Synchrotron Radiation Output from the ILC Undulator

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Outline

- The analytic tracking code
- Extension of the code to calculate synchrotron radiation
- Characterising the helical undulators from a field map
- Characterising the helical undulators from prototype measurements
- Synchrotron emmission from the helical undulator
- Efficiency of the code

Analytic Tracking Code

- Characterise an arbrtrary magnetic field in terms of it's multipole expansion and generalised gradients to produce an analytical description of field as a fuction of the longitudinal coordinate [a]
- Use the analytical expression in differential algebra or Lie algebra code to generate a Taylor or Lie (symplectic) map for the dynamics in the magnet.
- Evaluate the analytical expressions to perform a numerical integration giving a fast particle tracking code to describe the evolution of the canonical coordinates within the magnet.
- The C++ code that has been has been written has a modular structure which facilitates extending the code
- A Synchrotron Radiation Module is being implemented which calculates the synchrotron emission from a particle into an arbitrary observation point
- eg ILC Helical undulator

[a] Venturini and Dragt

Synchrotron Radiation Calculation

Accelerated charges radiate energy. The observed electric field [a] of the emitted radiation is:

$$\vec{E}(t) = \frac{e}{4\pi\epsilon_0 c}\left(\frac{c(1-|\vec{\beta}|^2)(\vec{n}-\vec{\beta})}{|\vec{R}|^2(1-\vec{n}\dot{\vec{\beta}})^3} + \frac{\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{|\vec{R}|(1-\vec{n}\dot{\vec{\beta}})^3}\right)_{RET}$$

$$\frac{d^2U}{d\Omega dt} = \epsilon_0 c E^2 r^2 = \vec{G}(t)^2$$

$$\vec{G}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\vec{G}(t)\exp(\imath\omega t)dt$$

$$\frac{d^2I}{d\Omega d\omega} = |\vec{G}(\omega)|^2 + |\vec{G}(-\omega)|^2$$

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[a] Jackson

Implementation

At each step of the tracking code, the sychrotron variables, X , $\dot{X}$, $\ddot{X}$ ($X = x, y, z$), are calculated by estimating the radius of curvature of the particle (using the two adjacent integration steps)

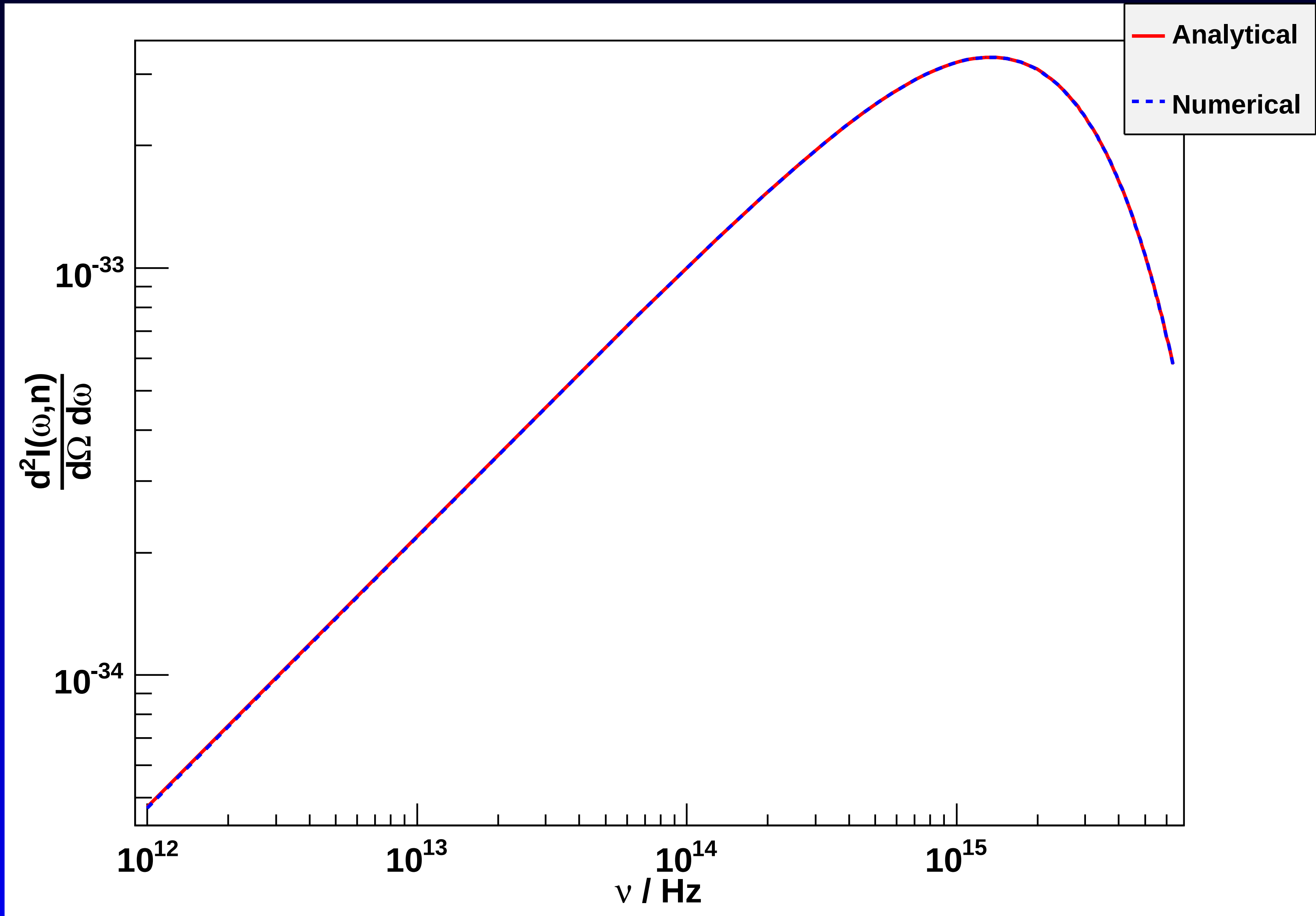
These variables are saved to a file and subsequently used to calculate the electric field at an arbitrary observation point.

At the end of the integration the differential intensity is found by performing a DFT on the electric field components.

