# One-loop Higgs plus four gluon amplitudes

***Full analytic results***

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Based on work done in collaboration with
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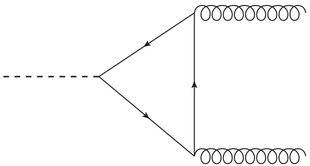
# Outline

- One of the main physics aims at the LHC is the discovery of the mechanism behind Electro-Weak symmetry breaking. For which the Higgs is the usually favoured candidate.
- At LHC energies the dominant source of Higgs production is through gluon fusion, which in the Standard Model is initiated by a top quark loop.
- As a result, $gg \to H + gg$ is a major source of background for Higgs searches through Vector-boson fusion.

# The Higgs plus gluon coupling in the large-$m_t$ limit



## The effective interaction

Integrating out the top-quark loop introduces a five dimensional effective operator.

(Wilczek,Djouadi,Spira,Zerwas,Dawson)

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} C H \,\mathrm{tr}(\mathrm{G}^{\mu\nu}\mathrm{G}_{\mu\nu})$$

To leading order in $\alpha_s$

$$C = \frac{\alpha_s}{6\pi v} \quad v = 246 \text{ eV}$$

The approximation is valid over a wide range of Higgs masses (Kramer,Laenen,Spira) and is a good approximation with increased number of jets provided $p_T < m_t$. (Del Duca,Kilgore,Oleari,Schmidt,Zeppenfeld)

## Selfdual and Anti-Selfdual Seperation (Dixon,Glover,Khoze)

One can split the gluon field strength tensor into selfdual (SD) and anti-selfdual (ASD) pieces

$$G^{\mu\nu}_{SD} = \frac{1}{2}(G^{\mu\nu} + {}^*G^{\mu\nu}) \quad G^{\mu\nu}_{ASD} = \frac{1}{2}(G^{\mu\nu} - {}^*G^{\mu\nu}) \quad {}^*G^{\mu\nu} = \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}$$

Introducing $\phi = 1/2(H + iA)$ leads to the following breakdown of the effective Lagrangian

$$\mathcal{L}^{\mathrm{eff}}_{H,A} = \frac{C}{2}\left(H\,\mathrm{tr}(\mathrm{G}^{\mu\nu}\mathrm{G}_{\mu\nu}) + \mathrm{A}\,\mathrm{tr}(\mathrm{G}^{\mu\nu}\mathrm{G}_{\mu\nu})\right)$$

$$\mathcal{L}^{\mathrm{eff}}_{H,A} = \frac{C}{2}\left(\phi\,\mathrm{tr}(\mathrm{G}^{\mu\nu}_{\mathrm{SD}}\mathrm{G}^{\mathrm{SD}}_{\mu\nu}) + \phi^\dagger\,\mathrm{tr}(\mathrm{G}^{\mu\nu}_{\mathrm{ASD}}{}^*\mathrm{G}^{\mathrm{ASD}}_{\mu\nu})\right)$$

Higgs amplitudes are recovered from the combination of $\phi$ and $\phi^\dagger$ amplitudes, which due to self-duality are more compact than if one had calculated the Higgs amplitude directly

# $\phi, \phi^\dagger$ Tree-level amplitudes and parity

## Parity of $\phi$ and $\phi^\dagger$ amplitudes (Dixon,Glover,Khoze)

Although Higgs amplitudes are made from the sum of $\phi$ and $\phi^\dagger$ amplitudes, in principle one need only calculate $\phi$ amplitudes since

$$A_n^{(m)}(\phi^\dagger, g_1^{\lambda_1}, \ldots, g_n^{\lambda_n}) = \left(A_n^{(m)}(\phi, g_1^{-\lambda_1}, \ldots, g_n^{-\lambda_n})\right)^* .$$

## MHV structure of $\phi$ and $\phi^\dagger$ amplitudes (Dixon,Glover,Khoze)

The simplest helicity amplitudes for $\phi +$ gluon amplitudes are identical in structure to the pure gluon case

$$A_n^{(0)}(\phi, 1^\mp, 2^+, \ldots, n^+) = 0$$

$$A_n^{(0)}(\phi, 1^+, \ldots, i^-, \ldots, j^-, \ldots, n^+) = \frac{\langle i\, j\rangle^4}{\langle 12\rangle\langle 23\rangle \ldots \langle (n-1)n\rangle\langle n1\rangle}$$

However $\sum_i p_i = -p_H$.

