

Higgs and Electroweak Physics

Sven Heinemeyer, IFCA (Santander)

St. Andrews, 08/2009

1. The SM and the Higgs
2. The Higgs in Supersymmetry
3. Experimental facts and fiction (from a theorist's view)

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Higgs and Electroweak Physics (I): The SM and the Higgs

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1. Higgs Theory
2. Electroweak Precision Observables
3. Properties of the SM Higgs boson

1. Higgs Theory

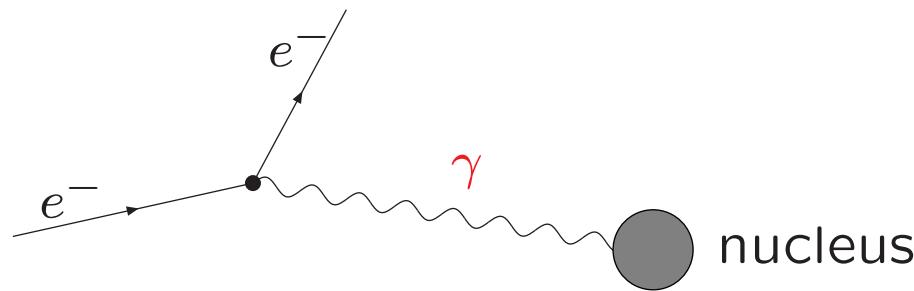
Standard Model (SM) of the electroweak and strong interaction

SM: Quantum field theory \Rightarrow interaction: exchange of field quanta

Construction principle of the SM: **gauge invariance**

Example: Quantum electro-dynamics (QED)

field quanta: photon A_μ



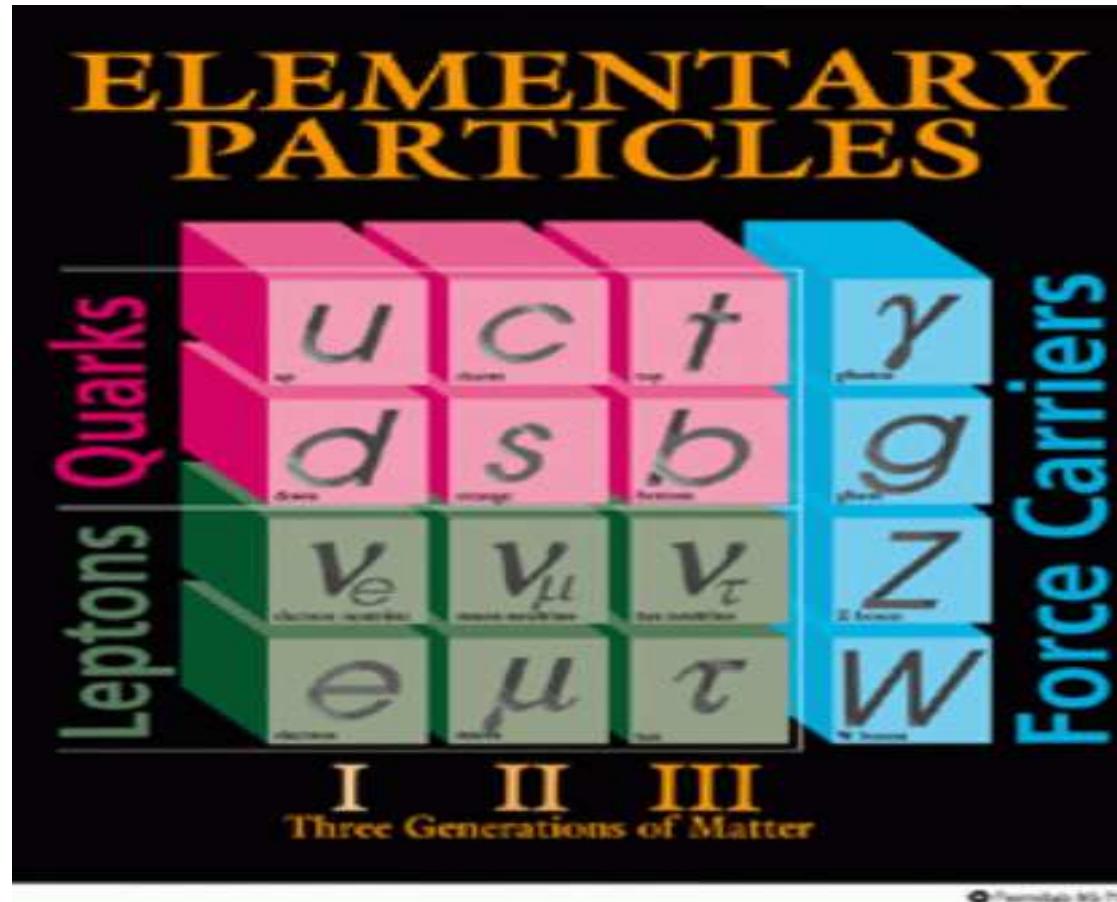
\mathcal{L}_{QED} invariant under **gauge transformation**:

$$\Psi \rightarrow e^{ie\lambda(x)}\Psi, A_\mu \rightarrow A_\mu + \partial_\mu\lambda(x)$$

mass term for photon: $m^2 A^\mu A_\mu$ not gauge invariant

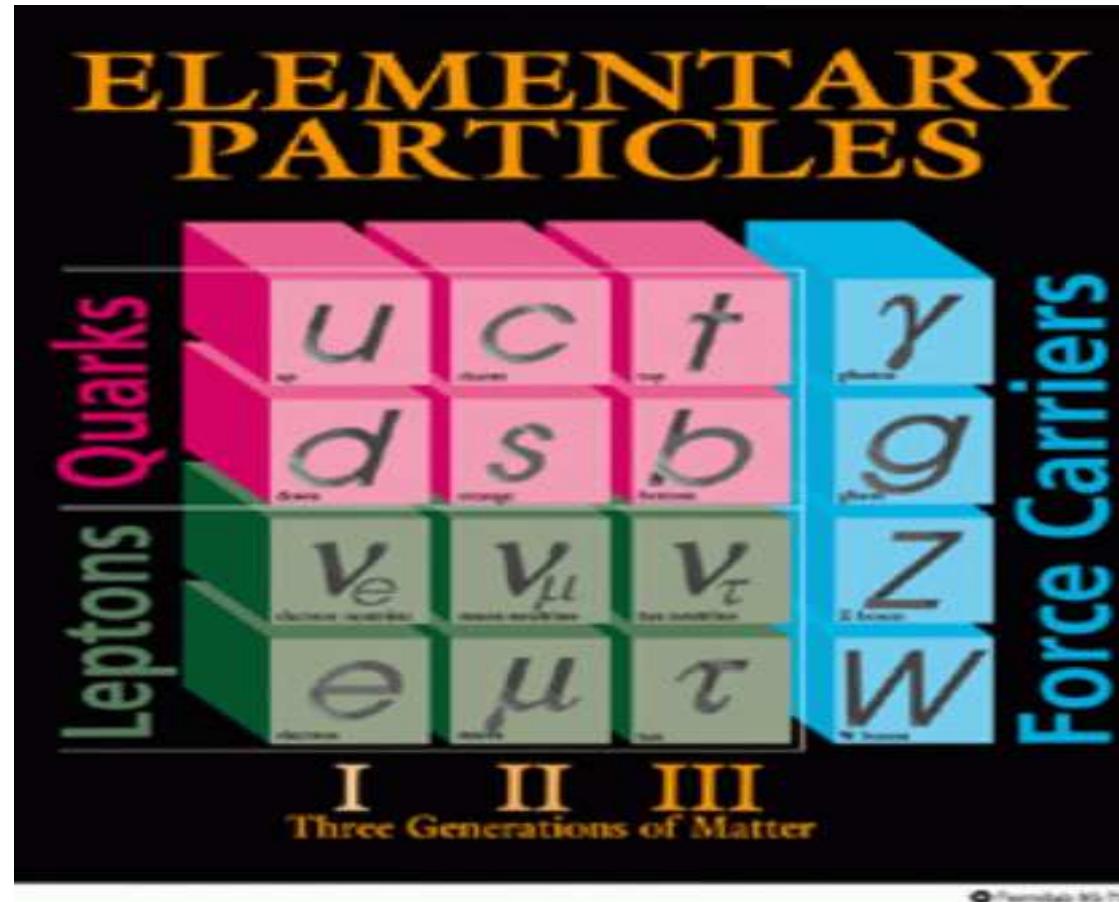
$\Rightarrow A_\mu$ is massless gauge field

Current status of knowledge: the Standard Model (SM)



⇒ all particles experimentally seen

Current status of knowledge: the Standard Model (SM)



⇒ all particles experimentally seen

⇒ but theory predicts massless gauge bosons . . .

