Constraints on Supersymmetry

- Absence of sparticles at LEP, Tevatron

  selectron, chargino > 100 GeV

  squarks, gluino > 300 GeV

- Indirect constraints

  Higgs > 114 GeV, b -> s γ

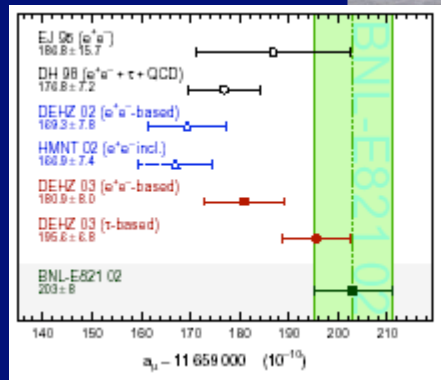
- Density of dark matter

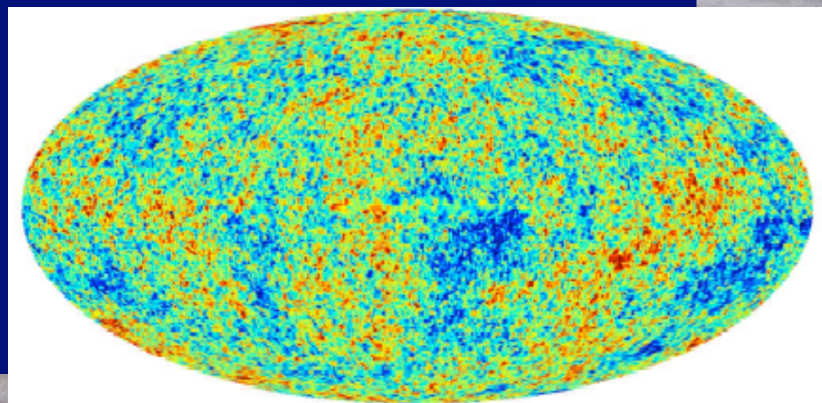
  lightest sparticle χ:

WMAP:

$0.094 < \Omega_\chi h^2 < 0.124$

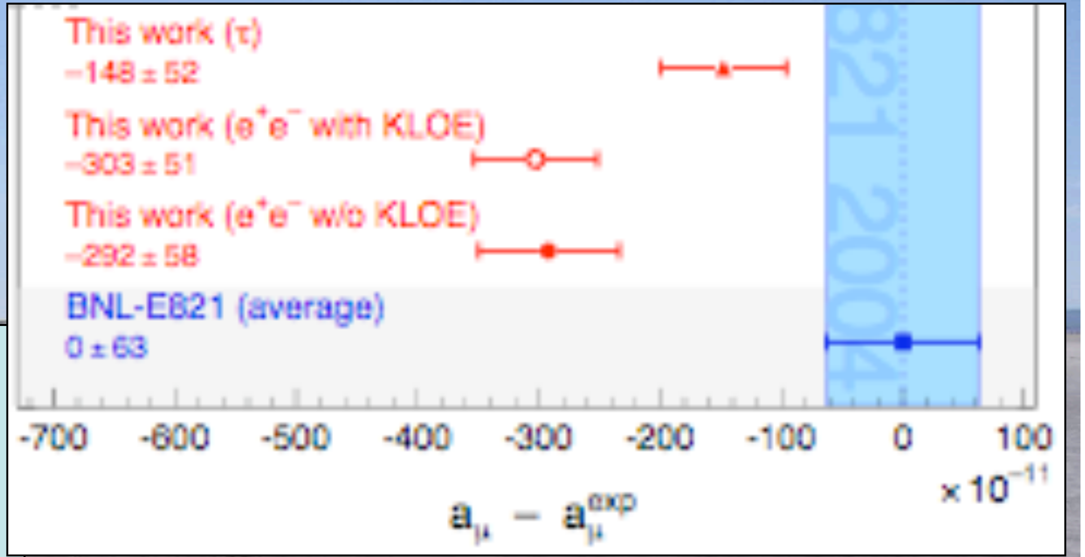
g_μ - 2

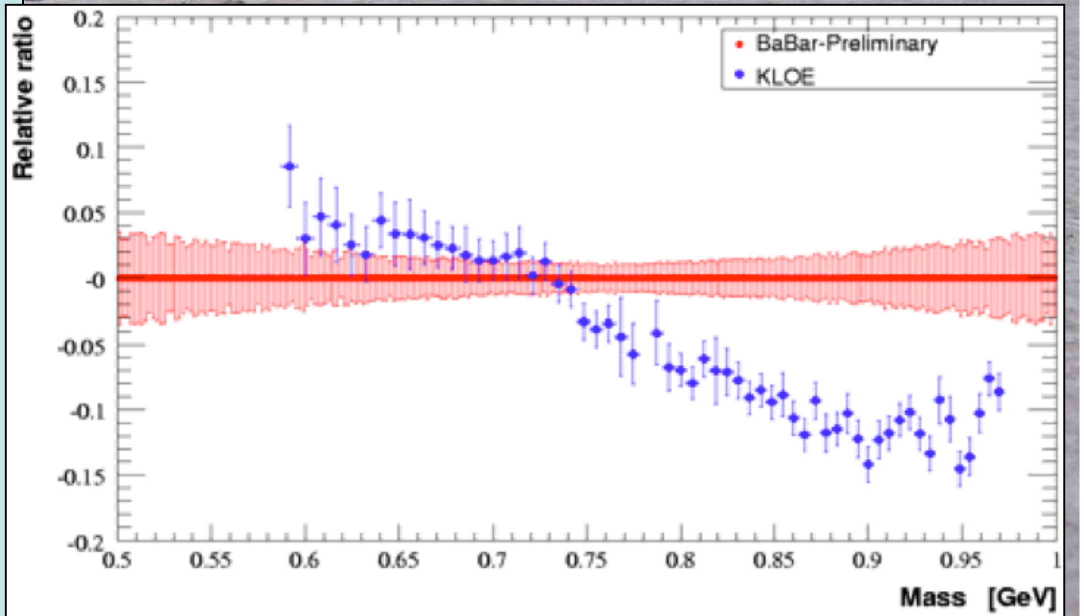




Quo Vadis g_μ - 2?

- Older e^+e^- data show discrepancy
  - now 3.4 σ
- Disagreement with τ decay data
  - Discrepancy ~ 2 σ
- New BABAR e^+e^- data apparently disagree with previous e^+e^- data
  - Agree with τ decay data
- Soon to be finalized?





Current Constraints on CMSSM

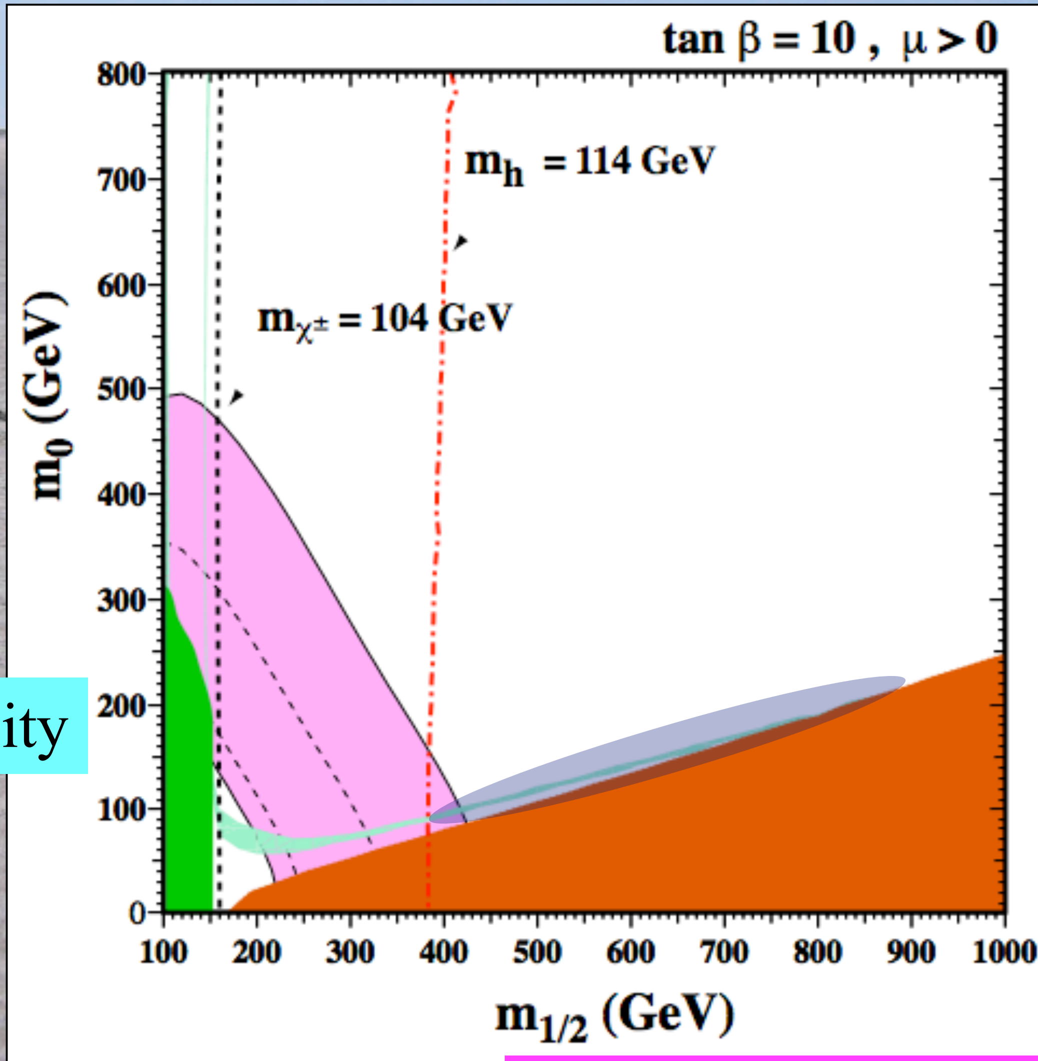
Assuming the lightest sparticle is a neutralino

Excluded because stau LSP

Excluded by b → s gamma

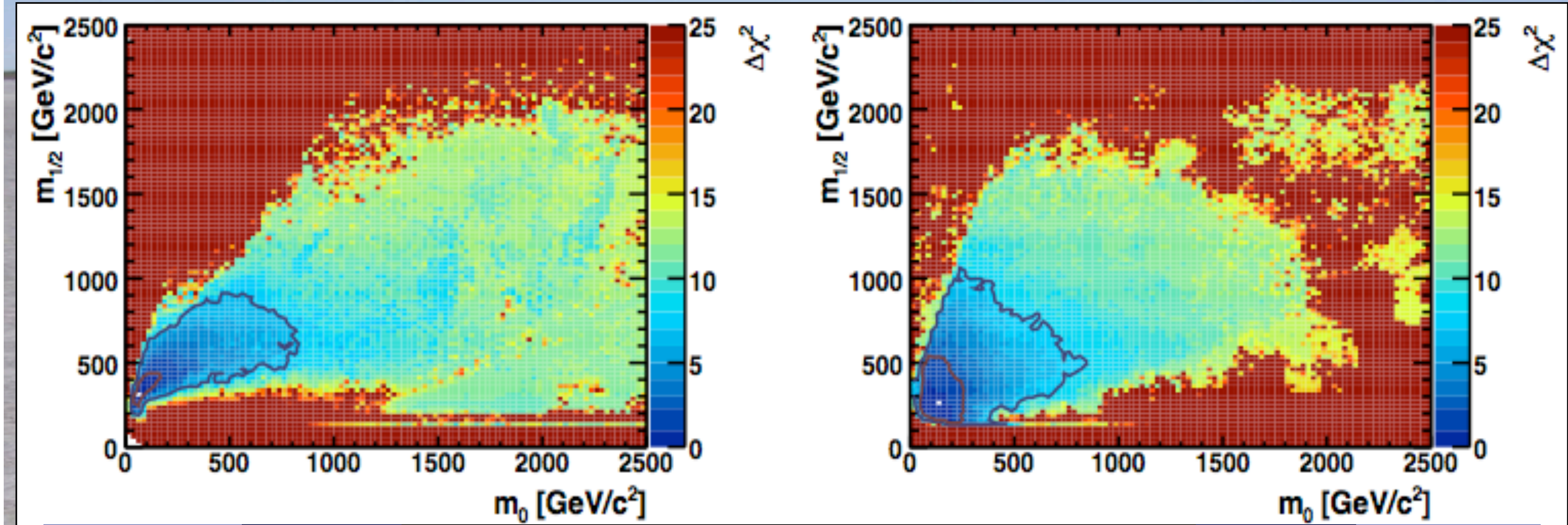
WMAP constraint on relic density

Preferred (?) by latest g - 2



JE + Olive + Santoso + Spanos

The (m_0, $m_{1/2}$) Planes in the CMSSM and the NUHM1



CMSSM Low (m_0, $m_{1/2}$) preferred **NUHM1**

[stau coannihilation region]

