Supersymmetry?

- Would unify matter particles and force particles
- Related particles spinning at different rates

0 - ½ - 1 - 3/2 - 2

Higgs - Electron - Photon - Gravitino - Graviton

(Every particle is a 'ballet dancer')

- Would help fix particle masses
- Would help unify forces
- Predicts light Higgs boson
- **Could provide dark matter for the astrophysicists and cosmologists**

A Bitino of Shistory

- 1967: Impossible to combine internal and external (Lorentz) symmetry – Coleman & Mandula
- 1971: Extend Poincaré symmetry using fernionic charges – Gol'fand & Likhtman
- 1971: Supersymmetry in 2 dimensions (for baryons in strings) – Neveu & Schwarz; Ramond
- 1973: First supersymmetric field theories in 4 dimensions: nonlinear for ν – Volkov & Akulov

renormalizable theories – Wess & Zumino

Why Supersymmetry (Susy)?

- Hierarchy problem: why is $m_W << m_P$?

  ($m_P \sim 10^{19}$ GeV is scale of gravity)
- Alternatively, why is

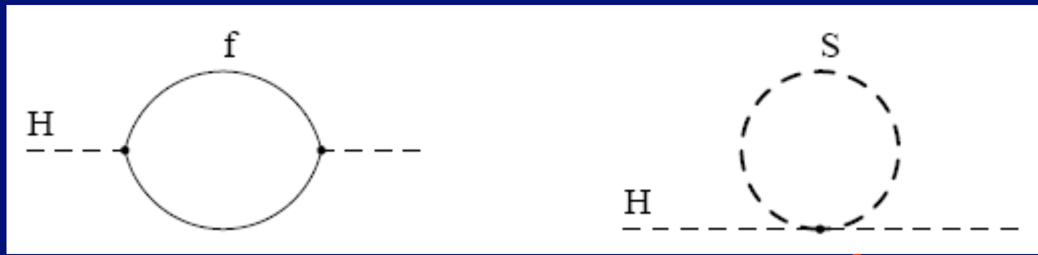
  $G_F = 1/\, m_W^2 >> G_N = 1/m_P^2$?
- Or, why is

  $V_{Coulomb} >> V_{Newton}$? $e^2 >> G\, m^2 = m^2 \,/\, m_P^2$
- Set by hand? What about loop corrections?

  $\delta m_{H,W}^2 = O(\alpha/\pi)\, \Lambda^2$
- Cancel boson loops $\Leftrightarrow$ fermions
- Need $| m_B^2 - m_F^2| < 1$ TeV^2

Loop Corrections to Higgs Mass^2

- Consider generic fermion and boson loops:



- Each is quadratically divergent: $\int^{\Lambda} d^4k/k^2$

$$\Delta m_H^2 = -\frac{y_f^2}{16\pi^2}[2\Lambda^2 + 6m_f^2 \ln(\Lambda/m_f) + ...]$$

