Lepton Flavor Violation in Supersymmetric Theories

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Lepton Flavor Violation

Motivation

- **There is no fundamental reason why lepton flavor should be conserved.**
- **Several SM extensions (GUT, technicolor, compositeness, SUSY), indeed, exploit the LFV possibility.**
- **Signal of LFV in charged lepton sector would be a clear signal of physics beyond the SM.**

Experimental sensitivity to LFV

Present bounds and future sensitivities for LFV processes are

LFV process	Present bound	Future sensitivity
BR($\mu \longrightarrow$ e γ)	1.2 x 10^{-11}	1.3 x 10^{-13}
BR($\tau \longrightarrow$ e γ)	1.1 x 10^{-7}	10^{-8}
BR($\tau \longrightarrow \mu\ \gamma$)	6.8 x 10^{-8}	10^{-8}
BR($\mu \longrightarrow$ 3 e)	1.0 x 10^{-12}	10^{-13}
BR($\tau \longrightarrow$ 3 e)	2.0 x 10^{-7}	10^{-8}
BR($\tau \longrightarrow$ 3 μ)	1.9 x 10^{-7}	10^{-8}

Outline

- **LFV in SM with three right-handed neutrino**
- **LFV in MSSM**
- **LFV in minimal supersymmetric seesaw model**
- **SUSY TeV B-L extension of the SM**
- **LFV in SUSY B-L**
- **Conclusions**

LFV in the SM

- **In standard model (neutrino are massless), lepton flavor is almost conserved.**

- **Now neutrino oscillation is observed:**
  ex) atmospheric neutrino: $\nu_\mu \longrightarrow \nu_\tau$
  solar neutrino: $\nu_e \longrightarrow \nu_\mu$

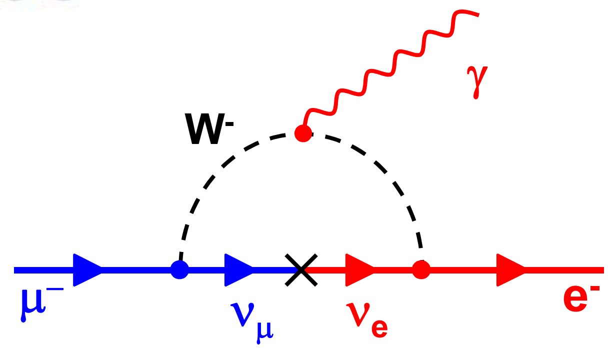
Tiny neutrino masses and large neutrino mixing angles

$$\Delta m_{23}^2 \sim 2 \times 10^{-3}\mathrm{eV}^2 \qquad \sin^2\theta_{\mu 3} \sim 0.4$$

$$\Delta m_{12}^2 \sim 8 \times 10^{-4}\mathrm{eV}^2 \qquad \sin^2\theta_{e2} \sim 0.3$$

- **In SM with massive neutrinos,**
  LFV is very tiny ~ $(m_{\nu i}^2 - m_{\nu j}^2) / M_W^2$

$$Br(\mu \to e\gamma) \sim \frac{\alpha}{4\pi}\left(\frac{m_\nu}{m_W}\right)^4 \sim 10^{-43}\left(\frac{m_\nu}{1\mathrm{eV}}\right)^4$$



→ LFV in the charged sector is forbidden in the Standard Model

LFV in MSSM

- Lepton flavor symmetry is accidental at low energy, and it may be violated beyond the SM.

- Naturalness problem with Higgs sector in SM implies that new physics will appear at TeV scale:

  – Supersymmetry (SUSY)
  – Extra Dimensions
  – Little Higgs

- In these models New particles and new interactions introduced $\Longrightarrow$ Source of flavor violation.

- No reason for no LFV interaction at TeV scale.

- Flavor-violation studies constrain SUSY breaking

- **MSSM Lagrangian:**

$$W = h_U Q_L U_L^c H_2 + h_D Q_L D_L^c H_1 + h_L L_L E_L^c H_1 + \mu H_1 H_2 .$$

- **Explicit soft supersymmetry breaking:**

$$-\mathcal{L}_{\rm SB} = \frac{1}{2}(m^2)_{ij}\,\phi_i^*\phi_j \;-\; \frac{1}{2}M_a\,\lambda^a\lambda^a \;+\; \frac{1}{6}A_{ijk}Y_{ijk}\,\phi_i\phi_j\phi_k \;+\; B\mu\,H_1H_2 + \text{h.c.}$$

- **MSSM contains new sources of flavor and CP beyond the CKM.**

- **In Supe-CKM basis all sources of FC are in the off-diagonal terms of sfermion mass matrix.**

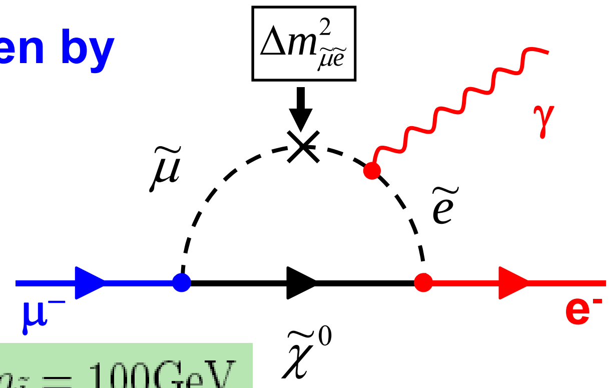
$$M_{\tilde{l}}^2 = \begin{pmatrix} \left(M_{\tilde{l}}^2\right)_{LL} & \left(M_{\tilde{l}}^2\right)_{LR} \\ \left(M_{\tilde{l}}^2\right)_{RL} & \left(M_{\tilde{l}}^2\right)_{RR} \end{pmatrix},$$