Focus-point region disfavoured

O.Buchmueller, JE et al: arXiv:0907.5568

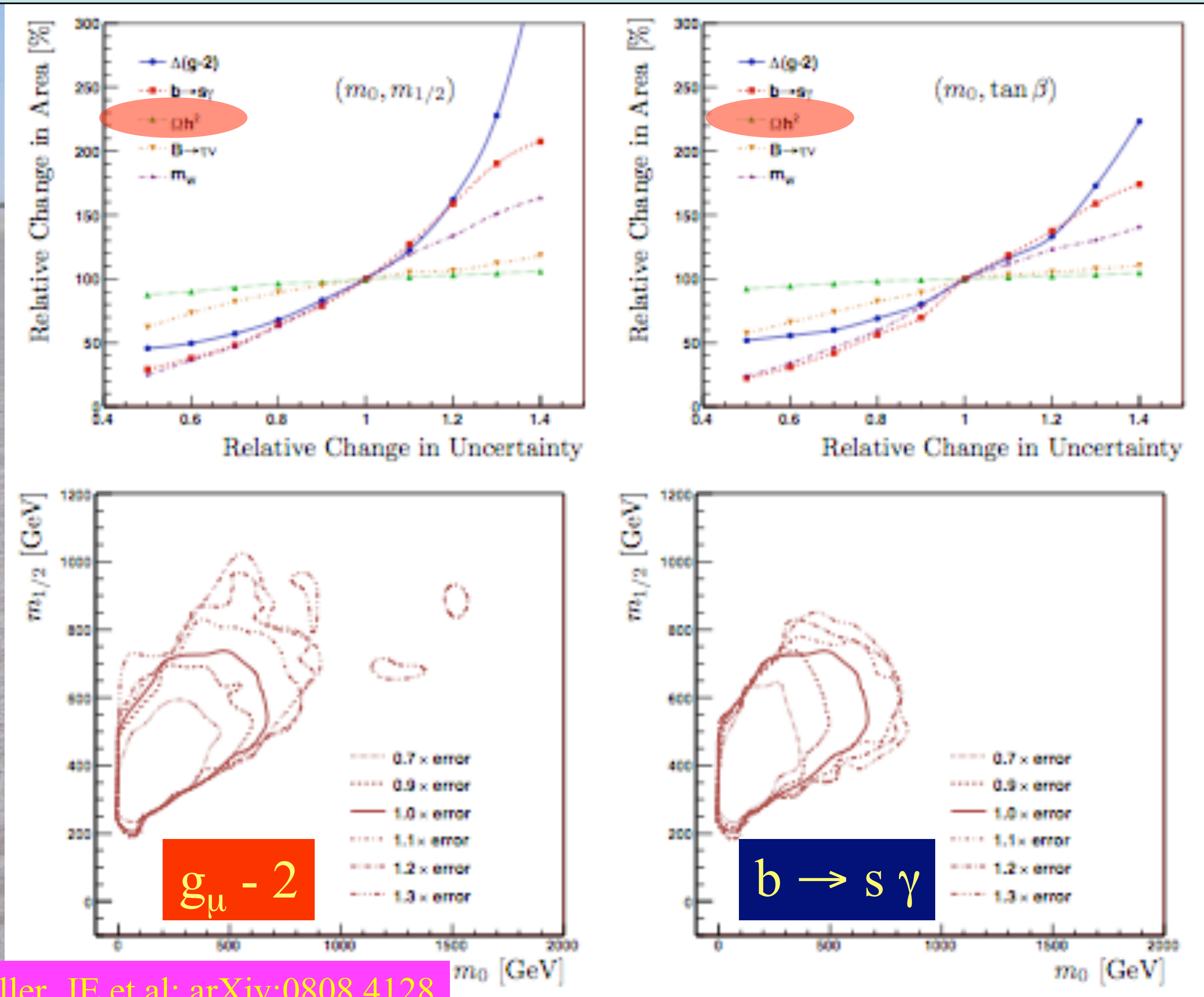
Contributions to the Global χ^2

Observable	Best CMSSM fit	Best NUHM1 fit	Best CMSSM FP fit
$(g-2)_\mu$	0.44	0.002	8.4
$\mathrm{BR}(B_u \to \tau\nu_\tau)$	0.20	0.41	0.85
M_W	0.53	0.08	1.5
$A_\ell(\mathrm{SLD})$	2.84	3.22	3.56
$A_{\mathrm{fb}}(b)(\mathrm{LEP})$	7.61	7.08	6.74
R_ℓ	0.96	1.01	1.05
$\mathrm{BR}^{\mathrm{SUSY}}_{b\to s\gamma}/\mathrm{BR}^{\mathrm{SM}}_{b\to s\gamma}$	1.16	0.001	0.95
M_h	0.17	0	0
χ^2_{tot}	20.6	18.5	29.8

Highlighted observables prefer stau coannihilation region over focus-point region, e.g., m_W

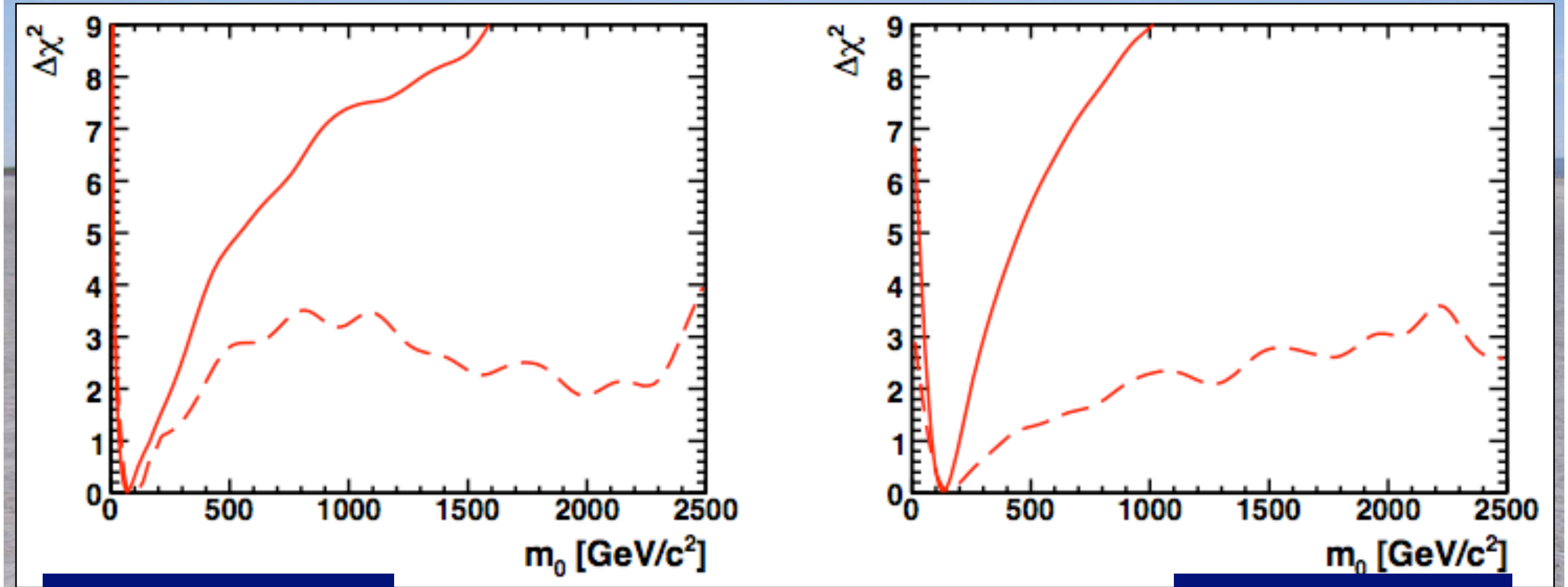
O.Buchmueller, JE et al: arXiv:0907.5568

Sensitivities to Constraints



O.Buchmueller, JE et al: arXiv:0808.4128

What Happens if g_μ - 2 Dropped?



