Hard probes for a strongly coupled plasma from AdS/CFT

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(lecture notes arXiv:0812.0500)

Outline

- Motivation :

  Hard probes for Heavy Ion Collisions at RHIC and LHC

  (see the talks by W. Zajc and A. Starinets for soft probes)

- Weak coupling: Partons and jets in perturbative QCD

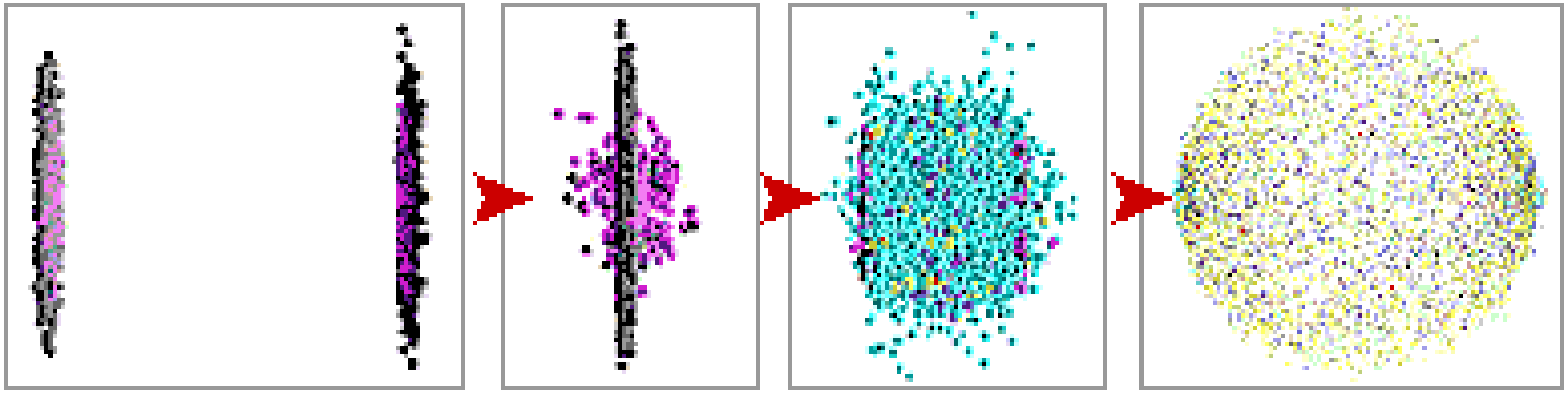
- Strong coupling: AdS/CFT Correspondence

- Finite–temperature plasma at strong coupling:
  - Deep inelastic scattering & Parton saturation
  - Energy loss & Momentum broadening

    (see also the talk by J. Casalderrey–Solana)

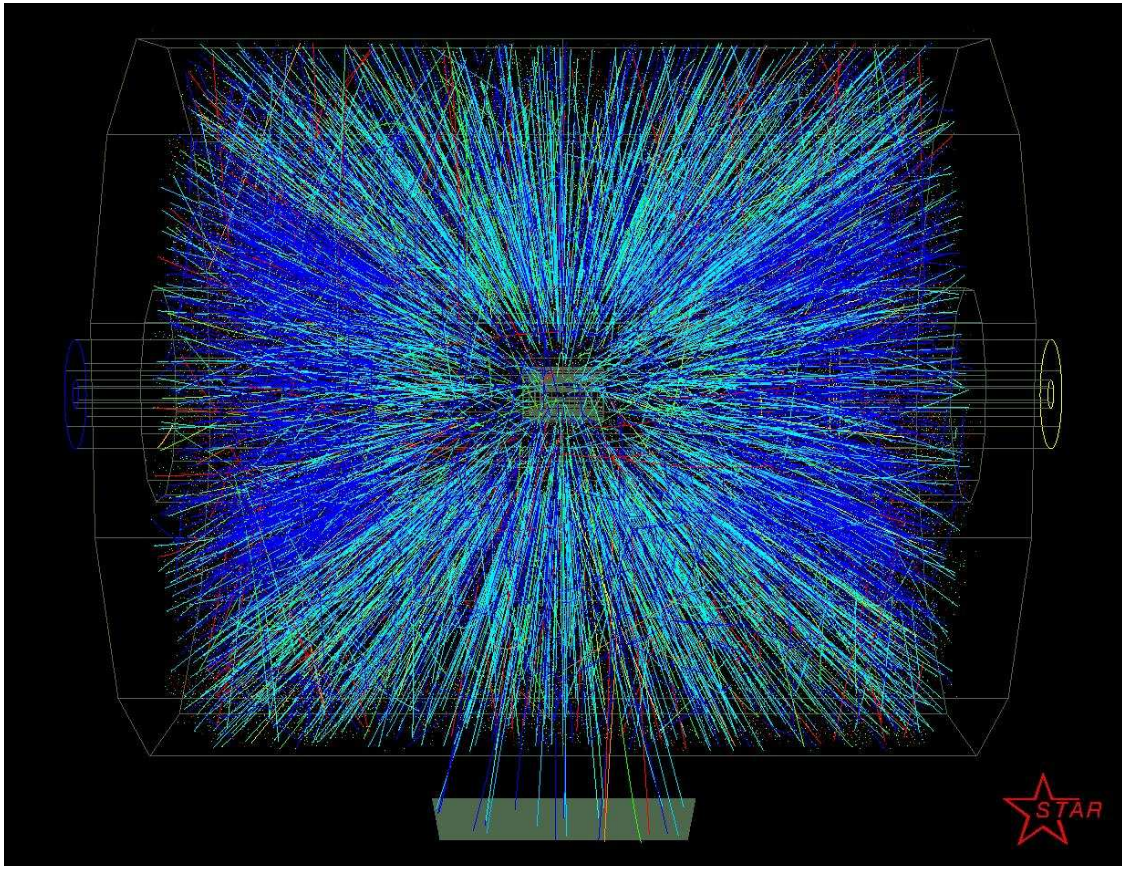
The Little Bang

- Ultrarelativistic heavy ion collisions @ RHIC and LHC



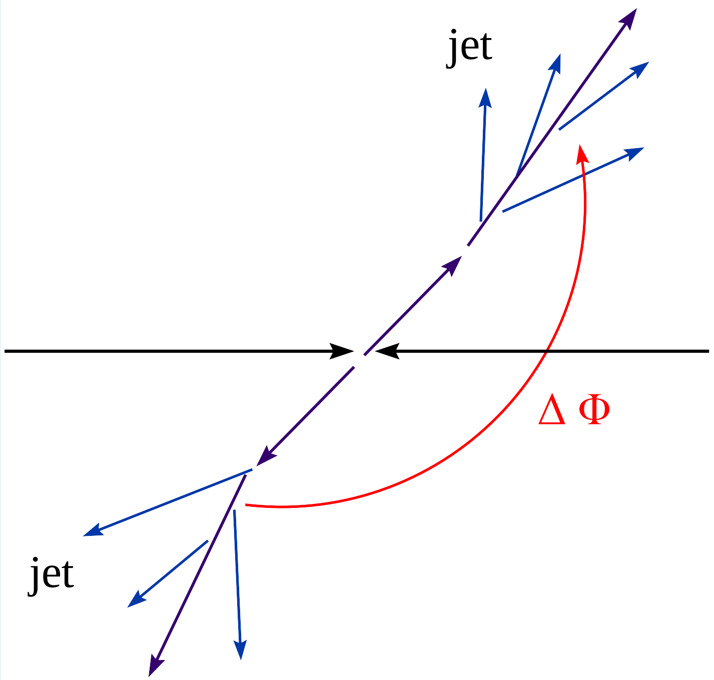
- Extremely complex phenomena
  - high density partonic systems in the initial wavefunctions
  - multiple interactions during the collisions
  - complicated, non-equilibrium, dynamics after the collision
  - expansion, thermalization, hadronisation
- Is there any place for strong–coupling dynamics ?

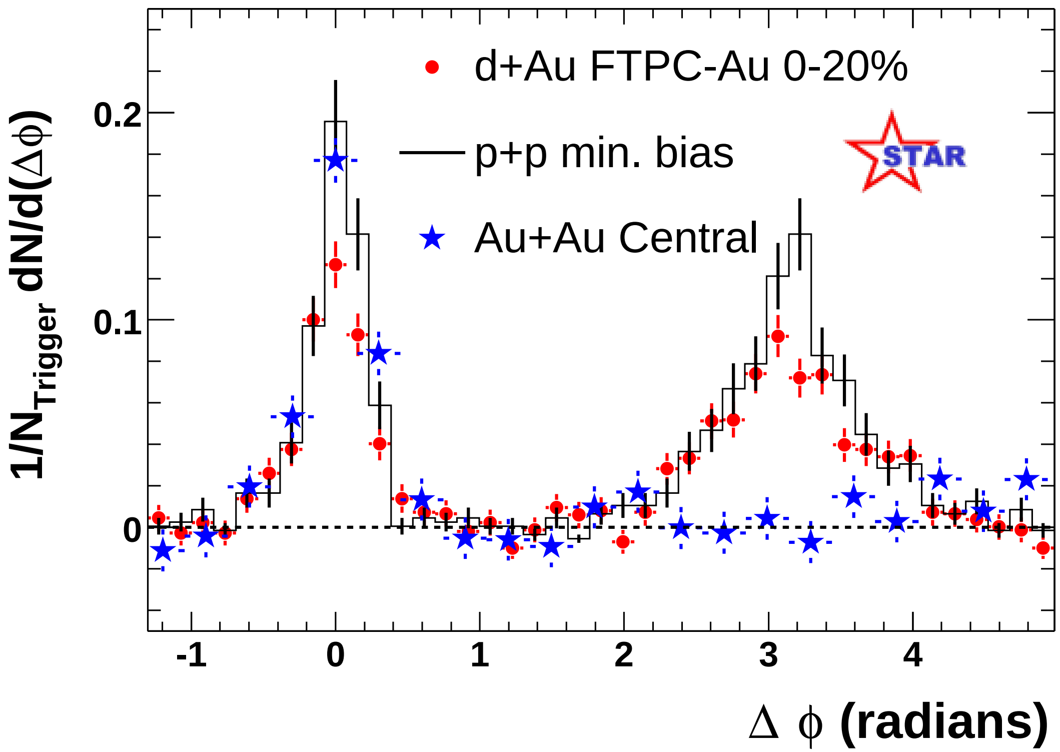
Hadron production at RHIC



- ~ 3000 hadrons in the final state vs. 400 nucleons in AA
- Most of them arise as hadronized partons
- Particle correlations are essential to disentangle phenomena

Jets in proton–proton collisions

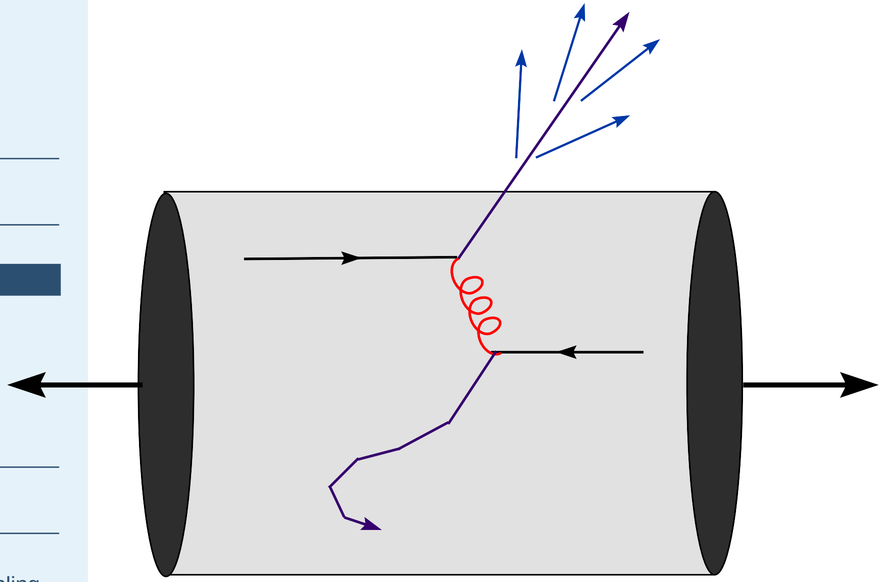


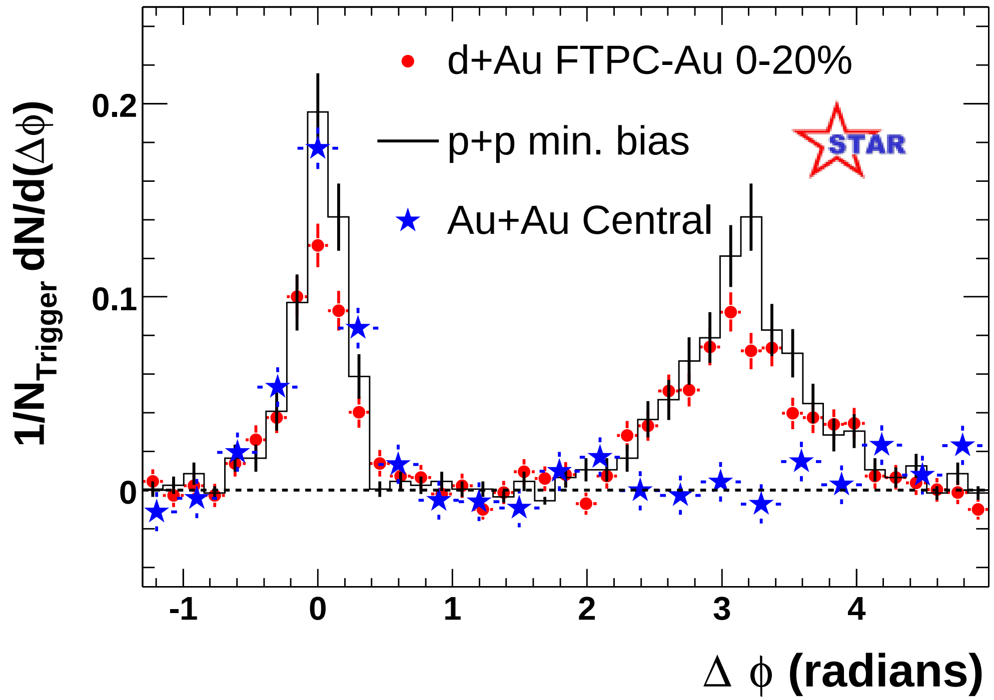


- Azimuthal correlations between the produced jets:

  a peak at $\Delta\Phi = 180°$

Nucleus–nucleus collision

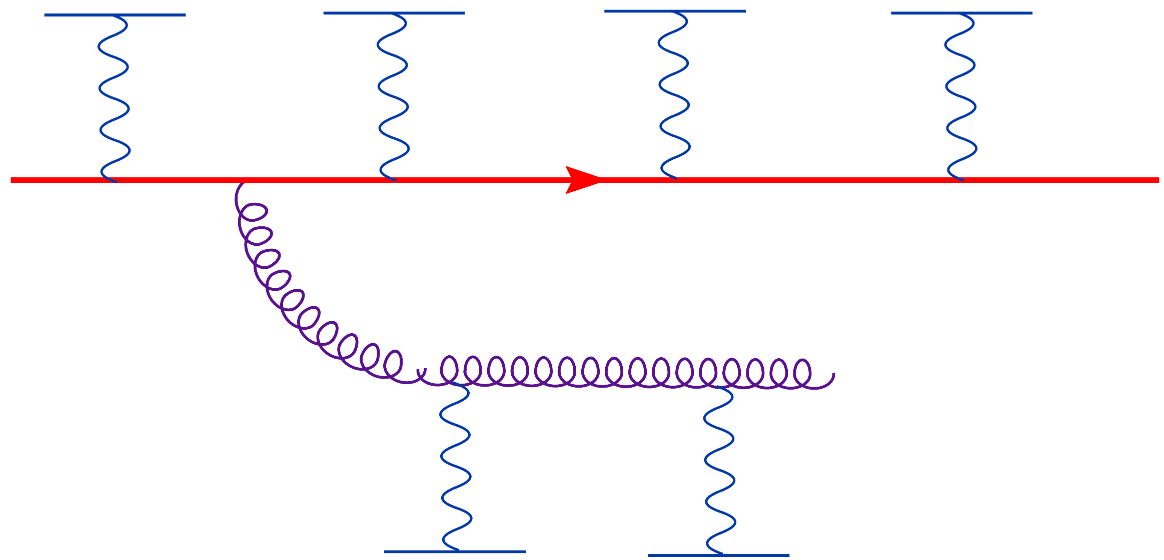




- The “away–side” jet has disappeared !
  absorbtion (or energy loss, or “jet quenching”) in the medium
- The matter produced in a heavy ion collision is opaque
  high density, strong interactions, ... or both

Energy loss at weak coupling

- Medium induced radiation

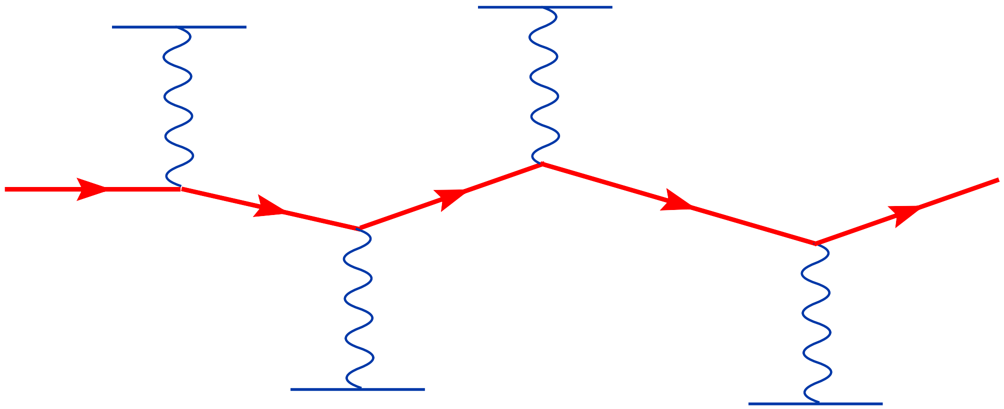


- A non–local process: gluon formation time $\Delta t \sim \omega/Q^2$

$$-\frac{\mathrm{d}E}{\mathrm{d}t} \simeq \alpha_s N_c \langle p_\perp^2 \rangle$$: relation to 'momentum broadening'