$$\begin{aligned} \left(M_{\tilde{l}}^2\right)_{LL} &= U_L m_L^2 U_L^\dagger + m_l^2 - \frac{m_Z^2}{2}(1-2\sin^2\theta_W)\cos 2\beta, \\ \left(M_{\tilde{l}}^2\right)_{RR} &= U_R (m_R^2)^T U_R^\dagger + m_l^2 + m_Z^2 \sin^2\theta_W \cos 2\beta, \\ \left(M_{\tilde{l}}^2\right)_{LR} &= \left(M_{\tilde{l}}^2\right)_{RL}^\dagger = -\mu\, m_l \tan\beta + \frac{v\cos\beta}{\sqrt{2}} U_L Y_l^{A*} U_R^\dagger, \end{aligned}$$

- **The flavor mixing is parameterized by the mass insertions** $\delta_{AB} = \Delta_{AB} / \tilde{m}^2$

- The branching ratio for the process $l_i \to l_j \gamma$ is given by

$$\mathrm{BR}(l_i \to l_j\gamma) = \frac{\alpha^3}{G_F^2}\frac{12\pi}{m_{\tilde{l}}^4}\left\{\left|M_3(x)\left(\delta^l_{ij}\right)_{LL} + \frac{m_{\tilde{\gamma}}}{m_l}M_1(x)\left(\delta^l_{ij}\right)_{LR}\right|^2 + L \leftrightarrow R\right\} \cdot \mathrm{BR}(l_i \to l_j \nu_i \bar{\nu}_j) \ .$$



Limits on the $\left|\delta^d_{ij}\right|$ from $l_j \to l_i\gamma$ decays for an average slepton mass $m_{\tilde{l}} = 100\mathrm{GeV}$

Masiero et al (97)

x	$\left\|\left(\delta^l_{12}\right)_{LL}\right\|$	$\left\|\left(\delta^l_{12}\right)_{LR}\right\|$	x	$\left\|\left(\delta^l_{13}\right)_{LL}\right\|$	$\left\|\left(\delta^l_{13}\right)_{LR}\right\|$	x	$\left\|\left(\delta^l_{23}\right)_{LL}\right\|$	$\left\|\left(\delta^l_{23}\right)_{LR}\right\|$
0.3	4.1×10^{-3}	1.4×10^{-6}	0.3	15	8.9×10^{-2}	0.3	2.8	1.7×10^{-2}
1.0	7.7×10^{-3}	1.7×10^{-6}	1.0	29	1.1×10^{-1}	1.0	5.3	2.0×10^{-2}
5.0	3.2×10^{-2}	3.8×10^{-6}	5.0	1.2×10^2	2.4×10^{-1}	5.0	22	4.4×10^{-2}

- **$\mu \to e\,\gamma$ impose stringent constraints on SUSY parameters**

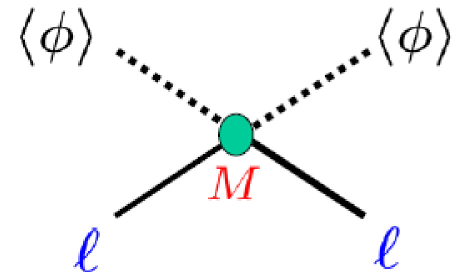
- **Proposal for SUSY flavor problem: Universality, Alignment, Decoupling**

- **LFV may probe pattern of SUSY breaking & constrain its origin**

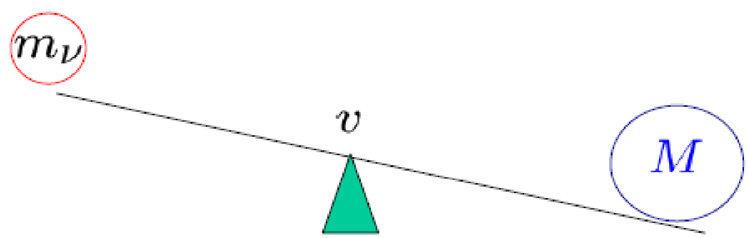
S.K., M. Gomez , Carvalho (01)

Neutrino Masses and Seesaw Mechanism

Effective operator: $\mathcal{L} = \frac{\phi\phi\ell\ell}{M} \rightarrow v\left(\frac{v}{M}\right)\nu\nu$



