Annual Theory Meeting

Durham, 2009

The QCD phase diagram at non-zero baryon density

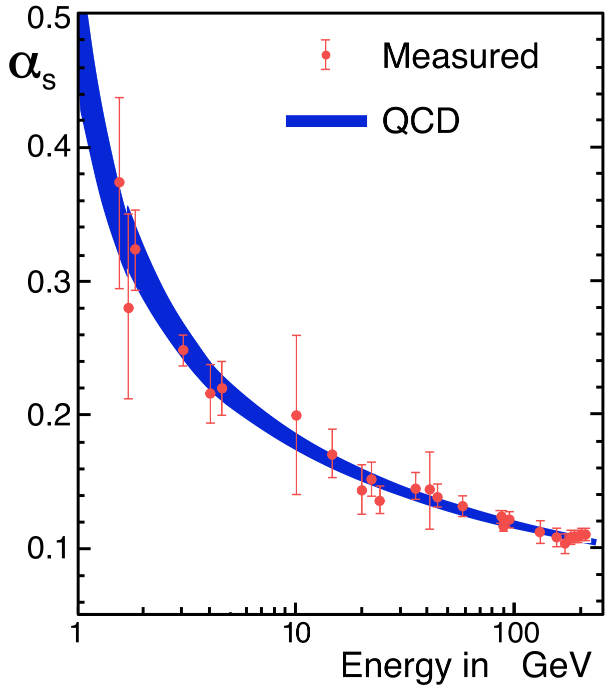
Owe Philipsen

Westfälische Wilhelms-Universität Münster

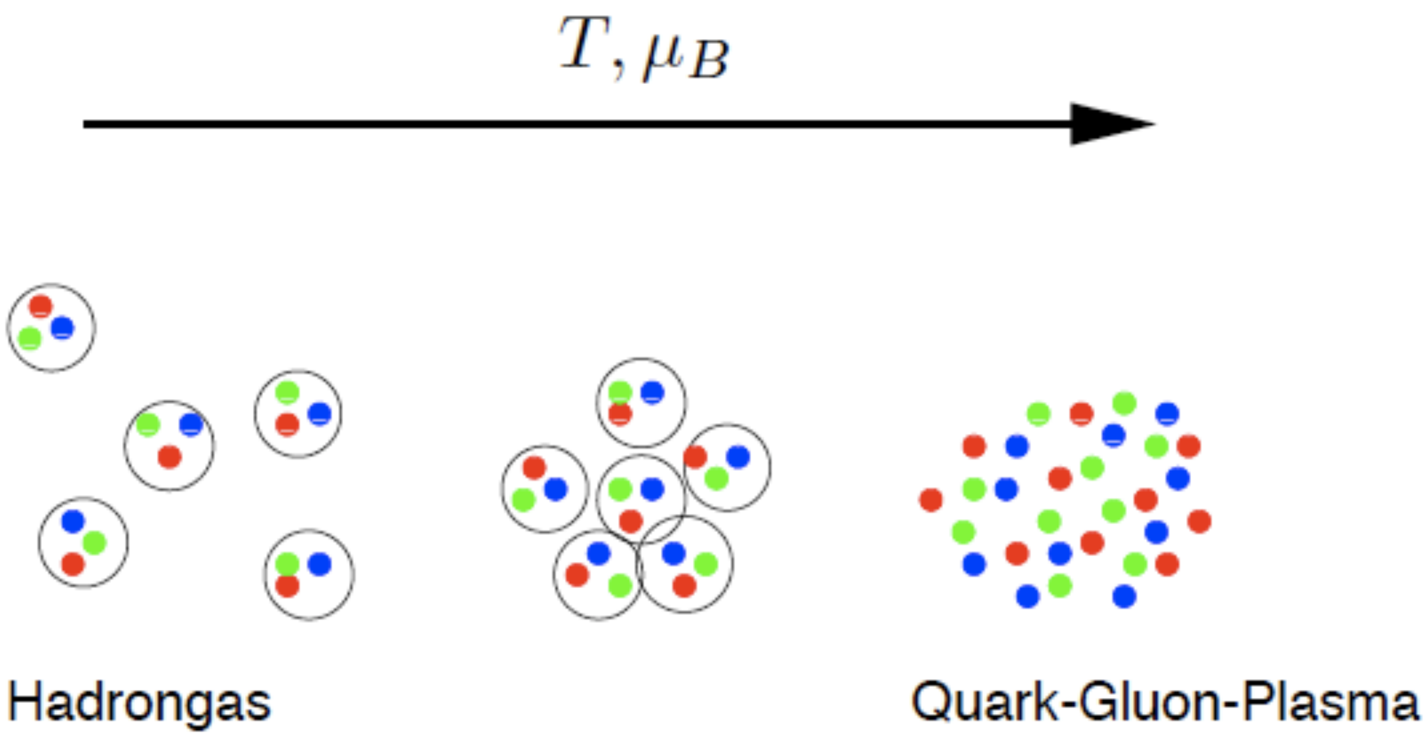
- Introduction
- Lattice techniques for finite temperature and density
- The phase diagram: the confusion before clarity?