Benchmarking

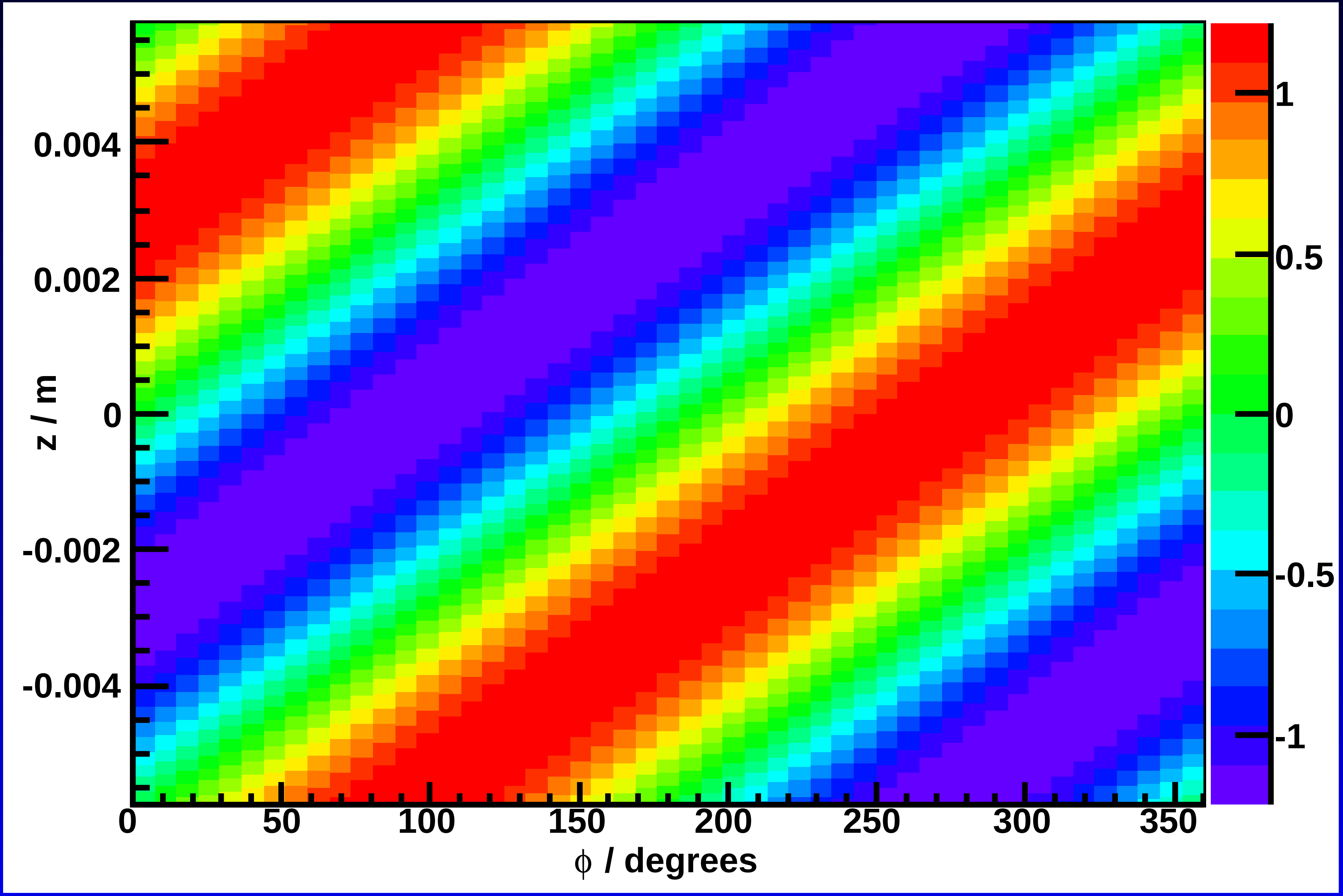
- Starting with a constant magnetic field ($B_y = 1$ T), model the field in terms of it's generalised gradients, produce analytical descriptions of the field and numerically integrate the motion of the particle through the field.
  - $E = 100$ MeV
  - R=33.3333 cm
  - L=10 cm
  - 10,000 integration steps
- At each integration step calculate the electric field observed at an observation point, perform a fourier transform on the field to produce the frequency distribution and compare with the analytical description

