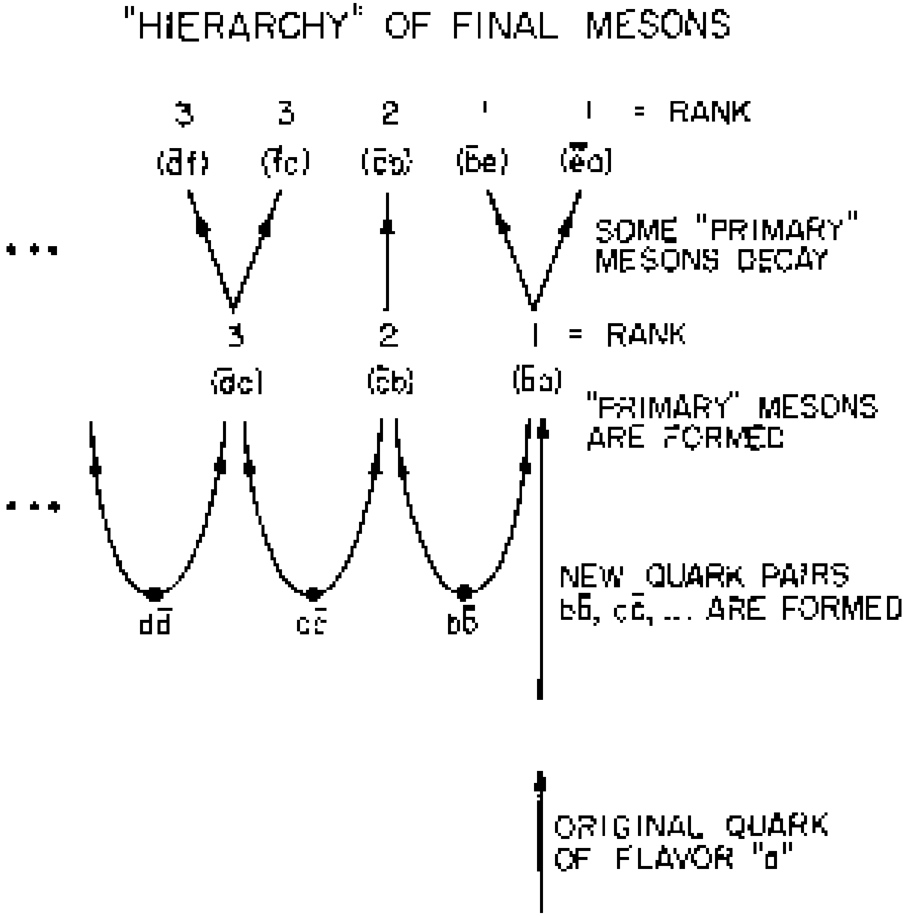


Fig. 1. Illustration of the "hierarchy" structure of the final mesons produced when a quark of type "a" fragments into hadrons. New quark pairs $b\bar{b}$, $c\bar{c}$, etc., are produced and "primary" mesons are formed. The "primary" meson $\bar{b}a$ that contains the original quark is said to have "rank" one and primary meson $\bar{c}b$ rank two, etc. Finally, some of the primary mesons decay and we assign all the decay products to have the rank of the parent. The order in "hierarchy" is *not* the same as order in momentum or rapidity.

$$f(z) = 1 - a + 3a(1-z)^2$$

$$a = 0.77$$

$$z = (E + p_z) \text{ fraction}$$

$$\mathrm{u\bar{u} : d\bar{d} : s\bar{s}} = 1 : 1 : 0.5$$

$$V : PS = 1 : 1$$

$$\exp(-p_\perp^2/2\sigma_q^2)\ d^2p_\perp$$

$$\sigma_q = 0.25 \text{ GeV}$$

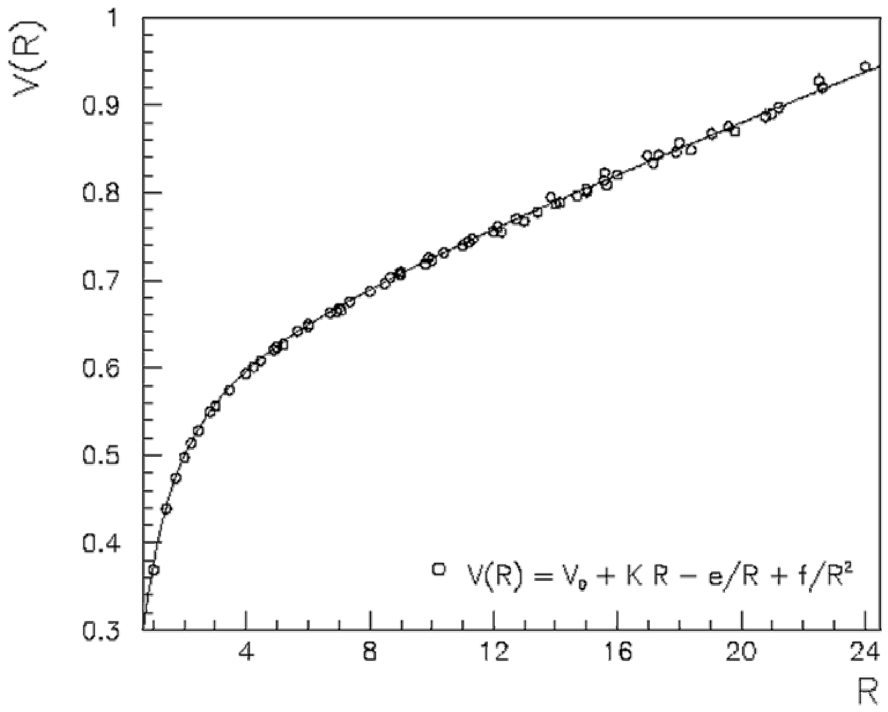
model for *one* jet

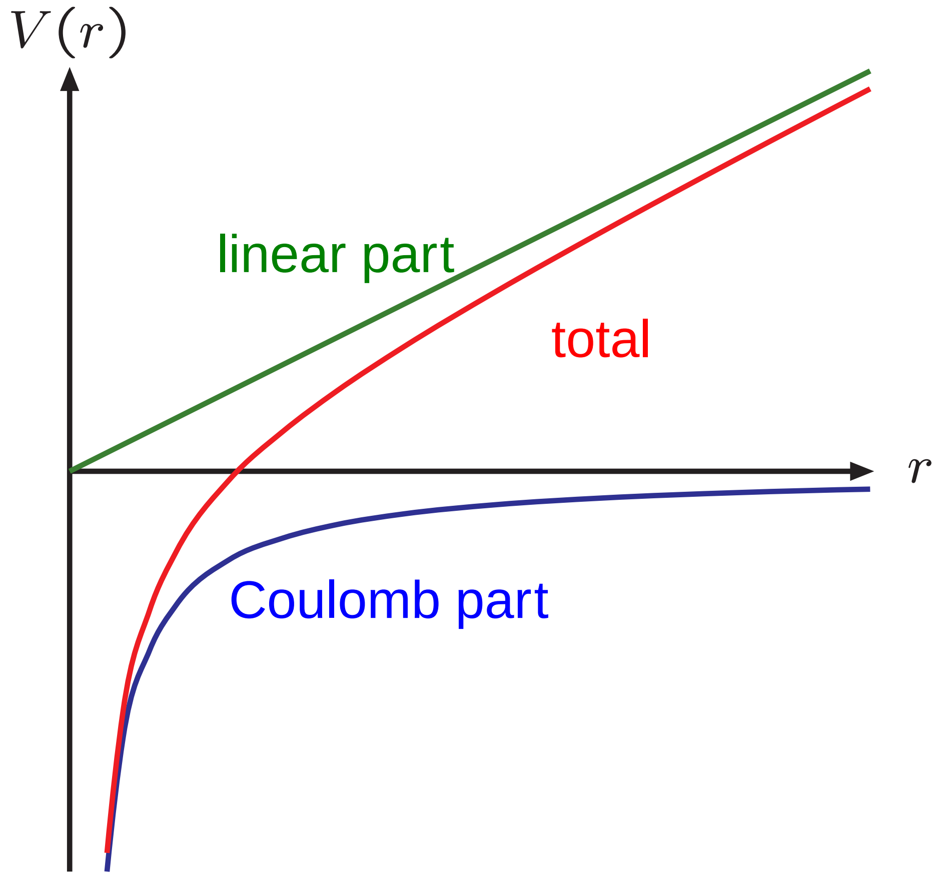
Confinement

Confinement = no free quarks

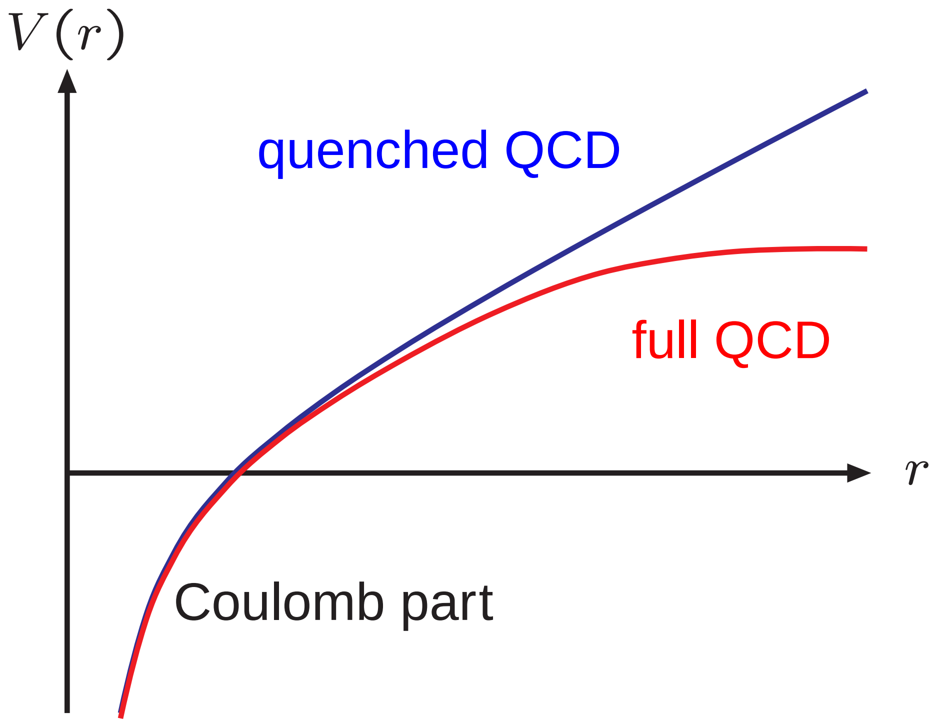
Linear confinement observed by Regge trajectories $m^2 - m_0^2 \propto J$.

