AdS/CFT and holographic superconductors

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[arXiv:0907.3796 (PRL 103:151601, 2009), arXiv:0912.0512]

and work in progress with Ben Withers

Plan of talk

- Brief review of Quantum Criticality and Superconductivity
- Brief review of the “AdS/CFT correspondence”
- Phenomenological approaches to holographic materials
- String/M Theory models of superconductors

Reviews

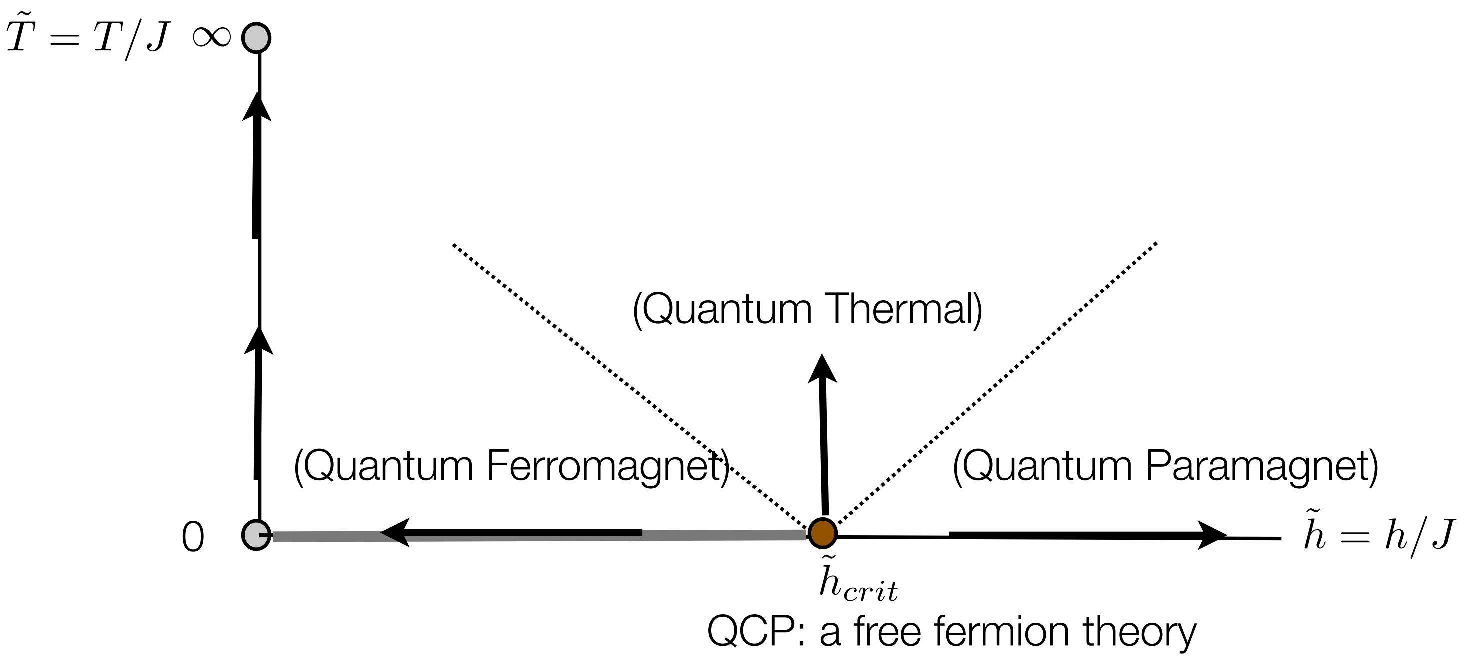
- Quantum phase transitions: Sachdev
- AdS/CFT & superconductors: Hartnoll 09, Herzog 09