Transverse momentum broadening

- Scattering off the plasma constituents



$$\frac{\mathrm{d}\langle p_\perp^2 \rangle}{\mathrm{d}t} \equiv \hat{q} \simeq \alpha_s N_c \, xg(x, Q^2)$$

- $xg(x, Q^2)$: gluon distribution per unit volume in the medium

$$xg(x,Q^2) \simeq n_q(T)\, xG_q + n_g(T)\, xG_g \quad \text{with} \quad n_{q,g}(T) \propto T^3$$

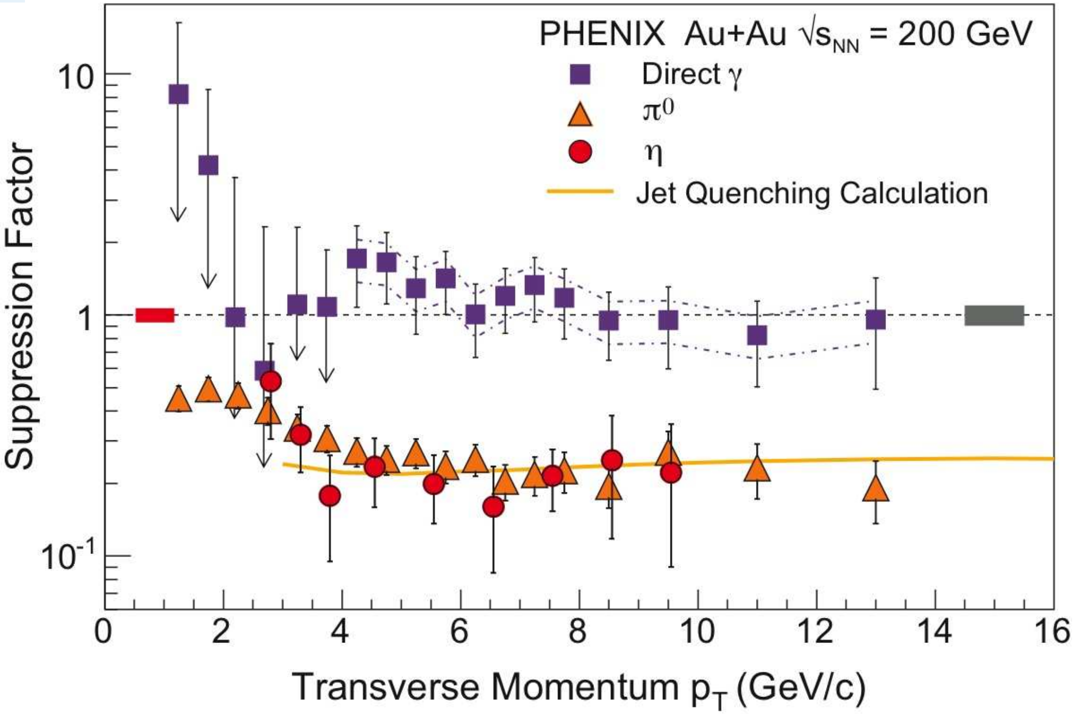
This requires parton evolution from T up to $Q \gg T$

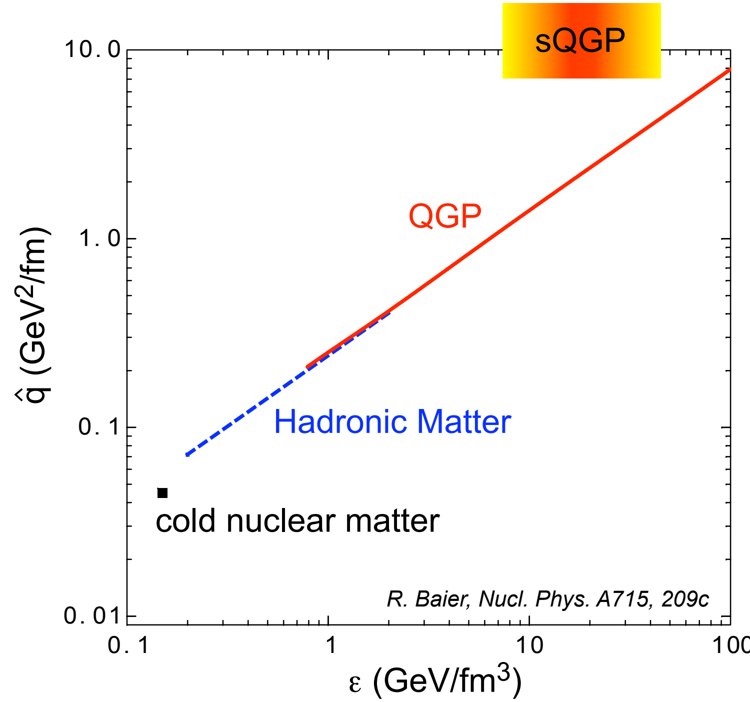
- "jet quenching parameter" $\hat{q}$: a local transport coefficient

Nuclear modification factor

- How to measure $\hat{q}$? Compare AA collisions at RHIC to pp

$$R_{AA}(p_\perp) \equiv \frac{Yield(A+A)}{Yield(p+p)\ \times\ A^2}$$

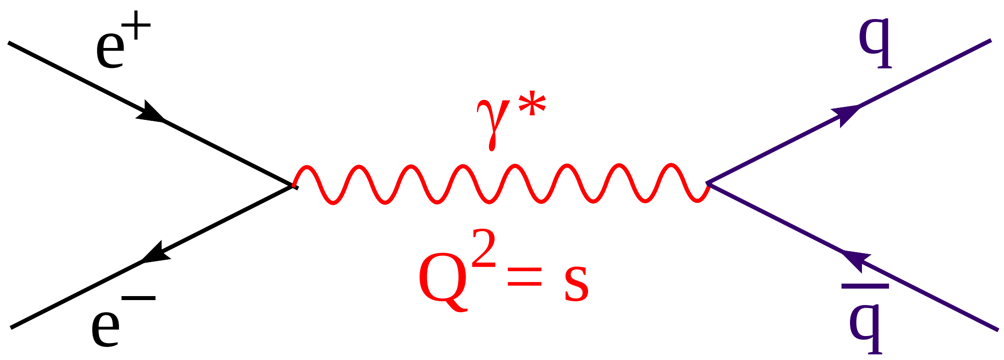




- RHIC data prefer a rather large value $\hat{q} \simeq 10$ GeV^2/fm, which seems (marginally) inconsistent with weak coupling

e^+e^- annihilation: Jets in pQCD

- How would a high–energy jet interact in a strongly coupled plasma ?
- How to produce jets in the first place ?
- Guidance from perturbative QCD: $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}$



- Decay of a time–like photon: $Q^2 \equiv q^\mu q_\mu = s > 0$

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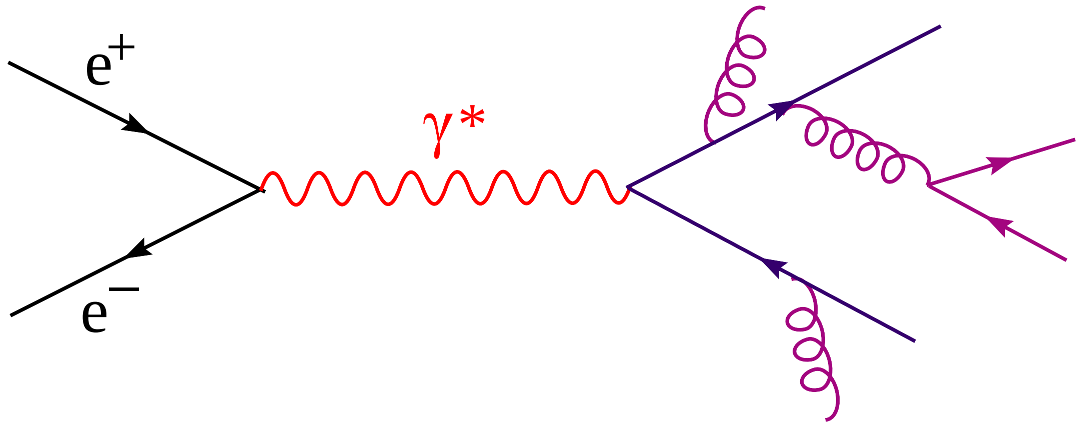
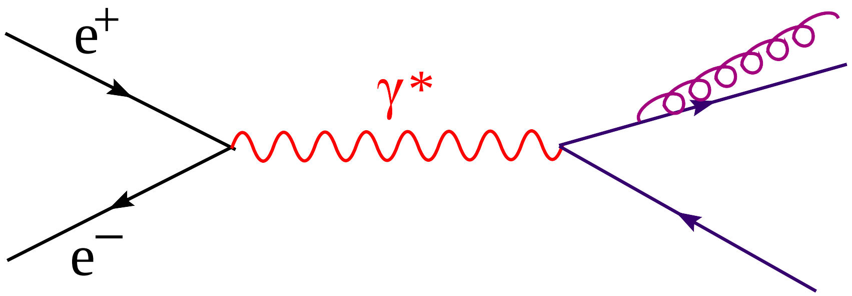
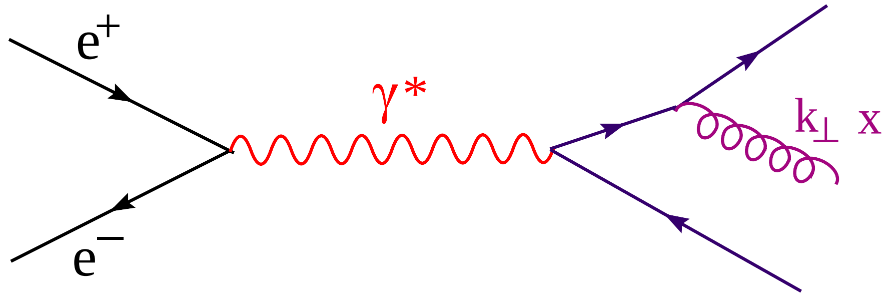


- The structure of the final state is determined by
  - parton branching & hadronisation

Parton branching at weak coupling



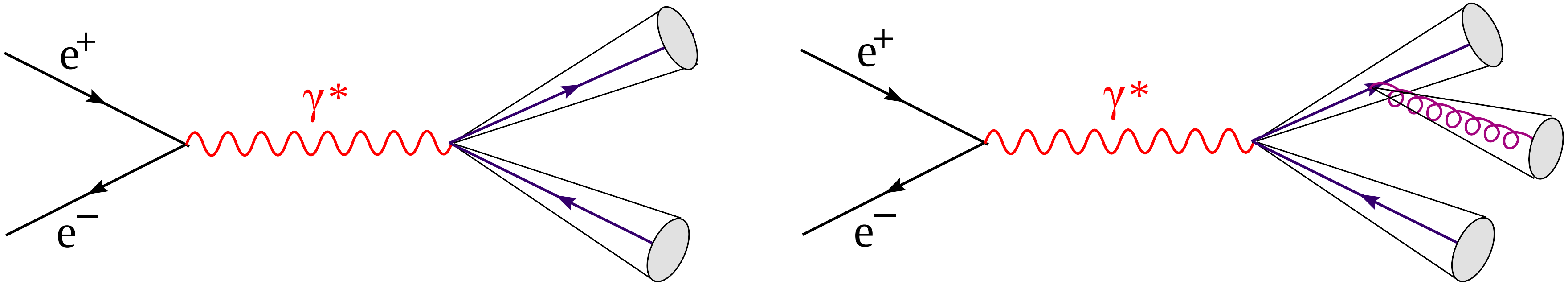


- Gluon 'formation time' : $\Delta t \sim k_\ell / k_\perp^2$
- Early partons are hard ($k_\perp \gg \Lambda_{\rm QCD}$) and hence perturbative
- Bremsstrahlung favors the emission of soft ($x \ll 1$) and collinear ($k_\perp^2 \ll s$) gluons ($k_\ell = x p_\ell$)

$$\mathrm{d}\mathcal{P}_{\rm Brem} \sim \alpha_s(k_\perp^2)\, N_c \,\frac{\mathrm{d}^2 k_\perp}{k_\perp^2}\,\frac{\mathrm{d}x}{x}$$

- Relatively simple final state !

Jets in perturbative QCD



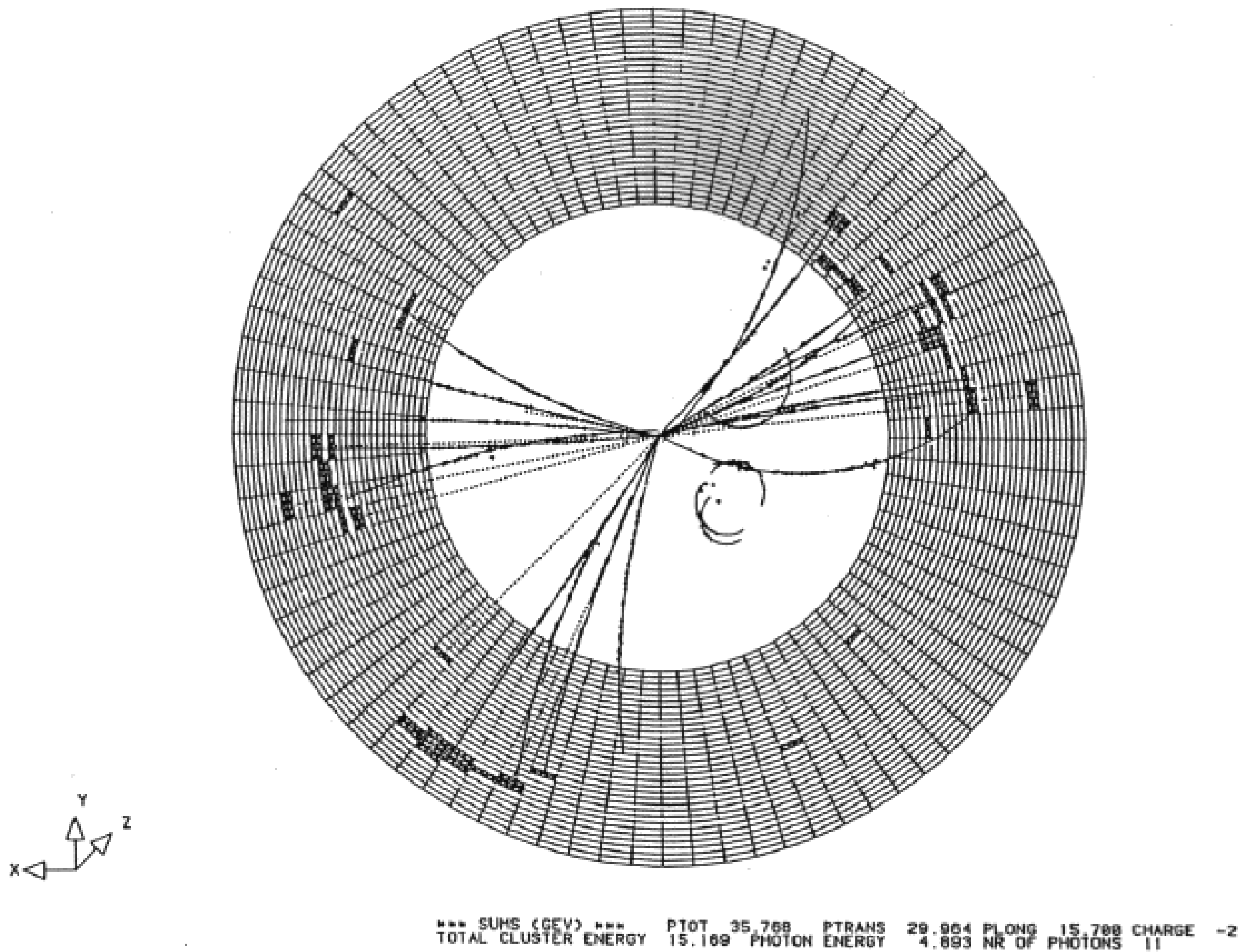
- Few, well collimated, jets
- e^+e^- cross–section computable in perturbation theory

$$\sigma(s) = \sigma_{\rm QED} \times \Big(3\sum_f e_f^2\Big)\left(1 + \frac{\alpha_s(s)}{\pi} + \mathcal{O}(\alpha_s^2(s))\right)$$

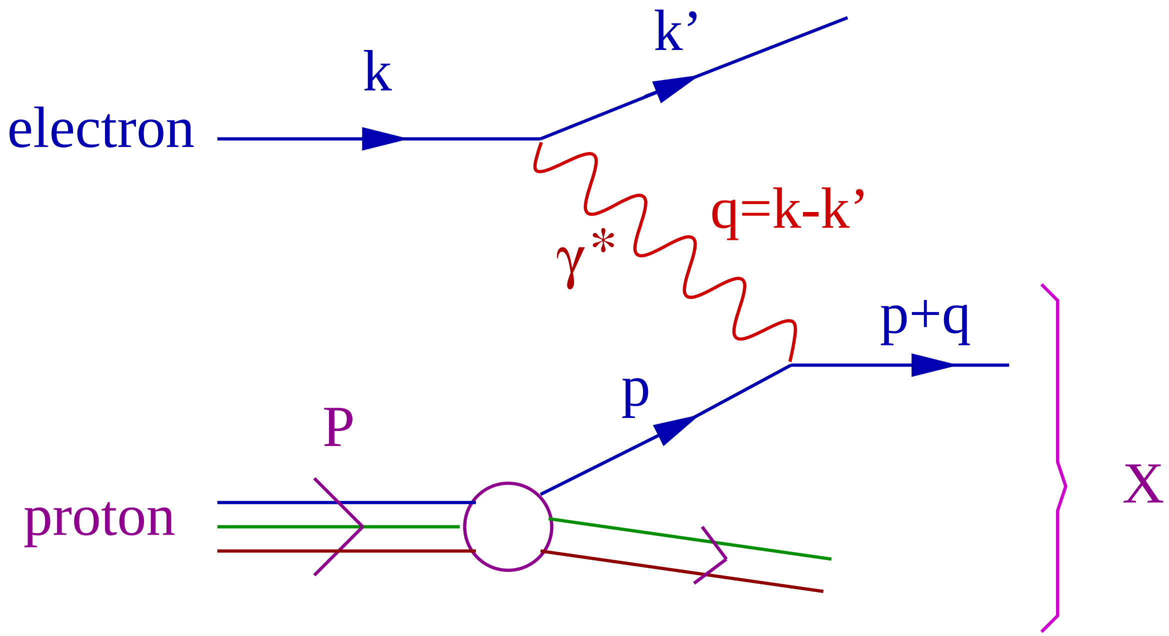
$\sigma_{\rm QED}$: cross–section for $e^+e^- \to \mu^+\mu^-$

