Lepton flavour violation in GUT theories

G.Ross, Coseners House, June 2009

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- Neutrino masses – Dirac or Majorana
- Rare lepton decays $\mu \to e\gamma, \tau \to \mu\gamma$ ....
- Testing the see saw

- Dirac or Majorana?

Dirac or Majorana?

$$\frac{f_{ij}}{\Lambda}\hat{\nu}_i \nu_j H^0 H^0 \quad \Rightarrow \quad \left(M_\nu\right)_{ij} = f_{ij}\frac{<H^0>^2}{\Lambda}$$

Weinberg 1979

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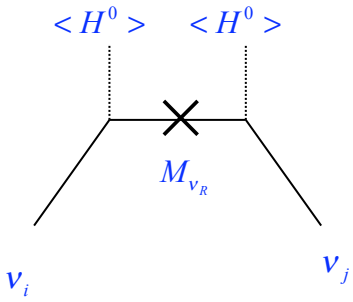
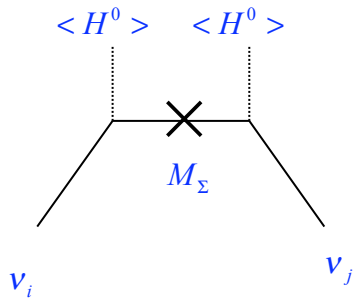
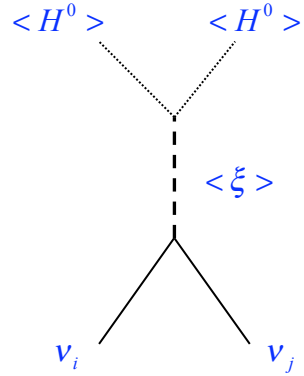
(A)	$\nu_i H^0 - l_i H^+$	(D)	$[\nu_i H^+, (\nu_i H^0 + l_i H^+)/\sqrt{2}, l_i H^0]$
(B)	$\nu_i l_j - l_i \nu_j$	(E)	$[\nu_i \nu_j, (\nu_i l_j + l_i \nu_j)/\sqrt{2}, l_i l_j]$
(C)	$H_1^+ H_2^0 - H_1^0 H_2^+$	(F)	$[H^+ H^+, \sqrt{2} H^+ H^0, H^0 H^0]$



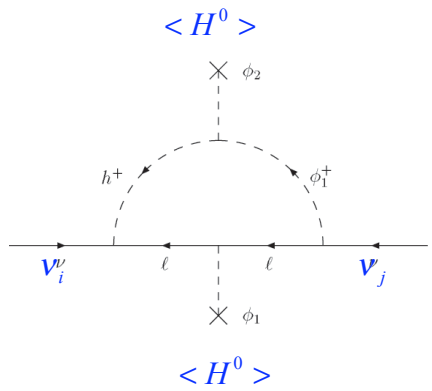
$(A)\times(A)$



$(D)\times(D)$



$(E)\times(F)$



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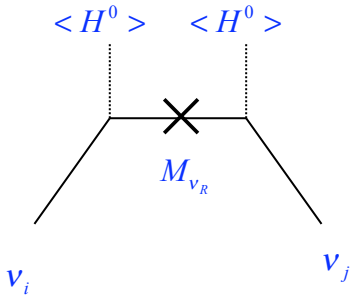
Minkowski mechanism



$(A)\times(A)$

$$m_{\nu_i} \sim \frac{<H^0>^2}{M_{\nu_R}}$$

$$\left(M_\nu = M_D^\nu \, M_M^{-1} \, M_D^{\nu T}\right)$$

$$m_{\nu_R} \sim 10^{13} GeV \left(\frac{H^0}{10^2 GeV}\right)^2$$

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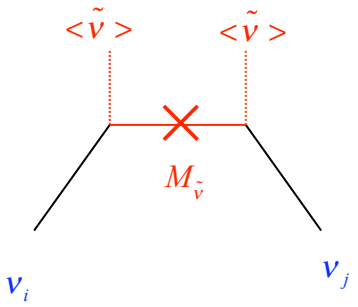
$SUSY \ \not{R}_P$