Thermal and quantum phase transitions

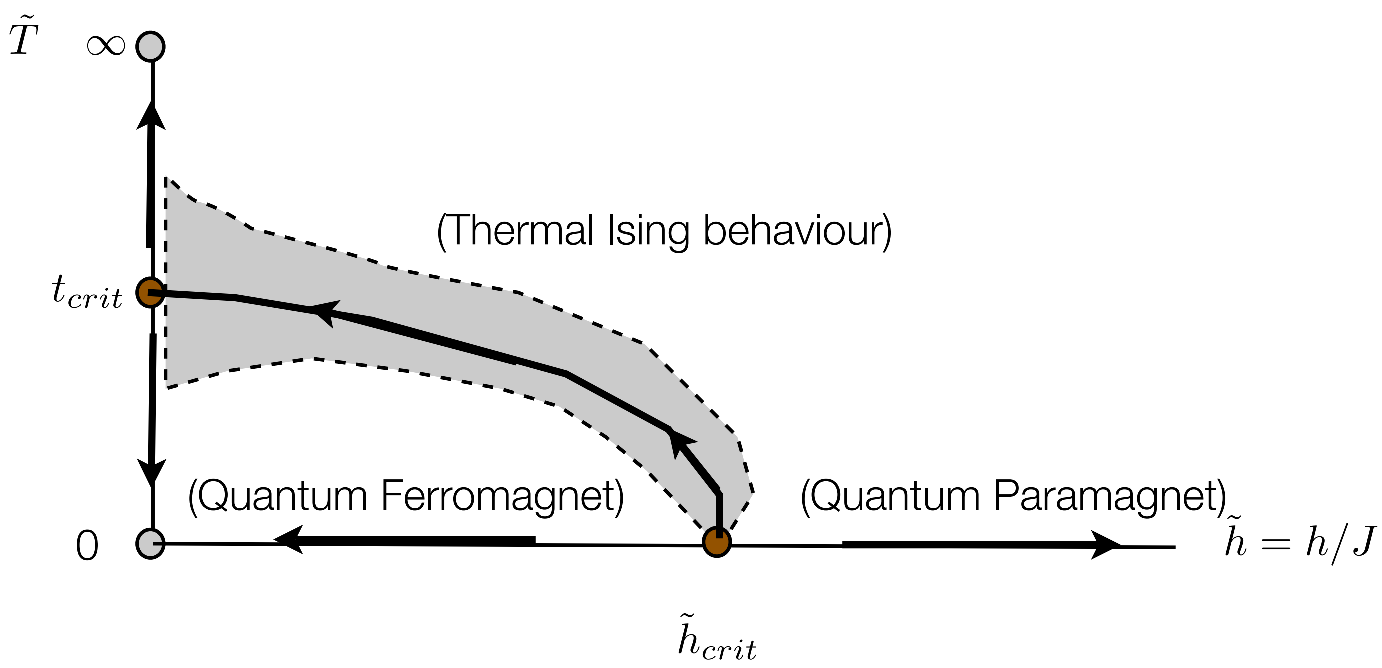
- At zero temperature a classical system cannot have second order phase transitions (as there are no fluctuations).
- However a quantum system has quantum fluctuations even at zero T. These may become correlated at long ranges even at zero T -- this is a critical quantum phase transition (or QCP).

Example: Ising model in d=1

- Quantum Ising model $\hat{H} = -J\sum_i \hat{s}_i^z \hat{s}_{i+1}^z - h\sum_i \hat{s}_i^x$



Ising d>1



$\tilde{T}$
∞
(Thermal Ising behaviour)
t_{crit}
(Quantum Ferromagnet)
(Quantum Paramagnet)
0
$\tilde{h} = h/J$
$\tilde{h}_{crit}$

So why are they interesting...?

- QCP `controls' phase diagram (should have 2 relevant directions).
- Interesting real world materials have properties (eg. superconductivity) that are conjectured to be described by a nearby QCP.

- We may reformulate a d-dim'l quantum system at finite temperature as a classical (1+d)-dim'l path integral in compact Euclidean time, radius 1/T.
- The anisotropy of (Euclidean) time and space means we expect Lifshiftz scaling in general, with scaling coefficient 'z'; $\xi \sim \xi_\tau^z \to \infty$
    - CFT for z = 1

- The 'Landau-Ginsberg-Wilson' theory at the fixed point is now includes time.

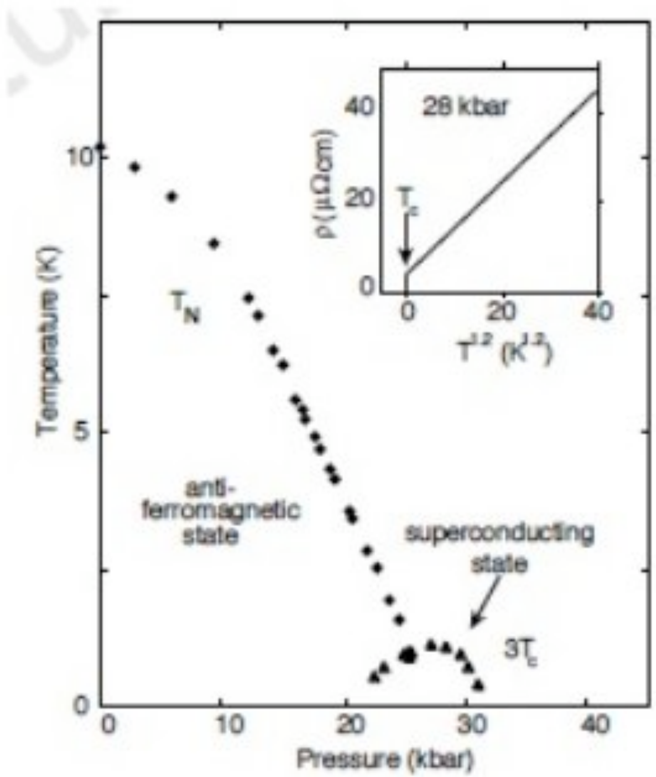
Superconducting metal films (~ 1+2 dim'l)

- Conventional superconductors
  - BCS theory: charged quasiparticles (Cooper pairs of electrons, eg. bound by phonon exchange) coupled to EM - broken U(1) -> superconductivity

- Non-conventional / non BCS
  - QCP *may* be important in understanding superconductor transition if it occurs nearby. Then expect a LGW theory with emergent local U(1) gauge field and associated charge current
  - A nearby QCP may indicate strongly coupled quantum behaviour is important.