Later confirmed e.g. by quenched lattice QCD

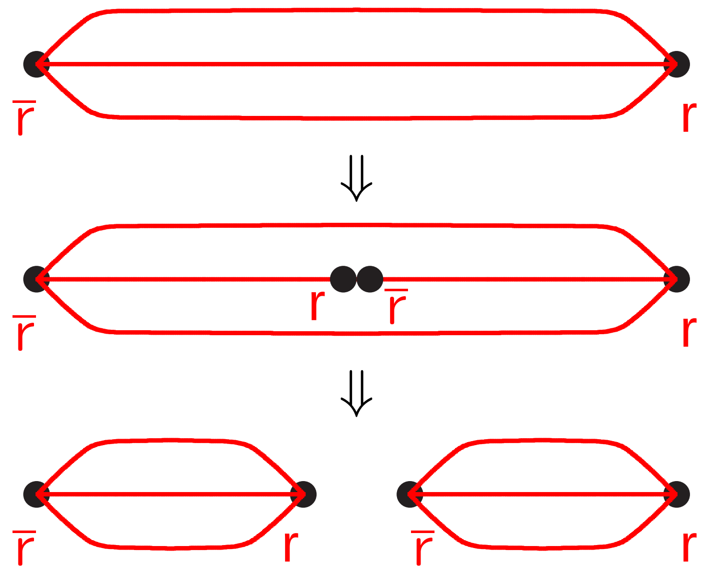




Real world (??, or at least unquenched lattice QCD)
$\Longrightarrow$ nonperturbative string breakings $gg\ldots \to q\bar{q}$



simplified colour representation:



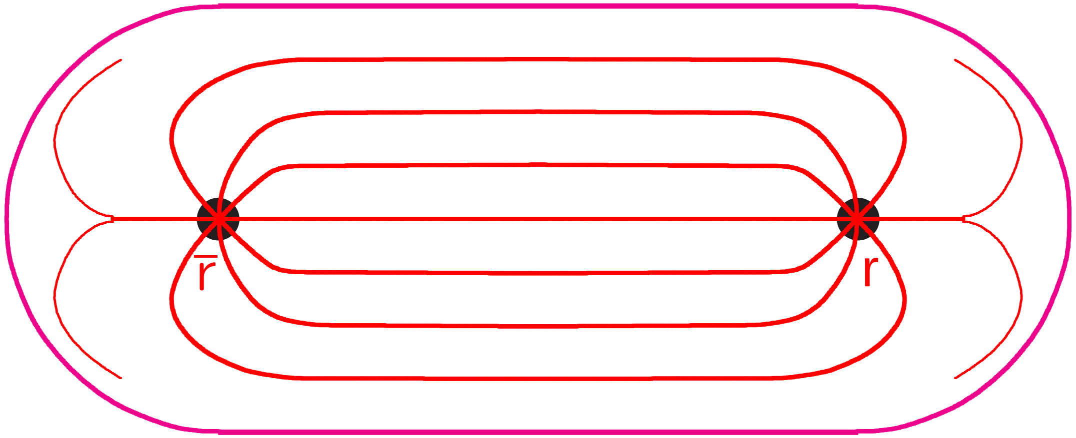
$$V(r) \approx -\frac{4}{3}\frac{\alpha_s}{r} + \kappa r \approx -\frac{0.13}{r} + r$$

(for $\alpha_s \approx 0.5$, r in fm and V in GeV)

$V(0.4\ \mathrm{fm}) \approx 0$: Coulomb important for internal structure of hadrons, not for particle production (?)

The Lund String Model (1977 -)

In QCD, for large charge separation, field lines seem to be compressed to tubelike region(s) $\Rightarrow$ **string(s)**



Gives linear confinement with string tension:

$$F(r) \approx \text{const} = \kappa \approx 1\ \text{GeV/fm} \quad \Longleftrightarrow \quad V(r) \approx \kappa r$$

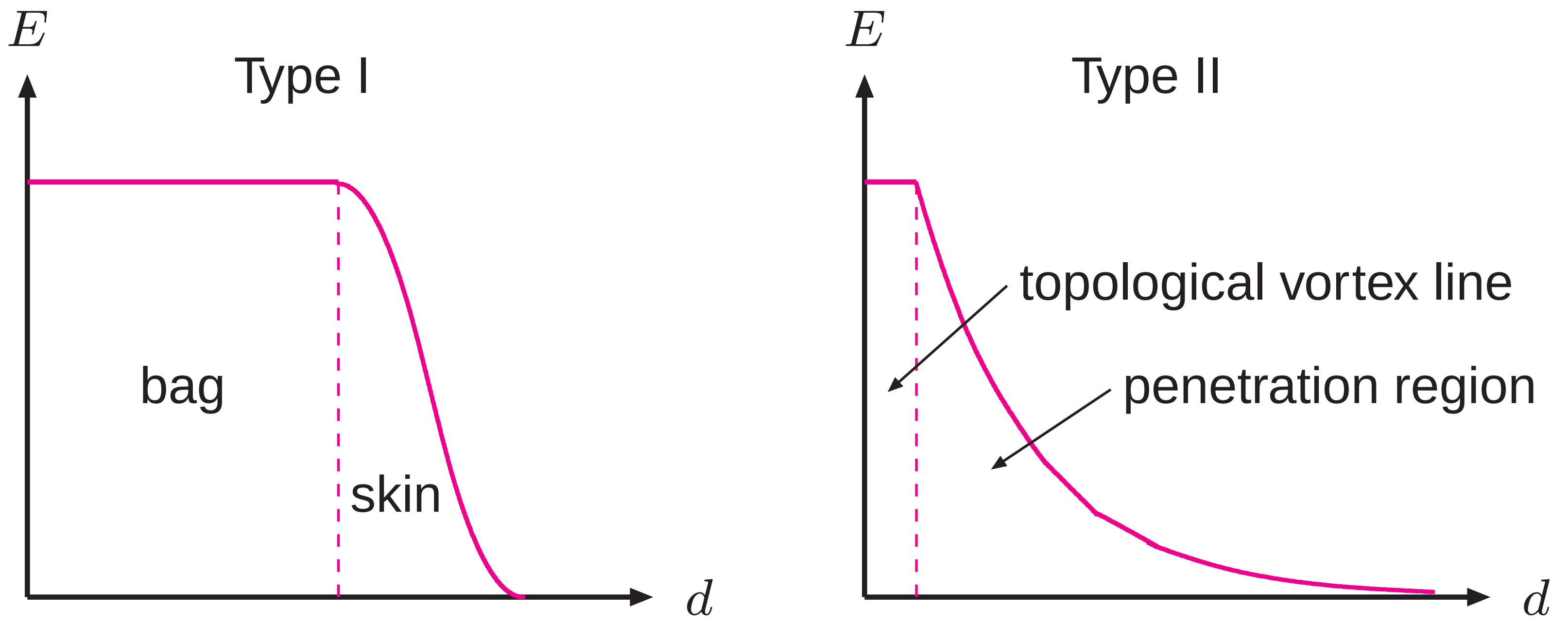
Separation of transverse and longitudinal degrees of freedom

$\Rightarrow$ simple description as 1+1-dimensional object

– a string with no transverse excitations –

with Lorentz covariant formalism

Analogy with superconductors



Details start to matter when many strings overlap (heavy ions, LHC):
bags lose separate identities more easily than vortex lines.
Little studied, evidence inconclusive: maybe in between?

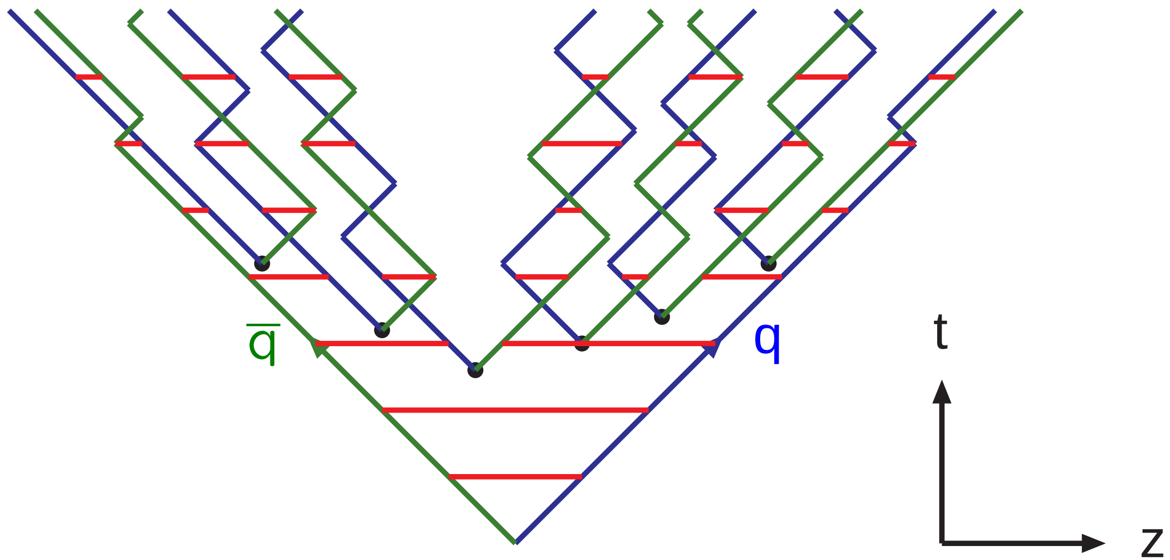
Whichever choice, key assumption is *uniformity*:
1+1-dimensional string parametrizes center of
translation-independent transverse profile

Lund model: repeated string breaks for large system
with pure $V(r) = \kappa r$, i.e. neglecting Coulomb part:

$$\left|\frac{\mathrm{d}E}{\mathrm{d}z}\right| = \left|\frac{\mathrm{d}p_z}{\mathrm{d}z}\right| = \left|\frac{\mathrm{d}E}{\mathrm{d}t}\right| = \left|\frac{\mathrm{d}p_z}{\mathrm{d}t}\right| = \kappa$$

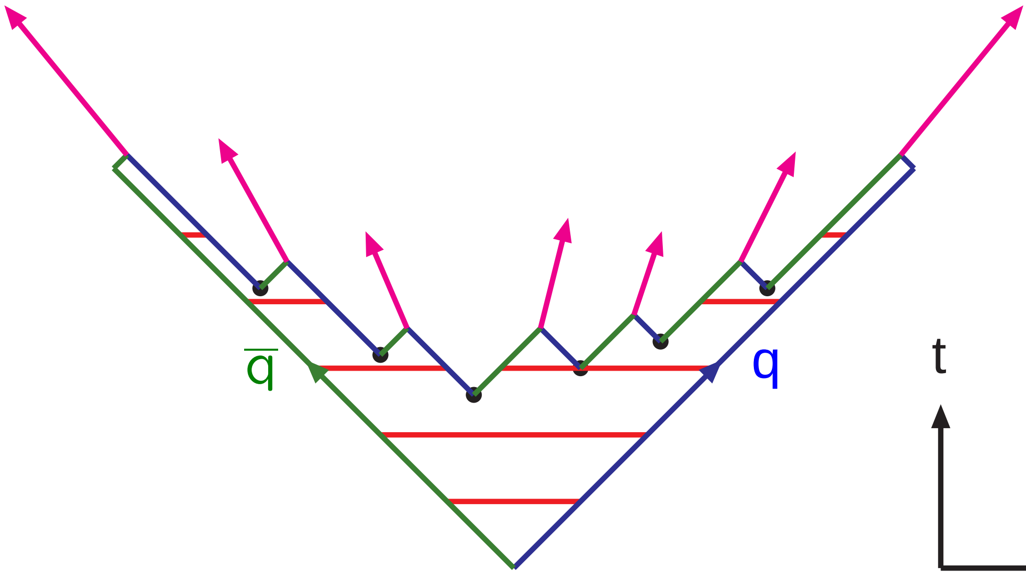
so energy–momentum quantities can be read off from space–time ones