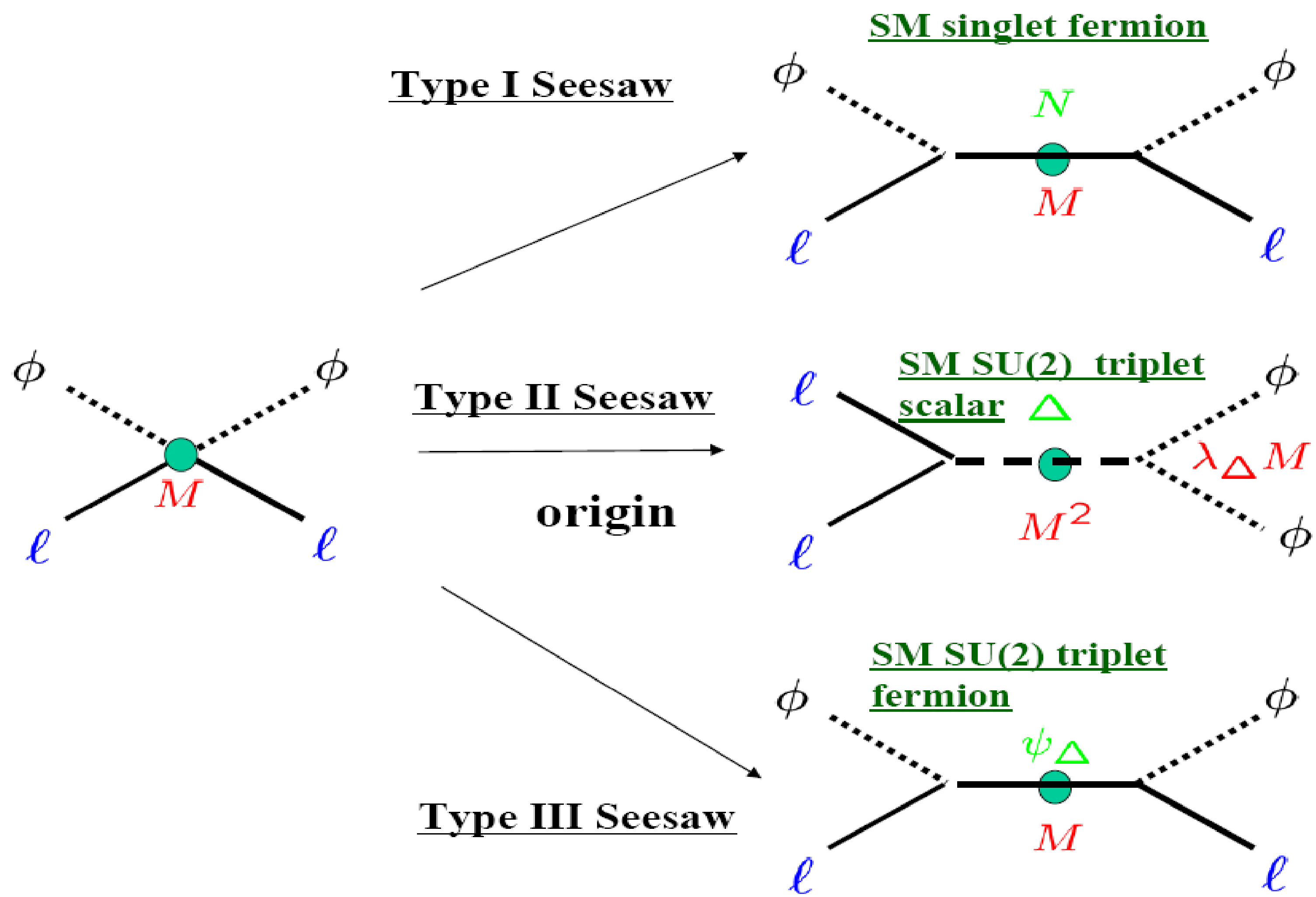
If the seesaw scale $M \gg v \rightarrow$ $m_\nu = v\left(\frac{v}{M}\right) \ll v$



Naturally, $m_\nu \sim \mathcal{O}(\sqrt{\Delta m_{12}^2}) - \mathcal{O}(\sqrt{\Delta m_{23}^2}) = 0.01 - 0.1 \text{ eV}$

→ $M \lesssim 10^{14}\text{ GeV} \ll M_{Pl}$

Three kinds of Seesaw Mechanism



LFV in SUSY Seesaw Models

- SUSY extension extensions of the SM + 3 ν_R :
  - Seesaw mechanism
  - Stabilize the hierarchy between NP and EW scale.

- **One of the implications of SUSY seesaw models is the prediction of sizable rates for LFV:**
  - **$l_j \to l_i$**
  - **$l_j \to 3\, l_i$**
  - **μ−e conversion in heavy nuclei.**

- **The leptonic superpotential of type-I SUSY seesaw is given by**

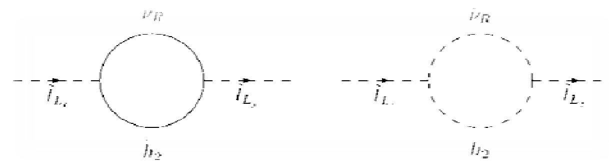
$$W = \hat{N}^c Y_\nu \hat{L} \hat{H}_2 + \hat{E}^c Y_l \hat{L} \hat{H}_1 + \frac{1}{2} \hat{N}^c m_M \hat{N}^c ,$$

N^c is the superfield ∍ ν_R and their scalar partner.
$Y_{l,\nu}$, are the lepton Yukawa couplings & m_M is the Majorana mass.

