Parton Model and perturbative QCD
Lecture III: The QCD improved parton model

65th Scottish Universities Summer School in Physics:

LHC Physics
August 2009

Keith Ellis

`ellis@fnal.gov`

Fermilab

Bibliography

R. K. Ellis, W.J. Stirling and B.R. Webber, *QCD and Collider Physics* (Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology)

Drell Yan: G. Altarelli, R. K. Ellis and G. Martinelli, *Large Perturbative Corrections To The Drell-Yan Process In QCD,* Nucl. Phys. B **157**, 461 (1979).

Heavy quark production: P. Nason, S. Dawson and R. K. Ellis, *The Total Cross-Section for the Production of Heavy Quarks in Hadronic Collisions,* Nucl. Phys. B **303**, 607 (1988).

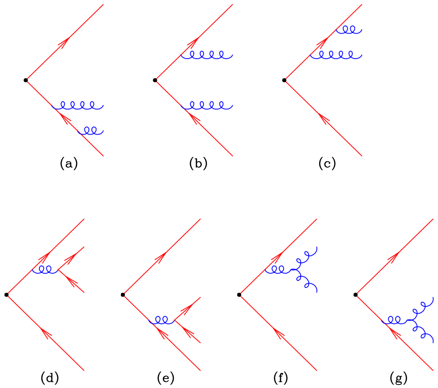
The QCD improved parton model

- Gauge group for QCD
- Hadron-hadron processes and factorization
- Parton luminosities
- W production
  - ⋆ DY cross section
  - ⋆ Subtraction method
- Top quark production
- NLO parton integrators
  - ⋆ Example: MCFM
  - ⋆ W + jet production
- Combining NLO results with parton showers

Gauge group for QCD

If we accept that there are three colours of quarks, what gauge groups would be acceptable.

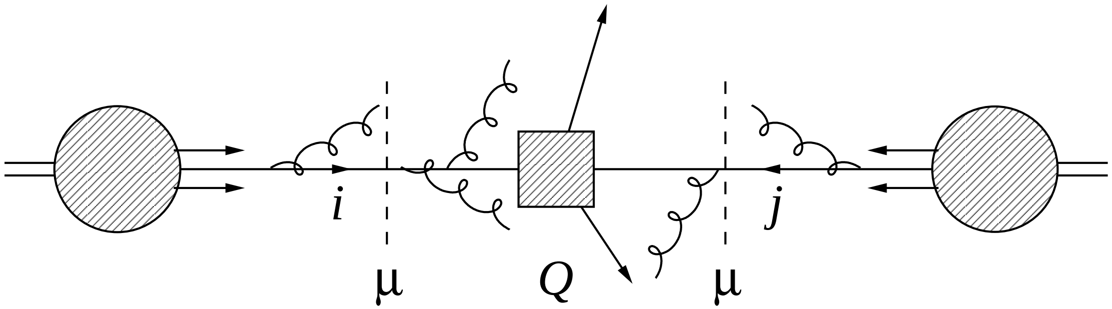
- Is U(3) an acceptable group? U(3) would lead to 9 gluons insted of eight. The ninth gluon $(a\bar{a} + b\bar{b} + c\bar{c})/\sqrt{3}$ would be invariant under U(3) transformations. It would therefore be a colour singlet; it would propagate without confinement leading to a long range strong force, in contrast with observation
- Colour factors have been measured by examining the angular structure of 4-jet events in e^+e^- collision.



- For example, the Delphi collaboration obtained the result, (CERN-PPE/97-112) $N_C/N_A = 0.38 \pm 0.1$, to be compared with the SU(3) (3/8-0.375) and U(3) (3/9=0.3333)
- O(3) with only three gluons is similarly excluded, in addition to its exclusion on the grounds that it would lead to binding of qq as well as $q\bar{q}$.

Hadron-hadron processes

- In hard hadron-hadron scattering, constituent partons from each incoming hadron interact at short distance (large momentum transfer Q^2).



- For hadron momenta P_1, P_2 ($S = 2P_1 \cdot P_2$), form of cross section is

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2 D_i(x_1, \mu^2) D_j(x_2, \mu^2) \hat{\sigma}_{ij}(\hat{s} = x_1 x_2 S, \alpha_S(\mu^2), Q^2/\mu^2)$$

where μ^2 is factorization scale and $\hat{\sigma}_{ij}$ is subprocess cross section for parton types i, j.

  - ⋆ Notice that factorization scale is in principle arbitrary: affects only what we call part of subprocess or part of initial-state evolution (parton shower).
  - ⋆ Unlike e^+e^- or ep, we may have interaction between spectator partons, leading to soft underlying event and/or multiple hard scattering.

Factorization of the cross section

Why does the factorization property hold and when it should fail? For a heuristic argument Consider the simplest hard process involving two hadrons

$$H_1(P_1) + H_2(P_2) \to V + X.$$

Do the partons in hadron H_1, through the influence of their colour fields, change the distribution of partons in hadron H_2 before the vector boson is produced? Soft gluons which are emitted long before the collision are potentially troublesome.
A simple model from classical electrodynamics. The vector potential due to an electromagnetic current density J is given by

$$A^\mu(t, \vec{x}) = \int dt' d\vec{x}' \, \frac{J^\mu(t', \vec{x}')}{|\vec{x} - \vec{x}'|} \, \delta(t' + |\vec{x} - \vec{x}'| - t) \; ,$$

where the delta function provides the retarded behaviour required by causality. Consider a particle with charge e travelling in the positive z direction with constant velocity β. The non-zero components of the current density are

$$\begin{aligned} J^t(t', \vec{x}') &= e\delta(\vec{x}' - \vec{r}(t')) \; , \\ J^z(t', \vec{x}') &= e\beta\delta(\vec{x}' - \vec{r}(t')), \;\; \vec{r}(t') = \beta t' \hat{z}, \end{aligned}$$

$\hat{z}$ is a unit vector in the z direction.

Vector potential

At an observation point (the supposed position of hadron H_2) described by coordinates x, y and z, the vector potential (after performing the integrations using the current density given above) is

$$\begin{aligned} A^t(t,\vec{x}) &= \frac{e\gamma}{\sqrt{[x^2+y^2+\gamma^2(\beta t-z)^2]}} \\ A^x(t,\vec{x}) &= 0 \\ A^y(t,\vec{x}) &= 0 \\ A^z(t,\vec{x}) &= \frac{e\gamma\beta}{\sqrt{[x^2+y^2+\gamma^2(\beta t-z)^2]}}, \end{aligned}$$

where $\gamma^2 = 1/(1-\beta^2)$. Target hadron H_2 is at rest near the origin, so that $\gamma \approx s/m^2$. Note that for large γ and fixed non-zero $(\beta t - z)$ some components of the potential tend to a constant independent of γ, suggesting that there will be non-zero fields which are not in coincidence with the arrival of the particle, even at high energy.
However at large γ the potential is a pure gauge piece, $A^\mu = \partial^\mu \chi$ where χ is a scalar function. Covariant formulation using the vector potential A has large fields which have no effect.
For example, the electric field along the z direction is

$$E^z(t,\vec{x}) = F^{tz} \equiv \frac{\partial A^z}{\partial t} + \frac{\partial A^t}{\partial z} = \frac{e\gamma(\beta t - z)}{[x^2+y^2+\gamma^2(\beta t-z)^2]^{\frac{3}{2}}}.$$

Parton luminosity

- A rough and ready way to estimate cross sections. Let us define the parton luminosity $\frac{dL_{ij}}{d\tau}$