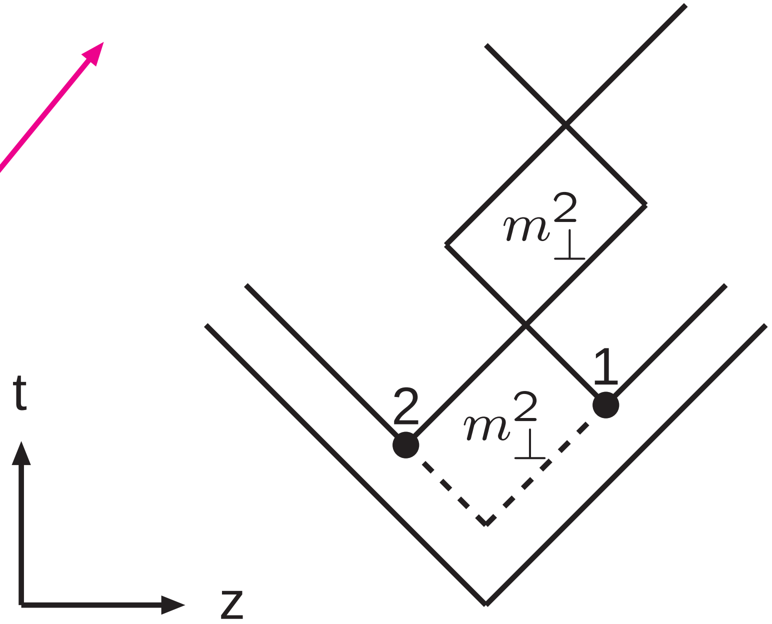
Motion of quarks and antiquarks in a $q\overline{q}$ system:



gives simple but powerful picture of hadron production

Fragmentation starts in the middle and spreads outwards:





but breakup vertices causally disconnected

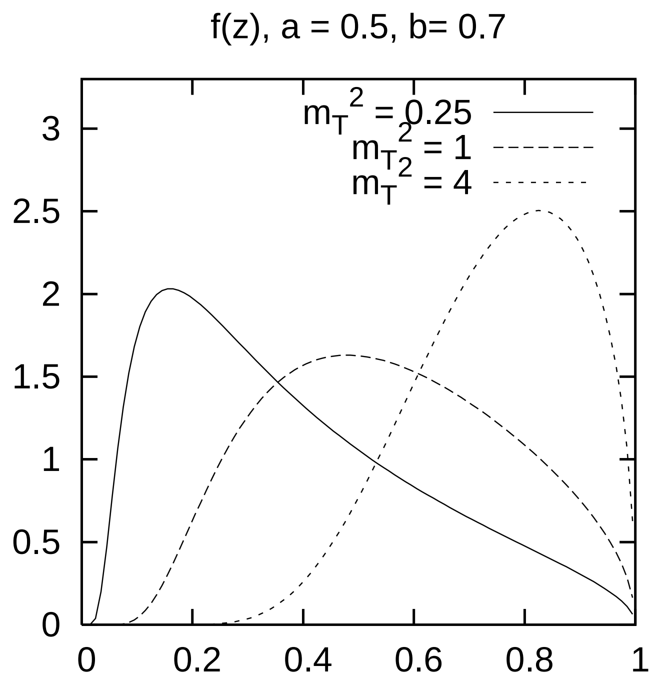
⇒ can proceed in arbitrary order

⇒ *left–right symmetry*

$$\begin{aligned}\mathcal{P}(1,2) &= \mathcal{P}(1) \times \mathcal{P}(1 \to 2)\\ &= \mathcal{P}(2) \times \mathcal{P}(2 \to 1)\end{aligned}$$

⇒ Lund symmetric fragmentation function

$f(z) \propto (1/z)\,(1-z)^a \exp(-b m_\perp^2/z)$



Interpretation

Can alternatively be written as matrix element times phase space:

$$
\begin{aligned}
\mathrm{d}\mathcal{P} &= |M|^2 \times \mathrm{d}(PS) \\
&= e^{-bA_{\mathrm{tot}}} \times \delta^{(2)}(\sum_i p_i - P_{\mathrm{tot}}) \prod_{i=1}^{n} N \, \mathrm{d}^2 p_i \, \delta(p_i^2 - m_i^2) \theta(E_i)
\end{aligned}
$$

where $M = \exp(i\xi A_{\mathrm{tot}})$ with $\xi = \kappa + ib/2$
by Wilson area law for confining field

Misleading similarity with Artru-Mennessier, since $\delta(p^2 - m^2)$ applied to $\exp(-bA_{\mathrm{allowed}})$ gives $f(z) = z^{a-1}, a > 0$, i.e. not symmetric.

$m_i \to m_{\perp i}$ given by physical mass spectrum + $p_\perp$
$N \leftrightarrow a$, a related to Regge trajectory intercept $\to a \approx 0.5$

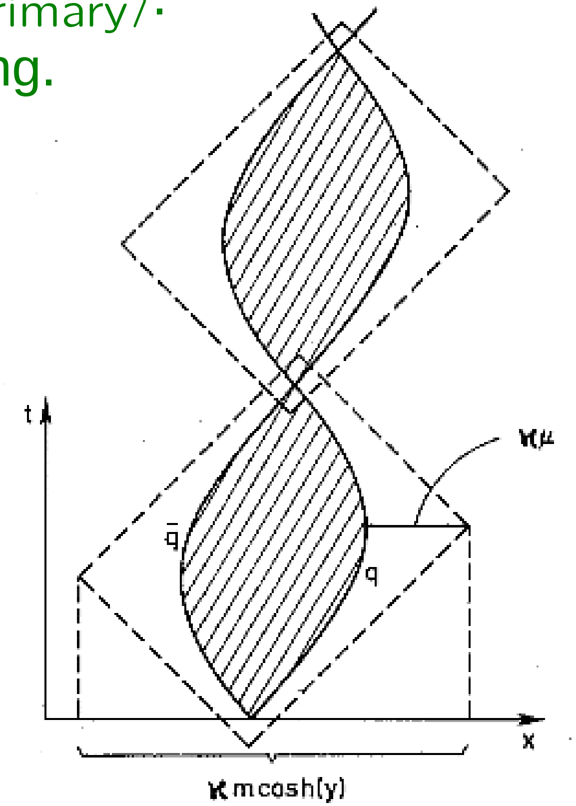
Introduce $\Gamma = (\kappa\tau)^2$ of breakup. Then, for large systems,

$$
\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}\Gamma} \propto \Gamma^a \, e^{-b\Gamma} \quad \Longrightarrow \quad \langle \Gamma \rangle = \frac{1+a}{b}
$$

e.g. $a = 0.5, b = 0.7$ GeV^{-2} gives $\langle \Gamma \rangle = 2$ GeV^2 or $\langle \tau \rangle = 1.5$ fm.

Comments and Extensions

If b fixed then larger $a \Rightarrow$ larger $\langle \Gamma \rangle \Rightarrow$ larger $\langle n_{\text{primary}} \rangle$.
Rapidity ordering is correlated with flavour ordering.
If $a, b \to \infty$ with a/b constant
then $\mathrm{d}\mathcal{P}/\mathrm{d}\Gamma \to \delta(\Gamma - a/b)$
which gives strict ordering.



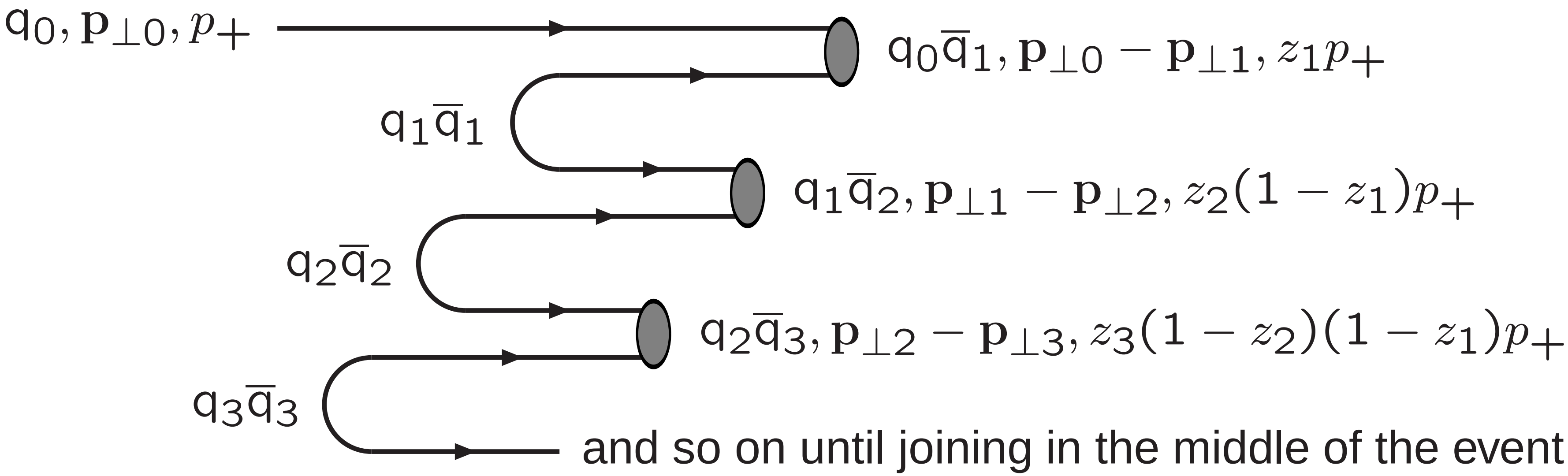
Bowler: massive quarks span reduced
string area relative to asymptotes
representing massless motion;
gives modification $z^{-bm_Q^2}$ to $f(z)$,
as required for good tunes to data.

UCLA model: take area law seriously, also for relative production of flavours:

$$\mathcal{P}_{\text{hadron}}(m_\perp^2) \propto \int_0^1 \frac{\mathrm{d}z}{z} (1-z)^a \exp\left(-b\frac{m_\perp^2}{z}\right)$$

$\Rightarrow$ large m suppressed
(basic idea; complete framework more sophisticated).

