Towards LHC Phenomenology beyond Leading Order

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IP³

Science & Technology Facilities Council

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The LHC . . .

. . . has been planned long time ago . . .

Linear Colliders also seem to have been supported . . .

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. . . because instead of hunting buffaloes, we are now hunting Higgs bosons . . .

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process	events/sec	
QCD jets $E_T > 150$ GeV	100	background
$W \to e\nu$	15	background
$t\bar{t}$	1	background
Higgs, $m_H \sim 130$ GeV	0.02	signal
gluinos, $m \sim 1$ TeV	0.001	signal

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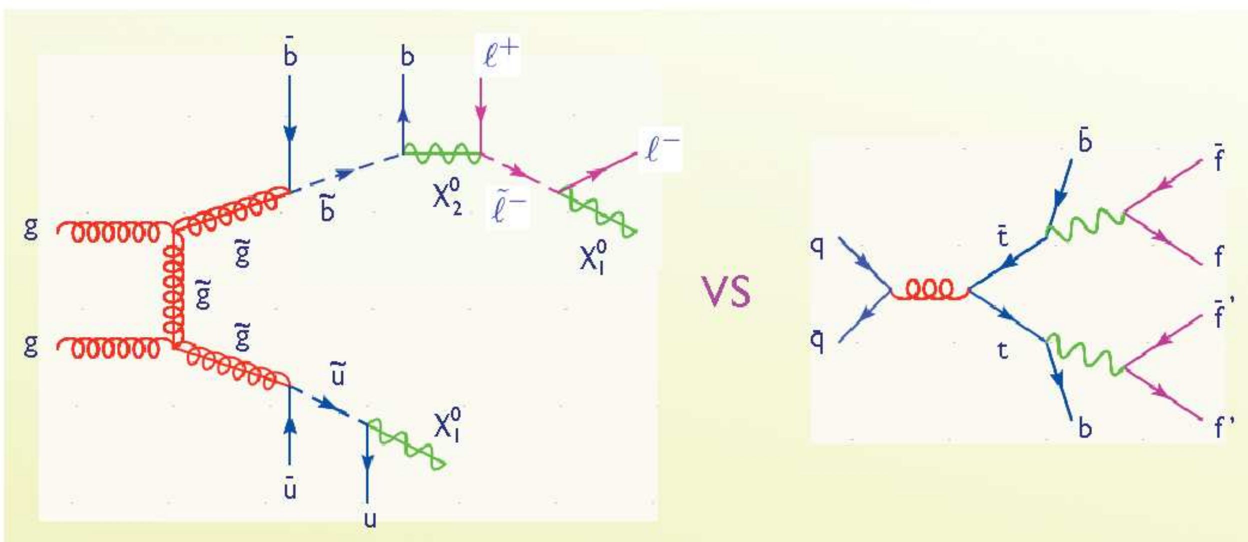
- "model discrimination phase":
  have to measure particle masses, widths, couplings, CP properties, . . .

  do we control the SM/MSSM predictions well enough?

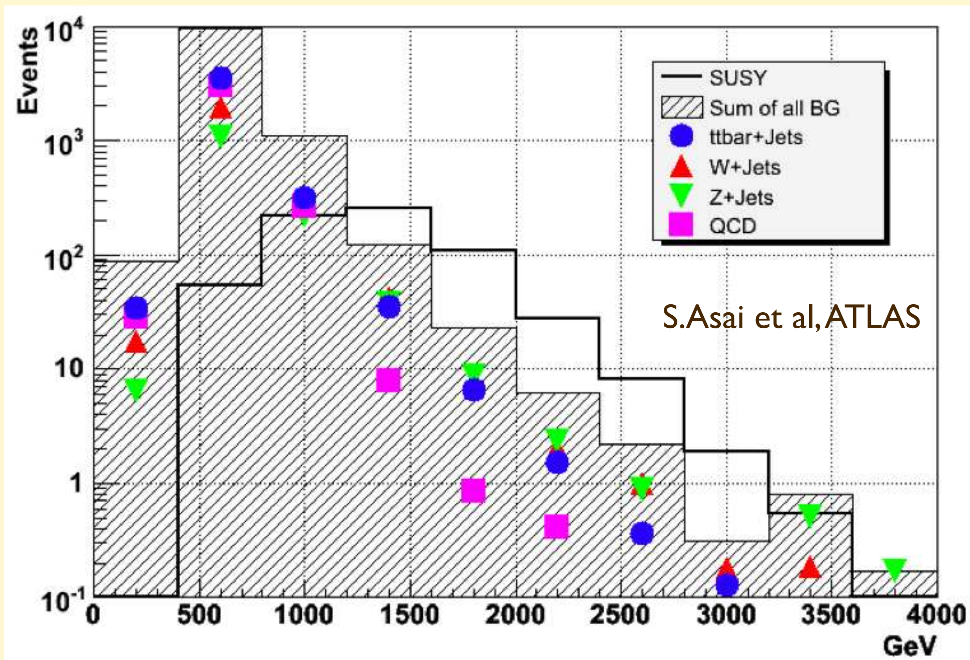
(heavy) SUSY particles:

- decay through cascades emitting quarks and leptons
- signatures: energetic jets and leptons, missing E_T
- QCD radiation generates additional hard jets



F. Maltoni

Overall result, after the complete detector simulation, etc....