Original work with Ph. de Forcrand (ETH/CERN)

QCD at high temperature/density: change of dynamics



asymptotic freedom $\alpha_s(p \to \infty) \to 0$

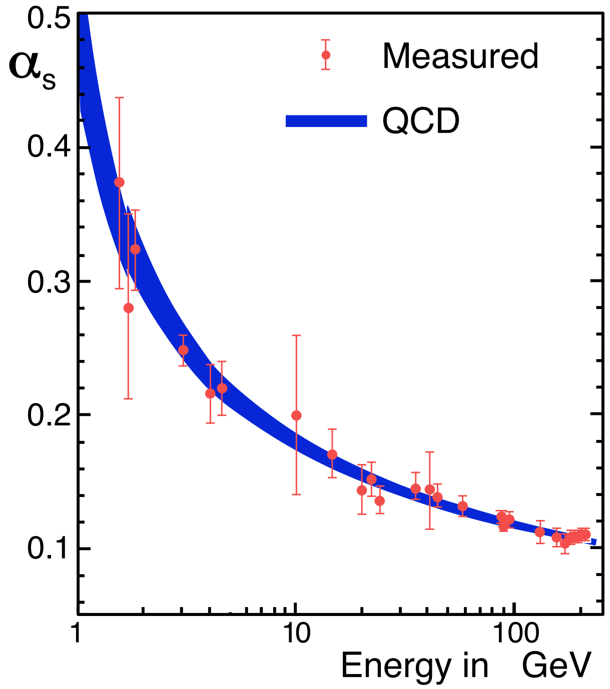


Chiral symmetry:	broken	(nearly) restored

Order parameters: $\langle \bar{\psi}\psi \rangle, \langle \psi\psi \rangle$

chiral condensate , Cooper pairs

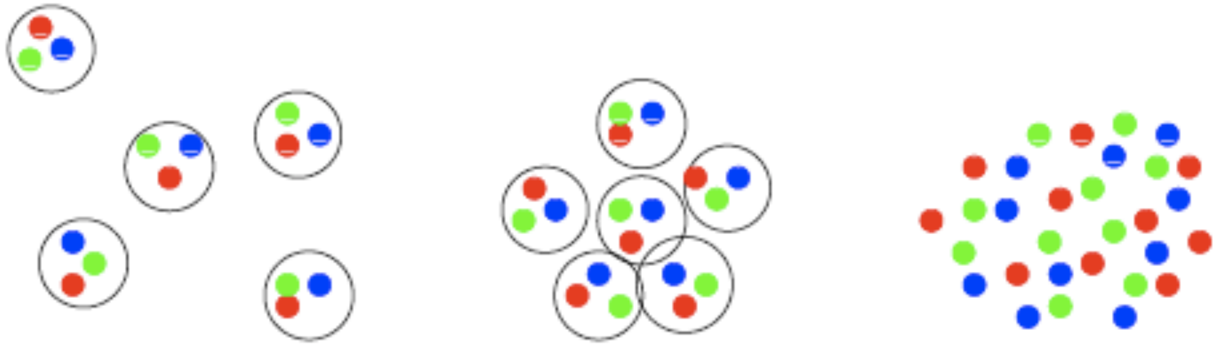
QCD at high temperature/density: change of dynamics



asymptotic freedom $\alpha_s(p \to \infty) \to 0$

T, μ_B

Phase transitions?



