Hard probes for a strongly coupled plasma from AdS/CFT

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Outline

- **Motivation:**
  Hard probes for Heavy Ion Collisions at RHIC and LHC
  (see the talks by W. Zajc and A. Starinets for soft probes)

- **Weak coupling:** Partons and jets in perturbative QCD

- **Strong coupling:** AdS/CFT Correspondence

- **Finite-temperature plasma at strong coupling:**
  - Deep inelastic scattering & Parton saturation
  - Energy loss & Momentum broadening
  (see also the talk by J. Casalderrey–Solana)
The Little Bang

- Ultrarelativistic heavy ion collisions @ RHIC and LHC

- Extremely complex phenomena
  - high density partonic systems in the initial wavefunctions
  - multiple interactions during the collisions
  - complicated, non-equilibrium, dynamics after the collision
  - expansion, thermalization, hadronisation

- Is there any place for strong–coupling dynamics?
~ 3000 hadrons in the final state vs. 400 nucleons in AA

Most of them arise as hadronized partons

Particle correlations are essential to disentangle phenomena
Azimuthal correlations between the produced jets:

a peak at $\Delta \Phi = 180^\circ$
The “away–side” jet has disappeared!
absorption (or energy loss, or “jet quenching”) in the medium

The matter produced in a heavy ion collision is opaque
high density, strong interactions, … or both
Energy loss at weak coupling

- Medium induced radiation

A non–local process: gluon formation time $\Delta t \sim \omega/Q^2$

$$-\frac{dE}{dt} \simeq \alpha_s N_c \langle p_\perp^2 \rangle : \text{relation to ‘momentum broadening’}$$
Transverse momentum broadening

- Scattering off the plasma constituents

\[
\frac{d\langle p_T^2 \rangle}{dt} \equiv \dot{q} \approx \alpha_s N_c xg(x, Q^2)
\]

- \( xg(x, Q^2) \): gluon distribution per unit volume in the medium

\[
xg(x, Q^2) \approx n_q(T) xG_q + n_g(T) xG_g \quad \text{with} \quad n_{q,g}(T) \propto T^3
\]

This requires parton evolution from \( T \) up to \( Q \gg T \)

- “jet quenching parameter” \( \dot{q} \): a local transport coefficient
Nuclear modification factor

How to measure $\hat{q}$? Compare $AA$ collisions at RHIC to $pp$

$$R_{AA}(p_{\perp}) \equiv \frac{Yield(A + A)}{Yield(p + p) \times A^2}$$

- RHIC data prefer a rather large value $\hat{q} \simeq 10 \text{ GeV}^2/\text{fm}$, which seems (marginally) inconsistent with weak coupling
$e^+e^-$ annihilation: Jets in pQCD

- How would a high–energy jet interact in a strongly coupled plasma?
- How to produce jets in the first place?

- Guidance from perturbative QCD: $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}$

- Decay of a time–like photon: $Q^2 \equiv q^\mu q_\mu = s > 0$
How would a high–energy jet interact in a strongly coupled plasma?

How to produce jets in the first place?

Guidance from perturbative QCD: $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}$

The structure of the final state is determined by

- parton branching & hadronisation
Parton branching at weak coupling

- Gluon ‘formation time’ : \( \Delta t \sim \frac{k_\ell}{k_\perp^2} \)
- Early partons are hard \((k_\perp \gg \Lambda_{\text{QCD}})\) and hence perturbative
- Bremsstrahlung favors the emission of soft \((x \ll 1)\) and collinear \((k_\perp^2 \ll s)\) gluons \((k_\ell = x p_\ell)\)

\[
\frac{d\mathcal{P}_{\text{Brem}}}{x} \sim \alpha_s(k_\perp^2) N_c \frac{d^2k_\perp}{x} \frac{dx}{x}
\]

- Relatively simple final state!
- Few, well collimated, jets

- $e^+e^-$ cross-section computable in perturbation theory

\[
\sigma(s) = \sigma_{\text{QED}} \times \left(3 \sum_f e_f^2\right) \left(1 + \frac{\alpha_s(s)}{\pi} + O(\alpha_s^2(s))\right)
\]