Problem:

Gauge fields Z, W^+, W^- are **massive**

explicite mass terms in the Lagrangian \Leftrightarrow breaking of gauge invariance

Solution: Higgs mechanism

scalar field postulated, mass terms from coupling to Higgs field

Higgs sector in the Standard Model:

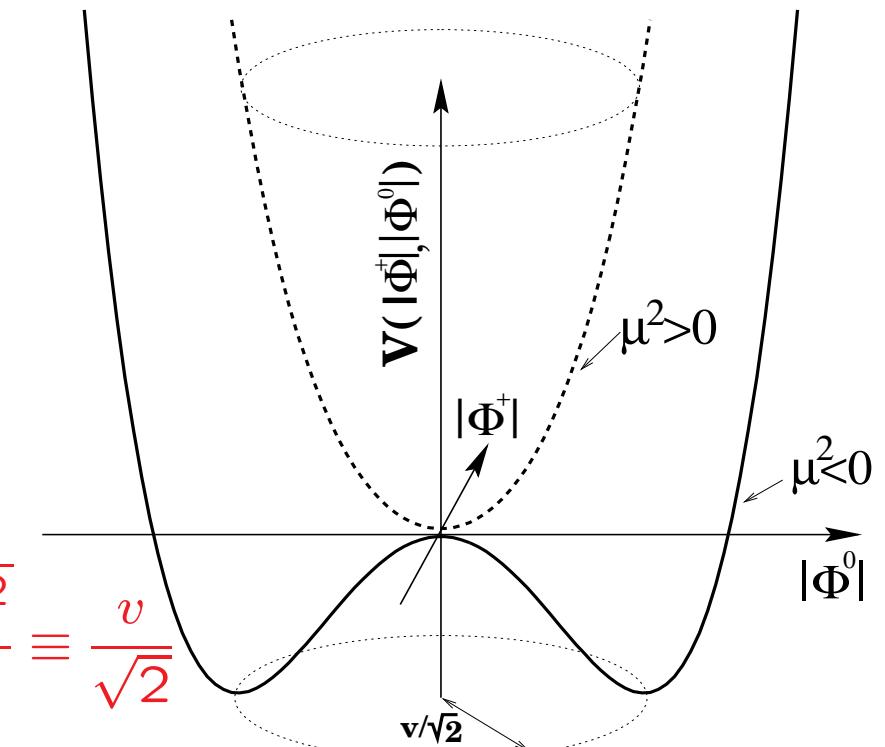
$$\text{Scalar SU(2) doublet: } \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Higgs potential:

$$V(\phi) = \mu^2 |\Phi^\dagger \Phi| + \lambda |\Phi^\dagger \Phi|^2, \quad \lambda > 0$$

$\mu^2 < 0$: Spontaneous symmetry breaking

minimum of potential at $|\langle \Phi_0 \rangle| = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$



$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \quad (\text{unitary gauge})$$

H : elementary scalar field, Higgs boson

Lagrange density:

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} = & (D_\mu \Phi)^\dagger (D^\mu \Phi) \\ & - g_d \bar{Q}_L \Phi d_R - g_u \bar{Q}_L \Phi_c u_R \\ & - V(\Phi) \end{aligned}$$

with

$$\begin{aligned} iD_\mu &= i\partial_\mu - g_2 \vec{W}_\mu - g_1 Y B_\mu \\ \Phi_c &= i\sigma_2 \Phi^\dagger \qquad Q_L \sim \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \Phi \sim \begin{pmatrix} 0 \\ v \end{pmatrix}, \Phi_c \sim \begin{pmatrix} v \\ 0 \end{pmatrix} \end{aligned}$$

Gauge invariant coupling to gauge fields

⇒ mass terms for gauge bosons and fermions

1.) $VV\Phi\Phi$ coupling:

$$V_{\text{wavy}} \rightarrow \text{wavy} + \text{wavy} \xrightarrow{*} v + \text{wavy} \xrightarrow{*} \text{wavy} + \dots$$

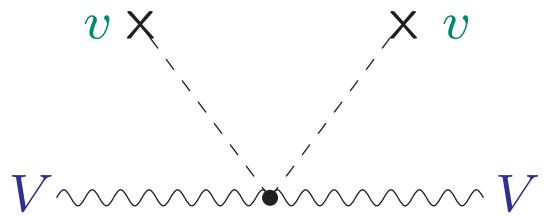
$$\frac{1}{q^2} \rightarrow \frac{1}{q^2} + \sum_j \frac{1}{q^2} \left[\left(\frac{gv}{\sqrt{2}} \right)^2 \frac{1}{q^2} \right]^j = \frac{1}{q^2 - M^2} : M^2 = g^2 \frac{v^2}{2}$$

2.) fermion mass terms: Yukawa couplings:

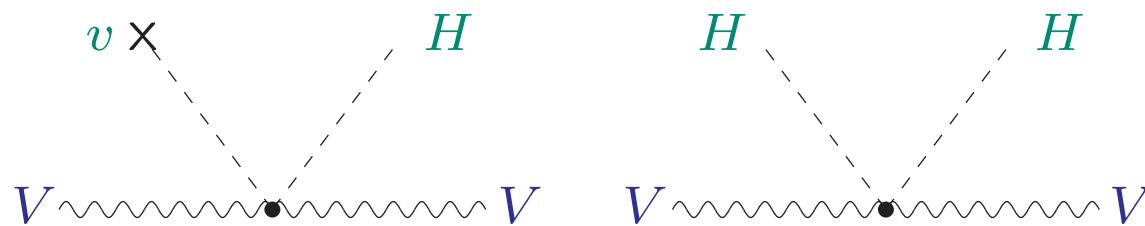
$$f \longrightarrow \longrightarrow \longrightarrow + \text{fermion} \xrightarrow{*} v + \text{fermion} \xrightarrow{*} \text{fermion} + \dots$$

$$\frac{1}{q} \rightarrow \frac{1}{q} + \sum_j \frac{1}{q} \left[\frac{g_f v}{\sqrt{2}} \frac{1}{q} \right]^j = \frac{1}{q - m_f} : m_f = g_f \frac{v}{\sqrt{2}}$$

1.) $VV\Phi\Phi$ coupling:



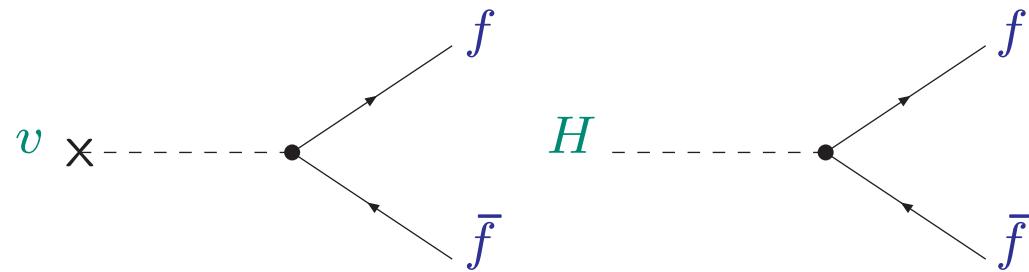
⇒ VV mass terms: $g_2^2 v^2 / 2 \equiv M_W^2$, $(g_1^2 + g_2^2)v^2/2 \equiv M_Z^2$



⇒ triple/quartic couplings to gauge bosons

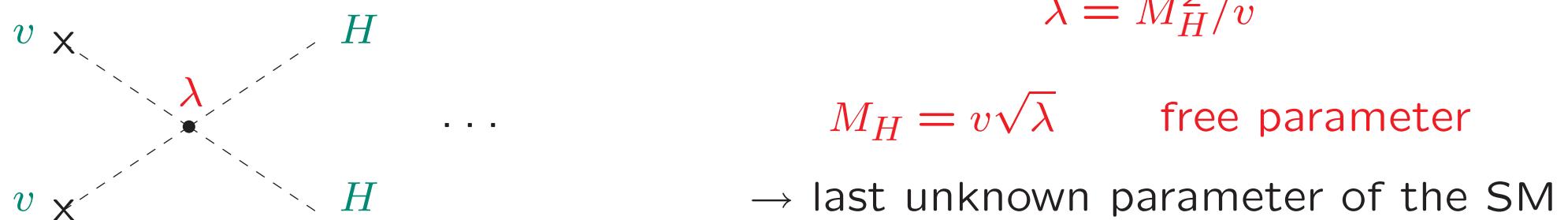
⇒ coupling \propto masses

2.) fermion mass terms: Yukawa couplings



$$m_f = v g_f \Rightarrow \text{coupling} \propto \text{masses}$$

3.) mass of the Higgs boson: self coupling



⇒ establish Higgs mechanism ≡ find the Higgs ⊕ measure its couplings

Another effect of the Higgs field:

Scattering of longitudinal W bosons: $W_L W_L \rightarrow W_L W_L$

$$\mathcal{M}_V = \text{Diagram showing two incoming } W \text{ bosons scattering into } \gamma, Z + \text{Diagram showing two incoming } W \text{ bosons scattering into } \gamma, Z + \text{Diagram showing two incoming } W \text{ bosons scattering into } W = -g^2 \frac{E^2}{M_W^2} + \mathcal{O}(1) \text{ for } E \rightarrow \infty$$

⇒ violation of unitarity

Contribution of a scalar particle with couplings prop. to the mass:

$$\mathcal{M}_S = \text{Diagram showing two incoming } W \text{ bosons scattering into } H + \text{Diagram showing two incoming } W \text{ bosons scattering into } H = g_{WWH}^2 \frac{E^2}{M_W^4} + \mathcal{O}(1) \text{ for } E \rightarrow \infty$$

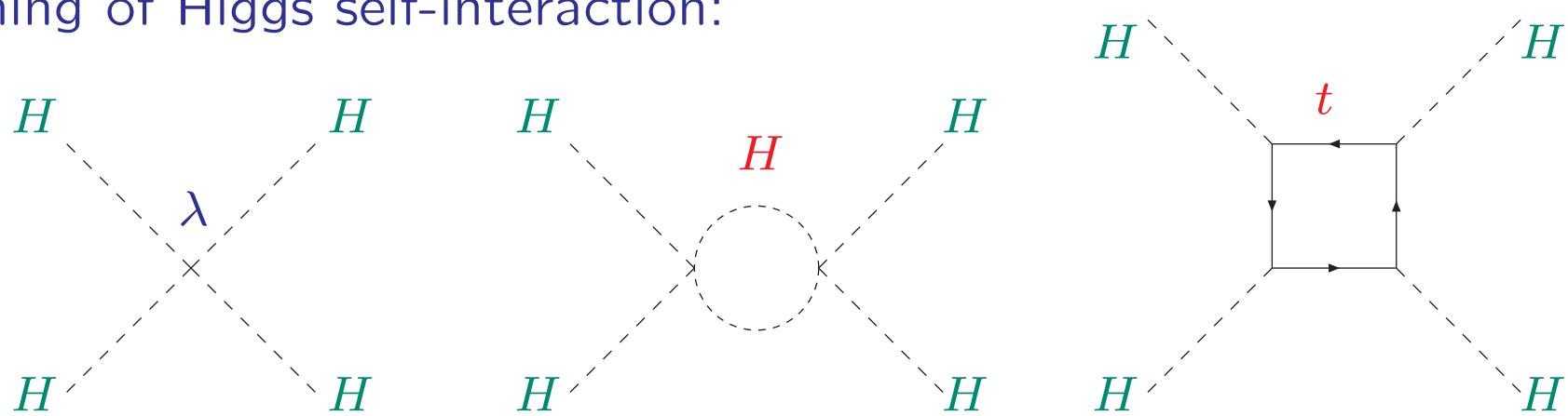
$$\mathcal{M}_{\text{tot}} = \mathcal{M}_V + \mathcal{M}_S = \frac{E^2}{M_W^4} (g_{WWH}^2 - g^2 M_W^2) + \dots$$

⇒ compensation of terms with bad high-energy behavior for

$$g_{WWH} = g M_W$$

What else do we know about the Higgs boson?