Hadrongas

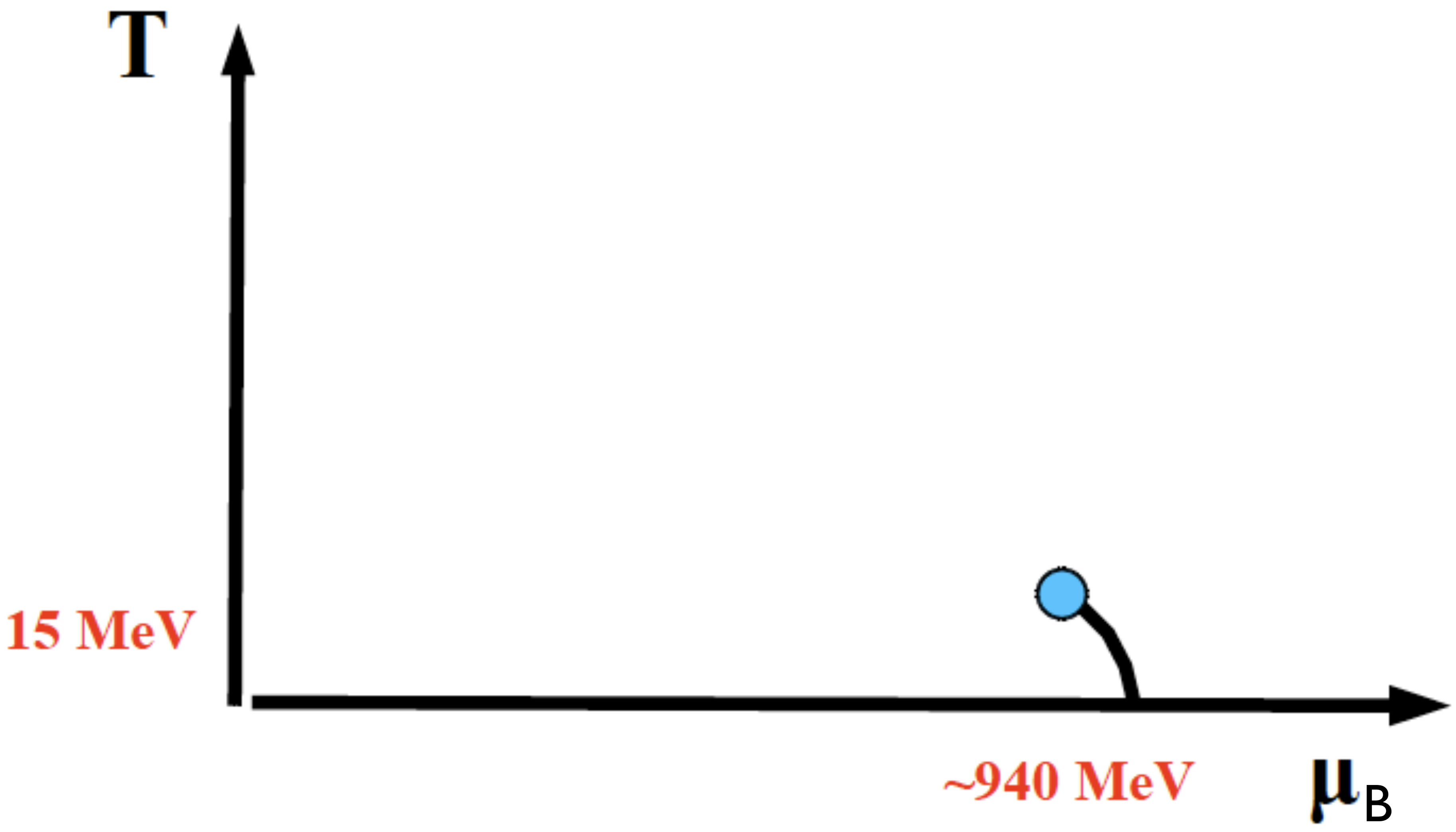
Quark-Gluon-Plasma

Chiral symmetry: broken (nearly) restored

Order parameters: $\langle \bar{\psi}\psi \rangle, \langle \psi\psi \rangle$

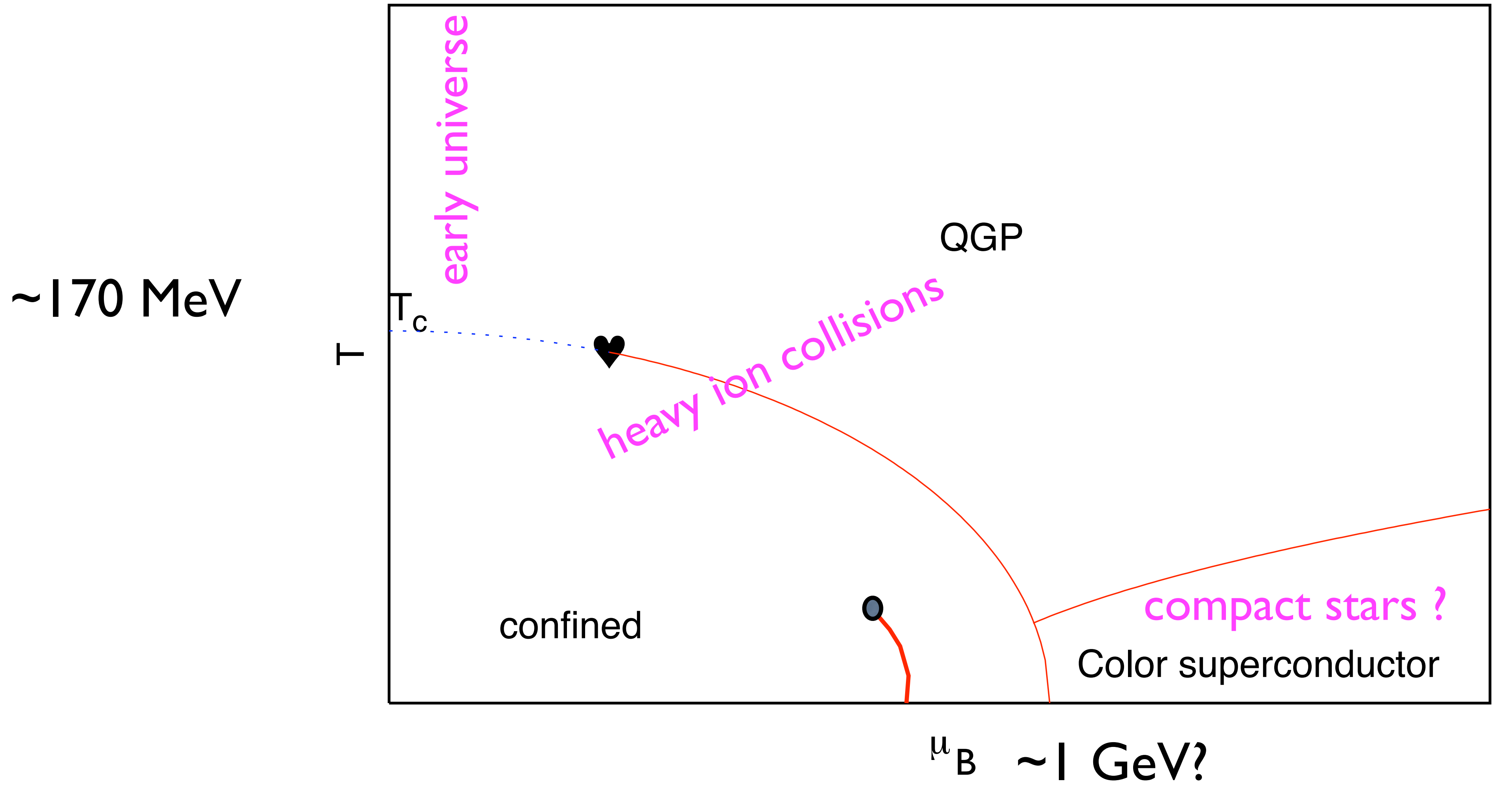
chiral condensate , Cooper pairs

The QCD phase diagram established by experiment:



Nuclear liquid gas transition, Z(2) end point

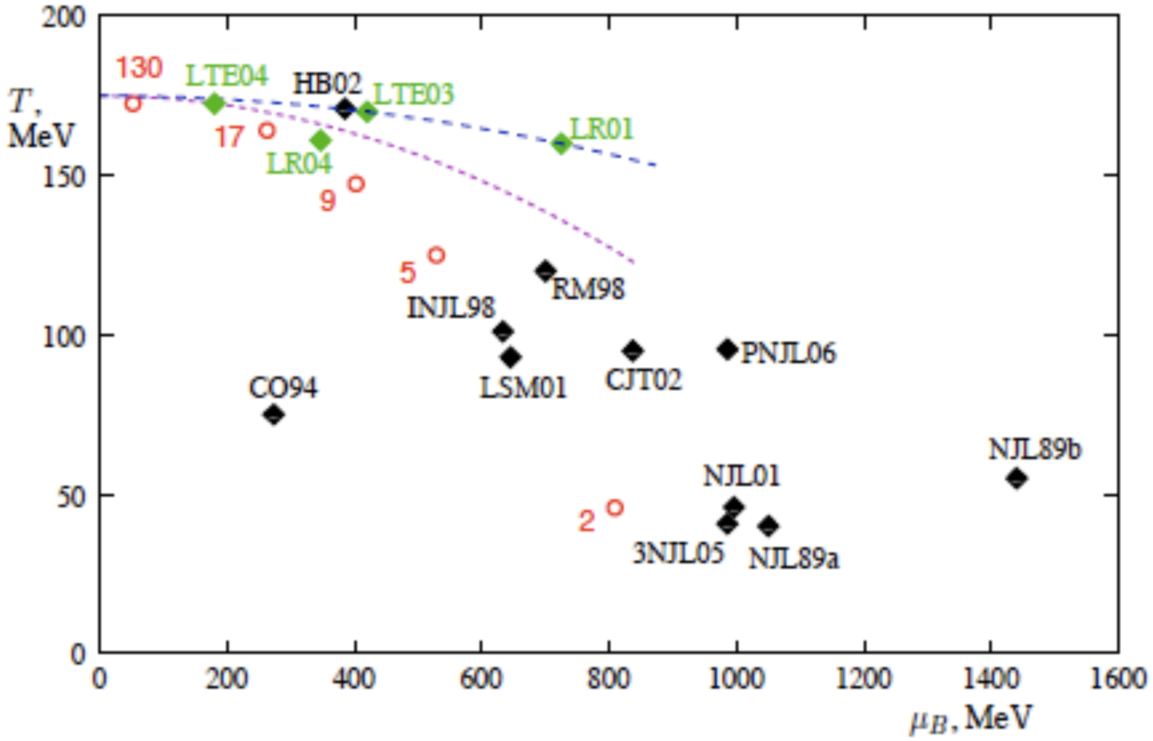
QCD phase diagram: theorist's view



Until 2001: no finite density lattice calculations, sign problem!

Expectation based on models: NJL, NJL+Polyakov loop, linear sigma models,
random matrix models, ...

Model predictions for critical end point (CEP)



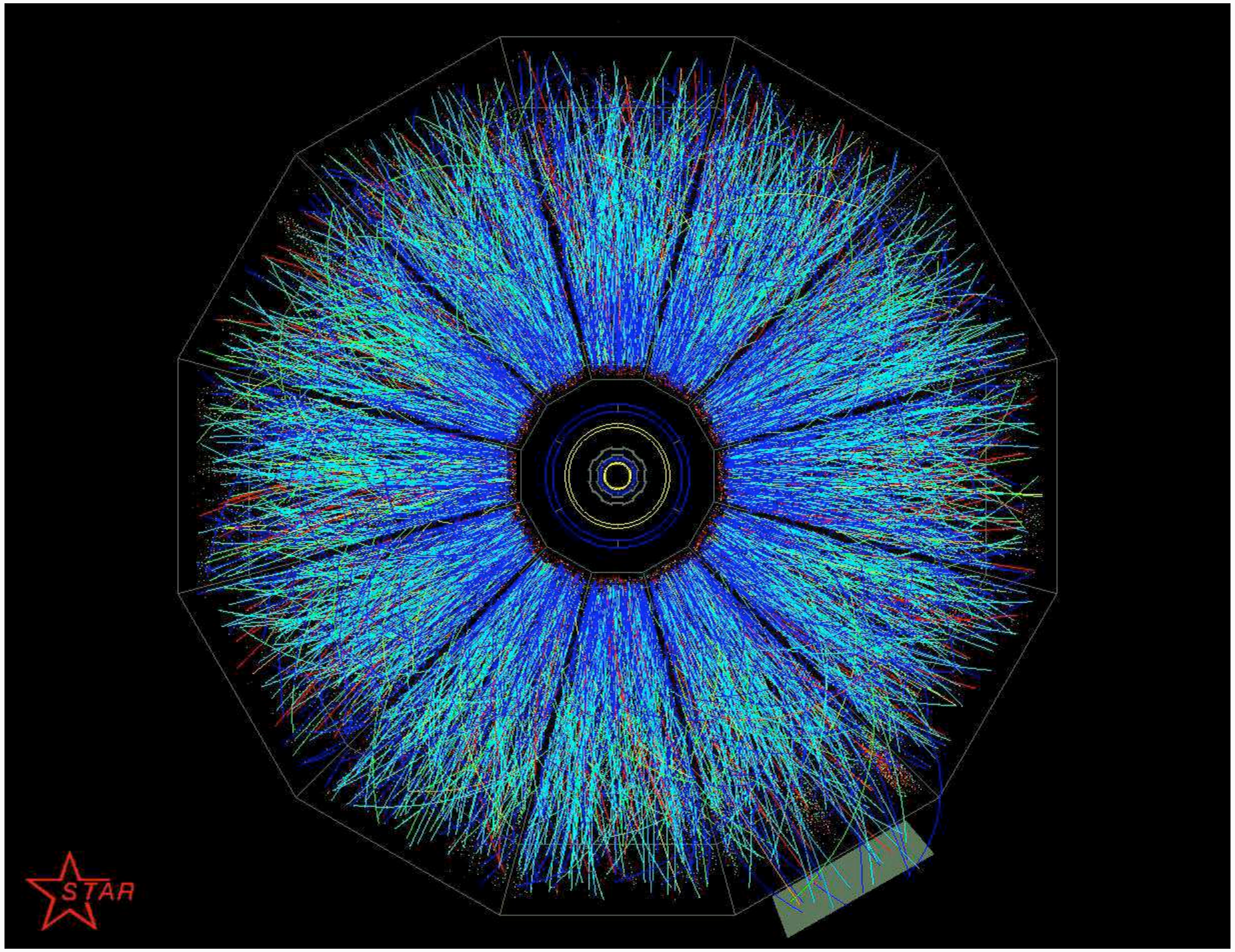
M. Stephanov, hep-lat/0701002

How to get funding for heavy ion programs:

How to get funding for heavy ion programs:

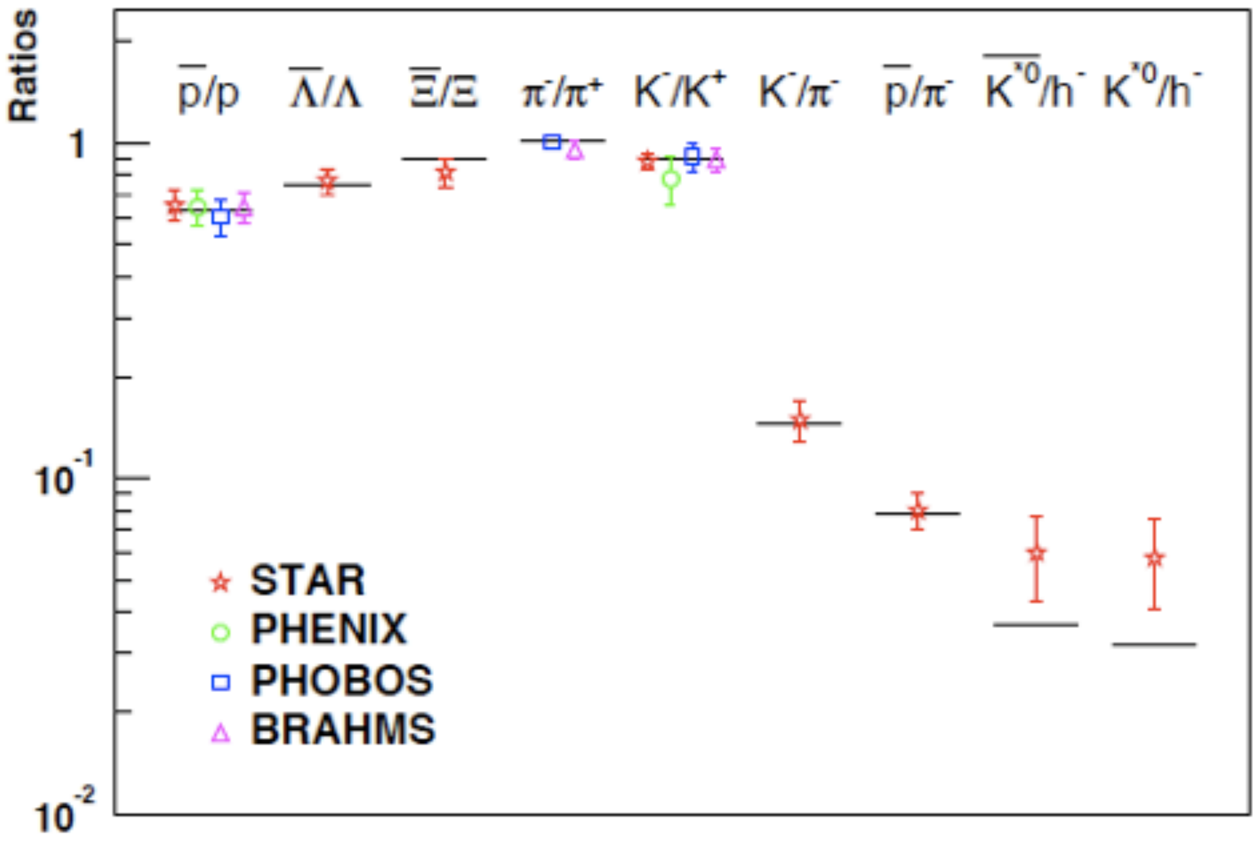
Free the Quarks!!!

