Threshold resummation for coloured heavy particles at the LHC

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IPPP Durham

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Based on M. Beneke, PF, C. Schwinn, **[arXiv:0907.1443[hep-ph]]**
and work in preparation

Pair production near threshold

IN THIS TALK: Pair-production of coloured heavy particles at LHC

$$p_i + p_j \longrightarrow HH^{'} + X \qquad H, H^{'} = \text{top, squarks, gluinos...}$$

Accurate determination of the cross section **phenomenologically important** (sensitivity to mass parameters, exclusion bounds, model discrimination...) and theoretically interesting due to **non-trivial colour exchange**

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Partonic cross section enhanced in the **threshold region**, $\beta \equiv \sqrt{1-(M_H+M_{H'})^2/\hat{s}} \rightarrow 0$:

- **Threshold logarithms**: $\sim \alpha_s^n \ln^m \beta^2 \Leftrightarrow$ soft-gluon exchange
- **Coulomb singularities**: $\sim (\alpha_s/\beta)^n \Leftrightarrow$ static interaction of non-relativistic particles

Enhanced terms can spoil convergence of perturbative series $\Rightarrow$ **RESUMMATION**

- **Normalisation** of the cross section (really important only for $m_H \gtrsim 1$ TeV)
- Generally observed to reduce dependence on the **factorisation-scale** (even for small masses...)

Factorisation and resummation

Theoretical basis for resummation is **factorisation** of **hard (H)** and **soft (S)** contributions near kinematic thresholds (for example $\hat{s} \sim Q^2$ in Drell-Yan)

$$\hat{\sigma} = H \otimes S$$

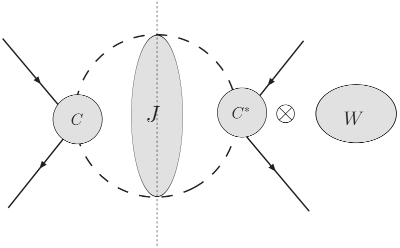
H: process-dependent hard function
S: soft function encoding long-distance effects ($\ni \log \beta^2$)

- **Resummation in Mellin space**
  [Sterman '87; Catani, Trentadue '89,...]
  - $H \otimes S \Rightarrow H(N)S(N)$
  - Threshold logs exponentiated by evolution equations for $S(N)$ and $H(N)$.
  - Numerical inversion of the Mellin transform (avoids **Landau pole**...)

- **Resummation in momentum space**
  [Becher, Neubert '06]
  - Based on factorisation in SCET
  - Resummation in momentum space via RG evolution equations

Factorisation of pair production in SCET

Effective theory description of pair production near threshold [Beneke, PF, Schwinn '09]:
SCET (collinear/soft modes) +**(p)NRQCD** (heavy non-relativistic fields)
$\Rightarrow$ arbitrary colour representations + factorisation of Coulomb corrections



$$\hat{\sigma}_{pp'}(\hat{s},\mu) = \sum_{i,i'} H_{ii'}(M,\mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E - \frac{\omega}{2})\, W^{R_\alpha}_{ii'}(\omega,\mu)$$

- Hard function $H_{ii'}$ depends on the precise nature of the physics model
- Process-independent soft function $W^{R_\alpha}_{ii'}$ ($\sim \alpha_s^n \log^m \beta^2$) $\Leftrightarrow$ **soft Wilson lines**
- Potential function J_{R_α} encodes Coulomb effects($\sim \alpha_s^n/\beta^n$)

Possible corrections at $O(\alpha_s^2 \log \beta^2)$ [Ferroglia et al. '09]

Colour structure of the factorisation formula

Factorised cross section has **non-trivial colour** structure

- Hard function is a matrix in colour space: $H_{ii'} \equiv H_{\{ab\}} c^{(i)}_{\{a\}} c^{(i')*}_{\{b\}}$
- Potential function $J_{\{k\}}$ projected over irreducible representations of the HH' system: $J_{\{k\}} = \sum_{R_\alpha} P^{R_\alpha}_{\{k\}} J_{R_\alpha}$, with $R \otimes R^{'} = \sum_\alpha R_\alpha$
- Soft function S replaced by a set of colour matrices $W^{R_\alpha}_{ii'}$

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Colour basis $c^{(i)}_{\{a\}}$ can be chosen such that $W^{R_\alpha}_{ii'}$ are diagonal to all orders in α_s
[Beneke, PF, Schwinn, arXiv:0907.1443[hep-ph]]