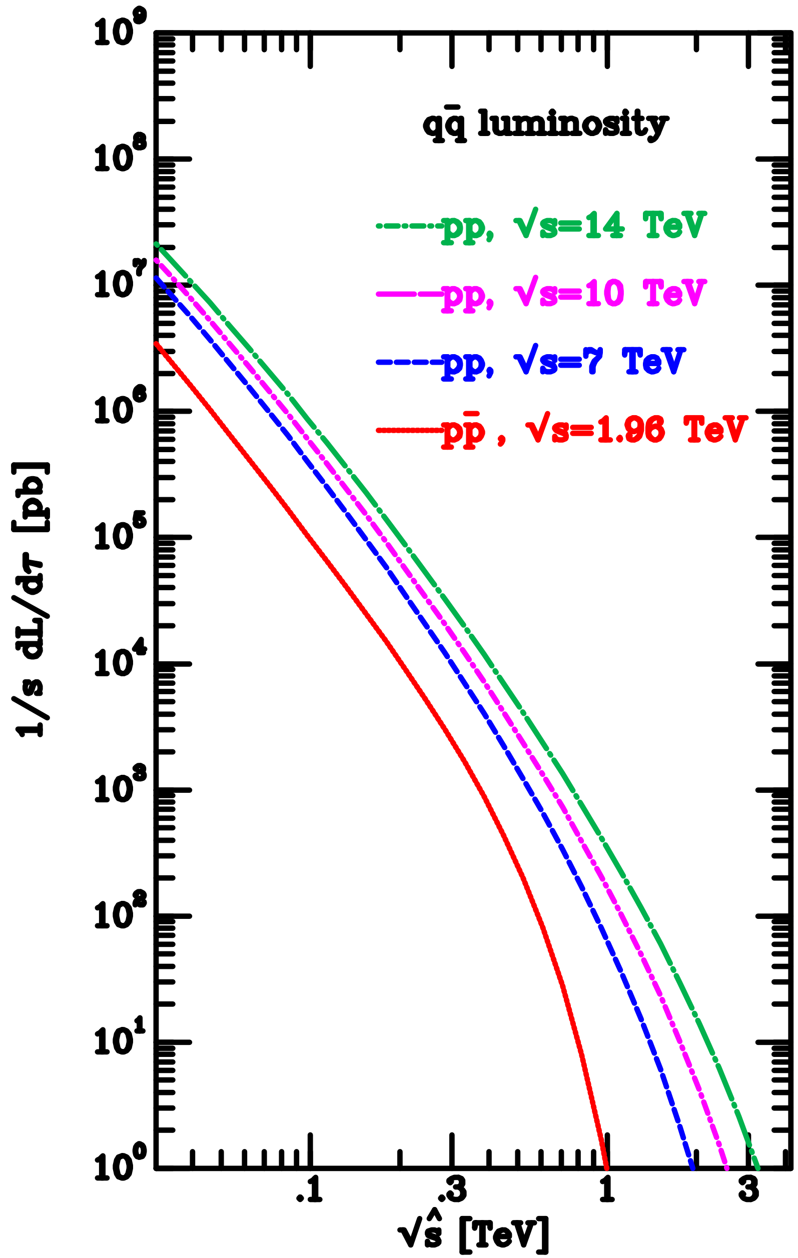
$$\tau \frac{dL_{ij}}{d\tau} = \frac{1}{1+\delta_{ij}} \int_0^1 dx_1 dx_2 \Big[\big(x_1 f_i(x_1,\mu^2)\ x_2 f_j(x_2,\mu^2) \big) + (1 \leftrightarrow 2) \Big] \delta(\tau - x_1 x_2).$$

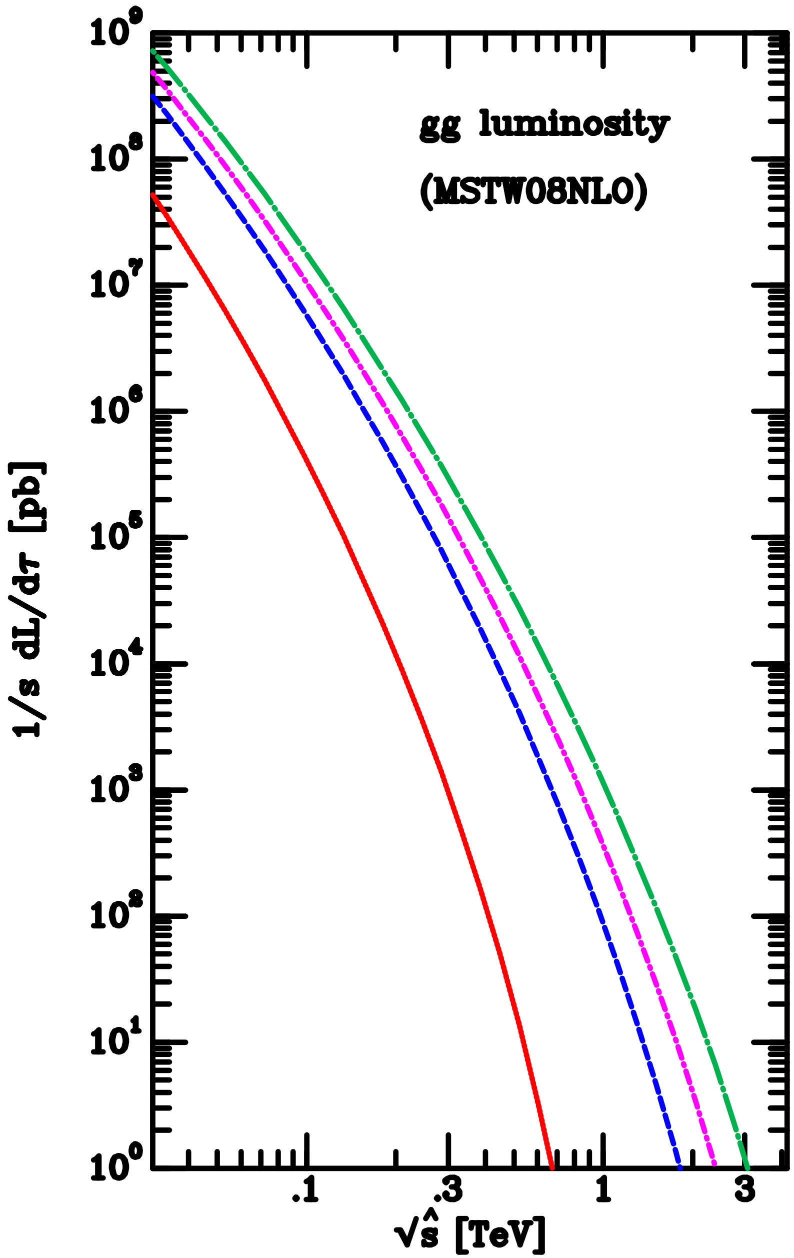
in terms of which we may write the hadronic cross section as

$$\sigma(s) = \sum_{\{ij\}} \int_{\tau_0}^1 \frac{d\tau}{\tau} \left[\frac{1}{s} \frac{dL_{ij}}{d\tau} \right] \left[\hat{s} \hat{\sigma}_{ij} \right],$$

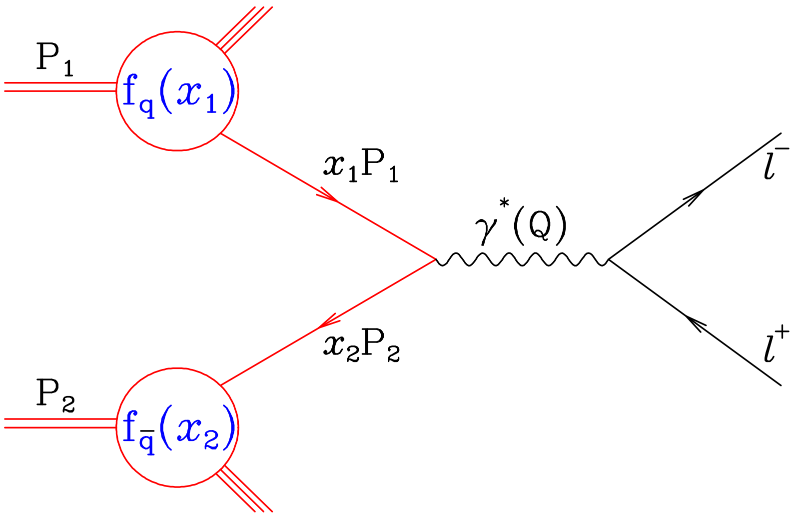
- the first object in square brackets has the dimensions of cross section,
- the second expression in square brackets $[\hat{s}\hat{\sigma}]$ is dimensionless.
- The production of a 200 GeV object, ($\sqrt{\hat{s}} = 0.2$ TeV) produced by gluon fusion at $\sqrt{s} = 7, 14$ TeV.
- Luminosities are 4×10^5 and 1.5×10^6 pb respectively.