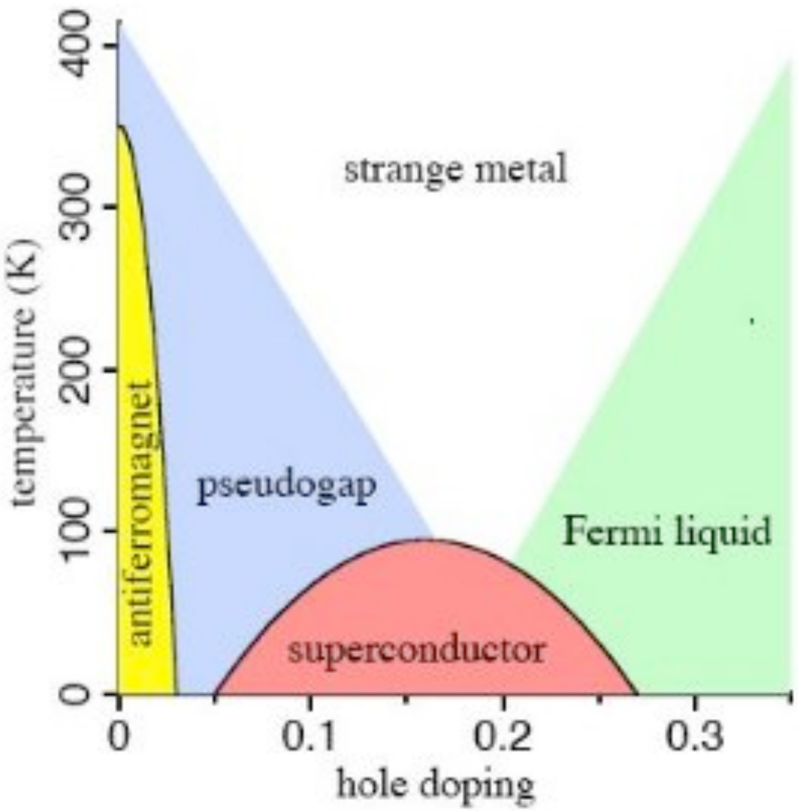
Superconductors and Quantum Criticality

- Heavy fermion systems: eg. $CePd_2Si_2$
- Existence of QCP is clear. However not clear that there is a weakly coupled quasiparticle picture - although might be BCS (d-wave magnetic pairing)



Superconductors and Quantum Criticality

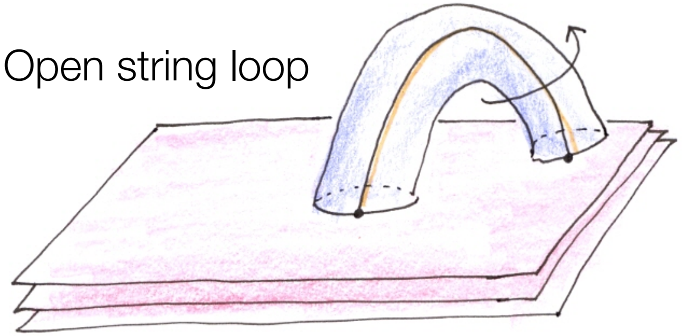
- Cuprates: eg. $La_{2-x}Sr_xCuO_4$
- Existence of QCP less certain



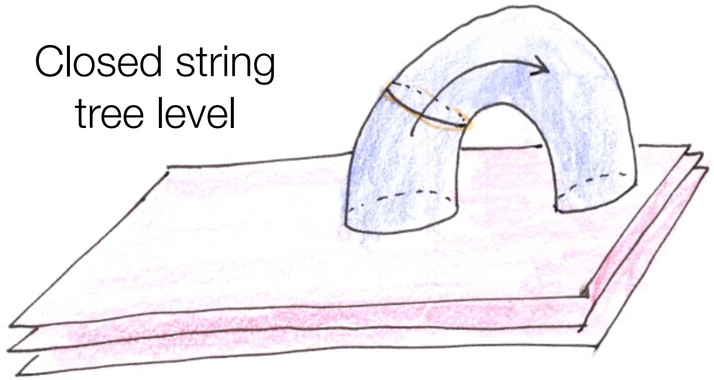
AdS/CFT: Decoupling of D3-branes

- D-branes are objects in open string perturbation theory that open strings end on.

Open string loop



Closed string
tree level



- Alternatively they are non-perturbative sources for closed strings eg. $T \sim 1/g_s$
- For N D3-branes the open string low energy description is maximally supersymmetric (1+3)-dim'l SU(N) YM, with $g^2_{YM} = g_s$. In fact this is a CFT, and the t'Hooft coupling is marginal $\lambda = N g^2_{YM}$.
- The closed string `decoupling' limit is where we focus on the same low energy excitations that for open strings give the SYM.

