Neutrino Mass Models and Lepton Flavor Violation

Ernest Ma
Physics and Astronomy Department
University of California
Riverside, CA 92521, USA

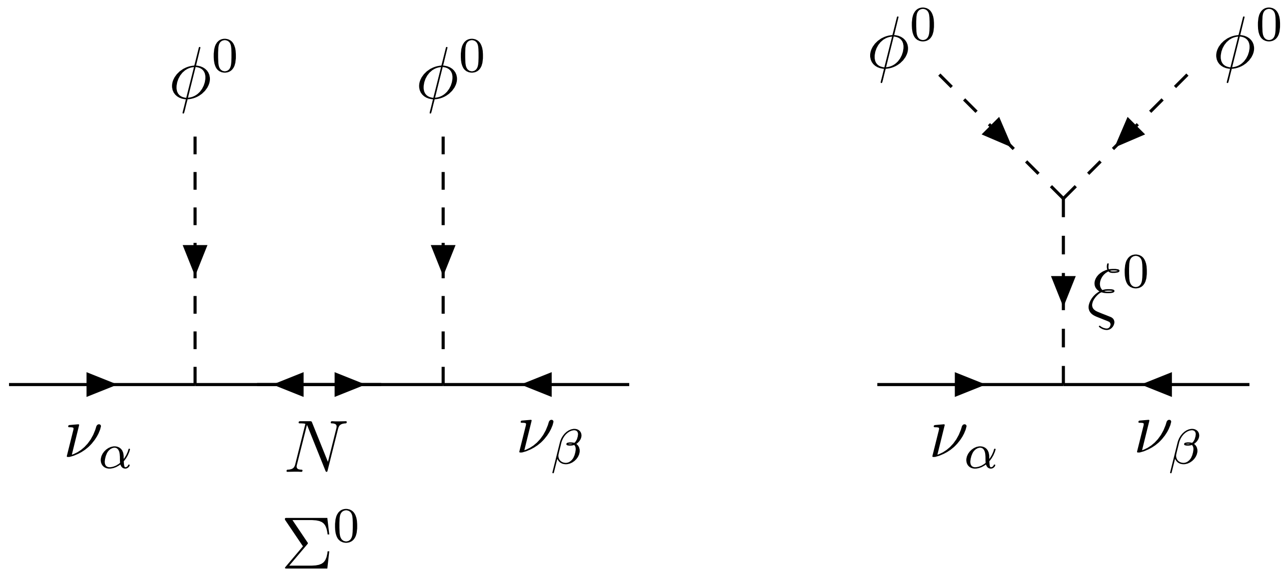
Contents

Introduction: Six Generic Mechanisms

Weinberg(1979): Unique dimension-five operator for Majorana neutrino mass in the standard model (SM):

$$\frac{f_{\alpha\beta}}{2\Lambda}(\nu_\alpha\phi^0 - l_\alpha\phi^+)(\nu_\beta\phi^0 - l_\beta\phi^+) \Rightarrow \mathcal{M}_\nu = \frac{f_{\alpha\beta}v^2}{\Lambda}.$$

