UNIVERSITY OF Southampton

Lattice QCD

Role of LQCD in the future programme

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Contents

Lattice landscape

Lattice kaon physics

- f_K/f_π and K_{l3}
- Neutral Kaon mixing: B_K

Heavy quarks

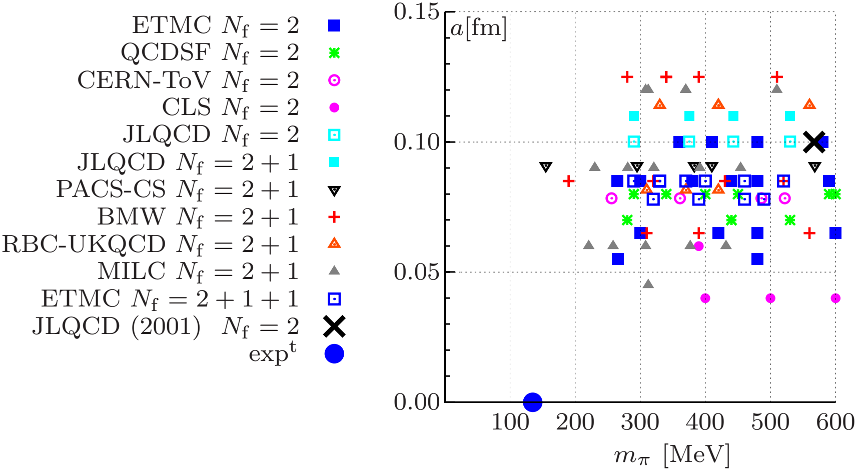
- Decay constants
- Semileptonic decays
- B mixing and decay
- Rare B decays
- Complementary approaches to b-physics

Stress-testing LQCD

Theoretical/computational/people

QCD and beyond

Lattice landscape



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Lattice kaon physics

- **High precision**
  - Unitarity of first row of CKM matrix and quark-lepton universality

$$f^{+}_{K^0\pi^-}, f_K/f_\pi$$

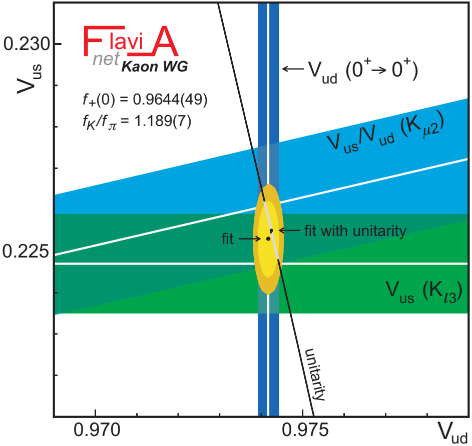
- **Precision**
  - CP violation in neutral kaon mixing in SM and beyond

B_K and BSM matrix elements

- **Exploratory**
  - $\Delta I = 1/2$ rule
  - Direct CP violation in $K \to \pi\pi$ and Re (ϵ'/ϵ)

$|V_{us}|$

- f_K/f_π and K_{l3} provide best lattice constraints on $|V_{us}|$
- Involve SU(3)-breaking: 20% in f_K/f_π, 4% in $f^+_{K\pi}(0)$
- Precision K_{l3} studies more recent, but improving rapidly
  - high precision possible because $f^+_{K\pi}(0) - 1$ is calculated
- Devil is in the systematic error estimation
- Around 1% precision now, aiming for 0.1% or better by 2012



f_K/f_π

		N_f	a/fm	$M_\pi L$	M_π/MeV	f_K/f_π
ETMC	TM	2	≥ 0.07	3.6	≥ 260	1.210(6)(15)(9)
NPLQCD	KS/DWF	2+1	0.13	3.7	≥ 290	$1.218(2)(^{11}_{24})$
MILC	KS	2+1	≥ 0.06	4	≥ 240	$1.197(3)(^{6}_{13})$
HPQCD	KS	2+1	≥ 0.09	3.8	≥ 250	1.189(2)(7)
RBC/UKQCD	DWF	2+1	0.11	4.6	≥ 330	1.205(18)(62)
ALV	KS/DWF	2+1	≥ 0.09	2.3	≥ 240	1.191(16)(17)
PACS-CS	NP-SW	2+1	0.09	2.3	≥ 160	1.189(20)
BMW	SW	2+1	≥ 0.065	≥ 4	≥ 190	1.18(1)(1)
PDG						1.193(6)