Parton luminosity plots





Lepton-pair production



- Mechanism for Lepton pair production, W-production, Z-production, Vector-boson pairs, . . .
- Collectively known as the Drell-Yan process.
- Colour average $1/N$.

$$\frac{d\hat{\sigma}}{dQ^2} = \frac{\sigma_0}{N} \, Q_q^2 \, \delta(\hat{s} - Q^2), \qquad \sigma_0 = \frac{4\pi\alpha^2}{3Q^2}, \quad \text{cf } e^+e^- \text{ annihilation.}$$

In the CM frame of the two hadrons, the momenta of the incoming partons are

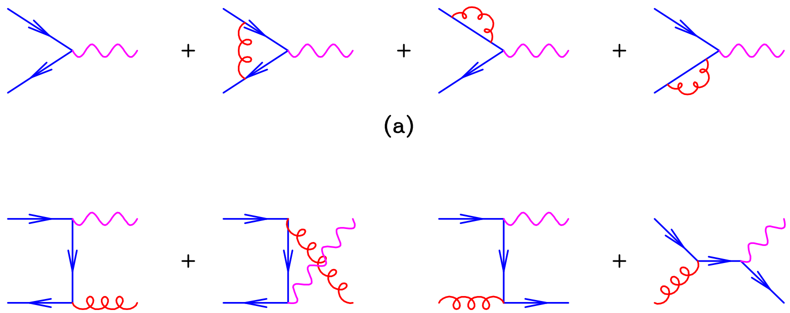
$$p_1 = \frac{\sqrt{s}}{2}(x_1, 0, 0, x_1), \quad p_2 = \frac{\sqrt{s}}{2}(x_2, 0, 0, -x_2) \; .$$

The square of the $q\bar{q}$ collision energy $\hat{s}$ is related to the overall hadron-hadron collision energy by $\hat{s} = (p_1 + p_2)^2 = x_1 x_2 s$. The parton-model cross section for this process is:

$$
\begin{aligned}
\frac{d\sigma}{dM^2} &= \int_0^1 dx_1 dx_2 \sum_q \{f_q(x_1) f_{\bar{q}}(x_2) + (q \leftrightarrow \bar{q})\} \frac{d\hat{\sigma}}{dM^2}(q\bar{q} \to l^+ l^-) \\
&= \frac{\sigma_0}{Ns} \int_0^1 \frac{dx_1}{x_1} \frac{dx_2}{x_2} \, \delta(1-z) \left[\sum_q Q_q^2 \{f_q(x_1) f_{\bar{q}}(x_2) + (q \leftrightarrow \bar{q})\} \right] .
\end{aligned}
$$

- For later convenience we have introduced the variable $z = \frac{Q^2}{\hat{s}} = \frac{Q^2}{x_1 x_2 s}$.
- The sum here is over quarks only and the $\bar{q}q$ contributions are indicated explicitly.

Next-to-leading order



(a) (b) (c)

- The contribution of the real diagrams (in four dimensions) is

$$|M|^2 \sim g^2 C_F \left[\frac{u}{t} + \frac{t}{u} + \frac{2Q^2 s}{ut} \right] = g^2 C_F \left[\left(\frac{1+z^2}{1-z} \right) \left(\frac{-s}{t} + \frac{-s}{u} \right) - 2 \right]$$

  where $z = Q^2/s$, $s + t + u = Q^2$.

- Note that the real diagrams contain collinear singularities, $u \to 0, t \to 0$ and soft singularities, $z \to 1$.
- The coefficient of the divergence is the unregulated branching probability $\hat{P}_{qq}(z)$.
- Ignore for simplicity the diagrams with incoming gluons.

- Control the divergences by continuing the dimensionality of space-time, $d = 4 - 2\epsilon$, (technically this is dimensional reduction). Performing the phase space integration, the total contribution of the real diagrams is

$$\sigma_R = \frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} c_\Gamma \Bigg[\Big(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{\pi^2}{3} \Big) \delta(1-z) - \frac{2}{\epsilon} P_{qq}(z)$$
$$- 2(1-z) + 4(1+z^2) \Big[\frac{\ln(1-z)}{1-z} \Big]_+ - 2 \frac{1+z^2}{(1-z)} \ln z \Bigg]$$

with $c_\Gamma = (4\pi)^\epsilon / \Gamma(1-\epsilon)$.

- The contribution of the virtual diagrams is

$$\sigma_V = \delta(1-z) \left[1 + \frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} c'_\Gamma \Big(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 6 + \pi^2 \Big) \right]$$

$$c'_\Gamma = c_\Gamma + O(\epsilon^3)$$

- Adding it up we get in dim-reduction

$$\sigma_{R+V} = \frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} c_\Gamma \Bigg[\Big(\frac{2\pi^2}{3} - 6\Big)\delta(1-z) - \frac{2}{\epsilon} P_{qq}(z) - 2(1-z)$$
$$+ \quad 4(1+z^2)\Big[\frac{\ln(1-z)}{1-z}\Big]_+ - 2\frac{1+z^2}{(1-z)}\ln z \Bigg]$$

- The divergences, proportional to the branching probability , are universal.
- We will factorize them into the parton distributions. We perform the mass factorization by subtracting the counterterm

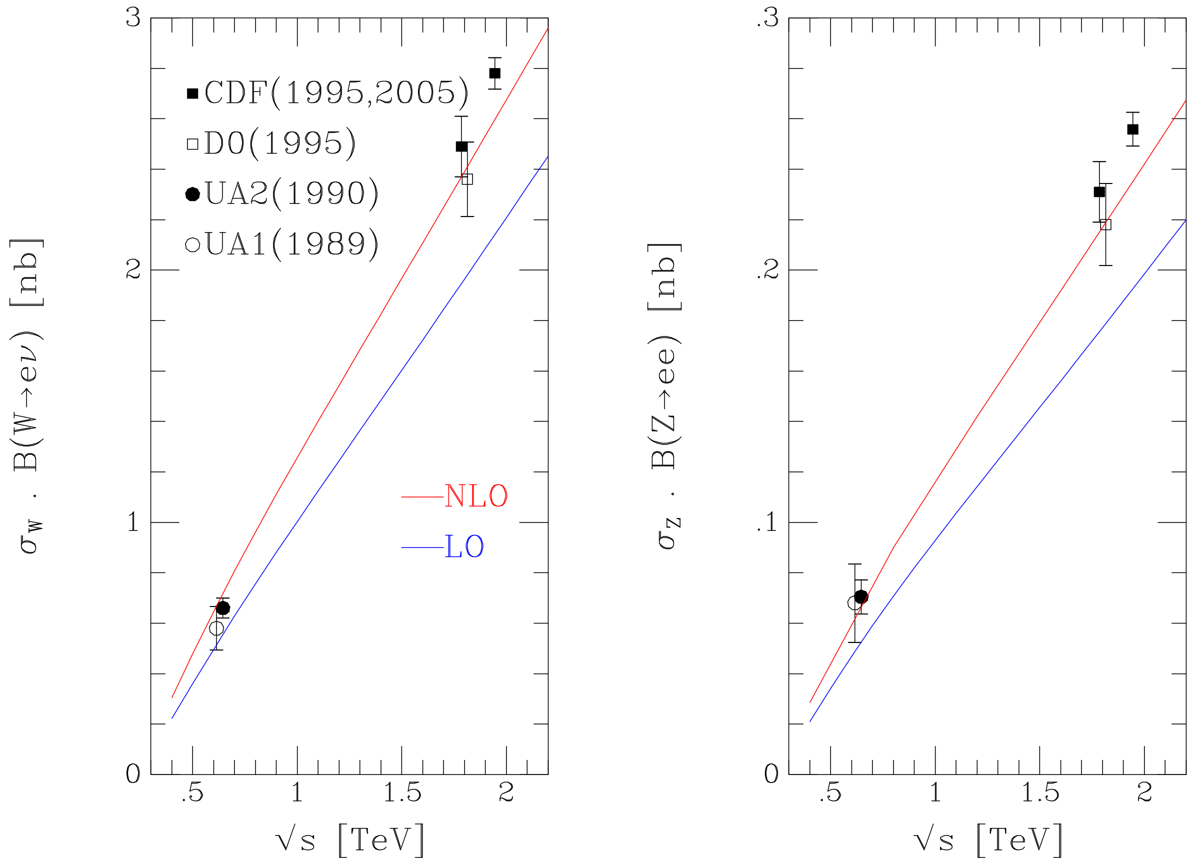
$$2\frac{\alpha_S}{2\pi} C_F \left[\frac{-c_\Gamma}{\epsilon} P_{qq}(z) - (1-z) + \delta(1-z)\right]$$