Ma(1998): Three tree-level realizations:
(I) fermion singlet N, (II) scalar triplet (ξ^{++}, ξ^+, ξ^0),
(III) fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^-)$
[Foot/Lew/He/Joshi(1989)];
and three generic one-loop realizations (IV), (V), (VI).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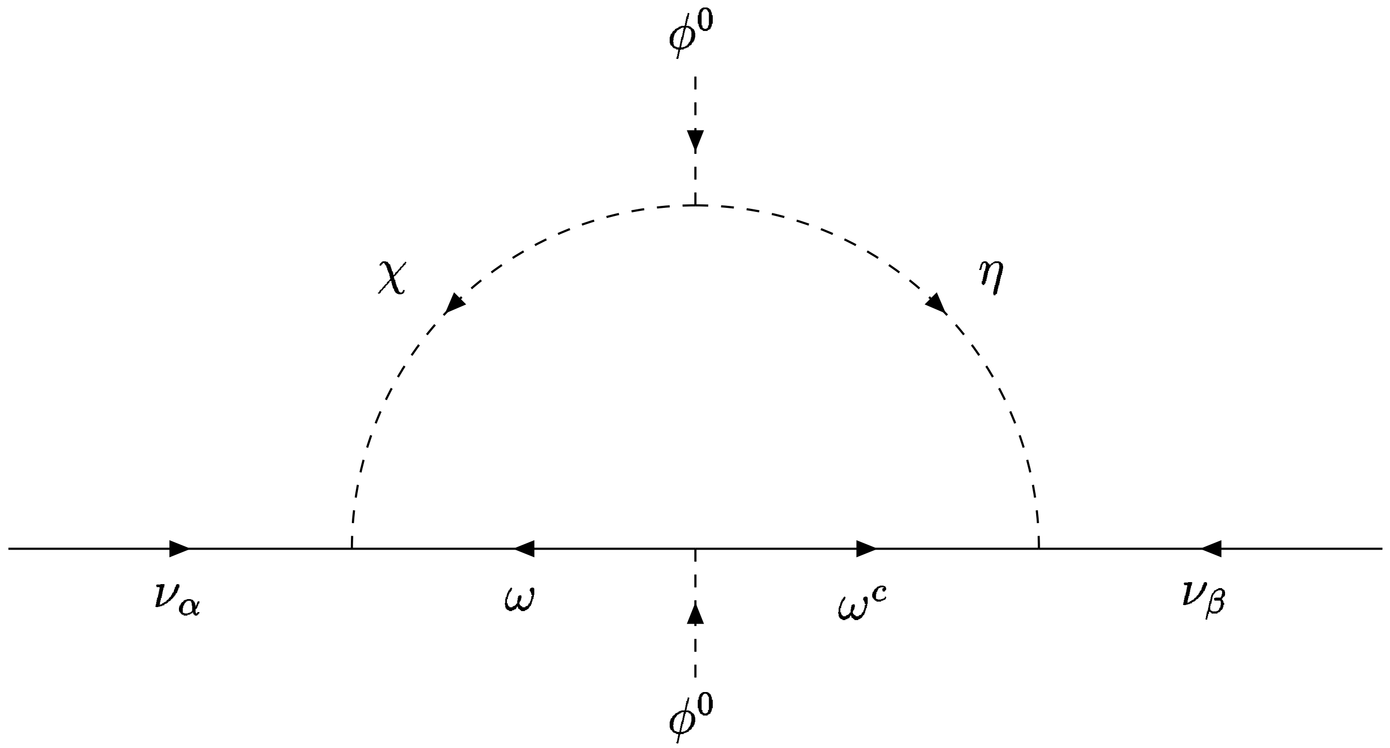