- **One can assume a basis where Y_l and m_M are diagonal.**

❑ **Neutrino flavor mixing radiatively induces slepton flavor mixing from GUT to $M_{\nu R}$ scale.**

❑ **Slepton off-diagonal mass corrections**



$$(\Delta m^{l}_{LL})^2{}_{ij} = \frac{-1}{8\pi^2}(3m_0^2 + A_0^2)(Y_\nu^{+}Y_\nu)_{ij}(\ln\frac{M_{GUT}}{M_R})$$

$$(\Delta m^{l}_{RR})^2{}_{ij} = 0$$

$$(\Delta m^{l}_{LR})^2{}_{ij} = \frac{-3}{8\pi^2}A_0 Y_e (Y_\nu^{+}Y_\nu)_{ij}(\ln\frac{M_{GUT}}{M_R})$$

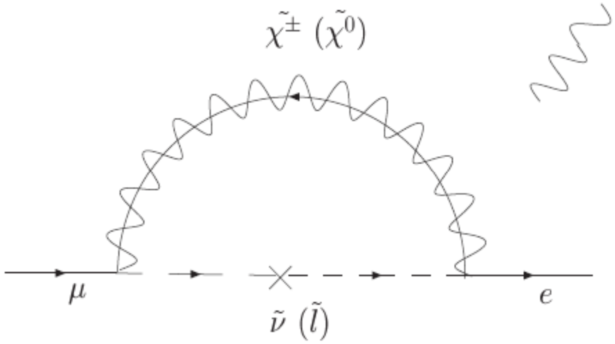
Even under universal scalar mass hypothesis, SUSY breaking slepton mass matrix is

$$(\tilde{m}_{\tilde{l}})_{ij} \simeq m_0^2\delta_{ij} + \Delta(\tilde{m}_{\tilde{l}})_{ij}$$

Flavor-dependent radiative correction

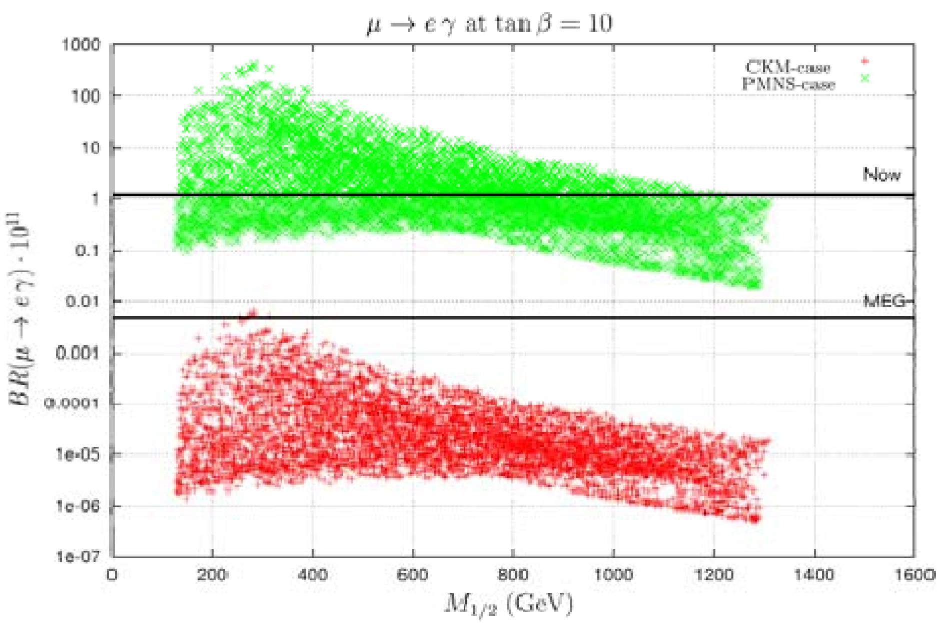
Low-energy SUSY has "memory" of all the multi-step RG occurring from such superlarge scale down to $M_W \Longrightarrow$ potentially large LFV

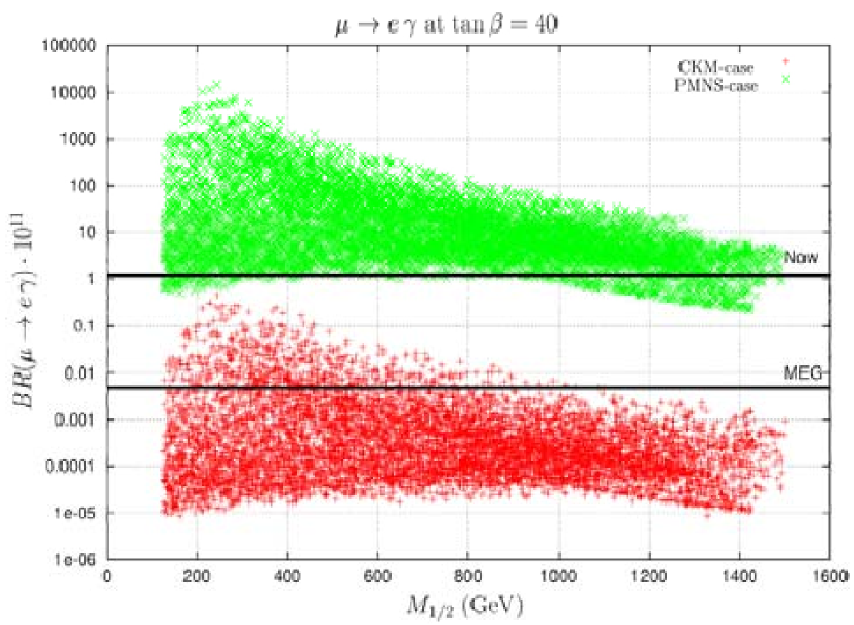
Example : SUSY Seesaw mechanism
Neutrino Yukawa coupling is flavor-violating.