CMSSM

NUHM1

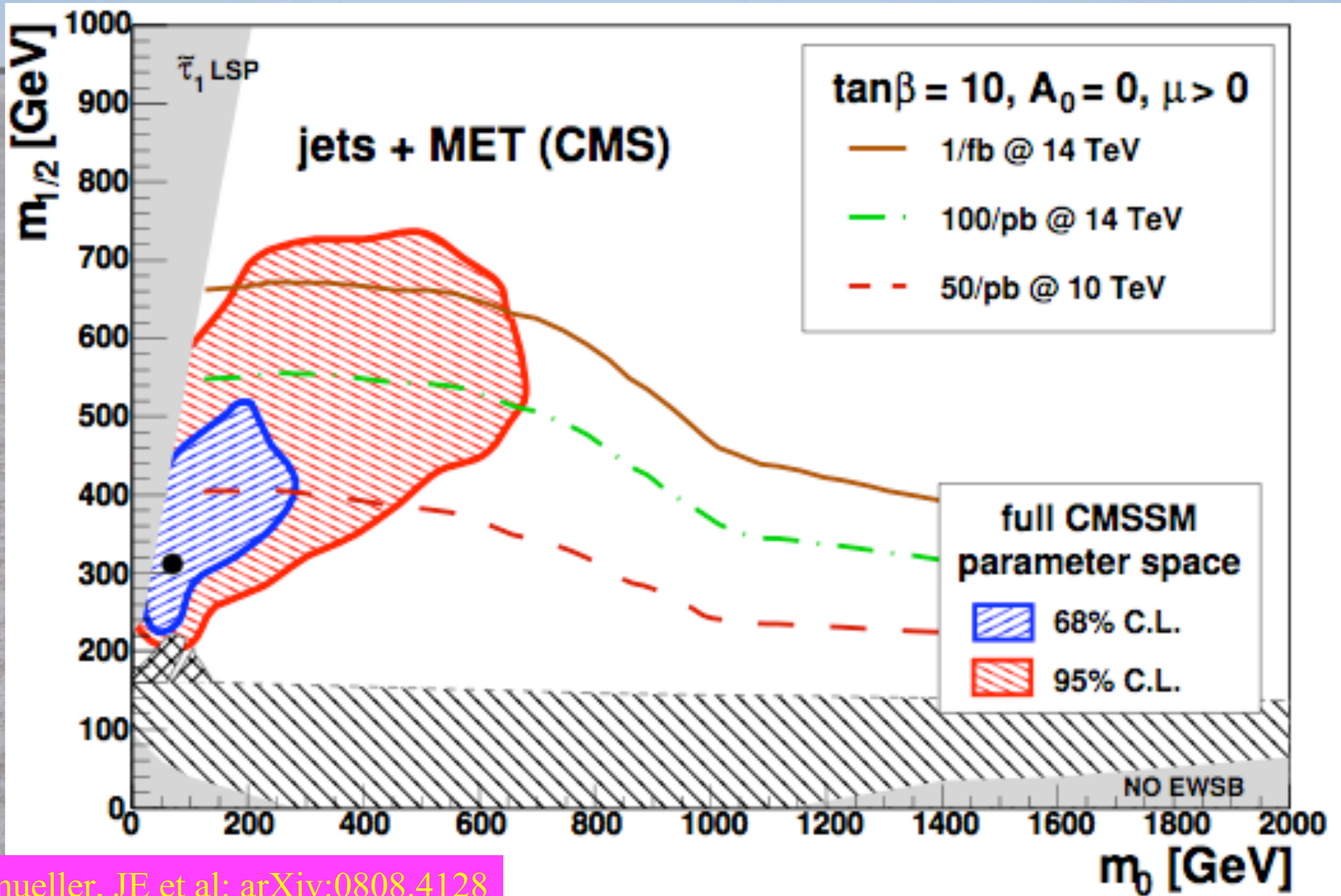
Solid lines: with g_μ - 2

Dashed lines: without g_μ - 2

Focus-point still disfavoured, e.g., by m_W

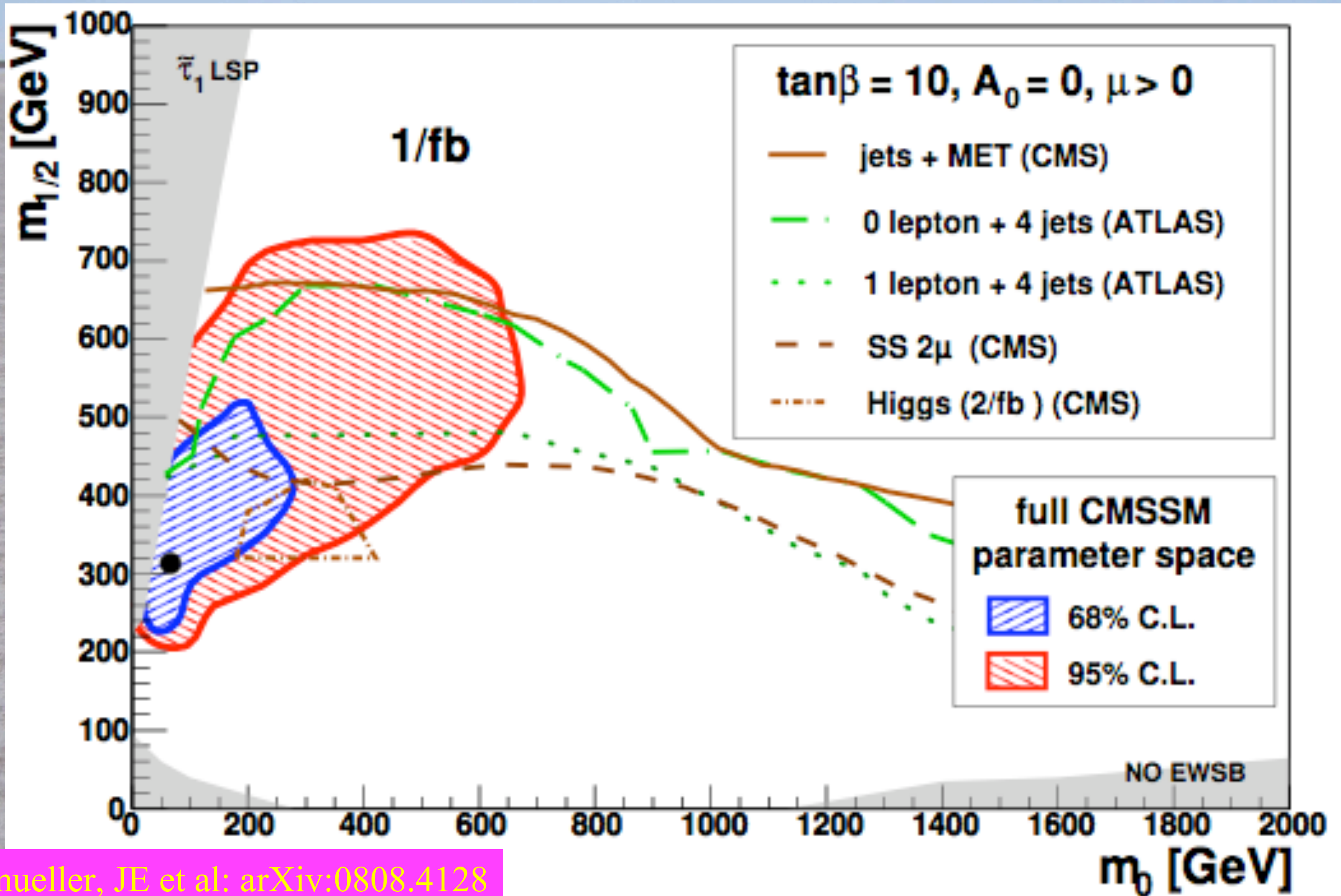
O.Buchmueller, JE et al: arXiv:0907.5568

How Soon Might the CMSSM be Detected?



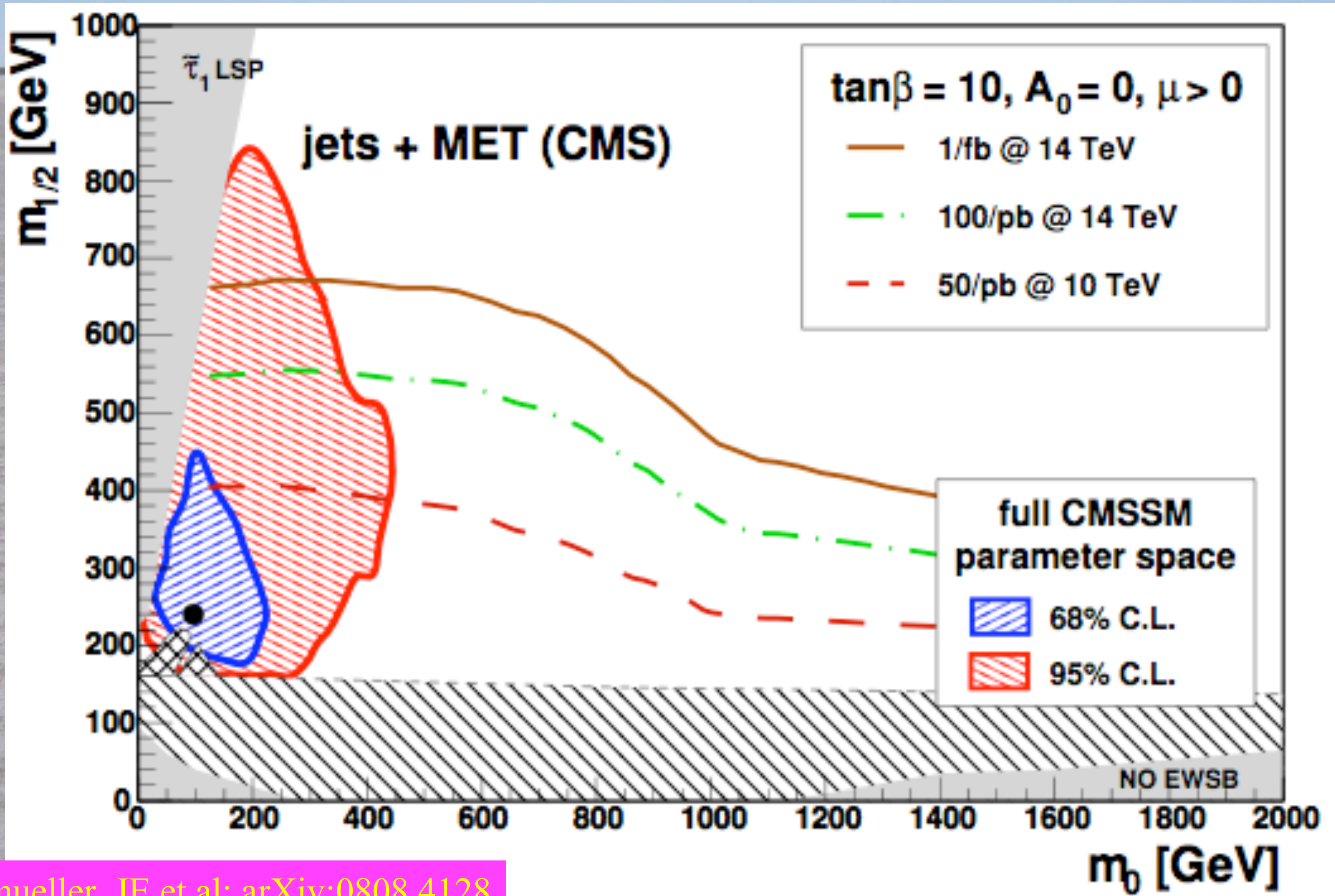
O.Buchmueller, JE et al: arXiv:0808.4128

CMSSM with 1/fb of LHC Data



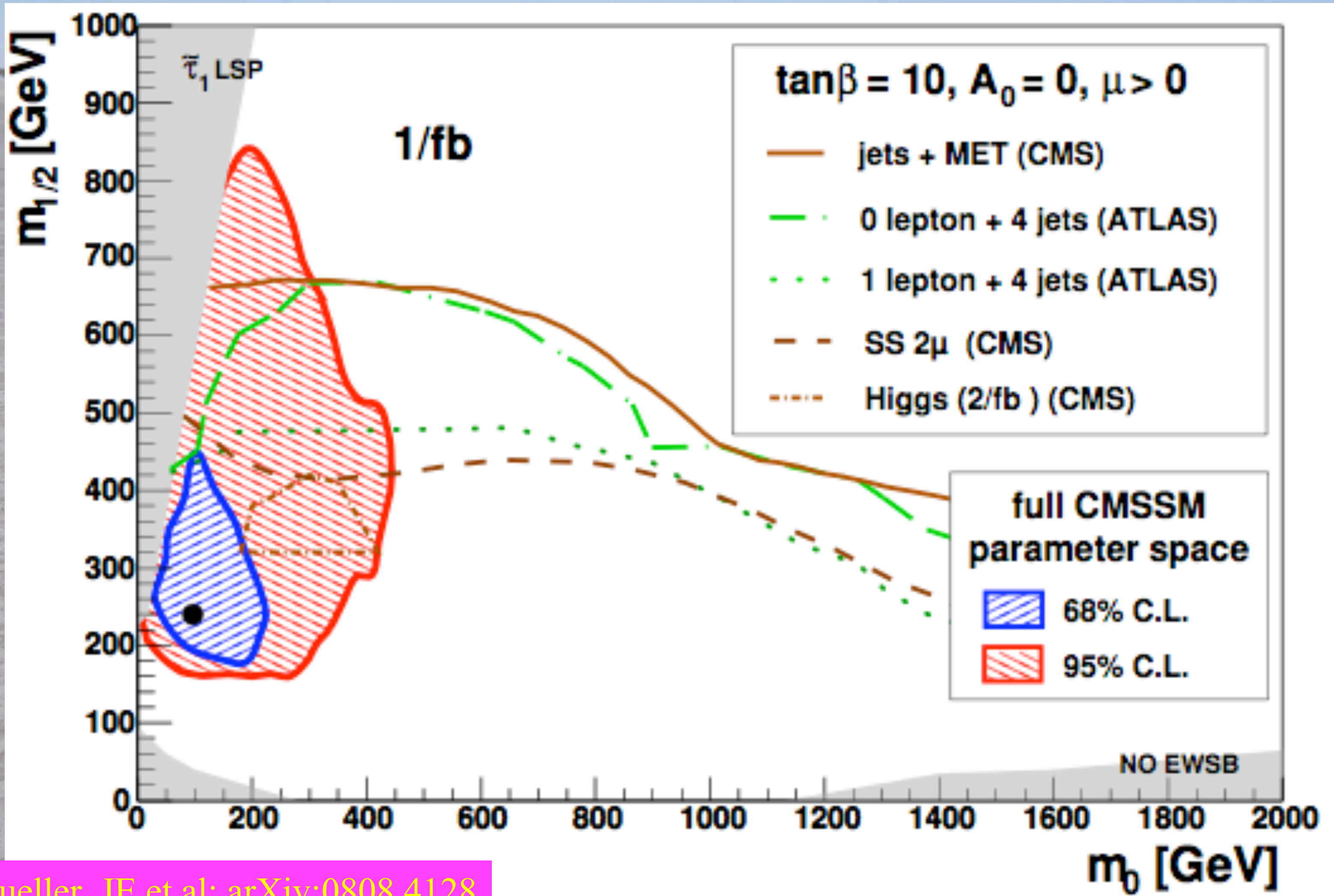
O.Buchmueller, JE et al: arXiv:0808.4128

How Soon Might the NUHM1 be Detected?



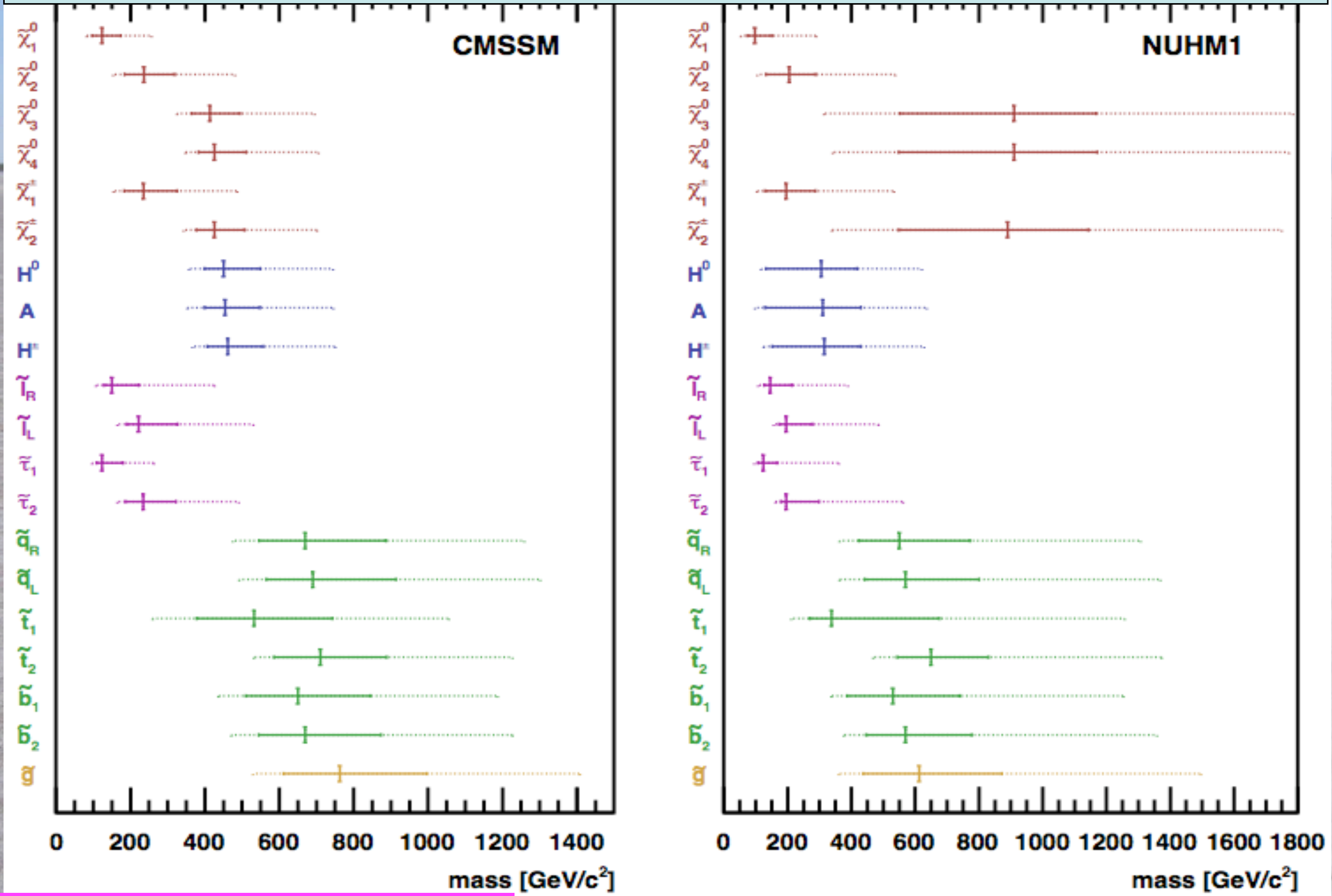
O.Buchmueller, JE et al: arXiv:0808.4128

