Constructing Viable SUSY Spectra: Handout

Matt Dolan

January 12, 2010

1 Part the First

Softsusy can be found in the softsusy-3.1 directory, whic you can get to by the change directory command

```
cd softsusy-3.1
```

,and should be already compiled for use. In this section we use the softpoint.x executable, which computes the spectrum of a single point in parameter space. There are a number of different ways to run this main program. We'll be using three of them to explore the spectra of gravity, gauge and anomaly mediated supersymmetry breaking. The UV boundary conditions and Renormalisation Group equations for these theories can be found in Martin's review of supersymmetry, which is included in the Softsusy folder.

- We'll start by playing around with some minimal supergravity (mSUGRA or CMSSM) points. The syntax for running Softsusy with mSUGRA boundary conditions is

  ```
  ./softpoint.x sugra m0 m12 A0 tanb unified sgnMu
  ```

  and then hitting return. *sugra* defines the mSUGRA boundary conditions. Then come the numerical inputs: *m0* is the scalar mass parameter at the GUT scale. It determines the initial values of the squark and slepton masses. *m12* is the gaugino mass parameter which governs the gluino, chargino and neutralino masses at the high scale. *A0* sets the initial conditions for the trilinear scalar couplings. *tanb* is the ratio of the two Higgs vevs, *unified* signifies that we want gauge coupling unification and *sgnMu* is the sign of the supersymmetric μ parameter, ± 1.
- A well studied point is the SPS1a point, which can be run by inputting

```
./softpoint.x sugra 100 250 -100 10 unified 1
```

You should see the output SUSY spectrum on your screen. It will be useful to redirect this output into a file, so that you can retrieve it for later use. Do this by running

```
./softpoint.x sugra 100 250 -100 10 unified 1 > Output.File
```

where *Output.File* is the name of the file you want the SUSY spectrum to land in, such as spectrum.txt or something. You can look at the file using either of

```
less Output.File
emacs Output.File &
```

less is a Unix file-viewing utility. It will display the contents of Output.File in the terminal. *emacs* is an all-powerful text editor which will allow you to change the file as well.

- Let's break down the output from softpoint.x at the SPS1s point. Ignoring the proprietary stuff at the top, the first interesting thing is the low energy Standard Model data, which appears between two lines of dashes and tells us the quark masses and W mass.

  After this comes the supersymmetry breaking mass parameters, output at M_Z. These are the matrices which appear in the soft supersymmetry breaking Lagrangian.

  After another line of dashes comes the physical MSSM spectrum, which is the most interesting part for us today. First is the masses of the five MSSM Higgses. The next important things are the three sneutrino masses, in order of family ($\tilde{\nu}_e$, $\tilde{\nu}_\mu$ and $\tilde{\nu}_\tau$). Then there are three matrices which give the masses of the light and right handed up squarks[1], the down squarks and the sleptons. The first row is left handed, the second right handed. Similarly, the columns of the mU matrix are for up, charm and top, the columns of mD are for down, strange and bottom and the columns of mE are for e, μ and τ.

  Then there are some mixing angles and the gluino mass, the two charginos and the four neutralinos. Finally comes the neutralino mixing matrix (not important for us today), and the identity of the LSP and its mass.

---

[1] Note the abuse of notation here. The squarks are scalars and have no handedness. The handedness refers to their quark partners.

You should find that the SPS1a point gives a fairly light SUSY spectrum: the lightest superparticle (the neutralino) mass is 97GeV, the gluino mass (important for sparticle production at the LHC) is about 600 GeV and the squark masses are all less than 600GeV. The lightest Higgs mass is 109GeV.

- Increase the scalar mass parameter $m0$ to 500, 1000 and 2000GeV,

  ```
  ./softpoint.x sugra 500 250 -100 10 unified 1
  ```

  and so on. You should find that the scalar particles increase in mass.

- Resetting m0 to 100GeV, increase the gaugino mass paramer $m_{1/2}$ to 500, 1000 and 2000GeV.

  ```
  ./softpoint.x sugra 100 500 -100 10 unified 1
  ```

  etc. Check that the gaugino masses increase. The scalar masses should also increase. Why is this? Be sure to keep an eye on the identity of the LSP as well.

- Finally increase $\tan\beta$ in increments of 10.

  ```
  ./softpoint.x sugra 100 250 -100 20 unified 1
  ```

  You should find changes mainly in the Higgs sector and third generation of scalars.