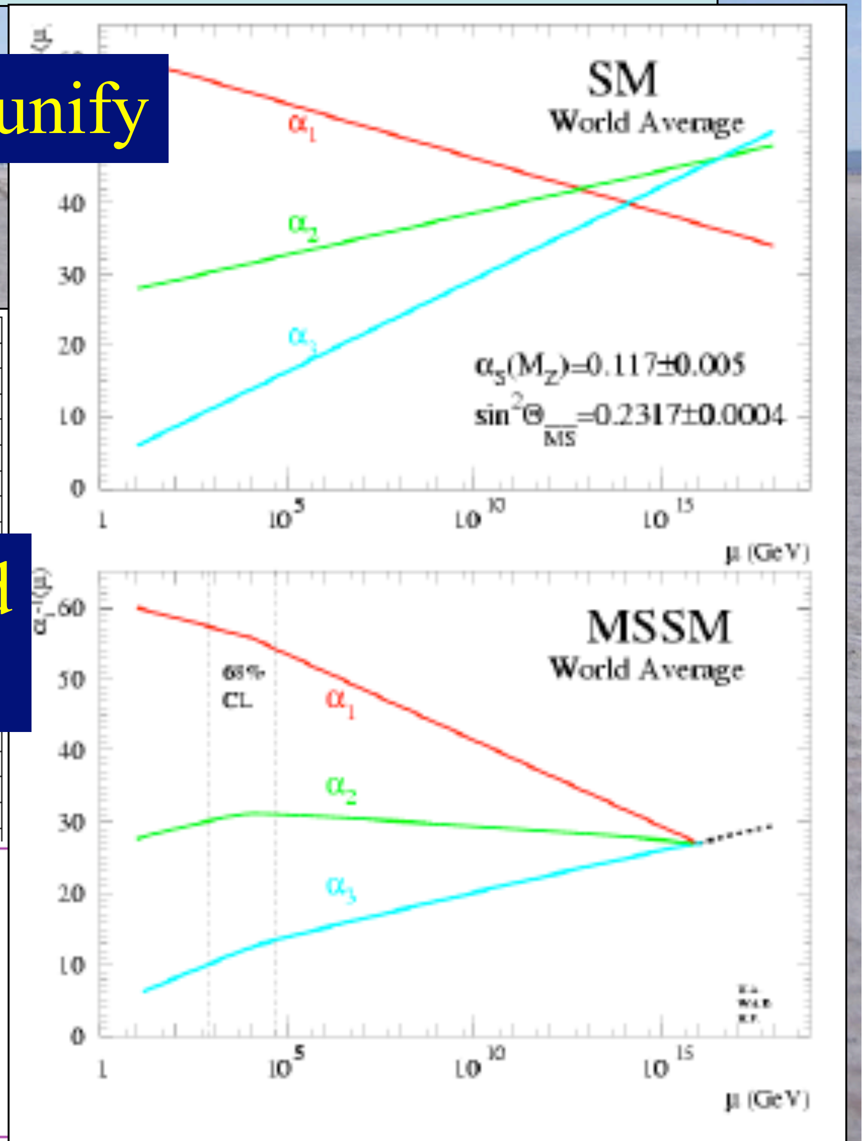
$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2}[\Lambda^2 - 2m_S^2 \ln(\Lambda/m_S) + ...]$$

- Leading divergence cancelled if

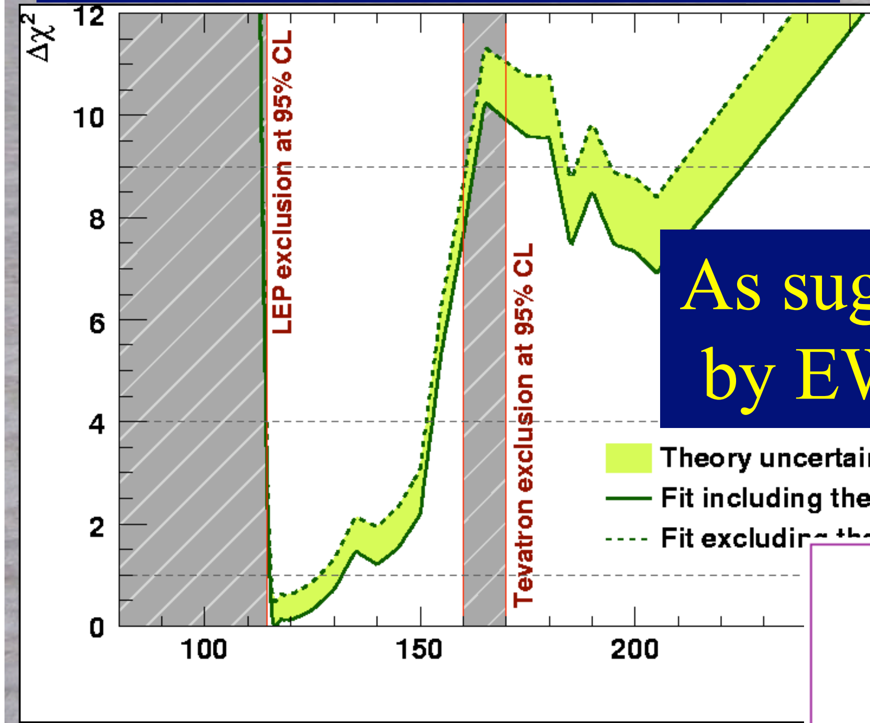
$\lambda_S = y_f^2$ x 2 **Supersymmetry!**

Other Reasons to like Susy

It enables the gauge couplings to unify



It predicts $m_H < 150$ GeV



As suggested by EW data

Provided by Fabiola Gianotti

Dark Matter in the Universe

Astronomers say that most of the matter in the Universe is invisible Dark Matter

‘Supersymmetric’ particles ?