$\sigma_{\text{QED}}$ : cross-section for $e^+e^- \rightarrow \mu^+\mu^-$

- Multi-jet ($n \geq 3$) events appear, but are comparatively rare
Deep inelastic scattering

- Space–like current: $Q^2 \equiv -q^\mu q_\mu \geq 0$ and $x \equiv \frac{Q^2}{2P \cdot q}$

- Physical picture: $\gamma^*$ absorbed by a quark excitation with
  - transverse size $\Delta x_\perp \sim 1/Q$
  - and longitudinal momentum $p_z = xP$
The proton structure function

\[ \sigma_{\gamma^* p}(x, Q^2) = \frac{4\pi^2 \alpha_{\text{em}}}{Q^2} F_2(x, Q^2) \]

- \( F_2(x, Q^2) \): ‘quark distribution’ = number of quarks with longitudinal momentum fraction \( x \) and transverse area \( 1/Q^2 \)
Parton evolution in pQCD

- Gluons are implicitly seen in DIS, via parton evolution

Bremsstrahlung favors the emission of gluons with $x \ll 1$
Partons at RHIC

Partons are actually ‘seen’ (liberated) in the high energy hadron–hadron collisions

- central rapidity: small–$x$ partons
- forward/backward rapidities: large–$x$ partons
Gluon Saturation: CGC

- When occupation number $\sim 1/\alpha_s \implies$ strong repulsion

$$n(x, Q^2) = \frac{\pi}{Q^2} \times \frac{xG(x, Q^2)}{\pi R^2} \sim \frac{1}{\alpha_s} \quad \text{when} \quad Q^2 \approx Q_s^2(x)$$
Gluon Saturation: CGC

The saturation momentum

\[ Q_s^2(x) \approx \alpha_s \frac{xG(x, Q_s^2)}{\pi R^2} \sim \frac{1}{x \lambda_s} \quad \text{with} \quad \lambda_s \sim 0.3 \]

For \( Q^2 < Q_s^2(x) \), the gluon occupation numbers saturate.
Hard probes in a strongly–coupled plasma

- Virtual photon: electromagnetic current $J_\mu$
- Thermal expectation value (retarded polarization tensor):
  \[
  \Pi_{\mu\nu}(q) \equiv \int \! d^4 x \, e^{-i q \cdot x} \, i\theta(x_0) \langle [J_\mu(x), J_\nu(0)] \rangle_T
  \]
- ‘Hard probe’: large virtuality $Q^2 \equiv |q^2| \gg T^2$
  - time–like current ($q^2 > 0$): jets
  - space–like current ($q^2 < 0$): DIS, partons
- Relativistic heavy quark: $M \gg T$ and $v \simeq 1$
  - energy loss
  - transverse momentum broadening
- Strong coupling $\implies$ AdS/CFT correspondence
Gauge theory side: CFT

- $\mathcal{N} = 4$ Supersymmetric Yang–Mills theory
  - color gauge group $\text{SU}(N_c)$
  - supersymmetry ($\text{fermions} \rightleftharpoons \text{bosons}$)
  - gluons, fermions, scalars ($\text{all in the adjoint repres.}$ !)
  - quantum conformal invariance ($\text{fixed coupling}$)
  - no confinement, no intrinsic scale

- Has this any relevance to QCD ??

- Perhaps better suited for QCD at finite temperature
  - deconfined phase ($\text{quark–gluon plasma}$)
  - quarks and gluons play rather similar roles
  - nearly conformal ($\text{small running–coupling effects}$)
Trace anomaly from lattice QCD

\[ \beta(g) \frac{dp}{dg} = \langle T^\mu_\mu \rangle = \mathcal{E} - 3p \]

\[ \frac{\mathcal{E} - 3p}{\mathcal{E}_0} \lesssim 10\% \text{ for any } T \gtrsim 2T_c \approx 400 \text{ MeV} \]
String theory side: $\text{AdS}$