Benchmarking the synchrotron calculation

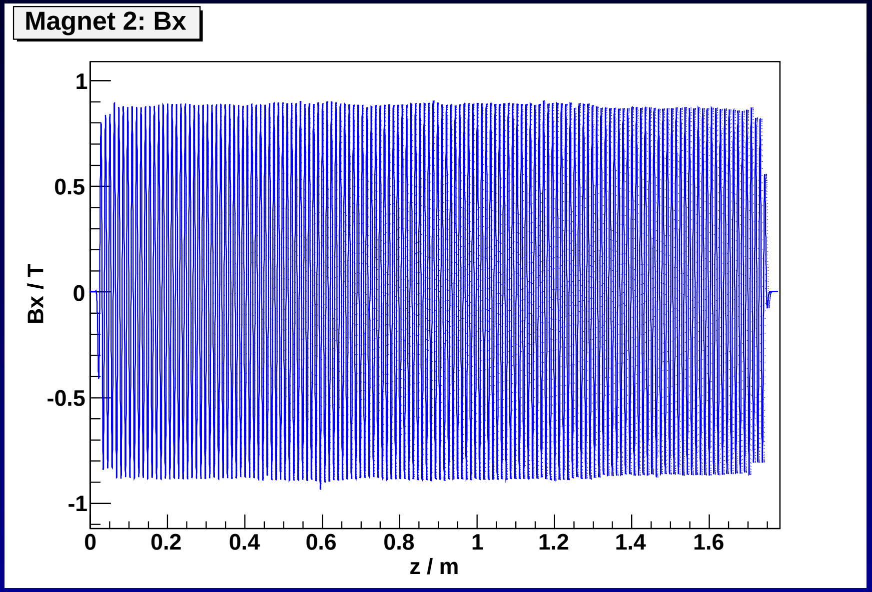


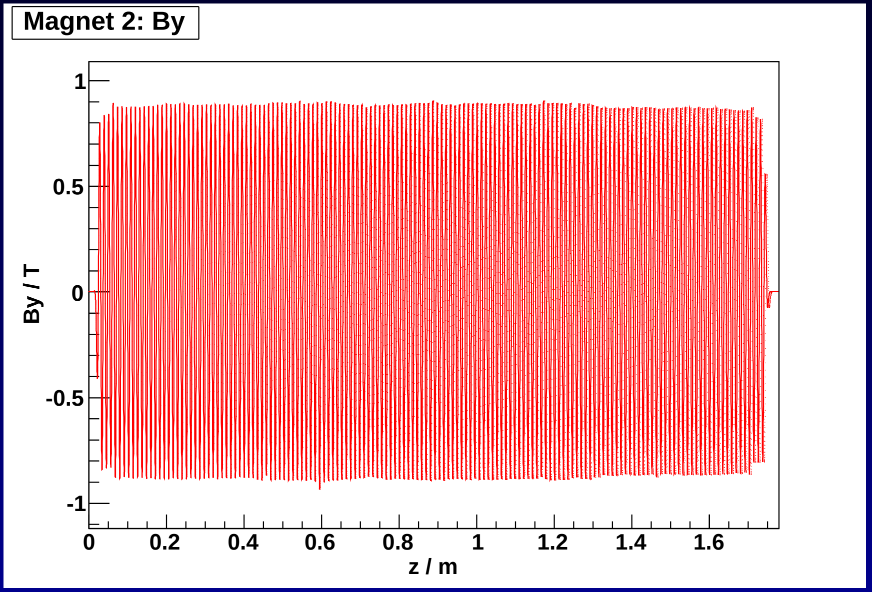
Accurate to $\sim 10^{-4}$

Field Map of the Helical Undulator [a]



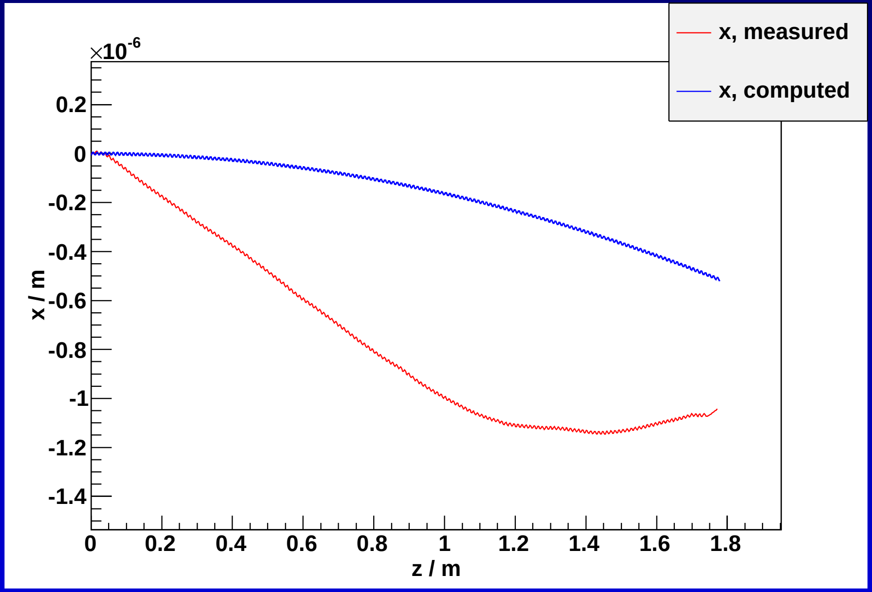
Measurements of the prototype undulator [a]

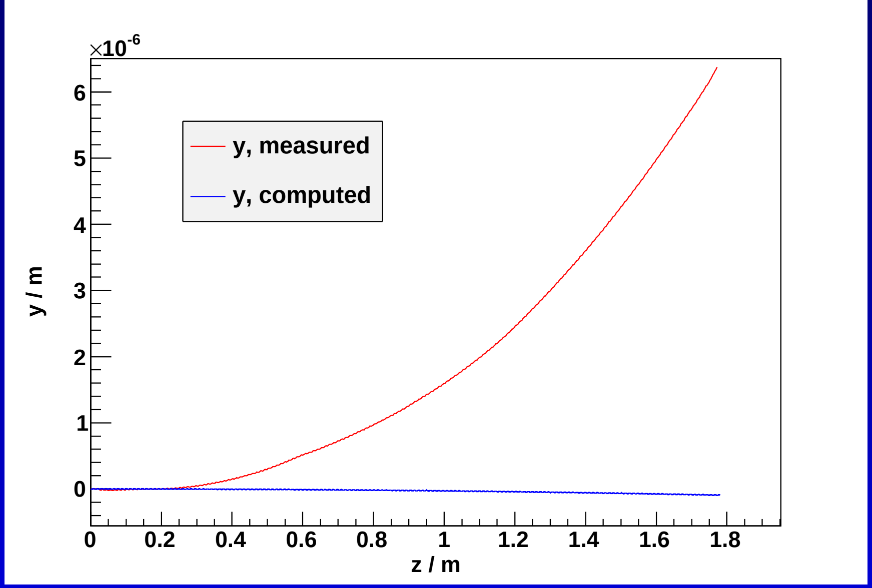




Particle Tracking Comparison

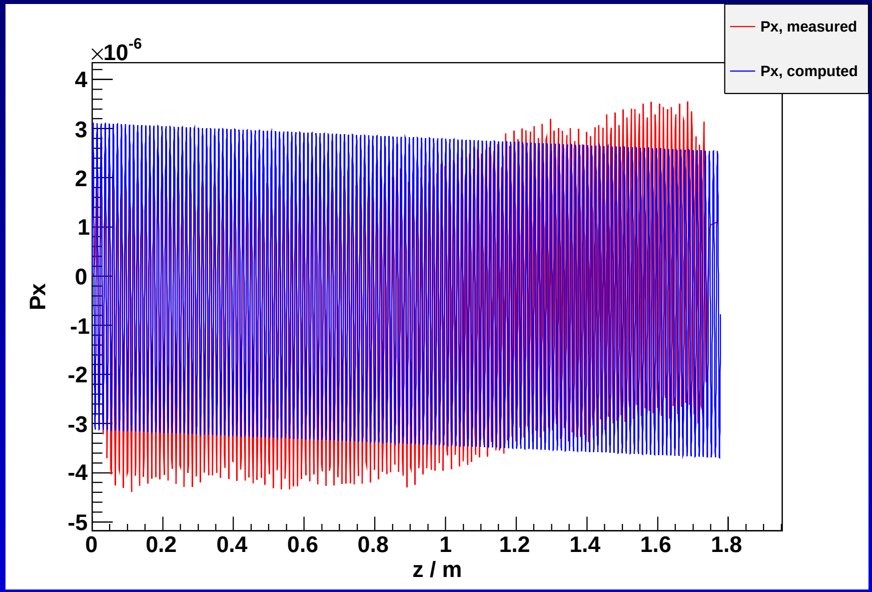
Tracking the dynamical variables x and y with a simulated field map and a measured fieldmap.