(The finite terms are necessary to get us to the $\overline{MS}$-scheme).

$$\hat{\sigma} = \frac{\alpha_S}{2\pi} C_F \left[\Big(\frac{2\pi^2}{3} - 8\Big)\delta(1-z) + 4(1+z^2)\Big[\frac{\ln(1-z)}{1-z}\Big]_+ - 2\frac{1+z^2}{(1-z)}\ln z + 2P_{qq}(z)\ln\frac{Q^2}{\mu^2}\right]$$

- Similar correction for incoming gluons.

Application to W, Z production



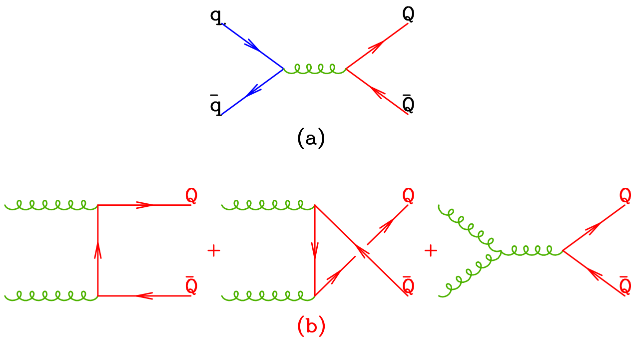
- Agreement with NLO theory is good.
- LO curves lie about 25% too low.
- NNLO results are also known and lead to a further modest (4%) increase at the Tevatron.

Heavy quark production, leading order

The leading-order processes for the production of a heavy quark Q of mass m in hadron-hadron collisions

$$\begin{array}{ll} (a) & q(p_1) + \overline{q}(p_2) \to Q(p_3) + \overline{Q}(p_4) \\ (b) & g(p_1) + g(p_2) \to Q(p_3) + \overline{Q}(p_4) \end{array}$$

where the four-momenta of the partons are given in brackets.



Process	$\overline{\sum}\lvert\mathcal{M}\rvert^2/g^4$
$q\,\overline{q} \to Q\,\overline{Q}$	$\frac{4}{9}\left(\tau_1^2 + \tau_2^2 + \frac{\rho}{2}\right)$
$g\,g \to Q\,\overline{Q}$	$\left(\frac{1}{6\tau_1\tau_2} - \frac{3}{8}\right)\left(\tau_1^2 + \tau_2^2 + \rho - \frac{\rho^2}{4\tau_1\tau_2}\right)$

$\overline{\sum}$ indicates averaged (summed) over initial (final) colours and spins. We have introduced the following notation for the ratios of scalar products:

$$\tau_1 = \frac{2p_1.p_3}{\hat{s}}, \quad \tau_2 = \frac{2p_2.p_3}{\hat{s}}, \quad \rho = \frac{4m^2}{\hat{s}}, \quad \hat{s} = (p_1 + p_2)^2.$$

- The short-distance cross section is obtained from the invariant matrix element in the usual way:

$$d\hat{\sigma}_{ij} = \frac{1}{2\hat{s}} \frac{d^3p_3}{(2\pi)^3 2E_3} \frac{d^3p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \overline{\sum} |\mathcal{M}_{ij}|^2 .$$

  The first factor is the flux factor for massless incoming particles. The other terms come from the phase space for $2 \to 2$ scattering.

- In terms of the rapidity $y = \frac{1}{2} \ln((E + p_z)/(E - p_z))$ and transverse momentum, p_T, the relativistically invariant phase space volume element of the final-state heavy quarks is

$$\frac{d^3p}{E} = dy \; d^2p_T \; .$$

The result for the invariant cross section may be written as

$$\frac{d\sigma}{dy_3 dy_4 d^2 p_T} = \frac{1}{16\pi^2 \hat{s}^2} \sum_{ij} x_1 f_i(x_1, \mu^2)\, x_2 f_j(x_2, \mu^2) \overline{\sum} |\mathcal{M}_{ij}|^2.$$

x_1 and x_2 are fixed if we know the transverse momenta and rapidity of the outgoing heavy quarks. In the centre-of-mass system of the incoming hadrons we may write

$$\begin{aligned} p_1 &= \frac{1}{2}\sqrt{s}(x_1, 0, 0, x_1) \\ p_2 &= \frac{1}{2}\sqrt{s}(x_2, 0, 0, -x_2) \\ p_3 &= (m_T \cosh y_3, p_T, 0, m_T \sinh y_3) \\ p_4 &= (m_T \cosh y_4, -p_T, 0, m_T \sinh y_4). \end{aligned}$$

Applying energy and momentum conservation, we obtain

$$x_1 = \frac{m_T}{\sqrt{s}}\left(e^{y_3} + e^{y_4}\right), x_2 = \frac{m_T}{\sqrt{s}}\left(e^{-y_3} + e^{-y_4}\right), \hat{s} = 2m_T^2(1 + \cosh \Delta y).$$

The quantity $m_T = \surd(m^2 + p_T^2)$ is the transverse mass of the heavy quarks and $\Delta y = y_3 - y_4$ is the rapidity difference between them.

In these variables the leading order cross section is

$$\frac{d\sigma}{dy_3 dy_4 d^2 p_T} = \frac{1}{64\pi^2 m_T^4 (1+\cosh(\Delta y))^2} \sum_{ij} x_1 f_i(x_1,\mu^2)\, x_2 f_j(x_2,\mu^2)\, \overline{\sum} |\mathcal{M}_{ij}|^2 .$$

Expressed in terms of m, m_T and Δy, the matrix elements for the two processes are

$$\overline{\sum}|\mathcal{M}_{q\overline{q}}|^2 = \frac{4g^4}{9}\Big(\frac{1}{1+\cosh(\Delta y)}\Big)\Big(\cosh(\Delta y) + \frac{m^2}{m_T^2}\Big) \ ,$$

$$\overline{\sum}|\mathcal{M}_{gg}|^2 = \frac{g^4}{24}\Big(\frac{8\cosh(\Delta y) - 1}{1+\cosh(\Delta y)}\Big)\Big(\cosh(\Delta y) + 2\frac{m^2}{m_T^2} - 2\frac{m^4}{m_T^4}\Big).$$

- As the rapidity separation Δy between the two heavy quarks becomes large

$$\overline{\sum}|\mathcal{M}_{q\overline{q}}|^2 \sim \text{ constant}, \quad \overline{\sum}|\mathcal{M}_{gg}|^2 \sim \ \exp \Delta y \ .$$

- The cross section is damped at large Δy and heavy quarks produced by $q\bar{q}$ annihilation are more closely correlated in rapidity those produced by gg fusion.