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$$m_{\nu_R} \sim 10^{13} GeV \left(\frac{H^0}{10^2 GeV}\right)^2$$

$$m_{\nu_i} \sim \frac{<\tilde{\nu}>^2}{M_N}$$

$$M_N \sim 10^3 GeV \left(\frac{<\tilde{\nu}>}{1MeV}\right)^2$$

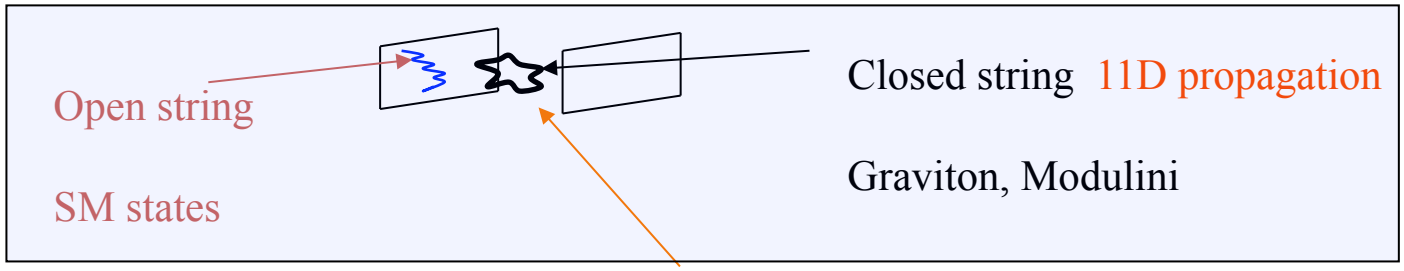
Dirac or Majorana?

$$h\,\bar{\nu}_L \nu_R H, \qquad m = h < H > \quad : \quad h < 10^{-12}\,?$$

Dirac or Majorana? $h\bar{\nu}_L \nu_R H, \qquad m = h < H > \quad : \quad h < 10^{-12}$?

- New space dimensions

Arkani Hamed, Dimopoulos, March Russell
Dienes,Farragi



Right-handed ν^s in the bulk

$$\lambda \int d^4x\, \nu_L(x) H(x) \nu_R(x, y=0)$$

$$\sum_n \frac{1}{\sqrt{2\pi R M_*}} \nu_{R,n}(x)\, e^{iny/R}$$

Flux spreading

$$m_\nu = \frac{\lambda < H > M_*}{M_{Planck}} \approx \lambda . 10^{-4} eV \left(\frac{M_*}{TeV} \right)$$

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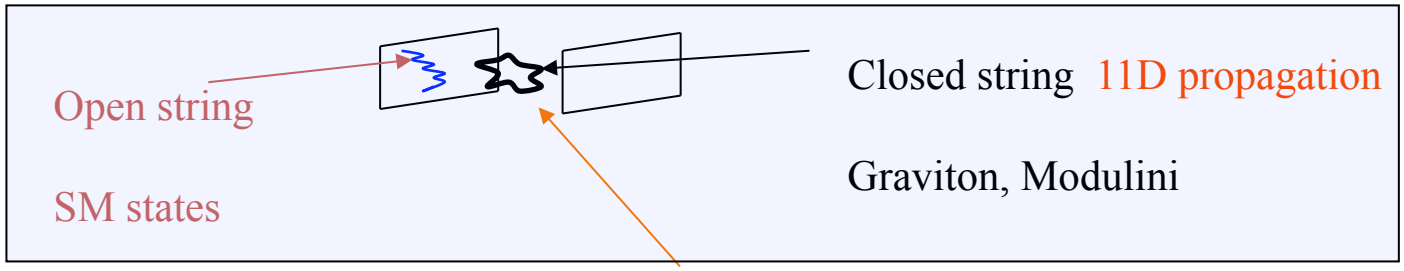

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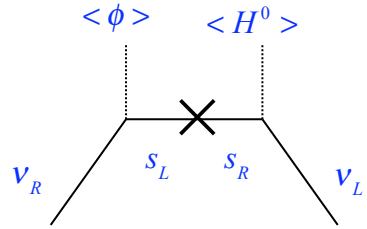
- In 4D small Dirac masses can arise through higher dimension operators

$$e.g. \quad K \supset \frac{F(\sigma,\sigma^*)}{M} L H_u \bar{N} + \frac{F'(\sigma,\sigma^*)}{M} L H_d^* \bar{N}$$

$$m_\nu \simeq \frac{\langle \phi \rangle}{M} < H >, \quad M \sim 10^{16} - 10^7 GeV$$

Abel, Dedes, Tamvakis

or



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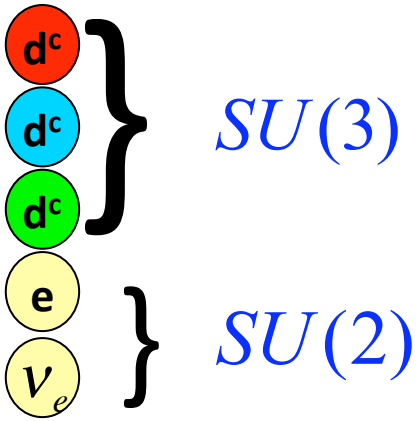
Cerdeno, Dedes, Underwood