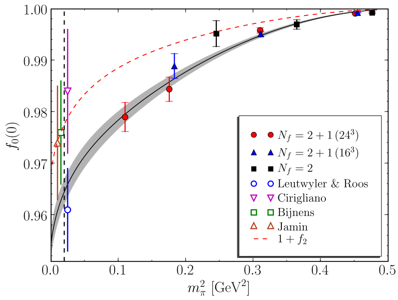
Summary table: Lellouch Lat08, Simula CKM2008, Boyle Kaon09

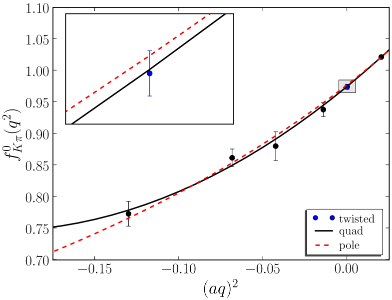
ETMC 0904.0954
MILC 0710.1118
HPQCD PRD73 (2006) 114502
RBC/UKQCD PRD78 (2008) 114509
PACS-CS Lat08 0811.2630
BMW Dürr Lat08

$K \to \pi l \nu$: $f^+(0)$

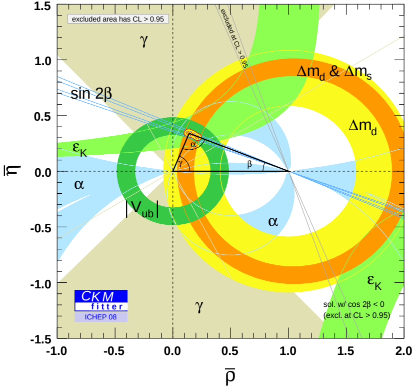
		N_f	a/fm	$M_\pi L$	M_π/MeV	$f^+(0)$
JLQCD 05	NP SW	2	0.09	≥ 5	≥ 550	0.967(6)
RBC 06	DWF	2	0.12	≥ 6	≥ 490	0.968(9)(6)
QCDSF 07	NP SW	2	0.08	≥ 6	≥ 590	0.965(2)(·)
ETMC 08 prel	tmW	2	0.09	≥ 4	≥ 260	0.956(3)(5)
FNAL/MILC 04	SQ	$2+1$				0.962(6)(9)
RBC/UKQCD 08	DWF	$2+1$	0.11	≥ 3	≥ 240	0.964(3)(4)(1)

Lellouch Lat08, Simula CKM2008, Boyle Kaon09





Neutral Kaon mixing: B_K



- $|\epsilon_K| = (2.232 \pm 0.007) \times 10^{-3}$ known to 0.3% precision
- $\delta B_K \approx 5\%$ from lattice calculations

In the standard analysis (Buras et al 1998)

$$|\epsilon_K| \propto B_K |V_{cb}|^2 \left[|V_{cb}|^2 (1 - \bar{\rho}) \eta_{tt} S(x_t) + \eta_{ct} S(x_c, x_t) - \eta_{cc} x_c \right]$$

...but $\delta |V_{cb}| \approx 2.5\%$ so $\delta |V_{cb}|^4$ also relevant

B_K

		N_f	a/fm	$M_\pi L$	M_π/MeV	$\hat{B}_K$
JLQCD	o'lap	2	0.12	2.7	≥ 290	0.734(5)(50)
ETMC	OS/TM	2	0.07	3.1	≥ 300	0.78(3)
HPQCD	KS	$2+1$	0.13	4.5	≥ 360	0.85(2)(18)
RBC/UKQCD	DWF	$2+1$	0.11	4.6	≥ 240	0.720(13)(37)
ALV	KS/DWF	$2+1$	≥ 0.09	≥ 2.5	≥ 230	0.724(8)(28)

Lellouch Lat08, Boyle Kaon09

JLQCD PRD77 (2008) 094503
ETMC Lat08 0810.2443
HPQCD PRD73 (2006) 114502
RBC/UKQCD PRL100 (2008) 032001
ALV arXiv:0905.3947