## The Spinor Helicity Formalism

The Spinor Helicity is a compact way of writing helicity amplitudes in terms of Weyl spinors. An on-shell massless momenta is written as

$$k_{\alpha\dot{\alpha}} = k_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$$

Spinor inner products are given by

$$\langle \lambda, \lambda' \rangle = \epsilon_{\alpha\beta}\lambda^\alpha \lambda'^\beta, \quad [\lambda, \lambda'] = -\epsilon_{\dot{\alpha}\dot{\beta}}\lambda^{\dot{\alpha}}\lambda'^{\dot{\beta}}$$

Products of four vectors have the following form

$$p^\mu k_\mu = \frac{1}{2}\langle pk \rangle [kp]$$

Polarisation vectors of gluons have the following representation

$$\epsilon^+_\mu(k,\eta) = \frac{\langle \eta|\gamma_\mu|k]}{\sqrt{2}\langle \eta k \rangle} \quad \epsilon^-_\mu(k,\eta) = -\frac{[\eta|\gamma_\mu|k\rangle}{\sqrt{2}[\eta k]}$$

# $\phi$ Amplitudes at tree-level and colour ordering

## Non-gluon like tree level amplitudes (Dixon,Glover,Khoze)

The remaining two helicity configurations needed to calculate $H + 4g$ at NLO are different to the pure glue case

$$A_n^{0)}(\phi, 1^-, 2^-, 3^-, 4^-) = \frac{m_H^4}{[12][23][34][41]}$$

$$A_n^{(0)}(\phi, 1^+, 2^-, 3^-, 4^-) =$$

$$\frac{m_\phi^4 \langle 24\rangle^4}{s_{124}\langle 12\rangle\langle 14\rangle\langle 2|p_\phi|3]\langle 4|p_\phi|3]} - \frac{\langle 4|p_\phi|1]^3}{s_{123}\langle 4|p_\phi|3][12][23]} + \frac{\langle 2|p_\phi|1]^3}{s_{134}\langle 2|p_\phi|3][14][34]}$$

## Colour Ordered amplitudes (Dawson,Kauffman,Del Duca,Frizzo,Maltoni)

We work with colour ordered amplitudes, knowing that all other colour configurations can be obtained by permutations of the leading colour amplitude.

# Previous calculations of $pp \to H + 2j$

- Tree level Large $m_t$ (Dawson,Kauffman,Desai,Risal)
- Tree level Exact $m_t$ (Del Duca,Kilgore,Oleari,Schmidt,Zeppenfeld)
- NLO Virtual (Campbell,Ellis,Giele,Zanderighi),
- NLO Real (Del Duca,Frizzo,Maltoni)
- Analytic one-loop $\phi$ amplitudes,
  $\phi++++$, $\phi-+++$ (Berger,Del Duca, Dixon )
  $\phi----$ (Badger,Glover)
  $\phi--++$ (Badger,Glover,Risager)
  $\phi-+-+$ (Glover,Mastrolia,CW)
  $\phi q\overline{q}-MHV$, $\phi q\overline{q}Q\overline{Q}$ (Dixon,Sofianatos)
- Leaving only $\phi-$NMHV and $\phi q\overline{q}-$NMHV amplitudes for complete analytic expression for $pp \to H + 2j$.

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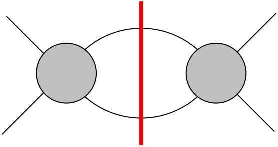
# Unitarity cuts of amplitudes



## The Optical Theorem

The optical theorem relates the discontinuity of a loop integral to the product of lower point amplitudes, via the unitarity of the S-matrix.

$$SS^\dagger = 1$$
$$\implies (1 + iT^\dagger)(1 - iT) = 1$$
$$\implies i(T - T^\dagger) = TT^\dagger$$

## The BDDK Method

Pioneered by Bern, Dixon, Dunbar and Kosower in the mid 90's the original unitarity method used the optical theorem to reconstruct coefficients of loop integrals from four-dimensional cuts.

By setting up systems of simultaneous equations in cuts of physical invariants, coefficients of basis integral functions, were able to be recovered.

Wide applications in SYM $\mathcal{N} = 4, 1$ which can be used to construct pure gluon amplitudes.