19

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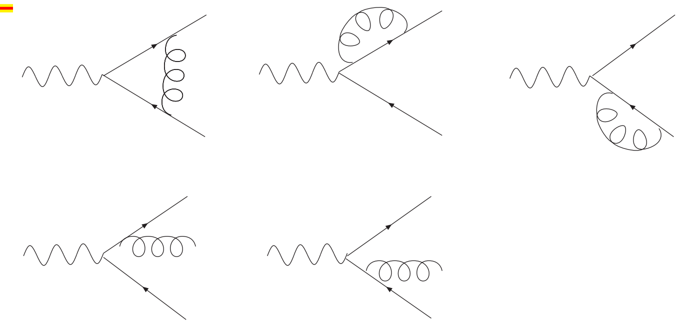
- now paradigm change:
  we are moving towards automated NLO tools

ingredients for m-particle observable at NLO



virtual part (one-loop integrals):

$$\mathcal{A}^V_{NLO} = A_2/\epsilon^2 + A_1/\epsilon + A_0$$

$$d\sigma^V \sim Re\left(\mathcal{A}^\dagger_{LO}\,\mathcal{A}^V_{NLO}\right)$$

real radiation part: soft/collinear emission of massless particles

$\Rightarrow$ need subtraction terms

$$\Rightarrow \int_{\text{sing}} d\sigma^S = -A_2/\epsilon^2 - A_1/\epsilon + B_0$$

$$\sigma^{NLO} = \underbrace{\int_{m+1}\left[d\sigma^R - d\sigma^S\right]_{\epsilon=0}}_{\text{numerically}} + \underbrace{\int_m \left[\underbrace{d\sigma^V}_{\text{cancel poles}} + \underbrace{\int_{\text{s}} d\sigma^S}_{\text{analytically}}\right]_{\epsilon=0}}_{\text{numerically}}$$

Modular structure

Tree Modules		One-Loop Module		IR Modules
$\lvert\mathcal{A}^{LO}\rvert^2$	$\oplus$	$2\,Re(\mathcal{A}^{LO\,\dagger}\mathcal{A}^{NLO,V})$	$\oplus$	$\sum_j \int_j \mathcal{S}_j$
$\lvert\mathcal{A}^{NLO,R}\rvert^2$			$\ominus$	$\sum_j \mathcal{S}_j$

↑

has been bottleneck in past years

progress

- more efficient techniques to calculate loop amplitudes
  - unitarity-based methods

    e.g. BlackHat, Rocket, CutTools, analytic, . . .
  - improved methods based on Feynman diagrams

    e.g. GOLEM, Denner et. al, . . .

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  LO tools can provide:
  - event generation
  - phase space integration
  - histogramming tools
  - subtraction terms for soft/collinear radiation

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  LO tools can provide:
  - event generation
  - phase space integration
  - histogramming tools
  - subtraction terms for soft/collinear radiation
- matching NLO amplitudes with parton showers
  collaboration with F. Krauss, F. Siegert, M. Schönherr

2009 status of NLO wishlist for LHC

$pp \to W\,W$ jet	Denner/Dittmaier/Kallweit/Uwer, Ellis/Campbell/Zanderighi
$pp \to Z\,Z$ jet	Binoth/Guillet/Karg/Kauer/Sanguinetti
$pp \to t\bar{t}\,b\bar{b}$	Bredenstein/Denner/Dittmaier/Pozzorini
$pp \to t\bar{t} + 2$ jets	
$pp \to Z\,Z\,Z$	Lazopoulos/Melnikov/Petriello, Hankele/Zeppenfeld
$pp \to V\,V\,V$	Binoth/Ossola/Papadopoulos/Pittau, Zeppenfeld et al.
$pp \to V\,V\,b\bar{b}$	
$pp \to V\,V + 2$ jets	VBF: Bozzi/Jäger/Oleari/Zeppenfeld, VBFNLO coll.
$pp \to W + 3$ jets	BlackHat coll.; Ellis/Giele/Kunszt/Melnikov/Zanderighi*
$pp \to b\bar{b}b\bar{b}$	Binoth/Guffanti/Guillet/Reiter/Reuter
$pp \to t\,\bar{t}$ jet	Dittmaier/Uwer/Weinzierl
$pp \to t\,\bar{t}\,Z$	Lazopoulos/McElmurry/Melnikov/Petriello
$pp \to b\,\bar{b}\,Z\,,\,b\,\bar{b}\,W$	Febres Cordero/Reina/Wackeroth