AdS/CFT as a computational tool

- For $N \to \infty$ string quantum corrections are suppressed. Hence the closed string description is a classical one of strings moving in a target space created by the D3-branes.
- The curvature of the space, is governed by: $\frac{R}{l_s} \sim \lambda^{1/4}$
- Hence for large λ supergravity well describes the theory. One finds the vacuum geometry is simply $AdS_5 \times S^5$. Deformations are asymptotic to this.
  - Finite temperature gravity = black holes
  - Real time dynamics involves solving the classical Einstein equations.
- Analysis of full supergravity is complicated. Can take a D-dim'l theory of gravity possibly + other fields, which admits an AdS vacuum and then look at deformations of this much simpler theory. One then says that this describes *some* (D-1) dim'l CFT.
- Whether such an approach is useful is ***unclear***.

AdS/CFT

- The geometry AdS_D we can understand simply in terms of the factor. This has a boundary, which is $\mathbb{R}^{1,D-2}$
- We may write the metric as; $$ds^2 = r^2 \eta_{\mu\nu} dx^\mu dx^\nu + \frac{dr^2}{r^2}$$
- Then $r \to \infty$ is the `boundary' of the geometry. The string theory (or supergravity fields) need boundary conditions there when we consider deforming about the vacuum.
- Ex. scalar (with conformal mass) deformations: $$h(r,x) = \frac{h^{(0)}(x)}{r} + \frac{h^{(1)}(x)}{r^2} + \ldots$$
- Specify `Dirichlet' data $h^{(0)}(x)$ and the solution then determines the `Neumann data' $h^{(1)}(x)$

AdS/CFT dictionary

Closed strings/gravity in AdS_D	$\longleftrightarrow$	CFT (SYM) on $\mathbb{R}^{1,D-2}$
\`Dirichlet' data for fields $\tilde{\mathcal{O}}^{(0)}(x)$	$\longleftrightarrow$	Sources for operator $\mathcal{O}(x)$
$g^{(0)}_{\mu\nu}(x)$		$T_{\mu\nu}$ ie. CFT lives on metric $g^{(0)}_{\mu\nu}(x)$
local Diffeo		Global Poincare invariance -> conservation
$A^{(0)}_{\mu}(x)$		$J_\mu(x)$
local U(1) gauge sym		Global U(1) -> current conservation
\`Neumann' data for fields $\tilde{\mathcal{O}}^{(1)}(x)$	$\longleftrightarrow$	Vevs for same operator $\mathcal{O}(x)$
		$< J_\mu >= A^{(1)}_{\mu}(x)$ $< T_{\mu\nu} >= g^{(1)}_{\mu\nu}(x)$
Response of $\tilde{\mathcal{O}}^{(1)}(x)$ given a delta function source in $\tilde{\mathcal{O}}^{(0)}(x)$	$\longleftrightarrow$	2-pt correlator $< \mathcal{O}(0)\mathcal{O}(x) >$

Model I: Transport

Policastro, Starinets, Son 01

- To study thermal plasma dynamics in a (D-1) dim'l CFT we write down a simple phenomenological model (which is a truncation of usual AdS/CFT):

$$S = \int d^D x \sqrt{g}\,(-\Lambda + R)$$

- Finite temperature: the vacuum is AdS-Schwarzschild

$$ds^2 = -g(r)dt^2 + \delta_{ij} dx^i dx^i + \frac{1}{g(r)} dr^2 \qquad\qquad g(r) = r^2 - \frac{\epsilon}{r}$$

- Deformations of this are supposed to describe the dynamics of the finite temperature CFT (ie. thermal strongly coupled plasma).
- At large scales (compared to energy density) hydrodynamics of a conformal fluid does indeed emerge from this model when classical gravity deformations are considered. *Bhattacharyya, Hubeny, Rangamani, Minwalla 07; Baier et al 07*
- One can calculate transport properties at any scale (not just hydro limit!)

Model II: Conductivity

Herzog, Kovtun, Sachdev 07
Hartnoll, Herzog 07

- To study conductivity we require a current so we add a local bulk U(1):

$$S = \int d^D x \sqrt{g} \left(-\Lambda + R - \frac{1}{4} F_{\mu\nu}^2 \right)$$

- The bulk operator $A_\mu(r,x)$ is dual to a conserved boundary current $J_\mu(x)$ charged under a global U(1)

- Take the ‘Dirichlet’ $A^{(0)} = \mu\, dt$ and then the `Neumann’ part $A^{(1)} =< J >$

- The vacuum is given by a charged AdS black hole: AdS-RN

$$ds^2 = -g(r)dt^2 + \delta_{ij}dx^i dx^i + \frac{1}{g(r)}dr^2 \qquad g(r) = 4r^2 - \frac{1}{r}\left(4r_+^3 + \frac{\alpha^2}{r}\right) + \frac{\alpha^2}{r^2}$$

$$A_t = \alpha \left(\frac{1}{r_+} - \frac{1}{r} \right)$$

- Hydro description is a charged fluid.

