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Possible Mechanism for Generating a Very Small Dirac Neutrino Mass

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Introduction

- NEUTRINO DO HAVE MASS [1]
- Is it a Majorana or a Dirac Particle (or A Mixed of Dirac and Majorana)?
- Why it is very small?

Majority of opinion: neutrino is a Majorana particle.

Majorana neutrino $\longrightarrow$ seesaw mechanism $\longrightarrow$ very small mass [2]

Could explain Leptogenesis (small $\Delta L = 2$) [3]

But

No conclusive evidence from the $0\nu\beta\beta$ decay [4]

Dirac neutrino can also support Leptogenesis [5]

So why not considering neutrino as a pure Dirac particle

A type-II seesaw-like mechanism

Assuming the existence of three Higgs scalars X, Y, and Z, with their SU(2)-rotated fields, $\tilde{X}, \tilde{Y}$, and $\tilde{Z}$.

The gauge group SU(2)$_L\times$ SU(2)$_R\times$ U(1)

(the coupling $g_L = g_R = g$ and g' respectively)

Using the following Yukawa coupling

$$-G_X\bar{\psi}_L X\psi_R - G_{\tilde{X}}\bar{\psi}_L\tilde{X}\psi_R + \mathrm{h.c.} \tag{1}$$

and a coupling among the three Higgs scalar particles

$$\mu(\epsilon_1 YXZ + \epsilon_2 Y\tilde{X}Z) + \mathrm{h.c.} \tag{2}$$

where G's are the Yukawa coupling constant, ϵ's are some coupling constant, and μ is some constant with a unit of mass. (No representation assign yet to X, Y, and Z, just a symbol to explain the diagram.)

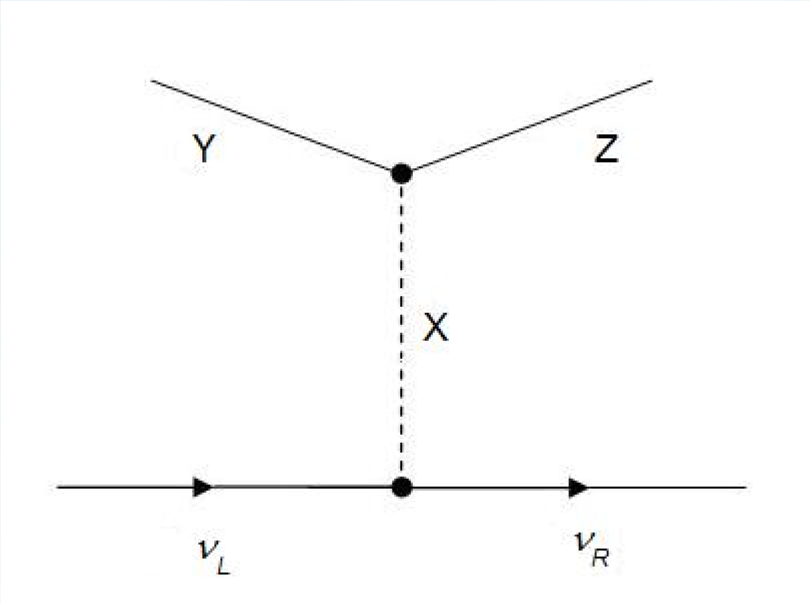

Figure 1: Diagram for a type-II seesaw-like mechanism

Possible Reps. assignment for X, Y, Z

1. The case where e_R, ν_R are singlets. Then X belongs to (**2**,1,1), a doublet of SU(2)$_L$. The Y and Z have two possibilities
   (a) Y will be a doublet of SU(2)$_L$ (**2**,1,1) and Z will be a singlet (or vise versa).
   (b) Y will be a doublet of SU(2)$_R$ (1,**2**,1) and Z belongs to (**2**,**2**,0) is a bidoublet (or vise versa).
2. The case where e_R, ν_R form a doublet of SU(2)$_R$ (1,**2**,-1). The X has to be a bidoublet (**2**,**2**,0). The Y and Z will be doublet of SU(2)$_R$ (1,**2**,-1) and SU(2)$_L$ (**2**,1,-1) respectively.

Case 1.a

The Higgs field contents of X and Y

$$X = \begin{pmatrix} X^+ \\ X^0 \end{pmatrix}_L ; \qquad Y = \begin{pmatrix} Y^0 \\ Y^- \end{pmatrix}_L . \tag{3}$$

The relevant Yukawa coupling for the neutrino and the charged lepton

$$-G_\nu \bar{\nu}_L X^{0*} \nu_R + \mathrm{h.c} \tag{4}$$

$$-G_e \bar{e}_L X^0 e_R + \mathrm{h.c} \tag{5}$$

The relevant three Higgs coupling

$$\mu \epsilon_1 Y^{0*} X^0 Z + \mu \epsilon_2 Y^{0*} X^{0*} Z + \mathrm{h.c.} \tag{6}$$