Applicability of perturbation theory?

- Consider the propagators in the diagrams.

$$\begin{aligned}(p_1+p_2)^2 = &\quad 2p_1.p_2 &= 2m_T^2\left(1+\cosh\Delta y\right)\ ,\\ (p_1-p_3)^2-m^2 = &\quad -2p_1.p_3 &= -m_T^2\left(1+e^{-\Delta y}\right)\ ,\\ (p_2-p_3)^2-m^2 = &\quad -2p_2.p_3 &= -m_T^2\left(1+e^{\Delta y}\right)\ .\end{aligned}$$

  Note that the propagators are all off-shell by a quantity of least of order m^2.

- Thus for a sufficiently heavy quark we expect the methods of perturbation theory to be applicable. It is the mass m (which by supposition is very much larger than the scale of the strong interactions Λ) which provides the large scale in heavy quark production. We expect corrections of order Λ/m
- This does not address the issue of whether the charm or bottom mass is large enough to be adequately described by perturbation theory.

Heavy quark production in $O(\alpha_S^3)$

In NLO heavy quark production m is the heavy quark mass.

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2 \; \hat{\sigma}_{ij}(x_1 x_2 S, m^2, \mu^2) F_i(x_1, \mu^2) F_j(x_2, \mu^2)$$

$$\hat{\sigma}_{i,j}(\hat{s}, m^2, \mu^2) = \sigma_0 c_{ij}(\hat{\rho}, \mu^2)$$

where $\hat{\rho} = 4m^2/\hat{s}, \bar{\mu}^2 = \mu^2/m^2, \sigma_0 = \alpha_S^2(\mu^2)/m^2$ and $\hat{s}$ in the parton total c-of-m energy squared. The coupling satisfies

$$\frac{d\alpha_S}{d\ln\mu^2} = -b_0 \frac{\alpha_S^2}{2\pi} + O(\alpha_S^3), \; b_0 = \frac{11N - 2n_f}{6}$$

$$c_{ij}\left(\rho, \frac{\mu^2}{m^2}\right) = c_{ij}^{(0)}(\rho) + 4\pi\alpha_S(\mu^2)\left[c_{ij}^{(1)}(\rho) + \overline{c}_{ij}^{(1)}(\rho)\ln\left(\frac{\mu^2}{m^2}\right)\right] + O(\alpha_S^2)$$

The lowest-order functions $c_{ij}^{(0)}$ are obtained by integrating the lowest order matrix elements

$$c_{q\overline{q}}^{(0)}(\rho) = \frac{\pi\beta\rho}{27}\left[(2+\rho)\right] ,$$

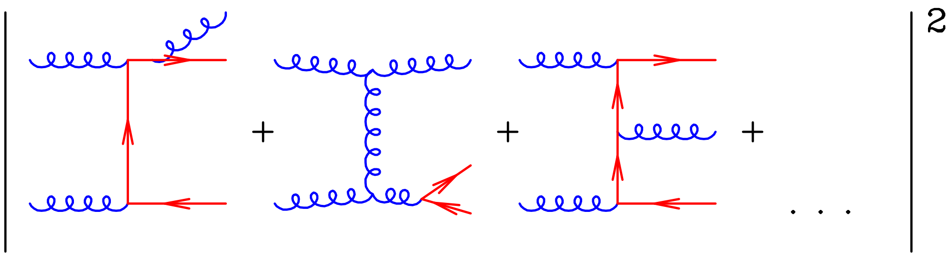
$$c_{gg}^{(0)}(\rho) = \frac{\pi\beta\rho}{192}\left[\frac{1}{\beta}\left[\rho^2+16\rho+16\right]\ln\left(\frac{1+\beta}{1-\beta}\right) - 28 - 31\rho\right] ,$$

$$c_{gq}^{(0)}(\rho) = c_{g\overline{q}}^{(0)}(\rho) = 0 ,$$

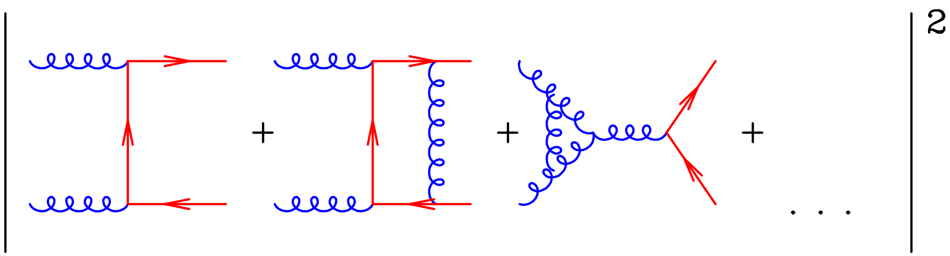
and $\beta = \sqrt{1-\rho}$.

- The functions $c_{ij}^{(0)}$ vanish both at threshold ($\beta \to 0$) and at high energy ($\rho \to 0$).
- Note that the quark-gluon process is zero in lowest order, but is present in higher orders.