We shall look for them with the LHC

More Shistory

- 1974: No-renormalization theorems –
  Ferrara, Iliopoulos, Wess & Zumino
- 1976: Discovery of supergravity –
  Freedman, van Nieuwenhuizen & Ferrara; Deser & Zumino
- 1979/1981: Relevance to hierarchy problem –
  Maiani, Witten
- 1983: Source of astrophysical dark matter –
  Goldberg; JE, Hagelin, Nanopoulos, Olive & Nanopoulos
- 1990: Superunification of gauge couplings –
  JE, Kelley & Nanopoulos; Langacker & Luo
- 1995: LEP data favour light Higgs boson

The Coleman-Mandula Catastrophe

- Known symmetry charges scalar: $Q\,|\mathrm{Spin}J> = \;|\mathrm{Spin}J>$

  and vector (Lorentz): P_μ $P_\mu^{(1)} + P_\mu^{(2)} = P_\mu^{(3)} + P_\mu^{(4)}$
- Consider a tensor charge: $\Sigma_{\mu\nu}$

  matrix elements: $< a|\Sigma_{\mu\nu}|a > = \;\alpha P_\mu^{(a)} P_\nu^{(a)} \;+\; \beta g_{\mu\nu}$
- Charge conservation: $P_\mu^{(1)} P_\nu^{(1)} + P_\mu^{(2)} P_\nu^{(2)} = P_\mu^{(3)} P_\nu^{(3)} + P_\mu^{(4)} P_\nu^{(4)}$
- Only possible solution: $P_\mu^{(1)} = \;P_\mu^{(3)} \text{ or } P_\mu^{(4)}$
- **No non-trivial scattering possible!**
- Crucial assumption: $< a|Q_\alpha|a > \neq \;0$
- Not true in supersymmetry!

Spinorology

- 2-component Weyl spinors for massless fermions: left-handed Ψ_L ψ_α
  right-handed Ψ_R $\overline{\psi}^{\dot\alpha}$
- Conjugate of Ψ_L is Ψ_R, and vice versa: $(\psi_\alpha)^* = \overline{\psi}_{\dot\alpha}$ $(\overline{\psi}^{\dot\alpha})^* = \psi^\alpha$
- Combine them to make 4-component Dirac spinor: $$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \begin{pmatrix} \psi_\alpha \\ \overline{\eta}^{\dot\alpha} \end{pmatrix}$$
- Conjugate: $\Psi^c = C\overline{\Psi}^T = \begin{pmatrix} \eta_\alpha \\ \overline{\psi}^{\dot\alpha} \end{pmatrix}$ $C = i\gamma^0\gamma^2$

Supersymmetry Algebra

- Simply stated:

$$
\begin{aligned}
Q|Boson> &= |Fermion> \\
Q|Fermion> &= |Boson>
\end{aligned}
$$

- Spinorial charges obey algebra:

$$
\begin{aligned}
[P^\mu, Q_\alpha] &= 0 = [P^\mu, \bar{Q}^{\dot{\alpha}}] \\
\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} &= 2(\sigma_\mu)_{\alpha\dot{\beta}} P^\mu \\
\{Q_\alpha, Q_\beta\} &= \{\bar{Q}^{\dot{\alpha}}, \bar{Q}^{\dot{\beta}}\} = 0
\end{aligned}
$$

- Only possible symmetry of S-matrix that combines particles of different spins
- Supermultiplets: chiral (0, 1/2), vector (1/2, 1)

Simplest Supersymmetric Field Theory

- Free scalar boson and free spin-1/2 fermion:

$$S = \int d^4x \; \mathcal{L}_{scalaire} + \mathcal{L}_{fermion}$$
$$\mathcal{L}_{scalaire} = -\partial^\mu \phi \, \partial_\mu \phi^*$$
$$\mathcal{L}_{fermion} = -i\psi^\dagger \bar{\sigma}^\mu \, \partial_\mu \psi$$