- Conductivity can be calculated at any scale (not only hydro limit).

- Much recent attention to Fermi surface: *Lee 08; Liu et al 09; Cubrovic et al 09 ; Faulkner et al 09*

Model III: Superfluidity in 1+2

Gubser 08
Hartnoll, Herzog, Horowitz 08

- To study superfluidity we require a 1+3 bulk theory with a charge current and associated local U(1):

$$S = \int d^4x\sqrt{g}\left(R - \frac{1}{4}F_{\mu\nu}^2 - |D\chi|^2 - V(|\chi|)\right)$$

$$\Lambda = -24, \; q = 2, \; m^2 = -8$$

$$V(|\chi|) = \Lambda + m^2\chi^2$$

$$D\chi = d\chi - iqA\chi$$

- Again the bulk operator A is dual to a global U(1) boundary current and we will take the boundary data $A^{(0)} = \mu\, dt$ and choose $\mu = 1$ (breaks scale)

- Now χ is a scalar charged under the bulk U(1). Dual to a complex scalar operator $\mathcal{O}_\chi$ with dimension $\Delta_\chi = 2$

- Then one finds $\chi(r) = \frac{\chi^{(0)}}{r} + \frac{\chi^{(1)}}{r^2} + \dots$ and we take Dirichlet data $\chi^{(0)} = 0$

$$< \mathcal{O}_\chi >= \chi^{(1)}$$

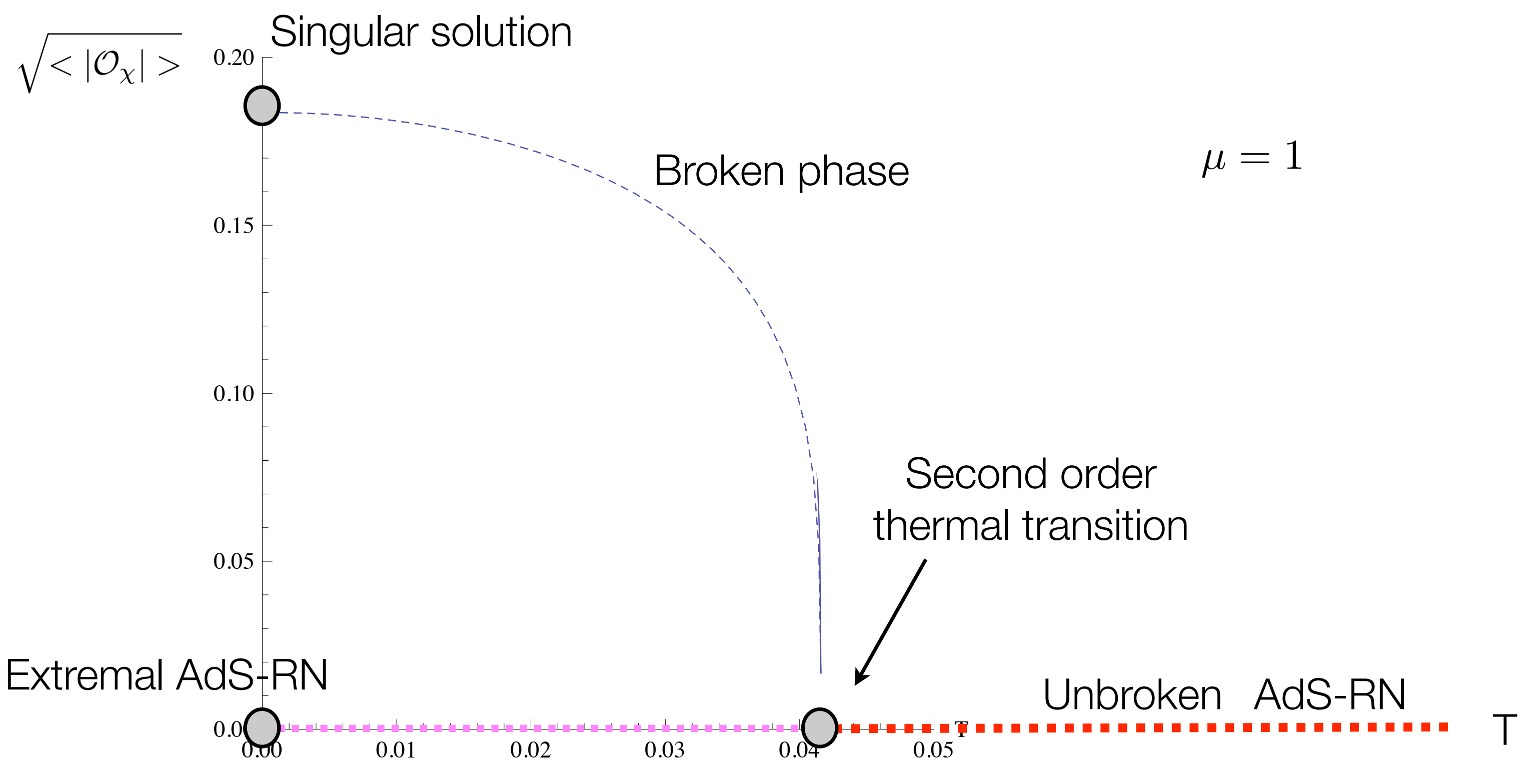
- `Normal phase' solution: AdS-RN soln before, $\chi = 0$