NUHM1 with 1/fb of LHC Data



O.Buchmueller, JE et al: arXiv:0808.4128

Spectra with likely Ranges

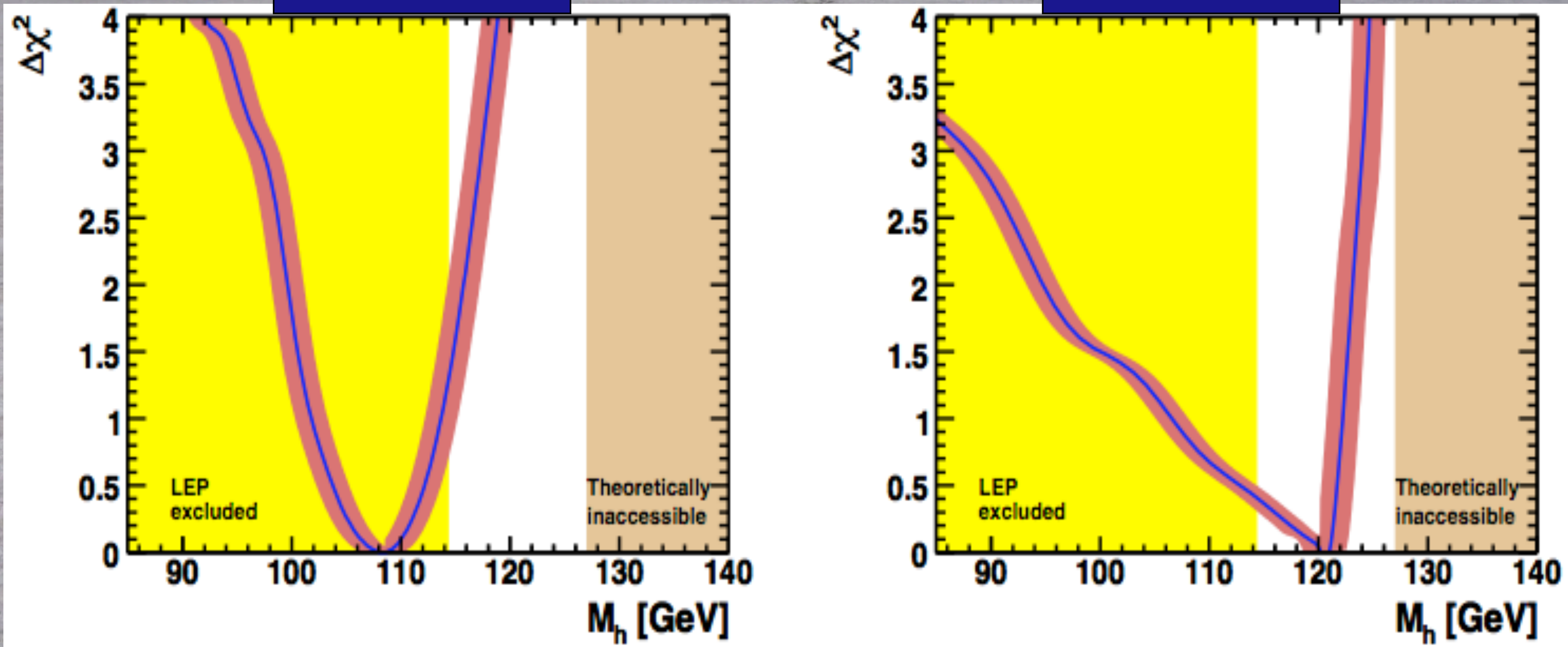


O.Buchmueller, JE et al: arXiv:0907.5568

Likelihood Function for Higgs Mass

CMSSM

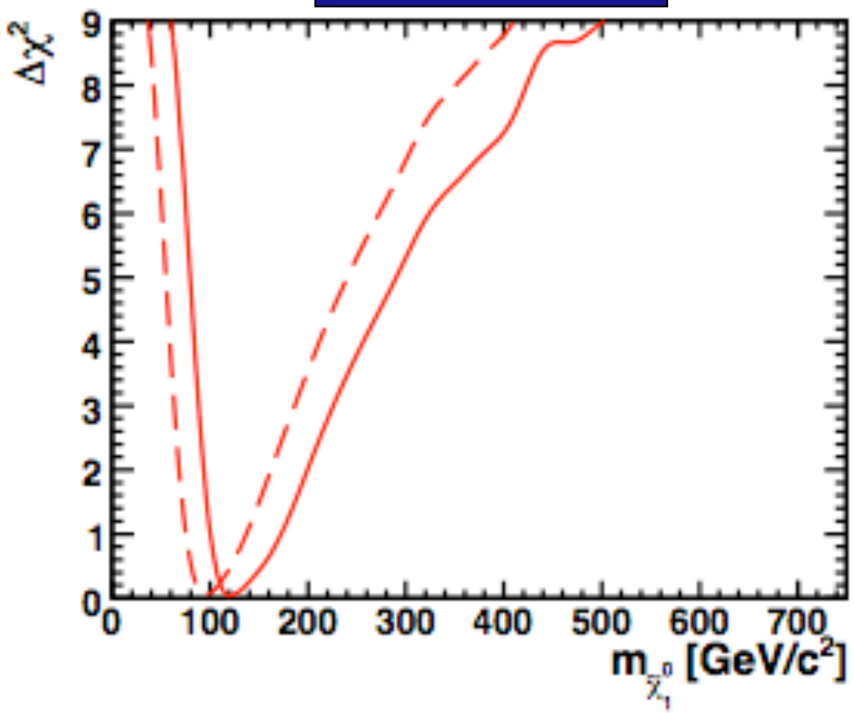
NUHM1



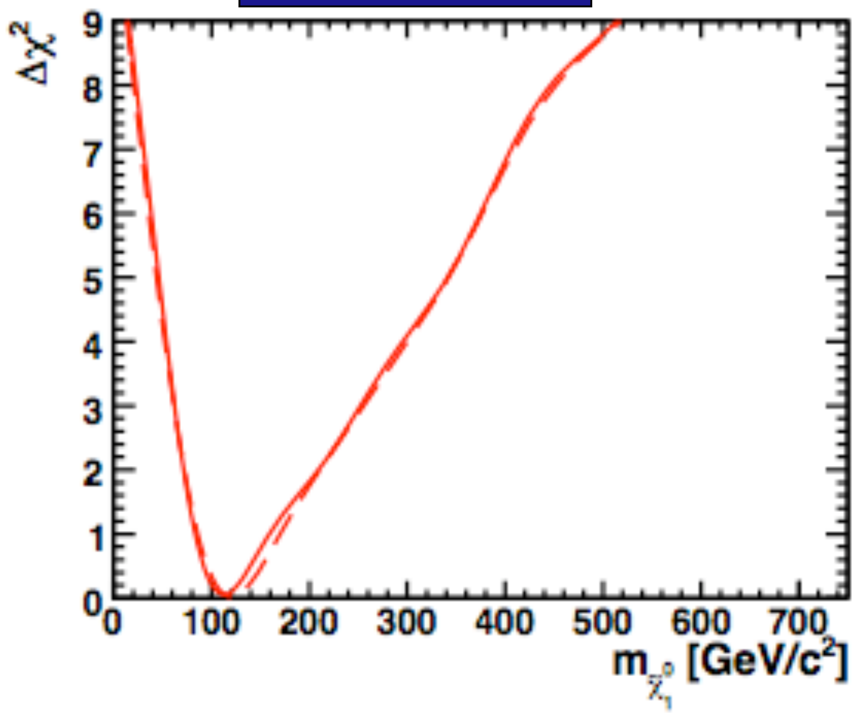
O.Buchmueller, JE et al: arXiv:0907.5568