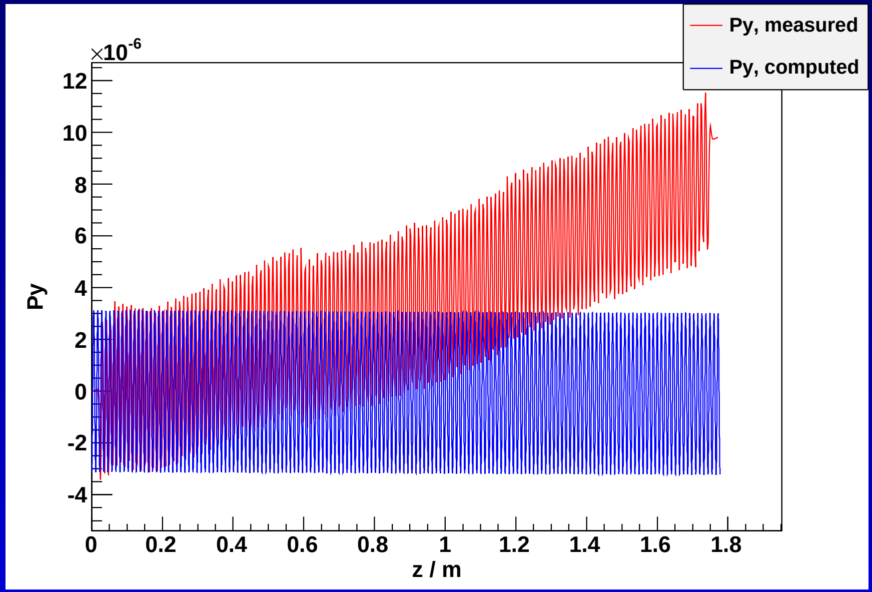




Particle Tracking Comparison

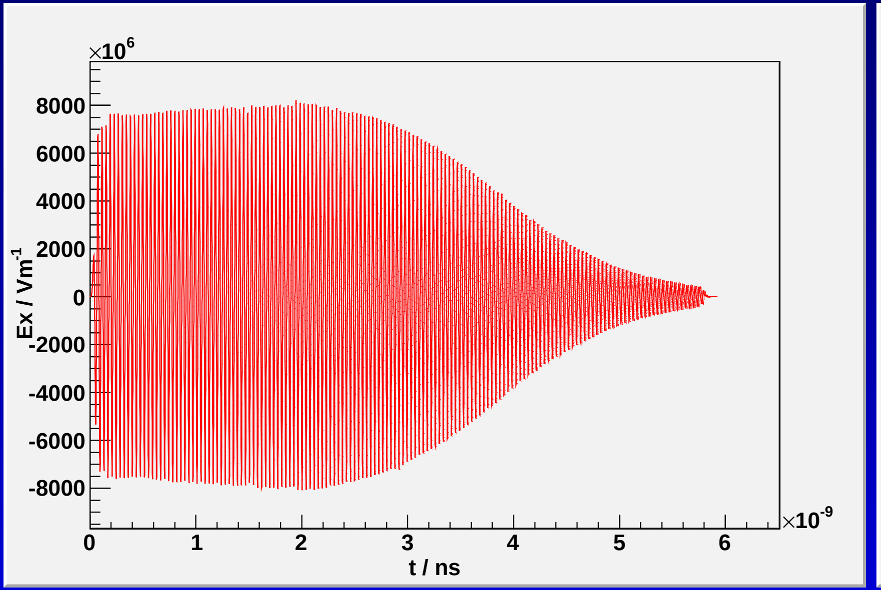
Tracking the dynamical variables Px and Py with a simulated field map and a measured fieldmap.

