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# Target Shock Wave Study



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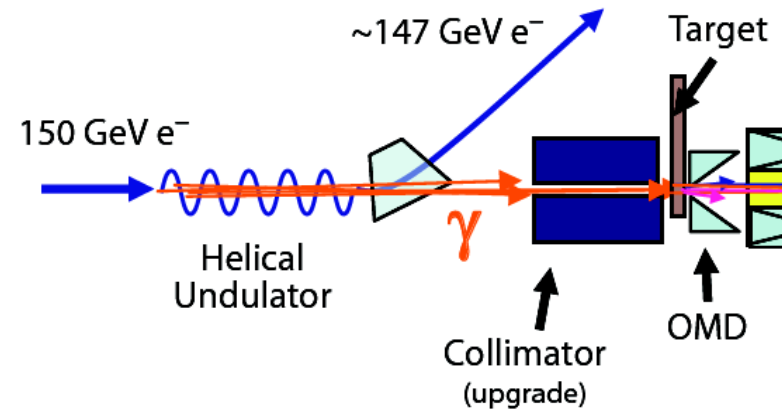
# Outline

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- Introduction
- Thermal shocks in target
- Energy deposition in target
- Simulations with FlexPDE
- Hydrodynamical model for temperature and pressure
- Summary

# Introduction

- Positron source, e.g. ILC RDR:



- Polarized  $\gamma$  on target  $\Rightarrow$  polarized  $e^+$
- Leading production process:  $e^+e^-$  pair creation
  - Quasi-classical approximations
  - Simulation with e.g. GEANT, FLUKA  $\rightarrow$  tested against data
- Possible problems: heat load, thermal shocks in target, ...
- Rotating wheel targets
- Prototype in Daresbury (Ti alloy) [Ian's talk]
- Alternatives: Liquid metals (Bi-Pb, Hg) [e.g. A.A. Mikhailichenko, CBN06-1, 2006]

# Thermal shocks in target

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- Rapid energy deposition of  $\gamma$  beam  $\Rightarrow$  pressure shock wave
- Hydrodynamical model [e.g. A.A. Mikhailichenko, CBN06-1, 2006]
  - $\rightarrow$  Temperature  $T = T(\vec{x}, t)$ , pressure  $P = P(\vec{x}, t)$   
described by hydrodynamical equations
- Simulations at LLNL and Cornell  
[talks at Argonne meeting, Sept. 2007, by T. Piggott and A.A. Mikhailichenko, respectively]
- Cornell simulations
  - FlexPDE
  - Large negative pressure at  $\mathcal{O}(10^{-10} \text{ s})$  at target exit
  - Results: *“Ti target not surviving with present margins”*

# Thermal shocks in target

## Hydrodynamic model behind simulations

E.g. for Cornell simulations [A.A. Mikhailichenko, CBN06-1, 2006, talk at Argonne meeting]

- Temperature:  $\nabla(k\nabla T) + \dot{Q} = \rho c_V \dot{T}$

$\dot{Q}(\vec{x}, t)$ : density of energy deposition;  $c_V$ : heat capacity

- Pressure:  $\ddot{P} - \nabla(c_0^2 \nabla P) = \Gamma / V_0 \dot{Q}$

$c_0$ : speed of sound;  $\Gamma = \Gamma(V) = V / c_V (\partial P / \partial T)_V$

- Gaussian distribution of energy deposition:

$$\dot{Q} = \sum_j \frac{2cQ_{\text{bunch}}}{\pi\sqrt{\pi}\sigma_z\sigma_{\perp}^2 l_T} \frac{z}{l_T} \exp\left(-\frac{(z + z_0 - c(t - jt_0))^2}{\sigma_z^2}\right) \exp\left(-\frac{r^2}{\sigma_{\perp}^2}\right)$$

$\int \dot{Q}(\vec{x}, t) dV dt = Q_{\text{bunch}}$ ;  $\sigma_z, \sigma_{\perp}$ : bunch dimensions;  $l_T$ : target thickness

- Density of energy distribution:  $Q(\vec{x}) = \int \dot{Q}(\vec{x}, t) dt$

# Energy deposition in target

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## Comparison with FLUKA simulations

[Zeuthen group: S. Riemann, A. Schälicke, A. Ushakov]

Include higher harmonics of undulator radiation

[A. Ushakov, talk at Zeuthen meeting, April 2008]

- $\gamma$ -beam intensity extends to larger  $r$  than for Gaussian distribution
- Energy is distributed in larger volume
- Gaussian approximation underestimates volume for  $K \sim 1$
- Better for  $K \lesssim 0.5$