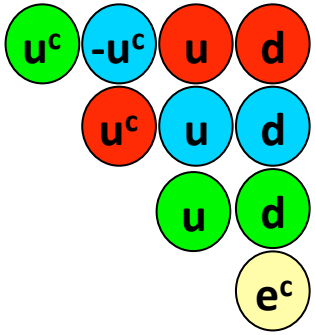
Grand Unification

$$SO(10) \supset SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$$

$(\bar{5})_L$:



$(10)_L$:



$$\nu_{e,L}{}^{c} \equiv \nu_{e,R}$$

$$(16)_L = (10)_L + (\bar{5})_L + (1)_L$$

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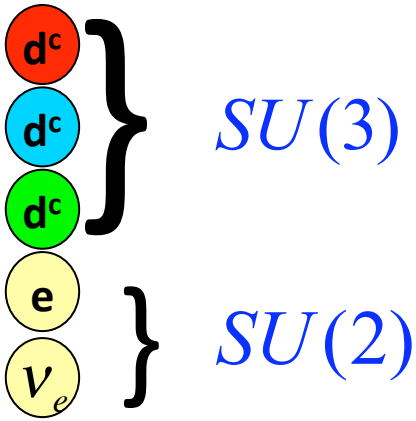

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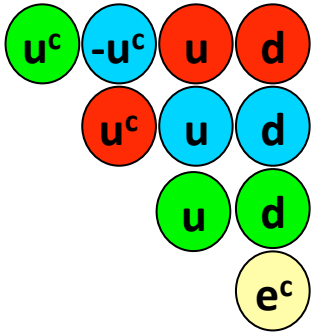

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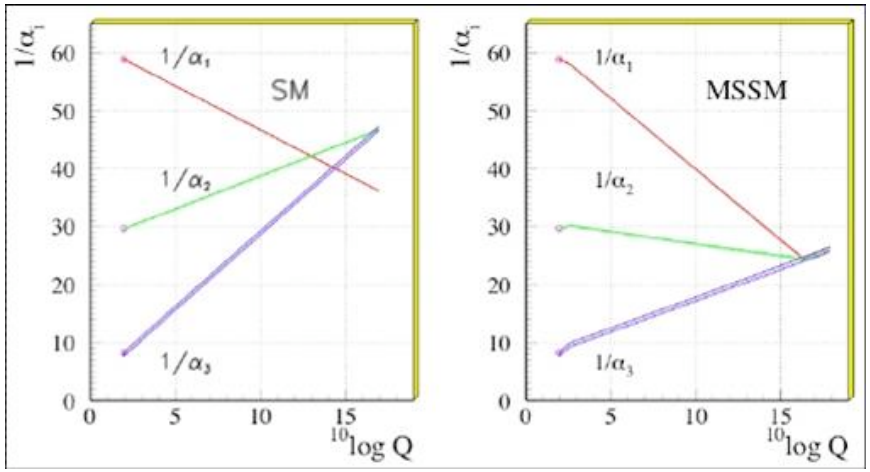
SUSY Grand Unification

Hierarchy problem : $m_{Higgs} << M_{GUT}$



Rare Lepton decays

$$\mu \to e\gamma, \tau \to \mu\gamma \; ....$$

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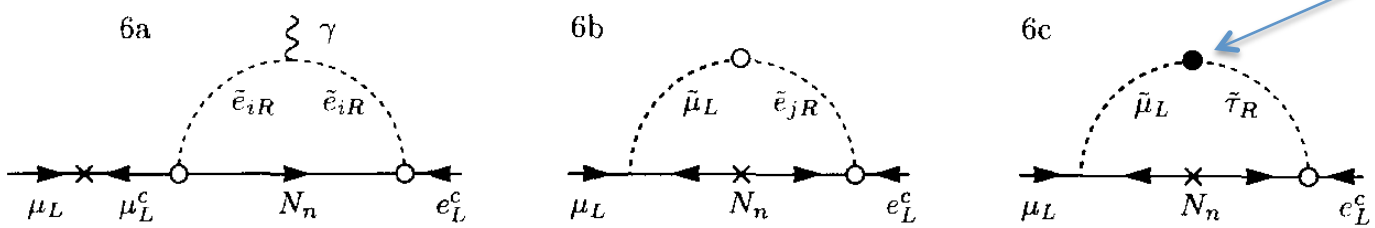
- Running from Planck scale

$$\frac{\Delta m_{ij}^2}{m_0^2} \simeq \frac{\lambda_i \lambda_j}{\pi^2} \ln\left(\frac{M_{Planck}}{M_G}\right)$$