- Type IIB string theory living in $D = 10$: $AdS_5 \times S^5$

$$ds^2 = \frac{R^2}{\chi^2}(-dt^2 + d\vec{x}^2) + \frac{R^2}{\chi^2}d\chi^2 + \frac{R^2}{\chi^2}d\Omega^2_5$$

- $0 \leq \chi < \infty$: ‘radial’, or ‘5th’, coordinate
- gauge theory lives at the Minkowski boundary $\chi = 0$

$\chi$ coordinates:
- $\chi$ is a ‘radial’ coordinate.
- Gauge theory lives at the Minkowski boundary $\chi = 0$.
- $\chi \sim L \sim 1/Q$.

$\chi$ is the coordinate in the higher-dimensional AdS space.

**AdS/CFT correspondence**
- Hard probes in a plasma
- CFT
- Trace anomaly

**String theory**
- Type IIB string theory
- Living in $D = 10$
- $AdS_5 \times S^5$

**AdS/CFT**
- $\chi$ coordinate
- Minkowski boundary
- Boundary $\chi = 0$
The Gauge/Gravity duality (Maldacena, 1997)

- **Gauge theory** has two parameters:
  - coupling constant $g$ (elementary charge)
  - number of colors $N_c$
  - weakly or strongly coupled depending upon $\lambda \equiv g^2 N_c$

- **String theory** has three parameters:
  - curvature radius of space $R$
  - string coupling constant $g_s$
  - string length $l_s$ (typical size of string vibrations)

- **Mapping of the parameters**:

  $$4\pi g_s = g^2, \quad (R/l_s)^4 = g^2 N_c$$

- **Strong ‘t Hooft coupling** (more properly, $N_c \to \infty$):

  $$\lambda \equiv g^2 N_c \gg 1 \quad \text{with} \quad g^2 \ll 1 \implies \text{classical (super)gravity}$$
\[ \mathcal{N} = 4 \text{ SYM at finite temperature} \quad \iff \quad \text{Black Hole in AdS}_5 \]

\[
ds^2 = \frac{R^2}{\chi^2} \left( -f(\chi) dt^2 + dx^2 \right) + \frac{R^2}{\chi^2 f(\chi)} d\chi^2 + R^2 d\Omega_5^2
\]

where \( f(\chi) = 1 - (\chi/\chi_0)^4 \) and \( \chi_0 = 1/\pi T = \text{BH horizon} \)

- A black hole has entropy and thermal (Hawking) radiation
**DIS off the Black Hole** *(Hatta, E.I., Mueller, 07)*

- **Abelian current** $J_\mu$ in 4D $\leftrightarrow$ **Maxwell wave** $A_\mu$ in $AdS_5$ BH
- $\text{Im} \Pi_{\mu\nu} \leftrightarrow$ absorption of the wave by the BH

**Maxwell equations in a curved space–time**

$$\partial_m \left( \sqrt{-g} g^{mn} g^{pq} F_{nq} \right) = 0$$

where 

$$F_{mn} = \partial_m A_n - \partial_n A_m$$
Relativistic heavy quark

- Heavy quark in 4D $\leftrightarrow$ a Nambu–Goto string in $AdS_5$ BH

  *Herzog, Karch, Kovtun, Kozcaz, and Yaffe; Gubser, 2006* ("trailing string")

- Energy loss $\leftrightarrow$ energy flux down the string

  \[
  \chi \cdot v \cdot 0^{1/2} \gamma^1 \frac{1}{\gamma^{1/2}T} \rightarrow V
  \]

- Nambu–Goto equations in a curved space–time
Physical interpretation

- **Rôle of the 5th dimension:** a reservoir of quantum fluctuations.
- **Radial penetration** $\chi$ of the wave packet in $AdS_5$ ↔ transverse size $L$ of the partonic fluctuation on the boundary.

![Diagram](image)

- **Space–like photon with virtuality $Q$:** The Maxwell wave penetrates up to a radial distance $\chi \sim 1/Q$. 