- `Broken phase' solution: $ds^2 = e^{-\beta(r)}g(r)dt^2 + \frac{dr^2}{g(r)} + r^2dx_i^2$ $\quad A = \phi(r)dt$

$$\chi(r) = \xi(r) \qquad \xi(r) \in \mathbb{R}$$

Model III: Superfluidity in 1+2

- Find for $T < T_c \sim 0.042$ broken solutions: black holes with scalar `hair'
- Can check that free energy is minimized by broken phase



Model III: Superfluidity in 1+2

- Phase structure: normal phase for $T > T_c$ broken phase for $T < T_c$
- We may phase rotate any solution; $\chi(r) = e^{i\theta}\xi(r)$
- One can perform a hydro expansion again. For $T > T_c$ one obtains charged fluid as before. However, for $T < T_c$ the phase is dynamic and gives rise to an additional Goldstone mode, θ , from the U(1) breaking giving a superfluid component.

in progress Sonner, Withers, TW

- Note that the difference between a superfluid and superconductor is simply whether the broken U(1) is local or global.
- Recent attention on Fermi surface *Gubser et al 09 ; Faulkner et al 09*

Balasubrahmanian, McGreevy 08
Herzog, Rangamani, Ross 08
Kachru et al 08

Models IV: $z \neq 1$

- Much recent progress in extending to $z = 2$ (Schrodinger sym) and general Lifshitz scaling.
- Black holes can be found with exotic asymptotic geometries
- Holographic dictionary mapping can be understood *Ross, Saremi 09*
- Superconductivity *Danielsson, Thorlacius 09*
- Fermi surface *Hartnoll, Polchinski, Silverstien, Tong 09*

Denef, Hartnoll 09 ; Gauntlett, Kim, Varela, Waldram 09 ; Gauntlett, Sonner, TW 09

See also Gubser, Herzog, Pufu, Tesileanu 09

M-Theory embedding

And also Basu et al, Erdmenger et al 08

- Take M2 or anti-M2 branes at tip of $\mathbb{R}^{1,2} \times CY_8$ where $ds^2(CY_8) = dr^2 + r^2 d(SE_7)^2$

$$d(SE_7)^2 = d(KE_6)^2 + \eta \otimes \eta \qquad \eta = d\psi + a \qquad da = 2J_{KE_6}$$

- In decoupling limit the geometry is, $AdS_4 \times SE_7$, with $G_4 = \pm vol(AdS_4)$

- For M2 branes have N=2 SCFT in 1+2 dim
    - Then for S^7 one finds $KE_6 = CP^3$ and enhanced susy N=8; CFT known
- For anti-M2 branes susy is broken; N=0 CFT in 1+2 dim
    - These vacuum solutions are known as `skew-whiffed'
    - They have been shown to be perturbatively stable
    - Note for S^7 actually the theory is supersymmetric

M-Theory embedding

Gauntlett, Sonner, TW 09

- Deformations can be reduced to a consistent truncation of the full 11-d supergravity. For anti-M2 branes we have found a 1+2 dim'l truncation;

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\Big[R - \frac{(1-h^2)^{3/2}}{1+3h^2}F_{\mu\nu}F^{\mu\nu} - \frac{3}{2(1-\frac{3}{4}|\chi|^2)^2}|D\chi|^2 - \frac{3}{2(1-h^2)^2}(\nabla h)^2 - \frac{24(-1+h^2+|\chi|^2)}{(1-\frac{3}{4}|\chi|^2)^2(1-h^2)^{3/2}}\Big] + \frac{1}{16\pi G}\int \frac{2h(3+h^2)}{(1+3h^2)}F\wedge F$$