Let us now consider a different scenario, gauge mediated supersymmetry breaking. In minimal gauge mediated supersymmetry breaking the soft supersymmetry breaking terms are defined at a messenger scale M_{mess} by

$$M_a = \frac{\alpha_a}{4\pi} \Lambda N_5 \tag{1}$$

for the gaugino masses and

$$m_0^2 = 2\Lambda^2 N_5 \sum_{a=1}^{3} C_a(i) \left(\frac{\alpha_a}{4\pi}^2 \right) \tag{2}$$

for the scalar mass squared. Λ is the supersymmetry breaking scale, N_5 is the number of messenger multiplets which communicate the SUSY breaking from the hidden to the visible sector and C_a is the quadratic Casimir operator for the gauge represenation in question.

- Before running Softsusy, you should be able to see that in mGMSB there is a distinct hierarchy between weakly and strongly interacting particles. Why?
- To run Softsusy with mGMSB boundary conditions the syntax is

  ```
  ./softpoint.x gmsb N5 Mmess Lambda 1 tanb sgnMu
  ```

  Reasonable ranges for these parameters are $1 \leq N_5 \leq 8$, $10^6 \leq M_{mess} \leq 10^{14}$GeV and $10^4 \leq \Lambda \leq 10^6$GeV. Run the point

  ```
  ./softpoint.x gmsb 2 1e10 1e5 1 10 1
  ```

- What is the dependence of the gaugino masses on the messenger scale M_{mess}. Why is this so?

The final model implemented in Softsusy is anomaly mediated SUSY breaking. We won't worry too much about the details of mAMSB in this tutorial.

Finally, it's also pretty easy to write new code for boundary conditions for your favourite model. This is discussed in the Softsusy manual. If you've made it through this far with time left, read this, and have a go at writing your own boundary conditions for your favorite model, so you can use them later on.

2 Part the Second: Viable Spectra and Constraints.

In this section we will be using softsusy.x, which is a program which has been hooked up to `micrOMEGAS` and scans through a two dimensional subspace of the full mSUGRA parameter space. We'll be exclusively considering mSUGRA in this section. softsusy.x outputs information about sparticle masses, and Standard Model and cosmological observables. You'll need to redirect the outut to a file for analysis and plotting by typing

```
./softsusy.x > Output.txt
```

,hitting return and leaving the program to run. The softsusy.x program has been compiled from the file main.cpp. To open and have a look at this file use emacs

```
emacs main.cpp &
```

or your own favorite editor. This will enable you to change the range of the parameter scan, and some of the other program variables such as the number of points in the scan. If you change something in main.cpp, you will need to save your changes and recompile the program, which can be done by typing

```
make
```

in the command line. It might be a good idea to make a copy of the original main.cpp before you do this:

```
cp main.cpp main_backup.cpp
```

Before galloping off and hacking my lovingly constructed program to destruction, let's consider the constraints implemented in the program and understand the results that you've just generated.

- **Neutral LSP** Not all points in mSUGRA parameter space have been created equal. What are the most basic requirements for realistic phenomenology? We're considering the R-parity preserving MSSM, so that the lightest supersymmetric particle (LSP) is stable. If the LSP is stable and makes up some component of the dark matter, it must be neutral. So charged LSPs should be rejected. Recall in Section 1 you saw that when m_0 is small and $m_{1/2}$ is large the lightest neutralino becomes heavier than the lightest stau.

- Using gnuplot (see the appendix), create a plot (or plots) of the total χ^2 in the m_0-$m_{1/2}$ plane, and identify the region excluded due to the charged LSP constraint. It should be a wedge shaped region on the side of your plot. (Note that in GMSB this constraint is moot, since the gravitino is naturally the LSP in that scenario.)

- **Direct Searches** We must also consider constraints from direct searches. The Tevatron and LEP colliders have been searching without success for SUSY particles. Any points in parameter space which have too light a spectrum should therefore be rejected. The regions affected by this are in the low $m_{1/2}$ and low m_0 regions, for obvious reasons. The best resource for sparticle masses is the Particle Data Group (PDG), which can be found online.

  A word of warning for the future: When applying direct search limits on sparticles, be very careful that the quoted limit is valid for the model you are investigating. For instance, in mSUGRA the mass limit on the lightest neutralino is 46 GeV. This is obtained by searching for charginos. This lets the experimenters put lower bound on the chargino mass. In mSUGRA though, the gaugino masses are determined by the universal gaugino mass parameter $m_{1/2}$, so one can translate this into a bound on the neutralino mass. The point is, without this gaugino universality the bound does not apply. In fact, Dreiner *et al.* have shown that a massless neutralino is completely consistent with all known physical bounds in this case!

We also include the LEP bound on the lightest Higgs, which is 114GeV. This is parametrised using data from the LEP experiment, which includes a small deviation consistent with a *circa* 115-6GeV Higgs.

- Find the region which is excluded by direct searches in your plots.