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upon integrating out X, we will have the following effective Lagrangian

$$\mu \frac{Y^{0*}Z}{M_X^2}((G_e\epsilon_1 + G_e\epsilon_2)\bar{e}_L e_R + \mu(G_\nu\epsilon_1 + G_\nu\epsilon_2)\nu_L\nu_R) + \mathrm{h.c.} \tag{7}$$

where M_X is the mass of the heavy X. When Y, Z acquired VEV, denoted by y and z respectively, we will have masses for the lepton

$$m_\nu = \mu G_\nu(\epsilon_1 + \epsilon_2)\frac{yz}{M_X^2} \tag{8}$$

$$m_e = \mu G_e(\epsilon_1 + \epsilon_2)\frac{yz}{M_X^2} \tag{9}$$

To have a small neutrino mass compared to the charged lepton mass we have to have $G_e >> G_\nu$. In this case we cannot break the parity unless we add additional left and right Higgs doublets with different VEV.

Case 1.b

The Higgs field contents of X, Y and Z

$$X = \begin{pmatrix} X^+ \\ X^0 \end{pmatrix}_L ; \qquad Y = \begin{pmatrix} Y^+ \\ Y^0 \end{pmatrix}_R . \tag{10}$$

$$Z = \begin{pmatrix} Z_1^0 & Z_1^+ \\ Z_2^- & Z_2^0 \end{pmatrix} . \tag{11}$$

The relevant Yukawa coupling for the neutrino and the charged lepton

$$-G_\nu \bar{\nu}_L X^{0*} \nu_R - G_e \bar{e}_L X^0 e_R + \mathrm{h.c} \tag{12}$$

The relevant three Higgs coupling

$$\mu \epsilon_1 X^{0*} Z_2^0 Y^0 + \mu \epsilon_2 X^{0*} Z_1^{0*} Y^0 + \mathrm{h.c.} \tag{13}$$

upon integrating out the X, we will have the following effective Lagrangian

$$\mu\left(\epsilon_1\frac{Y^{0*}Z_2^0}{M_X^2}+\epsilon_2\frac{Y^{0*}Z_1^{*0}}{M_X^2})(G_e\bar{e}_Le_R+G_\nu\nu_L\nu_R\right)+\mathrm{h.c.} \tag{14}$$

where M_X is the mass of the heavy X. When Y, Z acquired VEV, assuming that VEV of $< Z_1 > \approx < Z_2 > \approx z$, and the VEV of Y denoted by y, we will have masses for the lepton

$$m_\nu \approx \mu G_\nu(\epsilon_1+\epsilon_2)\frac{yz}{M_X^2} \tag{15}$$

$$m_e \approx \mu G_e(\epsilon_1+\epsilon_2)\frac{yz}{M_X^2} \tag{16}$$

Again to have a small neutrino mass compared to the charged lepton mass we have to have $G_e >> G_\nu$.

But unlike case 1.a, here we already have two doublet with different VEV. Because the VEV of X is small, the weak gauge bosons obtained masses from the Y and Z. The mass of the weak gauge bosons (in the matrix form)

$$\begin{array}{cc} & \begin{array}{cc} W_L^+ & W_R^+ \end{array} \\ \begin{array}{c} W_L^- \\ W_R^- \end{array} & \begin{pmatrix} \frac{1}{2}g^2z^2 & -\frac{1}{2}g^2z^2 \\ -\frac{1}{2}g^2z^2 & \frac{1}{4}g^2(2z^2+y^2) \end{pmatrix} \end{array}, \tag{17}$$

with the eigenvalues (with $y >> z$)

$$M_{W_1}^2 \approx g^2\left(\frac{1}{2}z^2 + \frac{1}{4}y^2\right) \qquad M_{W_2}^2 \approx g^2\left(\frac{1}{2}z^2 - \frac{z^4}{y^2}\right) \tag{18}$$

Thus we have a very massive right charged weak bosons $W_R^{\pm}$ and a less massive left charged weak bosons $W_R^{\pm}$ with a small mixing.

For the neutral gauge bosons,

$$\begin{array}{cc} & \begin{array}{ccc} W_{3L} & W_{3R} & B \end{array} \\ \begin{array}{c} W_{3L} \\ W_{3R} \\ B \end{array} & \begin{pmatrix} \frac{1}{2}g^2z^2 & -\frac{1}{2}g^2z^2 & 0 \\ -\frac{1}{2}g^2z^2 & \frac{1}{4}g^2(2z^2+y^2) & -\frac{1}{4}gg'y^2 \\ 0 & -\frac{1}{4}gg'y^2 & \frac{1}{4}g'^2y^2 \end{pmatrix} \end{array} \tag{19}$$

with the eigenvalues

$$M_{Z_R}^2 \approx \frac{1}{2}g^2z^2 + \frac{1}{4}(g^2 + g'^2)y^2 \tag{20}$$

$$M_{Z_L}^2 \approx \frac{1}{2}g^2z^2 - \frac{g^2(z^2 - (g'^2/2))z^2}{(g^2 + g'^2)y^2} \tag{21}$$

and one massless boson (foton) $M_A = 0$.