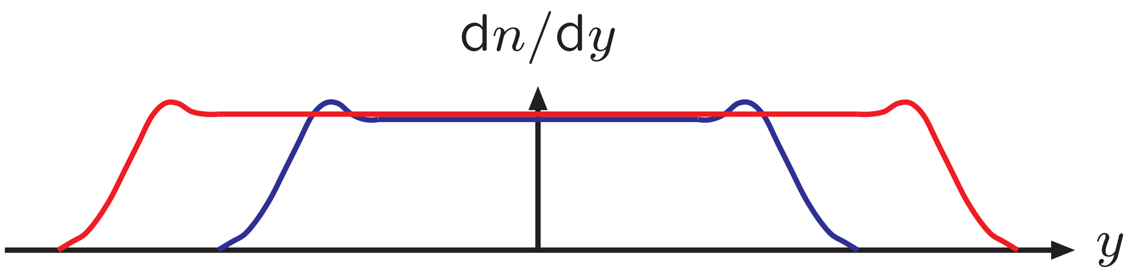
The iterative ansatz



and so on until joining in the middle of the event

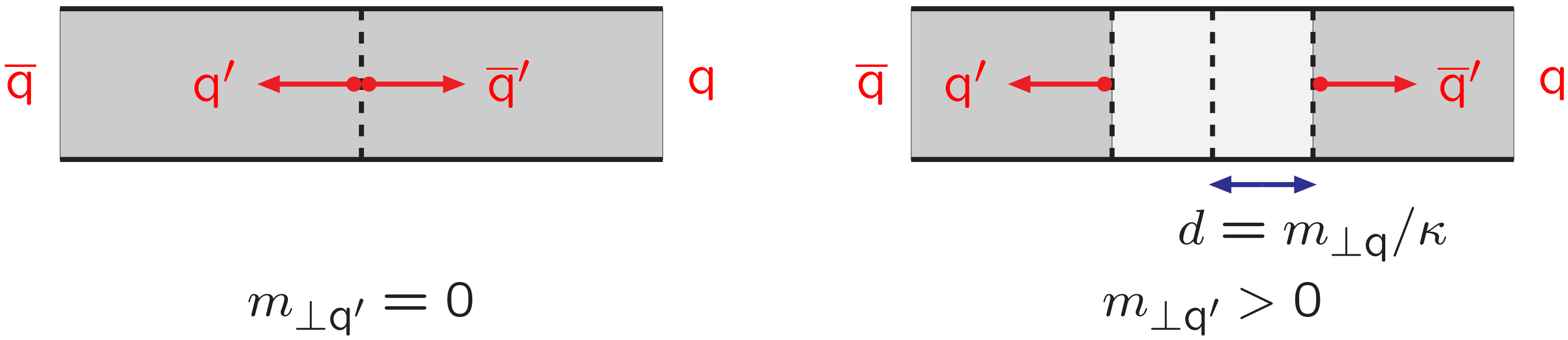
(in principle follows from model, in practice some work)

Scaling in lightcone $p_\pm = E \pm p_z$ (for $q\overline{q}$ system along z axis) implies flat central rapidity plateau + some endpoint effects:



$\langle n_{\mathrm{ch}} \rangle \approx c_0 + c_1 \ln E_{\mathrm{cm}}$, $\sim$ Poissonian multiplicity distribution

How does the string break?



String breaking modelled by tunneling:

$$\mathcal{P} \propto \exp\left(-\frac{\pi m_{\perp q}^2}{\kappa}\right) = \exp\left(-\frac{\pi p_{\perp q}^2}{\kappa}\right) \exp\left(-\frac{\pi m_q^2}{\kappa}\right)$$

1) common Gaussian $p_\perp$ spectrum

2) suppression of heavy quarks $u\overline{u} : d\overline{d} : s\overline{s} : c\overline{c} \approx 1 : 1 : 0.3 : 10^{-11}$

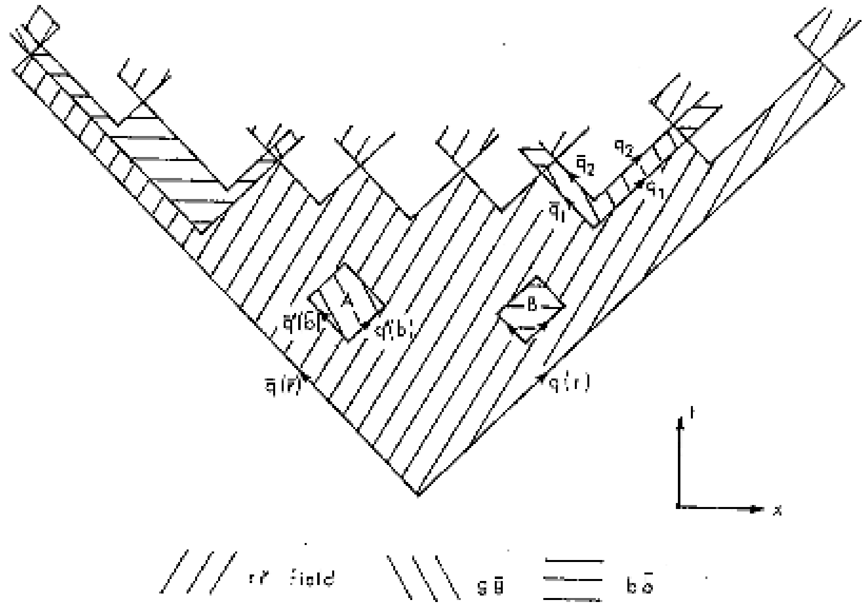
3) diquark $\sim$ antiquark $\Rightarrow$ simple model for baryon production

Hadron composition also depends on spin probabilities, hadronic wave functions, phase space, more complicated baryon production, . . .

$\Rightarrow$ “moderate” predictivity (many parameters!)

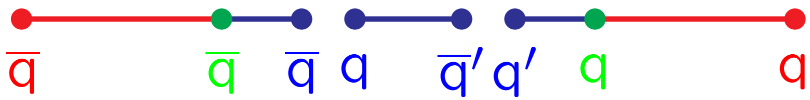
Baryon production

Meson production $\approx$ same colour everywhere.
Fluctuations with other colour $\rightarrow$ no net force.



i.e. r + g = $\bar{b}$
Baryon production as if diquark
when only one break
inside "wrong-colour" region:

Popcorn when several breaks:

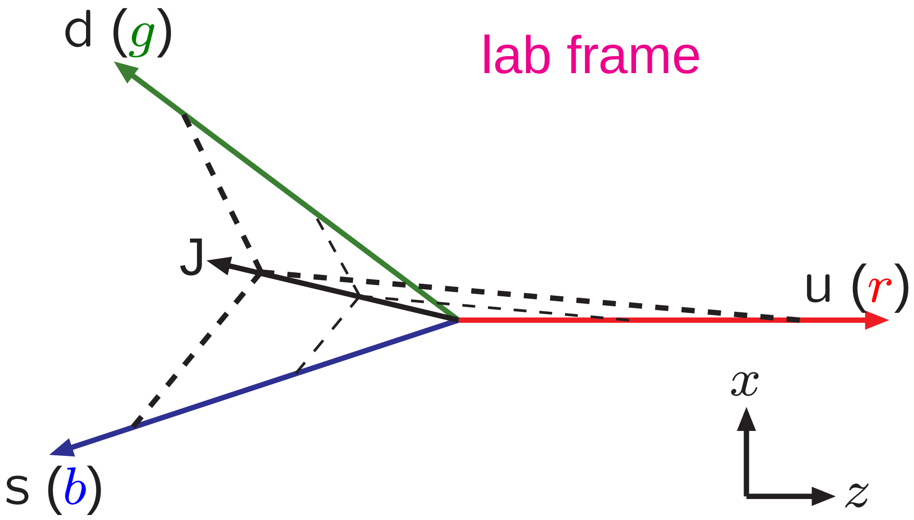


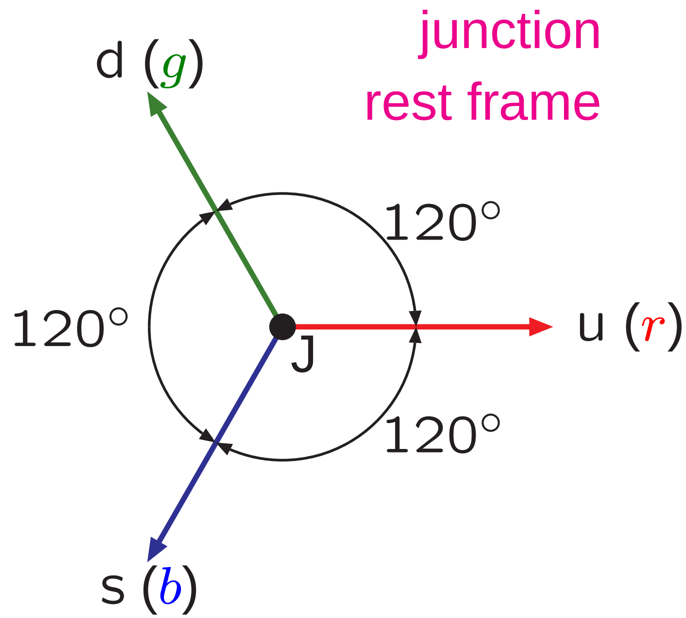
a can be flavour-dependent, $\mathrm{d}\mathcal{P}/\mathrm{d}\Gamma \propto \Gamma^{a_\alpha} \, e^{-b\Gamma}$,
e.g. $a_{\mathrm{qq}} > a_{\mathrm{q}}$ corresponding to larger formation time for diquarks.
Gives modified fragmentation function:

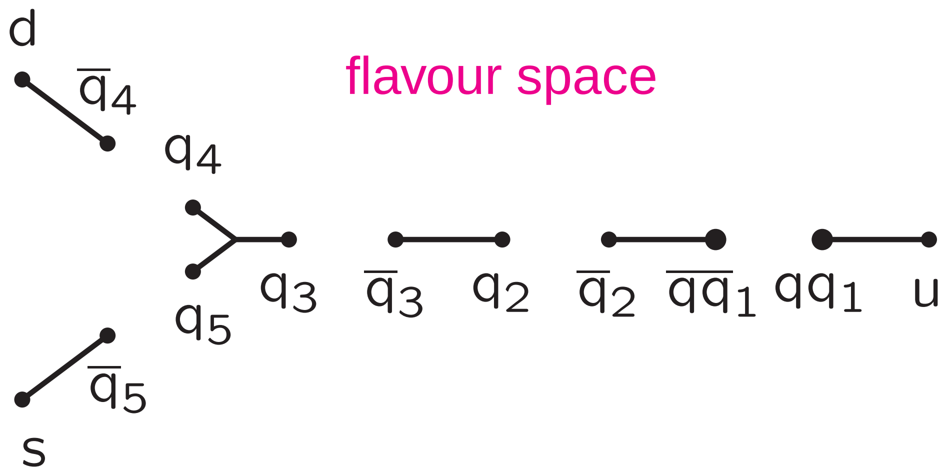
$$f(z) \propto \frac{1}{z} \, z^{a_\alpha} \left(\frac{1-z}{z} \right)^{a_\beta} \exp\left(-\frac{b m_\perp^2}{z} \right)$$