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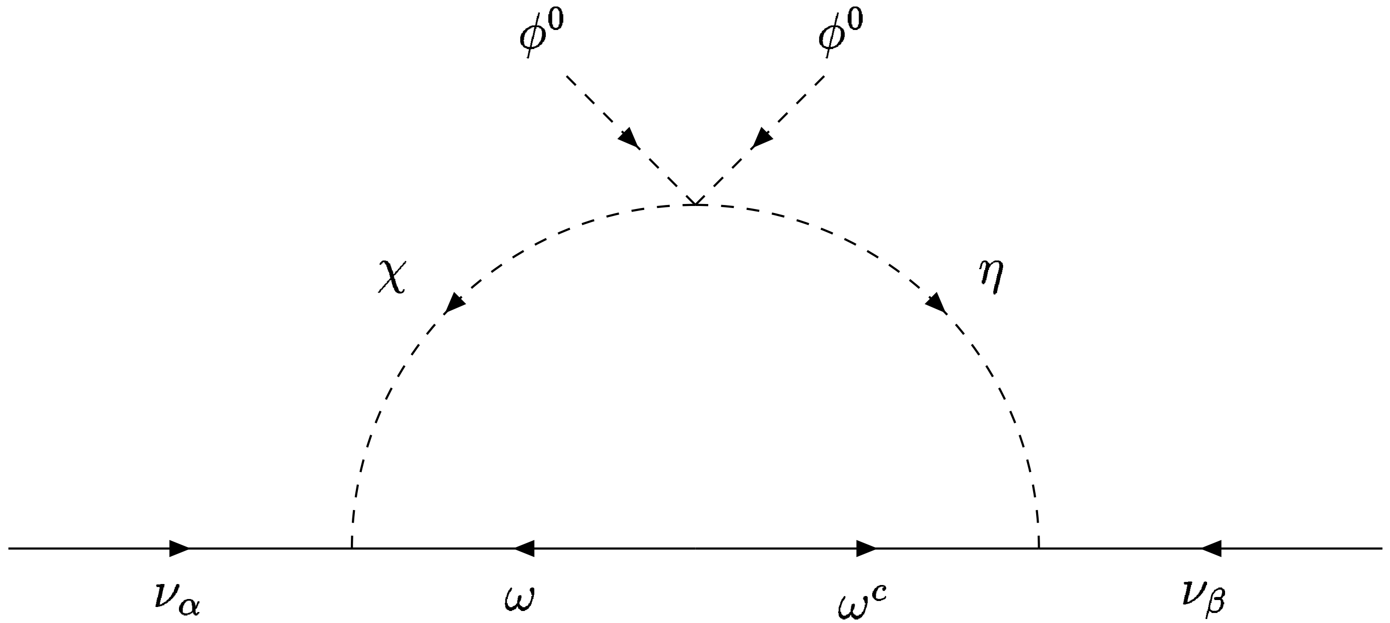
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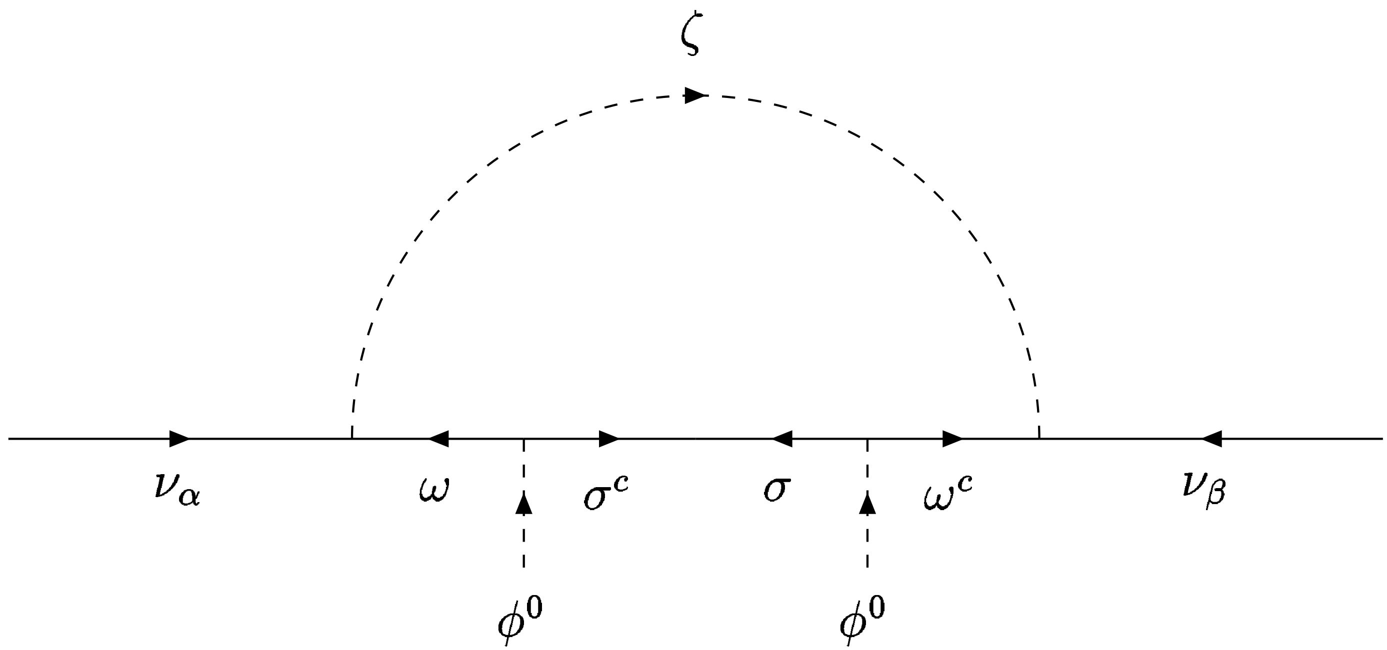
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In this talk, I assume