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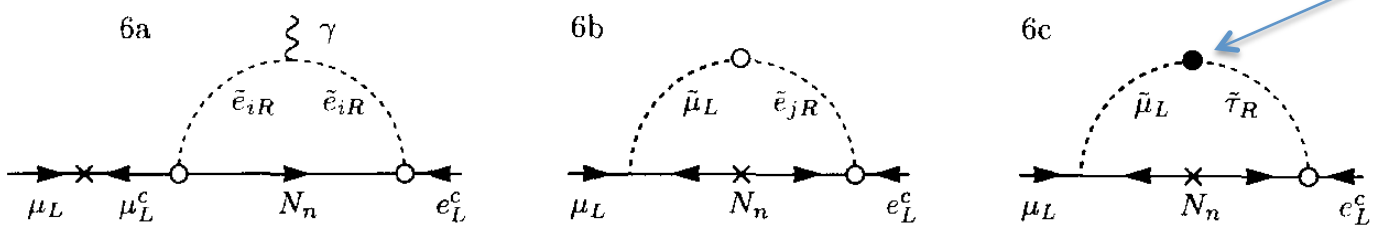


- Misalignment of soft terms

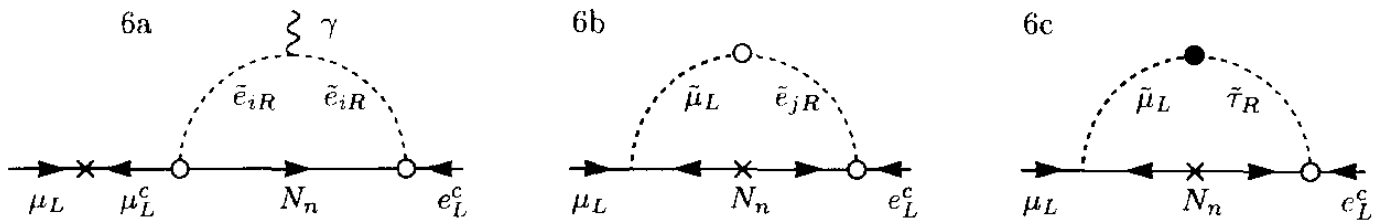
familon

e.g. $P = Y_{ij} Q_i q_j^c H_a \propto \left(\frac{\theta}{M}\right)^{\alpha(i,j)} Q_i q_j^c H_a$

Misalignment of Yukawa and A-terms?

$$A_{ij} Y_{ij} \tilde{Q}_i \tilde{q}_j^c H_a = (3+\alpha(i,j)) m_{3/2} Y_{ij} \tilde{Q}_i \tilde{q}_j^c H_a$$

- Running from the Planck scale $SU(5)$

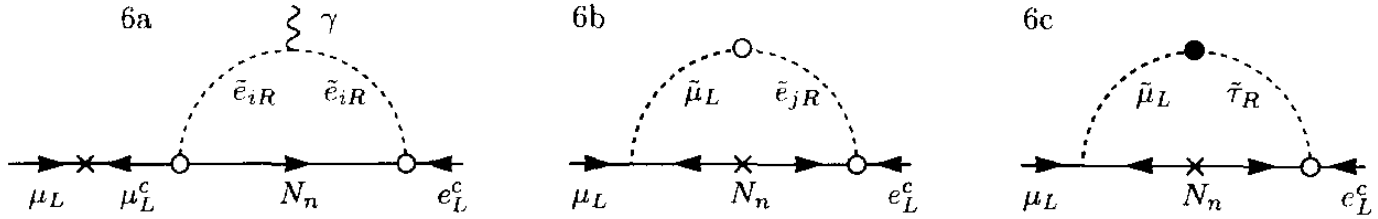


$$BR(\mu \to e\gamma) = 2.4 \times 10^{-12} \left(\frac{|V_{ts}|}{0.04}\frac{|V_{td}|}{0.01}\right)^2 \left(\frac{100GeV}{m_{\tilde{\mu}}}\right)^4 < 1.2 \times 10^{-11}$$

$$R(\mu \to e; Ti) = 5.1 \times 10^{-14} \left(\frac{|V_{ts}|}{0.04}\frac{|V_{td}|}{0.01}\right)^2 \left(\frac{100GeV}{m_{\tilde{\mu}}}\right)^4$$

$$B.R.(\tau \to \mu\gamma) \simeq 3.10^3 B.R.(\mu \to e\gamma) \simeq 7.2 \times 10^{-9}.. \qquad < 5.9 \times 10^{-8}$$