Running of Higgs self-interaction:



Renormalization group equation:

$$\frac{d\lambda}{dt} = \frac{3}{8\pi^2} \left[\lambda^2 + \lambda g_t^2 - g_t^4 + \frac{1}{16} (2g_2^4 + (g_2^2 + g_1^2)^2) \right], \quad t = \log \left(\frac{Q^2}{v^2} \right)$$

Two conditions:

- 1.) avoid Landau pole (for large $\lambda \sim M_H^2$)
- 2.) avoid vacuum instability (for small/negative λ)

1.) avoid Landau pole (for large $\lambda \sim M_H^2$)

$$\frac{d\lambda}{dt} = \frac{3}{8\pi^2} [\lambda^2]$$

$$\Rightarrow \lambda(Q^2) = \frac{\lambda(v^2)}{1 - \frac{3\lambda(v^2)}{8\pi^2} \log\left(\frac{Q^2}{v^2}\right)}$$

$$\lambda(\Lambda) < \infty \Rightarrow M_H^2 \leq \frac{8\pi^2 v^2}{3 \log\left(\frac{\Lambda^2}{v^2}\right)} \quad : \text{upper bound on } M_H$$

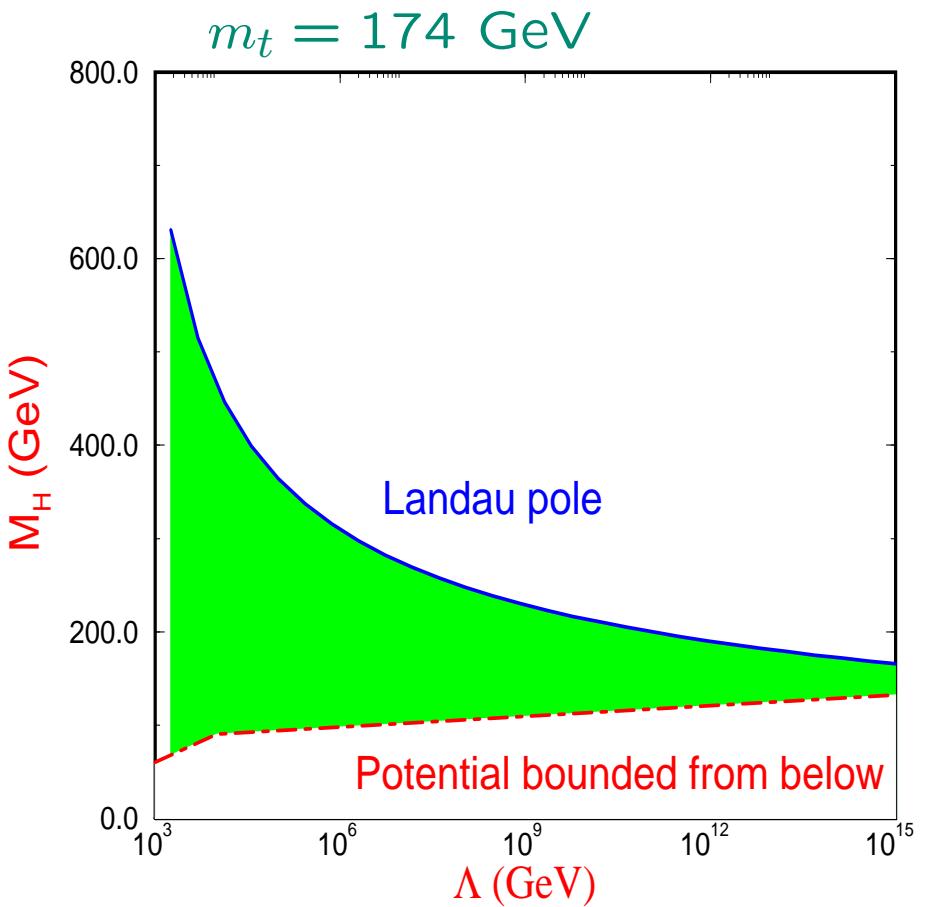
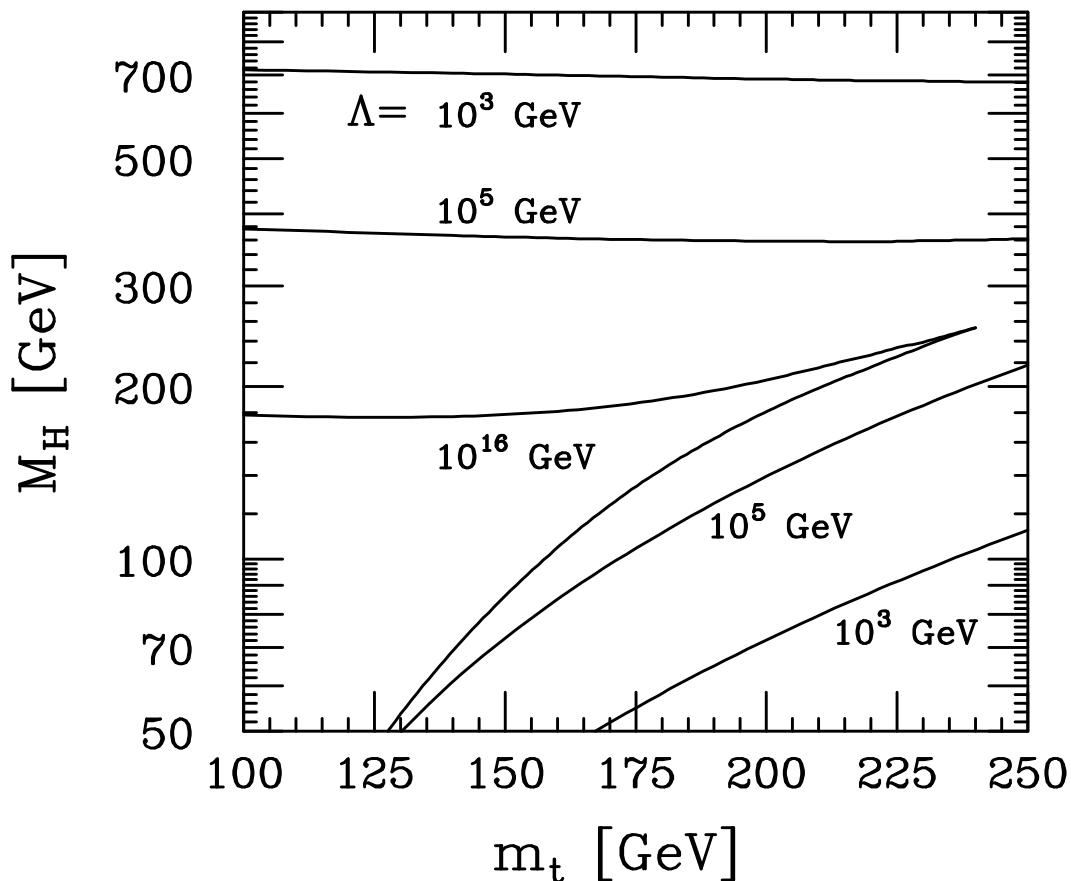
2.) avoid vacuum instability (for small/negative λ): $V(v) < V(0) \Rightarrow \lambda(\Lambda) > 0$

$$\frac{d\lambda}{dt} = \frac{3}{8\pi^2} \left[-g_t^4 + \frac{1}{16} (2g_2^4 + (g_2^2 + g_1^2)^2) \right]$$

$$\Rightarrow \lambda(Q^2) = \lambda(v^2) \frac{3}{8\pi^2} \left[-g_t^4 + \frac{1}{16} (2g_2^4 + (g_2^2 + g_1^2)^2) \right] \log\left(\frac{Q^2}{v^2}\right)$$

$$\lambda(\Lambda) > 0 \Rightarrow M_H^2 > \frac{v^2}{4\pi^2} \left[-g_t^4 + \frac{1}{16} (2g_2^4 + (g_2^2 + g_1^2)^2) \right] \log\left(\frac{\Lambda^2}{v^2}\right) : \text{lower bound}$$

Both limits combined:

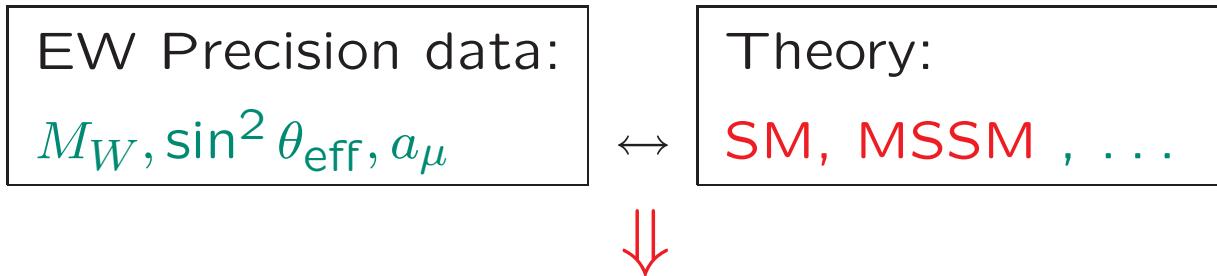


Λ : scale up to which the SM is valid

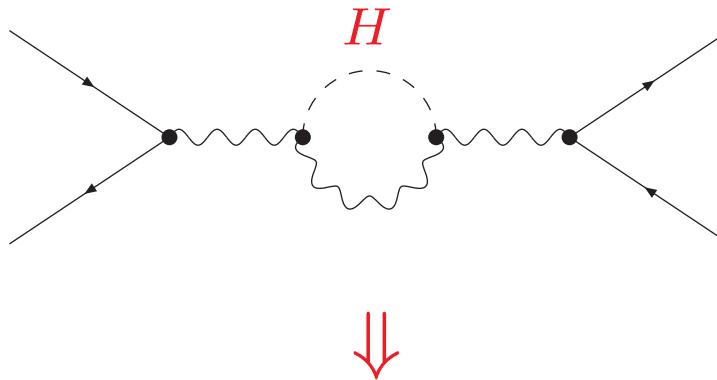
$$\Lambda = M_{\text{GUT}} \Rightarrow 130 \text{ GeV} \lesssim M_H \lesssim 180 \text{ GeV}$$

2. Electroweak Precision Observables (EWPO):

Comparison of electro-weak precision observables with theory:



Test of theory at quantum level: Sensitivity to loop corrections, e.g. H



SM: limits on M_H

Very high accuracy of measurements and theoretical predictions needed

Example: prediction of M_W , $\sin^2 \theta_{\text{eff}}$

A) Theoretical prediction for M_W in terms

of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} (1 + \Delta r)$$



loop corrections

Evaluate Δr from μ decay $\Rightarrow M_W$

One-loop result for M_W in the SM:

[A. Sirlin '80] , [W. Marciano, A. Sirlin '80]

$$\begin{aligned} \Delta r_{\text{1-loop}} &= \Delta \alpha - \frac{c_W^2}{s_W^2} \Delta \rho + \Delta r_{\text{rem}}(M_H) \\ &\sim \log \frac{M_Z}{m_f} \quad \sim m_t^2 \quad \log(M_H/M_W) \\ &\sim 6\% \quad \sim 3.3\% \quad \sim 1\% \end{aligned}$$

Example: prediction of M_W , $\sin^2 \theta_{\text{eff}}$

A) Theoretical prediction for M_W in terms

of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} (1 + \Delta r)$$



loop corrections

B) Effective mixing angle:

$$\sin^2 \theta_{\text{eff}} = \frac{1}{4 |Q_f|} \left(1 - \frac{\text{Re } g_V^f}{\text{Re } g_A^f} \right)$$

Higher order contributions:

$$g_V^f \rightarrow g_V^f + \Delta g_V^f, \quad g_A^f \rightarrow g_A^f + \Delta g_A^f$$

Comparison of SM prediction of M_W with direct measurements:

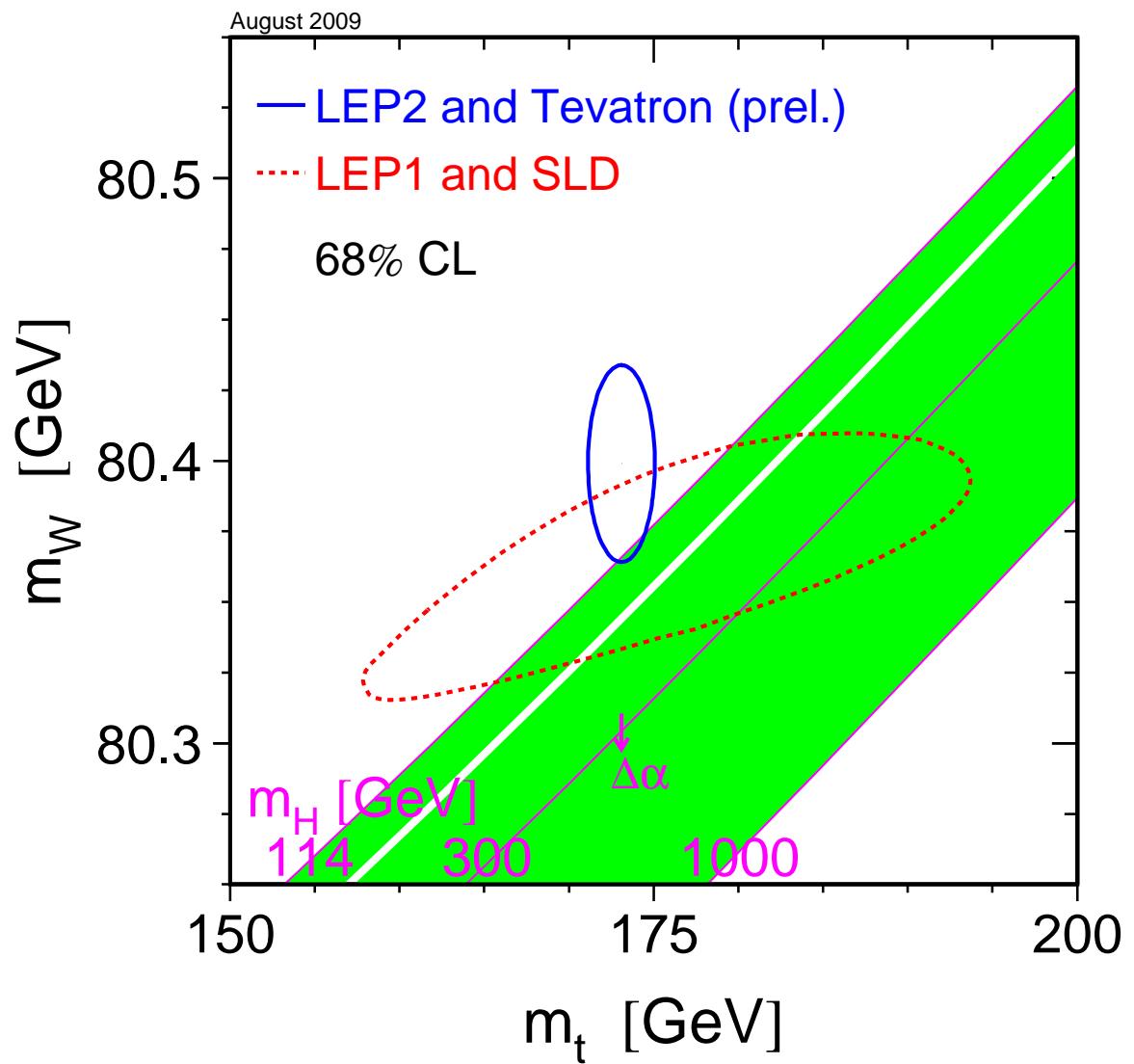
$$\Delta r = -\frac{11g_2^2}{96\pi^2} \frac{s_W^2}{c_W^2} \log\left(\frac{M_H}{M_W}\right)$$

general for EWPO:

$$\Delta \sim g_2^2 \left[\log\left(\frac{M_H}{M_W}\right) + g_2^2 \frac{M_H^2}{M_W^2} \right]$$

leading term: $\log(M_H)$

first term $\sim M_H^2$ with g_2^4



→ light Higgs boson preferred

[LEPEWWG '09]

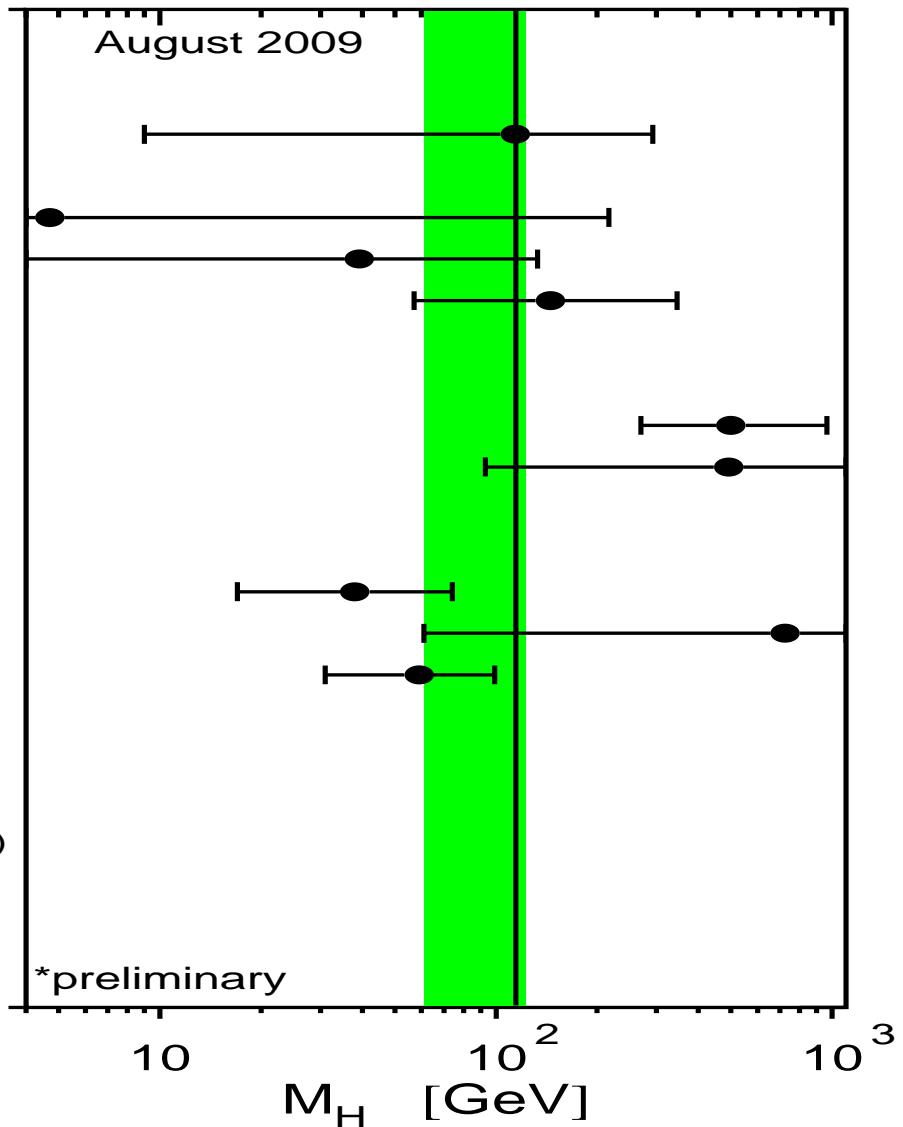
Results for M_H from other EWPO:

light Higgs preferred by:
 M_W , A_l^{LR} (SLD)

heavier Higgs preferred by:
 A_b^{FB} (LEP)
⇒ keeps SM alive

Γ_Z
 σ_{had}^0
 R_I^0
 $A_{\text{fb}}^{0,I}$
 $A_I(P_\tau)$
 R_b^0
 R_c^0
 $A_{\text{fb}}^{0,b}$
 $A_{\text{fb}}^{0,c}$
 A_b
 A_c
 $A_I(\text{SLD})$
 $\sin^2 \theta_{\text{eff}}^{\text{lept}}(Q_{\text{fb}})$
 m_W^*
 Γ_W^*

 $Q_W(\text{Cs})$
 $\sin^2 \theta_{\overline{\text{MS}}}(\text{e}^- \text{e}^-)$
 $\sin^2 \theta_W(\nu N)$
 $g_L^2(\nu N)$
 $g_R^2(\nu N)$



⇒ light Higgs boson preferred

[LEPEWWG '09]

Global fit to all SM data:

[LEPEWWG '09]

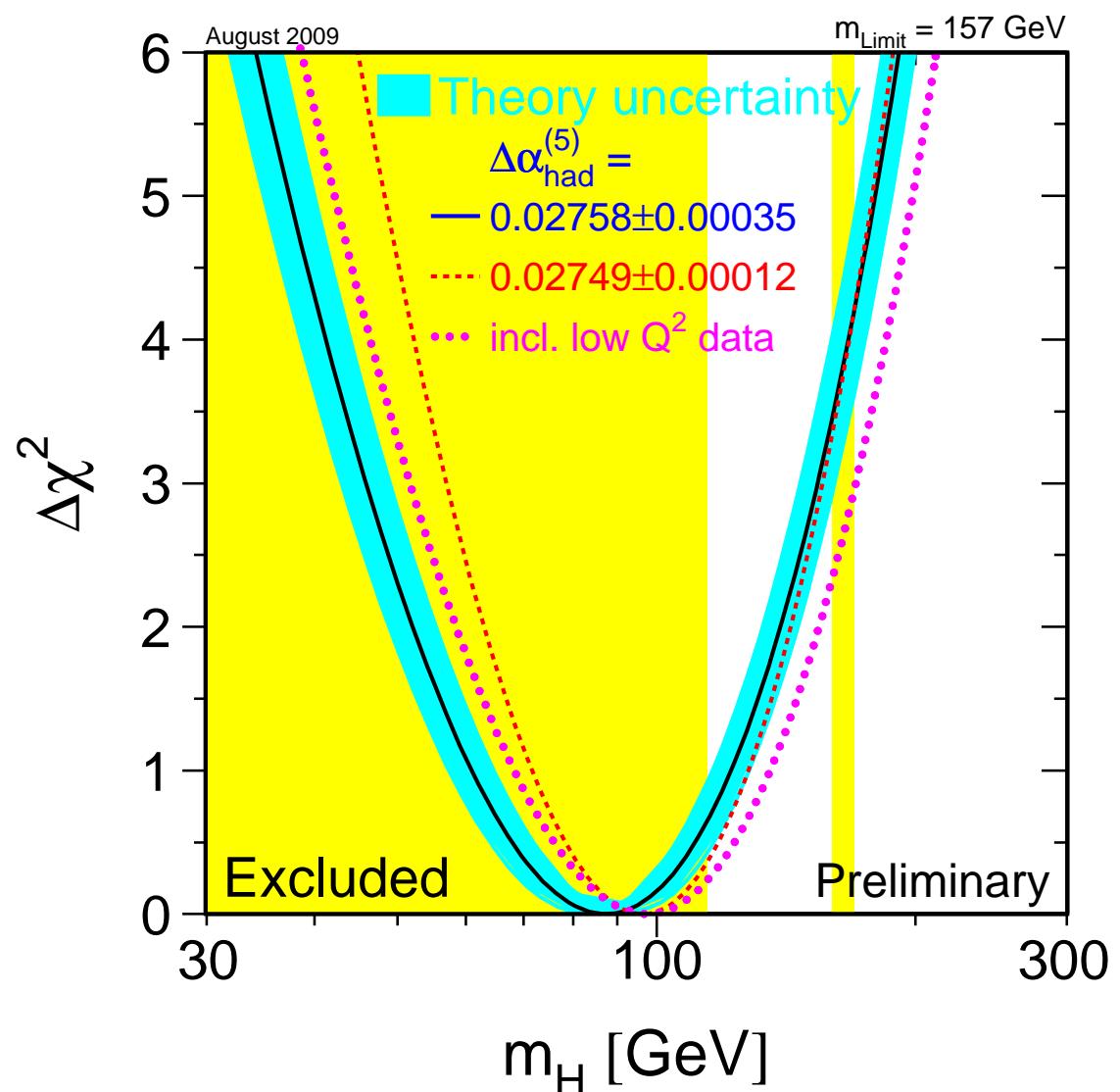
$$\Rightarrow M_H = 87^{+35}_{-26} \text{ GeV}$$

$M_H < 157$ GeV, 95% C.L.

Assumption for the fit:

SM incl. Higgs boson

\Rightarrow no confirmation of
Higgs mechanism



\Rightarrow Higgs boson seems to be light, $M_H \lesssim 160$ GeV

Global fit to all SM data incl. direct searches:

[*GFitter* '09]

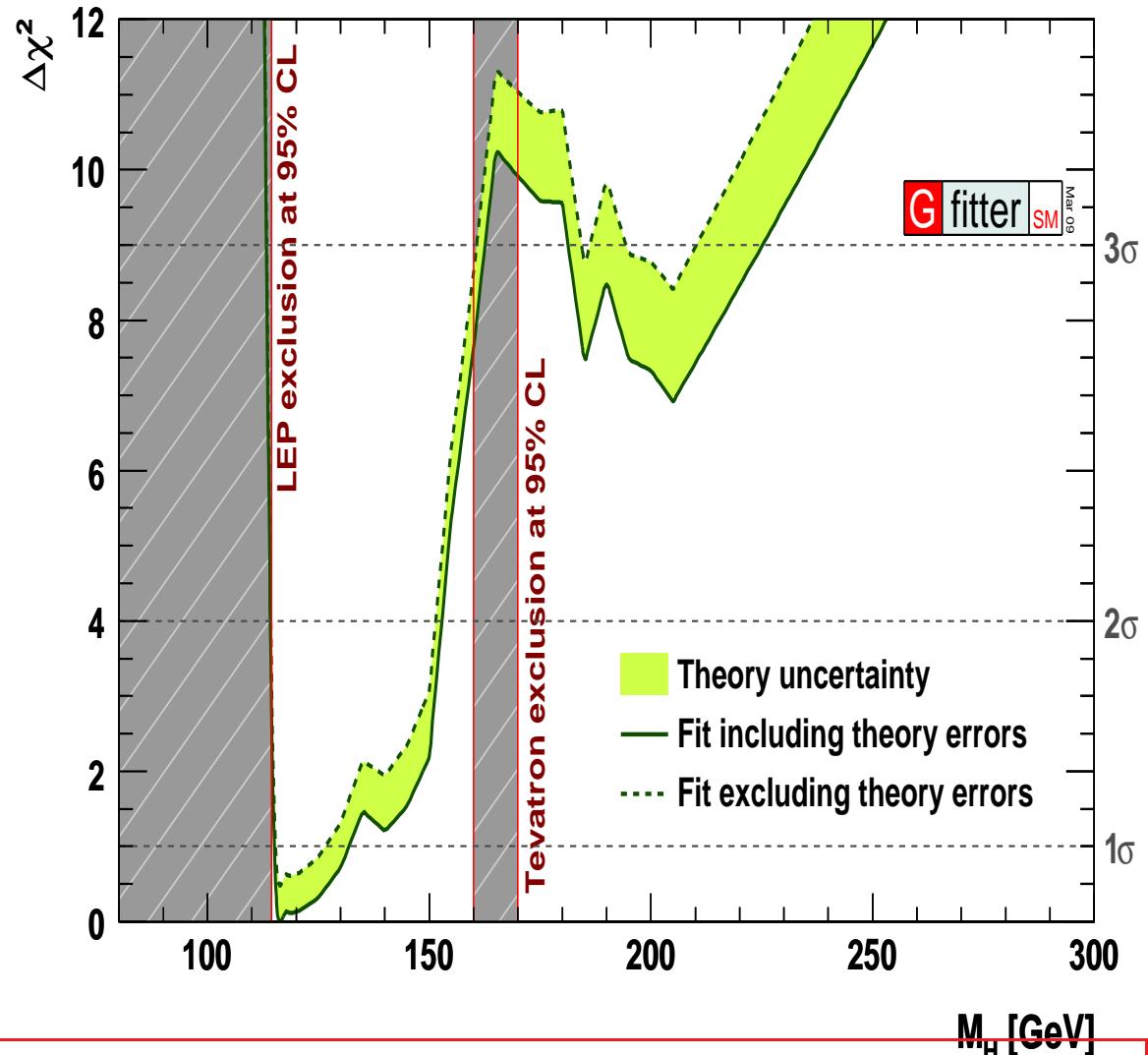
$$\Rightarrow M_H = 116.4^{+18.3}_{-1.4} \text{ GeV}$$

$$M_H < 152 \text{ GeV, 95% C.L.}$$

Assumption for the fit:

SM incl. Higgs boson

\Rightarrow no confirmation of
Higgs mechanism



\Rightarrow Higgs boson seems to be light, $M_H \lesssim 150$ GeV

Experimental errors of the precision observables:

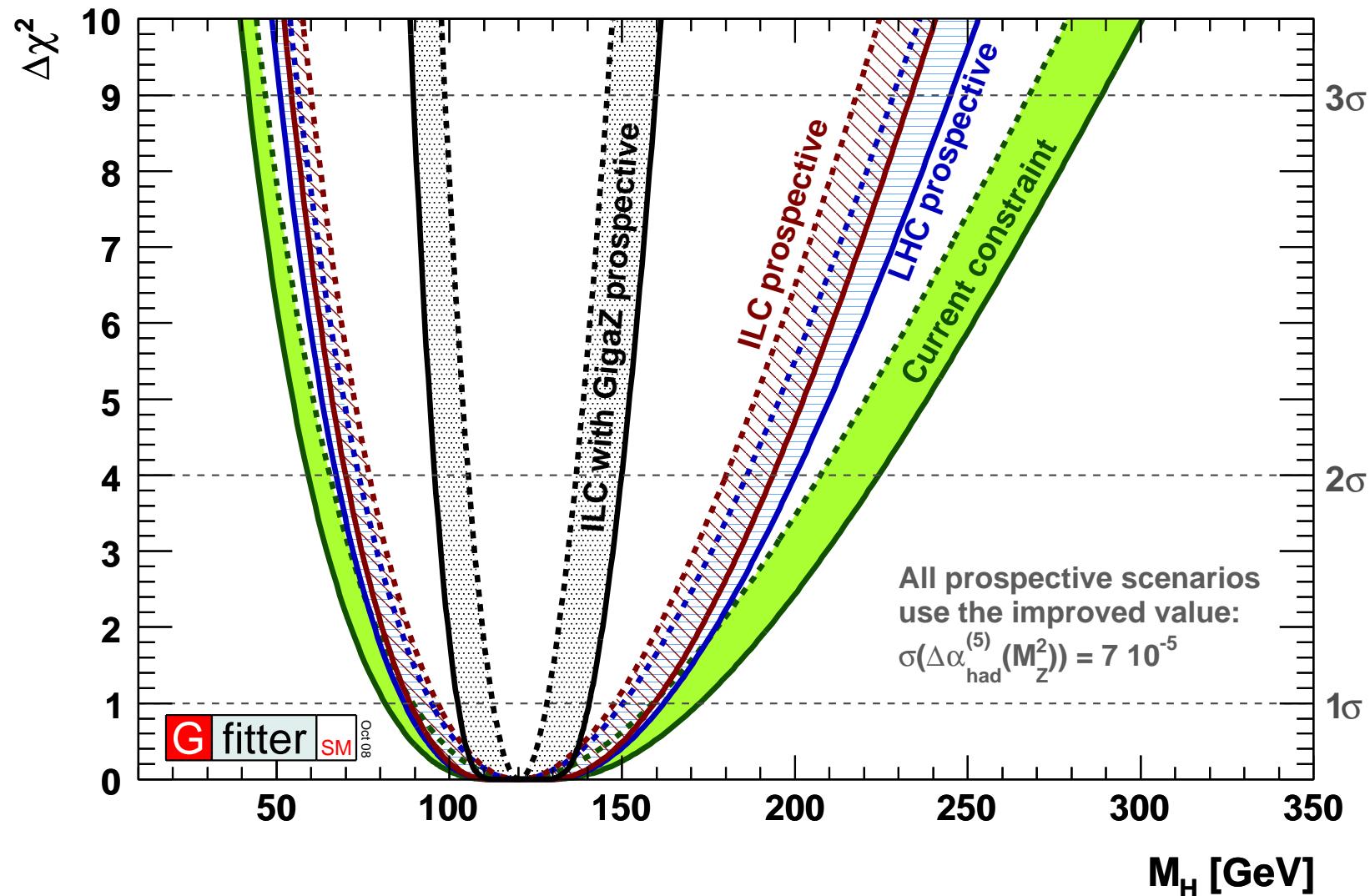
| | today | Tev./LHC | ILC | GigaZ |
|---|---------|----------|-----|-------|
| $\delta \sin^2 \theta_{\text{eff}} (\times 10^5)$ | 16 | 16 | – | 1.3 |
| δM_W [MeV] | 25 (23) | 15 | 10 | 7 |
| δm_t [GeV] | 1.3 | 1-2 | 0.2 | 0.1 |

Relevant SM parametric errors: $\delta(\Delta\alpha_{\text{had}}) = 5 \times 10^{-5}$, $\delta M_Z = 2.1$ MeV

| | $\delta m_t = 2$ | $\delta m_t = 1$ | $\delta m_t = 0.1$ | $\delta(\Delta\alpha_{\text{had}})$ | δM_Z |
|---|------------------|------------------|--------------------|-------------------------------------|--------------|
| $\delta \sin^2 \theta_{\text{eff}} [10^{-5}]$ | 6 | 3 | 0.3 | 1.8 | 1.4 |
| ΔM_W [MeV] | 12 | 6 | 1 | 1 | 2.5 |

Improvement in the Blue Band plot:

[*GFitter '09*]



(note: artificially $M_H^{\text{SM}} = 120 GeV)$

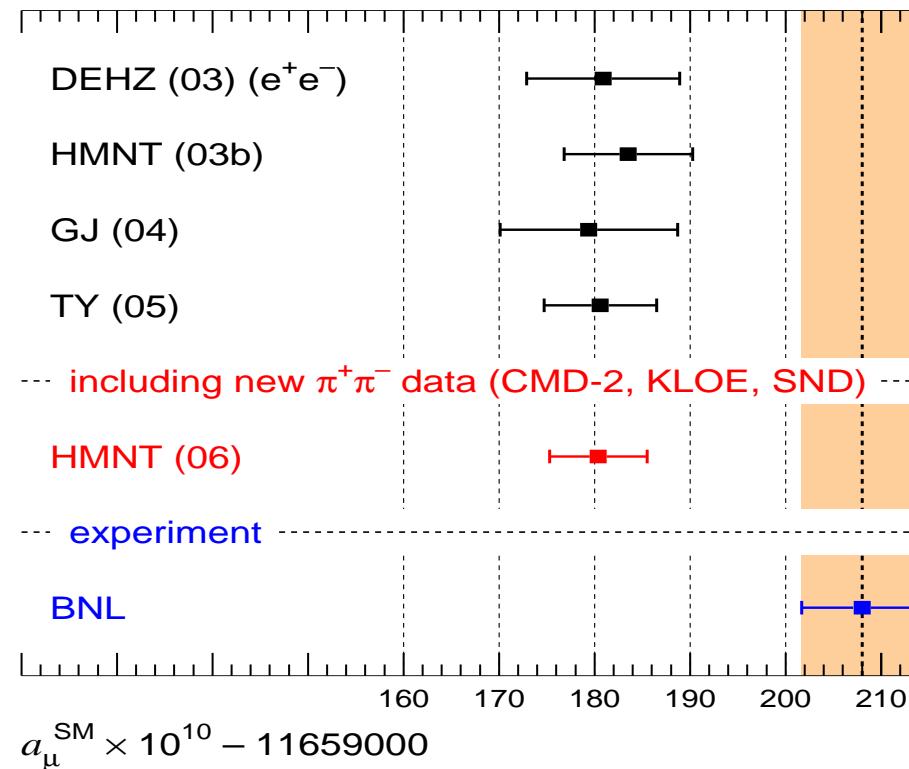
Another EWPO: the anomalous magnetic moment of the muon

$$a_\mu \equiv (g - 2)_\mu / 2$$

Overview about the current experimental and SM (theory) result:

[*g-2 Collaboration, hep-ex/0602035*]