Fragmentation of a junction topology

Encountered in R-parity violating SUSY decays $\tilde{\chi}_1^0 \rightarrow$ uds,
or when 2 valence quarks kicked out of proton beam



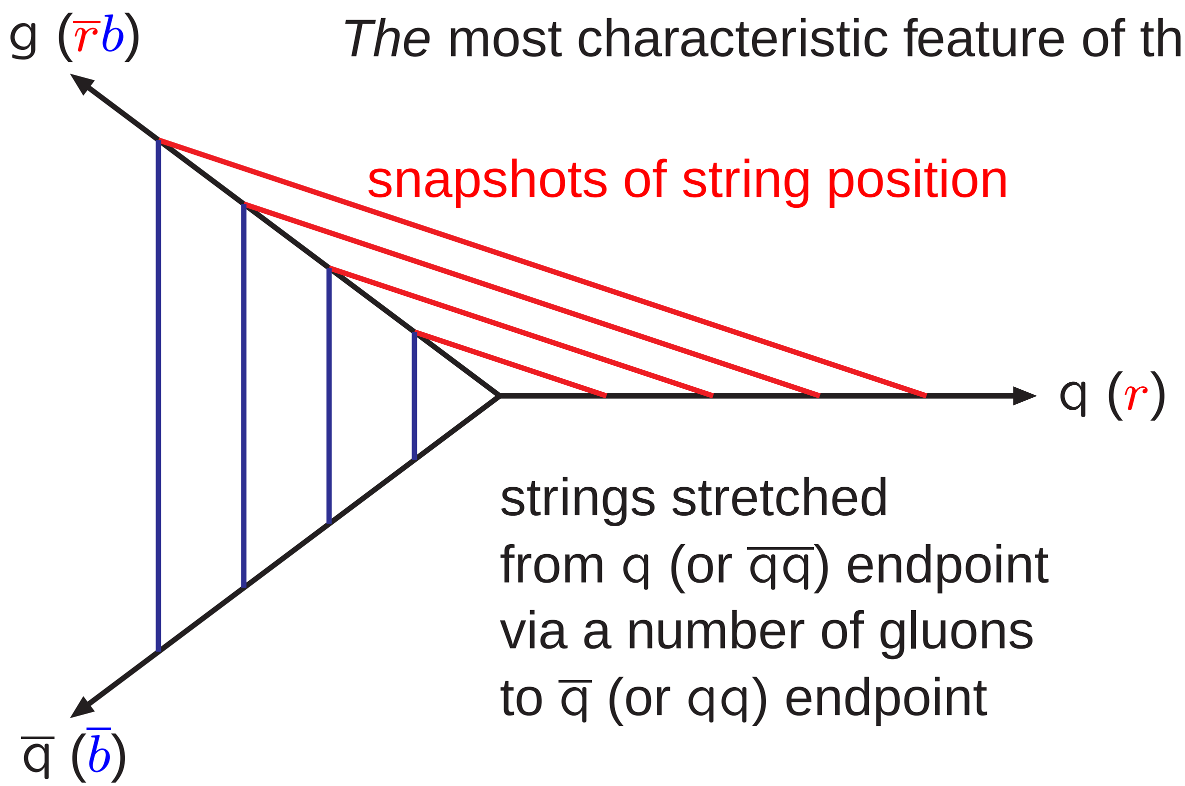




More complicated (but ≈solved) with gluon emission and massive quarks

The Lund gluon picture

The most characteristic feature of the Lund model



strings stretched
from q (or $\overline{\text{qq}}$) endpoint
via a number of gluons
to $\overline{\text{q}}$ (or qq) endpoint

Gluon = kink on string, carrying energy and momentum

Force ratio gluon/ quark = 2, cf. QCD $N_C/C_F = 9/4$, $\rightarrow$ 2 for $N_C \rightarrow \infty$

No new parameters introduced for gluon jets!, so:

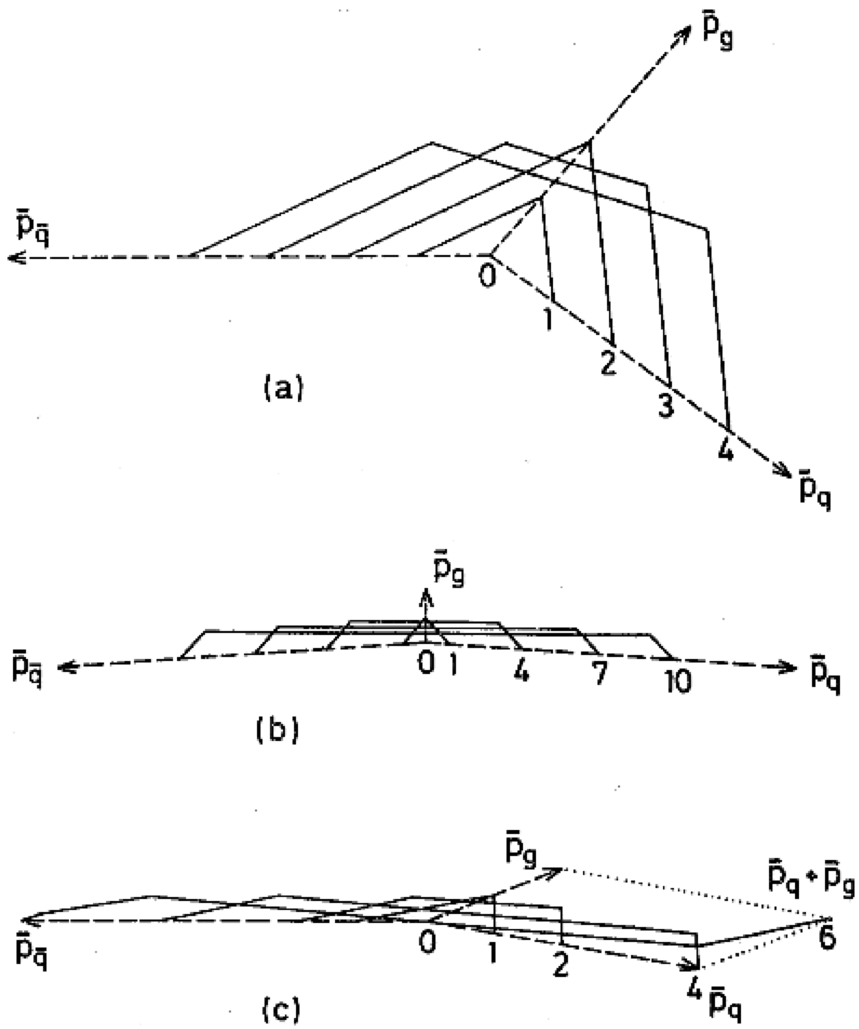
- Few parameters to describe energy-momentum structure!
- Many parameters to describe flavour composition!

Collinear and infrared safety

Complete string motion more complicated. New string region when gluon has lost all of its momentum, consisting of inflowing momentum from q and $\overline{\mathrm{q}}$.

For soft gluon this region appears early and for $E_{\mathrm{g}} \to 0$ the simple $\mathrm{q}\overline{\mathrm{q}}$ event is recovered.

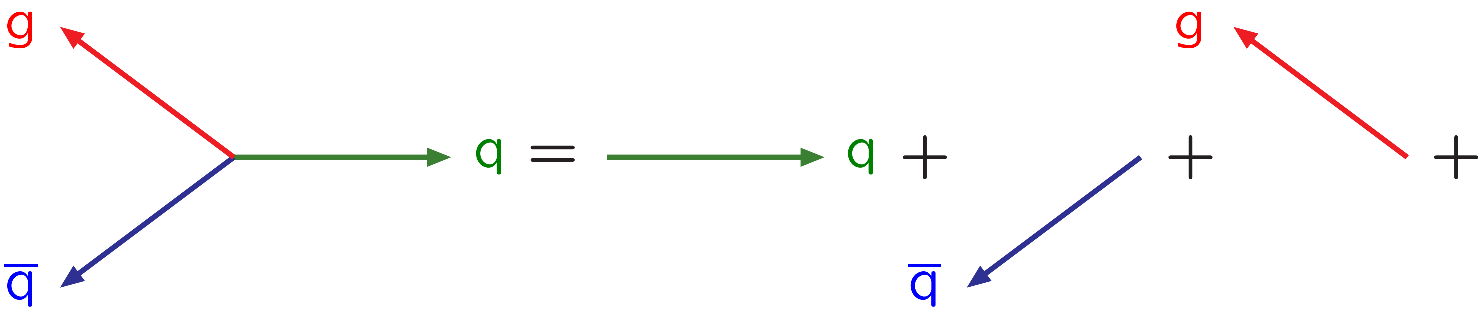
For collinear gluon the string end extends as far as the vector sum of momenta, so for $\theta_{\mathrm{qg}} \to 0$ again back to the simple $\mathrm{q}\overline{\mathrm{q}}$ event.



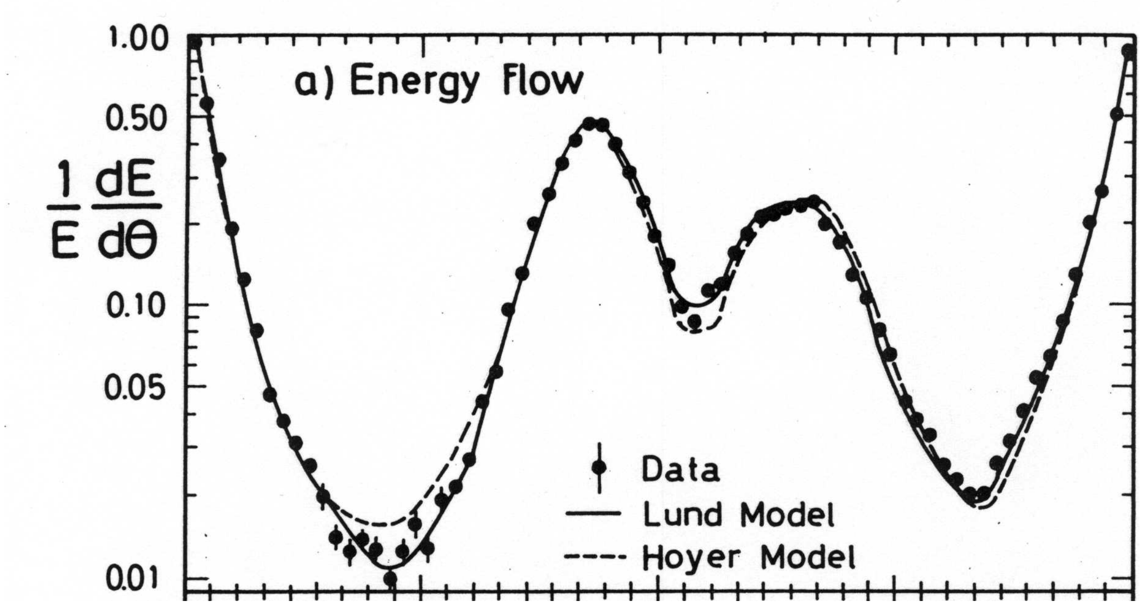
Same principles for arbitrary $\mathrm{qg}_1\mathrm{g}_2 \ldots \mathrm{g}_n\overline{\mathrm{q}}$ topology or $\mathrm{g}_1\mathrm{g}_2 \ldots \mathrm{g}_n$ closed gluon loop, but technically messy.