- **DM Relic Density** Demanding that the neutralino is the LSP is quite a weak constraints, given what other data are available. The COBE and WMAP satellites have measured the dark matter relic density to extremely high precision. As has been or will be explained in the lectures, we can use the Boltzmann equation to predict the neutralino relic density based on the structure of the SUSY spectrum (assuming a standard cosmology). This gives the single most important constraint on the MSSM spectrum. The sparticles produced in the early universe all decay to the neutralino, and the relic density is, generically speaking, far too high. Only when the spectrum has certain features is the relic density low enough to be phenomenologically viable. We'll constrain relic density using a two sided Gaussian, where

  $$\Omega_{DM} h^2 = 0.1143 \pm 0.02 \tag{3}$$

  The error here is dominated by the theoretical uncertainty in the relic density calculation. The experimental uncertainty is nearly an order of magnitude below this.

- Write down all the tree level diagrams you can think of which deplete the neutralino relic density. You can find a discussion of this in Martin hep-ph/9709356 which is awesome and which you should glance at before continuing.

- Plot the dark matter relic density and χ^2_{DM} in the m_0-$m_{1/2}$ plane. You should find that the majority of the parameter space fits this obversable terribly. What regions of parameter space have an acceptable $\Omega_{DM} h^2$? With reference to the diagrams in the Martin SUSY review, figure out what the depletion mechanism at work in these regions is.

- **The Muon Anomalous Magnetic Moment** Two important non-cosmological constraints are the anomalous magnetic moment of the muon $(g-2)_\mu$ and the rare branching ratio $BR(B \to X_s \gamma)$. The anomalous magnetic moment of the muon has been measured and calculated to an accuracy bordering on the ridiculous. The net result of a lot of people's work is that there is a discrepancy between the theoretical and experimental results which is

  $$\delta a_\mu = (29.2 \pm 8.6) \times 10^{-10}, \tag{4}$$

a 3.4σ discrepancy. The SM theory precision is about 4-loops, and the dominant SUSY contribution comes in at one loop, so that the supersymmetric contributions can account for the disagreement between theory and experiment. A nice review is Stockinger, hep-ph/0609168.

- Try to draw the Feynman diagrams for the most important of these one-loop contributions. The SUSY contributions are approximately given by

  $$(g-2)_\mu^{SUSY} = \frac{m_\mu^2 \mu \tan\beta}{16\pi^2}\left(g_1^2 F_1 M_1 + g_2^2 F_2 M_2\right) \tag{5}$$

  where m_μ is the muon mass and and $F_{1,2}$ are positive definite functions of the sparticle masses which scale as $1/M_{SUSY}^4$ in the limit of degenerate sparticles of mass M_{SUSY}.

- Plot the values of $\delta a_\mu = (g-2)_\mu^{SUSY}$ and $\chi^2_{\delta a_{mu}}$ in the m_0-$m_{m1/2}$ plane and compare with the formula above. In this light SUSY region the SUSY contributions are too large to be acceptable, while a heavy spectrum leads to a limit where the loop effects decouple. Somewhere in the middle there is a 'sweet spot' where supersymmetry is in good agreement with the data.

- **Flavor Changing Neutral Currents** The flavor changing neutral current fromthe decay of a bottom to a strange quark $BR(B \to X_s\gamma)$ is also quite constrained, from the BABAR and Belle experiments. There are similar one loop diagrams to write down here as in the anomalous magnetic moment case. Have a go at writing some down. A brief discussion of this process can be found in 0810.2874 (Altmannshofer and Wick) In this case the agreement with the Standard Model is quite good, so too light a spectrum is again unacceptable. A heavy spectrum leads to decoupling to the SM result.

- Plot the branching ratio and χ^2 for this process, and then the χ^2 off this process *and* the muon anomalous magnetic moment. Where does the favoured region lie?

  Both of the above quantities are predicted by `micrOMEGAS`. However, it is worth mentioning that there exist more dedicated and accurate programs for $BR(B \to X_S\gamma)$, for instance `SusyBSG`.

- **The Lightest Higgs** As mentioned earlier, the LEP experiment has put bounds on the existence of a Standard Model-like Higgs. This makes it difficult for the Higgs to be lighter than about 114GeV. We incorporate the constraints from those results in this program. The LEP experiment saw a small deviation consistent with the existence of a Higgs boson at about 115-116GeV. This leads to a slight preference for that region. Higgs masses higher than about 117GeV are of course consistent with experiment and are unconstrained.