# The one-loop basis and Rational terms

## The one-loop basis

An $n$-point one-loop integral can be written in the following form

$$A_n^{(1)} = \sum_i C_{4;i}\mathcal{I}_{4;i} + \sum_i C_{3;i}\mathcal{I}_{3;i} + \sum_i C_{2;i}\mathcal{I}_{2;i} + R. \qquad (1)$$

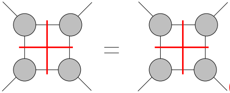
## The rational pieces

In (1) there is a piece $R$ which cannot be detected by four-dimensional cuts. These pieces arise from the cancellation of $\epsilon$ poles with $\mathcal{O}(\epsilon)$ pieces in the numerator.

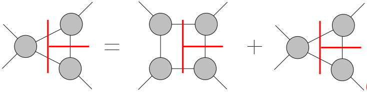
$$C_{j;i} * I_i = \left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon} + logs\right) * (C_0 + C_1\epsilon + C_2\epsilon^2 + \dots)$$

Only items in red are visible to four-dimensional cuts. Therefore need alternate approach to calculate full amplitude.

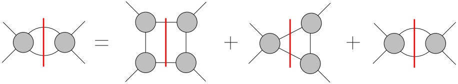
# Generalised Unitarity



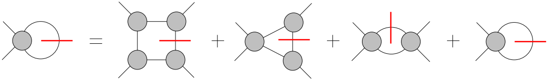
(Britto,Cachazo,Feng)



(Mastrolia,Forde,Bjerrum-Bohr,Dunbar,Perkins)

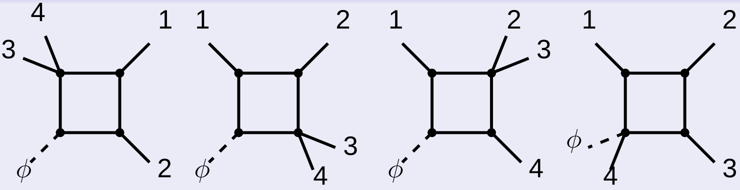


(Bern,Dixon,Dunbar,Kosower,Britto,Feng,
Mastrolia,Brandhuber,McNamara,Anastasiou,Forde,Badger,Bedford,Spence,Travaglini,Morgan,Kunszt)



(Glover,Britto,Feng,CW)

## Evaluation of Box Coefficients (Britto,Cachazo,Feng)




Each coefficient is determined by combining the product of four trees and solving each loop momenta for its on-shell solution.

$$\begin{aligned}\widehat{C}_{4;\phi 1|2|3|4}(\phi,1^+,2^-,3^-,4^-) &= A_4^{(0)}(\phi,\ell_1^-,1^+,\ell_2^-)A_3^{(0)}(\ell_2^+,2^-,\ell_3^-)\\ &\quad\times A_3^{(0)}(\ell_3^+,3^-,\ell_4^+)A_3^{(0)}(\ell_4^-,4^-,\ell_1^+)\\ &= \frac{s_{234}^3}{2\langle 1|p_\phi|2]\langle 1|p_\phi|4][23][34]}\end{aligned}$$

We find that there are no two-mass easy boxes. In the NMHV configuration,

## Evaluation of Triangle Coefficients (Forde)

The three cuts are not sufficient to freeze the loop momentum, have one free parameter $t$. Write loop momenta as

$$\ell^\mu = \frac{a^\mu_{-1}}{t} + a^\mu_0 + a^\mu_1 t$$

Such that the coefficient of a particular triangle is given by

$$C_{3;i} = -\mathrm{Inf}[A_1 A_2 A_3](t)\Big|_{t=0}$$

Where the $\mathrm{Inf}$ operation is defined as

$$\lim_{t\to\infty} (\mathrm{Inf}[A_1 A_2 A_3](t) - A_1(t) A_2(t) A_3(t)]) = 0$$

# Triple Cuts Example

## Evaluation of Triangle Coefficients (Forde)

The full loop parameterisation is given by

$$\ell^\mu = \alpha_{02} K_1^{\flat,\mu} + \alpha_{01} K_2^{\flat,\mu} + \frac{t}{2}\langle K_1^\flat|\gamma^\mu|K_2^\flat] + \frac{\alpha_{01}\alpha_{02}}{2t}\langle K_2^\flat|\gamma^\mu|K_1^\flat]$$