Case 2

The Higgs field contents of X, Y and Z

$$X = \begin{pmatrix} X_1^0 & X_1^+ \\ X_2^- & X_2^0 \end{pmatrix}. \tag{22}$$

$$Z = \begin{pmatrix} Z^0 \\ Z^- \end{pmatrix}_L ; \qquad Y = \begin{pmatrix} Y^0 \\ Y^- \end{pmatrix}_R . \tag{23}$$

The relevant Yukawa coupling for the neutrino and the charged lepton

$$-G_1 \bar{\nu}_L X_1^0 \nu_R - G_2 \bar{\nu}_L X_2^{0*} \nu_R + \mathrm{h.c} \tag{24}$$

$$-G_1 \bar{e}_L X_2^0 e_R - G_2 \bar{e}_L X_1^{0*} e_R + \mathrm{h.c} \tag{25}$$

The relevant three Higgs coupling are

$$\mu\epsilon_1 Y^{0*} X_1^0 Z^0 + \mu\epsilon_2 Y^{0*} X_2^{0*} Z^0 + \text{h.c.} \tag{26}$$

upon integrating out the X, we will have the following effective Lagrangian

$$\mu\Big((\epsilon_1 G_1 + \epsilon_2 G_2)\bar{\nu}_L \nu_R + (\epsilon_1 G_2 + \epsilon_2 G_1)\bar{e}_L e_R\Big)\frac{\bar{Y}^{0*} Z^0}{M_X^2} + \text{h.c.} \tag{27}$$

where M_X is the mass of the heavy X. When Y, Z acquired VEV, denoted by y and z respectively, we will have masses for the lepton

$$m_\nu \approx \mu(G_1\epsilon_1 + G_2\epsilon_2)\frac{yz}{M_X^2} \tag{28}$$

$$m_e \approx \mu(G_1\epsilon_2 + G_2\epsilon_1)\frac{yz}{M_X^2} \tag{29}$$

Thus in order to have a small neutrino mass (compared to the charged lepton mass), we can choose ϵ_1 and G_2 to be very small compared to ϵ_2 and G_1.

Assuming $(\epsilon_1, G_2) \propto 10^{-6}$, $(\epsilon_2, G_1) \propto 1$, $\mu \propto 1$ GeV, $y \propto 10^3$ GeV, $z \propto 10^2 GeV$, and $M_X \propto 10^4$ GeV, we will have $m_\nu \propto 1$ eV

Regarding the parity breaking, the mass of the weak gauge boson will come from the VEV of y and z. The mass of the weak gauge boson, in the matrix form is given by

$$\begin{array}{c c} & \begin{array}{cc} W_L^+ & W_R^+ \end{array} \\ \begin{array}{c} W_L^- \\ W_R^- \end{array} & \begin{pmatrix} \frac{1}{4}g^2 z^2 & 0 \\ 0 & \frac{1}{4}y^2 \end{pmatrix} \end{array}, \tag{30}$$

There is no mixing (but, actually there is a very small mixing due to a very small VEV of the bidoublet X)

$$\begin{array}{c c} & \begin{array}{ccc} W_{3L} & W_{3R} & B \end{array} \\ \begin{array}{c} W_{3L} \\ W_{3R} \\ B \end{array} & \begin{pmatrix} \frac{1}{4}g^2 z^2 & 0 & -\frac{1}{4}gg' z^2 \\ 0 & \frac{1}{4}g^2 y^2 & -\frac{1}{4}gg' y^2 \\ -\frac{1}{4}gg' z^2 & -\frac{1}{4}gg' y^2 & \frac{1}{4}g'^2(z^2 + y^2) \end{pmatrix} \end{array} \tag{31}$$

with the eigenvalues

$$M_{Z_R}^2 \approx \frac{1}{4}(g^2 + g'^2)y^2 + \frac{1}{8}(g^2 + g'^2)z^2 \tag{32}$$

$$M_{Z_L}^2 \approx \frac{1}{8}(g^2 + g'^2)z^2 \tag{33}$$

and one massless gauge (foton)

$$M_A = 0. \tag{34}$$

Koide Relation

The fact that the mass of the charged lepton above come from a seesaw-like mechanism is interesting, because there is a nice charged lepton relation by Koide long time ago [7]

$$\frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu}\sqrt{m_\tau})^2} = \frac{2}{3} \tag{35}$$

Koide himself suggest that the condition $m_i \propto v_i^2$ is required. This is similar to a seesaw mechanism.

Conclusion

- We can have a (type-II) seesaw mechanism with Dirac neutrino
- There are three possible mechanism for a type II seesaw-like mechanism, and the one with the intermediary field is a bidoublet (case 2) is the better one.
- Seesaw mechanism for charged leptons is supporting the Koide mass relation.

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THANK YOU