Independent fragmentation

Based on a similar iterative ansatz as string, but



minor
corrections
in middle



String effect
(JADE, 1980)
≈ coherence in
nonperturbative
context

Further numerous and detailed tests at LEP favour string picture . . .

. . . but much is still uncertain when moving to hadron colliders.

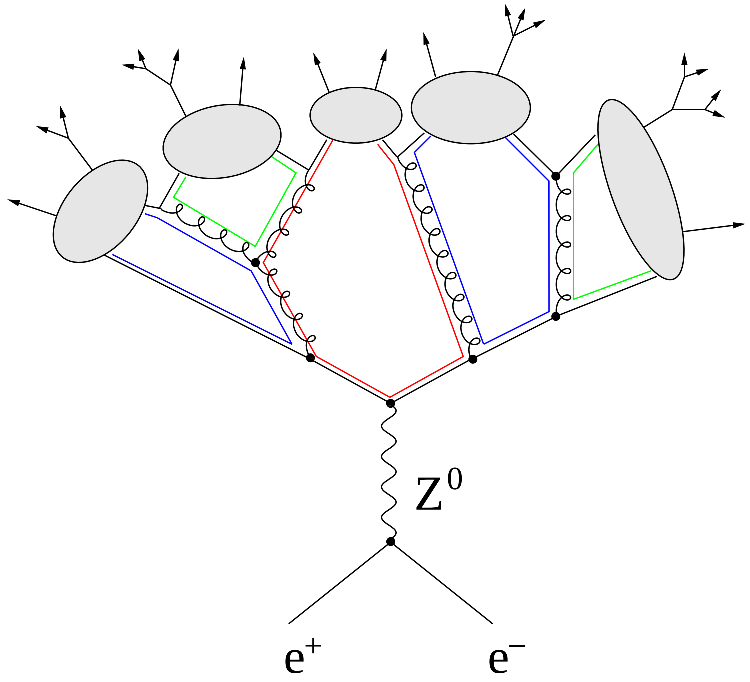
The HERWIG Cluster Model

"Preconfinement": colour flow is local
in coherent shower evolution

1) Introduce forced $\mathrm{g} \to \mathrm{q}\overline{\mathrm{q}}$ branchings

2) Form colour singlet clusters

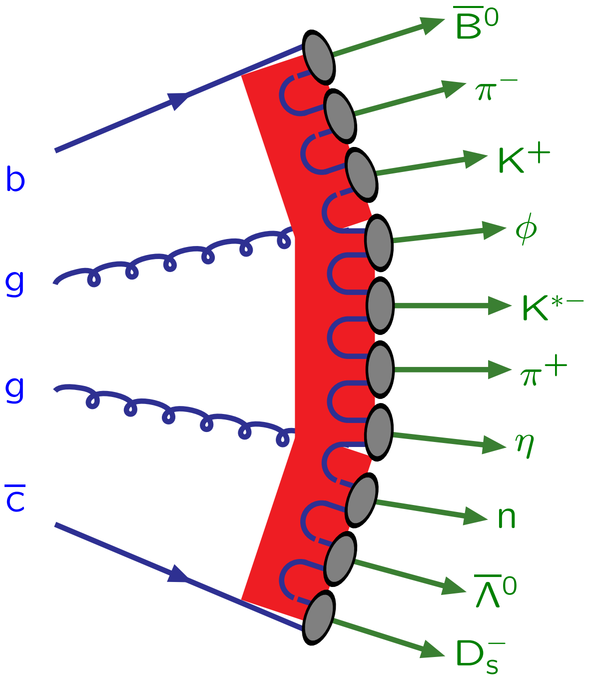
3) Clusters decay isotropically to 2
hadrons according to phase space
$W \sim (2s_1+1)(2s_2+1)(2p^*/m)$

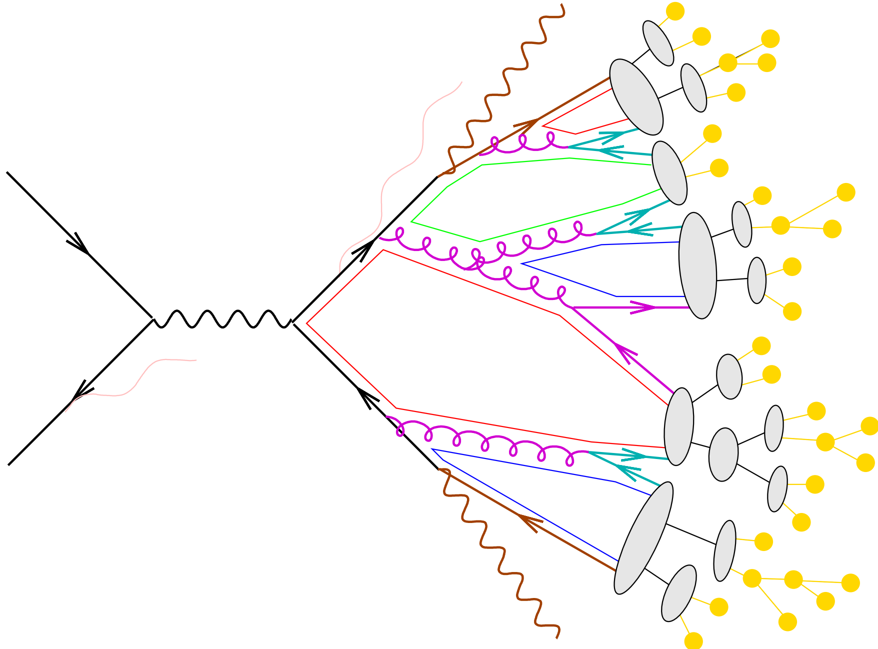


simple and clean, but . . .

- Tail to very large-mass clusters (e.g. if no emission in shower);
  if large-mass cluster $\to$ 2 hadrons then crazy "four-jet" events
  $\Longrightarrow$ split big cluster into 2 smaller along "string" direction; iterate;
  $\sim$ 15% of primary clusters are split, but give $\sim$ 50% of final hadrons
- Too soft charm/bottom spectra $\Longrightarrow$ anisotropic leading-cluster decay
- Charge correlations still problematic $\Longrightarrow$ all clusters anisotropic (?)
- Correlations between baryons and antibaryons $\Longrightarrow$ allow $\mathrm{g} \to \mathrm{qq} + \overline{\mathrm{qq}}$

String vs. Cluster





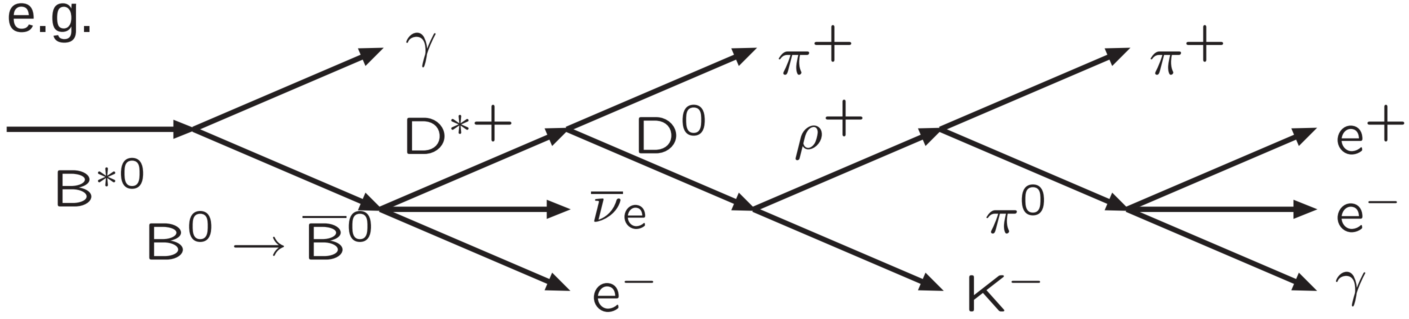
program	PYTHIA	HERWIG
model	string	cluster
energy–momentum picture	powerful predictive	simple unpredictive
parameters	few	many
flavour composition	messy unpredictive	simple in-between
parameters	many	few

"There ain't no such thing as a parameter-free *good* description"

Decays

Unspectacular/ungrateful but necessary:
this is where most of the final-state particles are produced!
Involves hundreds of particle kinds and thousands of decay modes.

e.g.

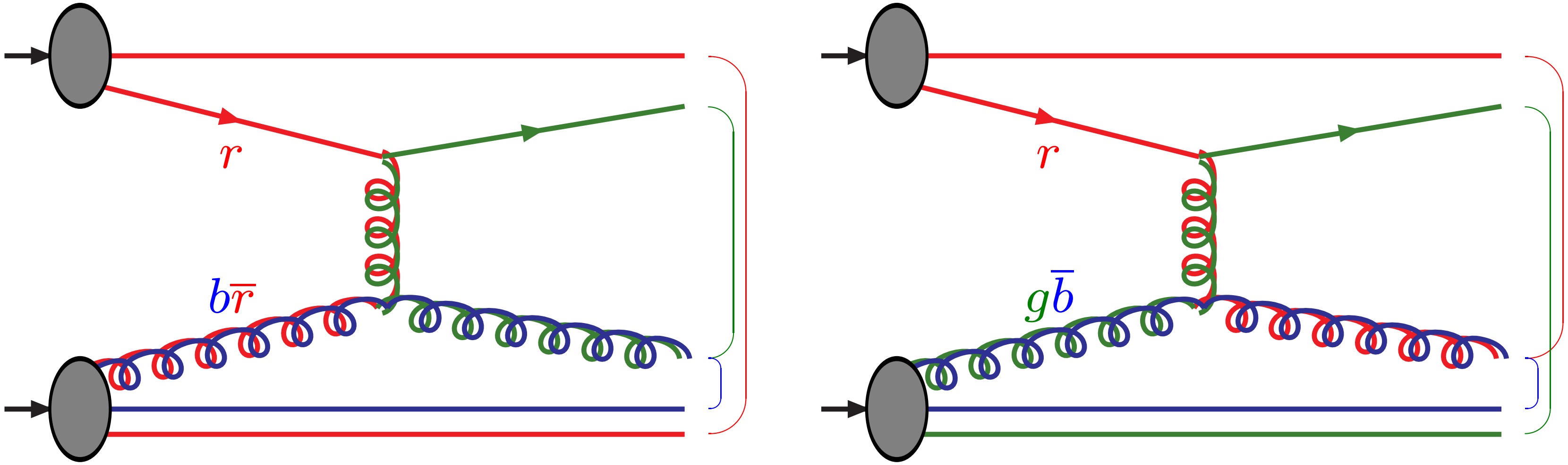


- $B^{*0} \to B^0\gamma$: electromagnetic decay
- $B^0 \to \overline{B}^0$ mixing (weak)
- $\overline{B}^0 \to D^{*+}\overline{\nu}_e e^-$: weak decay, displaced vertex, $|\mathcal{M}|^2 \propto (p_{\overline{B}}p_{\overline{\nu}})(p_e p_{D^*})$
- $D^{*+} \to D^0\pi^+$: strong decay
- $D^0 \to \rho^+ K^-$: weak decay, displaced vertex, ρ mass smeared
- $\rho^+ \to \pi^+\pi^0$: ρ polarized, $|\mathcal{M}|^2 \propto \cos^2\theta$ in ρ rest frame
- $\pi^0 \to e^+e^-\gamma$: Dalitz decay, $m(e^+e^-)$ peaked