$$K_1^\flat = \gamma\frac{\gamma K_1 - S_1 K_2}{\gamma^2 - S_1 S_2} \quad K_2^\flat = \gamma\frac{\gamma K_2 - S_2 K_1}{\gamma^2 - S_1 S_2}$$

$$\gamma_\pm(K_1, K_2) = K_1 \cdot K_2 \pm \sqrt{K_1 \cdot K_2^2 - K_1^2 K_2^2},$$

and $S_i = K_i^2$. For example a three mass triangle coefficient is given by

$$C_{3;\phi|12|34}(\phi, 1^+, 2^-, 3^-, 4^-) = \sum_{\gamma_\pm} \frac{m_\phi^4 \langle K_1^\flat 2\rangle^3 \langle 34\rangle^3}{\gamma(\gamma + m_\phi^2)\langle K_1^\flat 1\rangle\langle K_1^\flat 3\rangle\langle K_1^\flat 4\rangle\langle 12\rangle}$$

# Double Cuts

## Double Cuts by Stokes theorem (Mastrolia)

By combining the approaches of spinor integration (Britto,Feng,Mastrolia) which evaluates double cuts by residues, with a specific loop parameterisation

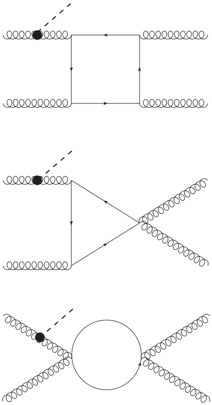
$$\ell_\mu = p_\mu + z\bar{z}q_\mu + \frac{z}{2}\langle q|\gamma_\mu|p] + \frac{\bar{z}}{2}\langle p|\gamma_\mu|q]$$

One can write the double cut as a contour integral

$$C_2 = \oint_{\bar{z}=z^*} dz \int d\bar{z} f(z,\bar{z}) \quad f(z,\bar{z}) = \frac{P(z,\bar{z})}{Q(z,\bar{z})}$$

Can integrate $z(\bar{z})$,

$$C_2 = \oint_{\bar{z}=z^*} dz F(z,\bar{z}) \quad F(z,\bar{z}) = \int d\bar{z} f(z,\bar{z}) = F^{\rm rat}(z,\bar{z}) + F^{\rm log}(z,\bar{z})$$
$$C_2 = \oint_{\bar{z}=z^*} dz F^{\rm rat}(z,\bar{z}) = \mathrm{Res}_z F(z,z^*)$$



### Evaluation by Feynman diagrams

Previous calculations have shown that the majority of the rational piece for Higgs $+$ gluon amplitudes arise from piece which depends on number of flavours $N_f$.

The remaining piece is zero for $H$ amplitudes, and a simple function of tree amplitudes for $\phi$. We find

that there are 739 Feynman diagrams, of which only 136 have a fermion loop and the worst contribution is a second rank box.

# Results

## Breakdown of results

The results are presented in the following way

$$A_4^{(1)}(H,1^{\lambda_1},2^{\lambda_2},3^{\lambda_3},4^{\lambda_4}) = ic_\Gamma(C_4(H,1^{\lambda_1},2^{\lambda_2},3^{\lambda_3},4^{\lambda_4}) + R_4(H,1^{\lambda_1},2^{\lambda_2},3^{\lambda_3},4^{\lambda_4})).$$

That is we have chosen to separate the rational and cut-constructible pieces. We further choose to spilt the cut-constructible pieces into divergent and finite pieces,

$$C_4(H,1^{\lambda_1},2^{\lambda_2},3^{\lambda_3},4^{\lambda_4}) = V_4(H,1^{\lambda_1},2^{\lambda_2},3^{\lambda_3},4^{\lambda_4}) + F_4(H,1^{\lambda_1},2^{\lambda_2},3^{\lambda_3},4^{\lambda_4}).$$

Here $V_4$ is determined by IR safety,

$$V_4(H,1^{\lambda_1},2^{\lambda_2},3^{\lambda_3},4^{\lambda_4}) = -A^{(0)}(H,1^{\lambda_1},2^{\lambda_2},3^{\lambda_3},4^{\lambda_4})\frac{1}{\epsilon^2}\left(\sum_{i=1}^{4}\left(\frac{\mu_R^2}{-s_{i(i+1)}}\right)^{\epsilon}\right).$$