- Multi–jet ($n \geq 3$) events appear, but are comparatively rare

3–jet event at OPAL (CERN)

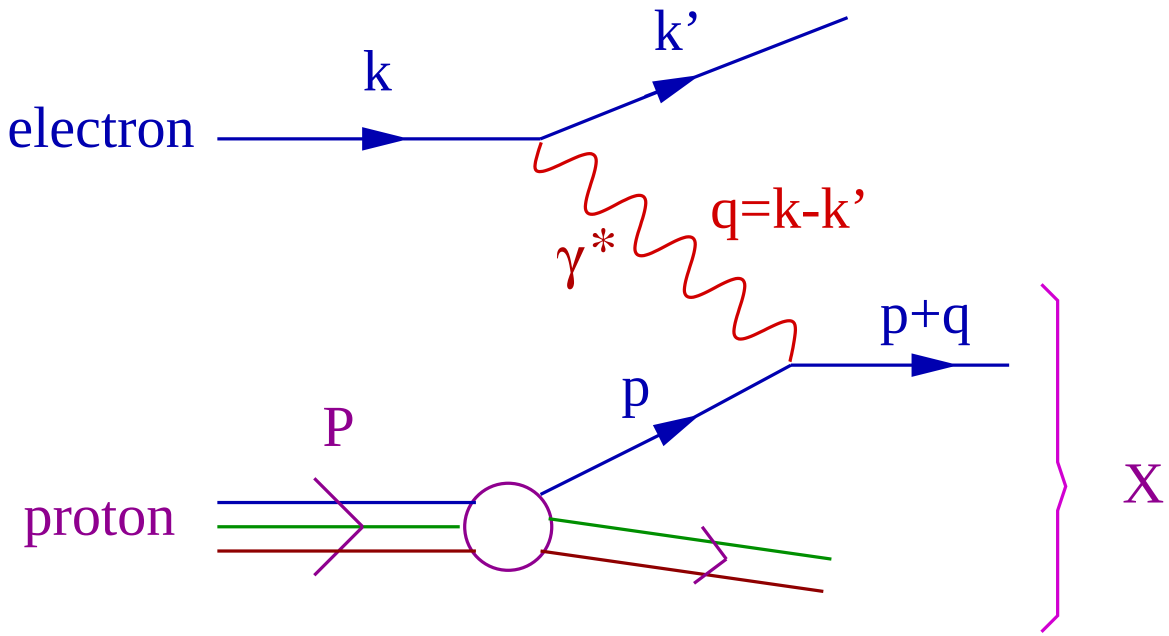


Deep inelastic scattering



- Space–like current: $Q^2 \equiv -q^\mu q_\mu \geq 0$ and $x \equiv \frac{Q^2}{2P\cdot q}$
- Physical picture: γ^* absorbed by a quark excitation with
  - transverse size $\Delta x_\perp \sim 1/Q$
  - and longitudinal momentum $p_z = xP$

The proton structure function

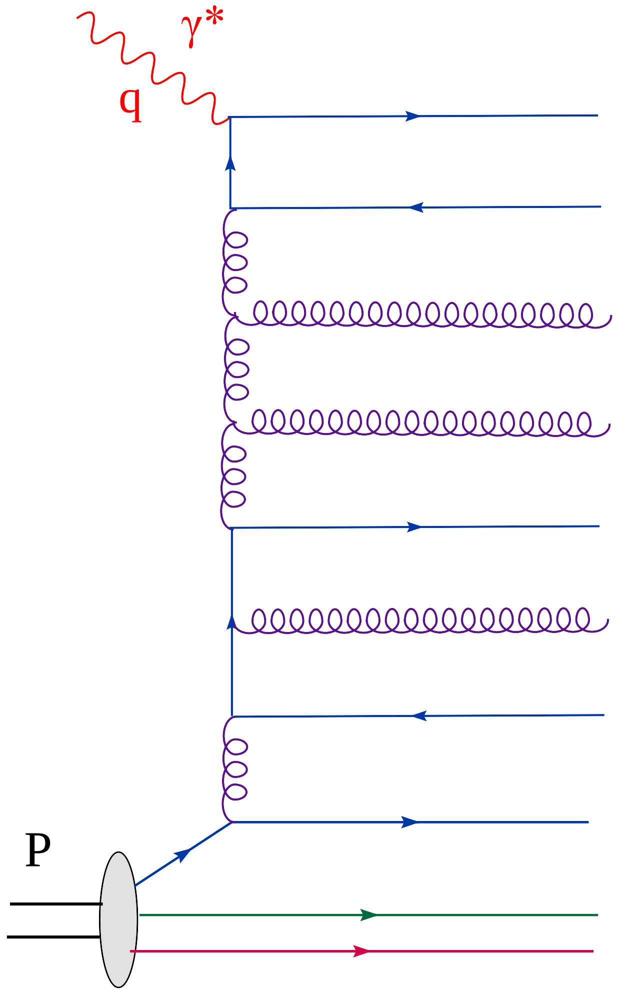


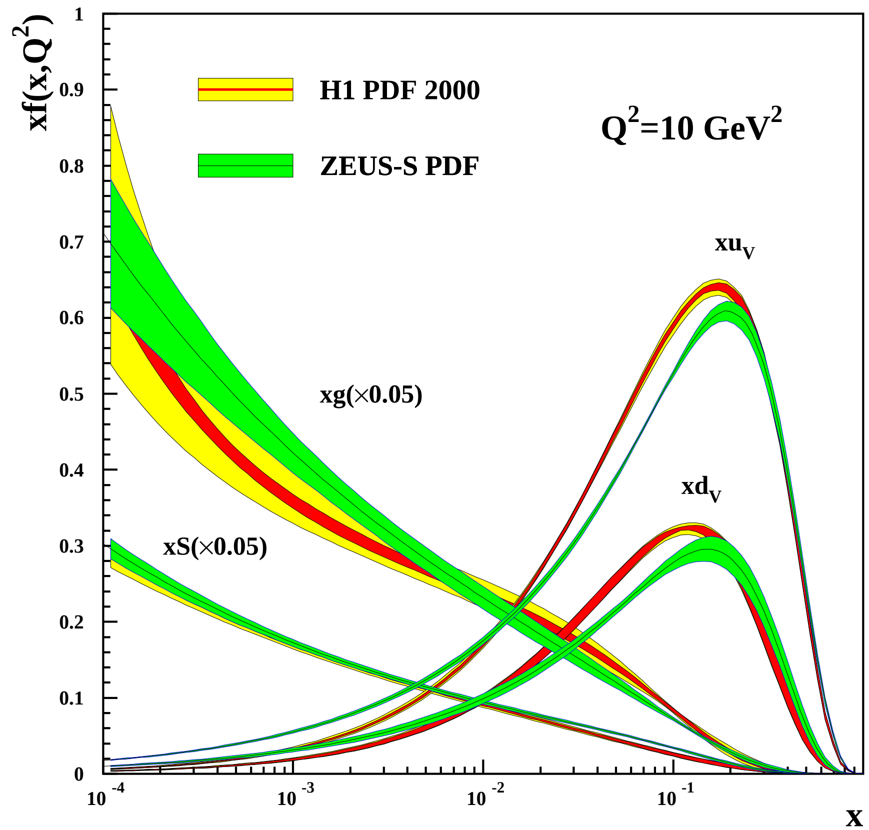
$$\sigma_{\gamma^* p}(x, Q^2) = \frac{4\pi^2 \alpha_{\rm em}}{Q^2} F_2(x, Q^2)$$

- $F_2(x, Q^2)$: 'quark distribution' = number of quarks with longitudinal momentum fraction x and transverse area $1/Q^2$

Parton evolution in pQCD

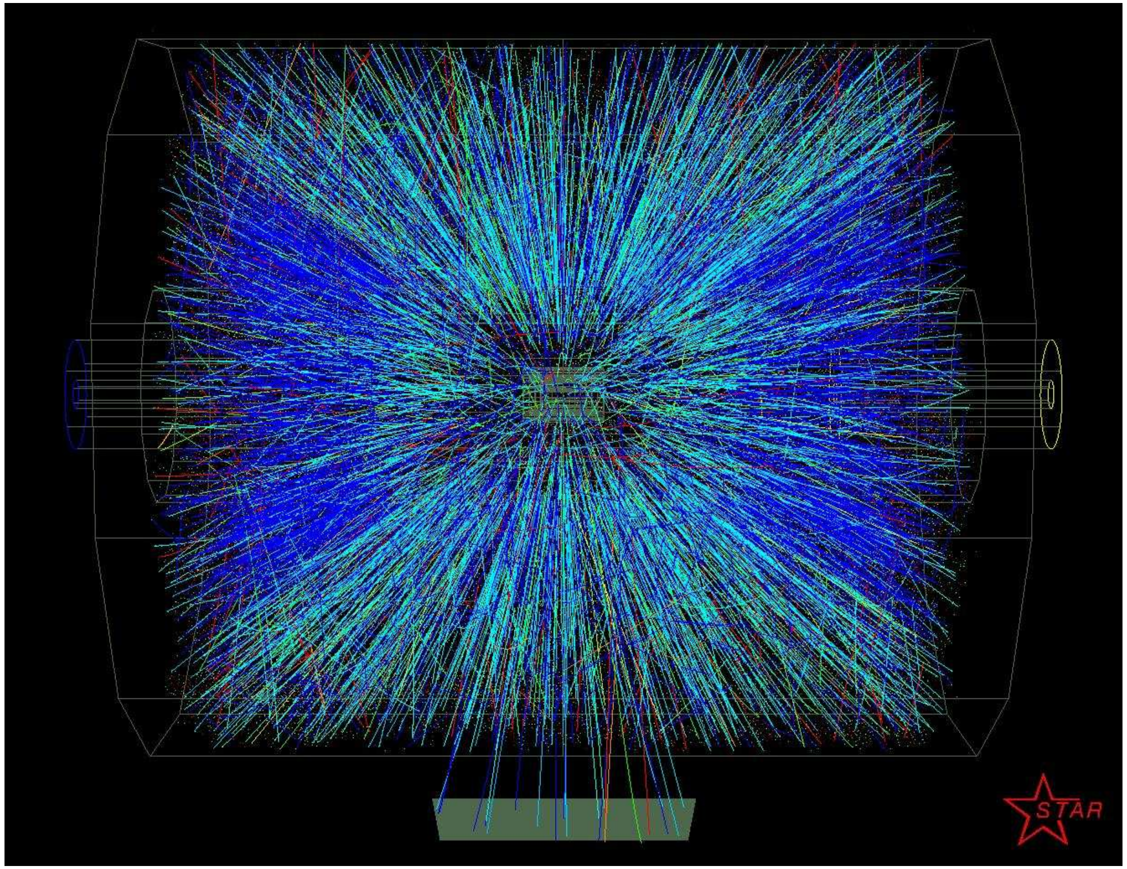
- Gluons are implicitly seen in DIS, via parton evolution





- Bremsstrahlung favors the emission of gluons with $x \ll 1$

Partons at RHIC

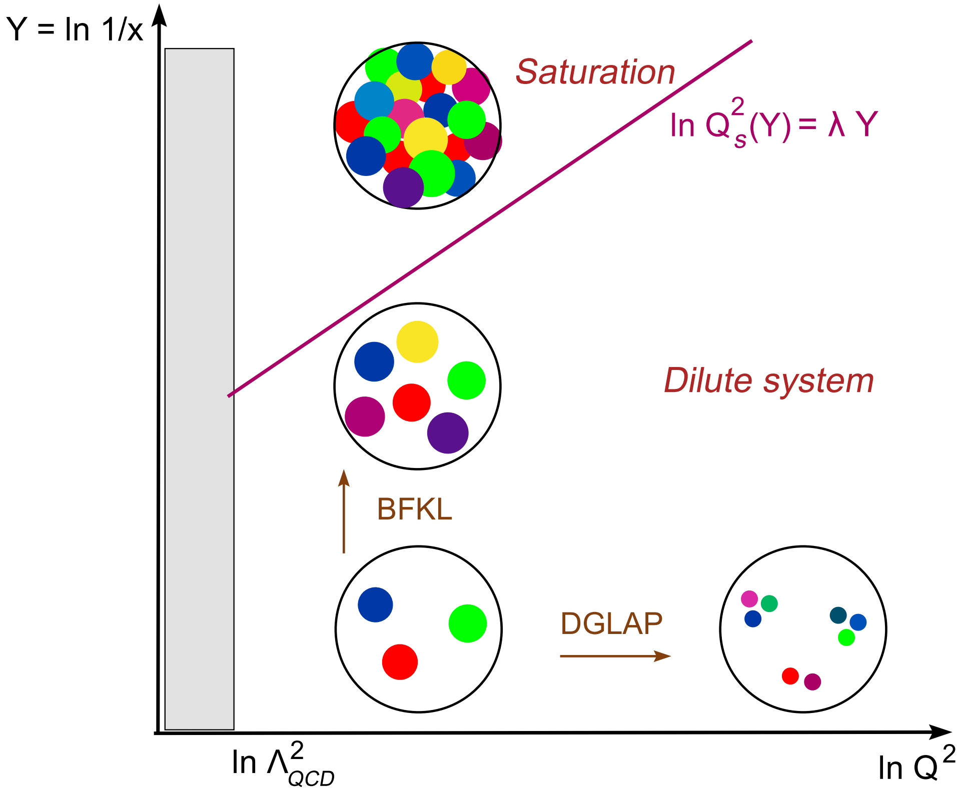


- Partons are actually 'seen' (liberated) in the high energy hadron–hadron collisions
  - central rapidity: small–x partons
  - forward/backward rapidities: large–x partons

Gluon Saturation: CGC

- When occupation number $\sim 1/\alpha_s \Longrightarrow$ strong repulsion

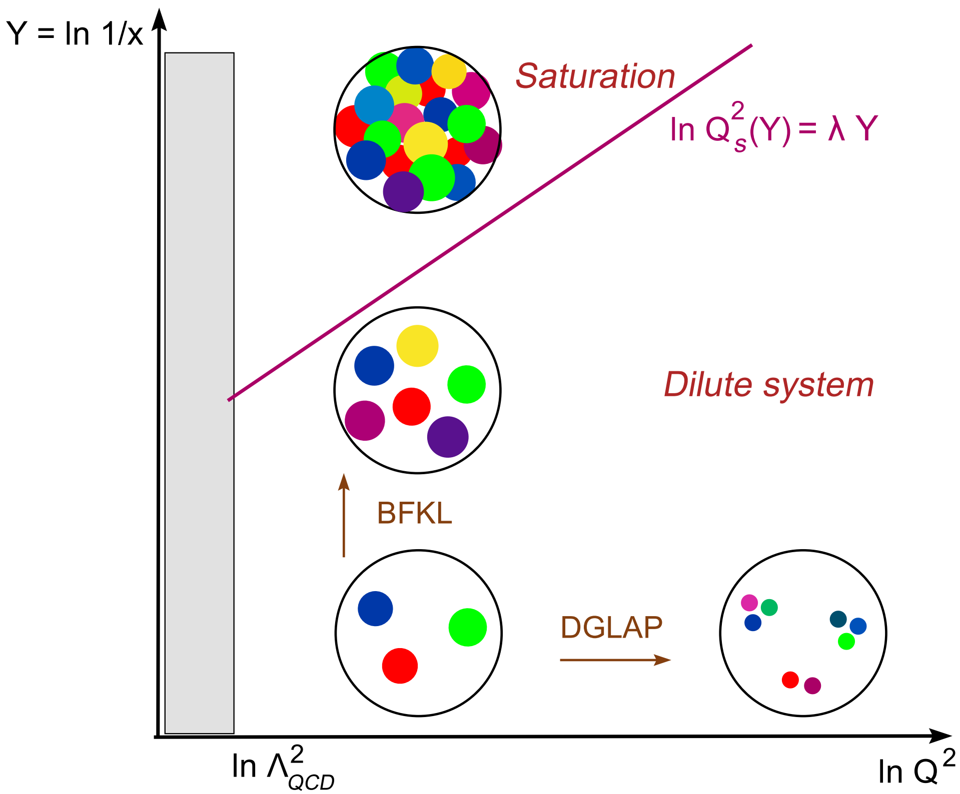
$$n(x,Q^2) = \frac{\pi}{Q^2} \times \frac{xG(x,Q^2)}{\pi R^2} \sim \frac{1}{\alpha_s} \quad \text{when} \quad Q^2 \simeq Q_s^2(x)$$



Gluon Saturation: CGC

- The saturation momentum

$$Q_s^2(x) \simeq \alpha_s \frac{xG(x, Q_s^2)}{\pi R^2} \sim \frac{1}{x^{\lambda_s}} \quad \text{with} \quad \lambda_s \sim 0.3$$



- For $Q^2 < Q_s^2(x)$, the gluon occupation numbers saturate

Hard probes in a strongly–coupled plasma

- Virtual photon : electromagnetic current J_μ
- Thermal expectation value (retarded polarization tensor) :