1- $Y_\nu = V^T_{CKM} Y^{diag}_\nu V_{CKM}$

2- $Y_\nu = U_{PMNS} Y^{diag}_\nu$





CALIBBI,FACCIA,Masiero, VEMPATI (04)

Vast Recent Literature on SUSY and Connection between Neutrino-oscillations, LFV rare decays

Examples:

Hisano, Moroi, Tobe, Yamaguchi (1995)

Hisano, Nomura, PRD 59, 116005 (1999)

Casas, Ibarra, hep-ph0103065

Lavignac, Masina, Savoy, hep-ph/0106245

Babu, Pati, hep-ph/0207289

Raidal, Strumia, PLB 553, 72 (2003)

Barr, hep-ph/0307372

Blažek, King, NPB 662, 359 (2003)

Masiero, Vempati, hep-ph/0209303

Petcov, Profumo, Takanishi, Yaguna (2004).

Antusch, Arganda, Herrero, Teixeira (2008)

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TeV Scale Seesaw

- origin of small neutrino mass?
- a smart explanation: seesaw mechanism (Yanagida, Gell-Man etal.,)

$$m_\nu \sim \frac{(\xi\langle H\rangle)^2}{M} \to M \sim \frac{10^{11}\text{GeV}}{(m_\nu/1\text{eV})} \text{ with } \xi \sim O(1)$$

- too heavy "new particle"
  - hopeless to test the seesaw mechanism directly

- Alternative? (testable model)
  - possible to lower the seesaw scale?

$$\text{M} \sim \text{O(1) TeV} \Rightarrow m_D \approx O(10^{-4})GeV, i.e., m_D \sim m_e$$

- **Light-heavy neutrino mixing is given by** $\theta = m_D / M \sim 10^{-7}$

TeV Scale B-L

S.K. (06)

- The tremendous success of gauge symmetry in describing the SM indicates that any extension of the SM should be through the extension of its gauge symmetry.

- **The minimal extension is based on the gauge group**

$$G_{B-L} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

- **This model accounts for the exp. results of the light neutrino masses**

- **New particles are predicted:**
  - Three SM singlet fermions (right handed neutrinos) (cancellation of gauge anomalies)
  - Extra gauge boson corresponding to B−L gauge symmetry
  - Extra SM singlet scalar (heavy Higgs)

- **These new particles have Interesting signatures at the LHC**

SUSY and B-L radiative symmetry breaking.

S.K., A. Masiero, 2007

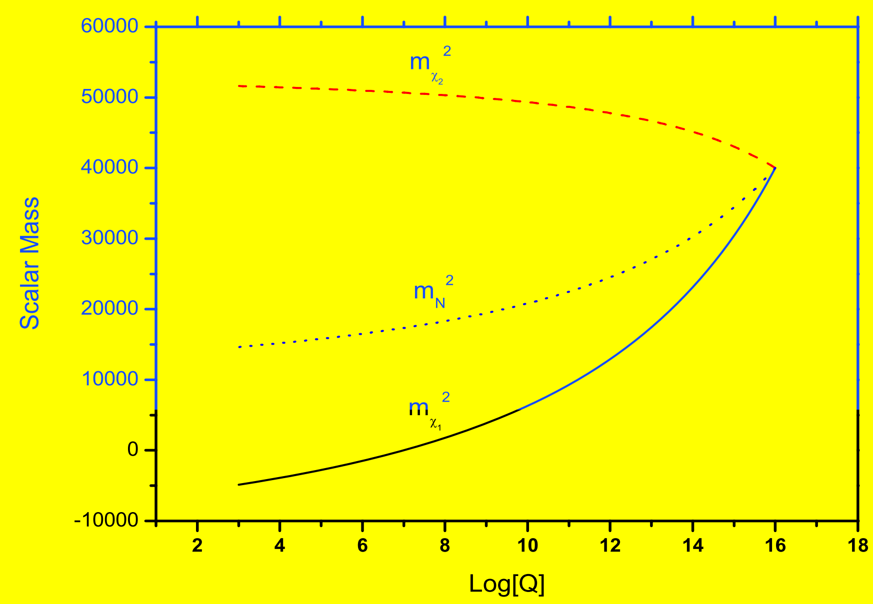
$$W = (h_U)_{ij} Q_i H_2 U_j^c + (h_D)_{ij} Q_i H_1 D_j^c + (h_L)_{ij} L_i H_1 E_j^c + (h_\nu)_{ij} L_i H_2 N_j^c + (h_N)_{ij} N_i^c N_j^c \chi_1 + \mu H_1 H_2 + \mu' \chi_1 \chi_2.$$

$$\frac{dm_{\chi_1}^2}{dt} = 6\tilde{\alpha}_{B-L} M_{B-L}^2 - 2\tilde{Y}_{N_3}\left(m_{\chi_1}^2 + 2m_{N_3}^2 + A_{N_3}^2\right),$$

$$\frac{dm_{N_3}^2}{dt} = \frac{3}{2}\tilde{\alpha}_{B-L} M_{B-L}^2 - \tilde{Y}_{N_3}\left(m_{\chi_1}^2 + 2m_{N_3}^2 + A_{N_3}^2\right).$$

$$\mu'^2 = \frac{m_{\chi_2}^2 - m_{\chi_1}^2 \tan^2\theta}{\tan^2\theta - 1} - \frac{1}{4} M_{Z_{B-L}}^2.$$

$$\sin 2\theta = \frac{2\mu_3^2}{\mu_1^2 + \mu_2^2}.$$



LFV in TeV SUSY B-L

- **The superpotential**

$$\hat{W} \sim |D^\mu \hat{H}_u|^2 + |D^\mu \hat{H}_d|^2 + |D^\mu \hat{\chi}_1|^2 + |D^\mu \hat{\chi}_2|^2 + \mu(\hat{H}_u \hat{H}_d) + \mu' \hat{\chi}_1 \hat{\chi}_2,$$

	l	N^c	E^c	Q	U^c	D^c	H_u	H_d	χ_1	χ_2
$SU(2)_L \times U(1)_Y$	$(2,-1/2)$	$(1,0)$	$(1,-1)$	$(2,1/6)$	$(1,2/3)$	$(1,-1/3)$	$(2,1/2)$	$(2,-1/2)$	$(1,0)$	$(1,0)$
$U(1)_{B-L}$	-1	-1	-1	$1/3$	$1/3$	$1/3$	0	0	-2	2

