Non-Abelian Discrete Flavor Symmetries from T^2/Z_N Orbifolds

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The flavor problem of the Standard Model

- What flavor is?
- Why do the parameters of the flavor sector, the fermion masses and mixing matrices, take the values they do?
- A popular and successful approach is to impose a non-abelian discrete flavor symmetry G_f to explain certain observed regularities.
- The nature of flavor is, in the context of flavor symmetries, therefore usually reduced to the question as to the origin of that symmetry.

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- The origin of discrete flavor symmetry may come from the Breaking of Continuous Flavor symmetry at High energy scale: $SU(3)$ ($SU(2)$, $SO(3)$) $\rightarrow G_f$. **However**, this requires the large representations of $SU(3)$ and **it is a highly non-trivial phenomenological task.**
- The second possibility for the origin of the discrete flavor symmetry is the Breaking of Space-time symmetry from extra-dimensions: Poincare 6d $\rightarrow$ Poincare 4d $\times$ G_f by orbifolding. (G. Altarelli, F. Feruglio, and Y. Lin '06)

General Information about Orbifolds

Two dimensional Torus

The 2-dimensional torus T^2 is obtained by identifying the opposite sides of a parallelogram:

$$\begin{aligned} (x_5, x_6) &\rightarrow (x_5, x_6) + \vec{e}_1 \\ (x_5, x_6) &\rightarrow (x_5, x_6) + \vec{e}_2 \ , \end{aligned} \tag{1}$$

where $\vec{e}_1 = (1, 0)$, $\vec{e}_2 = C(\cos(\alpha), \sin(\alpha))$ are the basis vectors of the torus.

General Information about Orbifolds

Orbifold T^2/Z_N

Orbifold can be obtained by modding out torus with the discrete group Z_N:

$$(x_5, x_6) \quad \longrightarrow \quad \Omega(x_5, x_6) \tag{2}$$

where Ω is the rotation generator of the discrete group Z_N.

All possible 2-dimensional Orbifolds

- T^2/Z_2, the basis vectors $(e_1 = (1,0), e_2 = (a,b))$ are arbitrary.
- T^2/Z_3, the basis vectors are fixed $(e_1 = (1,0), e_2 = (1/2, \sqrt{3}/2))$
- T^2/Z_4, the basis vectors are fixed $(e_1 = (1,0), e_2 = (0,1))$
- T^2/Z_6, the basis vectors are fixed $(e_1 = (1,0), e_2 = (1/2, \sqrt{3}/2))$

Symmetries from Orbifolding T^2/Z_N

- Choose the torus basis vectors and also modding out Z_N group
- Calculate the fixed points (3d-Branes where SM particles reside)
- Consider the symmetries that connect these fixed points (3d-Branes)
- Transform these symmetries to the non-abelian discrete flavor symmetries in 4 dimensions

Note that from now on we parametrize the torus by the complex number $z = x_5 + ix_6$.

T^2/Z_2

The Torus T^2 is defined by identifying the points in the complex plane related by

$$z \rightarrow z+1 \tag{3}$$

$$z \rightarrow z+\gamma. \tag{4}$$

In fact, the γ can be arbitrary, however, in order to get the flavor symmetry, we restrict it to be either $\gamma = e^{i\pi/3}$ (G. Altarelli, F. Feruglio, Y. Lin '06) which gives us S_4 (C.Hagedorn, M.Lindner, R.N.Mohapatra '06,...)flavor symmetry in the case that the space-time symmetry is Poincare and A_4 (E.Ma, G.Rajasekaran '01,...) for the proper Lorentz symmetry or if we choose $\gamma = e^{i\pi/2} = i$, we will get D_4 symmetry.