- The functions $c_{ij}^{(1)}$ are also known



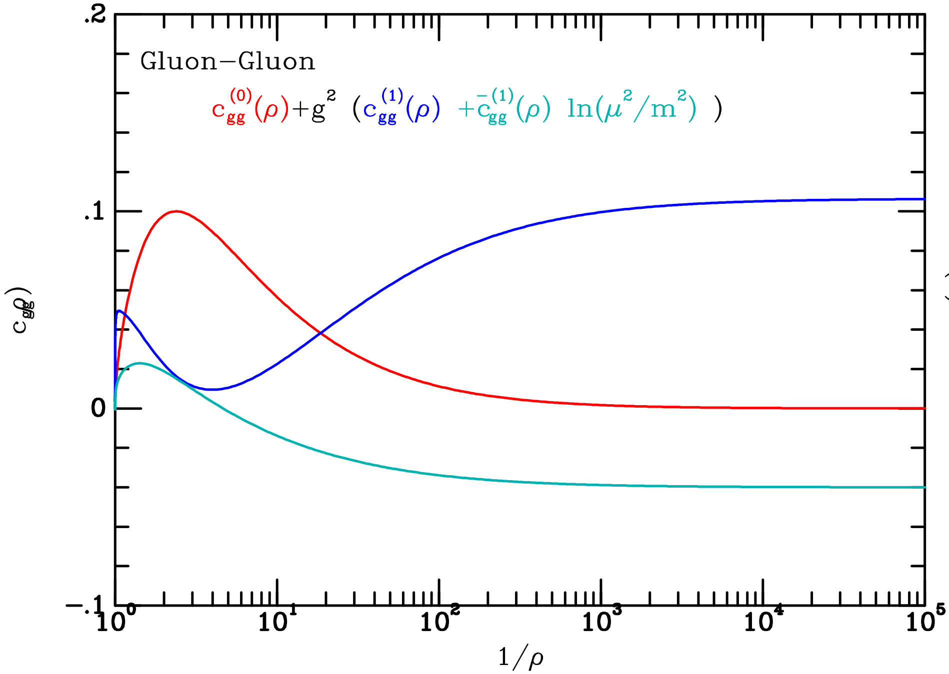
Real emission diagrams

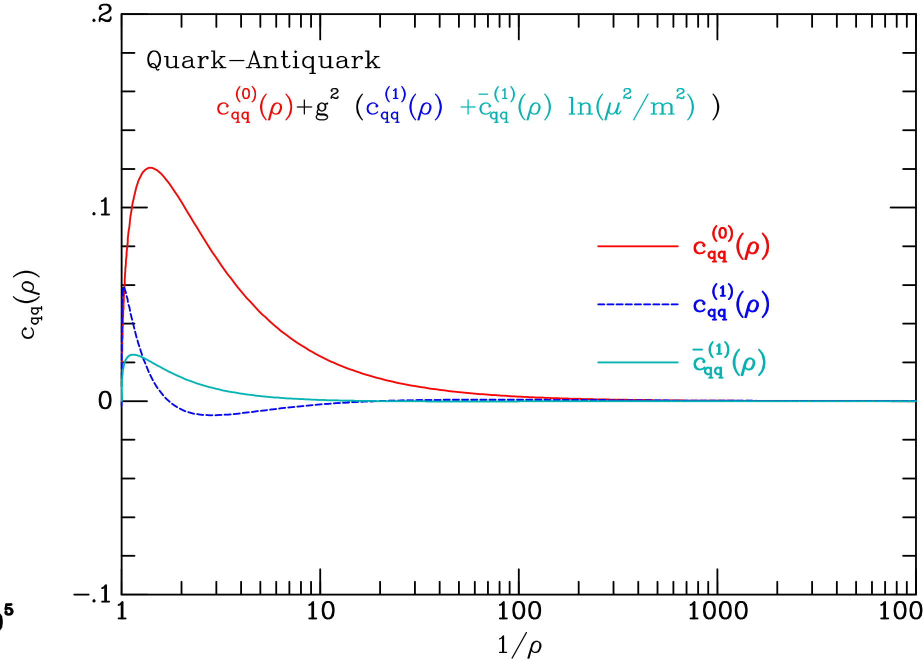


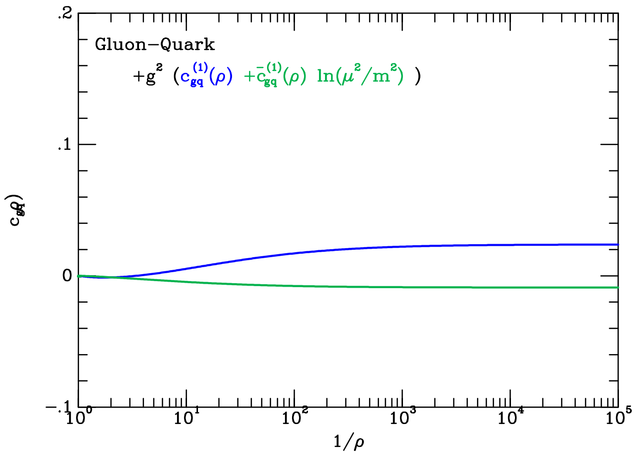
Virtual emission diagrams

- Examples of higher-order corrections to heavy quark production.
- In order to calculate the c_{ij} in perturbation theory we must perform both renormalization and factorization of mass singularities. The subtractions required for renormalization and factorization are done at mass scale μ.

Higher order results, $c_{ij}^{(1)}$







μ *dependence*

μ is an unphysical parameter. The physical predictions should be invariant under changes of μ at the appropriate order in perturbation theory. If we have performed a calculation to $O(\alpha_S^3)$, variations of the scale μ will lead to corrections of $O(\alpha_S^4)$,

$$\mu^2 \frac{d}{d\mu^2}\sigma = O(\alpha_S^4).$$

- The term $\overline{c}^{(1)}$, which controls the μ dependence of the higher-order perturbative contributions, is fixed in terms of the lower-order result $c^{(0)}$:

$$\begin{aligned}\overline{c}_{ij}^{(1)}(\rho) \quad = \quad & \frac{1}{8\pi^2}\Bigg[4\pi b c_{ij}^{(0)}(\rho) - \int_\rho^1 dz_1 \sum_k c_{kj}^{(0)}(\frac{\rho}{z_1})P_{ki}^{(0)}(z_1) \\ & - \int_\rho^1 dz_2 \sum_k c_{ik}^{(0)}(\frac{\rho}{z_2})P_{kj}^{(0)}(z_2)\Bigg].\end{aligned}$$

In obtaining this result we have used the renormalization group equation for the running coupling

$$\mu^2 \frac{d}{d\mu^2} \alpha_S(\mu^2) = -b\alpha_S^2 + \ldots$$

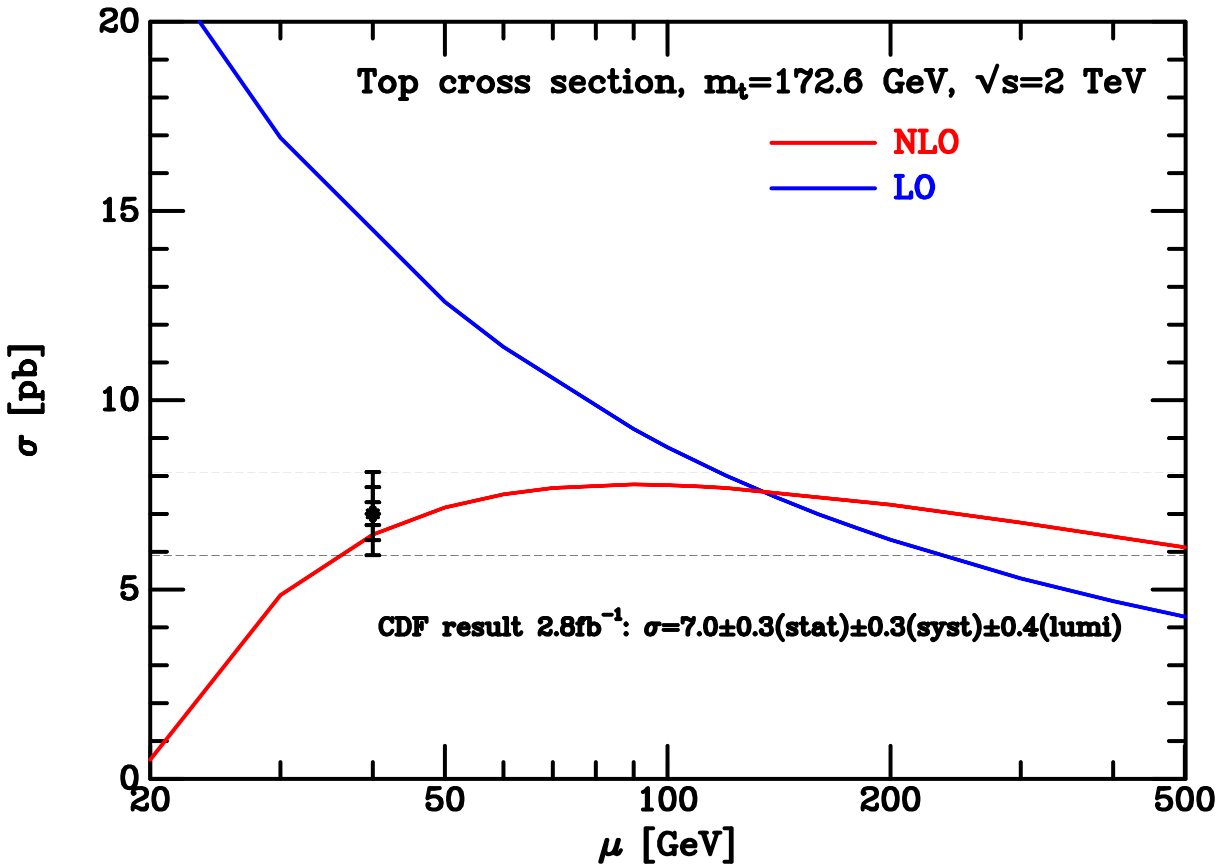
and the lowest-order form of the GLAP equation

$$\mu^2 \frac{d}{d\mu^2} f_i(x, \mu^2) = \frac{\alpha_S(\mu^2)}{2\pi} \sum_k \int_x^1 \frac{dz}{z} P_{ik}^{(0)}(z) f_k(\frac{x}{z}, \mu^2) + \ldots .$$