- Running from the Planck scale $SO(10)$



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$$\times \left(\frac{m_\tau}{m_\mu}\right)^2$$

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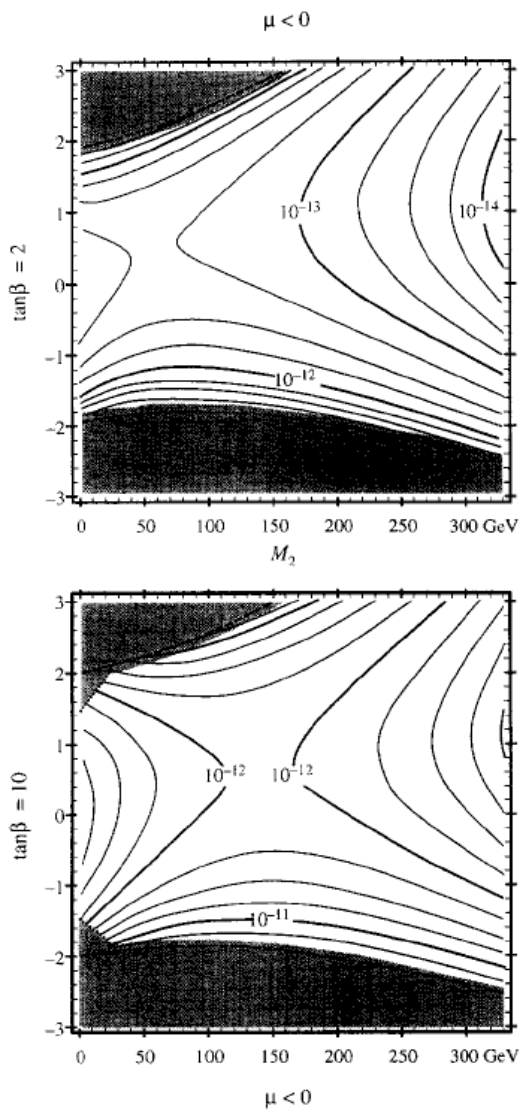
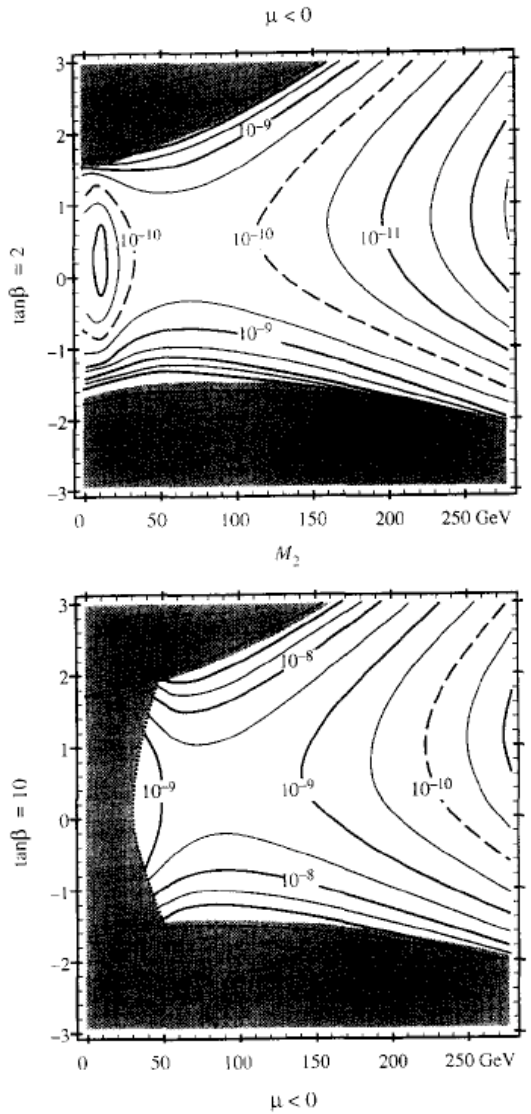