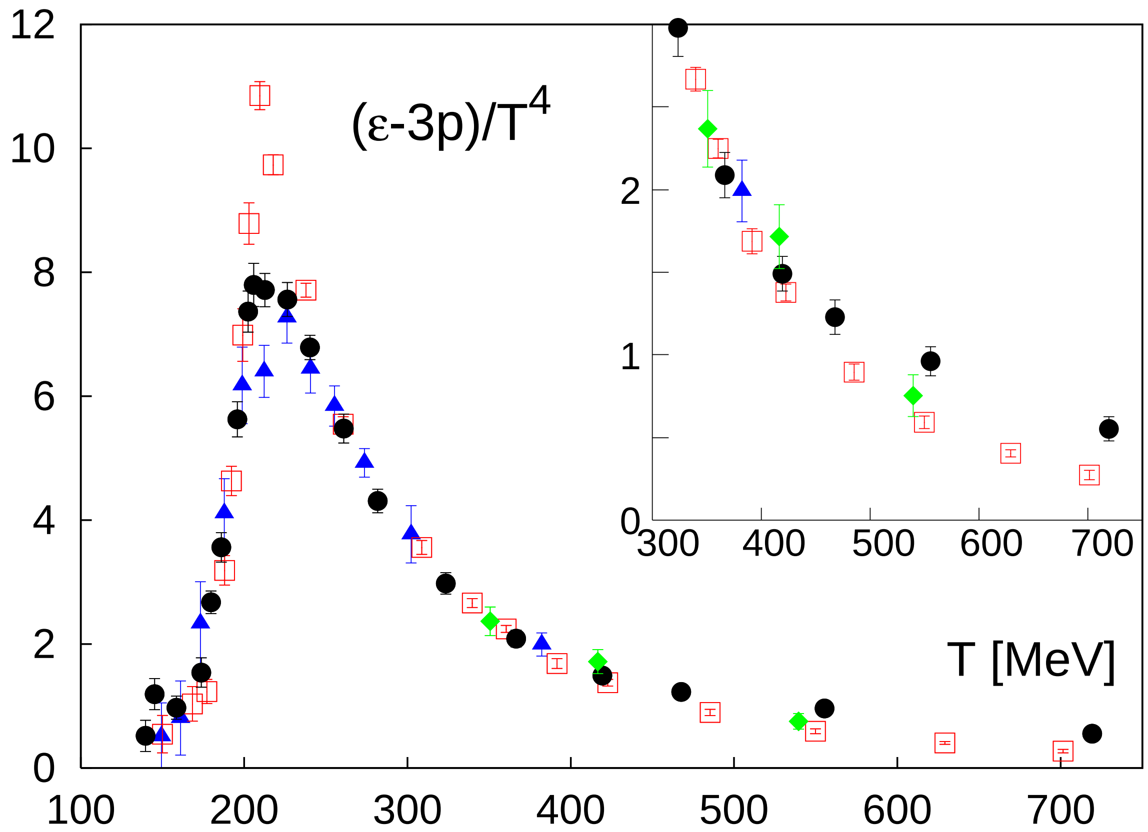
$$\Pi_{\mu\nu}(q) \equiv \int \mathrm{d}^4x \, \mathrm{e}^{-iq\cdot x} \, i\theta(x_0) \, \langle \, [J_\mu(x), J_\nu(0)] \, \rangle_T$$

- ‘Hard probe’ : large virtuality $Q^2 \equiv |q^2| \gg T^2$
  - time–like current ($q^2 > 0$) : jets
  - space–like current ($q^2 < 0$) : DIS, partons
- Relativistic heavy quark : $M \gg T$ and $v \simeq 1$
  - energy loss
  - transverse momentum broadening
- Strong coupling $\Longrightarrow$ AdS/CFT correspondence

Gauge theory side: CFT

- $\mathcal{N} = 4$ Supersymmetric Yang–Mills theory
  - color gauge group SU(N_c)
  - supersymmetry (fermions $\leftrightarrows$ bosons)
  - gluons, fermions, scalars (all in the adjoint repres. !)
  - quantum conformal invariance (fixed coupling)
  - no confinement, no intrinsic scale
- Has this any relevance to QCD ??
- Perhaps better suited for QCD at finite temperature
  - deconfined phase (quark–gluon plasma)
  - quarks and gluons play rather similar roles
  - nearly conformal (small running–coupling effects)

Trace anomaly from lattice QCD



$$\beta(g)\,\frac{\mathrm{d}p}{\mathrm{d}g} = \langle T^{\mu}_{\mu}\rangle = \mathcal{E} - 3p$$

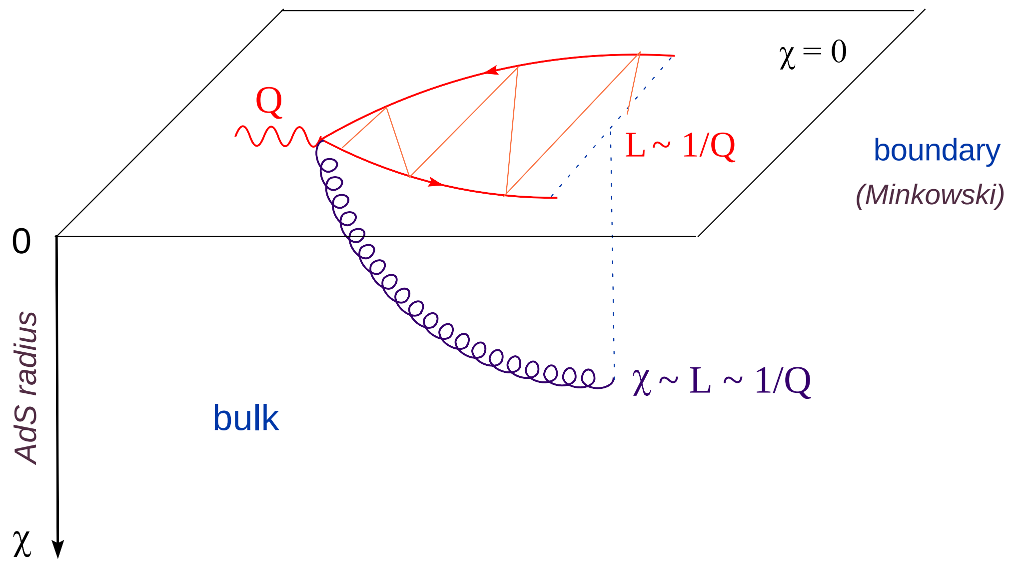
- $(\mathcal{E}-3p)/\mathcal{E}_0 \lesssim 10\%$ for any $T \gtrsim 2T_c \simeq 400$ MeV

String theory side: AdS

- Type IIB string theory living in $D = 10$: $AdS_5 \times S^5$

$$\mathrm{d}s^2 = \underbrace{\frac{R^2}{\chi^2}\underbrace{(-\mathrm{d}t^2 + \mathrm{d}\vec{x}^2)}_{\text{Minkowski}} + \frac{R^2}{\chi^2}\mathrm{d}\chi^2}_{AdS_5} + \underbrace{R^2\mathrm{d}\Omega_5^2}_{S^5}$$

- $0 \leq \chi < \infty$: ‘radial’, or ‘5th’, coordinate
- gauge theory lives at the Minkowski boundary $\chi = 0$



The Gauge/Gravity duality *(Maldacena, 1997)*

- Gauge theory has two parameters:
  - coupling constant g (elementary charge)
  - number of colors N_c
  - weakly or strongly coupled depending upon $\lambda \equiv g^2 N_c$
- String theory has three parameters:
  - curvature radius of space R
  - string coupling constant g_s
  - string length l_s (typical size of string vibrations)
- Mapping of the parameters :

$$4\pi g_s = g^2 , \qquad (R/l_s)^4 = g^2 N_c$$

- Strong 't Hooft coupling (more properly, $N_c \to \infty$) :

$$\lambda \equiv g^2 N_c \gg 1 \text{ with } g^2 \ll 1 \implies \text{classical (super)gravity}$$

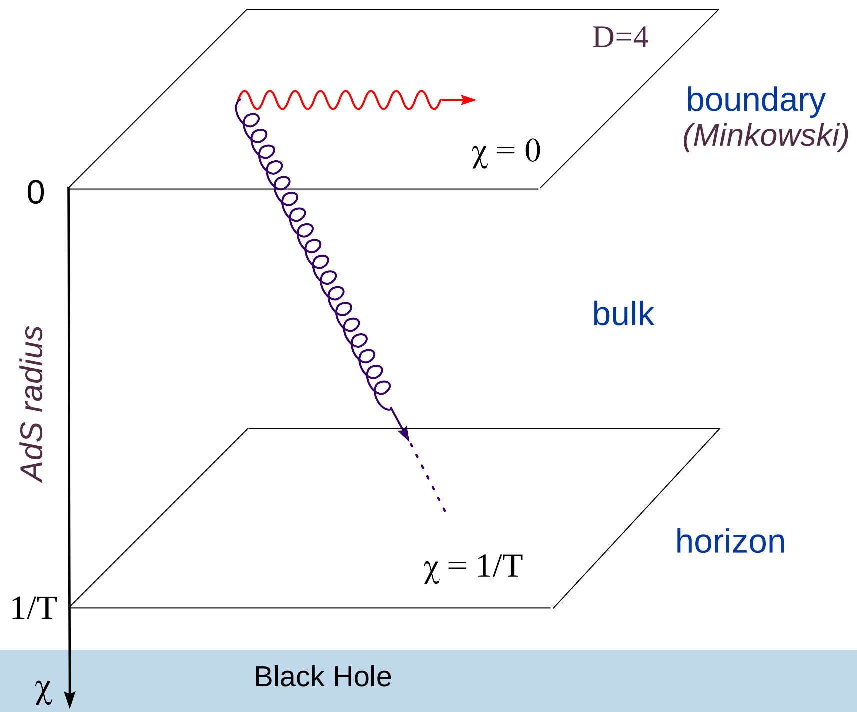
Heating AdS_5

- $\mathcal{N}=4$ SYM at finite temperature $\Longleftrightarrow$ Black Hole in AdS_5

$$\mathrm{d}s^2 = \frac{R^2}{\chi^2}\left(-f(\chi)\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2\right) + \frac{R^2}{\chi^2 f(\chi)}\mathrm{d}\chi^2 + R^2\mathrm{d}\Omega_5^2$$

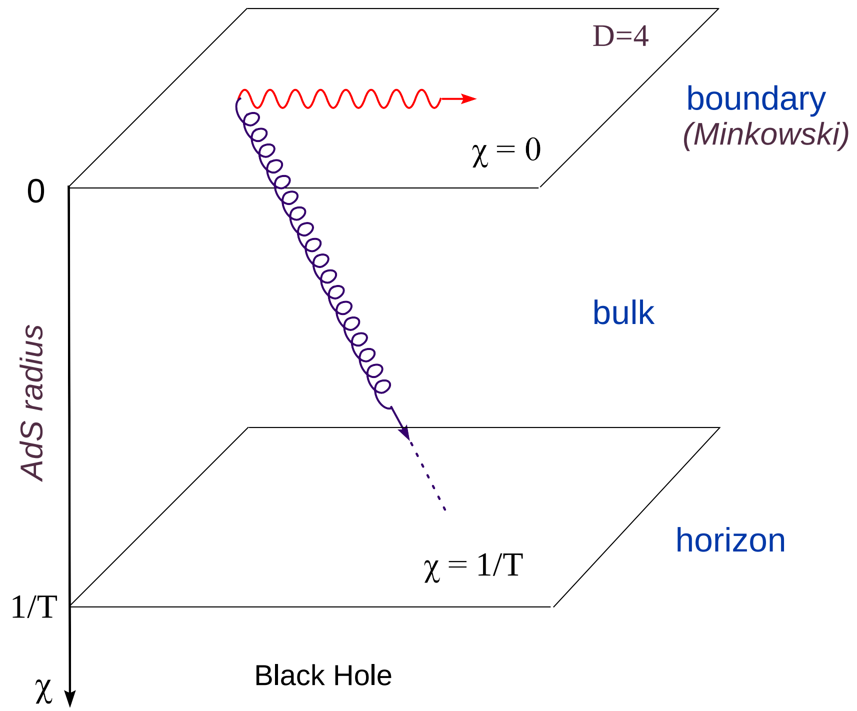
where $f(\chi) = 1-(\chi/\chi_0)^4$ and $\chi_0 = 1/\pi T$ = BH horizon

- A black hole has entropy and thermal (Hawking) radiation



DIS off the Black Hole *(Hatta, E.I., Mueller, 07)*

- Abelian current J_μ in 4D $\longleftrightarrow$ Maxwell wave A_μ in AdS_5 BH
- Im $\Pi_{\mu\nu}$ $\longleftrightarrow$ absorption of the wave by the BH



- Maxwell equations in a curved space–time

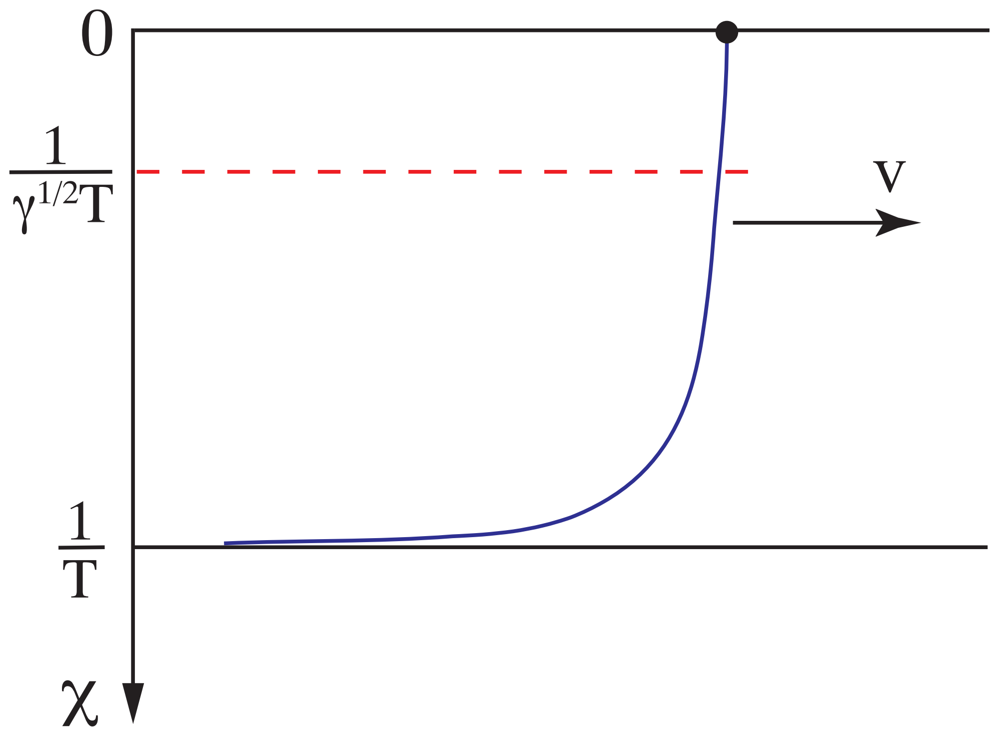
$$\partial_m\left(\sqrt{-g}g^{mn}g^{pq}F_{nq}\right) = 0 \qquad \text{where} \qquad F_{mn} = \partial_m A_n - \partial_n A_m$$

Relativistic heavy quark

- Heavy quark in 4D $\longleftrightarrow$ a Nambu–Goto string in AdS_5 BH

  Herzog, Karch, Kovtun, Kozcaz, and Yaffe; Gubser, 2006 ("trailing string")

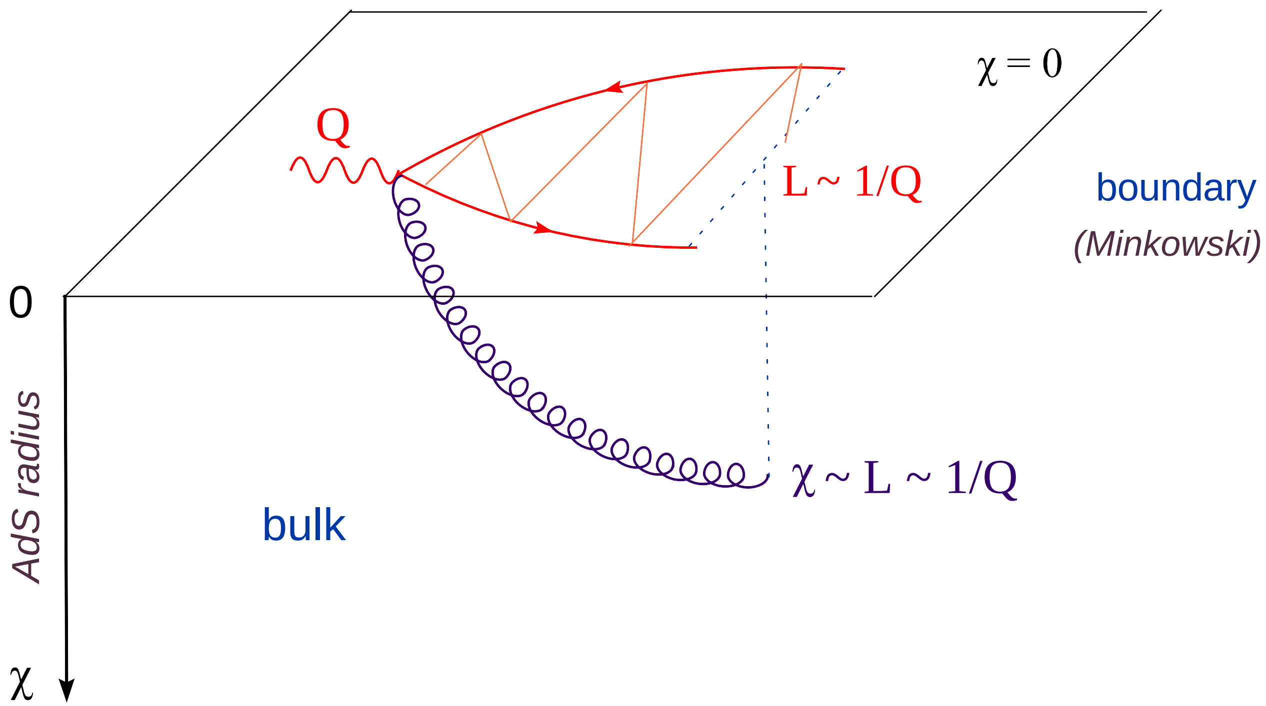
- Energy loss $\longleftrightarrow$ energy flux down the string