Physical interpretation:

- Rôle of the 5th dimension: a reservoir of quantum fluctuations.
- Radial penetration $\chi$ of the wave packet in $AdS_5$ ↔ transverse size $L$ of the partonic fluctuation on the boundary.

**Diagram:**

- `$Q$`:
- `$\chi = 0$`:
- `$L \sim 1/Q$`:
- `bulk`:
- `boundary` (Minkowski):
- `$\chi \sim L \sim 1/Q$`

New ideas in hadronization: Intersections between QCD, AdS/CFT and the QGP, IP3, Durham, April 15–17, 2009
Physical interpretation

- **Rôle of the 5th dimension:** a reservoir of quantum fluctuations.

- **Radial penetration** $\chi$ of the wave packet in $AdS_5$ ↔
  transverse size $L$ of the partonic fluctuation on the boundary

\[ q = (\omega, 0, 0, k) \sim t^{1/2} \quad L \sim 1/Q \]

- **A space–like photon cannot decay** (in the vacuum)

- **Virtual partonic fluctuations** with
  - transverse size $L \sim 1/Q$
  - and lifetime $\Delta t \sim \omega/Q^2$
Partonic fluctuation in the plasma

- A plasma at finite temperature: the space–like photon can decay due to the parton interactions in the medium

- Strong coupling:
  The potential barrier $\sim Q$ (energy–momentum conservation) disappears with increasing energy ($\omega$) or temperature ($T$)
Saturation momentum

- Gravitational interactions are proportional to the energy density in the wave ($\omega$) and in the plasma ($T$).

- The criterion for strong interaction within the plasma:

  $Q \lesssim \omega \frac{T^2}{Q^2} \times \frac{T^2}{\text{virtuality barrier}} \lesssim \text{plasma force}$

- The partonic fluctuation must live long enough to feel the effects of the plasma.

- High energy, or high $T$, or low $Q$:

  $Q \lesssim Q_s$ with

  $Q_s \sim (\omega T^2)^{1/3} \approx \frac{T}{x}$ where $x \equiv \frac{Q^2}{2\omega T}$

- Physics: the photon can decay due to the plasma force.
Medium induced parton branching

- The virtual photon disappears into the plasma via its decay (and not via thermal scattering)!

- ‘Quasi–democratic branching’ (Hatta, E.I., Mueller, 08)

\[ \omega_n \sim \frac{\omega_{n-1}}{2} \sim \frac{\omega}{2^n} \]

\[ \Delta t_n \sim \frac{\omega_n}{Q^2_n} \]

\[ \frac{\Delta Q_n}{\Delta t_n} \sim -T^2 \]

Lifetime of the current: \( \Delta t \sim \frac{\omega}{Q^2_s} \ll \frac{\omega}{Q^2} \implies \) no time to form jets
**Motivation**

**Partons and jets in pQCD**

**AdS/CFT correspondence**

**Hard probes at strong coupling**

**DIS at strong coupling**

- Dipole in a plasma
- Saturation momentum
- Branching at strong coupling
- Isotropy
- Parton saturation
- No Jets
- Meson melting

**Heavy Quark**

**Conclusions**

**Backup**

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**e^+e^- at strong coupling**

- A time-like current can decay already in the vacuum

- Infrared cutoff $\Lambda \rightarrow$ splitting continues down to $Q \sim \Lambda$

- In the COM frame $\rightarrow$ spherical distribution $\implies$ no jets!

  (similar conclusion by Hofman and Maldacena, 2008)

- Final state looks very different as compared to pQCD!
Parton saturation at strong coupling

- $Q > Q_s(x) = T/x : F_2(x, Q^2) \approx 0$
  $\implies$ no partons at high $Q^2$/large $x$

- $Q < Q_s(x) = T/x : F_2(x, Q^2) \sim xN_c^2Q^2$
  $\implies$ parton saturation with occupation numbers $\sim \mathcal{O}(1)$

- All partons have branched down to small values of $x$!
No forward jets!