Likelihood Function for Neutralino Mass

CMSSM



NUHM1

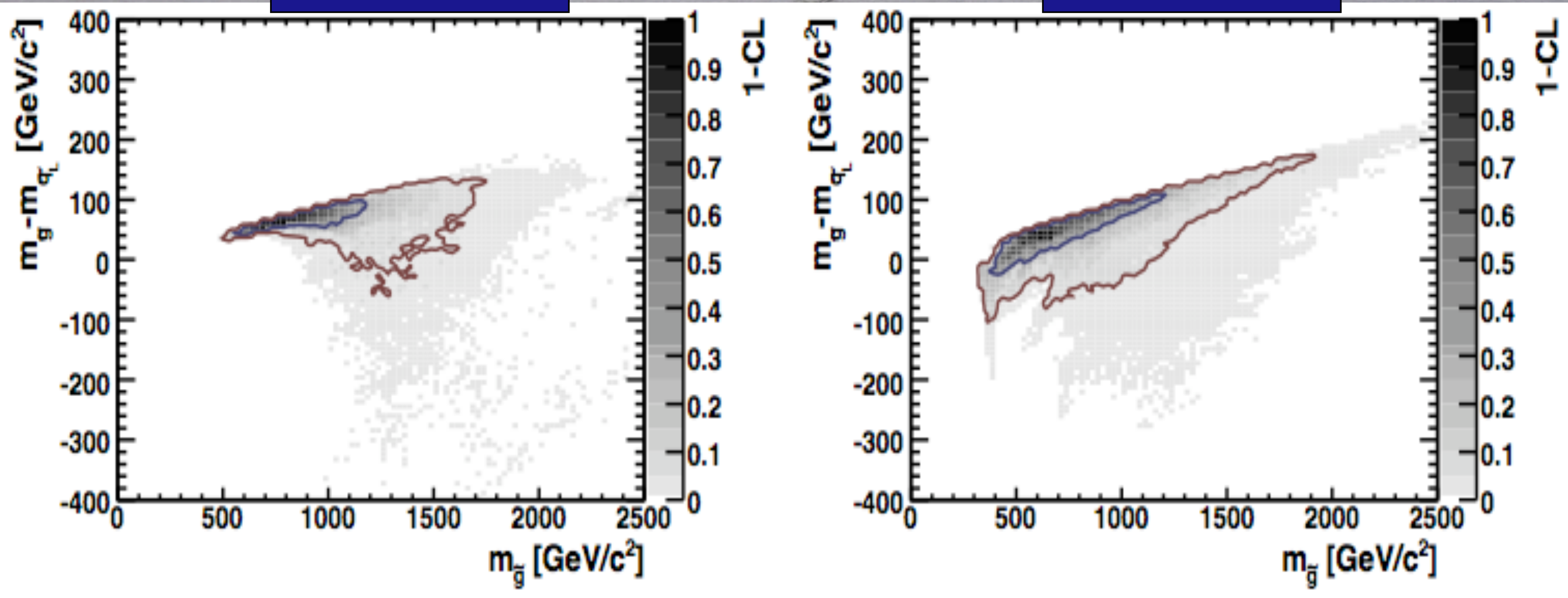


O.Buchmueller, JE et al: arXiv:0907.5568

Correlation between Gluino & Squark Masses

CMSSM

NUHM1

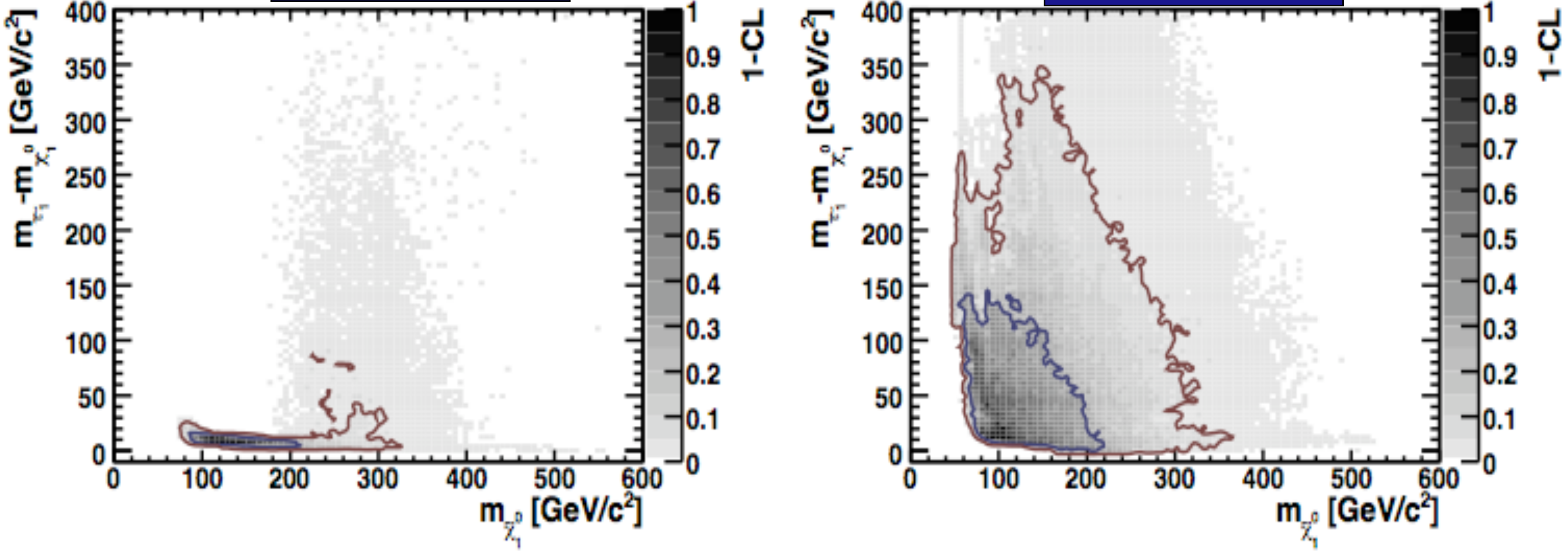


O.Buchmueller, JE et al: arXiv:0907.5568

Correlation between Neutralino & Stau Masses

CMSSM

NUHM1

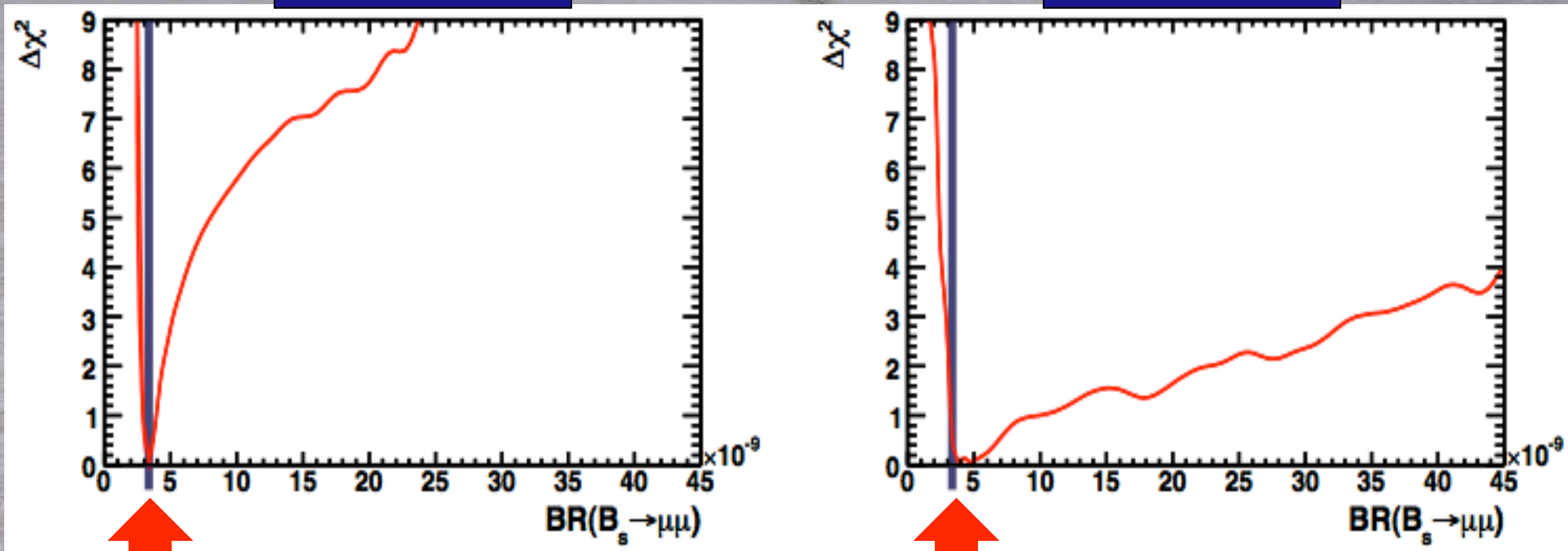


O.Buchmueller, JE et al: arXiv:0907.5568

Likelihood Function for $B_s \rightarrow \mu^+\mu^-$

CMSSM

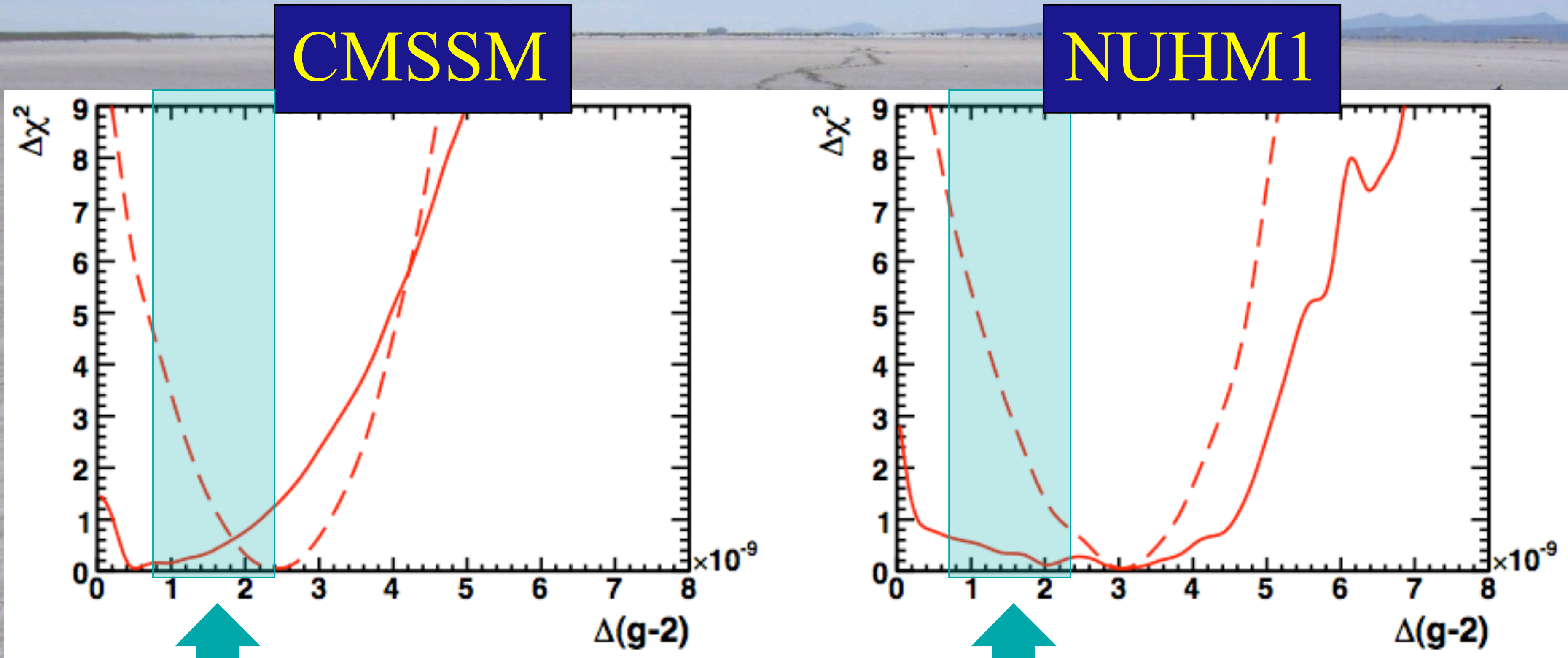
NUHM1



Standard Model prediction

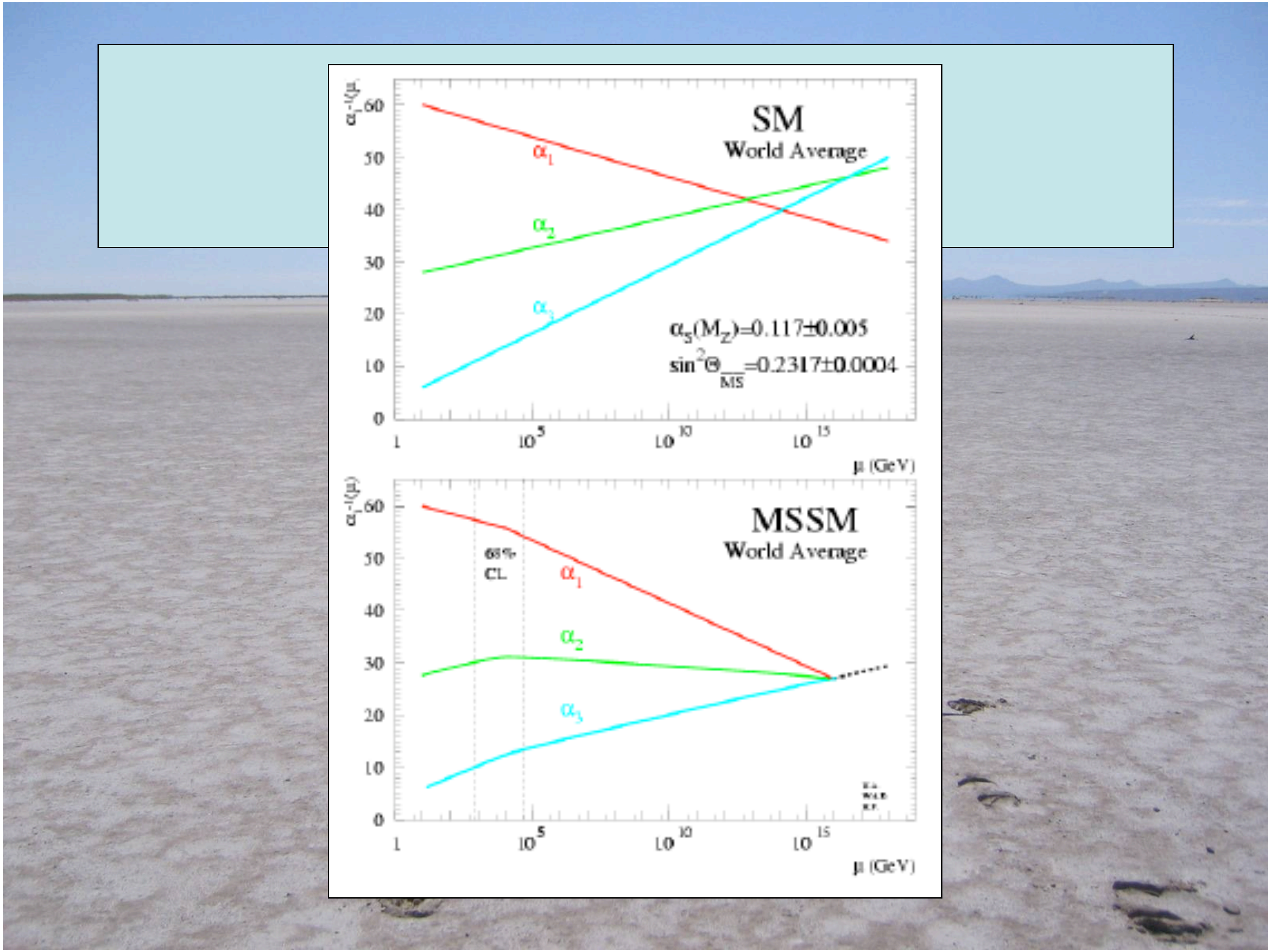
O.Buchmueller, JE et al: arXiv:0907.5568

Likelihood Function for g_μ- 2



Standard Model calculation using τ decay data

O.Buchmueller, JE et al: arXiv:0907.5568



SM
World Average
α_1
α_2
α_3
$\alpha_S(M_Z)$=0.117±0.005
$\sin^2\Theta_{\overline{MS}}$=0.2317±0.0004
$\alpha_i^{-1}(\mu)$
0 10 20 30 40 50 60
1 10^5 10^{10} 10^{15}
μ (GeV)
MSSM
World Average
68%
CL
α_1
α_2
α_3
$\alpha_i^{-1}(\mu)$
0 10 20 30 40 50 60
1 10^5 10^{10} 10^{15}
μ (GeV)

How to Grand Unify?