- Decompose initial-state and final-state product representations into **irreducible representations**: $\Rightarrow$ Clebsch-Gordan coefficients

$$r \otimes r' = \sum_\alpha r_\alpha \rightarrow C^{r_\alpha}_{\alpha a_1 a_2} \qquad\qquad R \otimes R' = \sum_\beta R_\beta \rightarrow C^{R_\beta}_{\alpha a_1 a_2}$$

- Identify pairs of equivalent initial- and final-state representations $P_i = (r_\alpha, R_\beta)$
- Construct colour basis and projectors

$$c^{(i)}_{\{a\}} = \frac{1}{\sqrt{\dim(r_\alpha)}} C^{r_\alpha}_{\alpha a_1 a_2} C^{R_\beta *}_{\alpha a_3 a_4} \qquad\qquad P^{R_\alpha}_{\{a\}} = C^{R_\alpha *}_{\alpha a_1 a_2} C^{R_\alpha}_{\alpha a_3 a_4}$$

Colour structure of the factorisation formula

EXAMPLE: $t\bar{t}$ **production in gluon fusion** $(8 \otimes 8 \to 3 \otimes \bar{3})$

- **Irreducible representations**:

$$8 \otimes 8 = 1 + 8_S + 8_A + 10 + \bar{10} + 27 \qquad\qquad 3 \otimes \bar{3} = 1 + 8$$

- **Pairs of equivalent representations**: $P_i = \{(1,1), (8_S, 8), (8_A, 8)\}$
- **Clebsch-Gordan coefficients**:

$$8 \otimes 8 \; : \; C^{(1)}_{a_1 a_2} = \frac{1}{\sqrt{N_C}} \delta_{a_1 a_2} \, , \qquad C^{(8_S)}_{\alpha a_1 a_2} = \frac{1}{2\sqrt{B_F}} D^{\alpha}_{a_2 a_1} \, , \qquad C^{(8_A)}_{\alpha a_1 a_2} = \frac{1}{2\sqrt{N_C}} F^{\alpha}_{a_2 a_1} \, , \quad \ldots$$

- **Colour basis**:

$$c^{(1)}_{\{a\}} = \frac{1}{\sqrt{N_C D_A}} \delta_{a_1 a_3} \delta_{a_2 a_4} \qquad c^{(2)}_{\{a\}} = \frac{1}{\sqrt{2 D_A B_F}} D^{\alpha}_{a_2 a_1} T^{\alpha}_{a_3 a_4} \qquad c^{(3)}_{\{a\}} = \sqrt{\frac{2}{N_C D_A}} F^{\alpha}_{a_2 a_1} T^{\alpha}_{a_3 a_4}$$

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- The basis $c^{(i)}_{\{a\}}$ diagonalises $W^{R_\alpha}_{ii'}$ to all orders in α_s (extends results for $t\bar{t}$, squarks, gluinos at one-loop [Kidonakis, Sterman '97; Kulesza, Motyka '08])
- $W^{R_\alpha}_{ii'}$ can be rewritten as the soft function of a single scalar in the representation R_α $\Rightarrow$ soft radiation emitted off the total colour charge of the pair

Resummation of logs in momentum space

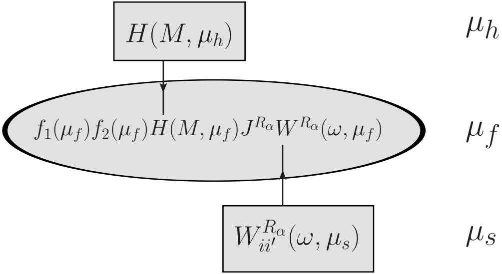
RG evolution equation for the soft function $W_i^{R\alpha}$
(generalisation of DY result [Becher, Neubert, Xu '07])

$$
\begin{aligned}
\frac{d}{d\log\mu_f} W_i^{R\alpha}(\omega,\mu_f) \quad = \quad & -2\left[\left(\Gamma^r_{\rm cusp}+\Gamma^{r'}_{\rm cusp}\right)\log\left(\frac{\omega}{\mu}\right)+2\gamma^{R\alpha}_{H,s}+2\gamma^r_s+\gamma^{r'}_s\right] W_i^{R\alpha}(\omega,\mu_f) \\
& -2\left(\Gamma^r_{\rm cusp}+\Gamma^{r'}_{\rm cusp}\right)\int_0^\omega d\omega' \, \frac{W_i^{R\alpha}(\omega',\mu_f)-W^{R\alpha}(\omega,\mu_f)}{\omega-\omega'}
\end{aligned}
$$

and similar for hard function $H_i(M,\mu_f)$

Resummation strategy

- Solve evolution equation in momentum space
- Evolve hard function H_i from $\mu_h \sim M_H$ to μ_f
- Evolve soft function $W_i^{R\alpha}$ from low scale μ_s (still to be defined) to μ_f.