# Simulations with FlexPDE

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- FlexPDE 6.07 available in Zeuthen [S. Riemann, A. Schälicke, A. Ushakov]
- Solves Partial Differential Equations (PDEs)
- 2-dim description of target via cylinder coordinates

```
COORDINATES XCYLINDER
```

```
...
```

```
BOUNDARIES
```

```
REGION 1
```

```
START(0,0)
```

```
LINE TO (15,0)
```

```
LINE TO (15,15)
```

```
LINE TO (0,15)
```

```
LINE TO CLOSE
```

→ Natural boundary conditions

# Simulations with FlexPDE

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- For  $P = 0$  on target surface ( $z = 0$  and  $z = 15$  mm)

```
BOUNDARIES
```

```
REGION 1
```

```
START(0,0)
```

```
LINE TO (15,0)
```

```
Value(p)=0
```

```
LINE TO (15,15)
```

```
Natural(p)=0
```

```
LINE TO (0,15)
```

```
Value(p)=0
```

```
LINE TO CLOSE
```

- Read in e.g. energy distribution from FLUKA simulation

```
Q0 = table('energy.tbl')
```



# Hydrodynamical model

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## Heat Equation

$$\nabla(k\nabla T) + \dot{Q} = \rho c_V \dot{T}$$

$\dot{Q}(\vec{x}, t)$ : density of energy deposition;  $c_V$ : heat capacity

- Heat flow in  $O(1 \text{ s}) \rightarrow$  energy deposition  $\dot{Q}$  instantaneous
- FlexPDE:

INITIAL VALUES

temp=t0 + cfac\*Q0/(C\*rho)

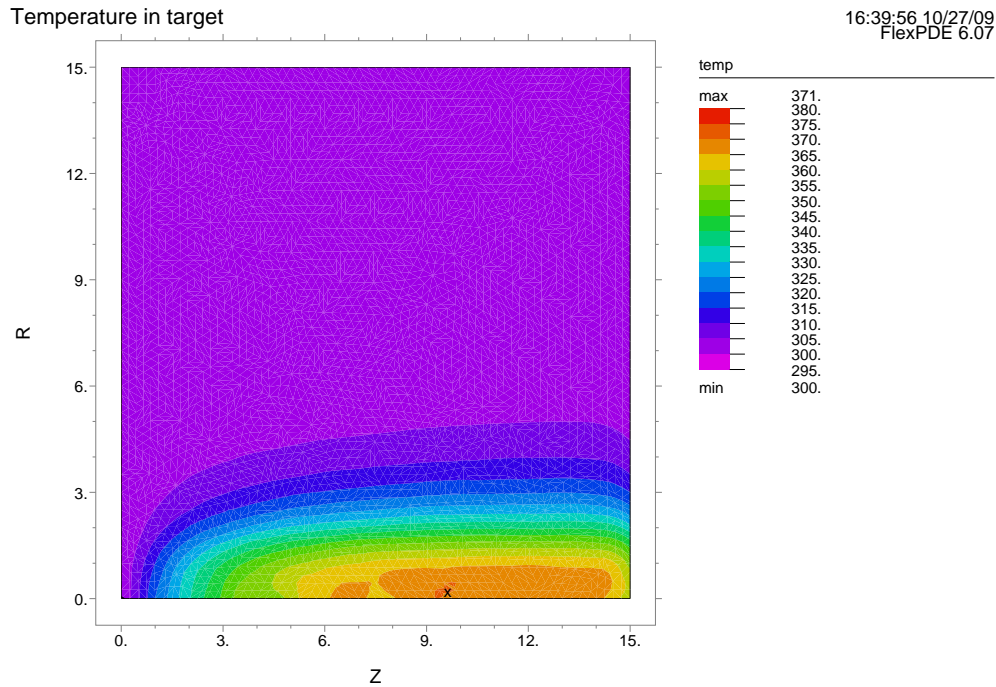
EQUATIONS { PDE's, one for each variable }