- Transform boson to fermion:

$$\delta\phi = \epsilon^\alpha \psi_\alpha \quad \text{et} \quad \delta\phi^* = \bar{\epsilon}_{\dot{\alpha}} \, \bar{\psi}^{\dot{\alpha}}$$
$$\Rightarrow \delta\mathcal{L}_{scalaire} = -\epsilon^\alpha \, (\partial^\mu \psi_\alpha) \, \partial_\mu \phi^* - \partial^\mu \phi \, \bar{\epsilon}_{\dot{\alpha}} \, (\partial_\mu \bar{\psi}^{\dot{\alpha}})$$

- Fermion to boson: $\delta\psi_\alpha = i(\sigma^\mu \epsilon^\dagger)_\alpha \, \partial_\mu \phi \quad \text{et} \quad \delta\bar{\psi}^{\dot{\alpha}} = -i(\epsilon \, \sigma^\mu)^{\dot{\alpha}} \, \partial_\mu \phi^*$
- Lagrangian changes by total derivative: action $A = \int d^4x \, L(x)$ invariant
- Supersymmetry: QQ = P $\phi \to \psi \to \partial\phi, \quad \psi \to \partial\phi \to \partial\psi$

Supersymmetry with Interactions

- General form: $L = L_0 - V(\phi^i, \phi_j^*) - \frac{1}{2} M_{ij}(\phi, \phi^*) \bar{\psi}^{ci} \psi^j$
- Variation includes: $\frac{\partial M}{\partial \phi^*} \psi^* \bar{\psi}^c \psi$ $\frac{\partial M_{ij}}{\partial \phi^k} \bar{E} \psi^k \bar{\psi}^{ci} \psi^j$
- Cannot cancel, so $M = M(\phi)$ symmetric $M_{ij} = \frac{\partial W}{\partial \phi^i \partial \phi^j}$
- Cancel variation in potential:

$$\frac{\partial V}{\partial \phi^i} \bar{E} \psi^I + (\mathrm{Herm.Conj.}) \rightarrow V = |\frac{\partial W}{\partial \phi^i}|^2$$

- Final form:

$$L = i\bar{\psi}_i \gamma_\mu \partial^\mu \psi^i + |\partial_\mu \phi^i|^2 - |\frac{\partial W}{\partial \phi^i}|^2 - \frac{1}{2} \frac{\partial^2 W}{\partial \phi^i \partial^j} \bar{\psi}^{ci} \partial \psi^j + (\mathrm{Herm.Conj.})$$

- Simple case:

$$W = \frac{\lambda}{3}\phi^3 + \frac{m}{2}\phi^2 \qquad L = i\bar{\psi}\gamma_\mu \partial^\mu \psi + |\partial_\mu \phi|^2 - |m\phi + \lambda\phi^2|^2 - m\bar{\psi}^c \psi - \lambda\phi\bar{\psi}^c \psi.$$

More Supersymmetric Field Theories

- Gauge bosons + adjoint spin-1/2 fermions = supersymmetric gauge theory
- Effective potential fixed by Yukawa, gauge couplings: $V = g^2\ \varphi^2\varphi^{*2} + y^2\ \varphi^2\varphi^{*2}$

  $\rightarrow$ prediction for Higgs mass

  $m_h < m_Z$ at tree level, loops $\delta m_h^2 \propto \frac{m_t^4}{m_W^2} \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right) + \ldots$
- Graviton minimally coupled to spin-3/2 fermion = supergravity

Minimal Supersymmetric Extension of Standard Model (MSSM)