• done • partial results * leading colour only

Interface

details worked out at Les Houches 2009 workshop on TeV colliders

Monte Carlo tool (MC) | One-Loop-Provider (OLP)

initialisation:

process info
CH summed

model parameters
fix scheme

$\cdots$

order →

← contract

copy/confirm

runtime:

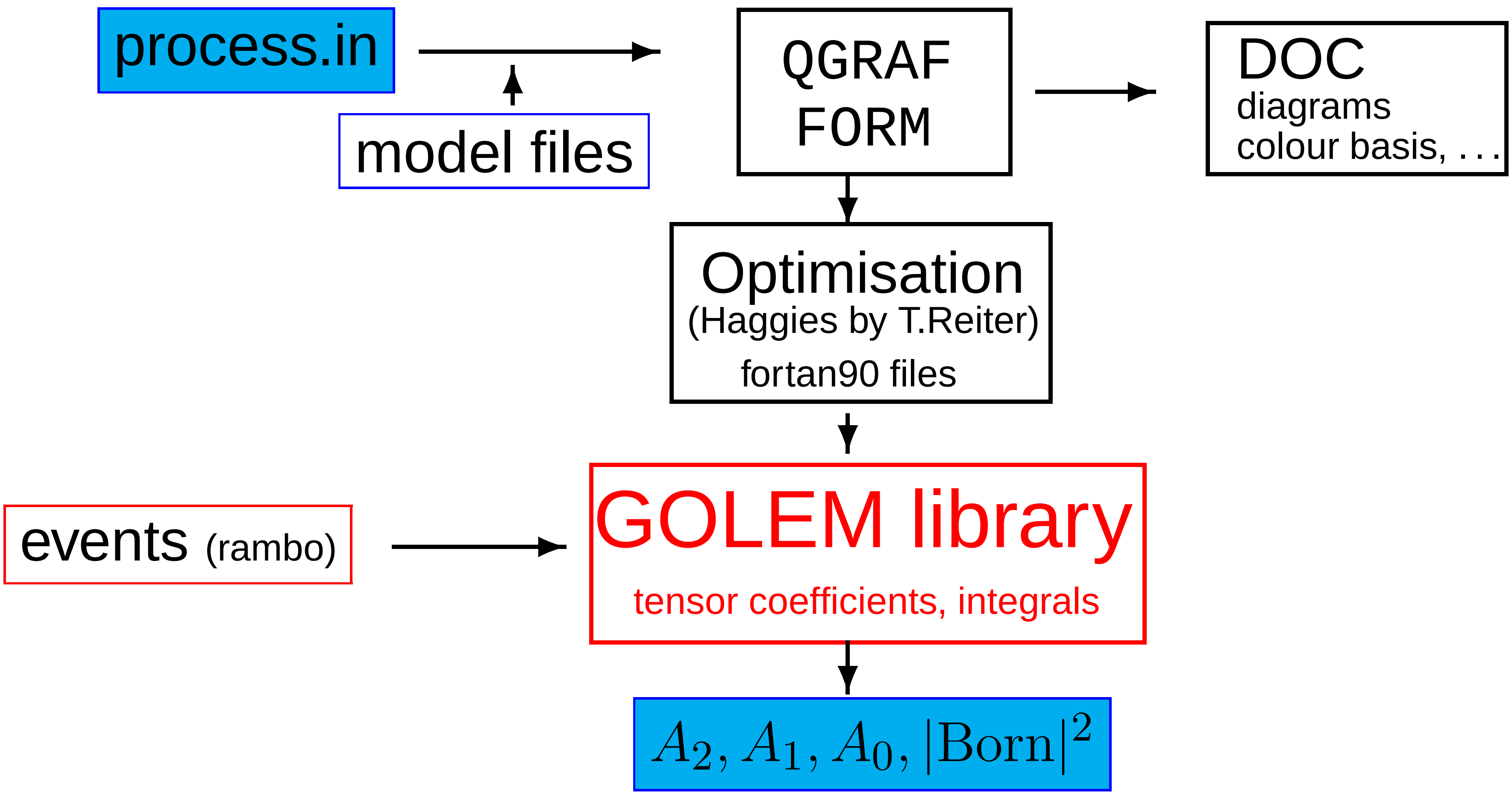
events ⇄ $A_2, A_1, A_0, |Born|^2$

standard interface

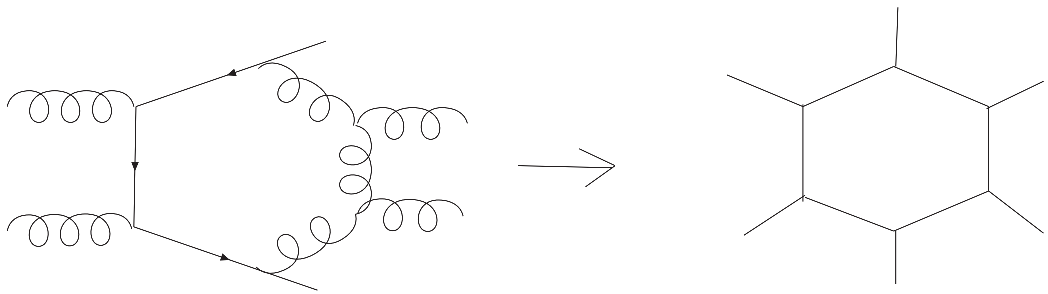
GOLEM

General One-Loop Evaluator of Matrix elements

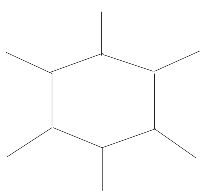
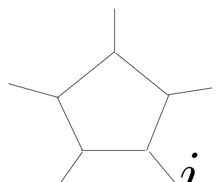
[Binoth, Cullen, Guillet, GH, Karg, Kauer, Pilon, Reiter, Rodgers, Wigmore]



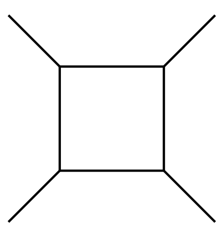
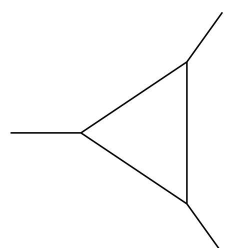
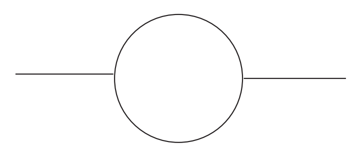
reduction to basis integrals



non-trivial tensor structure → scalar 6-point function + integrals with less legs


$= \sum_{i=1}^{6} b_i$

i . . . factorial growth in complexity !

reduction to set of basis integrals (4-, 3- and 2-point funcs.)

$\mathcal{A} = C_4$  $+ \, C_3$  $+ \, C_2$  $+ \, \mathcal{R}$

Golem strong points

- can deal with an arbitrary number of mass scales
  link LoopTools for finite massive boxes