div(K\*grad(temp)) = C\*rho\*dt(temp) { heat equation }

# Hydrodynamical model

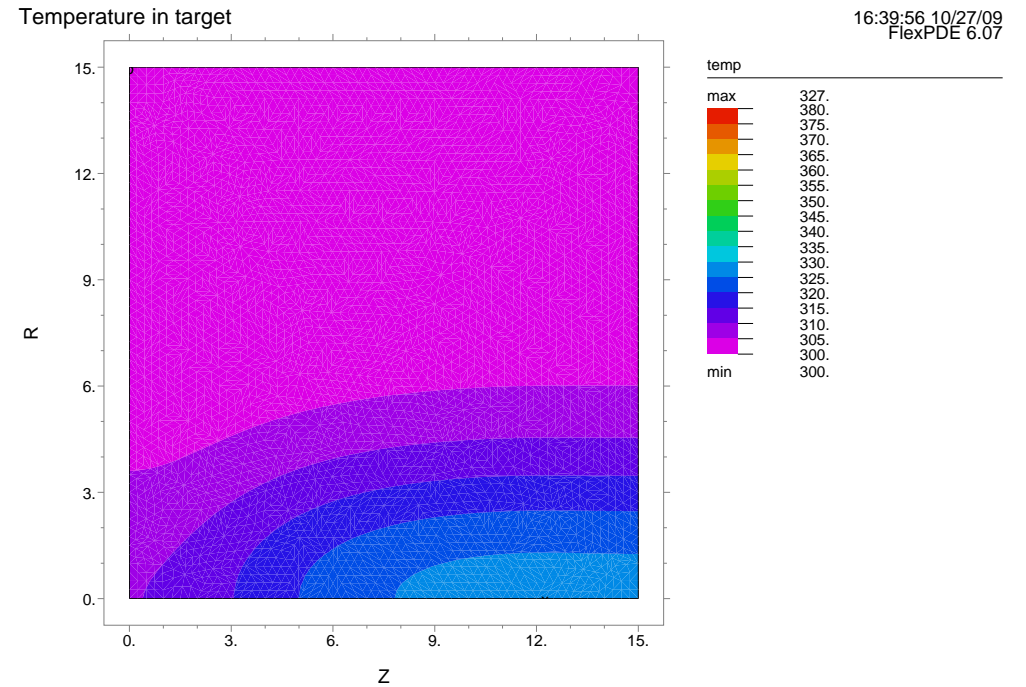
## Heat Equation

$t = 0$



temperature1: Cycle=0 Time= 0.0000 dt= 0.0100 P2 Nodes=1105 Cells=522 RMS Err= 1.  
Vol\_Integral= 3202838.

$t = 1 \text{ s}$



temperature1: Cycle=33 Time= 1.0000 dt= 0.0891 P2 Nodes=1184 Cells=561 RMS Err= 9.5e-5  
Vol\_Integral= 3202863.

for 100 bunches on 1 spot

# Hydrodynamical model

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## PDE for pressure

$$\ddot{P} - \nabla(c_0^2 \nabla P) = \Gamma/V_0 \dot{Q}$$

$c_0$ : speed of sound;  $\Gamma = \Gamma(V) = V/c_V (\partial P/\partial T)_V$

- 2nd time derivative  $\Rightarrow$  split in 2 PDE for FlexPDE:

EQUATIONS

$$p_t: \quad \text{dt}(p_t) - \text{div}(c_0 * \text{grad}(p)) = \text{Gamma} * \text{qdot}(t)$$

$$p: \quad p_t - \text{dt}(p) = 0$$

- Analyse pressure at time scale  $\mathcal{O}(10^{-10} \text{ s})$

- Use Gaussian approximation for  $\dot{Q}$ :

$$\text{qdot}(t) = \text{const} / \text{sigr}^2 * z * \text{Exp}(-(z - c * t)^2 / \text{sigz}^2) * \text{Exp}(-r^2 / \text{sigr}^2)$$