- Exploit logarithmic evolution of gauge couplings:

$$\frac{dg_a^2}{dt} = b_a \frac{g_a^4}{16\pi^2} + \ldots \quad \to \quad \frac{m_{GUT}}{m_W} = \exp\left(\mathcal{O}\left(\frac{1}{\alpha_{em}}\right)\right)$$

- Combination measurable at low energies:

$$\sin^2\theta_W(m_Z) = \frac{g'^2}{g_2^2 + g'^2} = \frac{3}{5}\frac{g_1^2(m_Z)}{g_2^2(m_Z) + \frac{3}{5}g_1^2(m_Z)} = \frac{1}{1+8x}[3x + \frac{\alpha_{em}(m_Z)}{\alpha_3(m_Z)}] \quad x \equiv \frac{1}{5}(\frac{b_2 - b_3}{b_1 - b_2})$$

- Values in SM and MSSM:

$$\begin{array}{rcccl}
\frac{4}{3}N_G - 11 & \leftarrow & b_3 & \rightarrow & 2N_G - 9 = -3 \\
\frac{1}{6}N_H + \frac{4}{3}N_G - \frac{22}{3} & \leftarrow & b_2 & \rightarrow & \frac{1}{2}N_H + 2N_G - 6 = +1 \\
\frac{1}{10}N_H + \frac{4}{3}N_G & \leftarrow & b_1 & \rightarrow & \frac{3}{10}N_H + 2N_G = \frac{33}{5} \\
\frac{23}{218} = 0.1055 & \leftarrow & x & \rightarrow & \frac{1}{7}\,.
\end{array}$$

- Experiment:

$$\alpha_{em} = \frac{1}{128};\ \alpha_3(m_Z) = 0.119 \pm 0.003,\ \sin^2\theta_W(m_Z) = 0.2315 \quad \to \quad \boxed{x = \frac{1}{6.92 \pm 0.07}}$$

MSSM Calculation

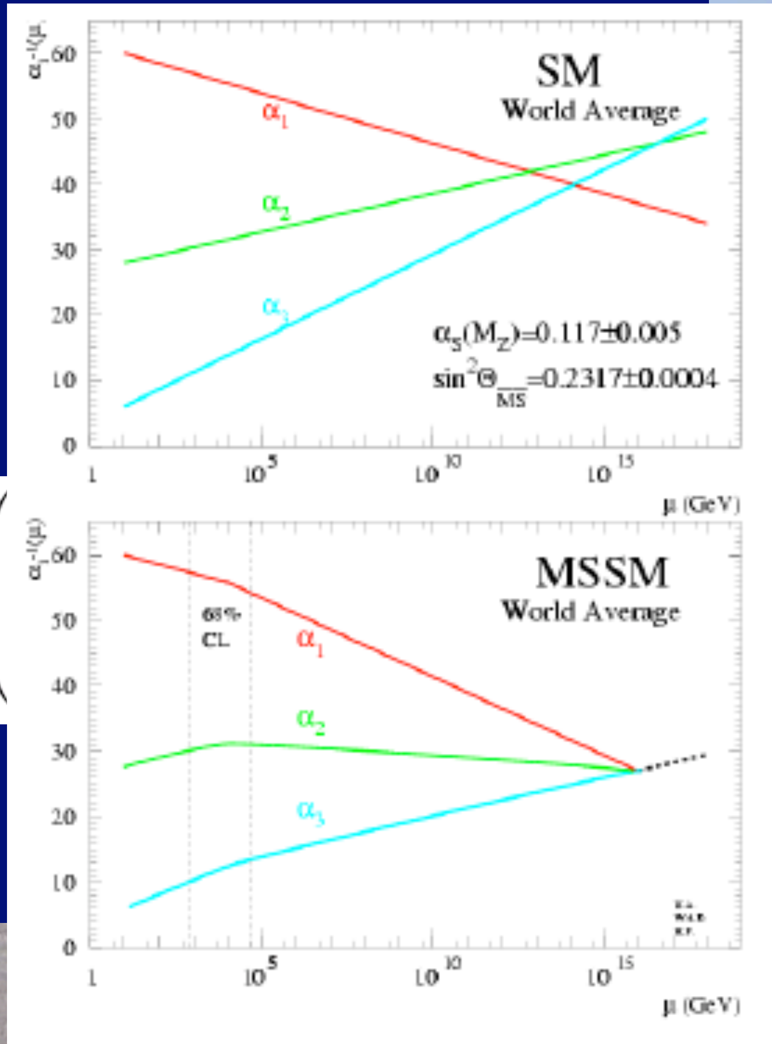
- At one loop:

$$b_i = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_g \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_H \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix}$$

- Two loops:

$$b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix} + N_g \begin{pmatrix} \frac{38}{15} & \frac{6}{5} & \frac{88}{15} \\ \frac{2}{5} & 14 & 8 \\ \frac{11}{5} & 3 & \frac{68}{3} \end{pmatrix} + N_H \Bigg($$

- Results are stable



Choice of GUT Group

- Should accommodate the known fermions:

$$(\nu, e)_L \in (1,2)\ ,\quad (u,d)_L \in (3,2)\ ,\quad e^c_L \in (1,1)\ ,\quad u^c_L\ ,\quad d^c_L \in (\bar{3},1)$$

- Need group with complex representations
- Preferably irreducible: $\sum_{q,\ell} Q_i = 3Q_u + 3Q_d + Q_e = 0$
- List of candidate groups of rank 4:

$$Sp(8)\ ,\quad SO(8)\ ,\quad SO(9)\ ,\quad F_4\ ,\quad SU(3)\times SU(3)\ ,\quad SU(5)$$

- BUT: real, real, real, real, $\Sigma_q Q_q \neq 0$, OK!

Particles in SU(5)

- Gauge bosons: $$\left(\begin{array}{ccc:cc} & & & \bar{X} & \bar{Y} \\ & g_{1,\ldots,8} & & \bar{X} & \bar{Y} \\ & & & \bar{X} & \bar{Y} \\ \hdashline X & X & X & & \\ & & & & W_{1,2,3} \\ Y & Y & Y & & \end{array}\right)$$

- Matter particles: $\underline{\bar{5}} = (\bar{3},1) + (1,2)\ ,\quad \underline{10} = (3,2) + (\bar{3},1) + (1,2)$

$$\bar{F} = \begin{pmatrix} d^c_R \\ d^c_Y \\ d^c_B \\ \cdots \\ -e^- \\ \nu_e \end{pmatrix}_L \ , \quad T = \left(\begin{array}{ccc:cc} 0 & u^c_B & -u^c_Y & -u_R & -d_R \\ -u^c_B & 0 & u^c_R & -u_Y & -d_Y \\ u^c_Y & -u^c_R & 0 & -u_B & -d_B \\ \hdashline u_R & u_Y & u_B & 0 & -e^c \\ d_R & d_Y & d_B & e^c & 0 \end{array}\right)_L$$

Higgs bosons in SU(5) GUT

- Adjoint 24-dimensional Higgs to break

  SU(5) → SU(3) x SU(2) x U(1) of SM

$$< 0|\Phi|0 >= \begin{pmatrix} 1 & 0 & 0 & \vdots & 0 & 0 \\ 0 & 1 & 0 & \vdots & 0 & 0 \\ 0 & 0 & 1 & \vdots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \vdots & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & \vdots & 0 & -\frac{3}{2} \end{pmatrix} \times \mathcal{O}(m_{GUT})$$

- 5-dimensional Higgs to break SU(2) x U(1)→ U(1)

$$< 0|\phi|0 >= (0,0,0,0,1) \times 0(m_W)$$

- Susy needed to prevent large GUT v.e.v. from leaking → small electroweak Higgs v.e.v.

Particle Masses in SU(5)

- Quarks and leptons in same GUT multiplet → relations between their masses
- Simple symmetry relations before renormalization

  e.g., $m_b = m_\tau$ in minimal SU(5) GUT
- Renormalized analogously to gauge couplings:

  non-susy case $\frac{m_b}{m_\tau} \simeq \left[\ln\left(\frac{m_b^2}{m_X^2}\right)\right]^{\frac{12}{33-2N_q}}$
- m_τ = 1.78 GeV used to predict m_b ~ 5 GeV a few weeks before its discovery!
- Different formula, similar number in susy SU(5)

Bigger GUT Models

- First look at groups of rank 5 with suitable complex representations
- Only suitable candidate is SO(10)
- Each generation in irreducible

$$16 = 10 + 5^* + 1 \text{ of } SU(5)$$

- Next step is rank 6: E_6 has suitable complex

$$27 = 16 + 10 + 1 \text{ of } SO(10)$$

Appears in String theory

Suitable for right-handed neutrino

New Interactions make Baryons Decay

- Exchanges of new X, Y bosons:

$$\left(\epsilon_{ijk} u_{R_k} \gamma_\mu u_{L_j}\right) \quad \frac{g_X^2}{8 m_X^2} \quad \left(2 e_R \ \gamma^\mu \ d_{L_i} + e_L \ \gamma^\mu \ d_{R_i}\right)$$

$$\left(\epsilon_{ijk} u_{R_k} \gamma_\mu d_{L_j}\right) \quad \frac{g_Y^2}{8 m_X^2} \quad \left(\nu_L \ \gamma^\mu \ d_{R_i}\right), \qquad G_X \equiv \frac{g_X^2}{8 m_X^2} \simeq G_Y \equiv \frac{g_Y^2}{8 m_Y^2}.$$

- Proton decay rate $\Gamma_B = c G_X^2 m_p^5$ lifetime: $\tau_p = \frac{1}{c} \frac{m_X^4}{m_p^5}$
- Preferred modes: $p \to e^+\pi^0 \ , \quad e^+\omega \ , \quad \bar{\nu}\pi^+ \ , \quad \mu^+ K^0 \ , \quad \ldots$
  $n \to e^+\pi^- \ , \quad e^+\rho^- \ , \quad \bar{\nu}\pi^0 \ , \quad \ldots$
- Estimate of X, Y masses: $m_X \simeq (1 \text{ to } 2) \times 10^{15} \times \Lambda_{QCD}$
- Lifetime too short:

$$\tau(p \to e^+\pi^0) \simeq 2 \times 10^{31\pm1} \times \left(\frac{\Lambda_{QCD}}{400 \text{ MeV}}\right)^4 \ y$$

exp't: $\boxed{\tau(p \to e^+\pi^0) > 1.6 \times 10^{33} \ y}$

Proton Decay in Supersymmetric SU(5)

- Increase in GUT scale:

$$m_X \simeq 10^{16}\ \mathrm{GeV}$$

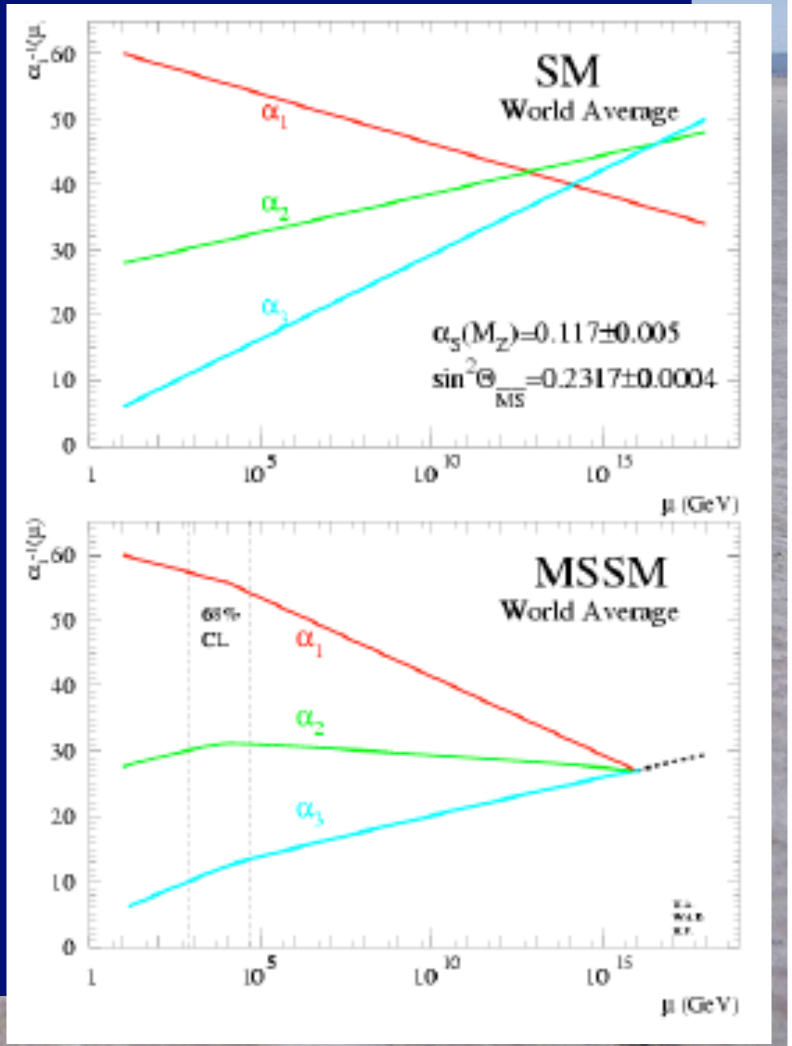
- X, Y exchanges OK
- Beware GUT Higgsinos:

$$G_X \to \mathcal{O}\left(\frac{\lambda^2 g^2}{16\pi^2}\right)\ \frac{1}{m_{\tilde{H}_3}\tilde{m}}$$

- Preferred decay modes:

$$p \to \bar{\nu}K^+\ ,\quad n \to \bar{\nu}K^0\ ,\quad \ldots$$

- Lifetime too short?
- Suppressed in some models



Neutrino Masses and Mixing

Why? Why not?