$SU(5), m_{\tilde{e}} = 300GeV$



$SO(10), m_{\tilde{e}} = 300GeV$

Barbieri, Hall, Strumia

- Misalignment of soft terms

$$(\hat{m}^2_{f,f^c})_{\overline{a}b} = m^2_{3/2}\left[\delta_{\overline{a}b}\left(k_0^{f,f^c} + l_0^{f,f^c}\frac{\langle X^\dagger X\rangle}{M^2_{\rm Pl}} - l_0^{f,f^c}\frac{F_{X^\dagger}F_X}{m^2_{3/2}M^2_{Pl}}\right)\right.$$
$$\left. + \sum_A \frac{\langle\phi^*_A\rangle_{\overline{a}}\langle\phi_A\rangle_b}{M^2_f}\left(k_A^{f,f^c}(1-x_Ax^*_A) - l_A^{f,f^c}\frac{F_{X^\dagger}F_X}{m^2_{3/2}M^2_{Pl}}\right) - k_4^{f,f^c}\delta_{\overline{a}b}\frac{\langle\Sigma^*\rangle\langle\Sigma\rangle}{M^2_\Sigma}x_\Sigma x_\Sigma\right]$$

FCNC

$$F_\phi = \frac{\partial W}{\partial\phi} + \phi^* W, \quad \langle W\rangle = m_{3/2}$$

Determined by Yukawa structure

$$A_{abc} \propto \left\langle F_X\left(\partial_X\frac{K_{\rm hid}}{M^2_{Pl}}\right)Y_{abc} + \sum_\Phi F_\Phi\partial_\Phi Y_{abc}\right.$$
$$\left. - \left(F_X(\tilde{K}^{-1})_{d\overline{e}}\partial_X\tilde{K}_{\overline{e}a}Y_{dbc} + \sum_\Phi F_\Phi(\tilde{K}^{-1})_{d\overline{e}}\partial_\Phi\tilde{K}_{\overline{e}a}Y_{dbc} + cyclic(a,b,c)\right)\right\rangle$$

Misalignment of Yukawa and A-terms?

e.g. $P = Y_{ij}Q_i q^c_j H_a \propto \left(\frac{\theta}{M}\right)^{\alpha(i,j)} Q_i q^c_j H_a$

$$A_{ij}Y_{ij}\tilde{Q}_i\tilde{q}^c_j H_a = (3+\alpha(i,j))m_{3/2}Y_{ij}\tilde{Q}_i\tilde{q}^c_j H_a$$

$F_{\phi,\Sigma} = m_{3/2}\langle\phi,\Sigma\rangle$, "natural" value

GGR, Vives

suppression factor $\left(\frac{m_{3/2}}{M_\phi}\right)^2$ possible

Antusch et al

A family symmetry model

Tri-bimaximal mixing

$$\Delta(27) \otimes SO(10) \otimes G \qquad (G = R \otimes U(1))$$

Varzielas, GGR

- $\psi_i^c, \psi_i \subset (16,3) \Rightarrow$ No mass while SU(3) unbroken

- Spontaneous symmetry breaking

c.f. Georgi-Jarskog

$$\bar{\phi}_3^i, \quad \bar{\phi}_{23}^i, \quad \bar{\phi}_{123}^i, \quad H_{45}$$

$$(1,\bar{3}) \qquad (1,\bar{3}) \qquad (1,\bar{3}) \qquad (45,1)$$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \varepsilon M, \quad \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \varepsilon^2 M, \quad M$$

- $$P_Y = \frac{1}{M^2} \bar{\phi}_3^i \psi_i \bar{\phi}_3^j \psi_j^c H + \frac{1}{M^3} \bar{\phi}_{23}^i \psi_i \bar{\phi}_{23}^j \psi_j^c H H_{45} + \frac{1}{M^2} \bar{\phi}_{23}^i \psi_i \bar{\phi}_{123}^j \psi_j^c H + \frac{1}{M^2} \bar{\phi}_{123}^i \psi_i \bar{\phi}_{23}^j \psi_j^c H$$

only terms allowed by G

Misalignment rates

EDMs

$$|\mathrm{Im}(\delta^u_{LR})_{11}| \approx 2\times10^{-7}\frac{A_0}{100\,\mathrm{GeV}}\left(\frac{500\,\mathrm{GeV}}{\langle\tilde{m}_u\rangle_{LR}}\right)^2\left(\frac{\bar{\varepsilon}}{0.13}\right)^3\left(\frac{\varepsilon}{0.05}\right)^2|y_1^f+y_2^f|\,|x_{123}-x_{23}-x_\Sigma|\sin\phi_1 \qquad \leq 10^{-6}\ |_{Expt}$$

$$|\mathrm{Im}(\delta^d_{LR})_{11}| \approx 5\times10^{-7}\frac{A_0}{100\,\mathrm{GeV}}\left(\frac{500\,\mathrm{GeV}}{\langle\tilde{m}_d\rangle_{LR}}\right)^2\left(\frac{\bar{\varepsilon}}{0.13}\right)^5\frac{10}{\tan\beta}|y_1^f+y_2^f|\,|x_{123}-x_{23}-x_\Sigma|\sin\phi_1,\ (1 \qquad \leq 10^{-6}\ |_{Expt}$$

$$|\mathrm{Im}(\delta^\ell_{LR})_{11}| \approx 2\times10^{-7}\frac{A_0}{100\,\mathrm{GeV}}\left(\frac{200\,\mathrm{GeV}}{\langle\tilde{m}_\ell\rangle_{LR}}\right)^2\left(\frac{\bar{\varepsilon}}{0.13}\right)^5\frac{10}{\tan\beta}|y_1^f+y_2^f|\,|x_{123}-x_{23}-x_\Sigma|\sin\phi_1, \qquad \leq 10^{-7}\ |_{Expt}$$