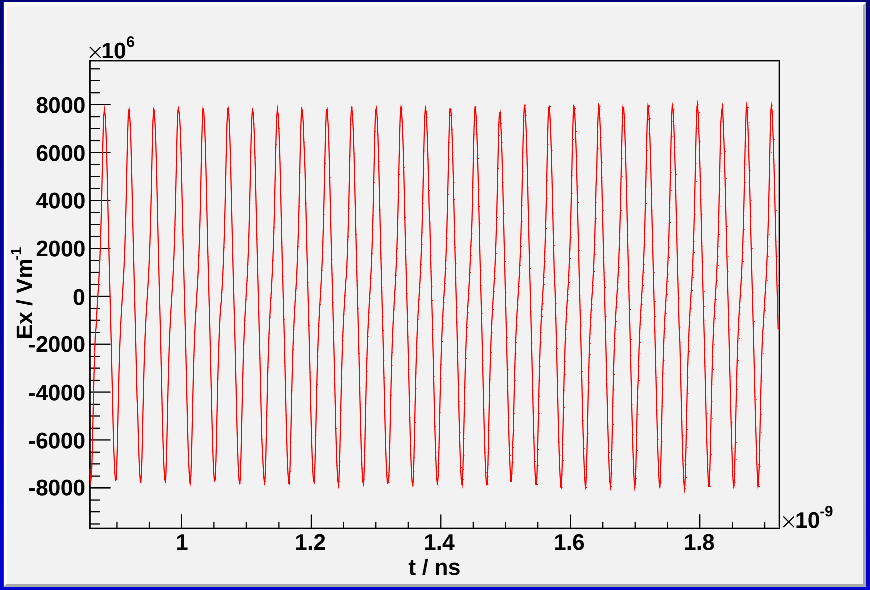




Electric Field

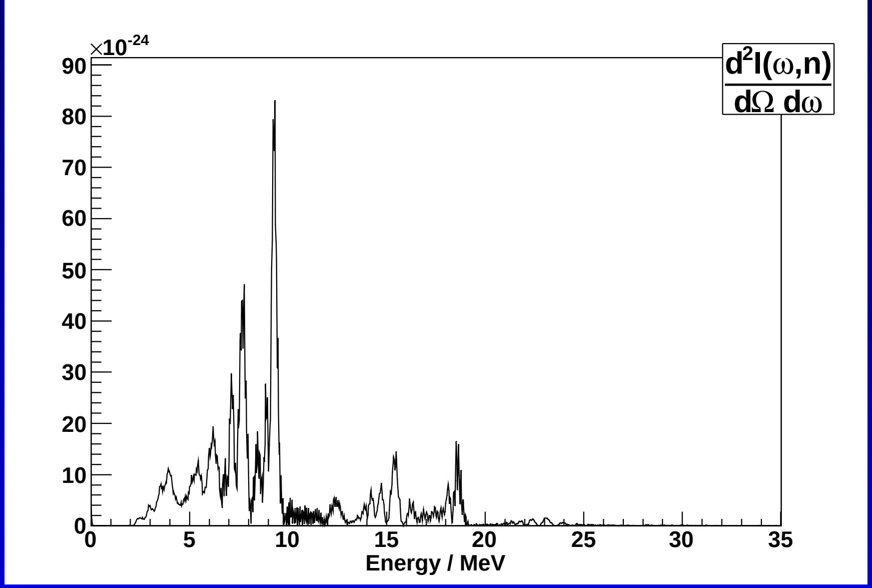
The electric field (Ex) observed on axis, 10 m from the end of the measured undulator fieldmap.

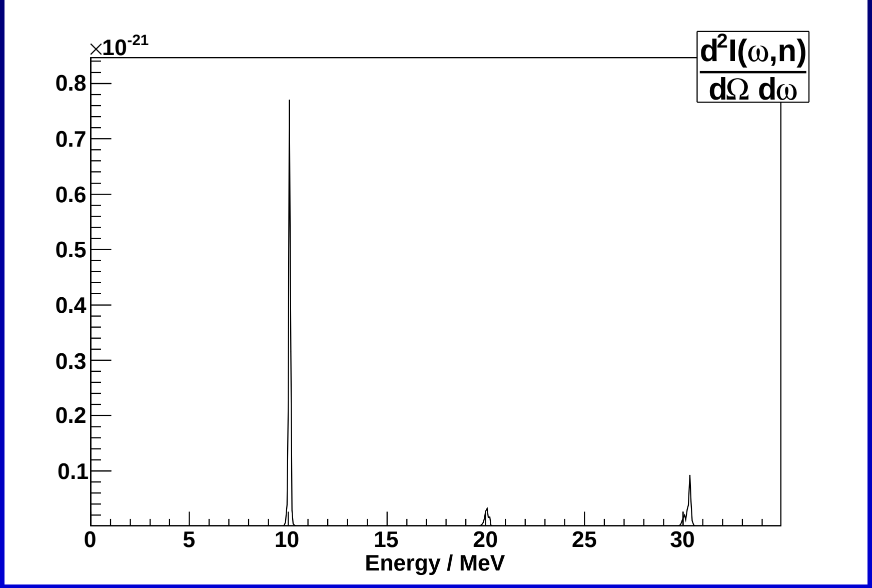




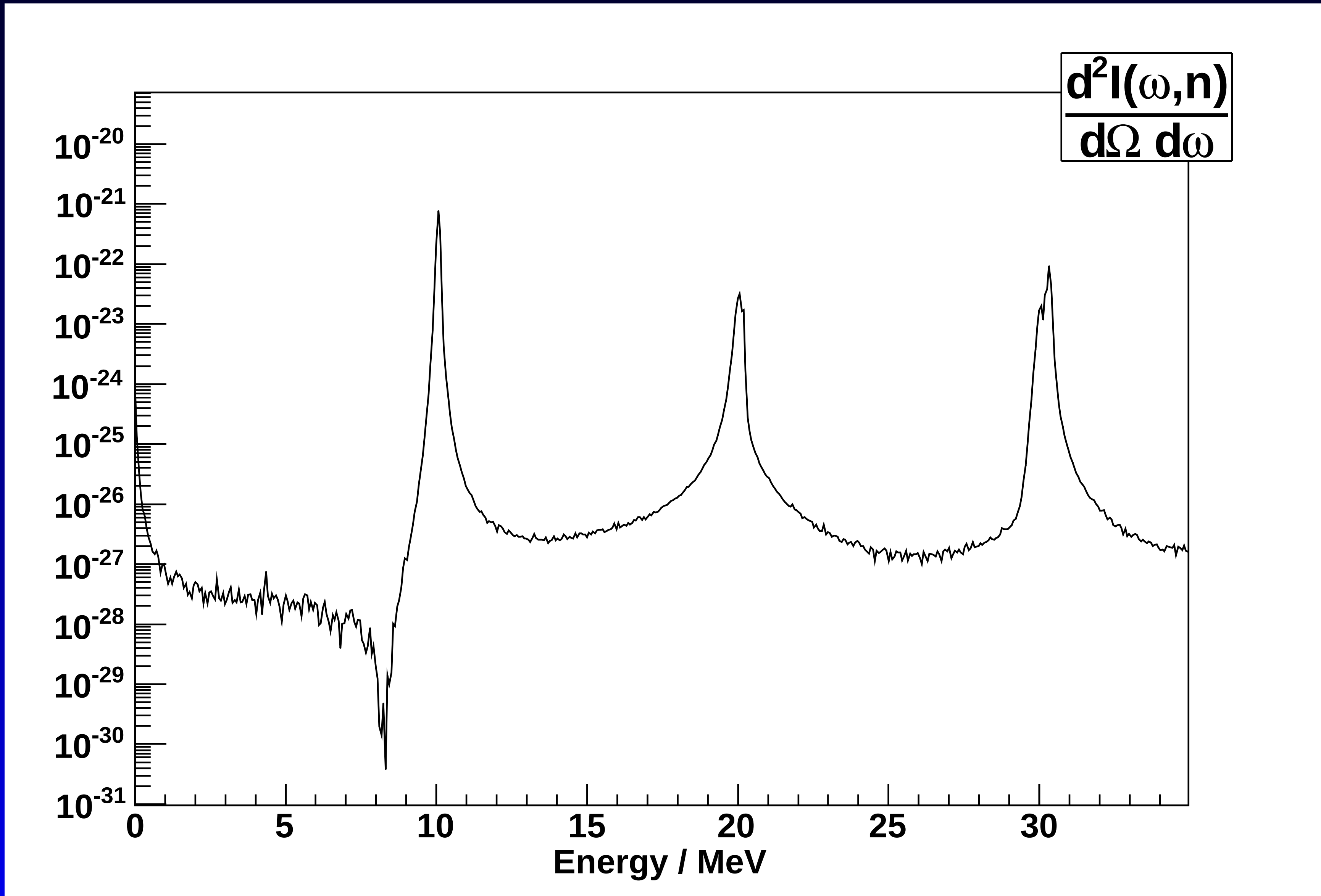
Synchrotron Energy Spectrum Comparison

The energy spectrum, calculated on axis, 10 m from the end of the simulated undulator field map (right) and the measured field map (left)