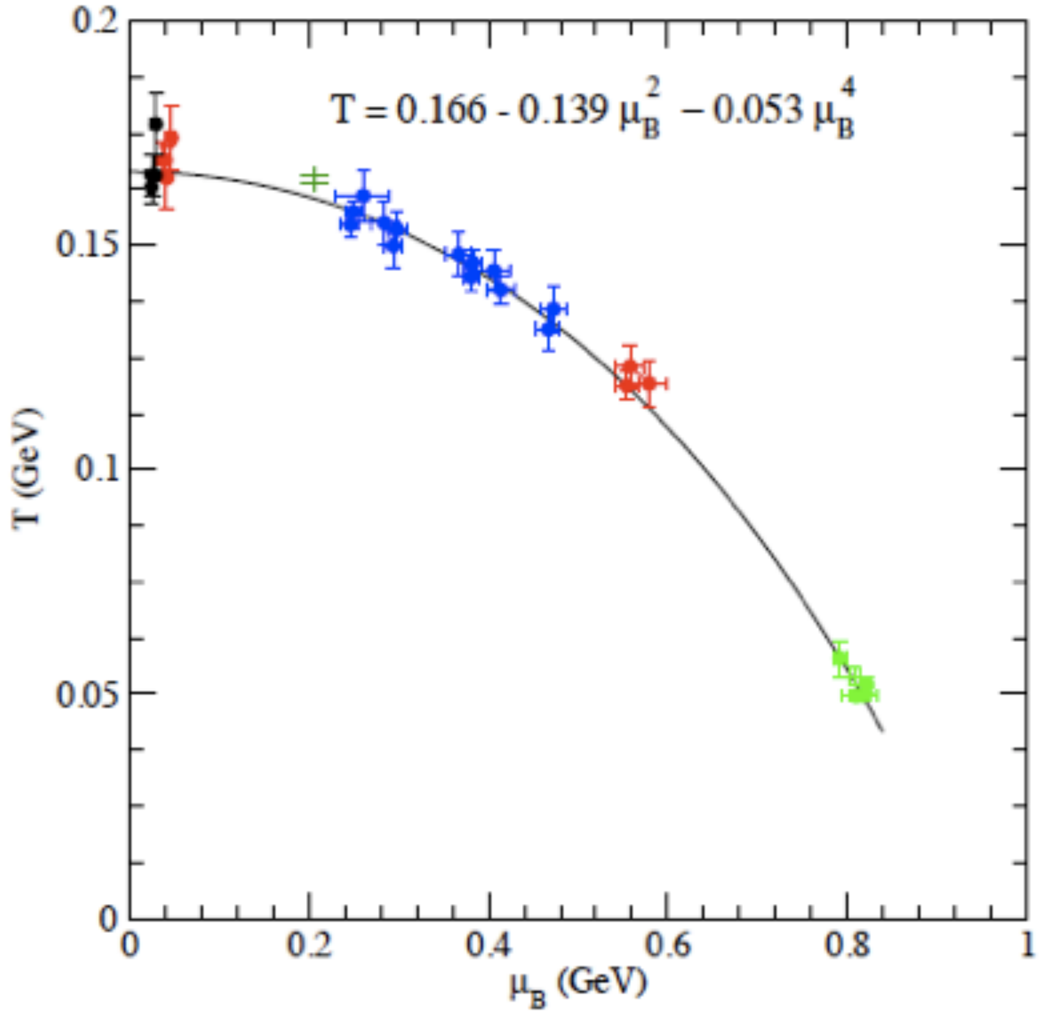
Thermal QCD in experiment



heavy ion collision experiments at RHIC, LHC, GSI.... have finite baryon density!

Phase boundary from hadron freeze-out?



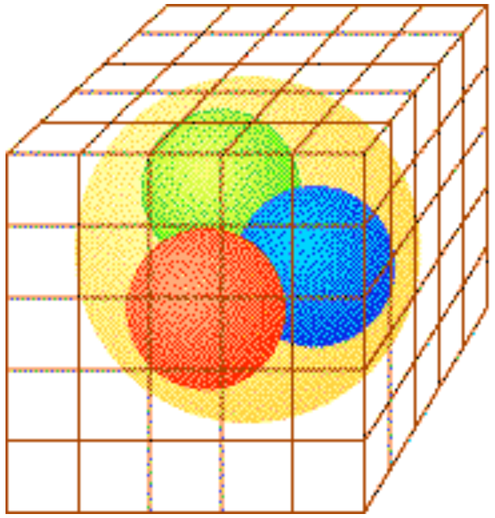


- At fixed collision energy $\sqrt{s}$, abundances well fitted by Boltzmann distribution (T, μ_B)
- T(freeze-out) $\leq$ T_c but very close ?