T^2/Z_2

The parity (Z_2) is defined by

$$z \to -z. \tag{5}$$

The fixed points are given by
$(z_1, z_2, z_3, z_4) = (1/2, (1+i)/2, i/2, 0)$.
The fixed points have two kinds of symmetries, namely, translation symmetry and rotation symmetry.
Translation symmetries:

$$S_1 : z \quad \to \quad z + 1/2, \tag{6}$$

$$S_2 : z \quad \to \quad z + i/2. \tag{7}$$

Rotation symmetry:

$$T_R : z \quad \to \quad \omega z, \tag{8}$$

where $\omega = e^{i\pi/2} = i$.

T^2/Z_2

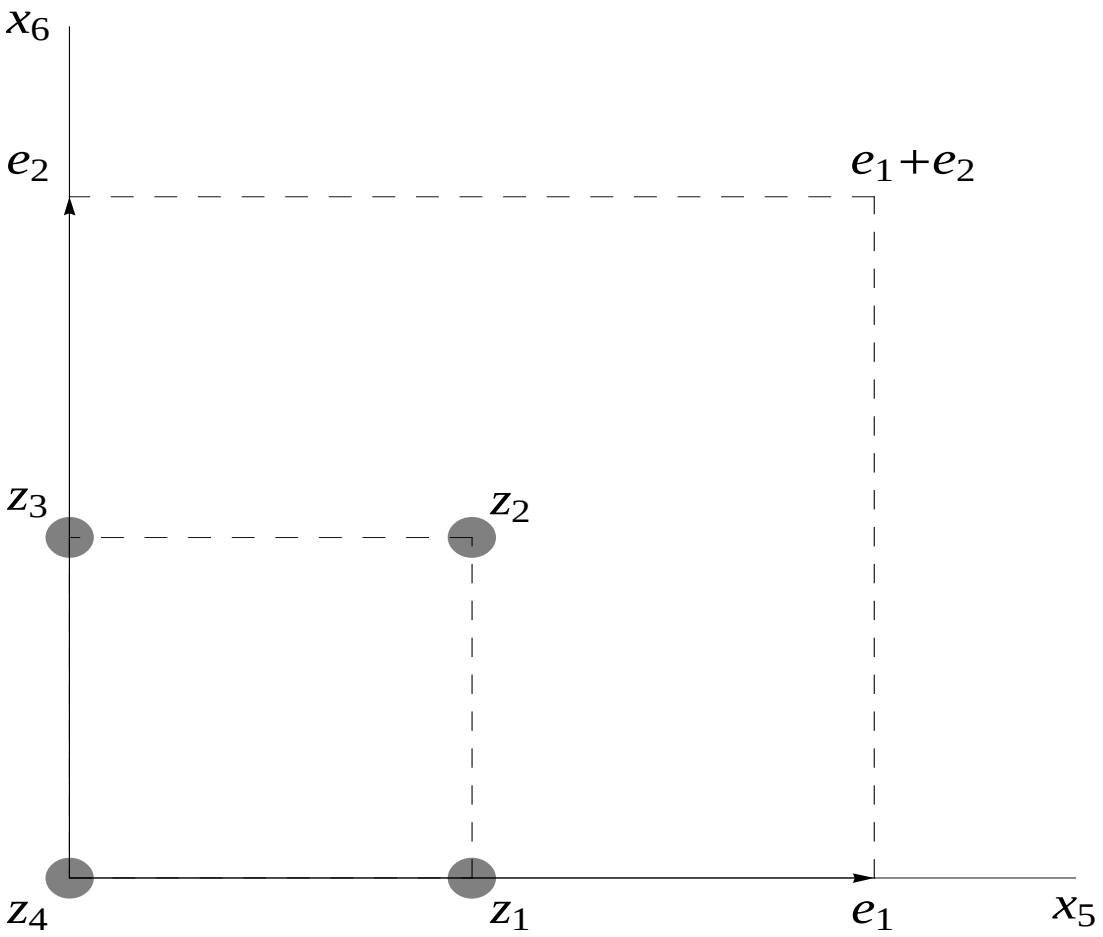
We can also write the symmetries in term of the interchanging of the fixed points,

$$S_1[(14)(23)] : (z_1, z_2, z_3, z_4) \rightarrow (z_4, z_3, z_2, z_1), \quad (9)$$
$$S_2[(12)(34)] : (z_1, z_2, z_3, z_4) \rightarrow (z_2, z_1, z_4, z_3), \quad (10)$$
$$T_R[(13)(2)(4)] : (z_1, z_2, z_3, z_4) \rightarrow (z_3, z_2, z_1, z_4). \quad (11)$$