(1) m_ν is Majorana and $(-)^L$ is conserved;

(2) New physics appear at the TeV scale.

There are two categories of neutrino mass models:

(A) m_ν comes from mixing, then $U_{l\nu}$ is not unitary, and Lepton Flavor Violation occurs at tree level;

(B) m_ν does not come from mixing, then $U_{l\nu}$ is unitary, and LFV usually occurs in one loop, but could also be at tree level.

Seesaw Without Mixing

Higgs triplet (Type II Seesaw):
$\xi^0 \nu_i \nu_j + \xi^+ (\nu_i l_j + l_i \nu_j)/\sqrt{2} + \xi^{++} l_i l_j$
$\Rightarrow m_\nu$ comes directly from $\langle \xi^0 \rangle$. From the terms
$M^2 \xi^\dagger \xi + f_1 (\Phi^\dagger \Phi)(\xi^\dagger \xi) + f_2 (\xi^\dagger \Phi)(\Phi^\dagger \xi) + [\mu \xi^\dagger \Phi \Phi + H.c.]$
in the Higgs potential,

$$\langle \xi^0 \rangle \simeq -\mu \langle \phi^0 \rangle^2 / (M^2 + (f_1 + f_2) \langle \phi^0 \rangle^2)$$

is naturally small, using either the canonical seesaw ($\mu \sim M$) or the inverse seesaw ($\mu << M$), where $\mu = 0 \Rightarrow (-)^L$ becomes L.

$\xi^{++} \to l_i^+ l_j^+$ is a great signature at the LHC and its decay branching fractions map out the relative entries of the neutrino mass matrix. [Ma/Raidal/Sarkar(2000)]

Unitarity of $U_{l\nu}$ is maintained, but important constraints exist from LFV:

$l_i \to l_j \gamma$ through ξ^{++}, ξ^{+} in one loop;
$l_i \to l_j l_k l_m$ through ξ^{++} at tree level.

With present knowledge of the neutrino mass matrix (approximate tribimaximal form), correlated predictions of this model at the TeV scale should be studied.

[Perhaps they have already been done?]

Radiative Seesaw from Dark Matter

Deshpande/Ma(1978): Add to the SM a second scalar doublet (η^+, η^0) which is odd under a new exactly conserved Z_2 discrete symmetry, then η^0_R or η^0_I is absolutely stable. This simple idea lay dormant for almost thirty years until Ma, Phys. Rev. D 73, 077301 (2006). It was then studied seriously in Barbieri/Hall/Rychkov(2006), Lopez Honorez/Nezri/Oliver/Tytgat(2007), Gustafsson/Lundstrom/Bergstrom/Edsjo(2007), and Cao/Ma/Rajasekaran, Phys. Rev. D 76, 095011 (2007).

Radiative Neutrino Mass:

Zee(1980): (IV)