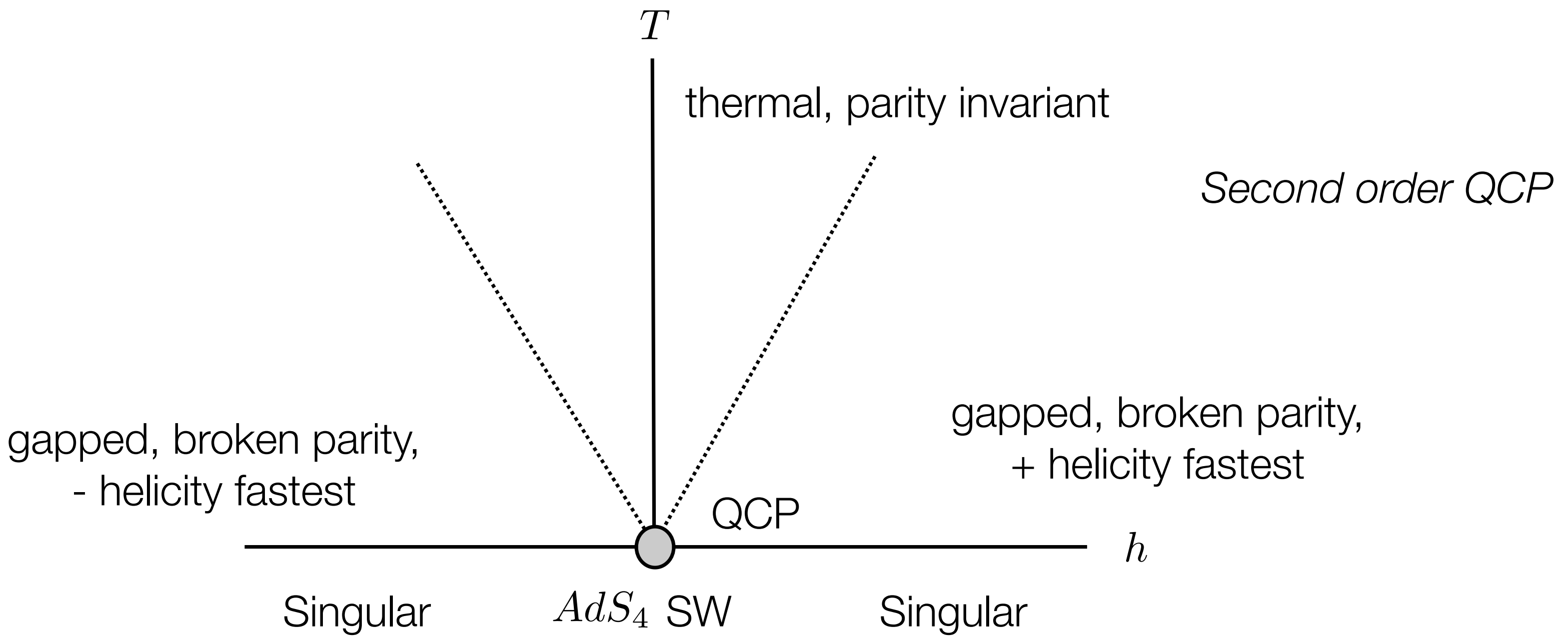
- Gravity, local U(1) gauge field, charged scalar $\Delta_\chi = 2$ and also a neutral relevant scalar h with $\Delta_h = 2$
- Then $h = \frac{h^{(0)}}{r} + \frac{h^{(1)}}{r^2} + \ldots$ and we fix Dirichlet data $h^{(0)} = const$

$$g \leftrightarrow T \qquad A \leftrightarrow J \qquad \chi \leftrightarrow \mathcal{O}_\chi \qquad h \leftrightarrow \mathcal{O}_h$$

- For $h = 0$, and to linear order in χ this reduces to phenomenological model

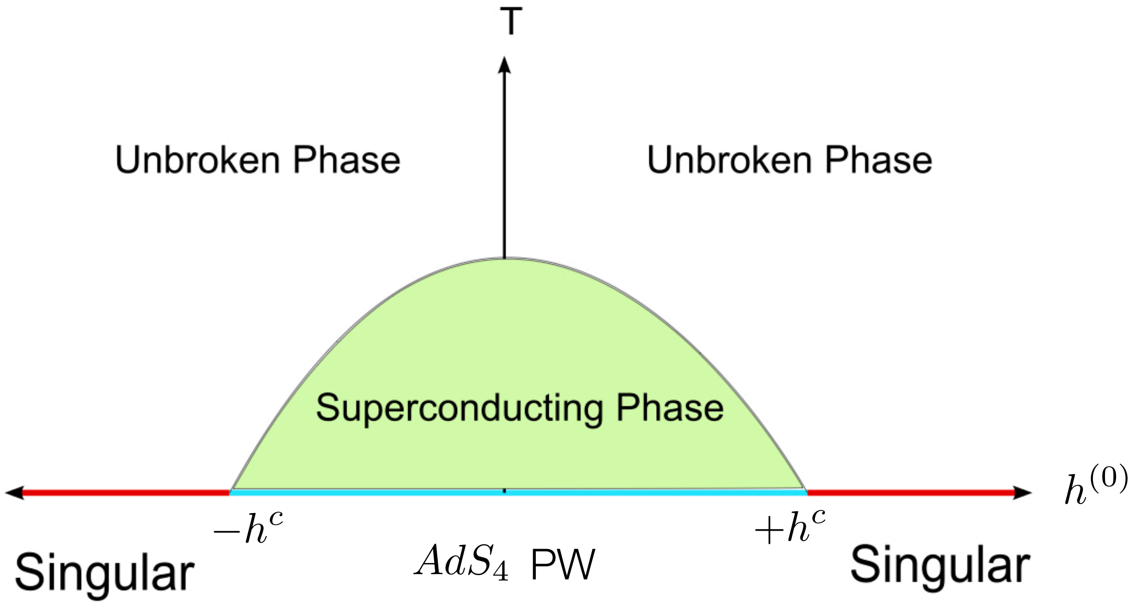
M-Theory behaviour: $\mu = 0$

- For $h^{(0)} \neq 0$ we find deformed AdS black holes with h scalar charge
  - As $T \to \infty$ these tend to AdS-Schwarzschild
  - For $T \to 0$ these become singular in the IR (finite distance, gap)
- The coupling $h\, F \wedge F$ means for $h \neq 0$ parity is broken