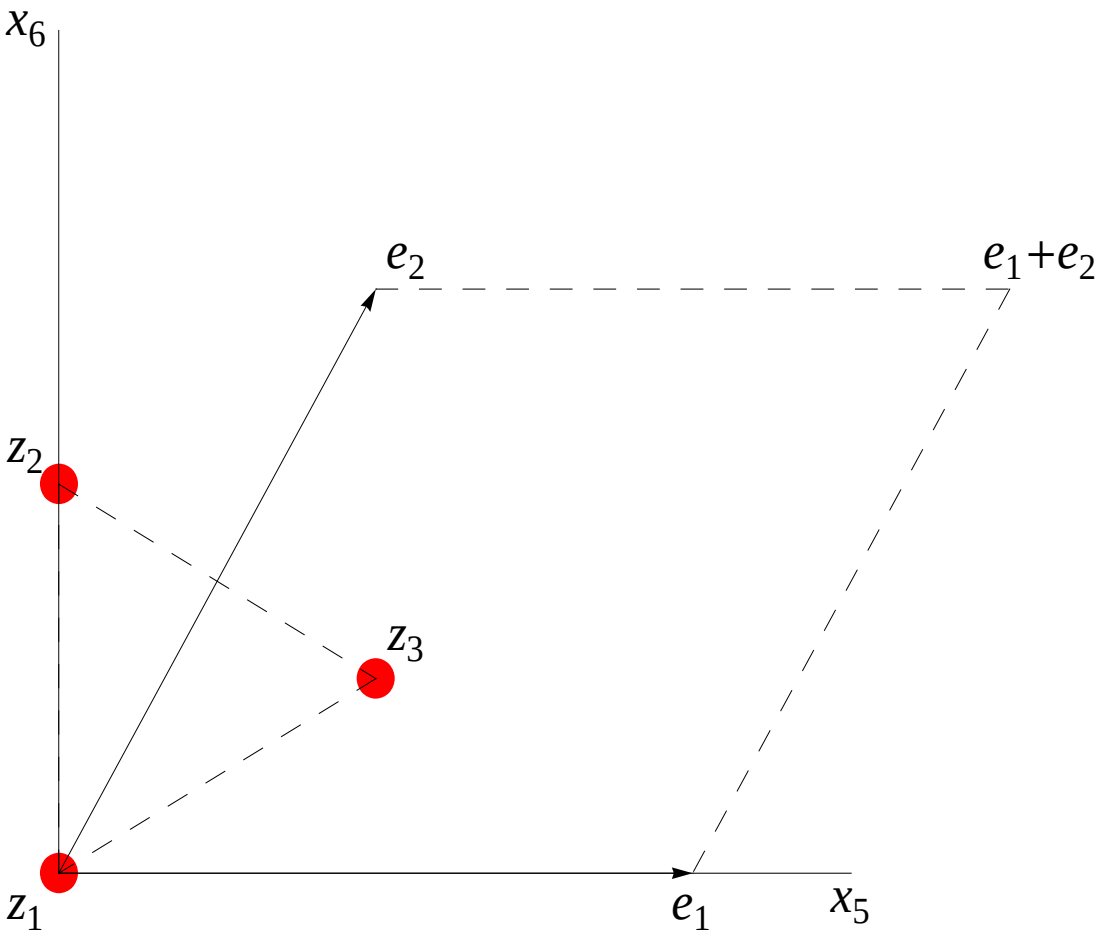
From these elements we can formulate the generators of the discrete group D_4,

$$A = [(13)(2)(4)][(14)(23)] = (1432), \quad (12)$$
$$B = [(12)(34)]. \quad (13)$$