- Time scale of bunch crossing target:  $l_T/c \sim 5 \cdot 10^{-11} \text{ s}$

# Hydrodynamical model

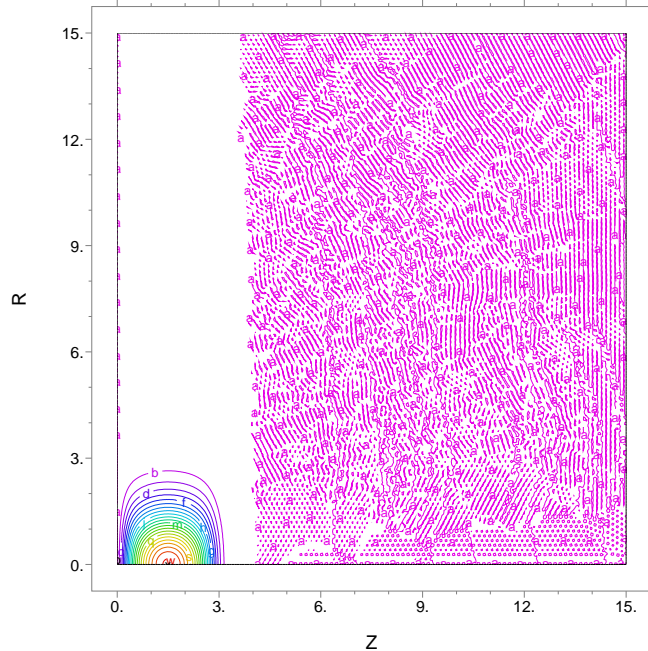
## PDE for pressure

$$t = 10^{-11} \text{ s}$$

Contours of  $P$  in a.u.

Heatflow and pressure in target with qdot

19:13:23 10/19/09  
FlexPDE 6.07

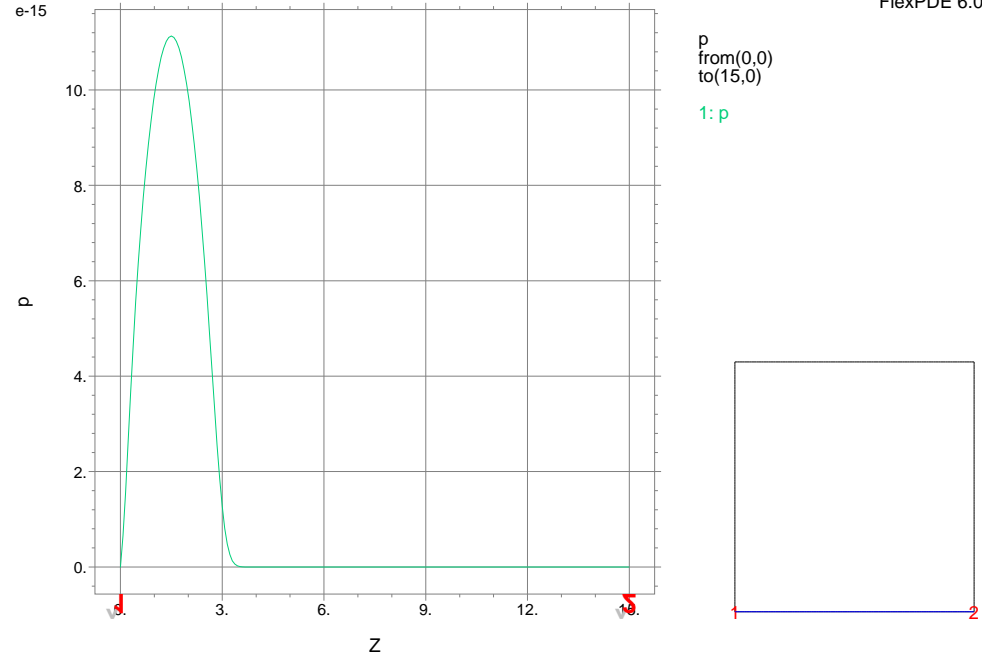


pressure\_qdot3: Cycle=106 Time= 1.0000e-11 dt= 8.8914e-14 P2 Nodes=44405 Cells=22002 RMS Err= 2.8e-6  
Vol\_Integral= 1.584657e-13

$P$  on beam axis

Heatflow and pressure in target with qdot

19:13:23 10/19/09  
FlexPDE 6.07



pressure\_qdot3: Cycle=106 Time= 1.0000e-11 dt= 8.8914e-14 P2 Nodes=44405 Cells=22002 RMS Err= 2.8e-6  
Surf\_Integral= 1.408875e-18