- colour does not add additional complexity
- rational parts are "for free"
- efficient use of recursive structure
  caching system
- projection onto helicity states
  exploit spinor helicity techniques, gauge cancellations, smaller building blocks
- collaboration has several independent programs
  $\Rightarrow$ strong checks
  alternatively based on FeynArts diagram generator,
  different levels of analytic simplifications
- can avoid spurious singularites from
  inverse Gram determinants

Gram determinants

- reduction $N \geq 5 \rightarrow N = 4$: inverse Gram determinants completely absent
- reduction of $N \leq 4$ tensor integrals: introduces spurious 1/det(G)

$$I_4^{n+2}(j) = \frac{1}{B} \left\{ b_j \, I_4^{n+2} + \frac{1}{2} \sum_k \mathcal{S}_{jk}^{-1} \, I_3^{n,\{k\}} - \frac{1}{2} \sum_{k \in S \setminus \{j\}} b_k \, I_3^{n,\{k\}}(j) \right\}$$

$$I_4^{n+2}(j_1, j_2) \sim \frac{1}{B^2} \; , \; I_4^{n+2}(j_1, j_2, j_3) \sim \frac{1}{B^3} \; \ldots$$

$$B = \det(G)/\det(\mathcal{S})\,(-1)^{N+1}$$

$$\mathcal{S}_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2 \quad ; \quad G_{ij} = 2\, r_i \cdot r_j$$

Gram determinants

solution:

do not reduce if B is small

Gram determinants

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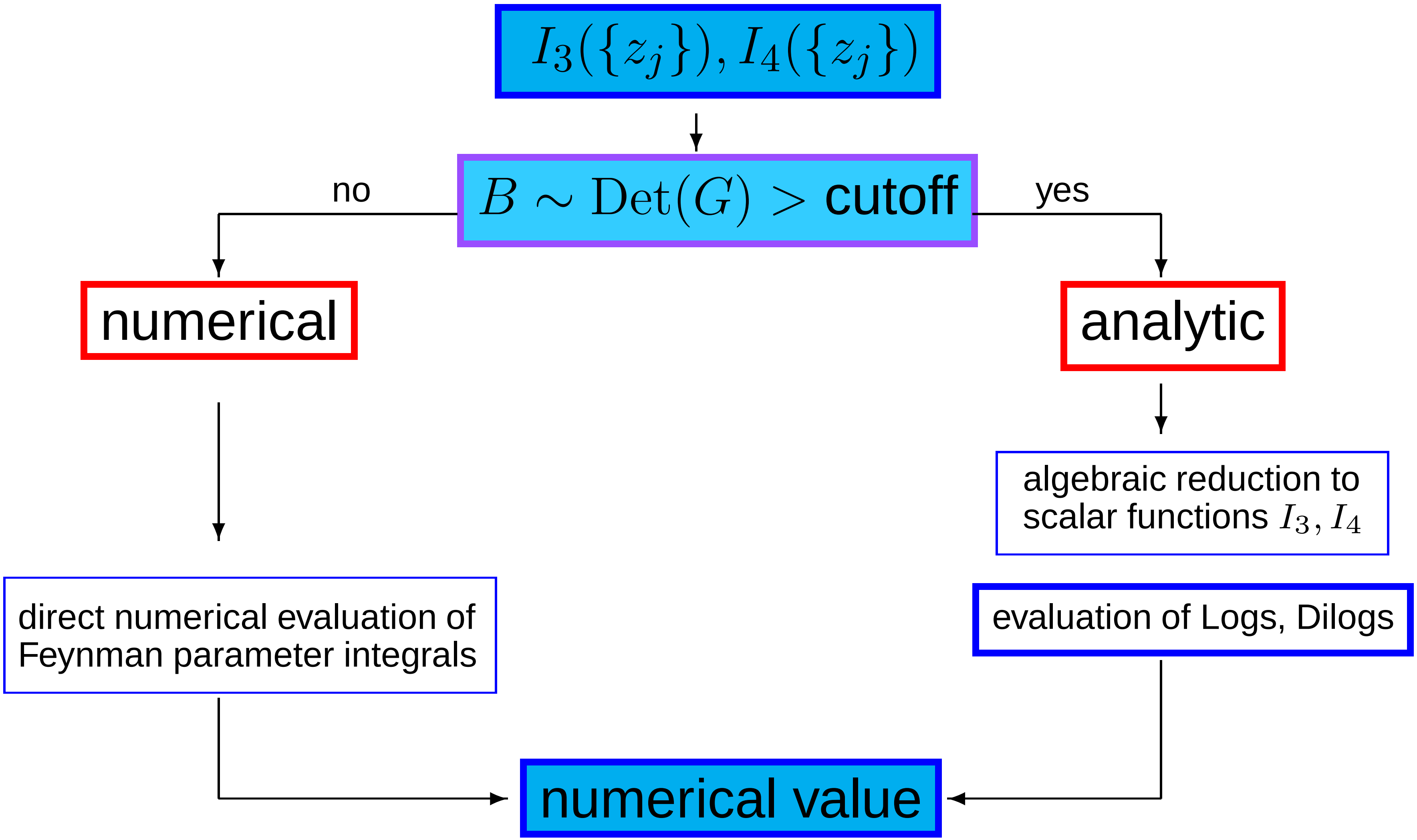
- calculate integral numerically
- special feature:

  1-dimensional integral representation

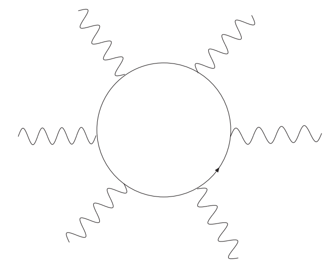
  $\Rightarrow$ numerical integration fast

treatment of 3-and 4-point integrals

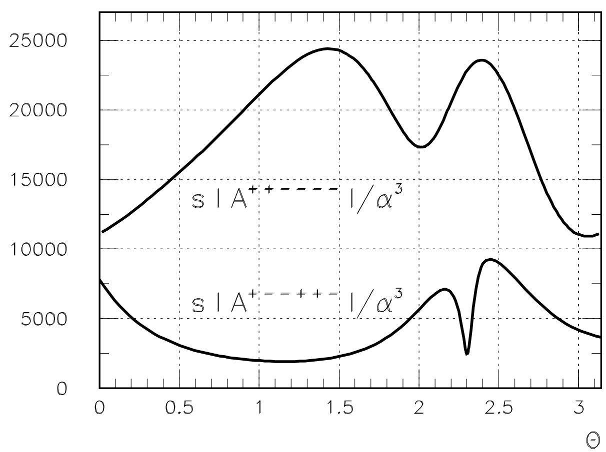


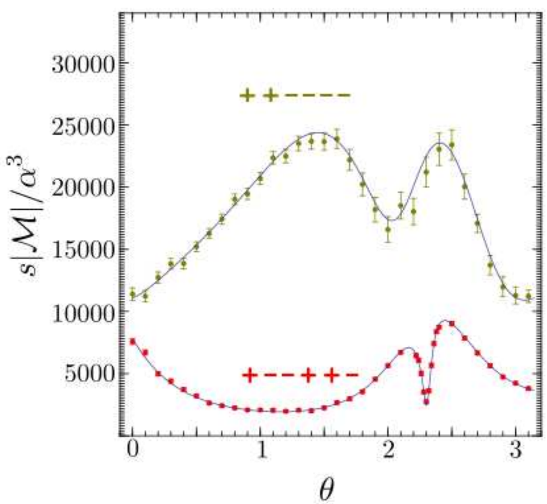
example six-photon amplitude

[Mahlon 94] (special helicity configurations only)
[Nagy, Soper 06; Gong, Nagy, Soper 08] (numerically)
[Binoth, Gehrmann, GH, Mastrolia 07]
[Ossola, Pittau, Papadopoulos 07]
[Bernicot, Guillet 08]