$\mu \to e\gamma$

$$|(\delta^\ell_{LR})_{12}| \approx 1\times10^{-4}\frac{A_0}{100\,\mathrm{GeV}}\frac{(200\ \mathrm{GeV})^2}{\langle\tilde{m}_l\rangle^2_{LR}}\frac{10}{\tan\beta}\left(\frac{\bar{\varepsilon}}{0.13}\right)^3|y_1|\,|x_{123}-x_{23}-x_\Sigma| \qquad \leq 10^{-5}\ |_{Expt}$$

Antusch, King, Malinsky, GGR
Calibbi,Jones-Perez,Vives

"Natural expectation"

$$(\delta^f_{LL})_{ij} = \frac{\left(m^2_{\tilde{f}LL}\right)_{ij}}{\langle m_{\tilde{f}}\rangle^2_{LL}}\,,\quad (\delta^f_{RR})_{ij} = \frac{\left(m^2_{\tilde{f}RR}\right)_{ij}}{\langle m_{\tilde{f}}\rangle^2_{RR}}\,,\quad (\delta^f_{LR})_{ij} = \frac{\left(m^2_{\tilde{f}LR}\right)_{ij}}{\langle m_{\tilde{f}}\rangle^2_{LR}}$$

Most stringent bounds
(Determined by A terms)

Misalignment rates

EDMs

$$|\mathrm{Im}(\delta^u_{LR})_{11}| \lesssim 2\times 10^{-8}\frac{A_0}{100\ \mathrm{GeV}}\left(\frac{500\,\mathrm{GeV}}{\langle\tilde{m}_u\rangle_{LR}}\right)^2\left(\frac{\bar{\varepsilon}}{0.15}\right)^3\left(\frac{\varepsilon}{0.05}\right)^3\sin\phi_1 \qquad \leq 10^{-6}\ |_{Expt}$$

$$|\mathrm{Im}(\delta^d_{LR})_{11}| \sim 2\times 10^{-7}\frac{A_0}{100\ \mathrm{GeV}}\left(\frac{500\,\mathrm{GeV}}{\langle\tilde{m}_d\rangle_{LR}}\right)^2\left(\frac{\bar{\varepsilon}}{0.15}\right)^6\frac{10}{\tan\beta}\sin\phi_1 \qquad \leq 10^{-6}\ |_{Expt}$$

$$|\mathrm{Im}(\delta^\ell_{LR})_{11}| \sim 6\times 10^{-8}\frac{A_0}{100\ \mathrm{GeV}}\left(\frac{200\,\mathrm{GeV}}{\langle\tilde{m}_e\rangle_{LR}}\right)^2\left(\frac{\bar{\varepsilon}}{0.15}\right)^6\frac{10}{\tan\beta}\sin\phi_1 \qquad \leq 10^{-7}\ |_{Expt}$$

$\mu \to e\gamma$

$$|(\delta^e_{LR})_{12}| \lesssim |(\delta^\ell_{LR})_{12}| \sim 3\times 10^{-5}\frac{A_0}{100\,\mathrm{GeV}}\frac{(200\,\mathrm{GeV})^2}{\langle\tilde{m}_l\rangle^2_{LR}}\frac{10}{\tan\beta}\left(\frac{\bar{\varepsilon}}{0.15}\right)^4 \qquad \leq 10^{-5}\ |_{Expt}$$

Antusch, King, Malinsky, GGR

Minimum possible $F_{H_{45}} \simeq 0$

$$(\delta^f_{LL})_{ij} = \frac{\left(m^2_{\tilde{f}\,LL}\right)_{ij}}{\langle m_{\tilde{f}}\rangle^2_{LL}}\,, \quad (\delta^f_{RR})_{ij} = \frac{\left(m^2_{\tilde{f}\,RR}\right)_{ij}}{\langle m_{\tilde{f}}\rangle^2_{RR}}\,, \quad (\delta^f_{LR})_{ij} = \frac{\left(m^2_{\tilde{f}\,LR}\right)_{ij}}{\langle m_{\tilde{f}}\rangle^2_{LR}}$$