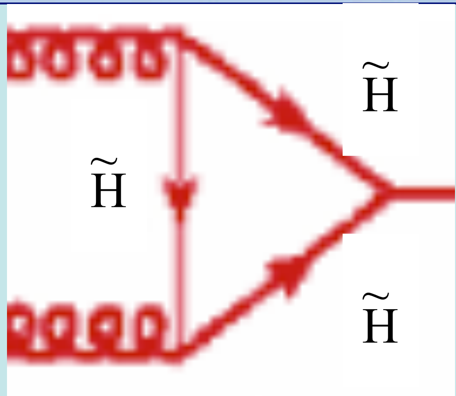
- Particles + spartners

$$\begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix} \; e.g., \; \begin{pmatrix} \ell \; (lepton) \\ \tilde{\ell} \; (slepton) \end{pmatrix} \; or \; \begin{pmatrix} q \; (quark) \\ \tilde{q} \; (squark) \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \; e.g., \; \begin{pmatrix} \gamma \; (photon) \\ \tilde{\gamma} \; (photino) \end{pmatrix} \; or \; \begin{pmatrix} g \; (gluon) \\ \tilde{g} \; (gluino) \end{pmatrix}$$

- 2 Higgs doublets, coupling μ, ratio of v.e.v.'s = $\tan \beta$
- Unknown supersymmetry-breaking parameters:
  - Scalar masses m_0, gaugino masses $m_{1/2}$,
  - trilinear soft couplings A_λ, bilinear soft coupling B_μ
- Often assume universality:
  - Single m_0, single $m_{1/2}$, single A_λ, B_μ: not string?
- Called constrained MSSM = CMSSM
- Minimal supergravity also predicts gravitino mass
  - $m_{3/2} = m_0$, $B_\mu = A_\lambda - m_0$

Why 2 Higgs Doublets?

- Cancel anomalous Higgsino triangle diagrams



- Superpotential must be analytic function of superfields:

$$W = \frac{\lambda}{3}\phi^3 + \frac{m}{2}\phi^2$$

  – Cannot use QU^cH, QD^cH^*
  – Must use QU^cH_u, QD^cH_d
- Two Higgs fields →
  – Coupling between them: $\mu H_u H_d$
  – Two different vev's, ratio tan β

Non-Universal Scalar Masses

- Different sfermions with same quantum #s?

  e.g., d, s squarks?

  disfavoured by upper limits on flavour-changing neutral interactions

- Squarks with different #s, squarks and sleptons?

  disfavoured in various GUT models

  e.g., $d_R = e_L$, $d_L = u_L = u_R = e_R$ in SU(5), all in SO(10)

- Non-universal susy-breaking masses for Higgses?

  No reason why not! NUHM

Renormalization of Susy Breaking Parameters

- After cancellation of quadratic divergences: renormalized logarithmically:

  gaugino masses: $d\, M_a/dt \sim \beta_a\, M_a$

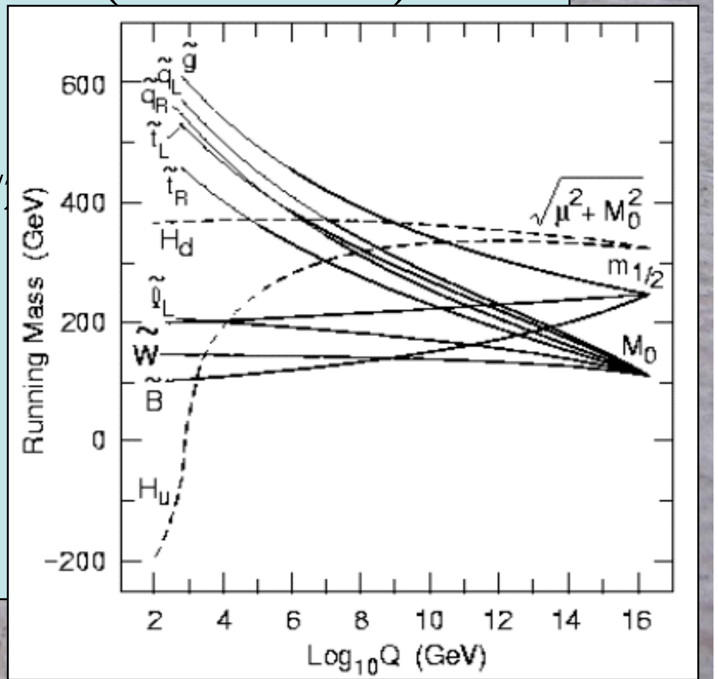
  scalar masses^2: $\frac{\partial m_{0_i}^2}{\partial t} = \frac{1}{16\pi^2}[\lambda^2(m_0^2 + A_\lambda^2) - g_a^2 M_a^2]$