- Last two well-renormalised calculations are in remarkable agreement
- Renormalisation error becoming dominant; statistical and extrapolation errors will be subleading
- Error $\sim 5\%$ now; aim for 1% error

B-meson decay constants

Unquenched results have 5–7% precision today (0902.1815, 0903.3598)

	FNAL/MILC	HPQCD
f_{B_s}/ MeV	243(11)	231(15)
f_B/ MeV	195(11)	190(13)
f_{B_s}/f_B	1.25(4)	1.226(26)

New techniques developed in quenched simulations: 3–4% error

- interpolate HQET plus relativistic ($\sim$ charm) (ALPHA JHEP 0802(2008)078)
- finite volume step scaling (Rome II NPB672(2003)372)
- combine step-scaling + HQET (JHEP 0801(2008)076)

	f_{B_s}/ MeV
static + relativistic	193(7)
fin-vol step scaling	192(6)(4)
combined	191(6)

Aim for 2% precision by end 2012

D-meson decay constants

	N_f	$f_D/$ MeV	$f_{D_s}/$ MeV	f_{D_s}/f_D
ETMC	2	197(7)(12)	244(4)(11)	1.24(4)(2)
FNAL/MILC	$2+1$	207(11)	249(11)	1.200(27)
HPQCD	$2+1$	207(4)	241(3)	1.162(9)
CLEOc		205.8(8.9)	259.5(7.3)	

- Best quoted errors already at 1–2% level
- Aim to reproduce with other light quark formulations and treatments of charm quark and get below 1% by 2012

Decay constants and new physics

- B, D and K all interesting
- New physics modifies coefficients of low-energy operators and introduces new operators
  - Example for f_{D_s}: exchange of particle with mass M generates (Dobrescu Kronfeld PRL100 (2008) 241802)

$$\frac{C_A^l}{M^2}\bar{s}\gamma_\mu\gamma_5 c\,\bar{\nu}_L\gamma^\mu l_L + \frac{C_P^l}{M^2}\bar{s}\gamma_5 c\,\bar{\nu}_L l_R + \mathrm{hc}$$

leading to $B(D_s \to l\nu) = R_{D_s} B(D_s \to l\nu)_{\mathrm{SM}}$ with

$$R_{D_s} = \left| 1 + \frac{C_A^l + \frac{C_P^l m_{D_s}^2}{m_l(m_c+m_s)}}{\sqrt{2}M^2 G_F V_{cs}^*} \right|^2$$

$(m_{H_\pm}, \tan\beta)$ constraints in 2HDMs

- $B \to \tau\nu$

$$R_B = \left| 1 - \frac{m_B^2}{m_{H_\pm}^2} \frac{\tan^2\beta}{1 + \epsilon_0 \tan\beta} \right|^2$$

  Hampered by f_B and V_{ub} uncertainty

- $D_s \to \tau\nu$ and $D_s \to \mu\nu$

$$R_{D_s} = \left| 1 + \frac{m_{D_s}^2}{m_{H_\pm}^2} \frac{1 - (m_s/m_c)\tan^2\beta}{(1 + m_s/m_c)(1 + \epsilon_0 \tan\beta)} \right|^2$$

  Most constraining at present (asuming V_{cs} known): less if f_{D_s} goes up to $\sim 250\,\text{MeV}$. Experimental errors dominate.

- K decays

$$\left| \frac{V_{us}^{K_{l2}}}{V_{us}^{K_{l3}}} \frac{V_{ud}^{0^+ \to 0^+}}{V_{ud}^{\pi_{l2}}} \right| = \left| 1 - \frac{m_K^2}{m_{H_\pm}^2} \left(1 - \frac{m_d}{m_s}\right) \frac{\tan^2\beta}{1 + \epsilon_0 \tan\beta} \right|$$

  f_K/f_π and $f_{K\pi}^+(0)$ input: 1% errors important for constraint

$|V_{cb}|$

- Luke's theorem and use of double ratios allow good precision
- Errors approaching 2% for quantities $|\mathcal{F}(1)|$ in $B \to D^*$ and $|\mathcal{G}(1)|$ for $B \to D$ from FNAL/MILC. Combine with experimental $|\mathcal{F}(1)V_{cb}|$ and $|\mathcal{G}(1)V_{cb}|$:

$$|V_{cb}| = \begin{cases} 38.2(0.6)_{\mathrm{expt}}(1.0)_{\mathrm{thy}} \times 10^{-3} & B \to D^* \\ 39.5(1.5)_{\mathrm{expt}}(0.9)_{\mathrm{thy}} \times 10^{-3} & B \to D \end{cases}$$