- Nambu–Goto equations in a curved space–time

Physical interpretation

- Rôle of the 5th dimension: a reservoir of quantum flucts.
- Radial penetration χ of the wave packet in AdS_5 $\longleftrightarrow$ transverse size L of the partonic fluctuation on the boundary



- Space–like photon with virtuality Q: The Maxwell wave penetrates up to a radial distance $\chi \sim 1/Q$

Physical interpretation

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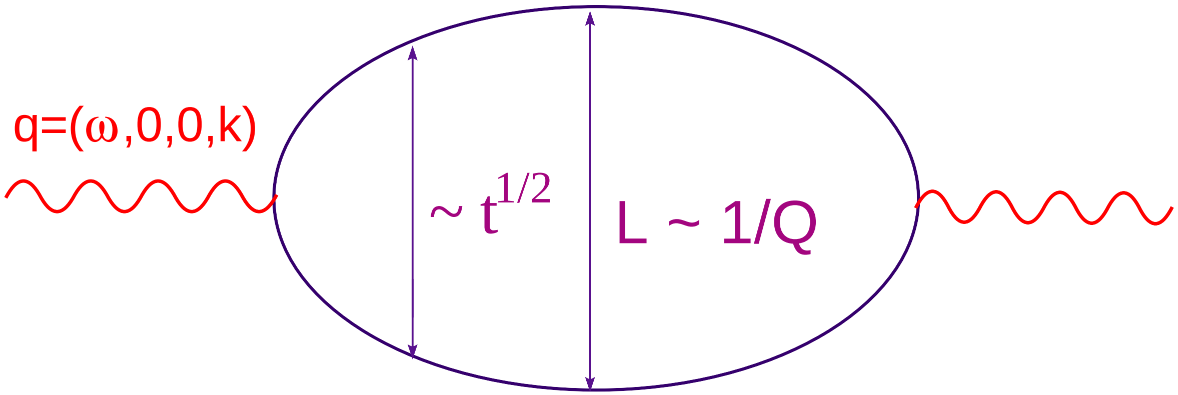
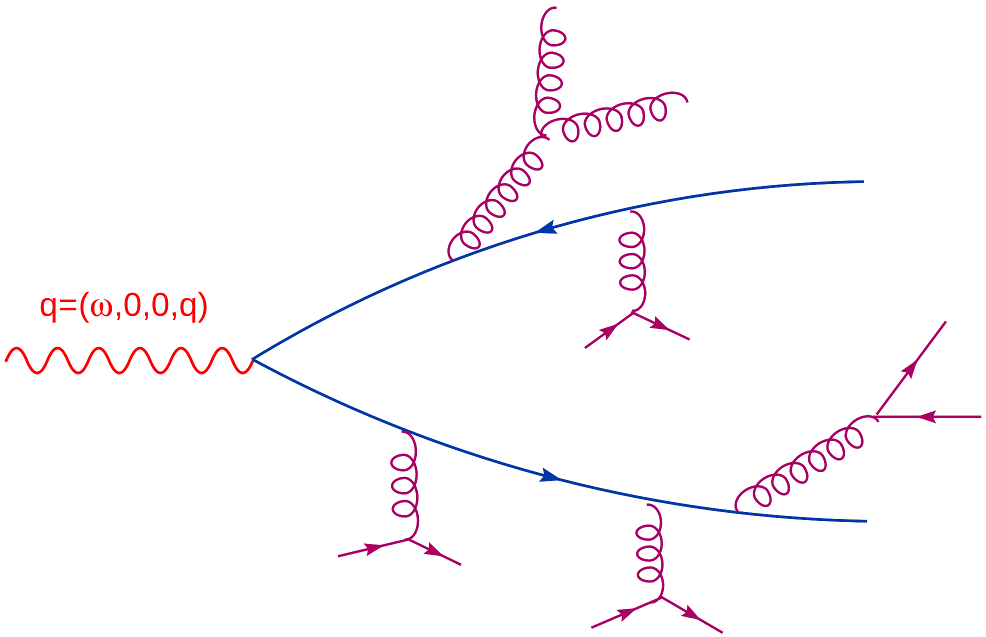


- A space–like photon cannot decay (in the vacuum)
- Virtual partonic fluctuations with
  - transverse size $L \sim 1/Q$
  - and lifetime $\Delta t \sim \omega/Q^2$

Partonic fluctuation in the plasma

- A plasma at finite temperature: the space–like photon can decay due to the parton interactions in the medium



- Strong coupling :

  The potential barrier $\sim Q$ (energy–momentum conservation) disappears with increasing energy (ω) or temperature (T)

Saturation momentum

- Gravitational interactions are proportional to the energy density in the wave (ω) and in the plasma (T)
- The criterion for strong interaction within the plasma

$$\underbrace{Q}_{\text{virtuality barrier}} \lesssim \underbrace{\frac{\omega}{Q^2}}_{\text{lifetime}} \times \underbrace{T^2}_{\text{plasma force}}$$

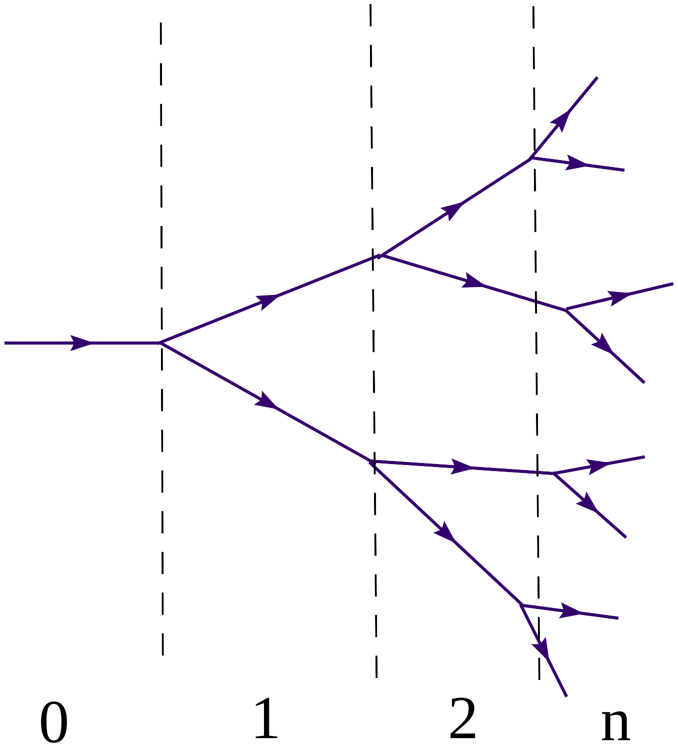
- The partonic fluctuation must live long enough to feel the effects of the plasma
- High energy, or high T, or low Q : $Q \lesssim Q_s$ with

$$Q_s \simeq (\omega T^2)^{1/3} \simeq \frac{T}{x} \quad \text{where} \quad x \equiv \frac{Q^2}{2\omega T}$$

- Physics: the photon can decay due to the plasma force

Medium induced parton branching

- The virtual photon disappears into the plasma via its decay (and not via thermal scattering) !
- 'Quasi–democratic branching' *(Hatta, E.I., Mueller, 08)*



$$\omega_n \sim \frac{\omega_{n-1}}{2} \sim \frac{\omega}{2^n}$$

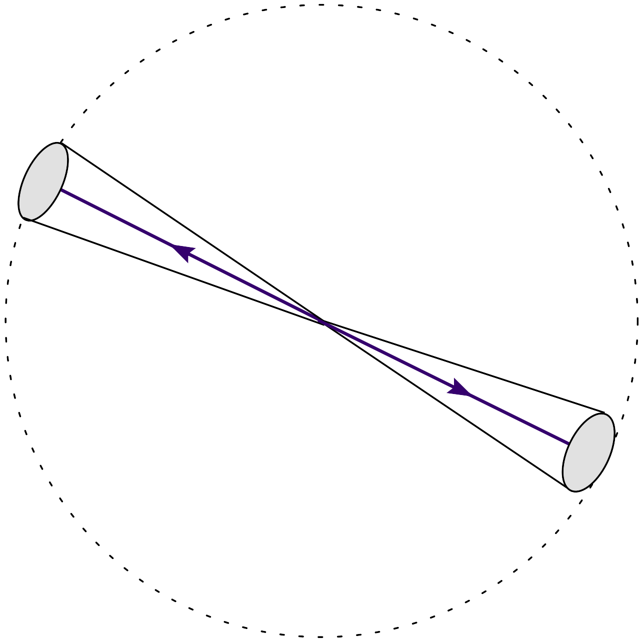
$$\Delta t_n \sim \frac{\omega_n}{Q_n^2}$$

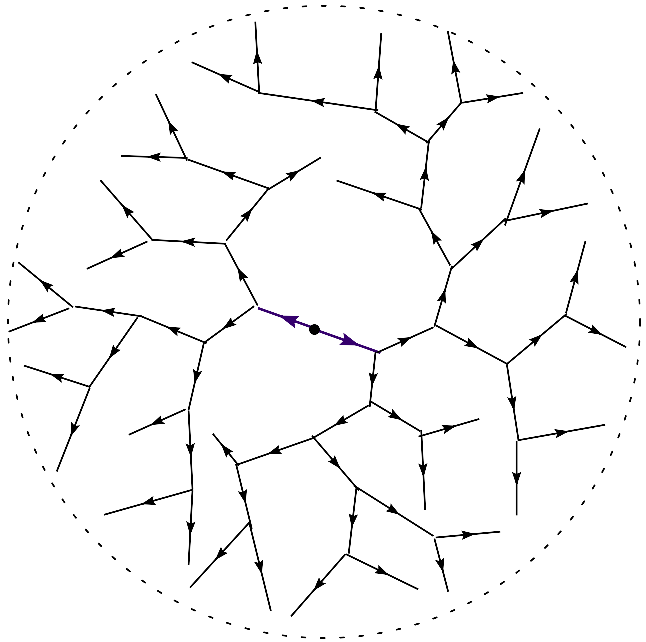
$$\frac{\Delta Q_n}{\Delta t_n} \sim -T^2$$

Lifetime of the current: $\Delta t \sim \dfrac{\omega}{Q_s^2} \ll \dfrac{\omega}{Q^2} \Longrightarrow$ no time to form jets

e^+e^- at strong coupling

- A time–like current can decay already in the vacuum





- Infrared cutoff $\Lambda \longrightarrow$ splitting continues down to $Q \sim \Lambda$
- In the COM frame $\longrightarrow$ spherical distribution $\Longrightarrow$ no jets !

  (similar conclusion by Hofman and Maldacena, 2008)
- Final state looks very different as compared to pQCD !

Parton saturation at strong coupling

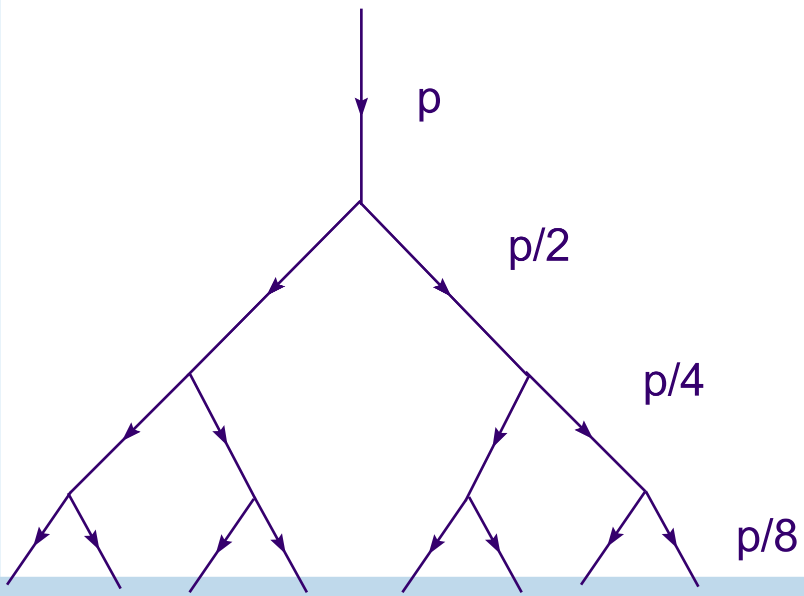
- $Q > Q_s(x) = T/x \ : \quad F_2(x, Q^2) \simeq 0$

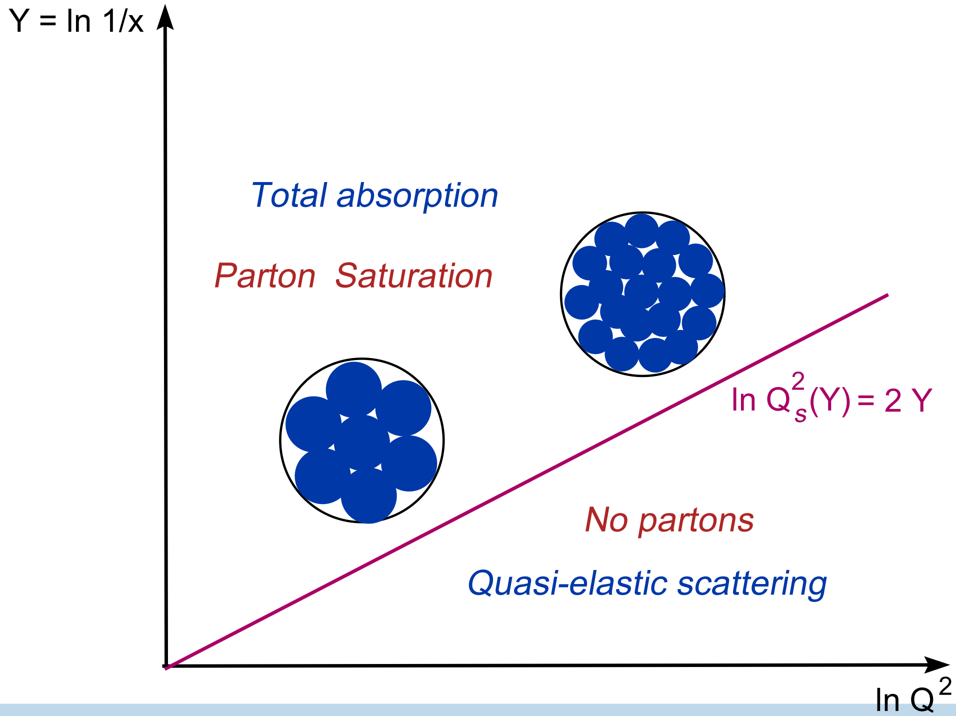
  $\Longrightarrow$ no partons at high Q^2/large x

- $Q < Q_s(x) = T/x \ : \quad F_2(x, Q^2) \sim xN_c^2 Q^2$

  $\Longrightarrow$ parton saturation with occupation numbers $\sim \mathcal{O}(1)$

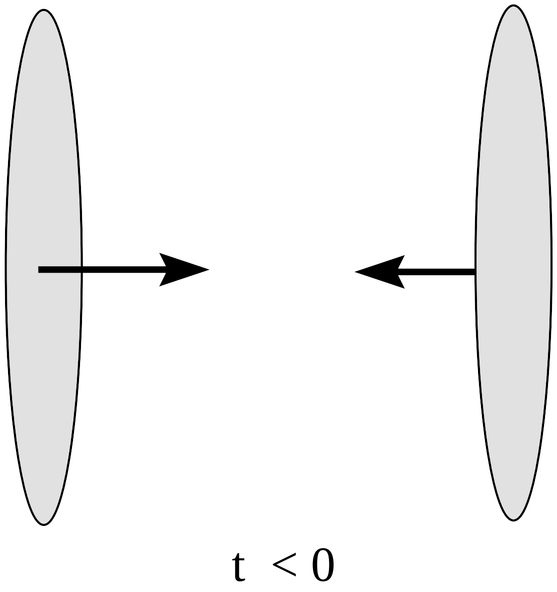
- All partons have branched down to small values of x !

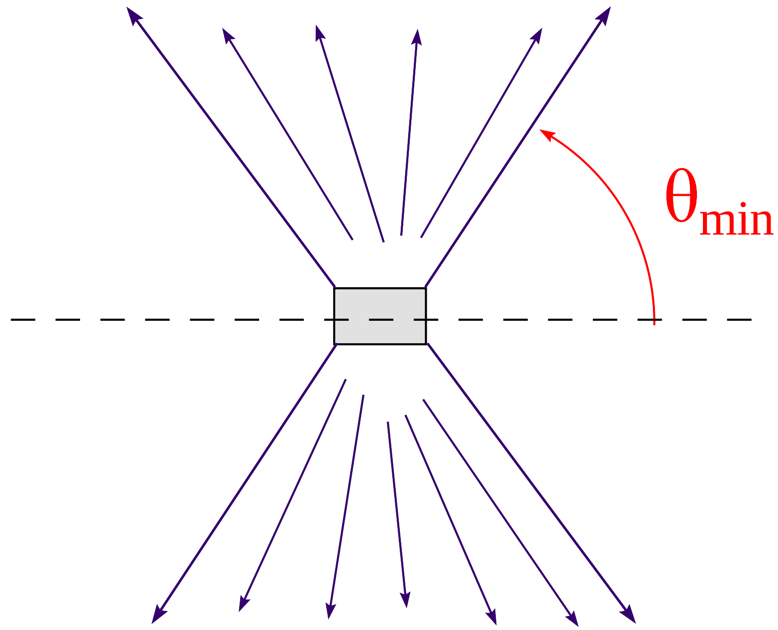




No forward jets !