- rational parts shown to be zero [Binoth, Guillet, GH 06]
- used both unitarity cuts and Golem

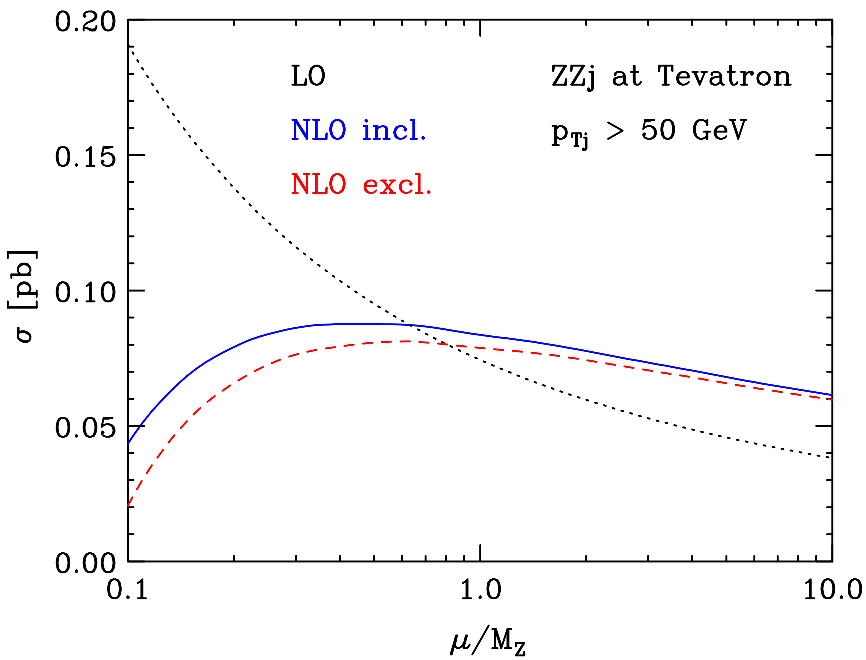


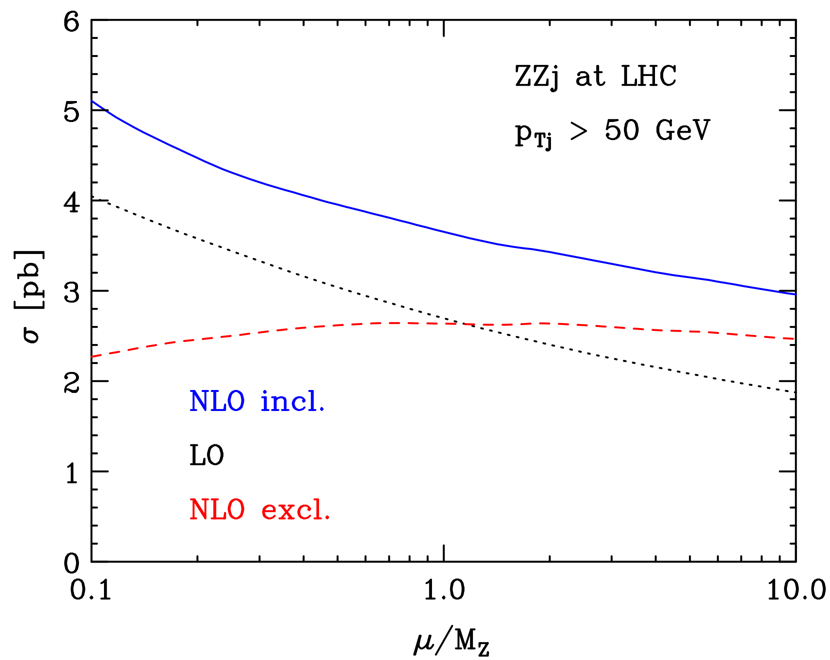


Gong, Nagy, Soper

ZZ + jet production: scale dependence

GOLEM collaboration + T. Gleisberg, plots by N. Kauer