# Hydrodynamical model

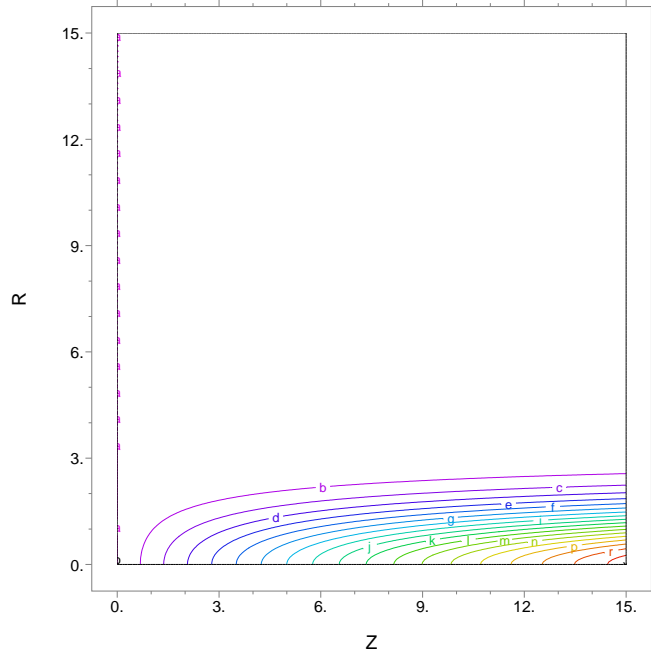
## PDE for pressure

$$t = 3 \cdot 10^{-10} \text{ s, natural boundary conditions}$$

Contours of  $P$  in a.u.

Heatflow and pressure in target with qdot

19:13:23 10/19/09  
FlexPDE 6.07



p	
max	5.56
s :	5.40
r :	5.10
q :	4.80
p :	4.50
o :	4.20
n :	3.90
m :	3.60
l :	3.30
k :	3.00
j :	2.70
i :	2.40
h :	2.10
g :	1.80
f :	1.50
e :	1.20
d :	0.90
c :	0.60
b :	0.30
a :	0.00
min	-9e-4

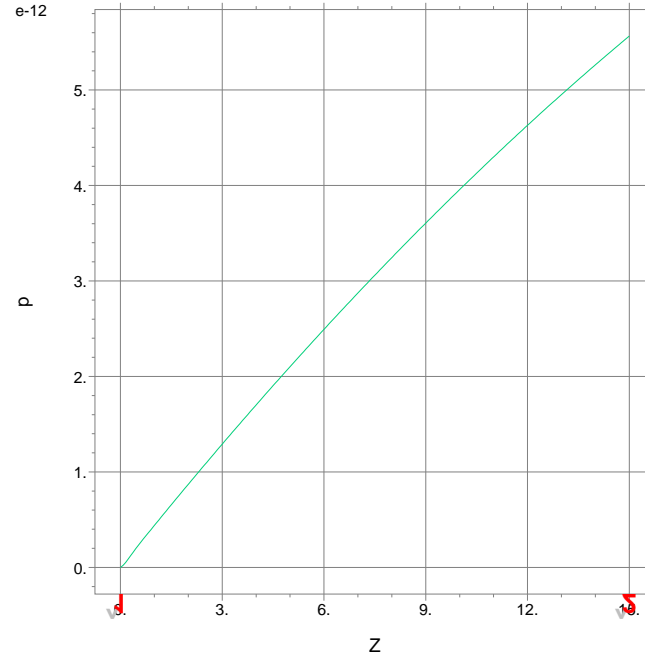
Scale = E-12

pressure\_qdot3: Cycle=3006 Time= 3.0000e-10 dt= 8.8914e-14 P2 Nodes=44405 Cells=22002 RMS Err= 3.e-14  
Vol\_Integral= 3.145502e-10

$P$  on beam axis

Heatflow and pressure in target with qdot

19:13:23 10/19/09  
FlexPDE 6.07



p  
from(0,0)  
to(15,0)  
1: p

pressure\_qdot3: Cycle=3006 Time= 3.0000e-10 dt= 8.8914e-14 P2 Nodes=44405 Cells=22002 RMS Err= 3.e-14  
Surf\_Integral= 2.796767e-15

# Hydrodynamical model

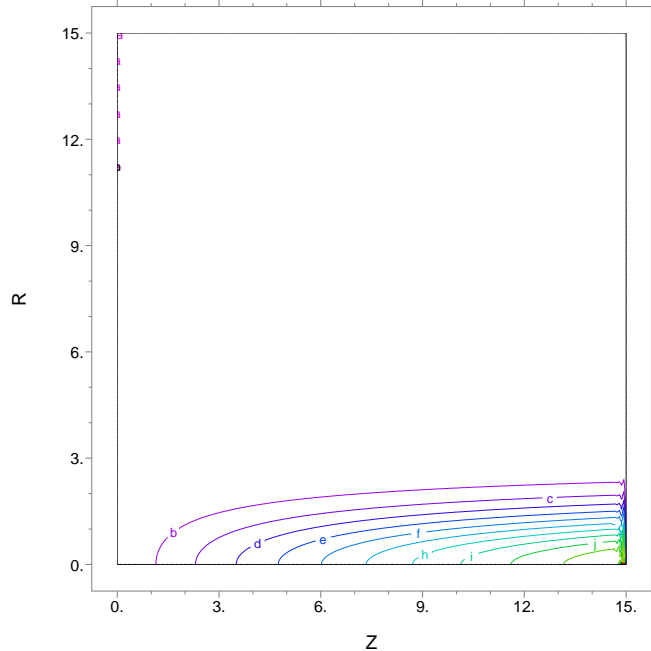
## PDE for pressure

$$t = 3 \cdot 10^{-10} \text{ s}, P = 0 \text{ for } z = 0, 15 \text{ mm}$$

Contours of  $P$  in a.u.