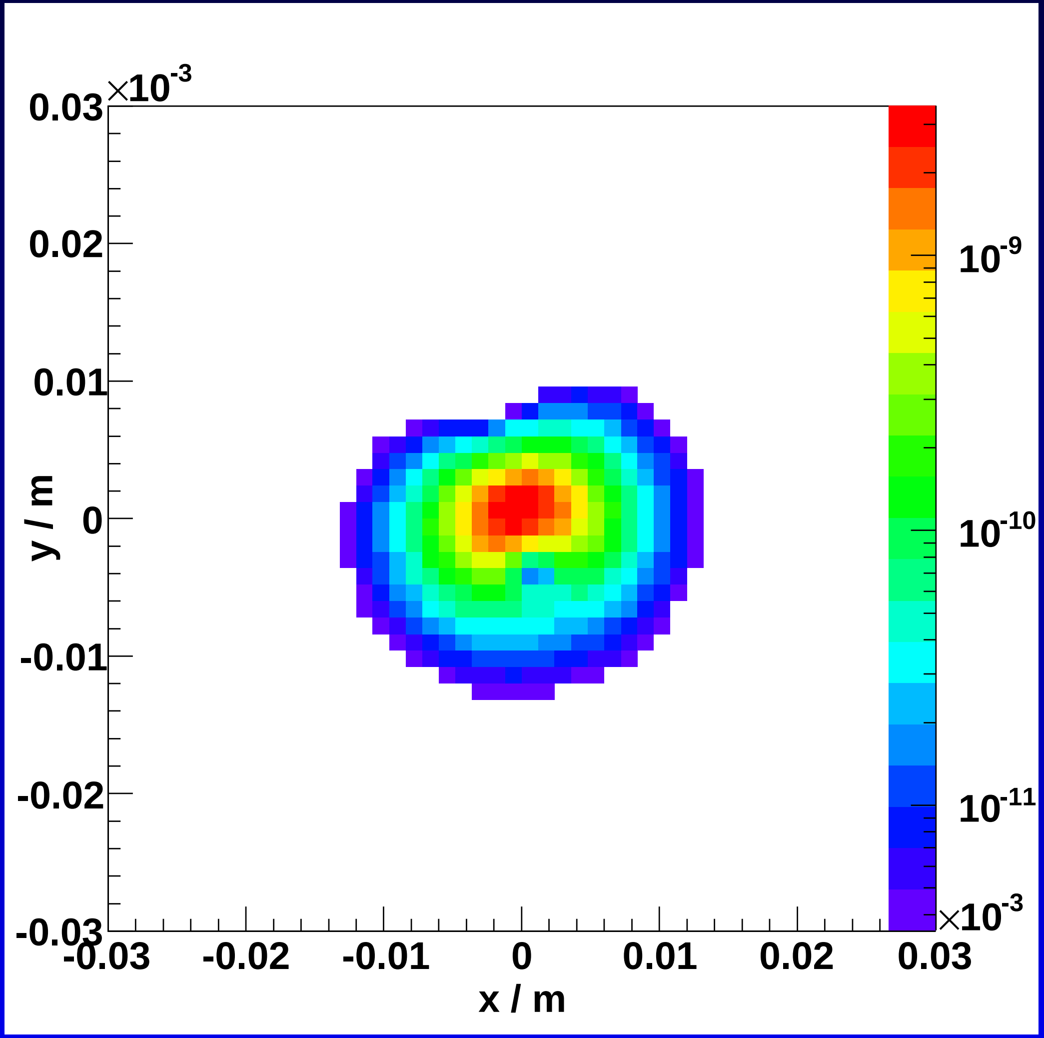
Synchrotron Energy Spectrum (simulated field map)



Energy emitted at the end of the undulator

Energy emitted into 1689 points at the end of the undulator

Problem with scaling (ignore the values)