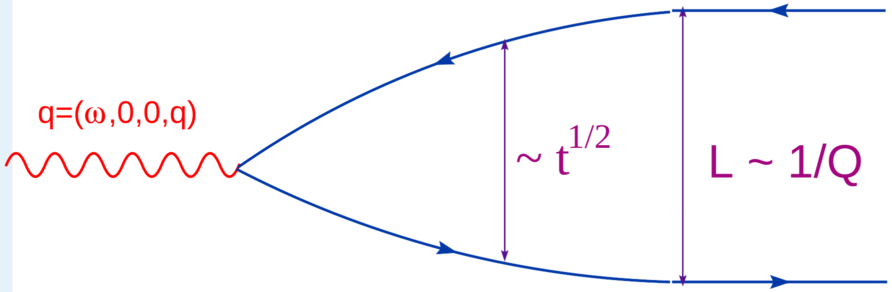
- No large–x partons $\Longrightarrow$ no forward/backward jets in a hadron–hadron collision at strong coupling

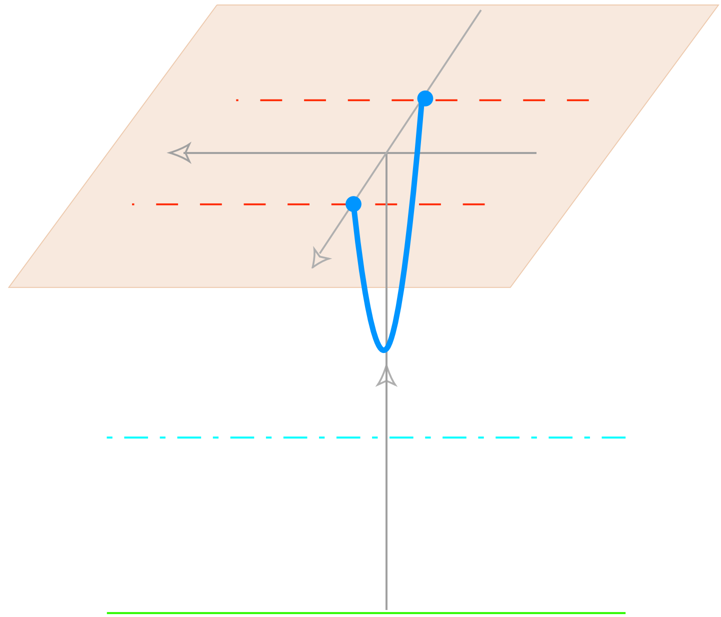




- ‘The Nightmare of CMS’

“No drag force on a small meson in the plasma”



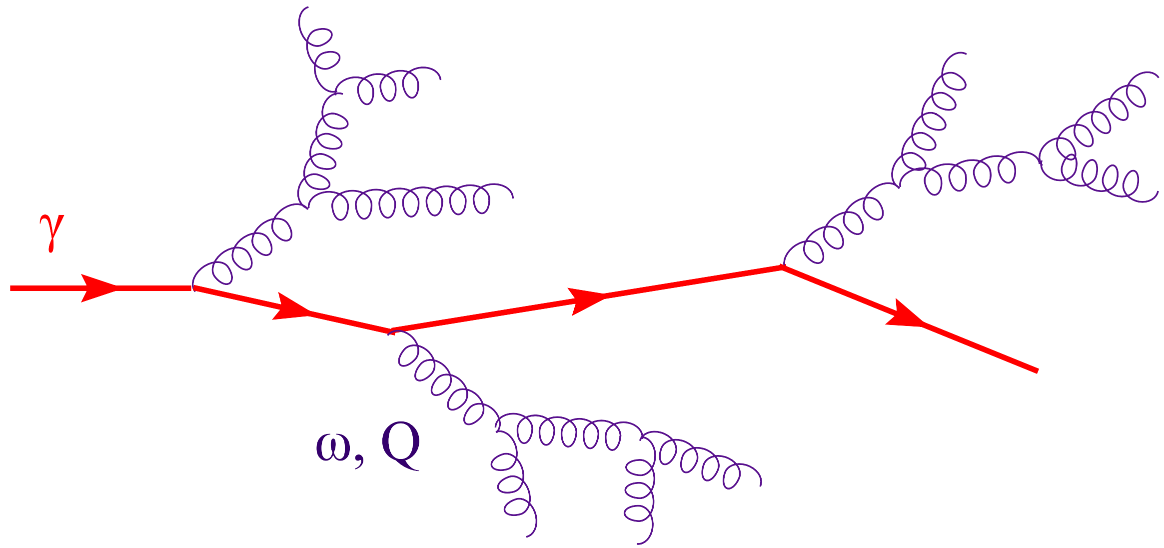


- Larger mesons melt in the plasma: critical size $L_s \sim 1/Q_s$

$$L_s \sim \frac{1}{Q_s} \quad \& \quad \gamma \sim \frac{\omega}{Q} \quad \Longrightarrow \quad L_s \sim \frac{1}{\sqrt{\gamma}T} = \frac{(1-v_z^2)^{1/4}}{T}$$

[Peeters, Sonnenschein, Zamaklar; Liu, Rajagopal, Wiedemann (06)]

Heavy Quark: Energy loss



- Virtual quanta with $Q \lesssim Q_s$ are absorbed by the plasma
- Parton formation time : $\Delta t \sim \omega/Q_s^2$

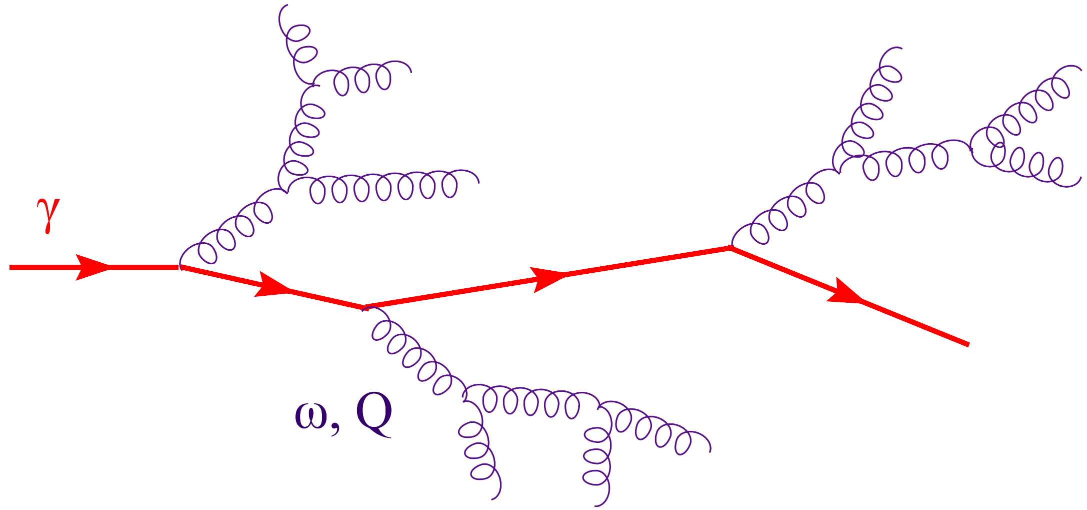
$$-\frac{\mathrm{d}E}{\mathrm{d}t} \simeq \sqrt{\lambda}\,\frac{\omega}{(\omega/Q_s^2)} \simeq \sqrt{\lambda}\,Q_s^2 \simeq \sqrt{\lambda}\,\gamma\,T^2$$

$$Q_s \simeq \frac{\omega}{Q_s^2}T^2 \simeq \frac{\gamma}{Q_s}T^2 \implies Q_s^2 \sim \gamma\,T^2$$

Herzog, Karch, Kovtun, Kozcaz, and Yaffe; Gubser, 2006 (trailing string)

Momentum broadening

- Fluctuations in the medium–induced emission process



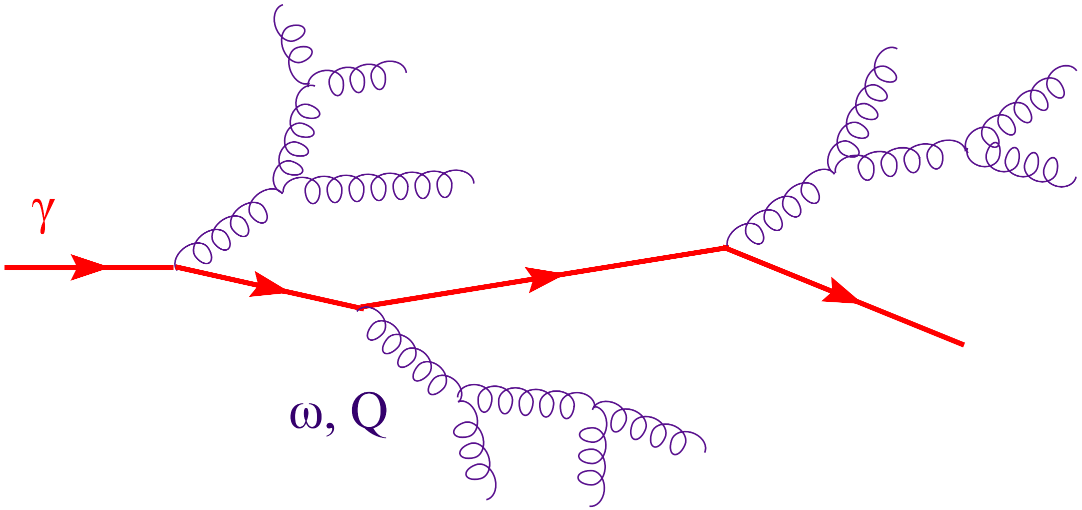
$$\frac{\mathrm{d}\langle p_T^2\rangle}{\mathrm{d}t} \sim \sqrt{\lambda}\,\frac{Q_s^2}{(\omega/Q_s^2)} \sim \sqrt{\lambda}\,\sqrt{\gamma}\,T^3$$

Casalderrey-Solana, Teaney; Gubser, 2006 (from trailing string)

- Non–local process: no meaningful 'jet quenching' transport coefficient ! *(A. Mueller et al, 2008)*

Momentum broadening

- Fluctuations in the medium–induced emission process



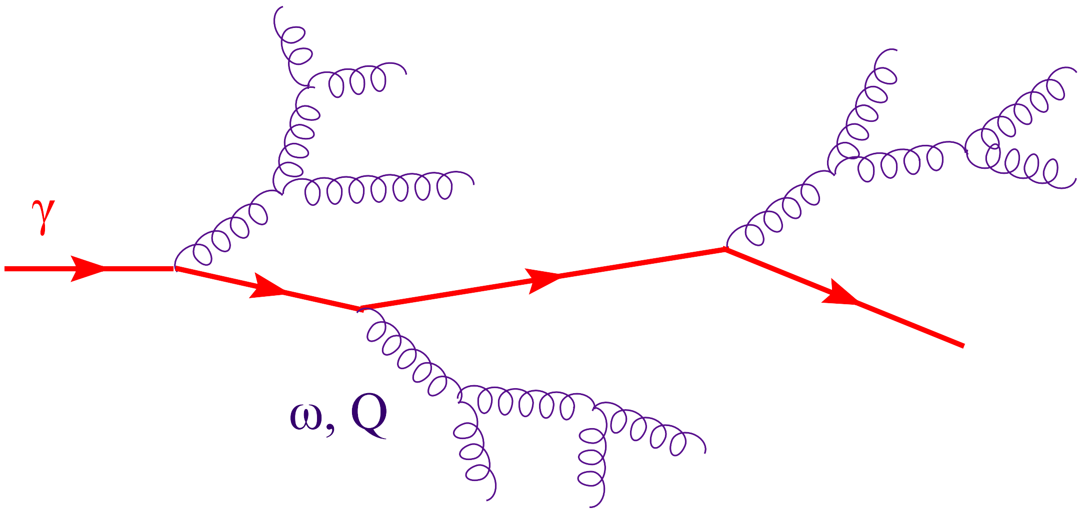
$$\frac{\mathrm{d}\langle p_T^2\rangle}{\mathrm{d}t} \sim \sqrt{\lambda}\,\frac{Q_s^2}{(\omega/Q_s^2)} \sim \sqrt{\lambda}\,\sqrt{\gamma}\,T^3$$

- Longitudinal broadening is parametrically larger !

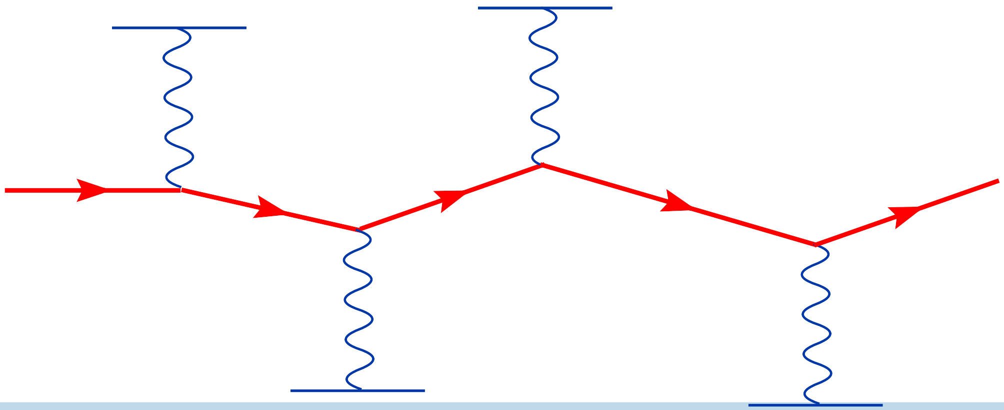
$$\frac{\mathrm{d}\langle p_L^2\rangle}{\mathrm{d}t} \sim \sqrt{\lambda}\,\frac{\omega^2}{(\omega/Q_s^2)} \sim \sqrt{\lambda}\,\sqrt{\gamma}\,\gamma^2\,T^3$$

Momentum broadening

- Strong coupling : fluctuations in the emission process

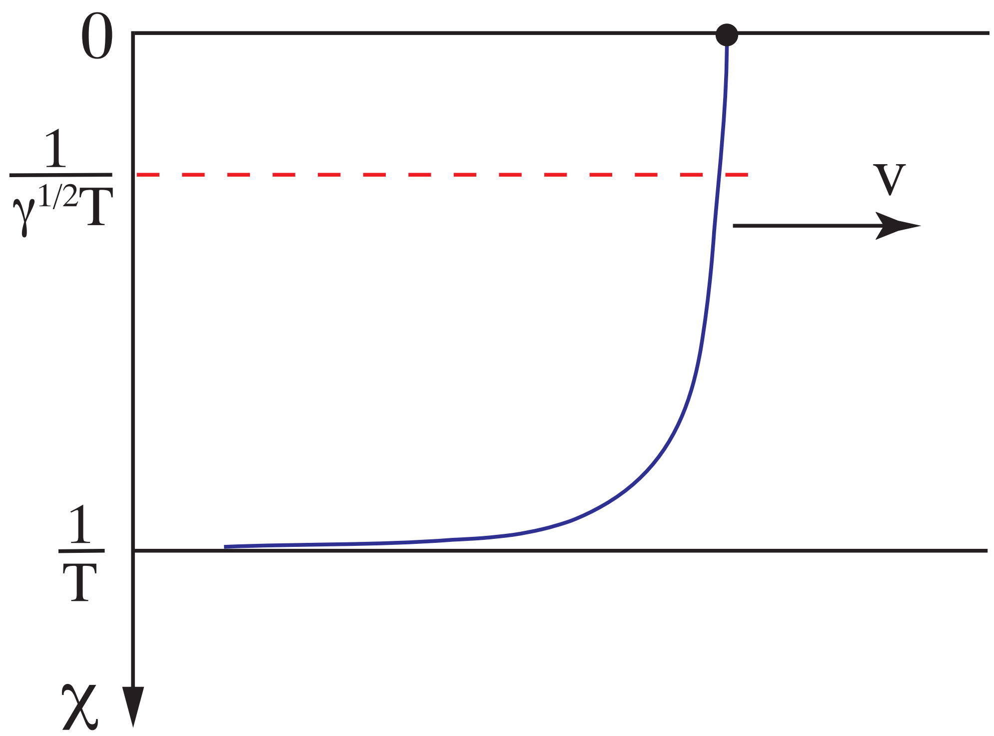


- pQCD : thermal rescattering (different physics !)



Stochastic trailing string

- How are quantum–mechanical (as opposed to thermal) fluctuations encoded in AdS/CFT ?