- **After Electroweak and $U(1)_{B-L}$ symmetry breaking, the neutral gaugino-higgsino mass matrix is given by**

$$\left(\begin{array}{cccc|ccc}
M_{1/2} & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W & 0 & 0 & 0 \\
0 & M_{1/2} & m_Z c_\beta c_W & -m_Z s_\beta c_W & 0 & 0 & 0 \\
-m_Z c_\beta s_W & m_Z s_\beta s_W M_1 & 0 & -\mu & 0 & 0 & 0 \\
m_Z c_\beta c_W & -m_Z s_\beta c_W & -\mu & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\mu' & (-2)m_{\chi_1} \\
0 & 0 & 0 & 0 & -\mu' & 0 & (+2)m_{\chi_2} \\
0 & 0 & 0 & 0 & (-2)m_{\chi_1} & (+2)m_{\chi_2} & M_{1/2}
\end{array}\right),$$

- **The LSP** $\chi_1^0 = V_{11}\tilde{B} + V_{12}\tilde{W}^3 + V_{13}\tilde{H}_d^0 + V_{14}\tilde{H}_u^0 + V_{15}\tilde{\chi}_1 + V_{16}\tilde{\chi}_2 + V_{17}\tilde{Z}_{B-L}.$

Sneutrino mass matrix

- The 6x6 sneutrino mass matrix is given by

$$\mathcal{L} = -\frac{1}{2}(\tilde{\nu'}^{\dagger}_L, \tilde{N'}^{c\dagger})\mathcal{M}^2 \begin{pmatrix} \tilde{\nu'}_L \\ \tilde{N'}^c \end{pmatrix},$$

- Where

$$\mathcal{M}^2 = \begin{pmatrix} M^2_{\tilde{\nu}^\dagger_L \tilde{\nu}_L} & M^2_{\tilde{\nu}^\dagger_L \tilde{\nu}_R} \\ M^2_{\tilde{\nu}^\dagger_R \tilde{\nu}_L} & M^2_{\tilde{\nu}^\dagger_R \tilde{\nu}_R} \end{pmatrix},$$

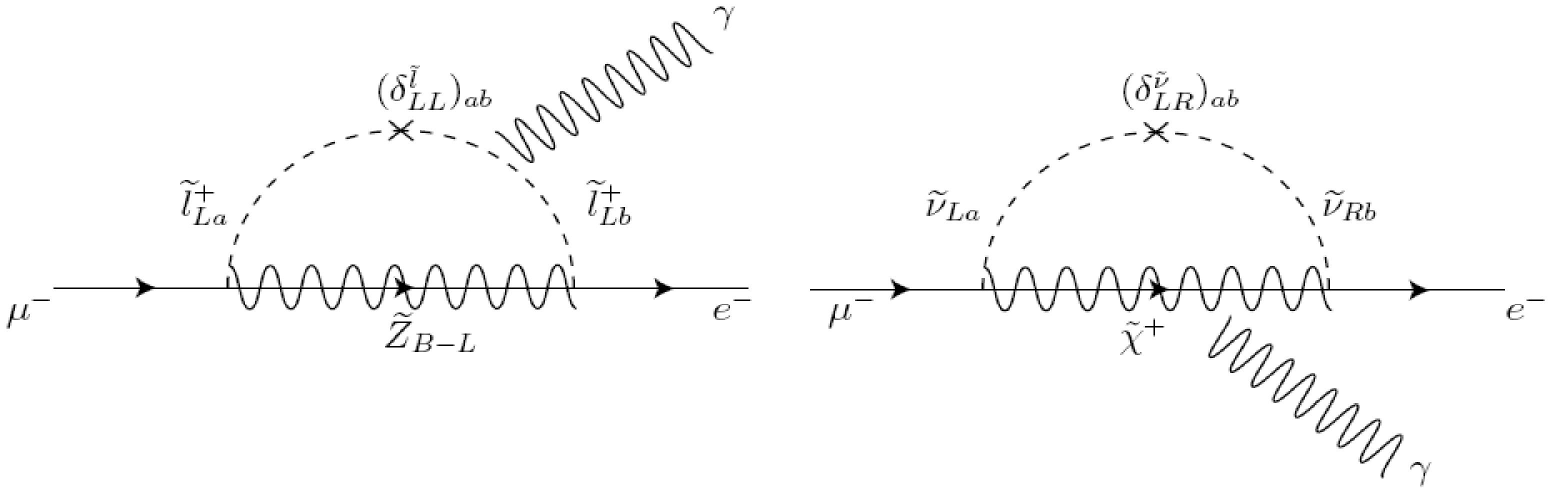
$$M^2_{\tilde{\nu}^\dagger_L \tilde{\nu}_L} = \tilde{m}^2_L + \frac{m_Z^2}{2}\cos 2\beta + \frac{1}{2}v^2 \sin^2\beta (Y^*_\nu Y^T_\nu),$$

$$M^2_{\tilde{\nu}^\dagger_R \tilde{\nu}_R} = \tilde{m}^2_N + M^2_N + \frac{1}{2}v^2 \sin^2\beta (Y^\dagger_\nu Y_\nu),$$

$$M^2_{\tilde{\nu}^\dagger_L \tilde{\nu}_R} = \frac{v}{\sqrt{2}} \sin\beta (Y_\nu)^* M_N,$$

Neutralino vs. Chargino

- **In SUSY B-L, additional contributions can be obtained**



- Neutralino contribution is proportional to *B-L gauge coupling squared ~ O(1)* and MI $(\delta^{l}{}_{LL})_{ab} \sim Y_\nu^2 \sim 10^{-12}$.