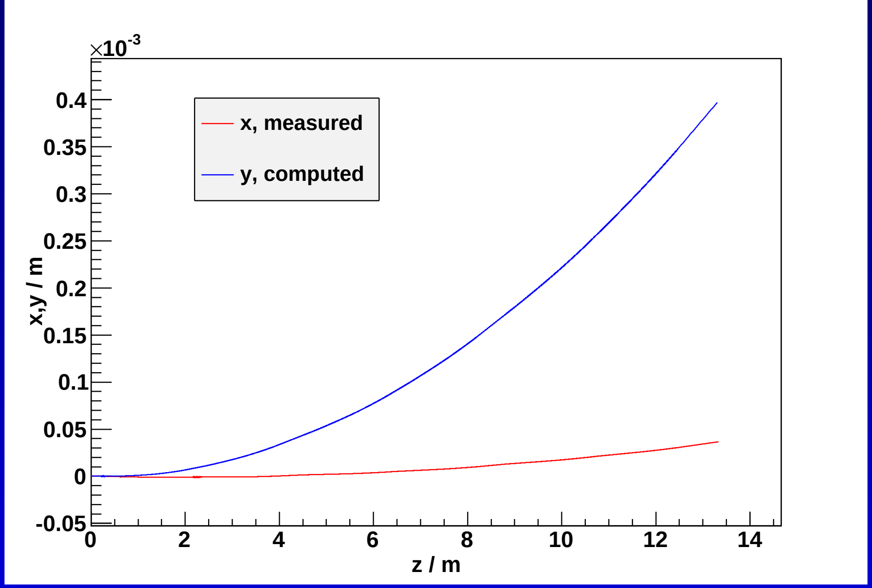


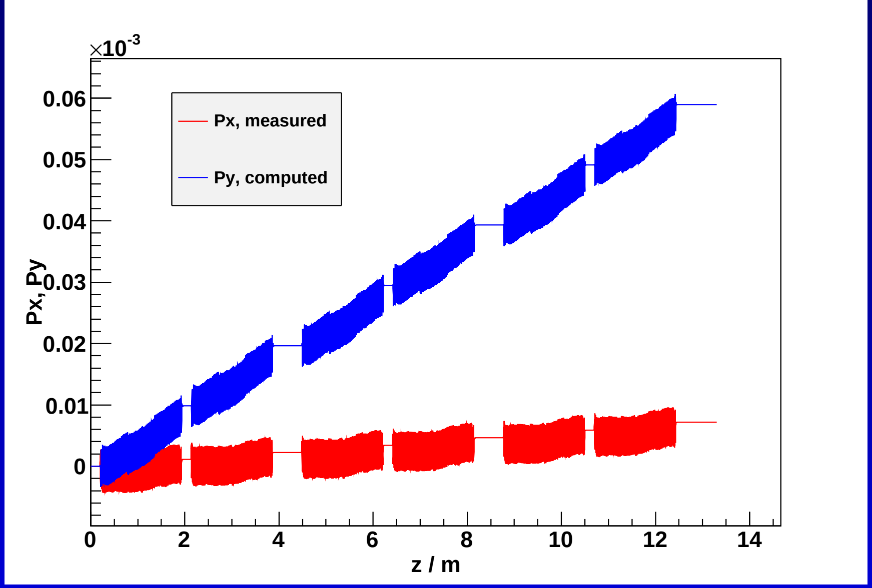
Undulator 6 cell lattice

GAP2	0.19	DRIF
UND	1.79	DRIF
GAP3	0.16	DRIF
UND	1.79	DRIF
GAP2	0.19	DRIF
GAP4	0.2	DRIF
GAP2	0.19	DRIF
UND	1.79	DRIF
GAP3	0.16	DRIF
UND	1.79	DRIF
GAP2	0.19	DRIF
GAP4	0.2	DRIF
GAP2	0.19	DRIF
UND	1.79	DRIF
GAP3	0.16	DRIF
UND	1.79	DRIF
GAP2	0.19	DRIF
GAP1	0.65	DRIF
QDU	0.25	QUAD
QDU	0.25	QUAD
GAP1	0.65	DRIF

Tracking through the lattice

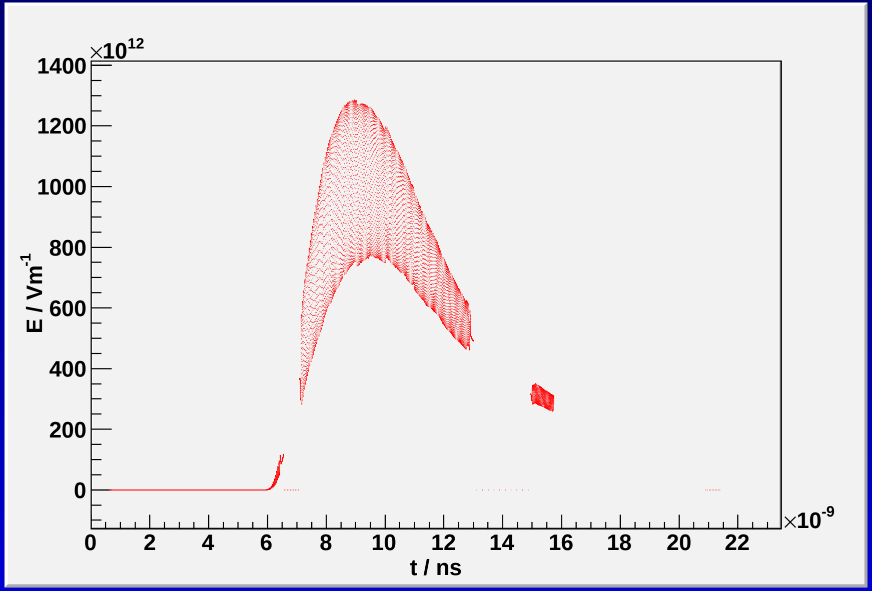
Evolution of the position and momenta variables in the lattice (note the systematic error sums)