- Application of step-scaling methods and twisted BCs have given 2% accuracy in $|\mathcal{G}(1)|$ from a nonperturbative calculation.
  - Quenched: extend to unquenched
  - Also calculated $\Delta^{B \to D}(\omega)$, parametrizing difference of $B \to D\tau\nu$ and $B \to De\nu$: independent of CKM, sensitive to NP
- Prospects good to aim for 0.5% theory error

$|V_{ub}|$

- Experimental results for $|V_{ub}|f^+_{B\pi}(q^2)$ in q^2 bins
- Combine with lattice $f^+_{B\pi}(q^2)$ using a parametrization of the form factor, preferably model-independent (eg based on analyticity and unitarity)
- Recent results

$$|V_{ub}| = \begin{cases} 3.40(20)(^{59}_{39}) \times 10^{-3} & \text{HPQCD quoted in 0903.3598} \\ 3.38(36) \times 10^{-3} & \text{FNAL/MILC 0811.3640} \end{cases}$$

- Approaching 10% accuracy for exclusive $|V_{ub}|$; aiming for 3% in late 2012.
- Discrepancy with inclusive $|V_{ub}|$
- $B \to \rho$ semileptonic decays could also be used

B mixing and decay

- B^0–$\bar{B}^0$ mixing
  - Most recent lattice results at 3 to 4%:

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}} = \begin{cases} 1.258(33) & \text{HPQCD 0902.1815} \\ 1.205(52) & \text{FNAL/MILC prelim 0903.3598} \end{cases}$$

  - HPQCD quote:

$$\frac{|V_{td}|}{|V_{ts}|} = \xi\sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}} = 0.214(1)_{\text{expt}}(5)_{\text{lattice}}$$

  - Aim for sub 1% precision in ξ by late 2012
- Lifetime differences

Rare B decays

$\langle P \vert \bar{q}\gamma^\mu b \vert B\rangle$	f^+, f^0	$\begin{cases} B \to \pi l \nu \\ B \to K l^+ l^- \end{cases}$
$\langle P \vert \bar{q}\sigma^{\mu\nu} q_\nu b \vert B\rangle$	f_T	$B \to K l^+ l^-$
$\langle V \vert \bar{q}\gamma^\mu b \vert B\rangle$ $\langle V \vert \bar{q}\gamma^\mu \gamma_5 b \vert B\rangle$	V A_0, A_1, A_2	$\begin{cases} B \to (\rho/\omega) l \nu \\ B \to K^* l^+ l^- \end{cases}$
$\langle V \vert \bar{q}\sigma^{\mu\nu} q_\nu b \vert B\rangle$ $\langle V \vert \bar{q}\sigma^{\mu\nu} \gamma_5 q_\nu b \vert B\rangle$	T_1 T_2, T_3	$\begin{cases} B \to K^* \gamma \\ B \to K^* l^+ l^- \end{cases}$

- Semileptonic cases good if can work at high q^2
- Vectors in final state more difficult for chiral extrapolation since not strongly-stable
- $B \to K^* \gamma$ biggest challenge: need $q^2 = 0$
  - Investigation of moving NRQCD to increase q^2 range

Lattice b-physics: complementary approaches

- Use relativistic quarks in charm region and extrapolate to b
- Effective theories
  - HQET: substantial progress in nonperturbative renormalisation, use of static-link fattening and inclusion of $O(\Lambda_{QCD}/m_b)$ corrections
  - NRQCD or Fermilab/Tsukuba (RHQ) actions
- Finite-volume and step-scaling approach of Rome-II group

$$\mathcal{O}(L_\infty) = \mathcal{O}(L_0)\frac{\mathcal{O}(L_1)}{\mathcal{O}(L_0)}\cdots\frac{\mathcal{O}(L_N)}{\mathcal{O}(L_{N-1})}$$