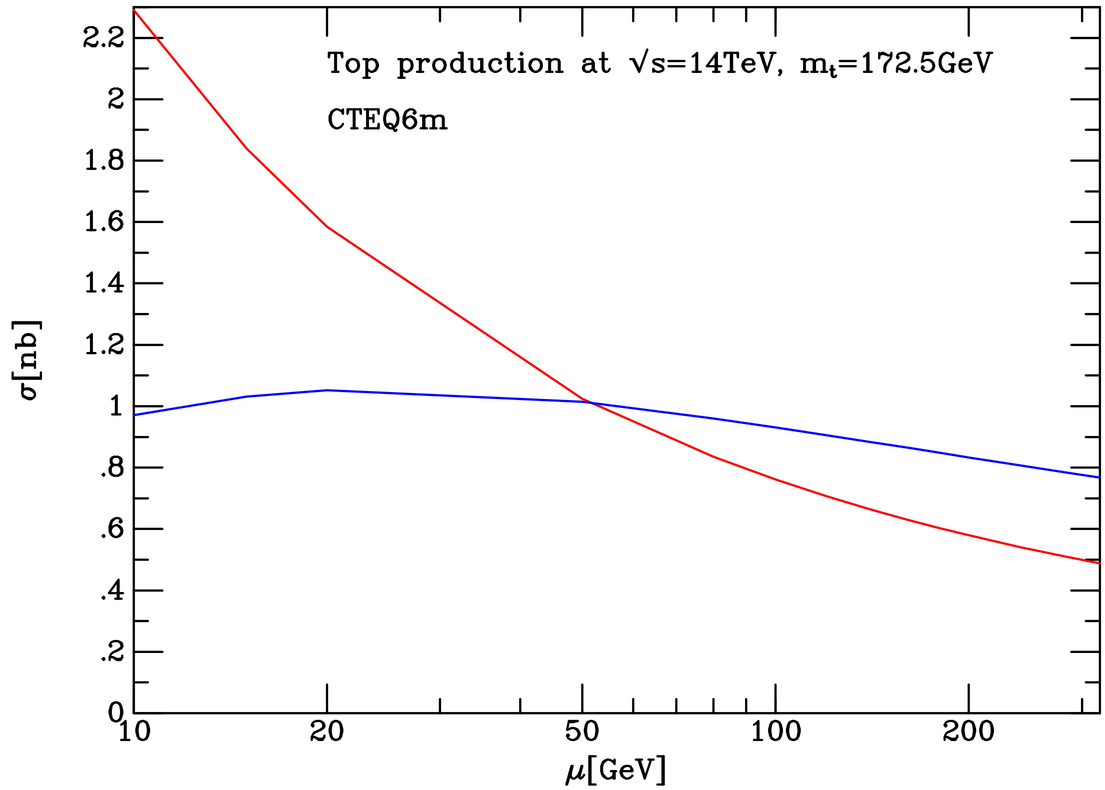
This illustrates an important point which is a general feature of renormalization group improved perturbation series in QCD. The coefficient of the perturbative correction depends on the choice made for the scale μ, but the scale dependence changes the result in such a way that the physical result is independent of that choice. Thus the scale dependence is formally small because it is of higher order in α_S. This does not assure us that the scale dependence is actually *numerically* small for all series. A pronounced dependence on the scale μ is a signal of an untrustworthy perturbation series.

Scale dependence in top production

- Inclusion of the higher order terms leads to a stabilization of the top cross section.



Top production at LHC



- At LHC top cross section is more than 100 times bigger than at Tevatron.

NLO QCD: Parton level integrators

- We would like to go beyond the results for the total cross section to get results for distributions.
- We have two separate divergent integrals which must be combined before numerical integration

$$\sigma_{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V$$

- Note that the jet definition can be arbitrarily complicated.

$$d\sigma^R = PS_{m+1}|\mathcal{M}_{m+1}|^2 F^J_{m+1}(p_1, \ldots p_{m+1})$$

We need to combine without knowledge of F^J.

- Two solutions: phase space slicing and subtraction.
- Illustrate with a simple one-dimensional example.

$$|\mathcal{M}_{m+1}|^2 \equiv \frac{1}{x}\mathcal{M}(x)$$

x is the energy of an emitted gluon.

- Divergences regularized in $d = 4 - 2\epsilon$ dimensions. Two solutions: phase space slicing and subtraction.
- Thus the full cross section in d dimensions is

$$\sigma = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F_1^J(x) + \frac{1}{\epsilon} \nu F_0^J$$

- Infrared safety: $F_1^J(0) = F_0^J$, KLN cancellation theorem, $\mathcal{M}(0) = \nu$

Phase space slicing

- Introduce arbitrary cutoff $\delta \ll 1$

$$\sigma = \int_0^\delta \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F_1^J(x) + \int_\delta^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F_1^J(x) + \frac{1}{\epsilon} \nu F_0^J \quad (1)$$

$$\simeq \int_0^\delta \frac{dx}{x^{1+\epsilon}} \nu F_0^J + \int_\delta^1 \frac{dx}{x} \mathcal{M}(x) F_1^J(x) + \frac{1}{\epsilon} \nu F_0^J \quad (2)$$

$$= \int_\delta^1 \frac{dx}{x} \mathcal{M}(x) F_1^J(x) + \ln(\delta) \nu F_0^J \quad (3)$$

(4)

- Procedure becomes exact for $\delta \to 0$ but numerical errors blow up. We have to compromise to find the best value of δ.
- Systematized by Giele-Glover-Kosower, JETRAD,DYRAD,EERAD

Subtraction method

- Exact identity

$$\sigma = \int_0^1 \frac{dx}{x^{1+\epsilon}}\left[\mathcal{M}(x)F_1^J(x) - \mathcal{M}(0)F_0^J\right] + \int_0^1 \frac{dx}{x^{1+\epsilon}}\nu F_0^J + \frac{1}{\epsilon}\nu F_0^J \quad (5)$$

$$= \int_0^1 \frac{dx}{x}\left[\mathcal{M}(x)F_1^J(x) - \mathcal{M}(0)F_0^J\right] + O(1)\nu F_0^J \quad (6)$$

- We have divided the problem into two separately finite integrals.
- Subtracted cross section must be valid everywhere in phase space.
- Subtraction method used in NLOJET++, MCFM