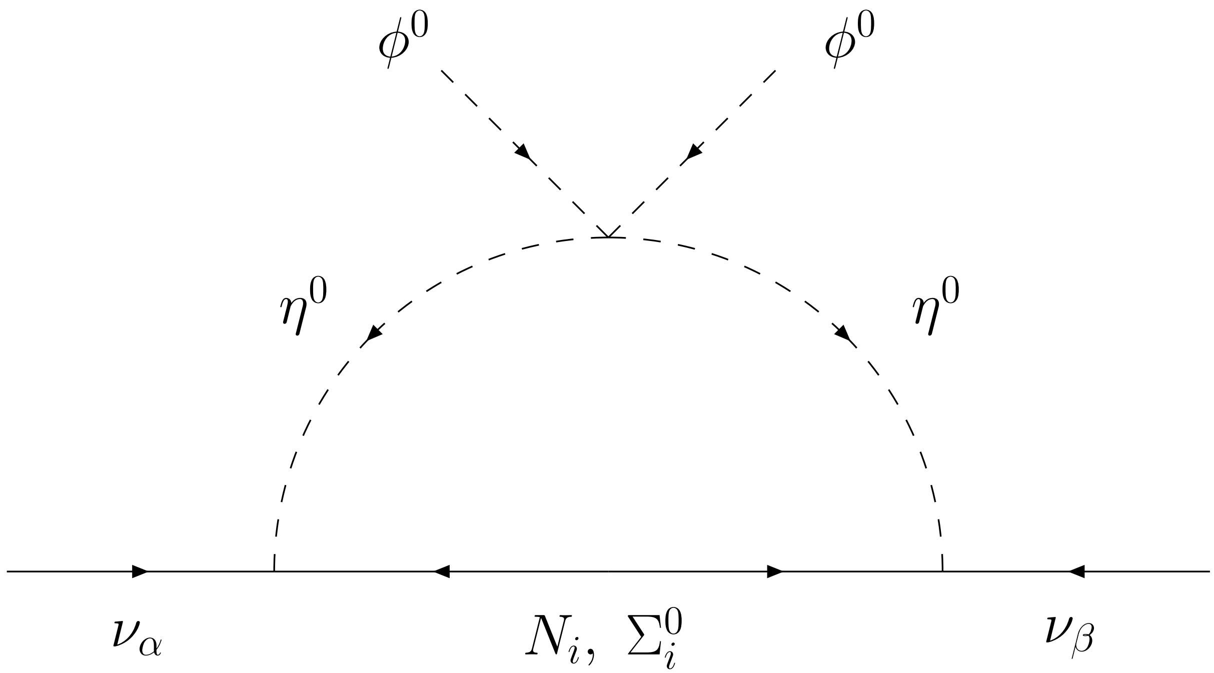
$\omega = (\nu, l), \omega^c = l^c$, $\chi = \chi^+, \eta = (\phi^+_{1,2}, \phi^0_{1,2}), \langle \phi^0_{1,2} \rangle \neq 0$.

Ma(2006): (V) [scotogenic = caused by darkness]

$\omega = \omega^c = N$ or Σ, $\chi = \eta = (\eta^+, \eta^0), \langle \eta^0 \rangle = 0$.

N or Σ interacts with ν, but they are not Dirac mass partners, because of the exactly conserved Z_2 symmetry, under which N or Σ and (η^+, η^0) are odd, and all SM particles are even. Using $f(x) = -\ln x/(1-x)$,

$$(\mathcal{M}_\nu)_{\alpha\beta} = \sum_i \frac{h_{\alpha i} h_{\beta i} M_i}{16\pi^2} [f(M_i^2/m_R^2) - f(M_i^2/m_I^2)].$$


ϕ^0
ϕ^0
η^0
η^0
ν_α
$N_i,\ \Sigma_i^0$
ν_β

Unitarity of $U_{l\nu}$ is again maintained, but one-loop LFV is a generic constraint on the $(\nu_i\eta^0 - l_i\eta^+)N_j$ couplings. This makes it difficult for N to be dark matter (although $\eta^0_{R,I}$ is still fine).

If $(\Sigma^+, \Sigma^0, \Sigma^-)$ is used instead of N, then Σ^0 itself may be a good dark-matter candidate.

There is no mixing between ν and Σ^0, or e^- and Σ^-, so the phenomenology here is different from that of the usual Type III seesaw. [See for example del Aguila/Aguilar-Saavedra (2009), Arhrib/Bajc/Ghosh/Han/Huang/Puljak/Senjanovic(2009).]

Double and Inverse Seesaws

Canonical (Type I) Seesaw:

$$\mathcal{M}_{\nu,N} = \begin{pmatrix} 0 & m_2 \\ m_2 & m_N \end{pmatrix}$$

$\Rightarrow m_\nu \simeq -m_2^2/m_N$ and $\nu - N$ mixing $\simeq m_2/m_N \simeq \sqrt{m_\nu/m_N} < 10^{-6}$ for $m_\nu < 1$ eV and $m_N > 1$ TeV.

Double Seesaw:

$$\mathcal{M}_{\nu,N,S} = \begin{pmatrix} 0 & m_2 & 0 \\ m_2 & 0 & m_1 \\ 0 & m_1 & m_S \end{pmatrix} .$$

Let $m_2 << m_N \simeq -m_1^2/m_S << m_1 << m_S$, then $m_\nu \simeq -m_2^2/m_N \simeq m_2^2 m_S/m_1^2$, and $\nu - N$ mixing $\simeq m_2/m_N \simeq m_2 m_S/m_1^2 \simeq m_\nu/m_2$, which is the same as in the canonical seesaw. The $\nu - S$ mixing $\simeq m_2/m_S$ is even more negligible.