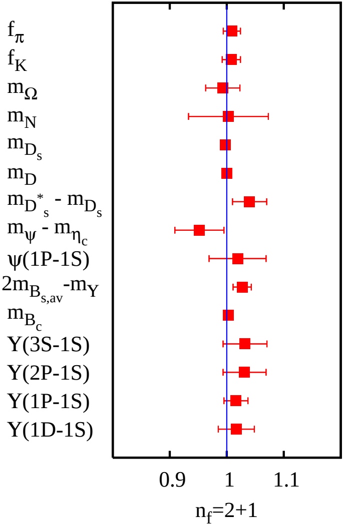
L_0 small enough to allow $a \approx 0.01$ fm and $L_N \sim L_\infty$ (last factor is 1 to required precision)

b-physics prognosis

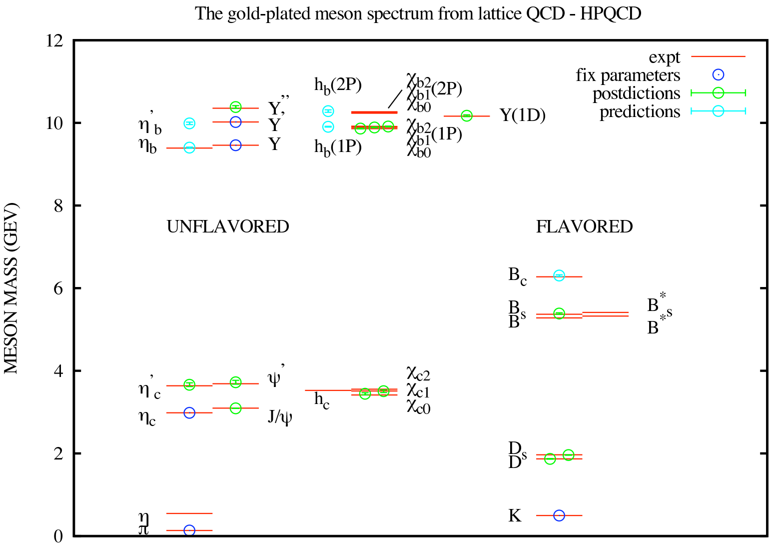
- *Best* results likely from combining extrapolation from $m_Q \approx m_c$ with effective theory results (including $\Lambda_{\mathrm{QCD}}/m_b$ corrections)
- Few % precision requires nonperturbative renormalisation
- Medium term: look for agreement between different approaches (HQET, NRQCD, Fermilab/Tsukuba) and study theoretical foundations
- Although quenched approximation has been banished from light quark physics, some heavy quark analysis still being developed using quenched ensembles: redo unquenched once methods established

Stress testing LQCD?

- Every new ensemble requires study of light hadron spectrum, f_K, f_π
- Many checks in spectroscopy, masses and splittings
- $\Delta I = 1/2$ rule and ϵ'/ϵ
  - LO ChPT with $K \to \pi$ and $K \to |0\rangle$ matrix elements is not sufficient
  - Need $K \to \pi\pi$ matrix elements: has spurred theoretical developments; computations underway
- $B \to M_1 M_2$ on the lattice needs new ideas
- Turn question around to test ChPT: LECs



Davies 0807.1402; see also MILC 0903.3598



plot from Davies HPQCD

Lattice calculations of LECs

- $SU(2)_L \times SU(2)_R$ vs $SU(3)_L \times SU(3)_R$
- SU(2) LECs (Necco 0901.4257; FLAG [Colangelo] Kaon09)
  - $\bar{\ell}_3$ and $\bar{\ell}_4$ from m_π^2 and f_π
  - $\bar{\ell}_6$ from pion electromagnetic form factor (charge radius)