# Notation

## Functions of log's

We use the following functions to express our result.

$$\begin{aligned}
\hat{L}_3(s,t) &= \frac{\log(s/t)}{(s-t)^3} + \frac{1}{2(s-t)^2}\left(\frac{1}{s}+\frac{1}{t}\right) \\
\hat{L}_2(s,t) &= \frac{\log(s/t)}{(s-t)^2} + \frac{1}{2(s-t)}\left(\frac{1}{s}+\frac{1}{t}\right) \\
\hat{L}_1(s,t) &= \frac{\log(s/t)}{(s-t)} \\
\hat{L}_0(s,t) &= \log(s/t)
\end{aligned}$$

## Box Combinations

We will also use the following combinations of finite box functions

$$\begin{aligned}
W^{(1)} &= \mathrm{F}^{1\mathrm{m}}_{4\mathrm{F}}(s_{23}, s_{34}; s_{234}) + \mathrm{F}^{2\mathrm{mh}}_{4\mathrm{F}}(s_{41}, s_{234}; m_H^2, s_{23}) + \mathrm{F}^{2\mathrm{mh}}_{4\mathrm{F}}(s_{12}, s_{234}; s_{34}, m_H^2) \\
W^{(2)} &= \mathrm{F}^{1\mathrm{m}}_{4\mathrm{F}}(s_{14}, s_{34}; s_{134}) + \mathrm{F}^{2\mathrm{mh}}_{4\mathrm{F}}(s_{12}, s_{134}; m_H^2, s_{34}) + \mathrm{F}^{2\mathrm{mh}}_{4\mathrm{F}}(s_{23}, s_{134}; s_{14}, m_H^2) \\
W^{(3)} &= \mathrm{F}^{1\mathrm{m}}_{4\mathrm{F}}(s_{12}, s_{14}; s_{124}) + \mathrm{F}^{2\mathrm{mh}}_{4\mathrm{F}}(s_{23}, s_{124}; m_H^2, s_{14}) + \mathrm{F}^{2\mathrm{mh}}_{4\mathrm{F}}(s_{34}, s_{124}; s_{12}, m_H^2)
\end{aligned}$$

We use color to denote quadruple, triple and double cuts

By far the most complicated helicity amplitude is the $H$-NMHV configuration.
(Badger,Glover,Mastrolia,CW)

$$
\begin{aligned}
F_4(H,1^+,2^-,3^-,4^-) \quad = \quad & \Bigg\{ \frac{s_{234}^3}{4\langle 1|p_H|2]\langle 1|p_H|4][23][34]} W^{(1)} \\
& + \left( \frac{\langle 2|p_H|1]^3}{2s_{134}\langle 2|p_H|3][34][41]} + \frac{\langle 34\rangle^3 m_H^4}{2s_{134}\langle 1|p_H|2]\langle 3|p_H|2]\langle 41\rangle} \right) W^{(2)} \\
& + \frac{1}{4s_{124}} \left( \frac{\langle 3|p_H|1]^4}{\langle 3|p_H|2]\langle 3|p_H|4][21][41]} + \frac{\langle 24\rangle^4 m_H^4}{\langle 12\rangle\langle 14\rangle\langle 2|p_H|3]\langle 4|p_H|3]} \right) W^{(3)} \\
& - \left( \sum_{\gamma=\gamma_\pm(p_H,p_1+p_2)} \frac{2m_H^4 \langle P_{12}^\flat 2\rangle^3 \langle 34\rangle^3}{\gamma(\gamma+m_H^2)\langle P_{12}^\flat 1\rangle\langle P_{12}^\flat 3\rangle\langle P_{12}^\flat 4\rangle\langle 12\rangle} \right) \mathrm{F}_3^{3\mathrm{m}}(m_H^2, s_{12}, s_{34}) \\
& + \left(1 - \frac{N_f}{4N_c}\right) \left( -\frac{\langle 3p_H 1]^2}{s_{124}[24]^2} \mathrm{F}_{4\mathrm{F}}^{1\mathrm{m}}(s_{12}, s_{14}; s_{124}) \right. \\
& \left. + \frac{4\langle 24\rangle\langle 3|p_H|1]^2}{s_{124}[42]} \hat{L}_1(s_{124}, s_{12}) - \frac{4\langle 23\rangle\langle 4|p_H|1]^2}{s_{123}[32]} \hat{L}_1(s_{123}, s_{12}) \right) \\
& + \left(1 - \frac{N_f}{N_c} + \frac{N_s}{N_c}\right) \left( \frac{[12][41]\langle 3|p_H|2]\langle 3|p_H|4]}{2s_{124}[24]^4} \mathrm{F}_{4\mathrm{F}}^{1\mathrm{m}}(s_{12}, s_{14}; s_{124}) \right. \\
& + \frac{2s_{124}\langle 24\rangle\langle 34\rangle^2[41]^2}{3[42]} \hat{L}_3(s_{124}, s_{12}) \\
& + \frac{\langle 34\rangle[41](3s_{124}\langle 34\rangle[41] + \langle 24\rangle\langle 3|p_H|1][42])}{3[42]^2} \hat{L}_2(s_{124}, s_{12}) + \dots
\end{aligned}
\tag{2}
$$