Resummation of logs in momentum space

Evolution of $W_i^{R_\alpha}$ driven by **anomalous dimensions** $\Gamma^r_{\rm cusp}$ and γ^r_s, $\gamma^{R_\alpha}_{H,s}$.

- **LL**: tree-level $W_i^{R_\alpha}$, 1-loop $\Gamma^r_{\rm cusp}$ $\Rightarrow$ resum $\alpha_s^n \ln^m \beta^2$, with $n+1 \leq m \leq 2n$
- **NLL**: tree-level $W_i^{R_\alpha}$, 2-loop $\Gamma^r_{\rm cusp}$, 1-loop γ^r_s, $\gamma^{R_\alpha}_{H,s}$ $\Rightarrow$ resum $\alpha_s^n \ln^n \beta^2$
- **NNLL**: 1-loop $W_i^{R_\alpha}$, 3-loop $\Gamma^r_{\rm cusp}$, 2-loop γ^r_s, $\gamma^{R_\alpha}_{H,s}$ $\Rightarrow$ resum terms suppressed by $\{\alpha_s, \beta\}$

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- $\Gamma^r_{\rm cusp}$ and γ^r_s known up to three loops [Moch, Vermaseren, Vogt '04/'05].
- Heavy-particle soft anomalous dimension $\gamma^{R_\alpha}_{H,s}$:
  - **1-loop**: from calculation of one-loop soft function (agrees with [Kulesza, Motyka '08])

$$\gamma^{(0),R_\alpha}_{H,s} = -2C_{R_\alpha}$$

  - **2-loop**: extracted from two loop results for HQET form factors [Korchemsky et al. '92; Kidonakis '09] using constraints from soft-collinear factorisation [Becher, Neubert '09]

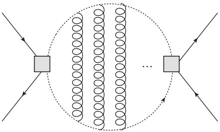
$$\gamma^{(1),R_\alpha}_{H,s} = -2C_{R_\alpha}\left[-C_A\left(\frac{98}{9} - \frac{2\pi^2}{3} + 4\zeta_3\right) + \frac{40}{9}T_f n_f\right]$$

Two-loop result agrees with [Czakon, Mitov, Sterman '09]

Resummation of Coulomb corrections

Near threshold exchange of Coulomb gluons between the pair H, H' is also kinematically enhanced: $\Delta\sigma^{\text{Coul},(1)}/\sigma^{\text{tree}} \sim \alpha_s/\beta \sim 1$

$\Rightarrow$ **Leading Coulomb corrections must be resummed to all orders as well**

$J_{R_\alpha}(E) \Leftrightarrow$ 

Resummation of Coulomb effects well understood from **quarkonia physics**:

$$J_{R_\alpha}(E) \quad \propto \quad -\frac{(2m_{\text{red}})^2}{4\pi}\text{Im}\left\{\sqrt{-\frac{E}{2m_{\text{red}}}} + \alpha_s(-C_{R_\alpha})\left[\frac{1}{2}\ln\left(-\frac{8\,m_{\text{red}}E}{\mu^2}\right) - \frac{1}{2} + \gamma_E + \psi\left(1 - \frac{\alpha_s(-C_{R_\alpha})}{2\sqrt{-E/(2m_{\text{red}})}}\right)\right]\right\}$$

Modified power counting: **LL** = $\left(\frac{\alpha_s}{\beta}\right)^k \alpha_s^n \log^m \beta^2 \quad n+1 \leq m \leq 2n$, etc...

Squark-antisquark production at the LHC

In the rest of this talk:

$$PP \to \tilde{q}\bar{\tilde{q}} + X$$

Apply **NLL soft resummation** and **Coulomb resummation** to total cross section for squark-antisquark production

$$\hat{\sigma}^{\text{Res}}_{\tilde{q}\bar{\tilde{q}}}(\hat{s}, \mu) = \sum_i H_i^{\text{tree}}(M, \mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E - \frac{\omega}{2})\, W_i^{R_\alpha,\text{NLL}}(\omega, \mu)$$

Resummed cross section is matched onto the full NLO result
[Zerwas et al., '96; Langenfeld, Moch '09]

$$\hat{\sigma}^{\text{match}}_{\tilde{q}\bar{\tilde{q}}}(\hat{s}, \mu_f) = \left[\hat{\sigma}^{\text{Res}}_{\tilde{q}\bar{\tilde{q}}}(\hat{s}, \mu_f) - \hat{\sigma}^{\text{Res}}_{\tilde{q}\bar{\tilde{q}}}(\hat{s}, \mu_f)\big|_{\text{NLO}}\right] + \hat{\sigma}^{\text{NLO}}_{\tilde{q}\bar{\tilde{q}}}(\hat{s}, \mu_f)$$

Scale choice

What is the natural scale choice for J_{R_α} and $W_i^{R_\alpha}$?