- There is no sacred symmetry to forbid m_ν
- The only sacred symmetries are EXACT gauge symmetries, e.g.,

  Q_{em} conserved

  $\leftrightarrow$ massless photon

  $\leftrightarrow$ U(1) gauge symmetry of SM
- No candidate gauge symmetry to forbid m_ν
- No massless gauge boson coupled to lepton # L
- Expect $m_\nu \neq 0$ in extensions of SM: GUTs, string

Models for Neutrino Masses

- Could be generated in Standard Model: using non-renormalizable interaction:
- $\frac{1}{M}\nu H \cdot \nu H \quad \rightarrow \quad m_\nu \nu \cdot \nu : \; m_\nu = \frac{\langle 0|H|0\rangle^2}{M}$
- Probably effective interaction due to exchange of massive fermion N = 'right-handed ν'
- Should then consider seesaw mass matrix:

$$(\nu_L, N)\begin{pmatrix} 0 & M_D \\ M_D^T & M \end{pmatrix}\begin{pmatrix} \nu_L \\ N \end{pmatrix}$$

- Does not need GUT, but M ~ $10^{10} – 10^{15}$ GeV
- Add singlet N to SU(5)? automatic in SO(10)

Bigger GUT Models

- First look at groups of rank 5 with suitable complex representations
- Only suitable candidate is SO(10)
- Each generation in irreducible representation

$$16 = 10 + 5^* + 1 \text{ of } SU(5)$$

- Next step is rank 6: E_6 has suitable complex

$$27 = 16 + 10 + 1 \text{ of } SO(10)$$

Suitable for
right-handed neutrino

Quantum Gravity & String

Some String History

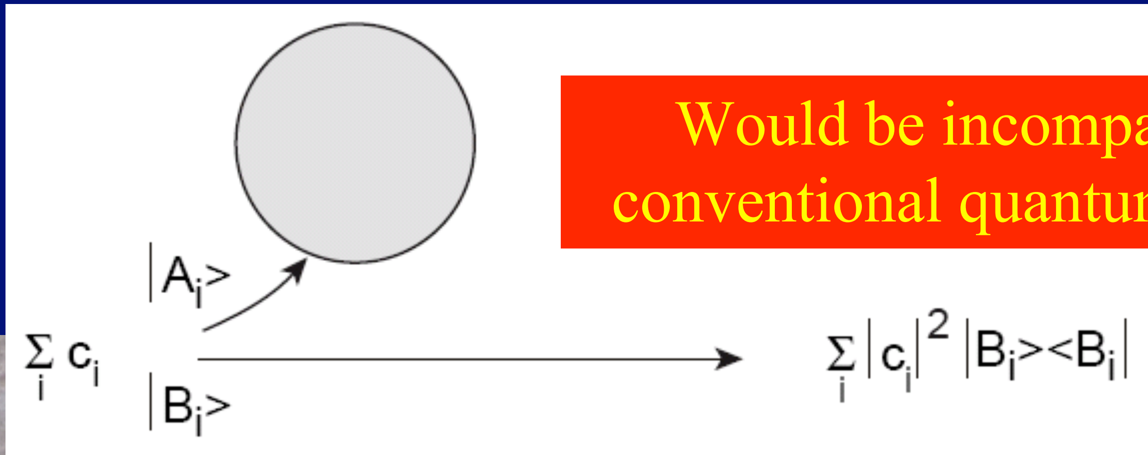
- **1969:** Discovery of string theory (Veneziano)
- **1971:** Superstring (Neveu + Scherk, Ramond)
- **1974:** Possible quantum theory of gravity (Scherk + Schwarz, Yoneya)
- **1984:** Superstrings free of anomalies (Green + Schwarz)
- **≥ 1985:** Realistic string models (many)
- **1990:** Large extra dimension? (Antoniadis)

Problems of Quantum Gravity

- Gravity grows with energy: $\sigma_G \sim E^2/m_P^4$
- Two-graviton exchange is infinite:

$$\int^{\Lambda \to \infty} d^4k \left(\frac{1}{k^2}\right) \leftrightarrow \int_{1/\Lambda \to 0} d^4x \left(\frac{1}{x^6}\right) \sim \Lambda^2 \to \infty$$

- **Gravity is a non-renormalizable theory**
- Pure states evolve to mixed states?



Would be incompatible with conventional quantum mechanics

String Theory

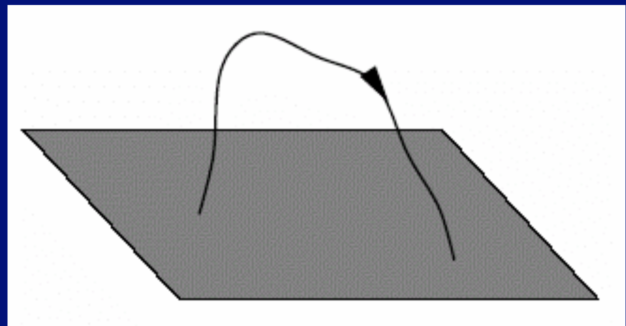
- Point-like particles → extended objects
- Simplest possibility: lengths of string
  - Open and/or closed
- Quantum consistency fixes # dimensions:
  - Bosonic string: 26, superstring: 10
- Must compactify extra dimensions, scale ~ $1/m_P$?
- Perturbative string unification scale:

$$M_{GUT} = O(g) \times \frac{m_P}{\sqrt{8\pi}} \simeq \text{few} \times 10^{17}\,\text{GeV}$$

Close to GUT scale, but larger?

Non-Perturbative String = M Theory

- Solitonic 'lumps' = balls of string: $m \propto \frac{1}{g_s}$
- Appear with various dimensions: 'D-branes'

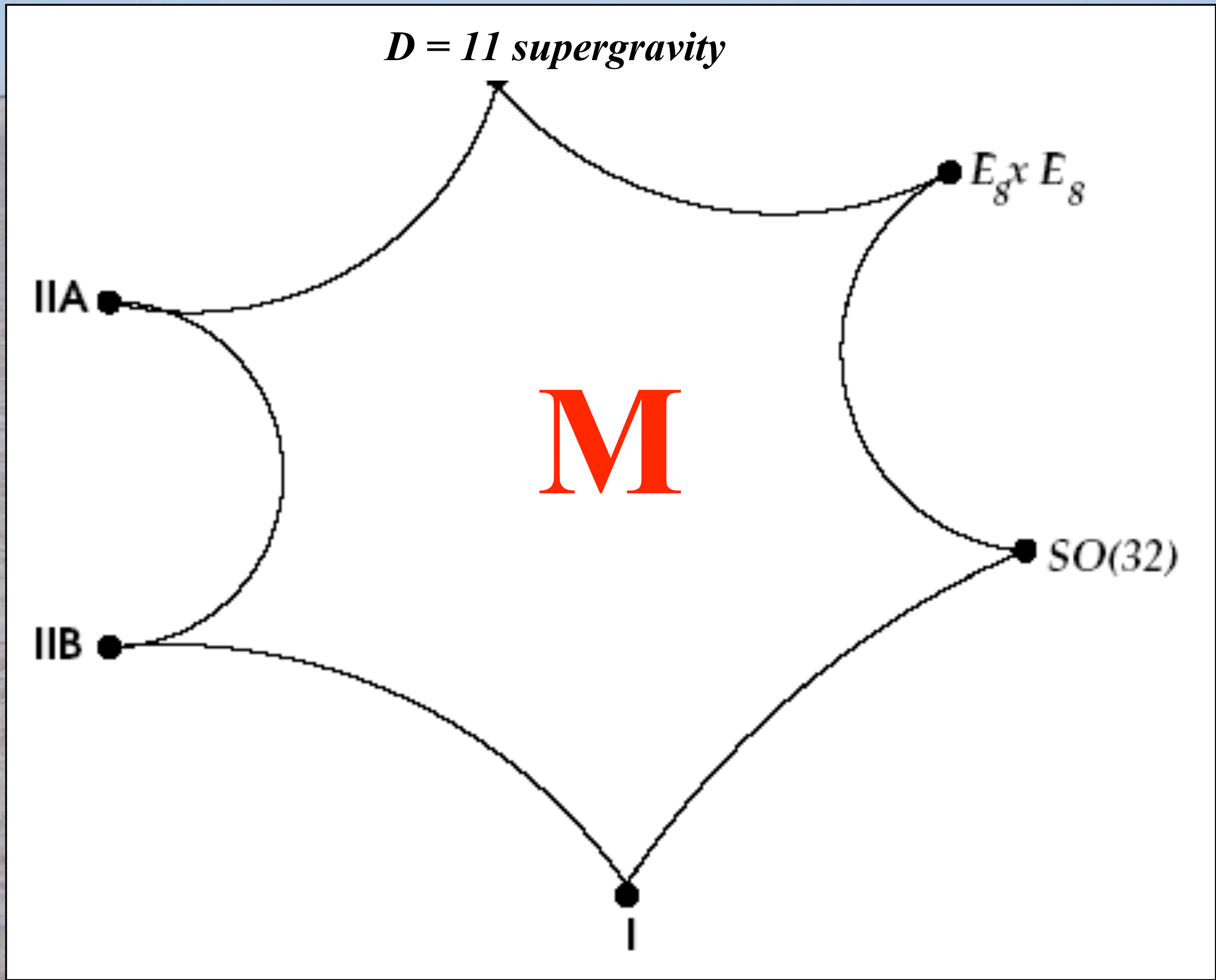


- Can regard string coupling as extra 'dimension'

  11-dimensional M theory

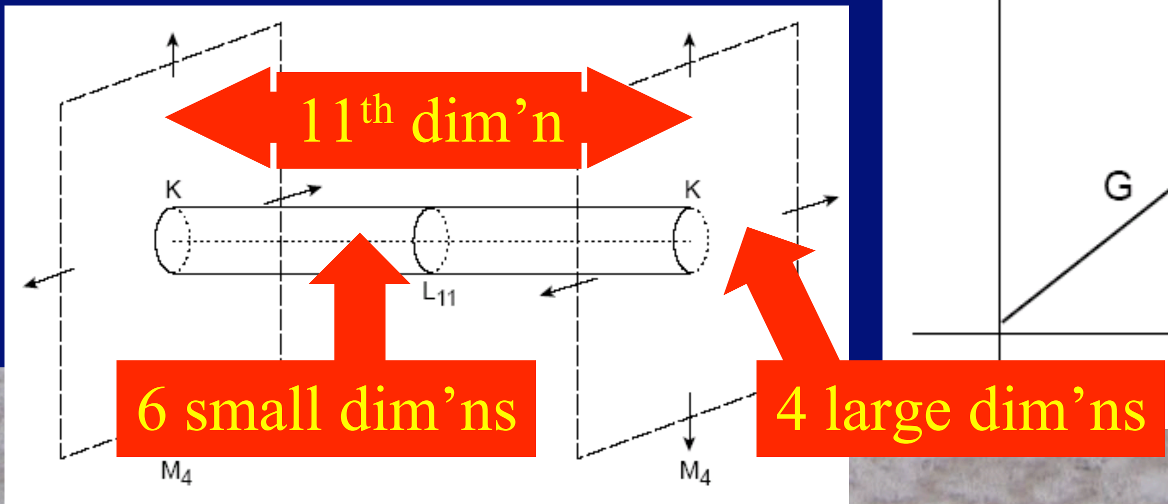
- Includes different string models in various limits
- New ways to get extra gauge symmetries

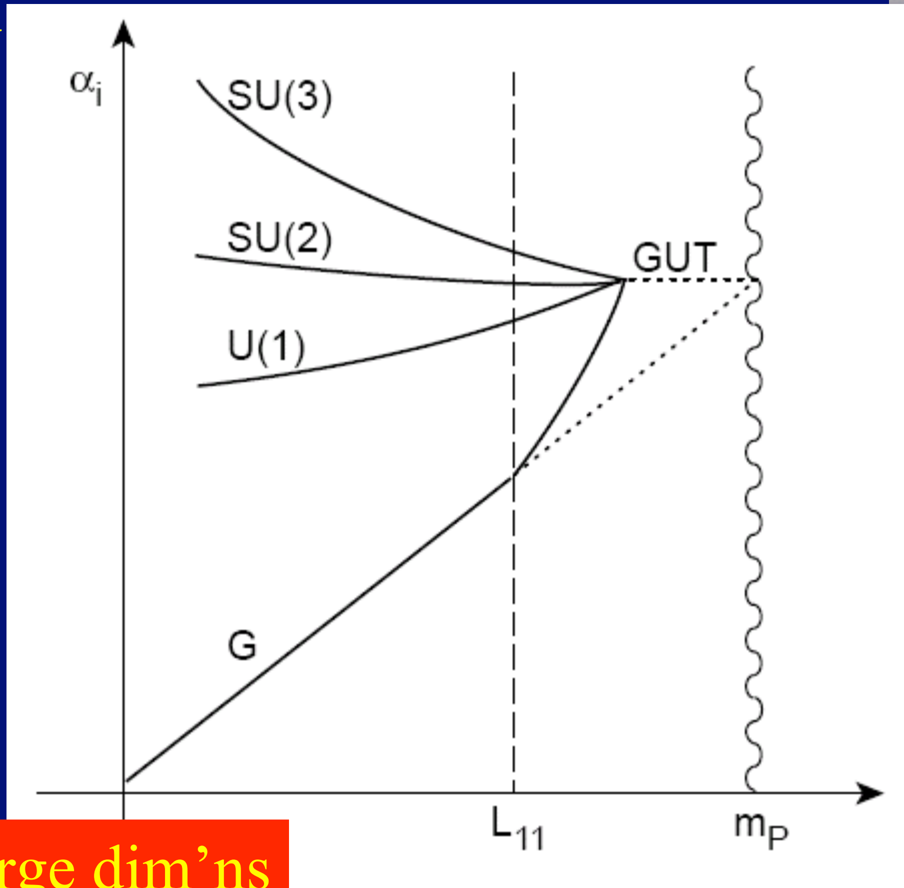
All Different String Theories are Related