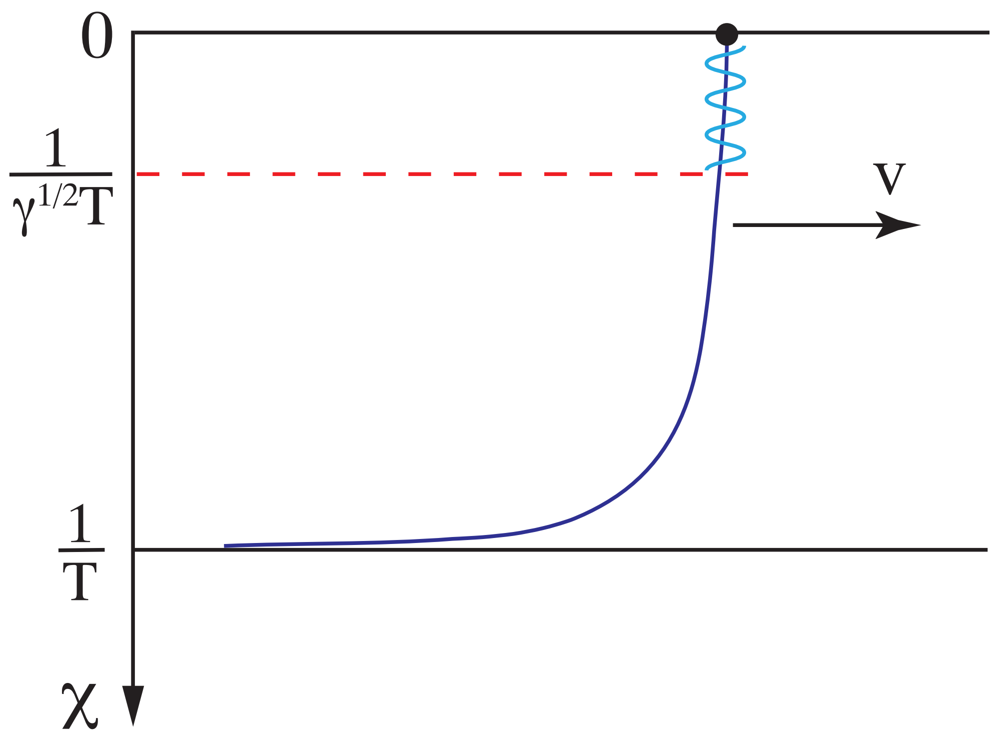


- World–sheet horizon at $\chi_s = 1/Q_s \sim 1/(\sqrt{\gamma}T) \ll 1/T$
- "Thermal fluctuations" with $T_{\rm eff} = \sqrt{\gamma}T$: Unruh temperature

Stochastic trailing string

- Langevin equation for the upper part of the string and the heavy quark

  (G. Giecold, E.I., A. Mueller, 2009)

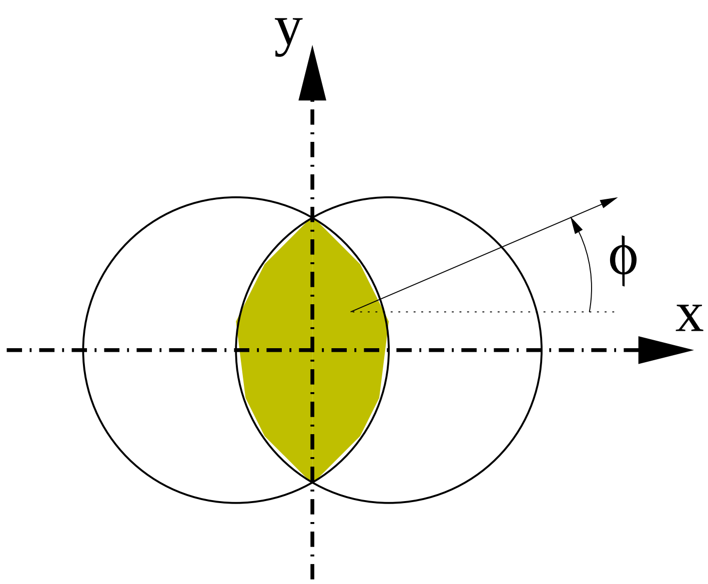


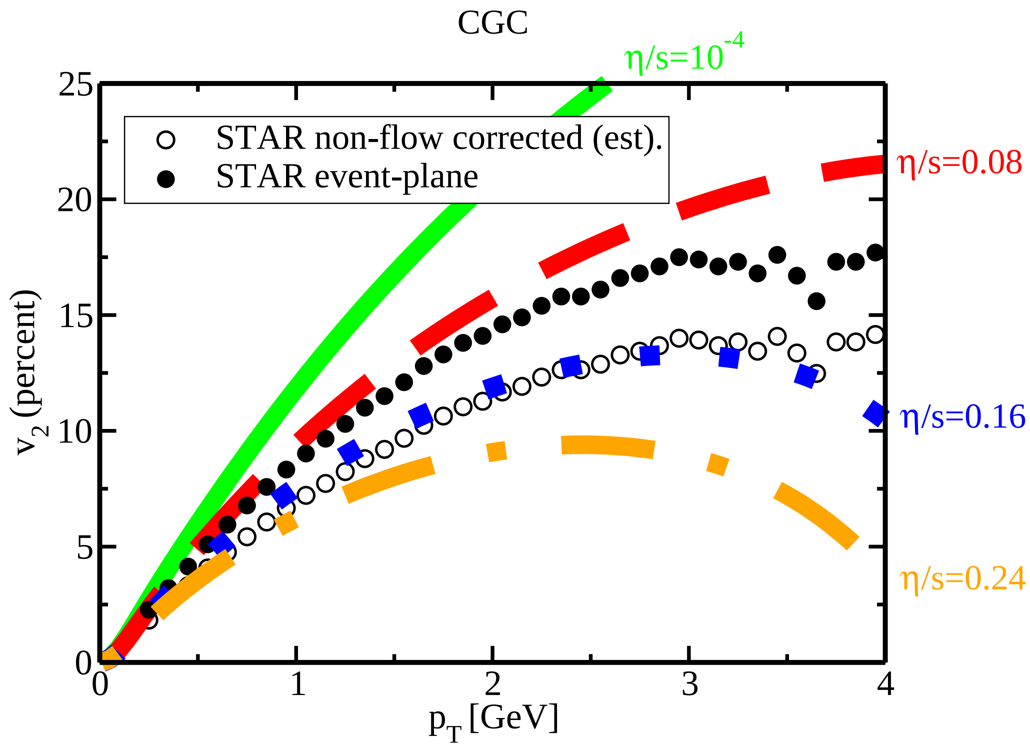
- Encodes both energy loss and momentum broadening

Conclusions

- Hard probes & high-energy physics appears to be quite different at strong coupling as compared to QCD
  - no jets in e^+e^- annihilation
  - no forward/backward particle production in HIC
  - different mechanism for jet quenching
- Are AdS/CFT methods useless for HIC ? Not necessarily so !
  - long–range properties (hydro, thermalization, etc) might stil be controlled by strong coupling
  - some observables receive contributions from several scales, from soft to hard: use AdS/CFT in the soft sector
  - most likely, the coupling is moderately strong, so it useful to approach the problems from both perspectives

Elliptic flow at RHIC: The perfect fluid





- Non–central AA collision: Pressure gradient is larger along x

$$\frac{\mathrm{d}N}{\mathrm{d}\phi} \propto 1 + 2v_2 \cos 2\phi\,, \qquad v_2 = \text{"elliptic flow"}$$

- Well described by hydrodynamical calculations with very small viscosity/entropy ratio: "perfect fluid"

Viscosity over entropy density ratio

- Viscosity/entropy density ratio at RHIC (in units of $\hbar$)

$$\frac{\eta}{s} = 0.1 \pm 0.1(\text{theor}) \pm 0.08(\text{exp}) \ \ [\hbar]$$

- Weakly interacting systems have $\eta/s \gg \hbar$
- Kinetic theory: viscosity is due to collisions among molecules

$$\eta \sim \rho\, v\, \ell = \text{mass density} \times \text{velocity} \times \underbrace{\text{mean free path}}_{\sim 1/g^4}$$

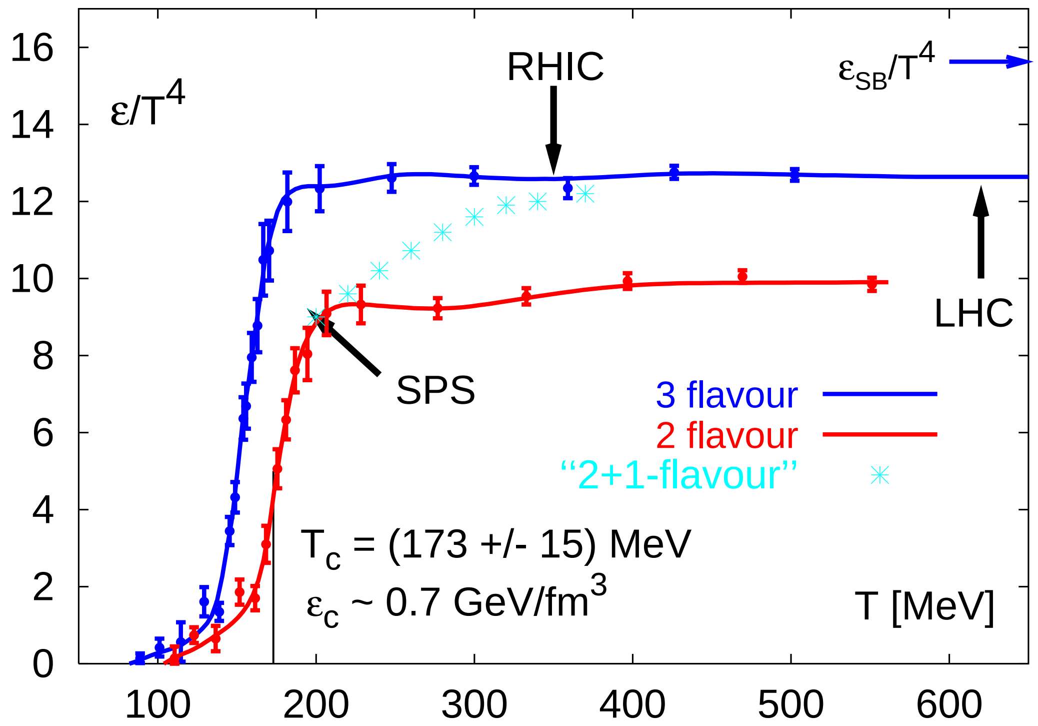
- Conjecture (from AdS/CFT) : *[Kovtun, Son, Starinets, 2003]*

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi} \qquad \text{[lower limit = infinite coupling]}$$

- The RHIC value is at most a few times $\hbar/4\pi$!

Heating QCD : Lattice results

- Energy density as a function of T (Bielefeld Coll.)



$$\mathcal{E}/\mathcal{E}_0 \approx 0.85 \quad \text{for} \quad T = 3T_c$$

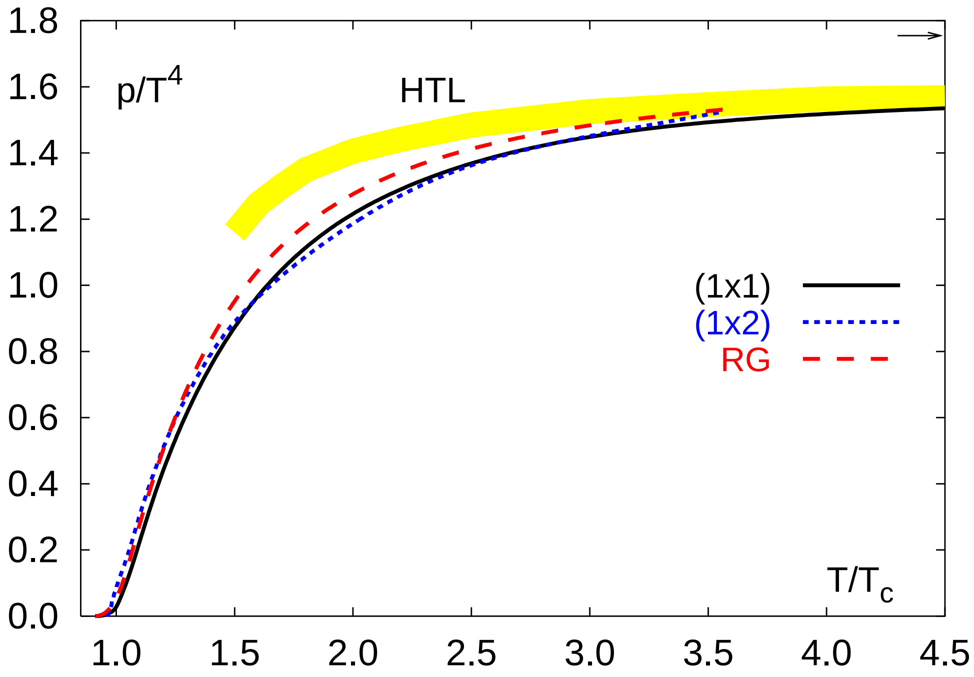
- Is this deviation from ideal gas small ? Or is it large ?
- AdS/CFT : $\mathcal{E}/\mathcal{E}_0 \to 3/4$ when $\lambda \to \infty$ ($\mathcal{N} = 4$ SYM)

Finite–T : Resummed perturbation theory

- This ratio $p/p_0 \approx 0.85$ can be also explained by resummed perturbation theory

  (collective phenomena: screening, thermal masses)

  (J.-P. Blaizot, A. Rebhan, E. Iancu, 2000)



- First principle calculation without free parameter

The ‘perfect fluid’

- Uncertainty principle applied to viscosity:

$$\eta \sim \rho v \lambda_f\,, \qquad S \sim n \sim \frac{\rho}{m}$$

$$\frac{\eta}{S} \sim m v \lambda_f \sim \hbar\,\frac{\text{mean free path}}{\text{de Broglie wavelength}} \gtrsim \hbar$$

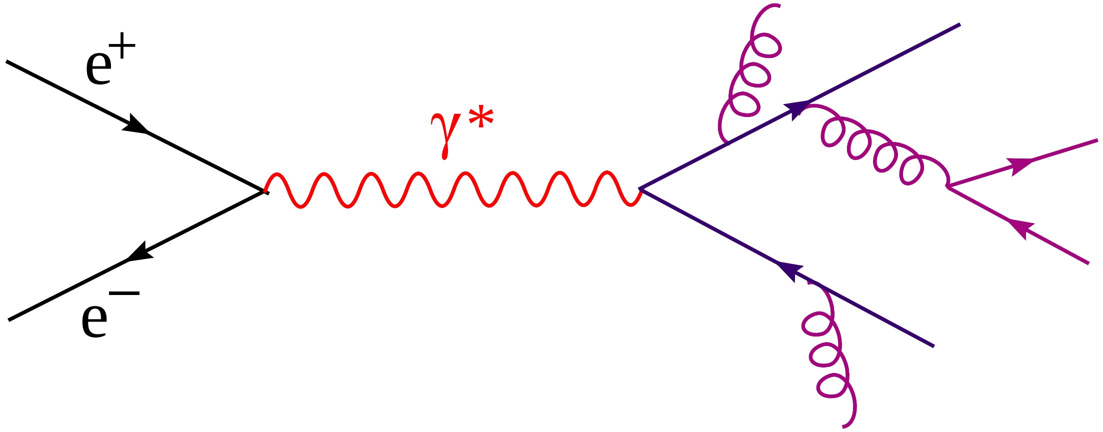
- Weakly interacting systems have $\eta/S \gg \hbar$
- Strongly coupled $\mathcal{N}=4$ SYM plasma

$$\frac{\eta}{S} \to \frac{\hbar}{4\pi} \qquad \text{when} \qquad \lambda \to \infty$$

(Policastro, Son, and Starinets, 2001)

- This bound is believed to be universal : $\eta/S \geq \hbar/4\pi$
- The data at RHIC are consistent with the lower limit being actually reached : ‘sQGP’

Jets



- ‘Multi–jet event’ : large emission angle & $x \sim \mathcal{O}(1)$

$$k_\perp \sim k \sim \sqrt{s} \implies \mathcal{P}_{\rm Brem} \sim \alpha_s(s) \ll 1$$

small probability for emitting an extra gluon jet !

- ‘Intra–jet activity’ : collinear and/or soft gluons

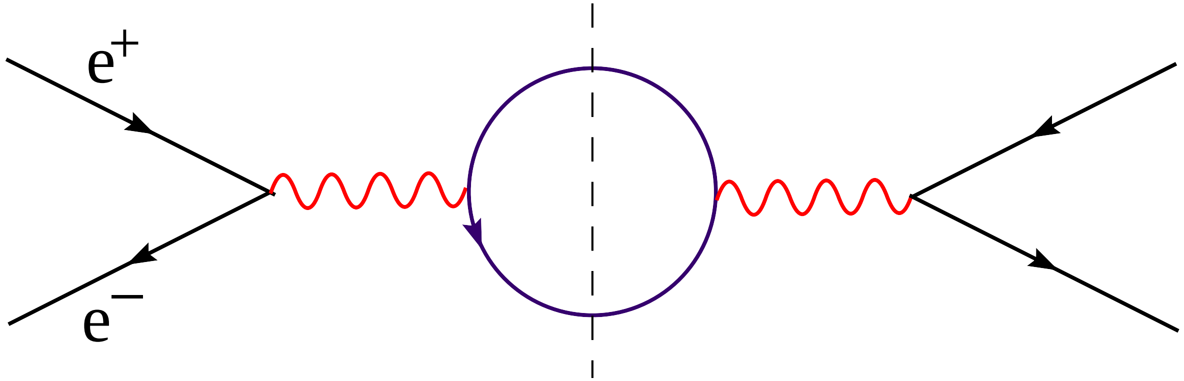
$$\Lambda_{\rm QCD} \ll k_\perp \ll k \ll \sqrt{s} \implies \mathcal{P}_{\rm Brem} \sim \alpha_s \ln^2 \frac{\sqrt{s}}{\Lambda_{\rm QCD}} \sim \mathcal{O}(1)$$

modifies particle multiplicity but not the number of jets

Optical theorem

- Total cross–section given by the optical theorem

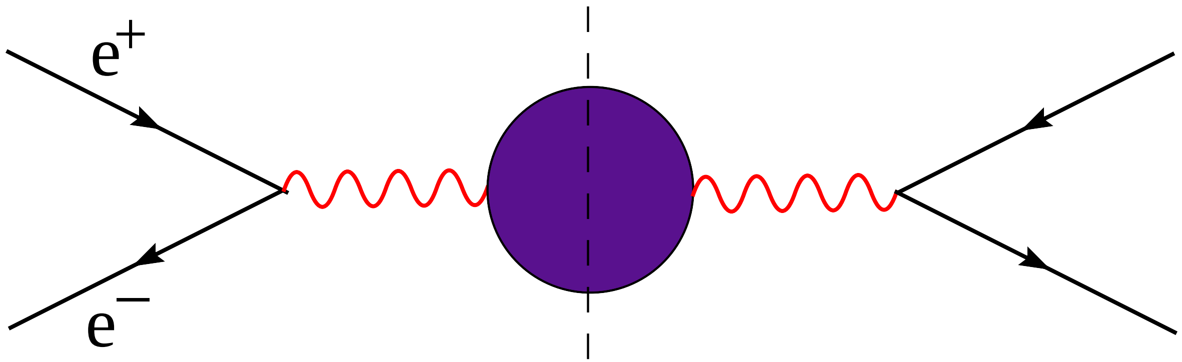
$$\sigma(e^+e^-) = \frac{1}{2s}\,\ell^{\mu\nu}\,\mathrm{Im}\,\Pi_{\mu\nu}(q)$$