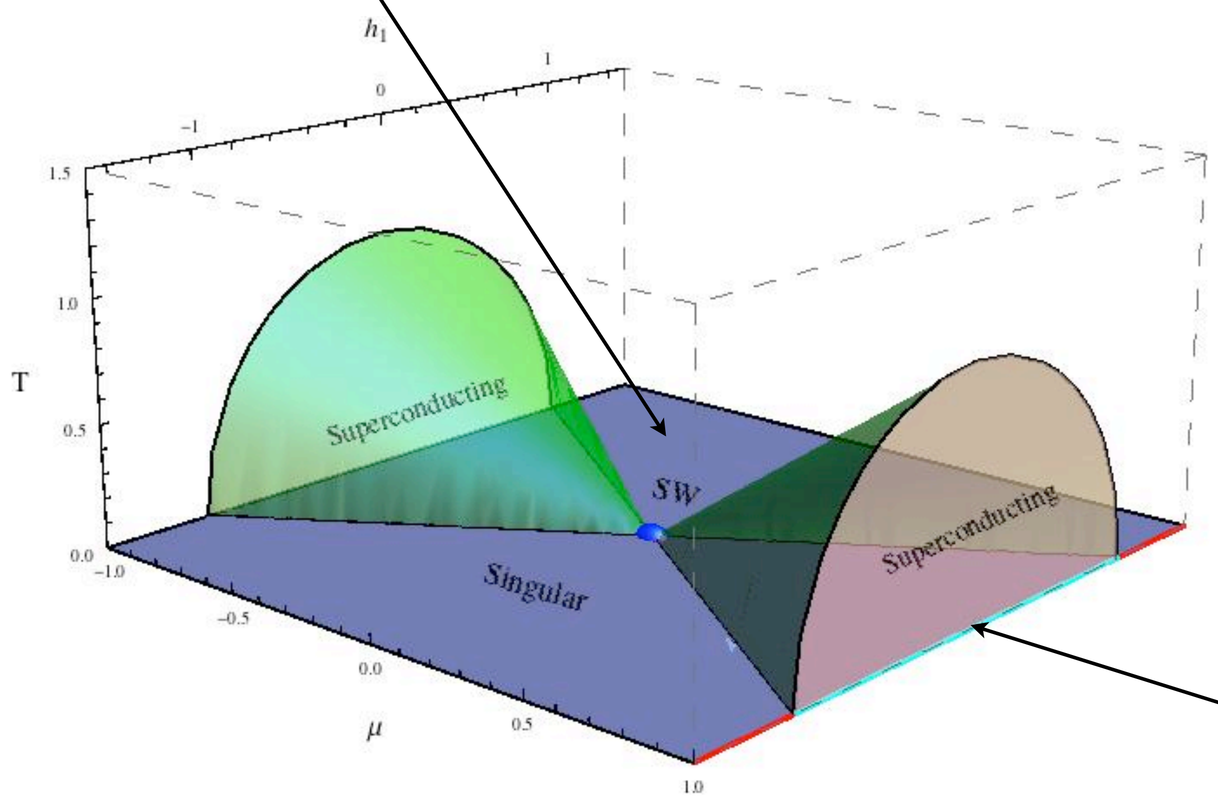
M-Theory behaviour: $\mu = 1$

- For $|h^{(0)}| < h^c \sim 0.35$ have deformed AdS-RN for $T > T_c(h^{(0)})$ and broken phase solutions for $T < T_c(h^{(0)})$
  - However $T_c(h^{(0)}) \to 0$ for $|h^{(0)}| \to h^c$
  - For $T \to 0$ solutions are AdS_4 with $\chi = \sqrt{2/3}$ in IR - Pope-Warner soln
- For $|h^{(0)}| > h^c$ have only deformed AdS-RN which are singular for $T \to 0$



Phase diagram

AdS_4 Skew-Whiffed QCP



Emergent IR
AdS_4 Pope-Warner

Summary

- 1+2 Quantum critical points with z=1 are described by relativistic CFTs.
  - Other scalings are possible in AdS/CFT too!
- Such QCPs, which are typically strongly coupled, may help describe the behaviour of exotic materials including high-T superconductors.
- AdS/CFT is an ideal tool which allows specific strongly coupled CFTs to be found, and in certain regimes to be ‘solved’ -- at least in the sense of thermodynamics, phase structure, transport.

- Do these CFTs describe real materials? -- probably not at present
- Do they describe materials near a QCP? -- unclear, but a possibility