- Scale μ_C for **Coulomb resummation** set by typical virtuality of a Coulomb gluon $\sqrt{|q^2|} \sim M\beta \sim M\alpha_s$

$$\Rightarrow \mu_C = \max\{2M\beta, 2M\alpha_s(\mu_C)\}$$

- **Soft function** $W_i^{R_\alpha}$ renormalised at the **low scale** μ_s and evolved to the factorisation scale μ_f $\Rightarrow$ Choose μ_s such that one-loop soft corrections to the hadronic cross section are minimised [Becher, Neubert, Xu '07]

$$\frac{\partial}{\partial \mu_s} \int dx_1 d_2 f(x_1) f(x_2) \Delta\hat{\sigma}^{S,(1)}(\hat{s}, \mu_s) = 0$$

This choice guarantees well-behaved perturbative expansion at the low scale μ_s

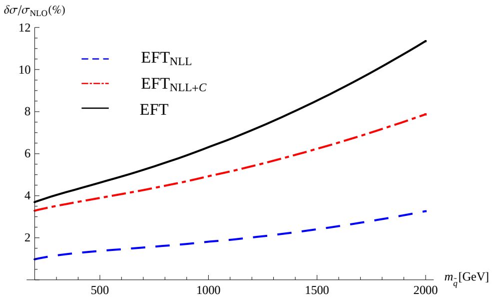
Squark-antisquark resummed cross section

Beneke, PF, Schwinn, PRELIMINARY

- **EFT_{NLL}**: NLL soft resummation, no Coulomb resummation
- **$\text{EFT}_{\text{NLL+C}}$**: NLL soft resummation **AND** Coulomb resummation. No soft/Coulomb interference
- **EFT**: NLL soft resummation + Coulomb resummation + soft/1st Coulomb interference

Setup:

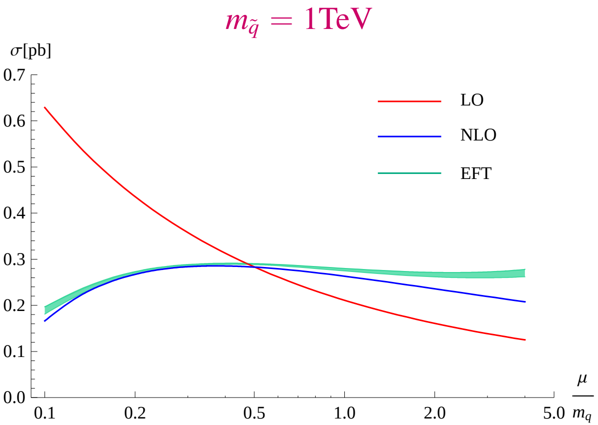
- PP@ 14 TeV
- MSTW2008 PDFs
- equal squark masses
- no stops
- $m_{\tilde{g}} = 1.25 m_{\tilde{q}}$
- $\mu_f = m_{\tilde{q}}$



EFT_{NLL} result agrees well with Kulesza, Motyka ’09

Factorisation-scale dependence

One of main motivations for resummation is **reduction of scale dependence** of NLO result:



Green band obtained by varying the **soft scale** μ_s by a factor 2 in both directions

Summary

- Factorisation formula for **soft** and **Coulomb** effects in pair-production near threshold
- Resummation of threshold logarithms directly in **momentum space** (analytic results, no Mellin inversion,...)
- General treatment of colour exchange:
  - $\Rightarrow$ Colour basis **diagonal to all orders** in α_s
  - $\Rightarrow$ Determination of **two-loop soft anomalous dimension** for arbitrary $SU(3)$ representation (necessary for NNLL resummation)
- Application of the results to **squark-antisquark production** at the LHC
  - $\Rightarrow$ NLL corrections $\sim 4 - 12\%$ for $m_{\tilde{q}} \sim 300\text{GeV} - 2\text{TeV}$
  - $\Rightarrow$ Significant reduction of factorisation-scale dependence