- The quark loop: The vacuum polarization tensor $\Pi_{\mu\nu}$ for a time–like photon (here, evaluated at one–loop order)
- This can be generalized to all–orders

Current–current correlator

$$\sigma(e^+e^-) = \frac{1}{2s}\,\ell^{\mu\nu}\,\mathrm{Im}\,\Pi_{\mu\nu}(q)$$



- $\Pi_{\mu\nu}$ = current–current correlator to all orders in QCD

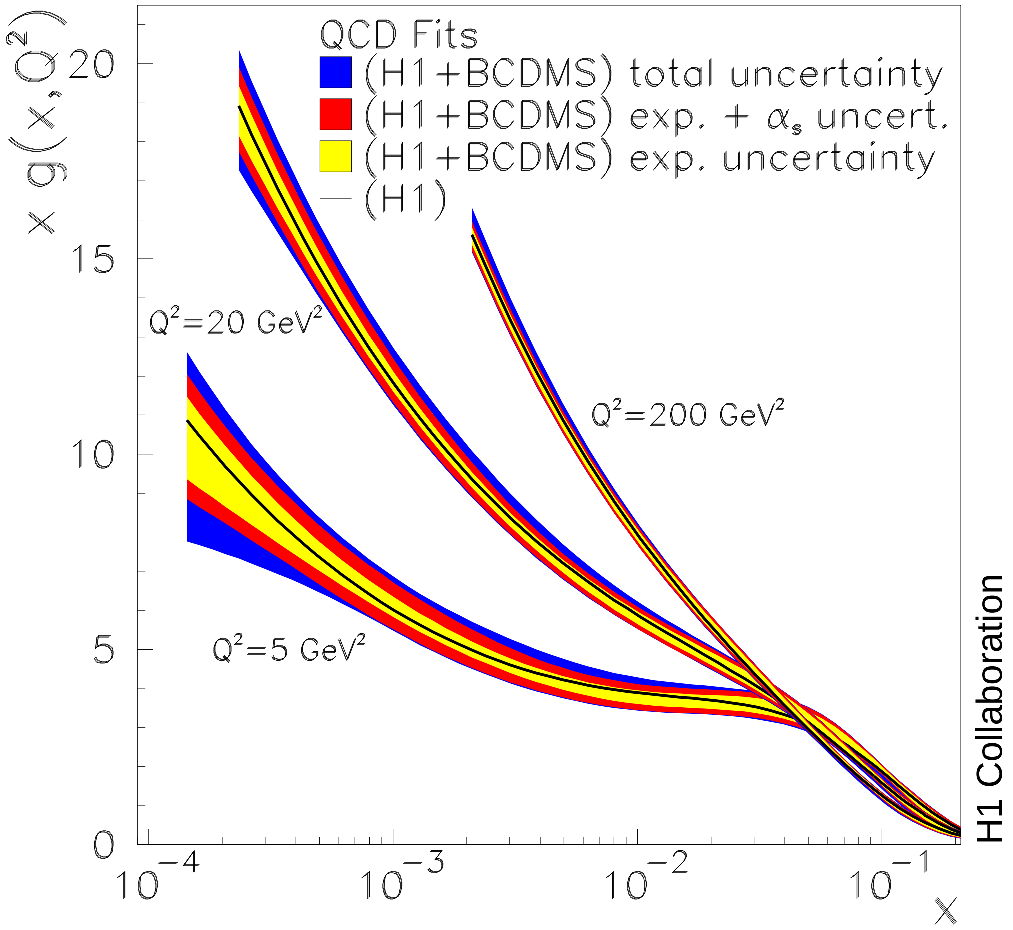
$$\Pi_{\mu\nu}(q) \equiv i\int \mathrm{d}^4x\,\mathrm{e}^{-iq\cdot x}\,\langle 0\,|\mathrm{T}\,\{J_\mu(x)J_\nu(0)\}\,|\,0\rangle$$

$$J^\mu = \sum_f e_f\,\bar{q}_f\,\gamma^\mu\,q_f\ :\ \text{quark electromagnetic current}$$

- Valid to leading order in α_{em} but all orders in α_s

Gluons at HERA

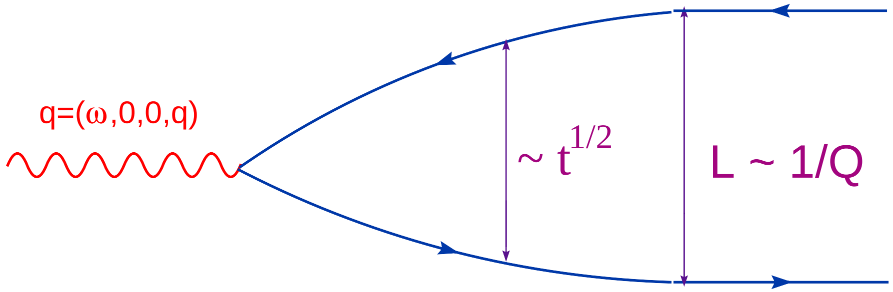
$xg(x,Q^2)$ = # of gluons with transverse area $\sim 1/Q^2$ and $k_z = xP$



▷ Rapid rise with $1/x$: $xg(x,Q^2) \sim 1/x^{\lambda}$ with $\lambda = 0.2 \div 0.3$

Screening length

- A small color dipole ('meson') with transverse size $L \ll 1/Q_s$ propagates through the strongly–coupled plasma with almost no interactions !



- Larger dipoles with $L \gtrsim 1/Q_s$ cannot survive in the plasma

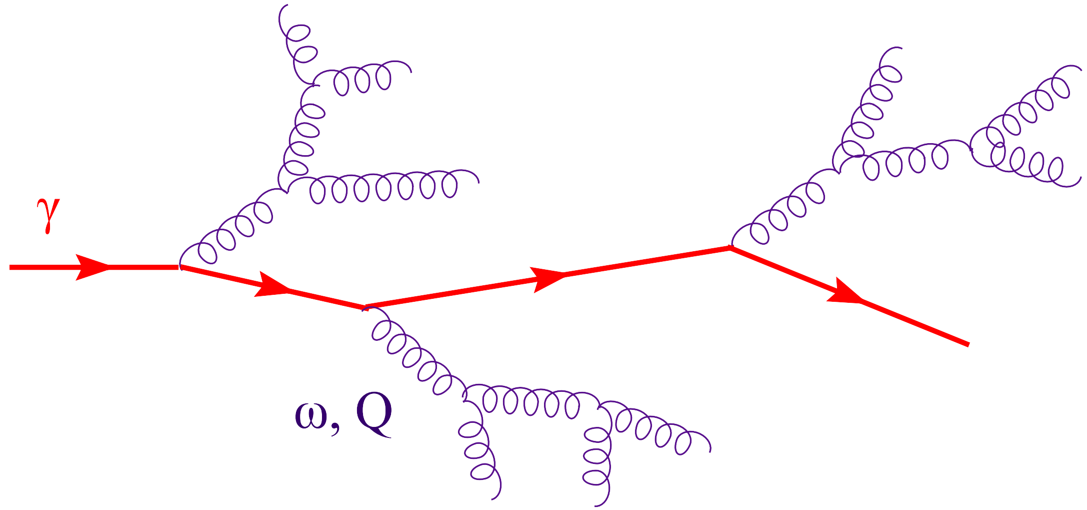
$$L_s \sim \frac{1}{Q_s} \quad \& \quad \gamma \sim \frac{\omega}{Q} \quad \Longrightarrow \quad L_s \sim \frac{1}{\sqrt{\gamma}\,T} \ll \frac{1}{T}$$

- The dipole lifetime is short on natural time scales:

$$\Delta t \sim \frac{\omega}{Q_s^2} \sim \frac{\sqrt{\gamma}}{T} \ll \frac{\gamma}{T}$$

Momentum broadening

- Fluctuations in the medium–induced emission process

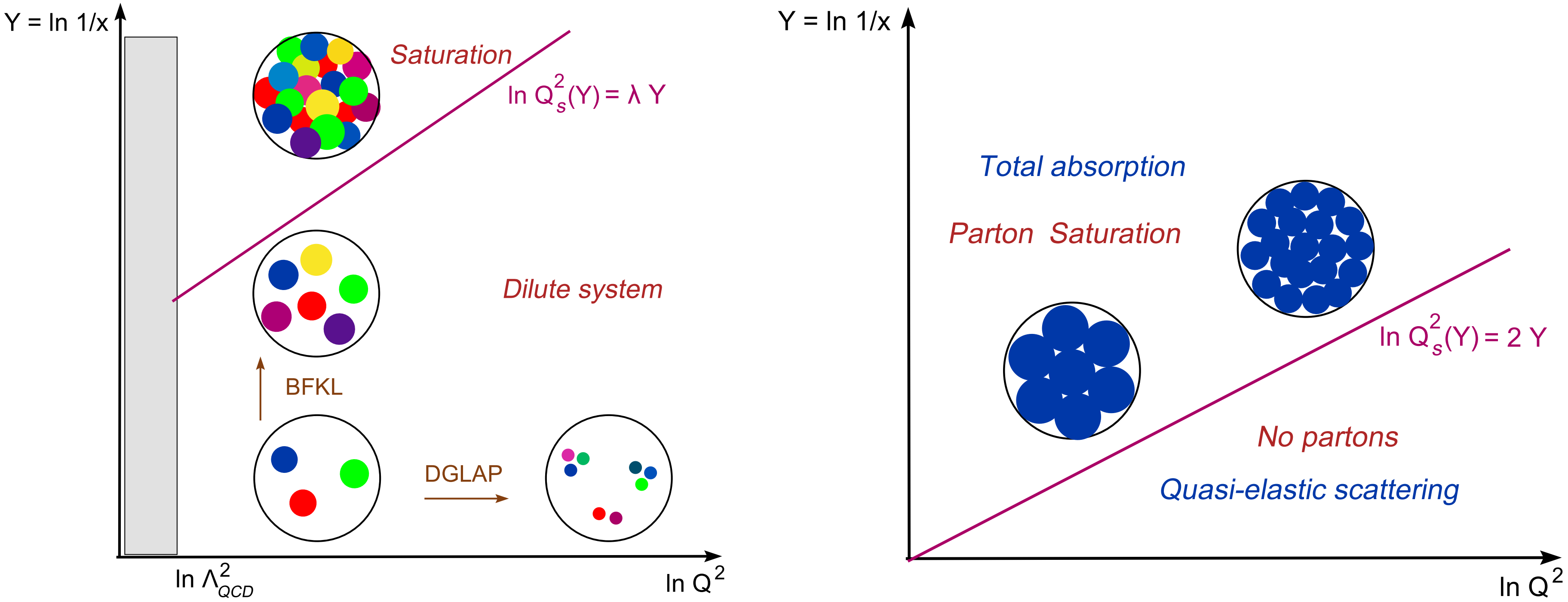


$$\frac{\mathrm{d}\langle p_T^2\rangle}{\mathrm{d}t} \sim \sqrt{\lambda}\,\frac{Q_s^2}{(\omega/Q_s^2)} \sim \sqrt{\lambda}\,\frac{Q_s^4}{\gamma Q_s} \sim \sqrt{\lambda}\sqrt{\gamma}\,T^3$$

$$\frac{\mathrm{d}\langle p_L^2\rangle}{\mathrm{d}t} \sim \sqrt{\lambda}\,\frac{\omega^2}{(\omega/Q_s^2)} \sim \sqrt{\lambda}\sqrt{\gamma}\,\gamma^2\,T^3$$

Casalderrey-Solana, Teaney; Gubser, 2006 (from trailing string)

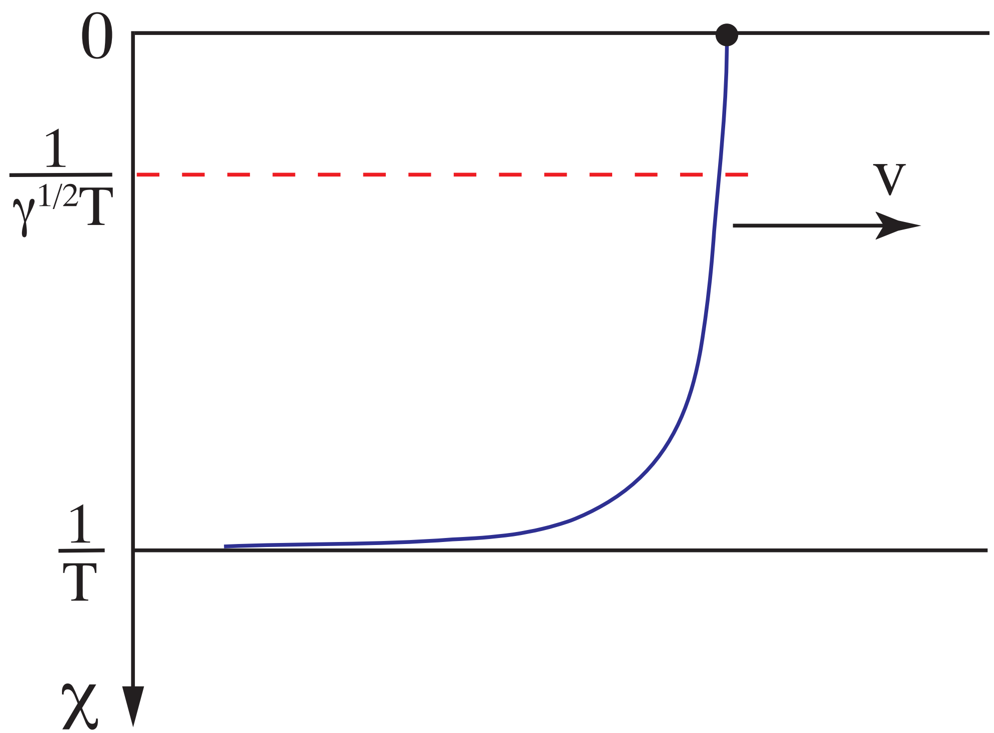
Saturation line: weak vs. strong coupling



- Saturation exponent : $Q_s^2(x) \propto 1/x^{\lambda_s} \equiv e^{\lambda_s Y}$
  - weak coupling (LO pQCD): $\lambda_s \approx 0.12\, g^2 N_c$
  - phenomenology & NLO pQCD: $\lambda_s \approx 0.2 \div 0.3$
  - strong coupling (plasma): $\lambda_s = 2$ (graviton)

Stochastic trailing string

- How are quantum–mechanical (as opposed to thermal) fluctuations encoded in AdS/CFT ?

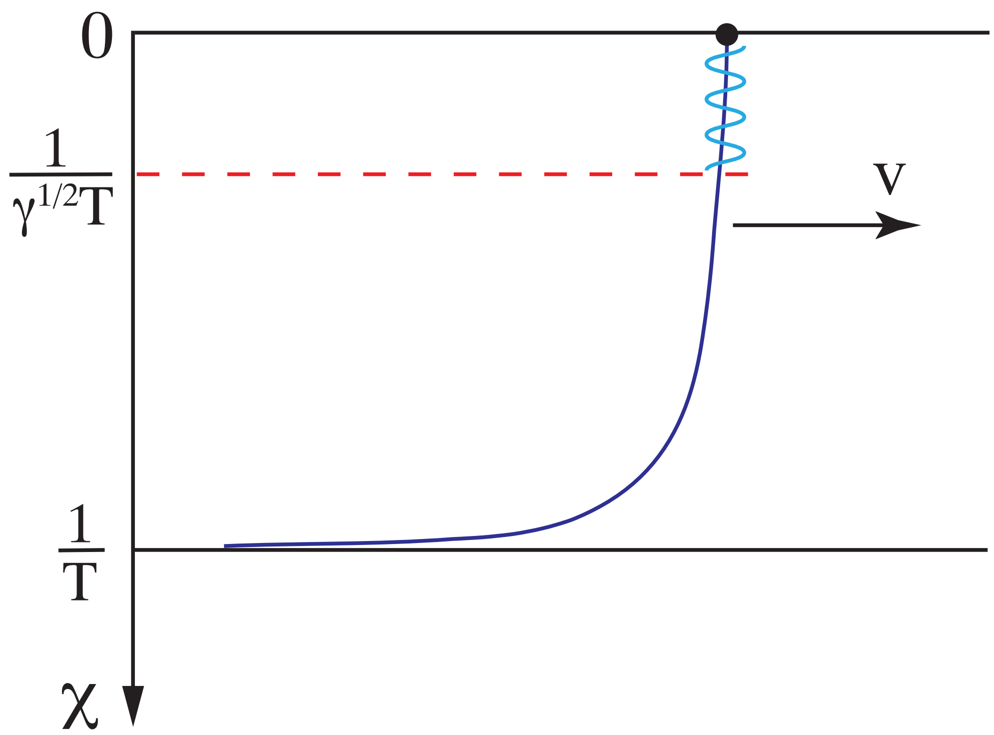


- World–sheet horizon at $\chi_s = 1/Q_s \sim 1/(\sqrt{\gamma}T) \ll 1/T$
- Hawking radiation (= thermal flucts.) plays no role (in contrast to a static string; cf. talk by Rangamani)

Stochastic trailing string

- Fluctuations on top of the world–sheet horizon χ_s

  $\Longrightarrow$ noise term on the ‘stretched horizon’ at $\chi = \chi_s + \epsilon$



- Langevin equation for the upper part of the string & the heavy quark *(G. Giecold, E.I., A. Mueller, 09)*
- Physics: Fluctuations in the parton cascades