Heatflow and pressure in target with qdot

15:18:00 10/20/09  
FlexPDE 6.07



p	
max	8.12
q:	8.00
p:	7.50
o:	7.00
n:	6.50
m:	6.00
l:	5.50
k:	5.00
j:	4.50
i:	4.00
h:	3.50
g:	3.00
f:	2.50
e:	2.00
d:	1.50
c:	1.00
b:	0.50
a:	0.00
min	0.00

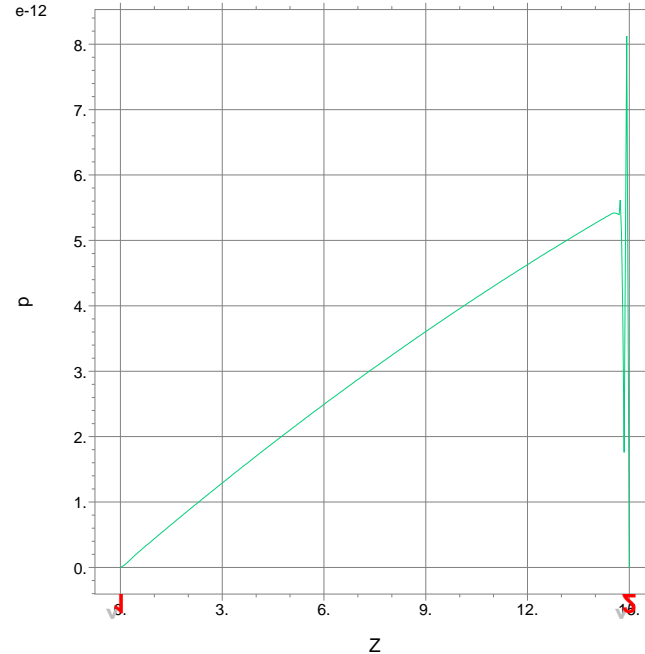
Scale = E-12

pressure\_qdot3: Cycle=3006 Time= 3.0000e-10 dt= 8.8914e-14 P2 Nodes=44405 Cells=22002 RMS Err= 3.e-14  
Vol\_Integral= 3.130920e-10

$P$  on beam axis

Heatflow and pressure in target with qdot

15:18:00 10/20/09  
FlexPDE 6.07



p  
from(0,0)  
to(15,0)  
1: p

pressure\_qdot3: Cycle=3006 Time= 3.0000e-10 dt= 8.8914e-14 P2 Nodes=44405 Cells=22002 RMS Err= 3.e-14  
Surf\_Integral= 2.777251e-15

# Hydrodynamical model

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## Work in progress

- Improvement of simulation
  - Boundary conditions
  - Moving mesh
- Timescales:  $\mathcal{O}(10^{-10} \text{ s}) \leftrightarrow \mathcal{O}(10^{-7} \text{ s})$  for pressure waves  
(speed of sound in  $\ddot{P} - \nabla(c_0^2 \nabla P) = \Gamma/V_0 \dot{Q}$ )
- Improve description of  $\dot{Q}$
- Improvement of hydrodynamical model

# Summary

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- Goal: Check possible pressure shock waves at target exit at  $\mathcal{O}(10^{-10} \text{ s})$
- Simulations with FlexPDE
- Hydrodynamical model
  - Heat equation:  $\nabla(k\nabla T) + \dot{Q} = \rho c_V \dot{T} \rightarrow$  time scale  $\mathcal{O}(1 \text{ s})$
  - Pressure:  $\ddot{P} - \nabla(c_0^2 \nabla P) = \Gamma/V_0 \dot{Q}$ 
    - $\rightarrow$  time scale  $\mathcal{O}(10^{-10} \text{ s})$  for  $\dot{Q}$
    - and  $\mathcal{O}(10^{-7} \text{ s})$  for pressure wave ( $c_0$ )
- $\rightarrow$  Work in progress