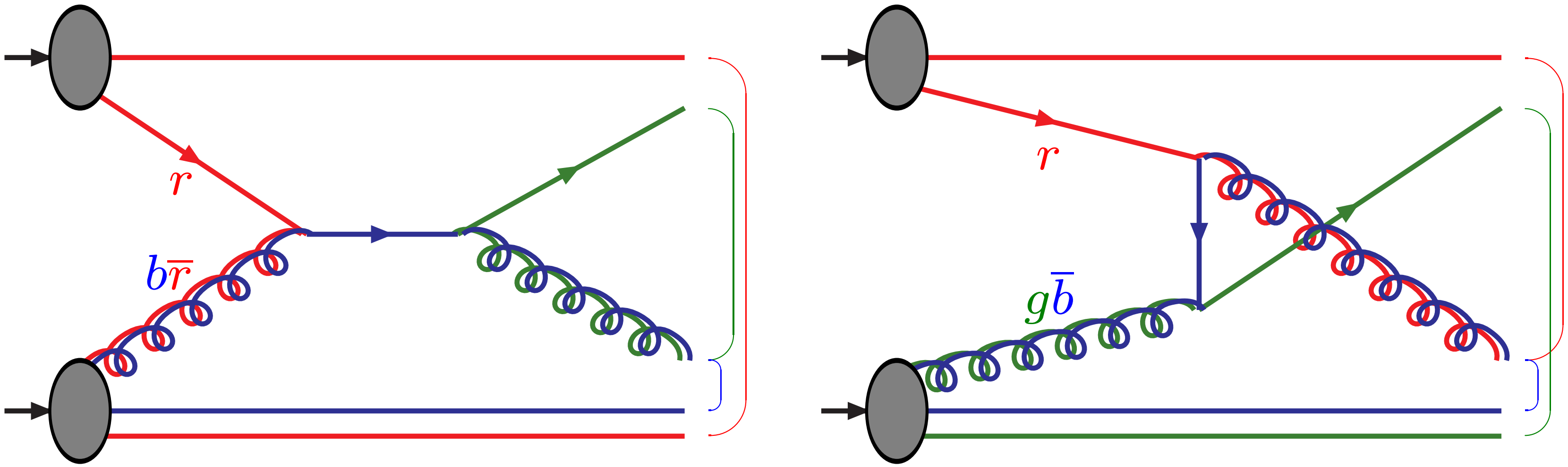
Dedicated programs, with special attention to polarization effects:
- EVTGEN: B decays
- TAUOLA: τ decays

Colour flow in hard processes

One Feynman graph can correspond to several possible colour flows, e.g. for $qg \to qg$:



while other $qg \to qg$ graphs only admit one colour flow:



so nontrivial mix of kinematics variables $(\hat{s}, \hat{t})$
and colour flow topologies I, II:

$$\begin{aligned} |\mathcal{A}(\hat{s},\hat{t})|^2 &= |\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t}) + \mathcal{A}_{\mathrm{II}}(\hat{s},\hat{t})|^2 \\ &= |\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t})|^2 + |\mathcal{A}_{\mathrm{II}}(\hat{s},\hat{t})|^2 + 2\,\mathcal{R}e\left(\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t})\mathcal{A}^*_{\mathrm{II}}(\hat{s},\hat{t})\right) \end{aligned}$$

with $\mathcal{R}e\left(\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t})\mathcal{A}^*_{\mathrm{II}}(\hat{s},\hat{t})\right) \neq 0$

$\Rightarrow$ indeterminate colour flow, while

- showers *should* know it (coherence),
- hadronization *must* know it (hadrons singlets).

Normal solution:

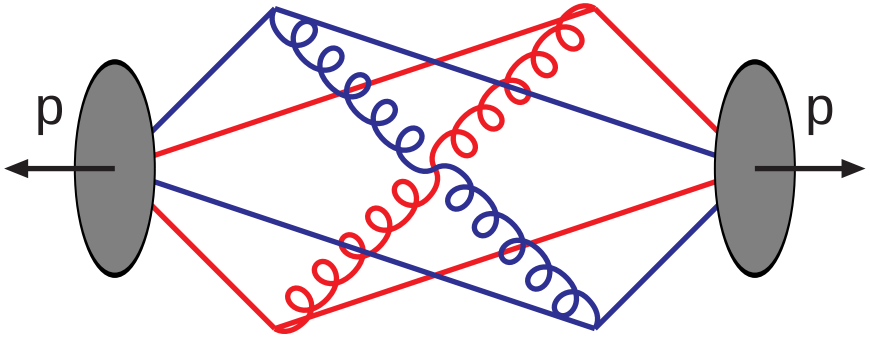
$$\frac{\text{interference}}{\text{total}} \propto \frac{1}{N_C^2 - 1}$$

so split I : II according to proportions in the $N_C \to \infty$ limit, i.e.

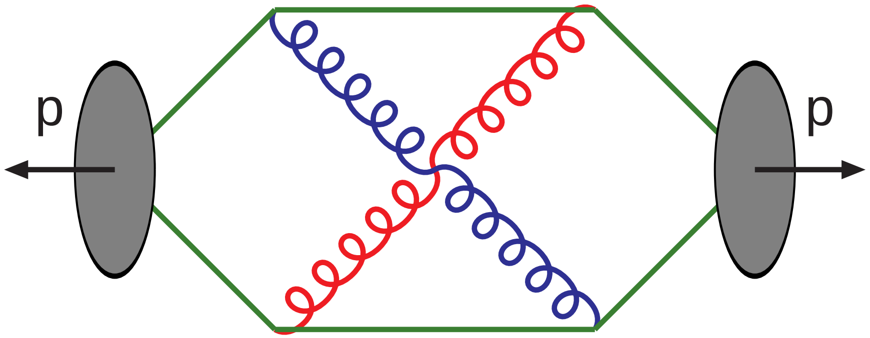
$$\begin{aligned} |\mathcal{A}(\hat{s},\hat{t})|^2 &= |\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t})|^2_{\text{mod}} + |\mathcal{A}_{\mathrm{II}}(\hat{s},\hat{t})|^2_{\text{mod}} \\ |\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t})|^2_{\text{mod}} &= |\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t}) + \mathcal{A}_{\mathrm{II}}(\hat{s},\hat{t})|^2 \left(\frac{|\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t})|^2}{|\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t})|^2 + |\mathcal{A}_{\mathrm{II}}(\hat{s},\hat{t})|^2}\right)_{N_C\to\infty} \\ |\mathcal{A}_{\mathrm{II}}(\hat{s},\hat{t})|^2_{\text{mod}} &= \ldots \end{aligned}$$

Colour correlations

$\langle p_\perp \rangle(n_{\text{ch}})$ is very sensitive to colour flow



long strings to remnants $\Rightarrow$ much n_{ch}/interaction $\Rightarrow \langle p_\perp \rangle(n_{\text{ch}}) \sim$ flat



short strings (more central) $\Rightarrow$ less n_{ch}/interaction $\Rightarrow \langle p_\perp \rangle(n_{\text{ch}})$ rising

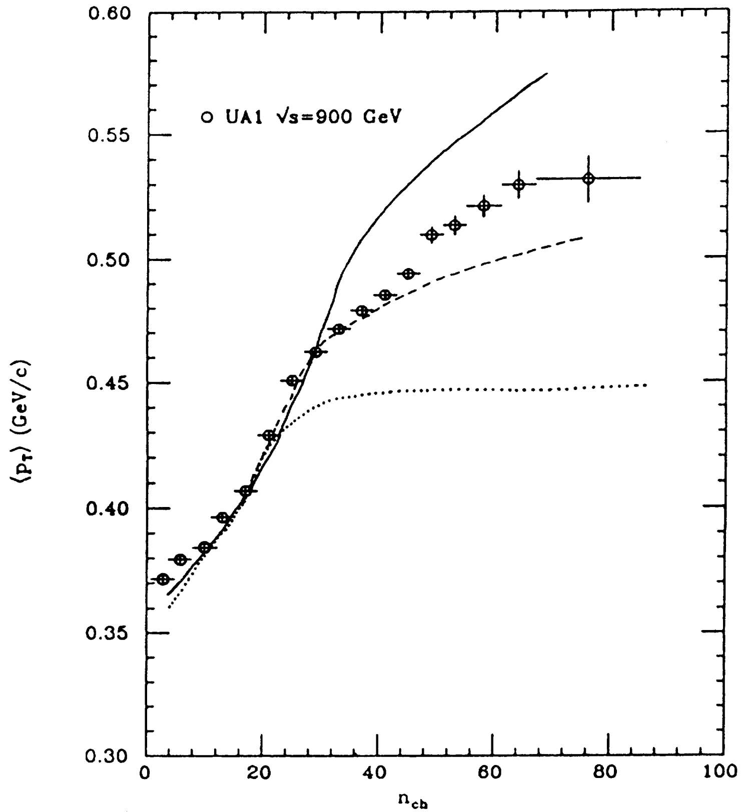
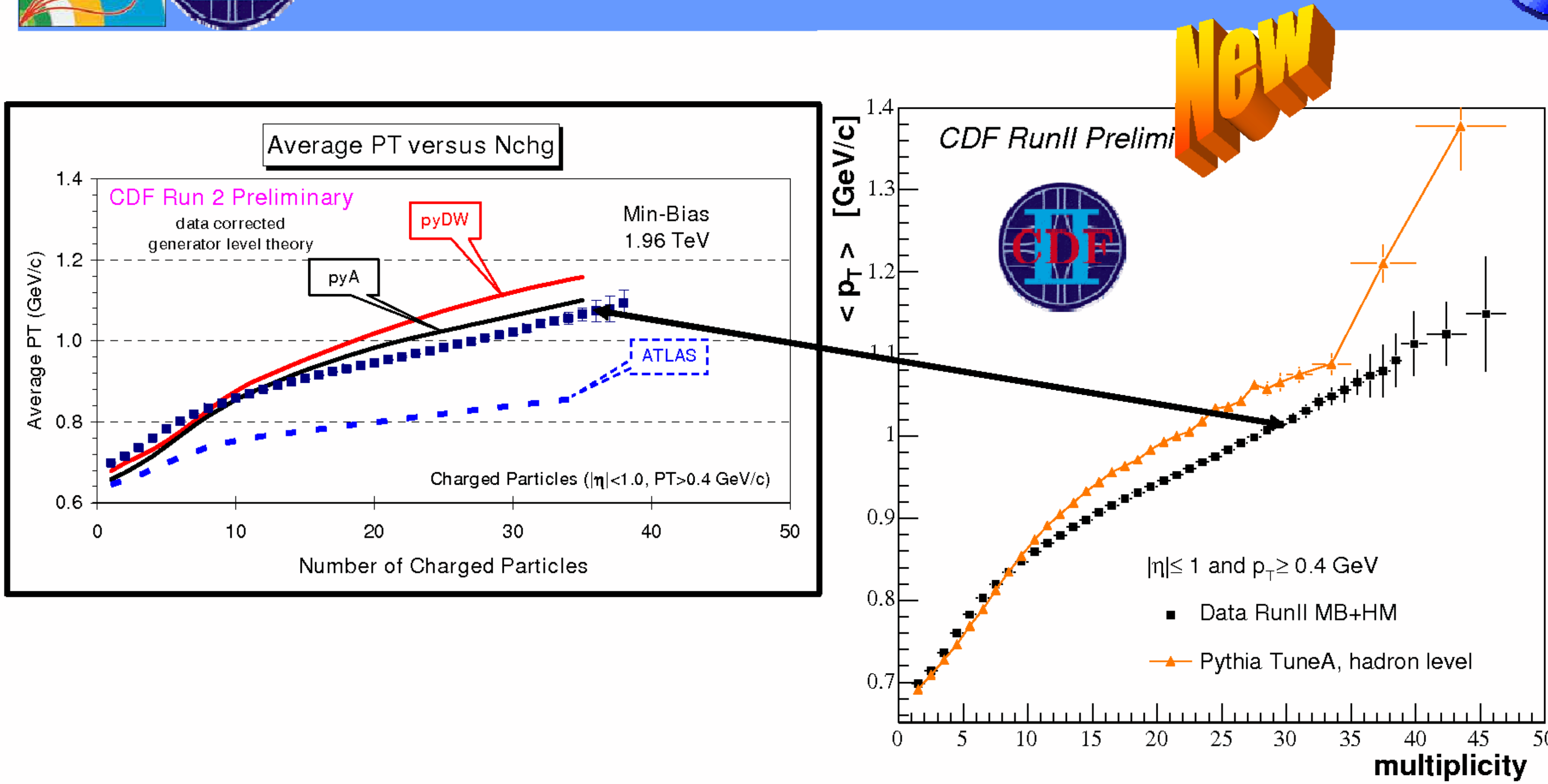