- No large-\(x\) partons \(\rightarrow\) no forward/backward jets in a hadron–hadron collision at strong coupling

\(\theta_{\text{min}}\)

- ‘The Nightmare of CMS’
“No drag force on a small meson in the plasma”

Larger mesons melt in the plasma: critical size $L_s \sim 1/Q_s$

\[
L_s \sim \frac{1}{Q_s} \quad & \quad \gamma \sim \frac{\omega}{Q} \quad \Rightarrow \quad L_s \sim \frac{1}{\sqrt{\gamma}T} = \frac{(1 - v_z^2)^{1/4}}{T}
\]

[Peeters, Sonnenschein, Zamaklar; Liu, Rajagopal, Wiedemann (06)]
Heavy Quark: Energy loss

- Virtual quanta with $Q \lesssim Q_s$ are absorbed by the plasma

- Parton formation time: $\Delta t \sim \omega/Q_s^2$

$$- \frac{dE}{dt} \simeq \sqrt{\lambda} \frac{\omega}{(\omega/Q_s^2)} \simeq \sqrt{\lambda} Q_s^2 \simeq \sqrt{\lambda} \gamma T^2$$

$$Q_s \simeq \frac{\omega}{Q_s^2} T^2 \simeq \frac{\gamma}{Q_s} T^2 \implies Q_s^2 \simeq \gamma T^2$$

Herzog, Karch, Kovtun, Kozcaz, and Yaffe; Gubser, 2006 (trailing string)
Momentum broadening

- **Fluctuations in the medium–induced emission process**

\[
\frac{d\langle p_T^2 \rangle}{dt} \sim \sqrt{\lambda} \frac{Q_s^2}{(\omega/Q_s^2)} \sim \sqrt{\lambda} \sqrt{\gamma} T^3
\]

*Casalderrey-Solana, Teaney; Gubser, 2006 (from trailing string)*

- **Non–local process**: no meaningful ‘jet quenching’ transport coefficient! *(A. Mueller et al, 2008)*
Momentum broadening

- Fluctuations in the medium–induced emission process

\[
\frac{d\langle p_T^2 \rangle}{dt} \sim \sqrt{\lambda} \frac{Q_s^2}{(\omega/Q_s^2)} \sim \sqrt{\lambda} \sqrt{\gamma} T^3
\]

- Longitudinal broadening is parametrically larger!

\[
\frac{d\langle p_T^2 \rangle}{dt} \sim \sqrt{\lambda} \frac{\omega^2}{(\omega/Q_s^2)} \sim \sqrt{\lambda} \sqrt{\gamma} \gamma^2 T^3
\]
Momentum broadening

- **Strong coupling**: fluctuations in the emission process

- **pQCD**: thermal rescattering (different physics!)

\[\gamma, \omega, Q\]
How are quantum–mechanical (as opposed to thermal) fluctuations encoded in AdS/CFT?

World–sheet horizon at $\chi_s = 1/Q_s \sim 1/(\sqrt{\gamma}T) \ll 1/T$

“Thermal fluctuations” with $T_{\text{eff}} = \sqrt{\gamma}T$ : Unruh temperature
Stochastic trailing string

- Langevin equation for the upper part of the string and the heavy quark

\[ \chi \]

\[ \frac{1}{\gamma^{1/2}T} \]

\[ 0 \]

\[ V \]

\[ \frac{1}{T} \]

- Encodes both energy loss and momentum broadening

\((G. \ Giecold, \ E.I., \ A. \ Mueller, \ 2009)\)
Conclusions

- **Hard probes & high-energy physics** appears to be quite different at strong coupling as compared to QCD
  - no jets in $e^+e^-$ annihilation
  - no forward/backward particle production in HIC
  - different mechanism for jet quenching

- **Are AdS/CFT methods useless for HIC?** Not necessarily so!
  - long–range properties (hydro, thermalization, etc) might still be controlled by strong coupling
  - some observables receive contributions from several scales, from soft to hard: use AdS/CFT in the soft sector
  - most likely, the coupling is moderately strong, so it useful to approach the problems from both perspectives
Elliptic flow at RHIC: The perfect fluid