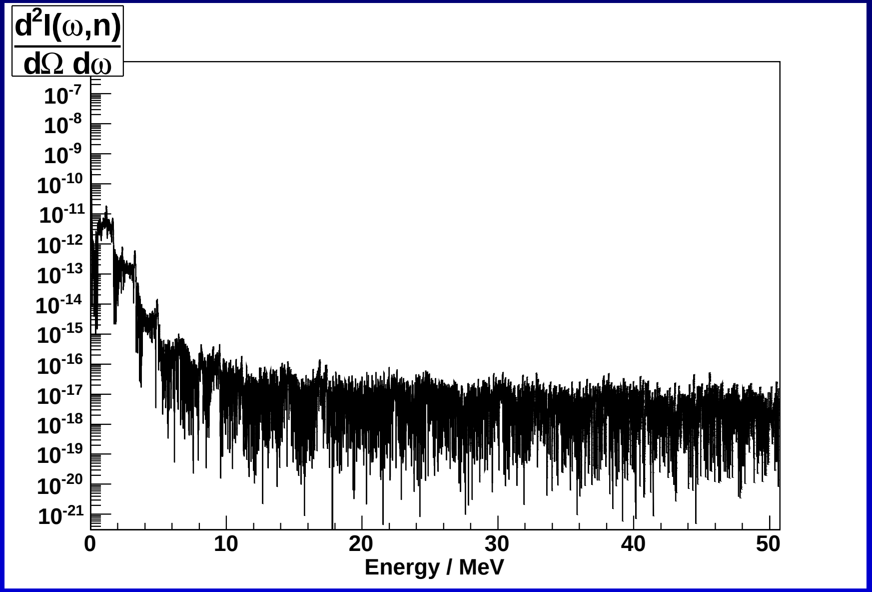




Electric fields and energy spectrum from the lattice

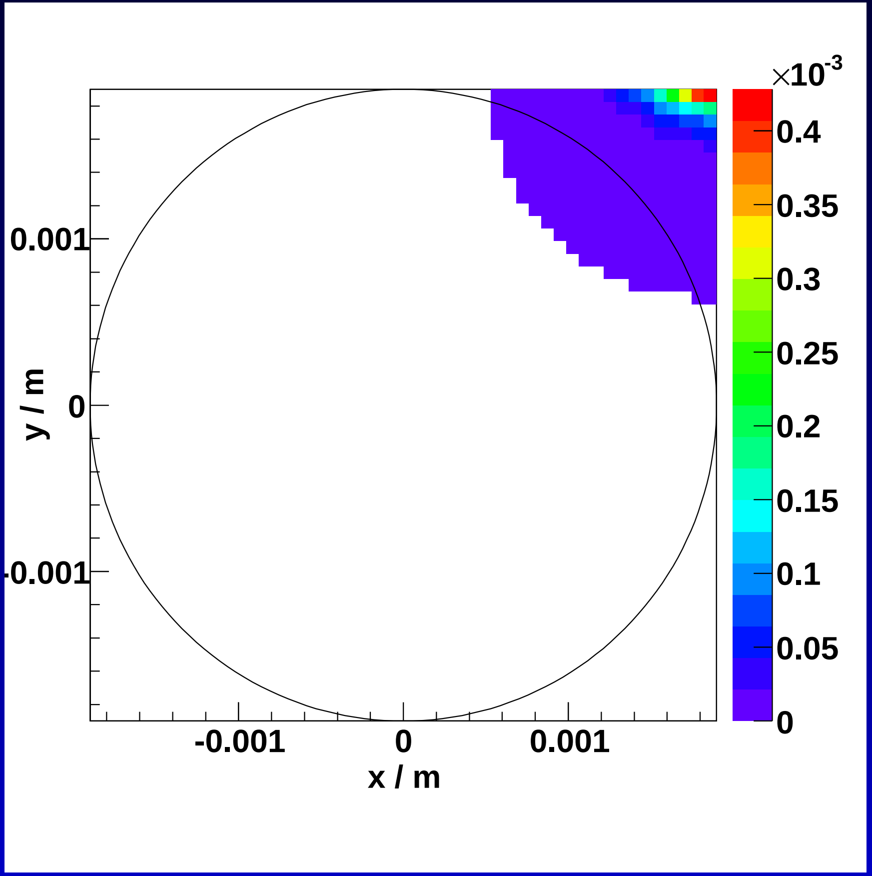
The electric field, Ex, and the energy spectrum calculated on-axis, at the end of the final undulator





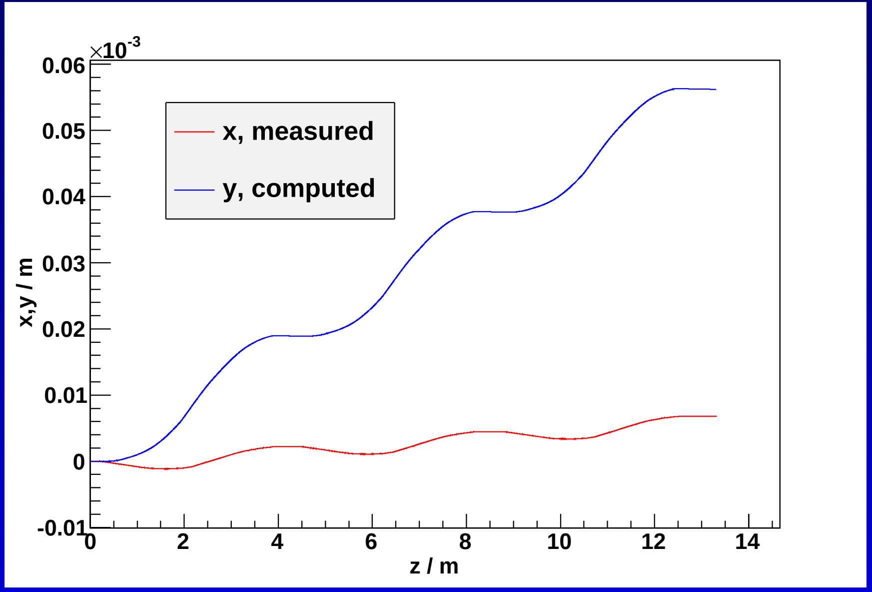
Power output at the lattice end

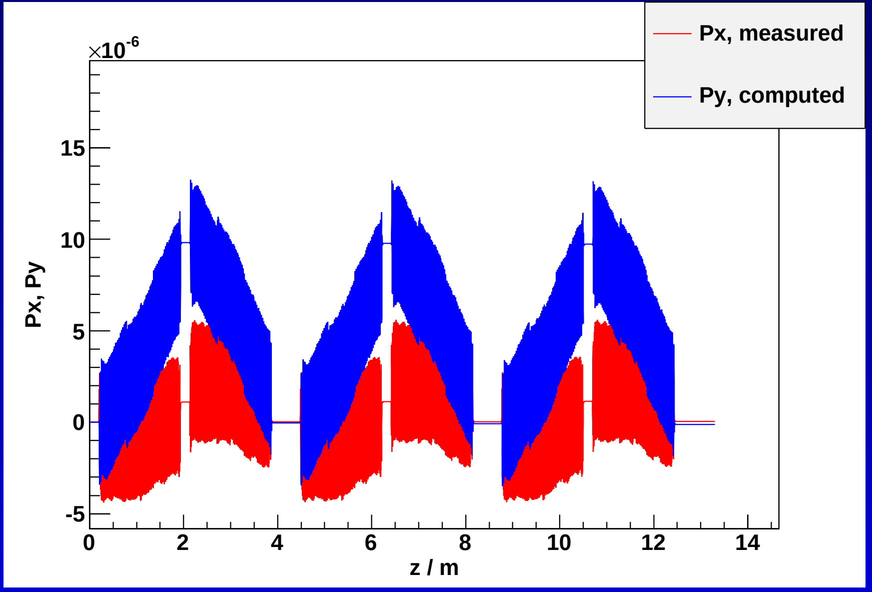
Power output at the end of the final undulator in the lattice (1,965 points)



Tracking through the lattice

Evolution of the position and momenta variables in the lattice (note the systematic error cancels)





Code Efficiency

- Analytic tracking code has been heavily optimised for speed
- Synchrotron extension has not been optimised at all so far
- Typical run times (2.66 GHz processor, 4Mb RAM)
  - Calculate analytic transfer matrices (100,000 steps per undulator) 1 hr 0” 9’
  - Numerical evaluation of the matrices for a particular trajectory and calculation of the synchrotron variables 9’ 11”
  - Calculating fields and power absorbed at an observation point $\sim$ 3”
  - Field interpolation and FFT - not benchmarked ($\sim \mathcal{O}$ seconds)
- Comparisons with other code [a]
- SPUR: (2m field map, 1225 points, 45 processors, (integration steps ???) 4 hrs
- SPECTRA: (2m field map, accuracy level 1 (???)) 68 hrs
- This work: (10,000 integration steps, 2500 points) 2 hrs 5’