NLO excl.: jet veto: no additional jets with $p_T > 50$ GeV

ZZ + jet production

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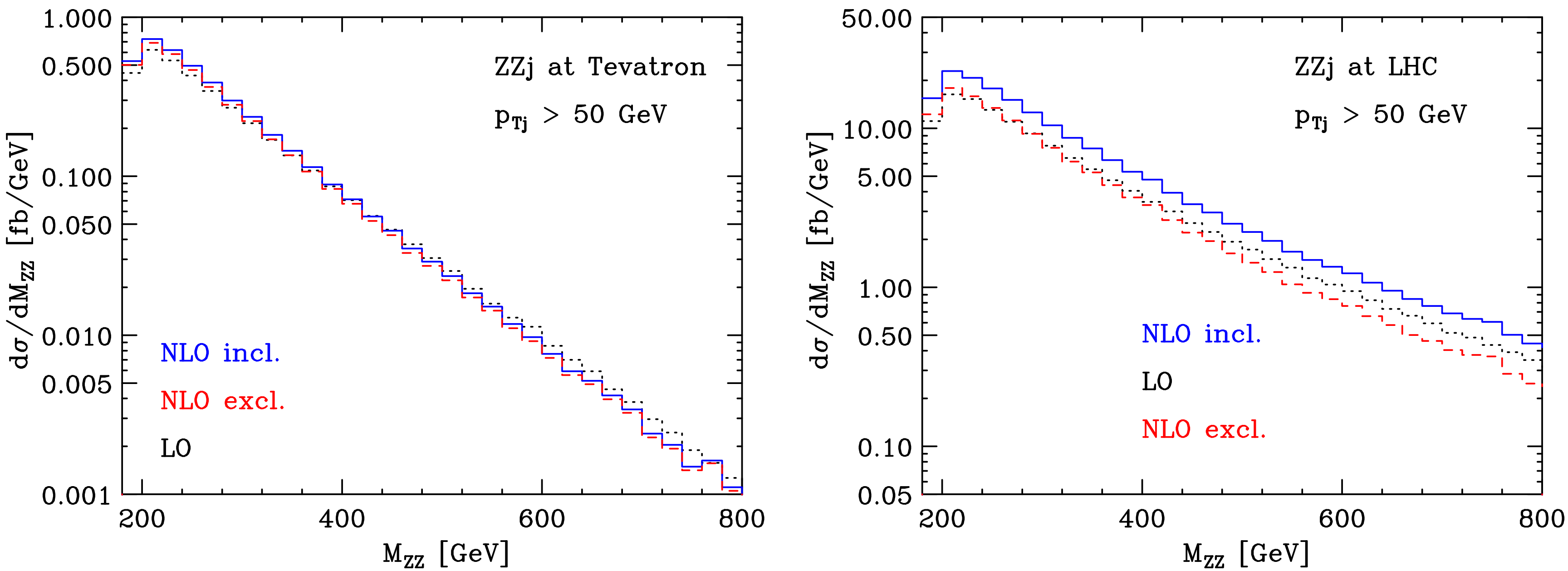


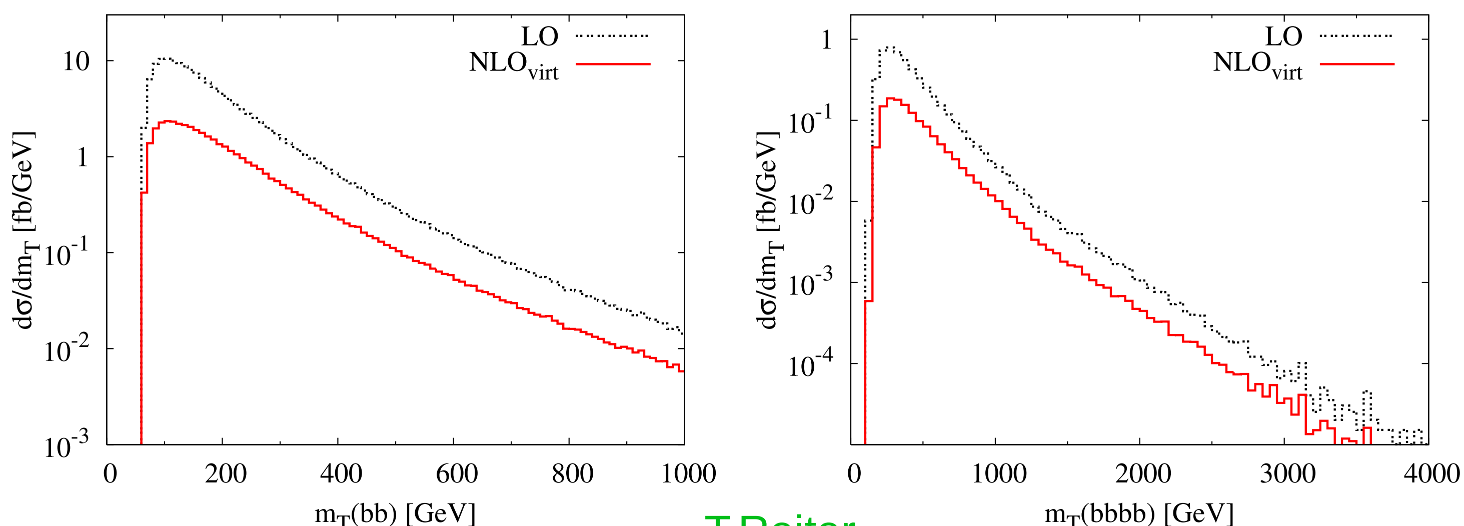
$pp \longrightarrow b\bar{b}b\bar{b}$ one-loop amplitude