	N_f	$\bar{\ell}_3$	$\bar{\ell}_4$	$\bar{\ell}_6$
CERN/TOV	2	3.0(5)(1)	4.1(1)($\cdot$)	
JLQCD/TWQCD	2	$3.38(^{56}_{47})$	$4.12(^{56}_{46})$	11.9(7)(10)
ETMC	2	3.42(30)	4.59(14)	
ETMC	2	3.2(4)(2)	4.4(1)(1)	14.9(12)(7)
PACS-CS	2+1	3.14(23)	4.04(19)	
RBC/UKQCD	2+1	3.13(33)(24)	4.43(14)(78)	12.2(9)
Gasser Leutwyler	phen	2.9(2.4)	4.3(9)	
Colangelo et al	phen		4.4(2)	
Bijnens et al 98				16.0(5)(7)

Theoretical

- UK groups linked to three international collaborations using three different formalisms
  - ETMC: twisted mass
  - HPQCD: staggered
  - RBC/UKQCD: domain wall

  collaborations leverage resources
- Nonperturbative renormalisation
- Chiral extrapolation: SU(2) versus SU(3)
- Making better use of configurations: stochastic propagators, wall sources, one-end trick for spatial averaging; multiple sources
- Working with arbitrary momenta: twisting and partial twisting
- Improved actions and operators
- Effective actions for heavy(ier) quarks: HISQ, static, NRQCD, RHQ

Computational cost

- Dramatic algorithmic improvement in C21: improved scaling
- Good idea of needed resources for target simulations
- Europe, US, Japan: access to 10–100 Tflops now, 0.5–1 Pflops 2010–2012
- HPC bid submitted April 2007; PPRP recommended $\approx$ £7M funding for UKQCD 21 months ago.
  - Would get us to $\sim$ 0.4 Pflops in 2011–2012
  - Still awaiting final outcome

Target light quark simulation aiming at 1% error	
N_{conf}	120
a	$1/20$ fm
a^{-1}	$\approx$ 4 GeV
m_l/m_s	$1/12$
m_π	200 MeV
L	4.5 fm
Vol	$90^3 \times 180$
Wilson	0.1 Pflops yr
DWF	1.5 Pflops yr

People power

- UK in credit from recent past (eg QCDOC) but now relying on manpower contributions and theoretical input
- Community needs access to machines to maintain international collaborations and to entice new blood

QCD and beyond

- New physics via matrix elements of additional quark and gluon operators at low energies
- hot and dense
  - QCD transport coefficients (viscosity, conductivity, ...) and validity of the hydrodynamic description at RHIC/ALICE
  - Nature of ultradense baryonic matter: deconfined, superconducting, can it be found in our universe? Difficult (sign problem) but indirect attacks via 2-colour QCD and complex Langevin simulation
- Lattice BSM: explore strongly-interacting QFTs that might help us interpret LHC results
  - SU(N) for any N with quarks in arbitrary representations: new technicolor, orientifold-planar equivalence (SUSY $\leftrightarrow$ non-SUSY at large N)
  - supersymmetry

Summary: targeted precision

Quantity	CKM	Hadronic ME	Error
f_K/f_π	$\lvert V_{us}\rvert$	f_K/f_π	$< 0.1\%$
$K \to \pi l \nu$	$\lvert V_{us}\rvert$	$f^+_{K\pi}(0)$	$< 0.1\%$
ϵ_K	$\mathrm{Im}\ V_{td}^2$	B_K	1%
$D_{(s)} \to l\nu$	$\lvert V_{cd}\rvert, \lvert V_{cs}\rvert$	$f_{D_{(s)}}$	$< 1\%$
$B \to l\nu$	$\lvert V_{ub}\rvert$	f_B	2%
Δm_d	$\lvert V_{td}\rvert$	$f_{B_d}\sqrt{B_{B_d}}$	2%
$\Delta m_d/\Delta m_s$	$\lvert V_{td}/V_{ts}\rvert$	ξ	0.8%
$B \to D^{(*)} l\nu$	$\lvert V_{cb}\rvert$	$F_{B\to D^{(*)} l\nu}$	0.5%
$B \to \pi l \nu$	$\lvert V_{ub}\rvert$	$f^+_{B\pi}(q^2)$	3%
$B \to [K^{(*)}, \rho][\gamma, l^+ l^-]$	$\lvert V_{td}/V_{ts}\rvert$	T_1 etc	4%