MCFM overview

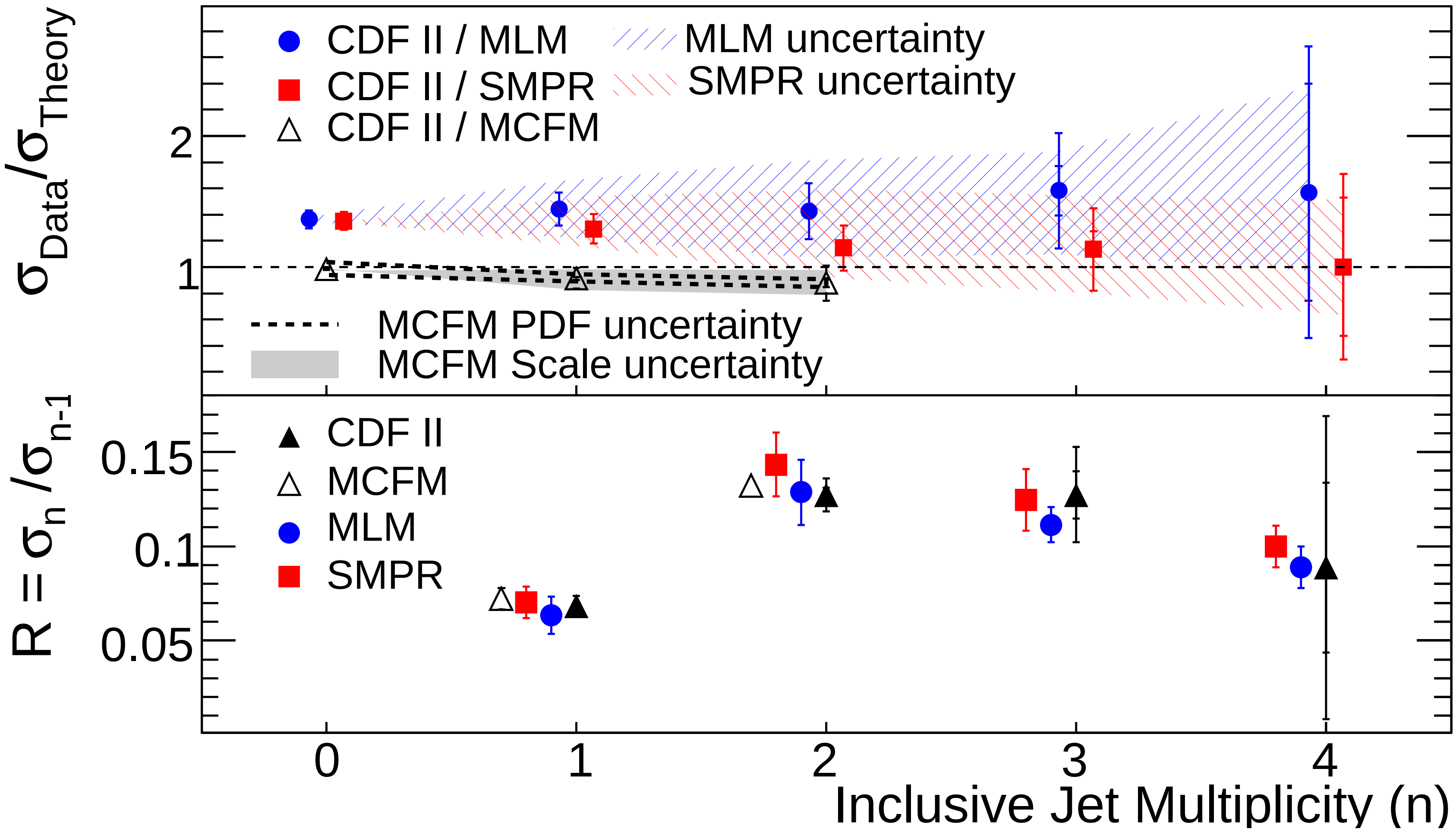
John Campbell and R.K. Ellis

- Parton level cross-sections predicted to NLO in α_S

$p\bar{p} \rightarrow W^{\pm}/Z$	$p\bar{p} \rightarrow W^{+} + W^{-}$
$p\bar{p} \rightarrow W^{\pm} + Z$	$p\bar{p} \rightarrow Z + Z$
$p\bar{p} \rightarrow W^{\pm} + \gamma$	$p\bar{p} \rightarrow W^{\pm}/Z + H$
$p\bar{p} \rightarrow W^{\pm} + g^{\star} (\rightarrow b\bar{b})$	$p\bar{p} \rightarrow Zb\bar{b}$
$p\bar{p} \rightarrow W^{\pm}/Z + 1$ jet	$p\bar{p} \rightarrow W^{\pm}/Z + 2$ jets
$p\bar{p}(gg) \rightarrow H$	$p\bar{p}(gg) \rightarrow H + 1$ jet
$p\bar{p}(VV) \rightarrow H + 2$ jets	$p\bar{p} \rightarrow t + X$
$pp \rightarrow t + W$	

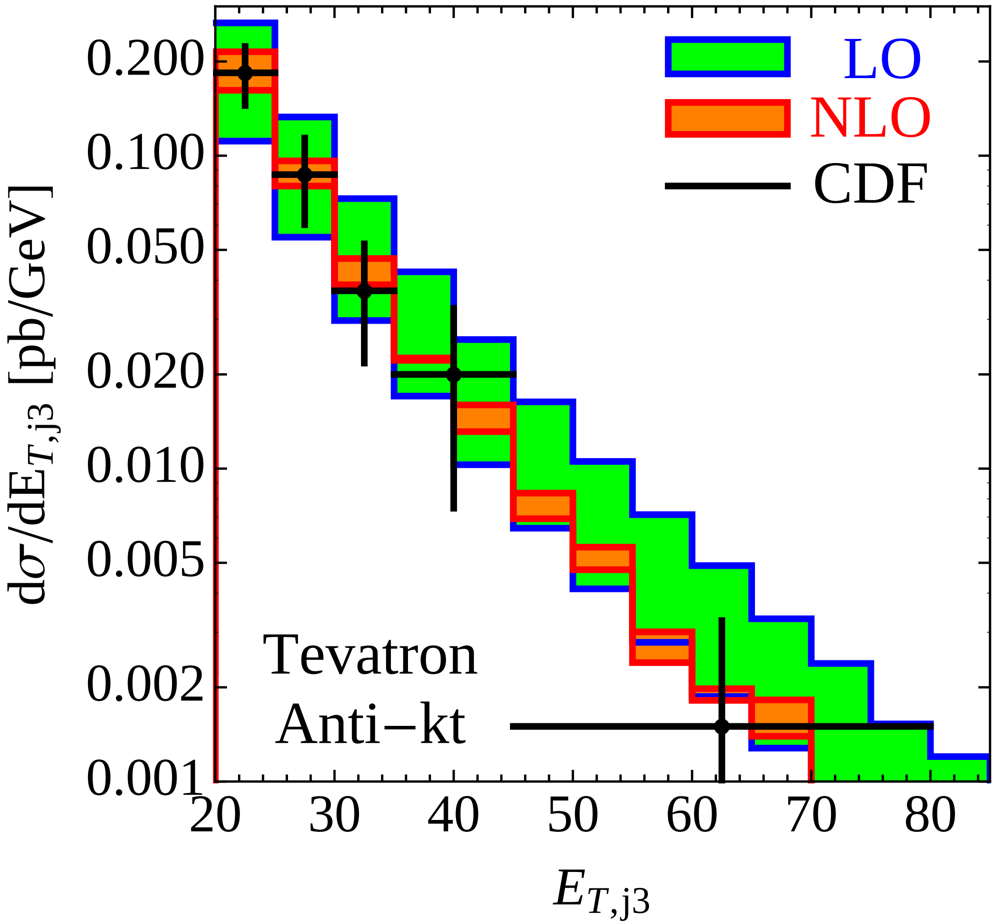
- $\oplus$ less sensitivity to μ_R, μ_F, rates are better normalized, fully differential distributions.
- $\ominus$ low particle multiplicity (no showering), no hadronization, hard to model detector effects
- $\star$ Based on the subtraction method

W + jet production



- Data is in agreement NLO prediction from MCFM
- Errors on tree level predictions (Alpgen+Herwig+MLM merging) and (Madgraph+Pythia+CKKW jet merging) are much larger.

New results on W + 3-jet production



- Leading color results, arXiv:0906.1445v1
- CDF Data is in agreement NLO prediction from MCFM
- See also Berger et al, arXiv:0907.1984

Conclusions

- The factorization property allows us to make predictions for processes at high energy.
- Crude information about cross section rates can be obtained from parton luminosities.
- Because of the factorization property, the QCD improved parton model gives a formalism which can be systematically improved by calculating higher orders in perturbation theory.
- The NLO formulation of QCD processes gives better information about normalization, and less dependence on unphysical scales.
- Residual scale dependence can give an *estimate* of the size of the uncalculated higher order terms.
- NLO is hence the first serious approximation in QCD.
- NLO calculation can be performed using the subtraction or slicing method to give information about distributions.
- A vigorous theoretical effort is underway to extend QCD results to more complicated multi-leg processes such as W+jet.