- Plot m_h and $\chi^2_{m_h}$, as well as $\chi^2_{m_h+\delta a_\mu+bs\gamma}$.
- Since this school is ostensibly about the dark universe, the spin-independent cross section between a neutralino and a proton/neutron are also saved. You will be able to compare these with the recently released results from the CDMSII experiment. The CDMSII result does not affect the region of parameter space we are looking at today. Plot the logarithm of the SI cross sections for these processes.

The final thing to do before moving on is to find the best-fit point. This is the point which has the lowest χ^2. Since the dark matter relic density is probably poorly fit in the two dimensional plane we are investigating, this number will probably be quite large. It may be more instructive to remove the contribution from the relic density from the χ^2_{total}.

Advanced: You can now change some of the parameters in the file main.cpp.

- For instance, you may choose to focus more on the light supersymmetry or heavy supersymmetric regions, or zoom in on the parts of parameter space which best fit the dark matter constraint and investigate the spectra there. Remember to save and recompile the program before running it again.
- You might try adapting the program to scan over $\tan\beta$, for a fixed value of m_0.
- *A priori* there is no reason for the neutralino to constitute the entire dark matter relic density. After all why shouldn't the dark sector be as complicated as the visible sector? On the other hand, if the relic density is higher than the WMAP bound something is certainly wrong with the model. To take this into account adapt the constraint on the dark matter to be a one-sided Gaussian: if the relic density prediction $\Omega_{DM}h^2$ is less than or equal to the WMAP central value the χ^2 should be zero. If it is greater than the WMAP bound then $\chi^2_\Omega = (c-p)^2/\sigma^2$ where c is the central value, p is the prediction for that point from Micromegas and σ is the error on the prediction.
- If you are feeling more adventurous, read through some of the Softsusy manual, and try to change main.cpp to scan over the Λ-M_{mess} or Λ-$\tan\beta$ planes in GMSB.

Appendix A

This lists the entries of each node printed out by softsusy.x for the plotting.

1. m_0
2. $m_{1/2}$
3. χ^2_{total}
4. $\Omega_{DM}h^2$
5. $\chi^2(\Omega_{DM}h^2)$
6. $(g-2)_\mu^{SUSY}$
7. $\chi^2_{(g-2)^{SUSY}}$
8. $BR(B \rightarrow X_s\gamma)$
9. $\chi^2_{bs\gamma}$
10. m_h (GeV)
11. $\chi^2_{m_h}$
12. σ_{SI}^{proton}
13. $\sigma_{SI}^{neutron}$
14. $m_{\tilde{\chi}_1^0}$
15. $m_{\tilde{\chi}_2^0}$
16. $m_{\tilde{\chi}_1^\pm}$
17. $m_{\tilde{\chi}_2^\pm}$
18. $m_{\tilde{g}}$
19. $m_{\tilde{q}_L}$
20. $m_{\tilde{q}_R}$
21. $m_{\tilde{t}_1}$
22. $m_{\tilde{b}_1}$

23. $m_{\tilde{\nu}_1}$

24. $m_{\tilde{e}_R}$

25. m_A

Appendix B: Gnuplot

Gnuplot is an enormously useful plotting program which it is well worth learning how to use. gnuplot will run in the terminal, and produce plots in new windows. To open gnuplot run

```
gnuplot
```

There is extensive help available from inside gnuplot which can be accessed by typing

```
help [command]
```

where the (optional)

$$command$$

is the command about which you would like to learn. There are also extensive tutorials on the web. This Appendix will teach you the bones of making two plots from the scans in Section 2 of this tutorial. First we will set pm3d environment, and set the shape of the plot to be square

```
set pm3d map
set size square
```

Say we have a file, data.txt which has rows corresponding to points in parameter space and columns corresponing to data derived from those points. We want to investigate the variation of an observable, in the 4th column say, over the parameter space. If we've saved the scanned points in the 1st and 2nd columns of our file, then to make a three dimensional plot projected onto a two dimensional plane type

```
splot 'data.txt' u ($1):($2):($4) w pm3d notit
```

This tells gnuplot to make a multidimensional plot (splot) *using* the 1^{st}, 2^{nd} and 4^{th} columns as the x, y and z axes with pm3d and no title. The dollar sign means the column entries are being treated as variables, and therefore you can plot functions of them as well. For instnace, you could plot the total χ^2 without including the dark matter constraint:

```
splot 'data.txt' u ($1):($2):($3-$5) w pm3d notit
```

or the logarithm of the spin-independ proton-neutralino cross-section, which ranges over several orders of magnitude

```
splot 'data.txt' u ($1):($2):(log10($12)) w pm3d notit
```

It's also possible to make 3d dimensional interactive plots using pm3d, which you can rotate on the screen. To do this you need to unset the map option:

```
unset view
set style data pm3d
```

and use the same splot commands as previously.

p