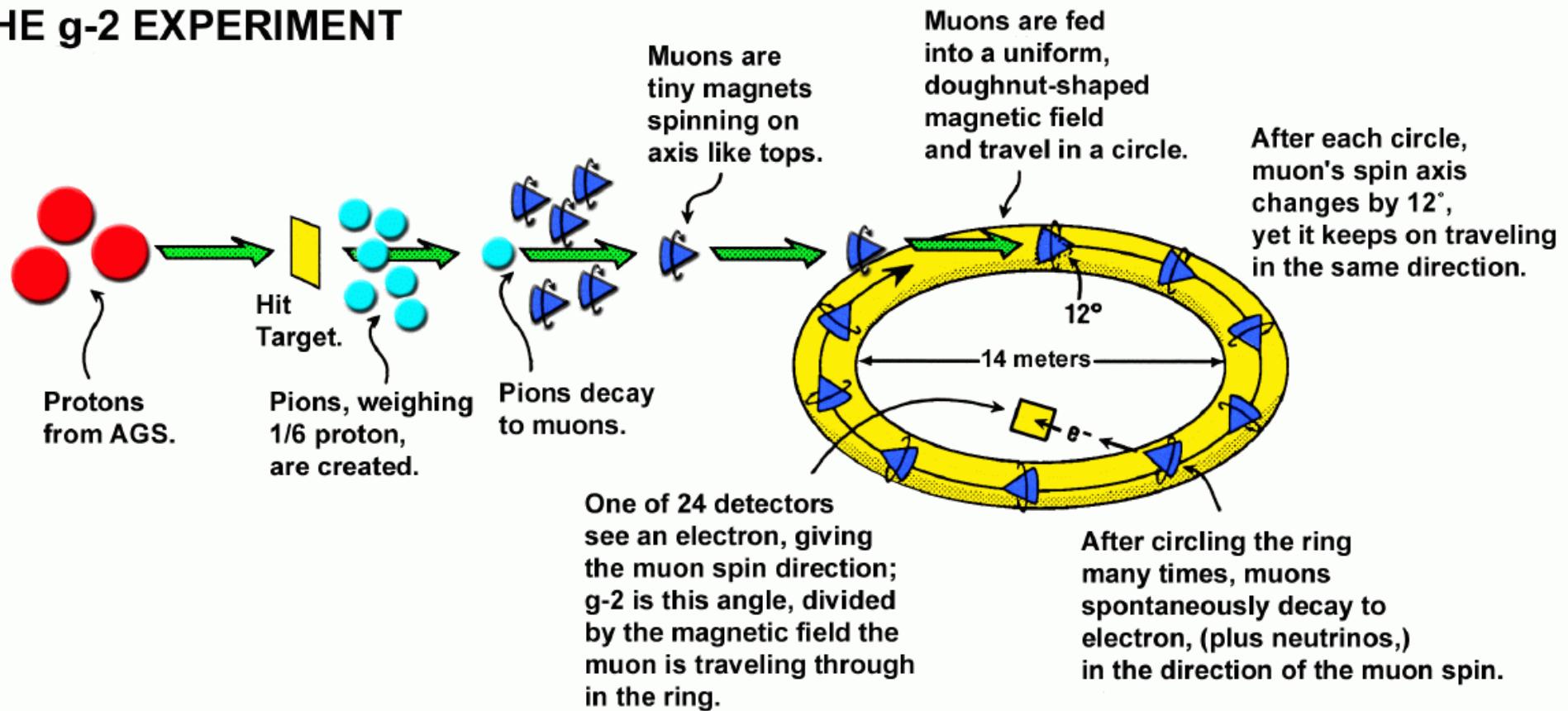
$\rightarrow T$



$$a_\mu^{\text{exp}} - a_\mu^{\text{theo,SM}} \approx (28 \pm 8) \times 10^{-10} : 3.4 \sigma$$

The $(g - 2)_\mu$ experiment:

LIFE OF A MUON: THE g-2 EXPERIMENT



Coupling of muon to magnetic field : $\mu - \mu - \gamma$ coupling

$$\bar{u}(p') \left[\gamma^\mu F_1(q^2) + \frac{i}{2m_\mu} \sigma^{\mu\nu} q_\nu F_2(q^2) \right] u(p) A_\mu \quad F_2(0) = a_\mu$$

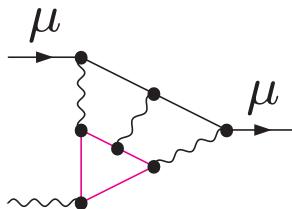
Current status of $(g - 2)_\mu$:

Experiment:

- 2001 - 2006: very stable development
- final error: 6×10^{-10} , still statistically dominated

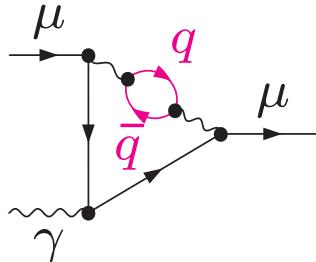
Theory:

- the **light-by-light** contribution:



2002: sign error discovered; since then stabilized

- the **hadronic vacuum** contribution:



problems with the τ data \Rightarrow hardly used anymore

(status 06/2009)

new 'direct' e^+e^- data from **CMD-II** and **SND**

\Rightarrow agrees well (also with old e^+e^- data)

new **radiative return** data from **KLOE** and B-factories

\Rightarrow agrees well with e^+e^- data

new SM evaluations, based on new exp data for a_μ^{had} :

$$a_\mu(\text{Exp-SM}) = \left\{ \begin{array}{ll} [\text{HMNT '06}] & 28(8) \\ [\text{DEHZ '06}] & 28(8) \\ [\text{FJ '07}] & 29(9) \\ [\text{MRR '07}] & 29(9) \end{array} \right\} \times 10^{-10}$$

better agreement between evaluations, more precise,
larger deviation from exp than ever before

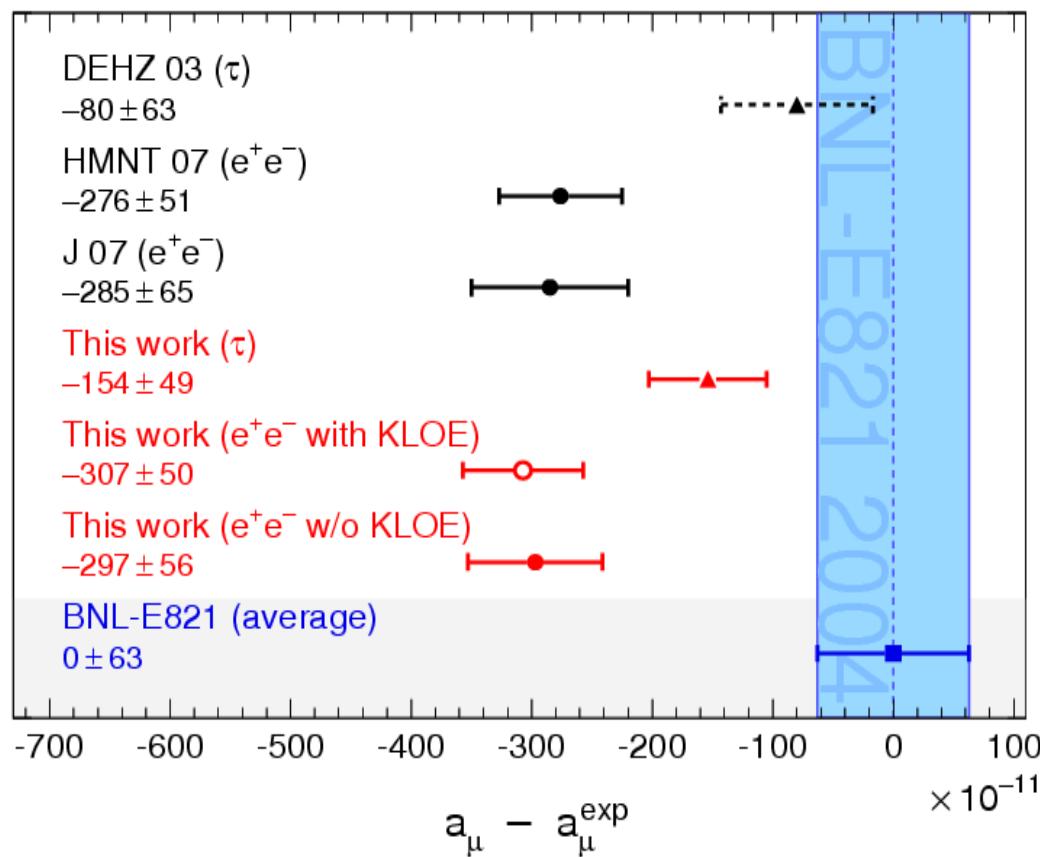


3σ deviation has now been definitely established

New development for τ data:

[M. Davier, A. Höcker et al. '09]

Re-evaluation of τ data: improved evaluation of iso-spin breaking effects
⇒ shift in τ data :



Now: 1.9σ deviation! ⇒ still tbc!

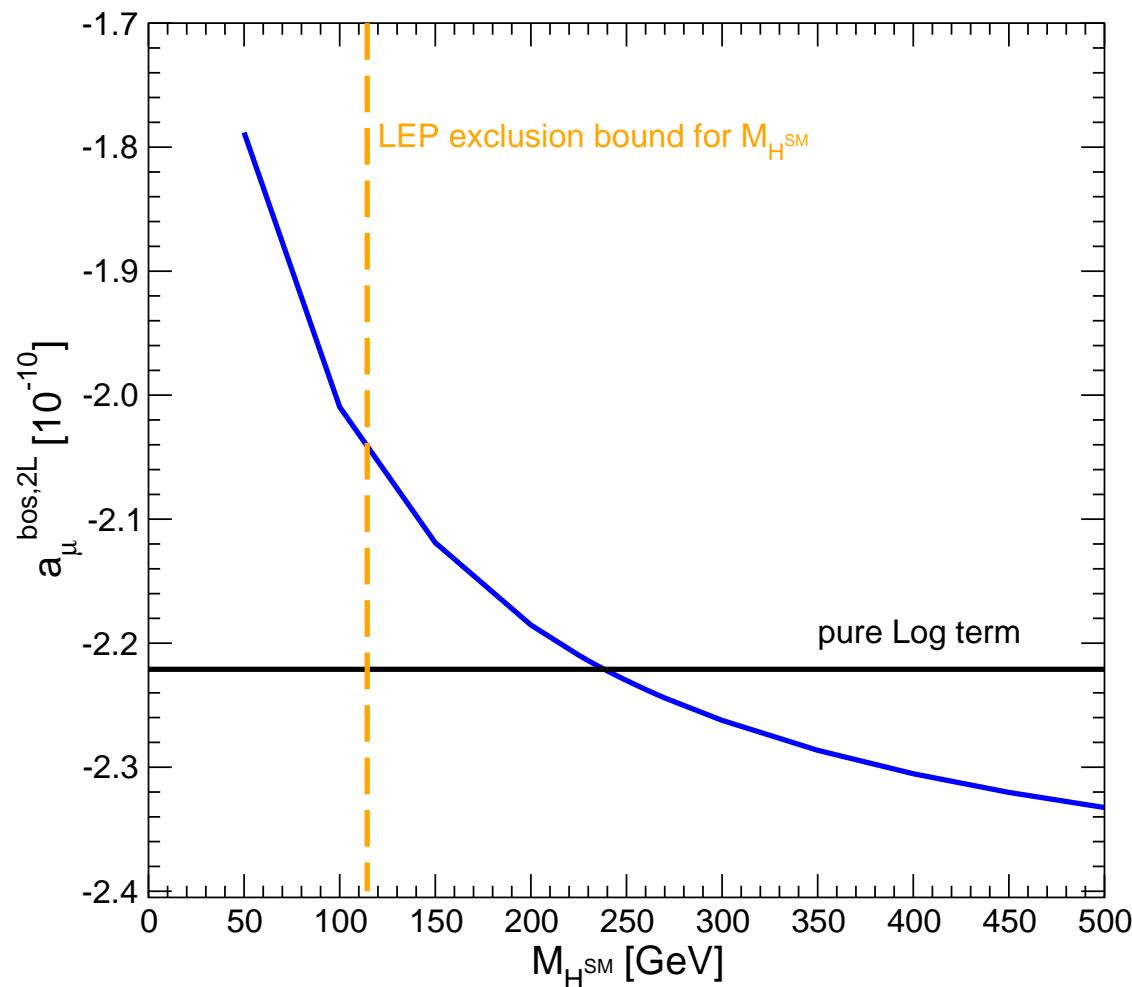
If correct: ⇒ new average of all data possible . . .