FIG. 27. Average transverse momentum of charged particles in $|\eta| < 2.5$ as a function of the multiplicity. UA1 data points (Ref. 49) at 900 GeV compared with the model for different assumptions about the nature of the subsequent (nonhardest) interactions. Dashed line, assuming $q\bar{q}$ scatterings only; dotted line, gg scatterings with "maximal" string length; solid line gg scatterings with "minimal" string length.

Min-Bias Correlations



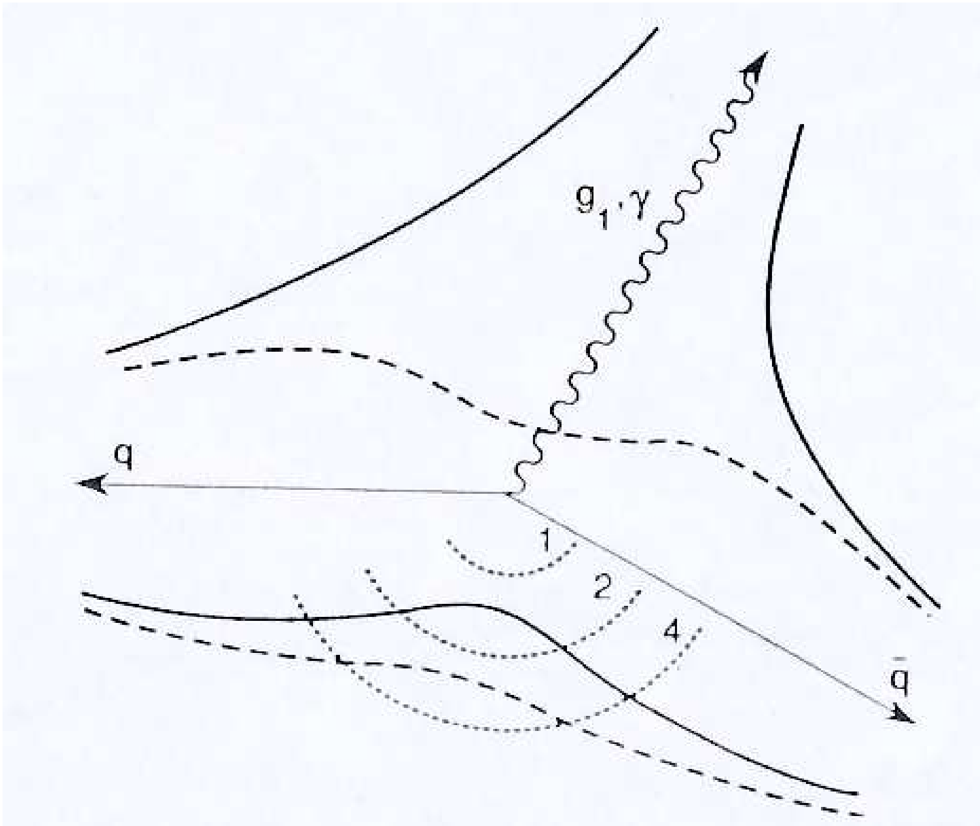
- **Data at 1.96 TeV on the average p_T of charged particles versus the number of charged particles (p_T > 0.4 GeV/c, |η| < 1) for "min-bias" collisions at CDF Run 2. The data are corrected to the particle level and are compared with PYTHIA Tune A at the particle level (*i.e.* generator level).**

The Dipole Picture

(dipole and antenna used interchangeably)

Lund picture “derived” in pQCD in terms of **dipole** radiation pattern:

around $q\bar{q}g$ and $q\bar{q}\gamma$



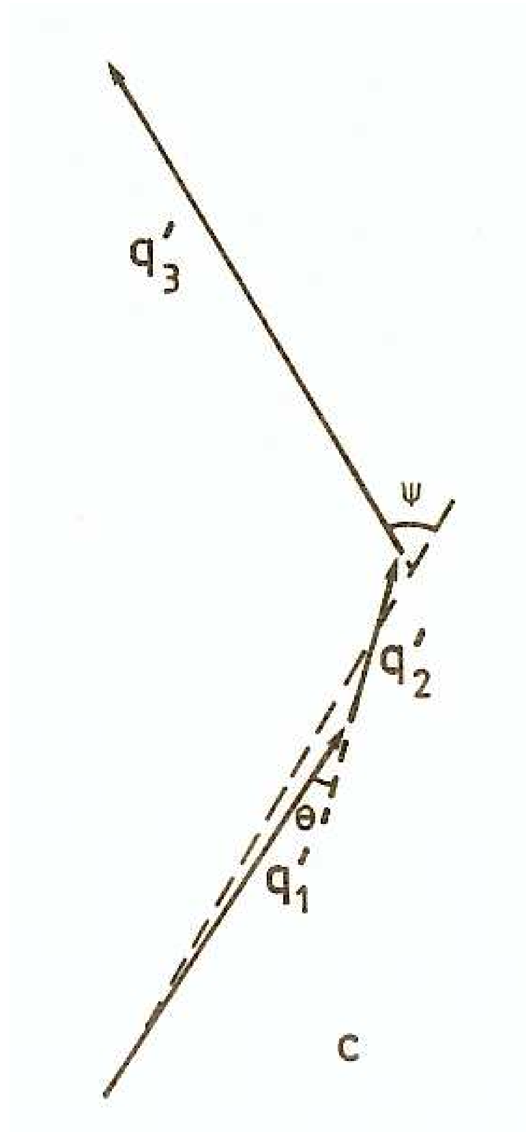
the “**Leningrad dipole**” (now St. Petersburg) (introduced 1985)

(Ya.I. Azimov, Yu.L. Dokshitzer, V.A. Khoze, S.I. Troyan)

G. Gustafson (1986):
A chain of dipoles offers
dual description to
a colour-ordered set of gluons.

Formulate a parton cascade
in terms of
dipole $\rightarrow$ **dipole** $+$ **dipole**
instead of g $\rightarrow$ g g.

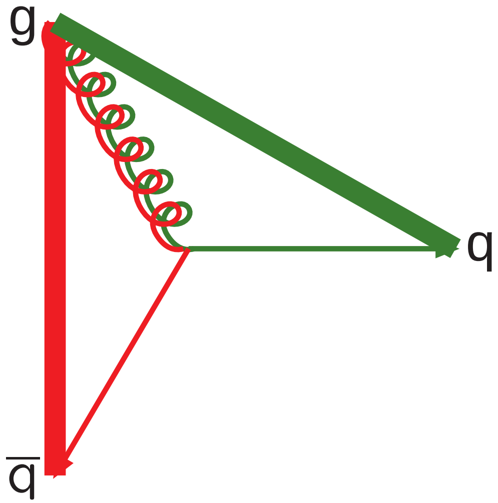
Transverse-momentum-ordered
dipole showering
properly takes into account coherence,
equivalently with angular ordering.
Partons always on shell.



natural match perturbative dipole shower
and nonperturbative string fragmentation

Shower Algorithms

Two main trends:
- use $p_\perp$ as evolution variable
- dipole kinematics = radiator + recoiler



Lund string
↓
Leningrad antenna
(Azimov, Dokshitzer, Khoze, Troyan)
↓
Lund dipole (Gustafson)

NLO ME's

ARIADNE (Lönnblad)

Catani–Seymour dipole

traditional showers

LDCMC

Nagy, Soper

PYTHIA 6.3,8.1

Krauss, Schumann (→ SHERPA)

Dinsdale, Ternick, Weinzierl (→ ?)

Winter, Krauss (→ SHERPA)

VINCIA (Giele, Kosower, Skands)

Outlook

- No (promising) new fragmentation frameworks in last 25 years
- String model best bet (?), but too many "materials constants"
  $\star$ will lattice QCD one day be able to help?
  $\star$ mass dependence goes part of the way (UCLA model)
- Cluster model also has evolved towards many parameters
  $\Rightarrow$ there is no few-parameter *good* description

Many unsolved issues, especially:
multiple interactions $\Rightarrow$ *dense-packing of strings* $\Rightarrow$ collective effects?

- Higher colour representations (colour ropes)
- Colour reconnections (= colour exchange between q's and g's)?
- Bose–Einstein correlations?
- Partial formation of Quark-Gluon Plasma (QGP)?
- Rescattering of hadrons $\Rightarrow$ strangeness content, collective flow?

LHC studies may provide hints in which direction to go
but data may also be too messy to provide straight answers.
Don't expect any quick fixes!