Inverse Seesaw [Mohapatra/Valle(1986)]:

$$\mathcal{M}_{\nu,N,S} = \begin{pmatrix} 0 & m_2 & 0 \\ m_2 & m_N & m_1 \\ 0 & m_1 & m_S \end{pmatrix}.$$

The limit $m_S = m_N = 0 \Rightarrow \nu$ and S have $L = 1$ and N has $L = -1$. Thus m_s, m_N may be naturally small.

Let $m_S << m_2 << m_1 << m_1^2/m_N$, then $m_\nu \simeq (m_2^2/m_1^2)m_S$, which is of the same form as in the double seesaw, and $\nu - N$ mixing remains $\simeq m_\nu/m_2$, i.e. negligible, but dramatically, $\nu - S$ mixing $\simeq m_2/m_1$ which can be big, and unitarity violation and LFV may be observable. [Malinsky/Ohlsson/Xing/Zhang(2009).] To illustrate, let $m_2 \sim 10$ GeV, $m_1 \sim 1$ TeV, $m_S \sim 10$ keV, then $m_\nu \sim 1$ eV and $\nu - S$ mixing $\sim 10^{-2}$.

To enforce the form of the inverse seesaw with m_S a naturally small two-loop radiative effect, consider [Ma(2009)] $SO(10) \to SU(5) \times U(1)_\chi \to$

$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\chi$, where $Q_\chi = 5(B-L) - 4Y$. Then $(u,d), u^c, e^c$ have $Q_\chi = 1$, $(\nu, e), d^c$ have $Q_\chi = -3$, N^c has $Q_\chi = 5$, and (ϕ^+, ϕ^0) has $Q_\chi = -2$. Add scalar singlets $\eta_1 \sim 1$, $\eta_2 \sim 2$, and fermion singlets $S_3 \sim -3$, $S_2 \sim 2$, $S_1 \sim -1$. This particle content $\Rightarrow$ conserved lepton parity: $\nu, e, e^c, N^c, S_3, S_1, \eta_1$ are odd and S_2, η_2 are even. Hence

$$\mathcal{M}_{\nu, N^c, S_3, S_1} = \begin{pmatrix} 0 & m_2 & 0 & 0 \\ m_2 & 0 & m_1 & 0 \\ 0 & m_1 & 0 & 0 \\ 0 & 0 & 0 & m_1' \end{pmatrix},$$

where m_1 and m'_1 come from $\langle\eta_2\rangle$, and $\langle\eta_1\rangle = 0$.
S_2 gets a one-loop radiative Majorana mass from m'_1 through $(S_2 S_1 \eta_1^\dagger)^2(\eta_1\eta_1\eta_2^\dagger)$.
Then S_3 gets a two-loop radiative scotogenic Majorana mass, i.e. m_S, through $(S_3 S_2 \eta_1)^2(\eta_1^\dagger\eta_1^\dagger\eta_2)$.

Bonus: S_2 is a dark-matter candidate.

In general, the inverse seesaw mechanism is better motivated with an extra gauge symmetry at the TeV scale. This predicts a Z' boson, which would be easy to find at the LHC if kinematically allowed.

Linear and Lopsided Seesaws

Linear Seesaw [Malinsky/Romao/Valle(2005)]:

$$\mathcal{M}_{\nu,N,S} = \begin{pmatrix} 0 & m_2 & m_2' \\ m_2 & 0 & m_1 \\ m_2' & m_1 & 0 \end{pmatrix},$$

so that $m_\nu \simeq -2m_2m_2'/m_1$. However, the two singlets may be rotated by $\theta = \tan^{-1}(m_2'/m_2)$, and this becomes the matrix of the inverse seesaw, with $m_S = -m_1 \sin 2\theta \simeq -2m_1m_2'/m_2$, resulting in $m_\nu \simeq (m_2/m_1)^2 m_S = -2m_2m_2'/m_1$ as expected.