Restrictions on M_H from a_μ ?

⇒ Higgs enters only at the two-loop level

Example for M_H dependence:

[S.H., D. Stöckinger, G. Weiglein '04]



⇒ no restrictions on M_H (but wait for tomorrow! :-)

3. Properties of the SM Higgs boson

1.) Decay to fermions:

coupling:

$$g_{f\bar{f}H} = [\sqrt{2} G_\mu]^{1/2} m_f$$

decay width:

$$\Gamma(H \rightarrow f\bar{f}) = N_c \frac{G_\mu M_H}{4\sqrt{2}\pi} m_f^2(M_H^2) \left(1 - 4\frac{m_f^2}{M_H^2}\right)^{3/2}$$

with N_c = number of colors

Bulk of QCD corrections for decays to quarks are mapped into

$$m_q^2(\text{pole}) \rightarrow m_q^2(M_H^2)$$

Dominant decay process: $H \rightarrow b\bar{b}$

2.) Decay to heavy gauge bosons ($V = W, Z$):

coupling:

$$g_{VVH} = 2 \left[\sqrt{2} G_\mu \right]^{1/2} M_V^2$$

on-shell decay width ($M_H > 2M_V$):

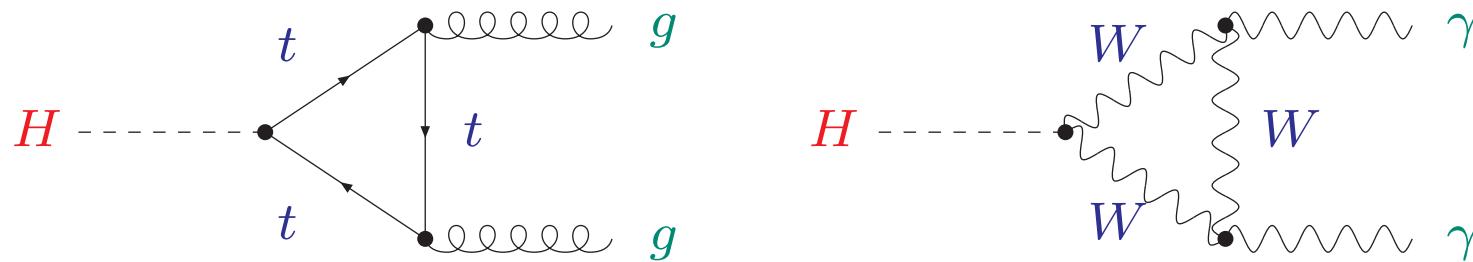
$$\Gamma(H \rightarrow VV) = \delta_V \frac{G_\mu M_H^3}{16 \sqrt{2} \pi} \left(1 - 4 \frac{M_V^2}{M_H^2} + 12 \frac{M_V^4}{M_H^4} \right) \left(1 - 4 \frac{M_V^2}{M_H^2} \right)^{1/2}$$

with $\delta_{W,Z} = 2, 1$

off-shell decay width ($M_H < 2M_V$):

$$\Gamma(H \rightarrow VV^*) = \delta'_V \frac{3G_\mu^2 M_H}{16 \pi^3} M_V^4 \times \text{Integral}$$

3.) Decay to massless gauge bosons (gg , $\gamma\gamma$):



$$\Gamma(H \rightarrow gg) = \frac{G_\mu \alpha_s^2(M_H^2) M_H^3}{36 \sqrt{2} \pi^3} \left[1 + C \frac{\alpha_s(\mu)}{\pi} \right]$$

via the top quark loop with

$$C = \frac{215}{12} - \frac{23}{6} \log \left(\frac{\mu^2}{M_H^2} \right) + \mathcal{O}(\alpha_s)$$

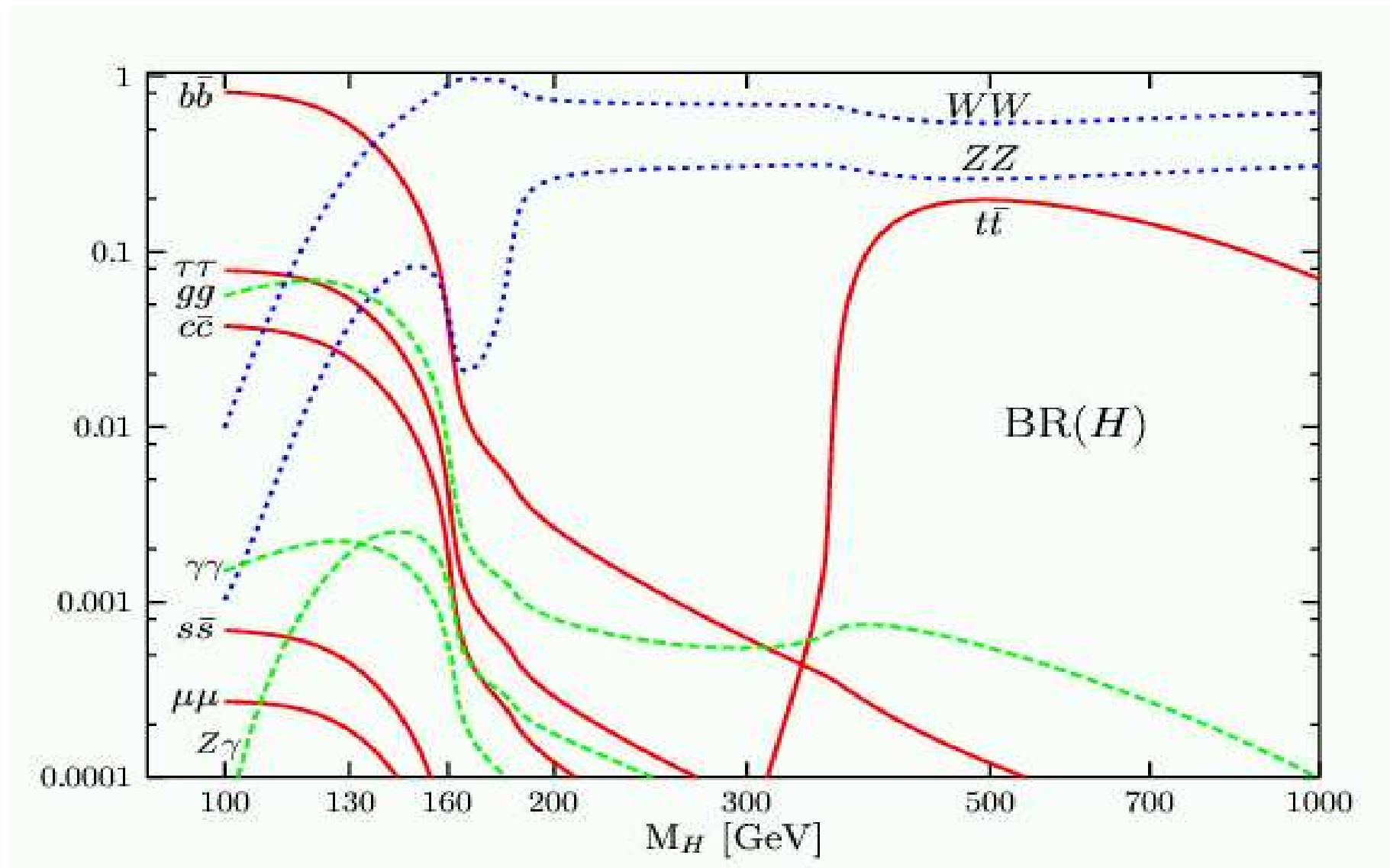
\Rightarrow huge QCD corrections

$$\Gamma(H \rightarrow \gamma\gamma) = \frac{G_\mu \alpha^2 M_H^3}{128 \sqrt{2} \pi^3} \left| \frac{4}{3} e_t^2 - 7 \right|^2$$

via the top quark and W boson loop

Overview of the branching ratios:

[taken from [hep-ph/0503172](#)]



The total SM Higgs boson width:

[taken from [hep-ph/0503172](#)]