- Assuming universal input parameters (CMSSM)
- Solutions at low energy scales Q:

$$M_a\,(Q) = (\alpha_a/\ \alpha_{GUT})\ m_{1/2}$$

$$m_{0_i}^2 = m_0^2 + C_i m_{1/2}^2$$

- Gluino heavier than photino, wino
- Squarks heavier than sleptons



Mass Renormalizations

- **Assuming universality at the GUT scale**
- Gaugino masses:
  - $M_a = (\alpha_a / \alpha_{GUT})\, m_{1/2}$, e.g., $\rightarrow M_2 / M_3 = \alpha_2 / \alpha_3$
- Squark and slepton masses:
  - Squark mass^2: $m_0^2 + 6\, m_{1/2}^2$
  - Left-handed slepton mass^2: $m_0^2 + 0.5\, m_{1/2}^2$
  - Right-handed slepton mass^2: $m_0^2 + 0.15\, m_{1/2}^2$
- Minimal flavour violation (MFV):
  - Flavour mixing of squarks and sleptons induced by CKM, neutrino mixing

Electroweak Symmetry Breaking

- Could be triggered by renormalization effects:

$$\frac{\partial m_{0_i}^2}{\partial t} = \frac{1}{16\pi^2}[\lambda^2(m_0^2 + A_\lambda^2) - g_a^2 M_a^2]$$

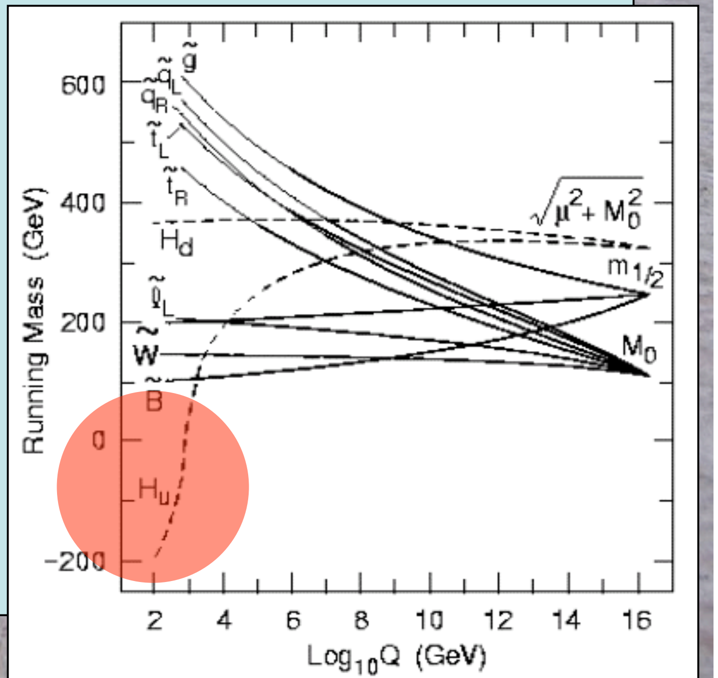
- Driven by large Yukawa coupling of top quark:

$$\frac{m_W}{m_P} = exp(\frac{-\mathcal{O}(1)}{\alpha_t}) : \ \alpha_t \equiv \frac{\lambda_t^2}{4\pi}$$

- Higgs mass^2 → negative
- Electroweak scale

  naturally ~ 100 GeV

  for m_t ~ 60 to 200 GeV



Lightest Supersymmetric Particle

- Stable in many models because of conservation of R parity:

  $R = (-1)^{2S - L + 3B}$

  where S = spin, L = lepton #, B = baryon #

- Particles have R = +1, sparticles R = -1:

  Sparticles produced in pairs

  Heavier sparticles → lighter sparticles

- Lightest supersymmetric particle (LSP) stable

Possible Nature of LSP

- No strong or electromagnetic interactions

  Otherwise would bind to matter

  Detectable as anomalous heavy nucleus

- Possible weakly-interacting scandidates

  Sneutrino

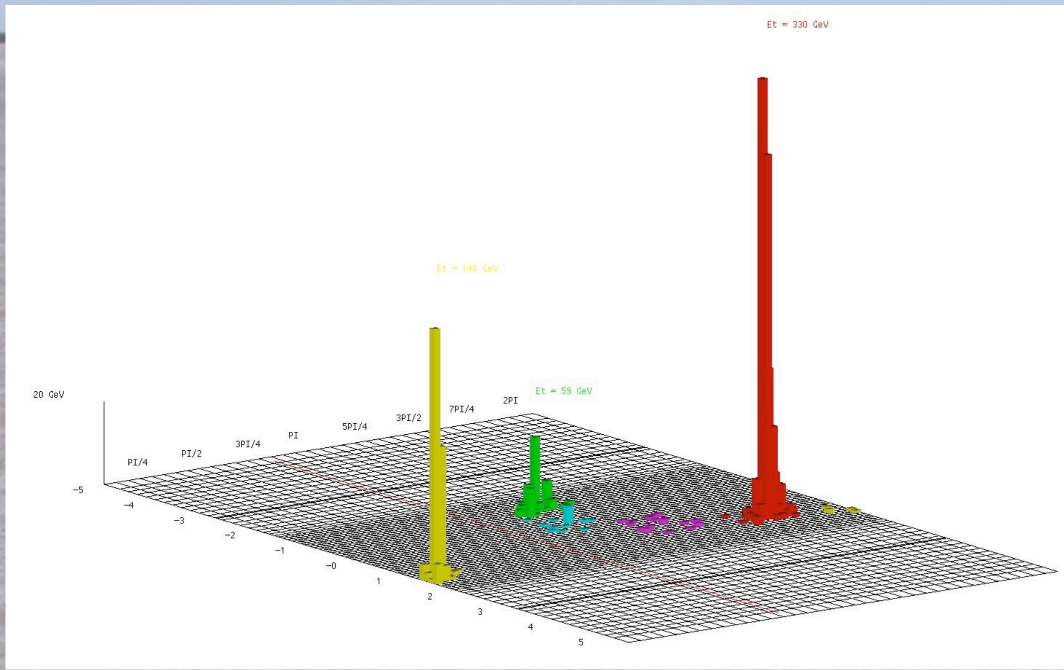
  (Excluded by LEP, direct searches)

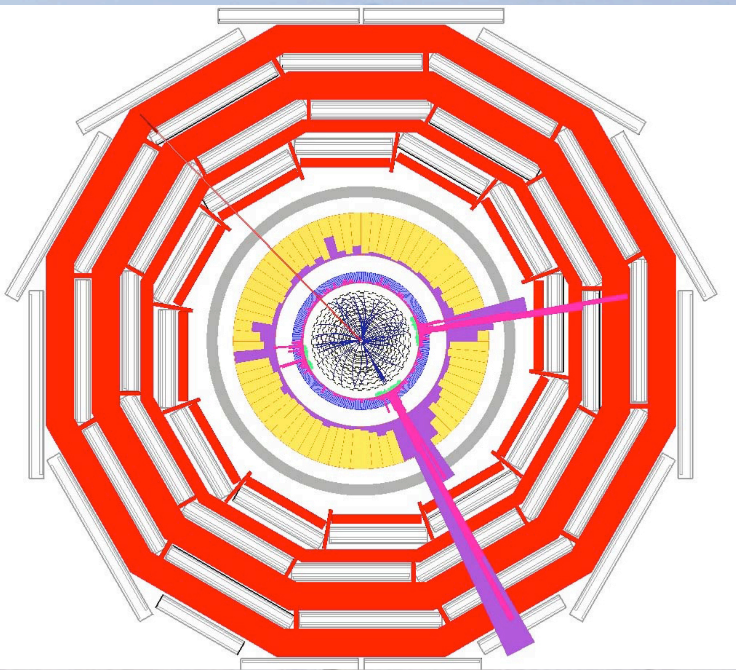
  Lightest neutralino χ (partner of Z, H, γ)

  Gravitino

  (nightmare for detection)

Classic Supersymmetric Signature





Missing transverse energy
carried away by dark matter particles