Lopsided Seesaw [Ma(2009)]:

$$\mathcal{M}_{\nu,N,S} = \begin{pmatrix} 0 & m_2 & 0 \\ m_2 & m_N & m_1 \\ 0 & m_1 & m_S \end{pmatrix}.$$

Let $m_2 << m_N$ and $m_1^2/m_N << m_S << m_1 << m_N$. Then $m_\nu \simeq -m_2^2/m_N$ as in the canonical seesaw and $\nu - N$ mixing is again negligible, but $\nu - S$ mixing is now $m_1 m_2/m_S m_N < m_2/m_1$ and can be big. For example, let $m_2 \sim 1$ GeV, $m_N \sim 10^9$ GeV, $m_1 \sim 10$ GeV, $m_S \sim 1$ keV, then $m_\nu \sim 1$ eV, and $\nu - S$ mixing is 10^{-2}.

This scenario is easily supported by a $U(1)_N$ model [Ma(1996)], where $Q_N = 6Y_L + T_{3R} - 9Y_R$, from the $SU(3)_C \times SU(3)_L \times SU(3)_R$ decomposition of E_6.

Here $(u,d), u^c, e^c$ have $Q_N = 1$, $(\nu, e), d^c$ have $Q_N = 2$, N^c has $Q_N = 0$, and S has $Q_N = 5$. Two Higgs doublets $(\phi_1^0, \phi_1^-) \sim -3$ and $(\phi_2^+, \phi_2^0) \sim -2$ are needed.

Add Higgs singlets $\chi_1 \sim -5$ and $\chi_2 \sim 10$, then it is natural for $\langle \chi_2 \rangle << \langle \chi_1 \rangle$ through the $\chi_1^2 \chi_2$ term in the Higgs potential. This means that m_N is an invariant mass, m_1 comes from $\langle \chi_1 \rangle$ and m_S from $\langle \chi_2 \rangle$ as desired.

Conclusion

Neutrino mass models with new physics at the TeV scale, such as U(1) gauge symmetries, are very well suited to have significant effects in the unitarity violation of $U_{l\nu}$ and Lepton Flavor Violation. The SM singlet (S) that the neutrino mixes with may be at the TeV scale if it is the inverse seesaw or the linear seesaw. It may also be a light particle (sterile neutrino), as in the lopsided seesaw.

Discussion on seesaw texture:

It has been claimed that large mixing can occur in Type I seesaw if there is a special texture for m_D, i.e. 2 approximately zero mass eigenvalues for 3 families. To understand what this really means, consider 2 families for simplicity. The most general 4×4 mass matrix spanning (ν_1, ν_2, N_1, N_2) is

$$\begin{pmatrix} 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_2 \\ m_1 & 0 & M_1 & M_3 \\ 0 & m_2 & M_3 & M_2 \end{pmatrix}.$$

The texture hypothesis is equivalent to setting $m_1 = M_1 = 0$. It is clear that ν_1 and $\nu_2' = (M_3\nu_2 - m_2 N_1)/\sqrt{M_3^2 + m_2^2}$ are massless, allowing thus large $\nu_2 - N_1$ mixing.

For $0 \neq m_1 << m_2$ and $0 \neq M_1 << M_{2,3}$, the 2×2 reduced mass matrix is

$$\mathcal{M}_\nu = \begin{pmatrix} m_1^2 M_2/M_3^2 & -m_1 m_2/M_3 \\ -m_1 m_2/M_3 & m_2^2 M_1/M_3^2 \end{pmatrix},$$

resulting in canonical, linear , and inverse seesaws.

It is argued that these zeros are protected by chiral symmetry. This is obviously not true, because the one-loop diagram connecting ν_2 to itself through N_2 and the SM Higgs boson is infinite (from $M_2 \neq 0$).

To maintain this zero as a symmetry limit, $M_2 = 0$ is required. Lepton number conservation is then implied, with $L = 1$ for N_1 and $L = -1$ for N_2. In this case, $(\mathcal{M}_\nu)_{11}$ is negligible, and for $m_2/M_3 \sim 10^{-2}$, we need $m_1 < 100$ eV, and $M_1 < 10$ keV.

Conclusion: The texture idea (alone) for small masses and large mixing has no natural limit.