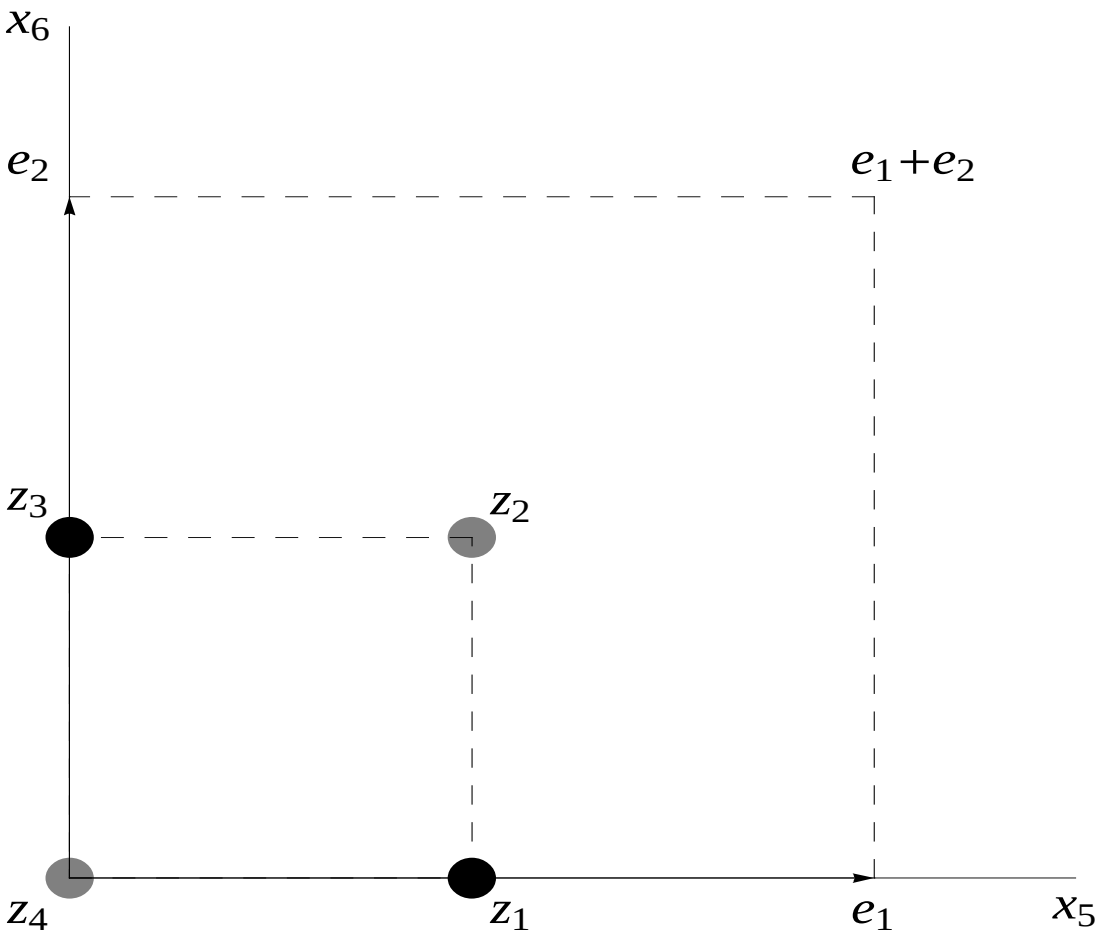
T^2/Z_2



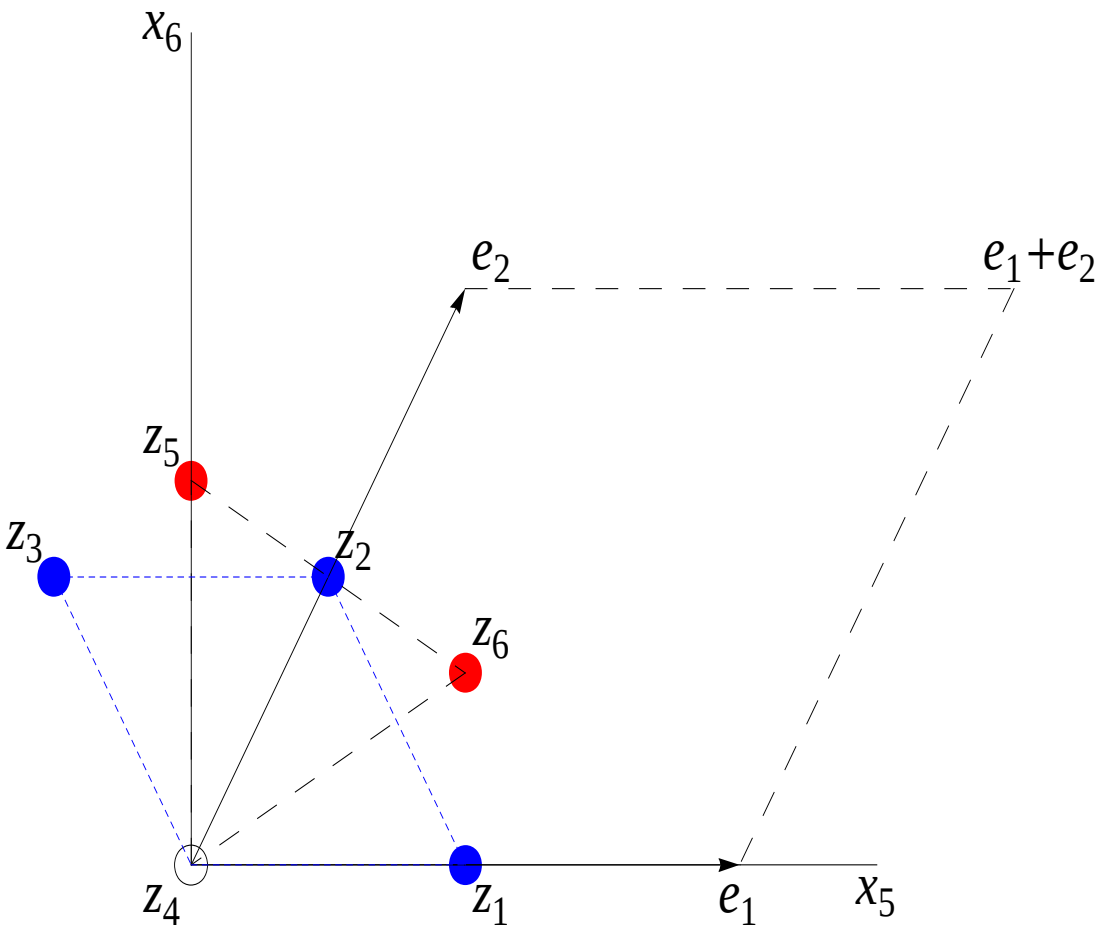
T^2/Z_3



T^2/Z_4



T^2/Z_6



Symmetries from Orbifolding

- For T^2/Z_2, the flavor symmetries are A_4, S_4, D_4
- For T^2/Z_3, the flavor symmetry is $D_3 \simeq S_3$
- For T^2/Z_4, the flavor symmetry is D_4
- For T^2/Z_6, the flavor symmetry is $D_3 \times Z_2 \simeq D_6$

Conclusions

- The breaking of the Poincare symmetry 6d to 4d, $R^4 \times T^2/Z_N \to R^4 \times G_f$, gives us the non-abelian discrete flavor symmetries such as $S_4, A_4, D_3 \simeq S_3, D_4, D_6 \simeq D_3 \times Z_2$, which are all popular groups for flavored model building.
- Bonus: The Vacuum alignments of the flavon can be achieved by Orbifolding (T.Kobayashi, Y.Omura, K.Yoshioka,'08).
- In this approach, we see a connection between flavor symmetry in 4d and space-time symmetry in extra dimensions.