Most stringent bounds
(Determined by A terms)

Discussion

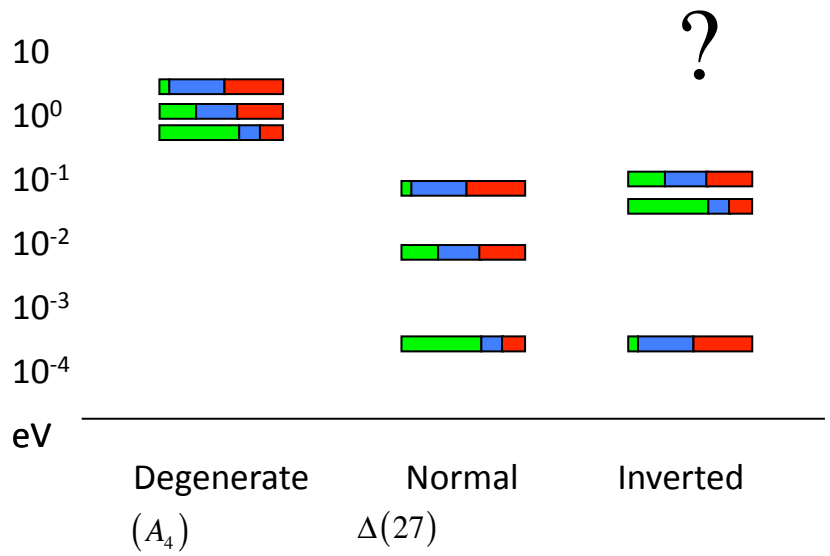
- How can flavour mixing be implemented?

Family symmetry

Discussion

- How can flavour mixing be implemented? Family symmetry

  ...implications of mass hierarchy measurement $\Delta(27)$ model strongly favours normal hierarchy



Discussion

- How can flavour mixing be implemented? Family symmetry

  ...implications of mass hierarchy measurement $\Delta(27)$ model strongly favours normal hierarchy

  ...precision for θ_{13}, δ_{CP} ..

$$\sin^2\theta_{12} \approx \frac{1}{3} \pm 0.03$$

$$\sin^2\theta_{23} \approx \frac{1}{2} \pm 0.03$$

$$\sin\theta_{13} \approx \sqrt{\frac{m_e}{2m_\mu}} = 0.053 \pm 0.05 \quad (3 \pm 1^o)$$

King; GGR, Varzielas

$$\longrightarrow \quad \theta_{12} + \frac{1}{\sqrt{2}}\frac{\theta_C}{3}\cos(\delta - \pi) \simeq 35.26 \pm 2^o$$

$3\cos(\delta - \pi)$

From charged lepton mixing

Antusch, King

Discussion

- How can flavour mixing be implemented?

  ...implications of mass hierarchy measurement

  ...precision for θ_{13}, δ_{CP} ..

- How do these theories inscribe into a more general theory

  $\Delta(27)$ family model fits easily with underlying $SO(10)$ GUT

Discussion

- How can flavour mixing be implemented?

  ...implications of mass hierarchy measurement

  ...precision for θ_{13}, δ_{CP} ..

- How do these theories inscribe into a more general theory

- Is it possible to discriminate between GUT and TeV-scale see-saws?

  Predictions for rare lepton decays

  (SUSY) see-saw parameters may be measurable

See saw parameters

Masses + Mixing Angles	6	(15)	(8)
CP violating phases	3	(6)	(3)

$$\kappa = Y_\nu^T M^{-1} Y_\nu$$

Insufficient information

$$P = Y_\nu^\dagger Y_\nu$$

RGE for sleptons (SUSY)

$$\left(m^2_{\tilde{\ell},\tilde{\nu}}\right)_{ij} \simeq (\text{diagonal part})_{\tilde{\ell},\tilde{\nu}} - \frac{1}{8\pi^2}(3m_0^2 + A_0^2)(\mathbf{Y}_\nu^\dagger)_{ik}(\mathbf{Y}_\nu)_{kj} \log \frac{M_X}{M_k}$$

In *principle* all parameters determined by neutrino structure and radiative corrections (assuming degenerate sleptons at GUT scale)

In *practice* off diagonal elements induce rare lepton flavour violation, measurable…

on diagonal elements difficult due to other contributions from gauge and charged lepton Yukawa

Davidson, Ibarra

Sequential see saw and symmetry (texture zero) simplifies structure…

Discussion

- How can flavour mixing be implemented?

  …implications of mass hierarchy measurement

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