- Non–central $AA$ collision: Pressure gradient is larger along $x$
  \[
  \frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi, \quad v_2 = \text{“elliptic flow”}
  \]

- Well described by hydrodynamical calculations with very small viscosity/entropy ratio: “perfect fluid”
Viscosity over entropy density ratio

- **Viscosity/entropy density ratio at RHIC** (in units of $\hbar$)
  \[
  \frac{\eta}{s} = 0.1 \pm 0.1(\text{theor}) \pm 0.08(\text{exp}) \ [\hbar]
  \]

- Weakly interacting systems have $\eta/s \gg \hbar$

- **Kinetic theory**: viscosity is due to collisions among molecules
  \[
  \eta \sim \rho v \ell = \text{mass density} \times \text{velocity} \times \text{mean free path} \sim \frac{1}{g^4}
  \]

- **Conjecture (from AdS/CFT)**: \cite{Kovtun:2003wp}
  \[
  \frac{\eta}{s} \geq \frac{\hbar}{4\pi} \quad \text{[lower limit = infinite coupling]}
  \]

- The RHIC value is at most a few times $\hbar/4\pi$!
Energy density as a function of $T$ (Bielefeld Coll.)

\[ \frac{\varepsilon}{T^4} \]

- RHIC
- LHC
- SPS

$T_c = (173 \pm 15) \text{ MeV}$

$\varepsilon_c \sim 0.7 \text{ GeV/fm}^3$

$\frac{\varepsilon}{\varepsilon_0} \approx 0.85$ for $T = 3T_c$

Is this deviation from ideal gas small? Or is it large?

AdS/CFT: $\frac{\varepsilon}{\varepsilon_0} \to \frac{3}{4}$ when $\lambda \to \infty$ ($\mathcal{N} = 4$ SYM)
Finite–$T$ : Resummed perturbation theory

- This ratio $p/p_0 \approx 0.85$ can be also explained by resummed perturbation theory
  (collective phenomena: screening, thermal masses)
  
  ($J.-P. Blaizot, A. Rebhan, E. Iancu, 2000$)

First principle calculation without free parameter
The ‘perfect fluid’

- Uncertainty principle applied to viscosity:

\[ \eta \sim \rho v \lambda_f , \quad S \sim n \sim \frac{\rho}{m} \]

\[ \frac{\eta}{S} \sim m v \lambda_f \sim \frac{\hbar}{\text{mean free path}} \quad \frac{\hbar}{\text{de Broglie wavelength}} \gtrsim \hbar \]

- Weakly interacting systems have \( \eta/S \gg \hbar \)

- Strongly coupled \( \mathcal{N} = 4 \) SYM plasma

\[ \frac{\eta}{S} \rightarrow \frac{\hbar}{4\pi} \quad \text{when} \quad \lambda \rightarrow \infty \]

(Policastro, Son, and Starinets, 2001)

- This bound is believed to be universal: \( \eta/S \geq \hbar/4\pi \)

- The data at RHIC are consistent with the lower limit being actually reached: ‘sQGP’
‘Multi–jet event’: large emission angle & $x \sim \mathcal{O}(1)$

\[ k_\perp \sim k \sim \sqrt{s} \implies \mathcal{P}_{\text{Brem}} \sim \alpha_s(s) \ll 1 \]

small probability for emitting an extra gluon jet!

‘Intra–jet activity’: collinear and/or soft gluons

\[ \Lambda_{\text{QCD}} \ll k_\perp \ll k \ll \sqrt{s} \implies \mathcal{P}_{\text{Brem}} \sim \alpha_s \ln^2 \frac{\sqrt{s}}{\Lambda_{\text{QCD}}} \sim \mathcal{O}(1) \]

modifies particle multiplicity but not the number of jets
Optical theorem

- Total cross-section given by the optical theorem

\[ \sigma(e^+e^-) = \frac{1}{2s} \ell^{\mu\nu} \text{Im} \Pi_{\mu\nu}(q) \]

- The quark loop: The vacuum polarization tensor \( \Pi_{\mu\nu} \) for a time-like photon (here, evaluated at one-loop order)