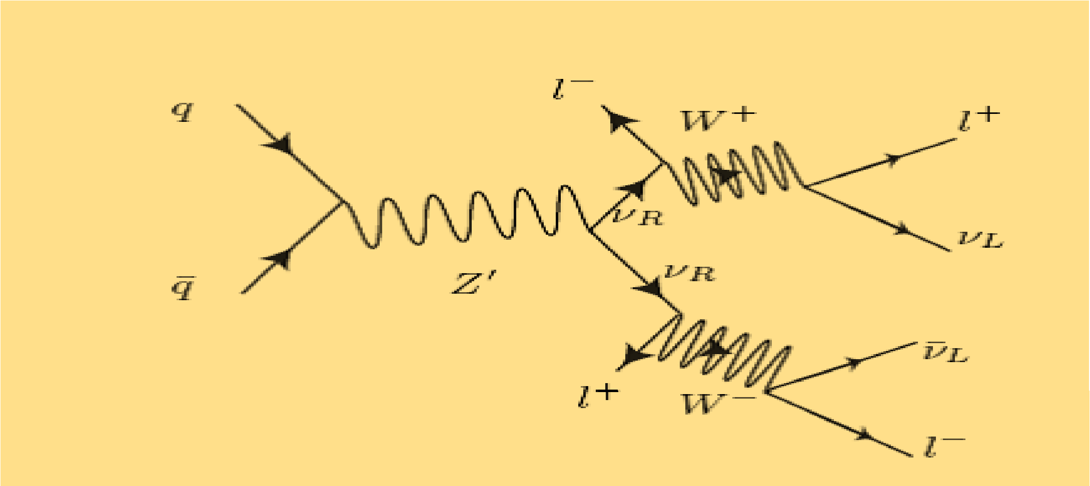
- **Chargino contribution is proportional to B-L gauge x Y_ν and $(\delta^{\nu}{}_{LL})_{ab} \sim v\, M_N\, Y_\nu / m^2 \sim 10^{-5}$.**

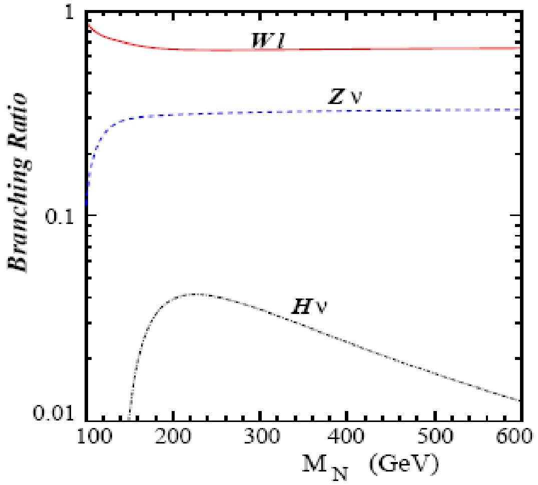
- **LFV is very tiny in SUSY model with TeV scale seesaw.**

LFV at LHC

S.K., K. Huitu, et al (2008)

$$\mathcal{L}_{int.} \sim -g'' C_\mu [(\overline{\nu_R})_i \gamma^\mu (\nu_R)_i + b_{ij} \overline{(\nu_L)^c}_i \gamma^\mu (\nu_R)_j + h.c.] + \frac{g_2}{\sqrt{2}} [W_\mu^- l_i^+ \gamma^\mu U_{ij} (\nu_L)_j + b_{ij} W_\mu^- l_i^+ \gamma^\mu (\nu_R)_j^c + h.c.],$$



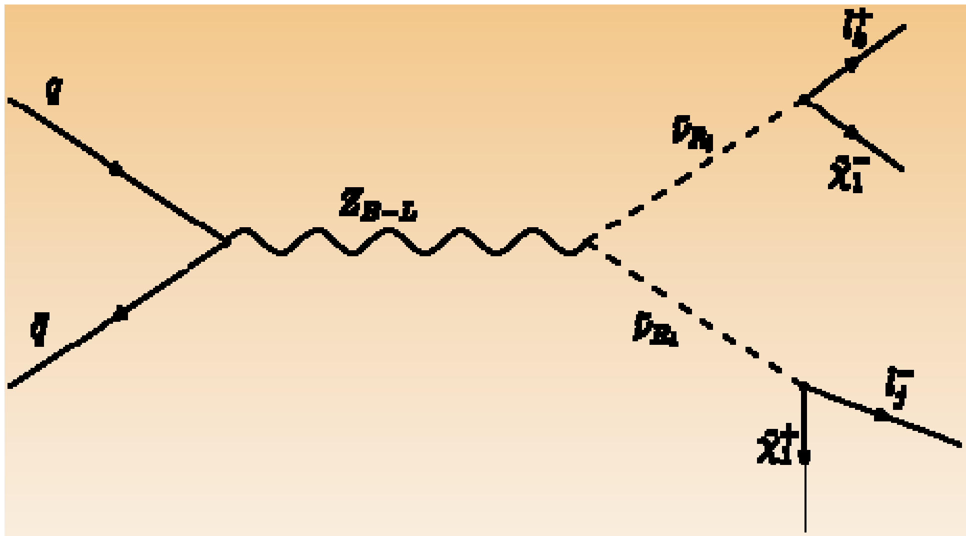


The decay modes, which go through the Higgs H or H' and Z boson, can be neglected compared to the main mode $\nu_R \rightarrow W\,l$.