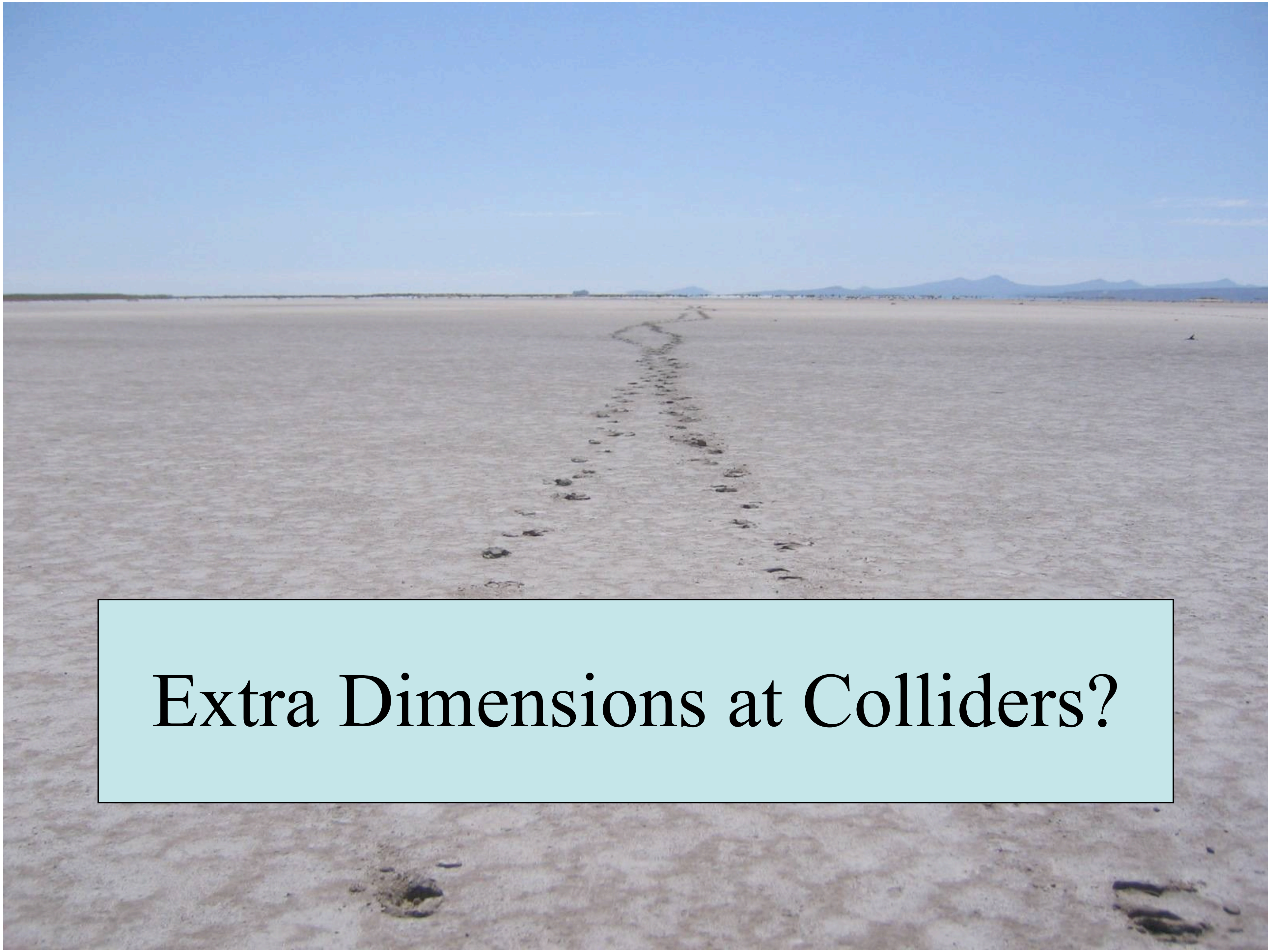


Scenario for String Unification

- E\If extra dimension below GUT scale: gravity grows faster with energy
- Unify at 10^{16} GeV?
- E.g., in M theory with large 11^{th} dimension







Extra Dimensions at Colliders?

How large could extra Dimensions be?

- 1/TeV?

  could break supersymmetry, electroweak

- micron?

  can rewrite hierarchy problem

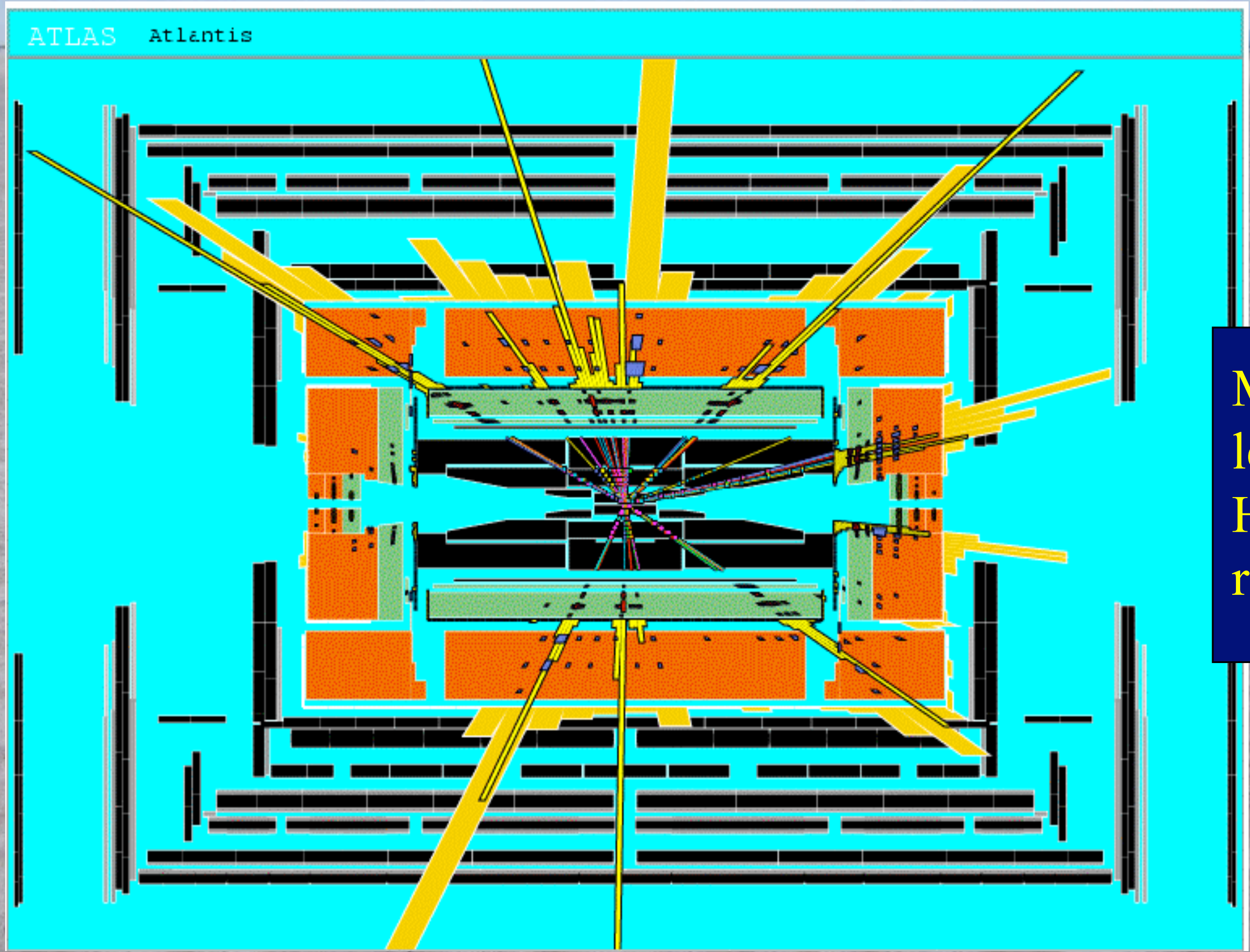
- Infinite?

  warped compactifications

- **Look for black holes, Kaluza-Klein excitations @ colliders?**

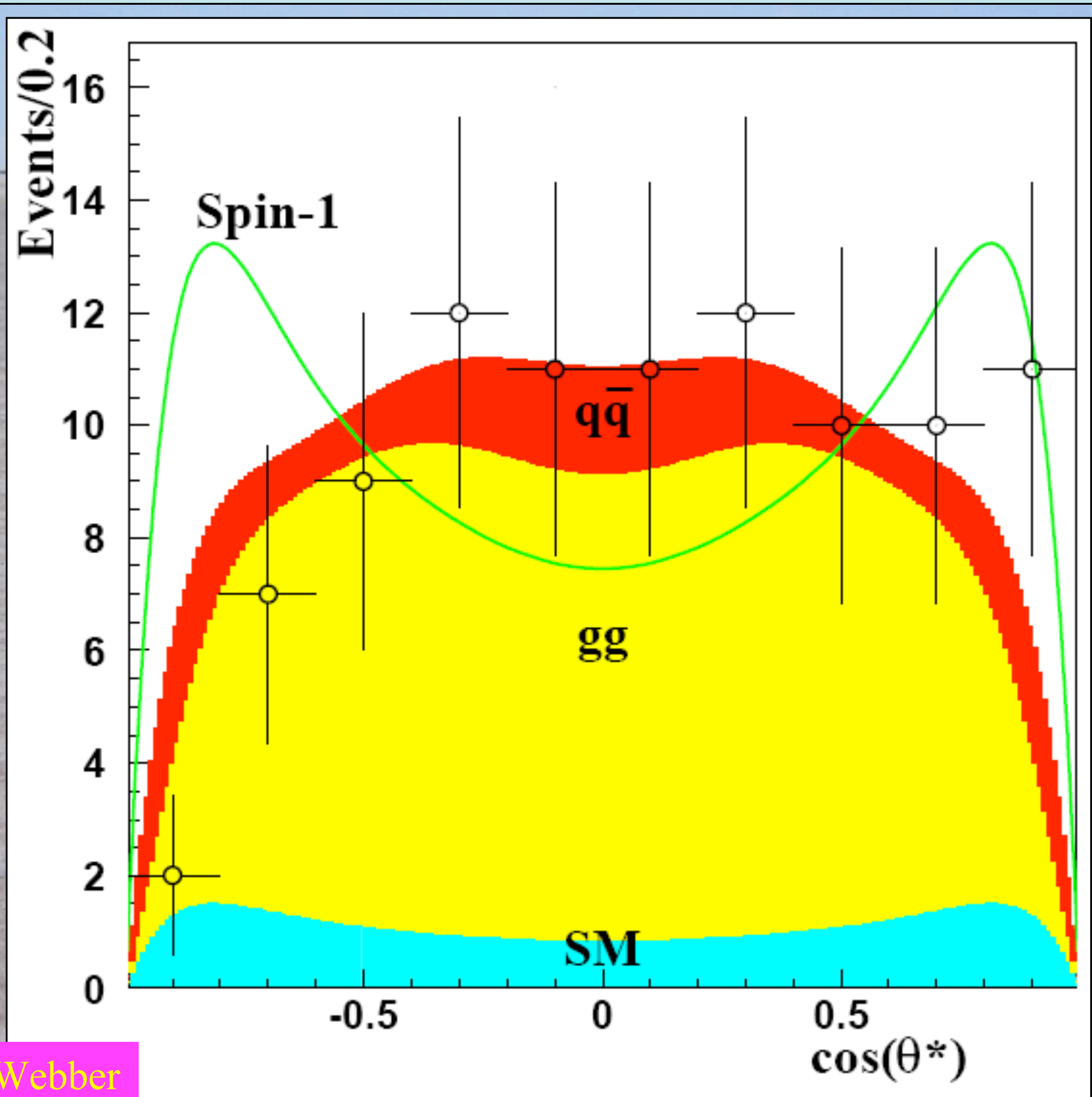
And if gravity becomes strong at the TeV scale …

Black Hole Production at LHC?



Multiple jets, leptons from Hawking radiation

Identifying Graviton Resonance @ LHC



Cambridge: al et Webber

Summary

- The origin of mass is the most pressing in particle physics
- Needs a solution at energy < 1 TeV

  Higgs? Supersymmetry? Extra Dimensions?

  LHC will tell!
- Lots of speculative ideas for other physics beyond the Standard Model

  Grand unification, strings, branes, …
- Hints provided by neutrinos

  How else can one test these speculations?