- This can be generalized to all-orders
Current–current correlator

\[ \sigma(e^+ e^-) = \frac{1}{2s} \ell^{\mu\nu} \text{Im} \Pi_{\mu\nu}(q) \]

\[ \Pi_{\mu\nu}(q) \equiv i \int d^4 x \ e^{-i q \cdot x} \langle 0 | T \{ J_{\mu}(x) J_{\nu}(0) \} | 0 \rangle \]

\[ J^\mu = \sum_f e_f \bar{q}_f \gamma^\mu q_f : \text{quark electromagnetic current} \]

- \( \Pi_{\mu\nu} = \) current–current correlator to all orders in QCD
- Valid to leading order in \( \alpha_{\text{em}} \) but all orders in \( \alpha_s \)
Gluons at HERA

\[ xg(x, Q^2) = \text{# of gluons with transverse area} \sim 1/Q^2 \text{ and } k_z = xP \]

\[ xg(x, Q^2) = x \frac{1}{x} = \frac{1}{x^2} \text{ over a wide range of } x. \]

\[ \frac{1}{x} \]}

- Rapid rise with \( 1/x \): \( xg(x, Q^2) \sim 1/x^\lambda \) with \( \lambda = 0.2 \div 0.3 \)
A small color dipole (‘meson’) with transverse size $L \ll 1/Q_s$ propagates through the strongly–coupled plasma with almost no interactions!

Larger dipoles with $L \gtrsim 1/Q_s$ cannot survive in the plasma:

$$L_s \sim \frac{1}{Q_s} \quad \& \quad \gamma \sim \frac{\omega}{Q} \implies L_s \sim \frac{1}{\sqrt{\gamma} T} \ll \frac{1}{T}$$

The dipole lifetime is short on natural time scales:

$$\Delta t \sim \frac{\omega}{Q_s^2} \sim \frac{\sqrt{\gamma}}{T} \ll \frac{\gamma}{T}$$
Momentum broadening

- Fluctuations in the medium–induced emission process

\[
\frac{d\langle p_{T}^2 \rangle}{dt} \sim \sqrt{\lambda} \frac{Q_s^2}{(\omega/Q_s^2)} \sim \sqrt{\lambda} \frac{Q_s^4}{\gamma Q_s} \sim \sqrt{\lambda} \sqrt{\gamma} T^3
\]

\[
\frac{d\langle p_{L}^2 \rangle}{dt} \sim \sqrt{\lambda} \frac{\omega^2}{(\omega/Q_s^2)} \sim \sqrt{\lambda} \sqrt{\gamma} \gamma^2 T^3
\]

Casalderrey-Solana, Teaney; Gubser, 2006 (from trailing string)
Saturation line: weak vs. strong coupling

Saturation exponent: \( Q_s^2(x) \propto \frac{1}{x^{\lambda_s}} \equiv e^{\lambda_s Y} \)

- weak coupling (LO pQCD): \( \lambda_s \approx 0.12 g^2 N_c \)
- phenomenology & NLO pQCD: \( \lambda_s \approx 0.2 \div 0.3 \)
- strong coupling (plasma): \( \lambda_s = 2 \) (graviton)
How are quantum–mechanical (as opposed to thermal) fluctuations encoded in AdS/CFT?

World–sheet horizon at $\chi_s = 1/Q_s \sim 1/(\sqrt{\gamma}T) \ll 1/T$

Hawking radiation (= thermal flucuts.) plays no role
(in contrast to a static string; cf. talk by Rangamani)
Stochastic trailing string

- Fluctuations on top of the world–sheet horizon $\chi_s$
  $\implies$ noise term on the ‘stretched horizon’ at $\chi = \chi_s + \epsilon$

- Langevin equation for the upper part of the string & the heavy quark  
  \[ \frac{1}{\gamma^{1/2}T} \]

- Physics: Fluctuations in the parton cascades  
  \( (G. \ Giecold, \ E.I., \ A. \ Mueller, \ 09) \)