$q\bar{q} \longrightarrow b\bar{b}b\bar{b}$ completed
[Ph.D. thesis of Thomas Reiter, Dec. '08]

$gg \longrightarrow b\bar{b}b\bar{b}$ virtual part completed
[Binoth, Guillet, Reiter '09]

results for finite combination

$|\mathcal{A}_{\mathrm{LO+NLOvirt}}|^2$ − UV counterterms − IR subtraction terms



T.Reiter

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- we are moving towards automated tools for NLO predictions

  GOLEM approach:
  - setup valid for massive and massless particles
  - keeps spin information
  - numerically robust as inverse Gram determinants can be avoided
  - combination with parton shower in progress
  - tensor integral library publicly available at http://lappweb.in2p3.fr/lapth/Golem/golem95.html

backup slides

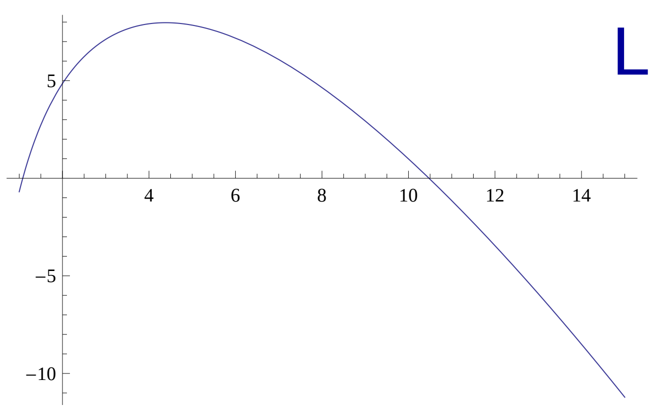
asymptotic complexity

- unitarity based methods:
  complexity of colour ordered amplitudes:

  $$\tau_{\text{tree}} \times \tau_{\text{cuts}} \sim N^4 \times \binom{N}{5} \xrightarrow{\text{N large}} N^9$$

- Feynman diagram reduction:

  $$\tau_{\text{diagrams}} \times \tau_{\text{form factors}} \sim 2^N \times \Gamma(N)$$



Log$(N^9/(\Gamma(N)\,2^N))$

form factor representation

$$I_N^{n,\mu_1\ldots\mu_r}(S) =$$

$$\sum_{l_1\cdots l_r\in S} p_{l_1}^{\mu_1}\cdots p_{l_r}^{\mu_r}\, A_{l_1\ldots,l_r}^{N,r}(S)$$

$$+ \sum_{l_1\cdots l_{r-2}\in S} \left[g^{\cdot\cdot} p_{l_1}^{\cdot}\cdots p_{l_{r-2}}^{\cdot}\right]^{\{\mu_1\cdots\mu_r\}} B_{l_1\ldots,l_{r-2}}^{N,r}(S)$$

$$+ \sum_{l_1\cdots l_{r-4}\in S} \left[g^{\cdot\cdot} g^{\cdot\cdot} p_{l_1}^{\cdot}\cdots p_{l_{r-4}}^{\cdot}\right]^{\{\mu_1\cdots\mu_r\}} C_{l_1\ldots,l_{r-4}}^{N,r}(S)$$

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important: more than two metric tensors $g^{\mu\nu}$ never occur !

for $N \geq 6$: simultaneous reduction of rank r and number of legs N

$$I_N^{n,\mu_1\ldots\mu_r}(S) = -\sum_{j\in S} \mathcal{C}_{j6}^{\mu_1}\, I_{N-1}^{n,\mu_2\ldots\mu_r}(S\setminus\{j\})$$

$$\mathcal{S}_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2$$

Gram determinants

to avoid spurious 1/det(G) terms: do not reduce

golem95:
define dimensionless quantity $\hat{B} = B \times$ (largest entry of S)

if $\hat{B} < \hat{B}^{\rm cut}$: switch to direct numerical evaluation
(default: $\hat{B}^{\rm cut} = 0.005$)

file demo_detg.f90 contains example where $\hat{B} \to 0$
in rank 3 box integral $I_4^{n+2}(1,2,2;S)$ with two massive legs

Real part for B $\rightarrow$ 0