# $H+---$

$$+\left(\frac{2s_{124}\langle 34\rangle^2[41]^2}{\langle 24\rangle[42]^3}-\frac{\langle 24\rangle\langle 3|p_H|1]^2}{3s_{124}[42]}\right)\hat{L}_1(s_{124},s_{12})$$
$$+\frac{\langle 3|p_H|1](4s_{124}\langle 34\rangle[41]+\langle 3|p_H|1](2s_{14}+s_{24})}{s_{124}\langle 24\rangle[42]^3}\hat{L}_0(s_{124},s_{12})$$
$$-\frac{2s_{123}\langle 23\rangle\langle 34\rangle^2[31]^2}{3[32]}\hat{L}_3(s_{123},s_{12})+\frac{\langle 23\rangle\langle 34\rangle[31]\langle 4|p_H|1]}{3[32]}\hat{L}_2(s_{123},s_{12})$$
$$+\frac{\langle 23\rangle\langle 4|p_H|1]^2}{3s_{123}[32]}\hat{L}_1(s_{123},s_{12})\bigg)\bigg\}+\Big\{(2\leftrightarrow 4)\Big\}$$

and rational pieces

$$R_4(H,1^+,2^-,3^-,4^-)=\Bigg\{\left(1-\frac{N_f}{N_c}+\frac{N_s}{N_c}\right)\frac{1}{2}\,\frac{\langle 23\rangle\langle 34\rangle\langle 4|p_H|1][31]}{3s_{123}\langle 12\rangle[21][32]}-\frac{\langle 3|p_H|1]^2}{s_{124}[42]^2}$$
$$+\frac{\langle 24\rangle\langle 34\rangle\langle 3|p_H|1][41]}{3s_{124}s_{12}[42]}-\frac{[12]^2\langle 23\rangle^2}{s_{14}[42]^2}-\frac{\langle 24\rangle(s_{23}s_{24}+s_{23}s_{34}+s_{24}s_{34})}{3\langle 12\rangle\langle 14\rangle[23][34][42]}$$
$$+\frac{\langle 2|p_H|1]\langle 4|p_H|1]}{3s_{234}[23][34]}-\frac{2[12]\langle 23\rangle[31]^2}{3[23]^2[41][34]}\Bigg\}+\Big\{(2\leftrightarrow 4)\Big\}$$

# Conclusions

- Strong need for fast and efficient evaluation of Higgs phenomenology at the LHC, best achieved by evaluation of compact formulae.
- The Higgs is produced dominantly at the LHC through gluon fusion, which in the standard model proceeds through a top quark loop. Calculations can be drastically simplified by working in an effective theory in which the mass of the top tends to infinity.
- The evaluation of Higgs helicity amplitudes can be further simplified by considering the Higgs as a real part of a complex scalar field $\phi$. This field couples to the self-dual piece of the gluon field strength tensor, and produces compact helicity amplitudes.
- The helicity amplitudes for the process $0 \to H + 4g$ are now known analytically, and complete analytic control of $pp \to H + 2j$ requires only one remaining helicity amplitude, $A_4^{(1)}(\phi, 1_q^-, 2^-, 3^-, 4_{\bar{q}}^{\pm})$. This builds upon an earlier numeric code, which has proven too slow for effective use in NLO tools such as MCFM.

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