These decays are very clean with four hard leptons: $l_i^+ l_j^- l_k^+ l_m^-$ in the final states and large missing energy due to the associated neutrinos:

LFV at LHC (continue)

- **Sneutrino-pair production at LHC**



$$q\bar{q} \rightarrow Z_{B-L} \rightarrow \tilde{\nu}_{R_i}\tilde{\nu}_{R_i} \rightarrow l_j^- l_k^+ + \tilde{\chi}_m^+ \tilde{\chi}_n^-$$
$$\rightarrow \mu^- e^+ + \tilde{\chi}_1^+ \tilde{\chi}_1^- \rightarrow \mu^- e^+ + 4\ jet + mis.energy$$

Right-handed sneutrino

- **The mixing in the right-handed sneutrino mass matrix is of order v′.**
- **In the basis** $(\tilde{\nu}_L, \tilde{\nu}_L^*, \tilde{N}^c, \tilde{N}^{c*})$

$$\mathcal{M}^2 \simeq \begin{pmatrix} \tilde{m}_L^2 + \frac{1}{2}m_Z^2 \cos 2\beta & 0 & v_2 Y_\nu^\dagger M_N & -v_2\ U_{MNS}^\dagger.(Y_\nu^A)^\dagger \\ 0 & \tilde{m}_L^2 + \frac{1}{2}m_Z^2 \cos 2\beta & -v_2\ U_{MNS}^T.(Y_\nu^A)^T & v_2 Y_\nu^T M_N \\ v_2 Y_\nu M_N & -v_2 (Y_\nu^A)^* U_{MNS}^* & \tilde{m}_N^2 + M_N^2 & -v_1'\ (Y_N^A)^* + v_2' Y_N \mu' \\ -v_2 (Y_\nu^A) U_{MNS} & v_2 Y_\nu^* M_N & -v_1'\ Y_N^A + v_2'\ Y_N \mu'^* & \tilde{m}_N^2 + M_N^2 \end{pmatrix}$$

- **The sneutrino mass matrix is diagonalized by:** $\Gamma_{\tilde{\nu}} \mathcal{M}^2 \Gamma_{\tilde{\nu}}^\dagger = \mathcal{M}_{diag}^2.$
- **Therefore:** $(\tilde{\nu}_{phy})_i = (\Gamma_{\tilde{\nu}})_{ij} \tilde{\nu}_j\ , \qquad i, j = 1, 2..., 12.$
- **The lightest right-handed sneutrinos can be the LSP (DM candidate)**
- **Pair-charginos produced via γ and Z-boson, which decay to:** $\tilde{\chi}^+ \to \ell^+ \tilde{N}_R.$
- **This leads to a signature: pp → $\ell^+_i \ell^-_j$ + missing energy.**

- **If lightest sneutrino is not LSP then**

$$\Gamma(\tilde{\nu} \to \tilde{\chi}_j^0 \nu) = \frac{g^2 |Z_{iZ}|^2 m_{\tilde{\nu}}}{32\pi \cos^2\theta_W} B(m^2_{\tilde{\chi}_j^0}/m^2_{\tilde{\nu}}) \,,$$

- **OR**

$$\Gamma(\tilde{\nu} \to \tilde{\chi}^+ \ell^-) = \frac{g^2 |V_{11}|^2 m_{\tilde{\nu}}}{16\pi} B(m^2_{\tilde{\chi}^+}/m^2_{\tilde{\nu}}) \,,$$

- **Sneutrino-antisneutrino oscillation can be observed at the LHC by studying a charge asymmetry of the leptons in the final states.**

- **If** $x_{\tilde{\nu}} \equiv \dfrac{\Delta m_{\tilde{\nu}}}{\Gamma_{\tilde{\nu}}} \gtrsim 1,$ **a measurable same sign dilepton signal is expected.**

- **The probability of a same sign dilepton signal is**

$$P(\ell^+\ell^+) + P(\ell^-\ell^-) = \chi_{\tilde{\nu}} [BR(\tilde{\nu} \to \ell^{\pm} + X)]^2 \,,$$

- **Here** $\chi_{\tilde{\nu}} \equiv x^2_{\tilde{\nu}}/[2(1 + x^2_{\tilde{\nu}})]$ **is the integrated oscillation probability.**

Conclusions

- **Signal of LFV in charged lepton sector would be a signal of SUSY models with heavy right-handed neutrions.**

- **In general the experimental limits of BR($\mu \longrightarrow e\gamma$) impose stringent constraints on SUSY parameter space.**

- **LFV is very tiny in SUSY model with TeV scale seesaw as in SUSY B-L extension of the SM.**

- **TeV Majorana neutrino masses imply the existence of $\Delta L = 2$ phenomena, which leads to sneutrino-antisneutrino mixing.**