Braun-Munzinger et al

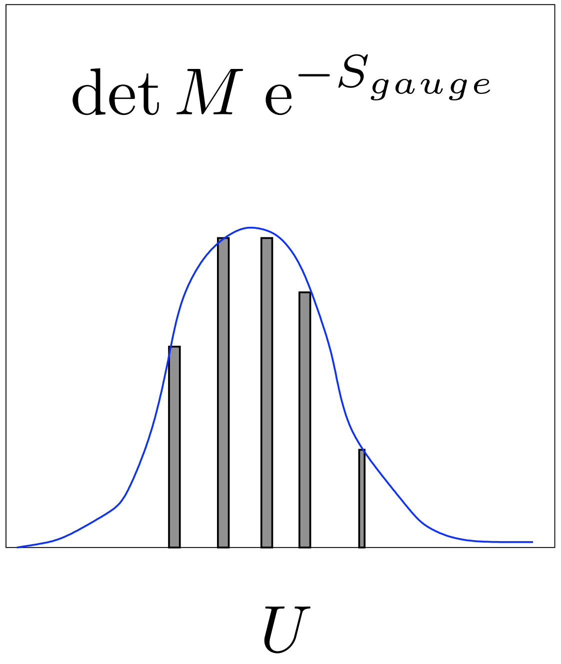
Theory: The Monte Carlo method

QCD partition fcn: $Z = \int DU \prod_f \det M(\mu_f, m_f; U)\, \mathrm{e}^{-S_{gauge}(\beta;U)}$



links=gauge fields

lattice spacing a<< hadron << L !
thermodynamic behaviour, large V !



➡ typically $> 10^8 - 10^{10}$ dim. integral

Monte Carlo, importance sampling

$T = \frac{1}{aN_t}$

Continuum limit: $N_t \to \infty, a \to 0$

Here: $N_t = 4, 6$

$a \sim 0.3, 0.2$ fm

Light fermions expensive, cost$(\det M) \sim \frac{1}{m_q^n}$, here staggered fermions

How to measure p.t., critical temperature

deconfinement/chiral phase transition $\longrightarrow$ quark gluon plasma

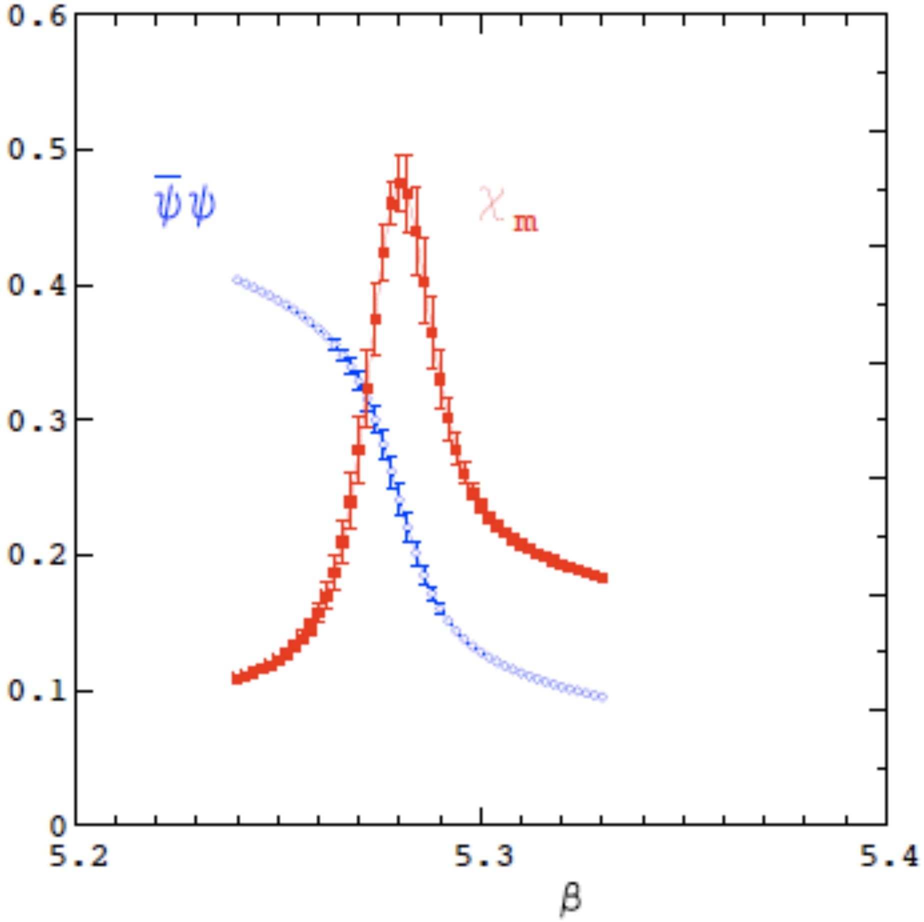
"order parameter":

chiral condensate $\langle\bar{\psi}\psi\rangle$

generalized susceptibilities:

$$\chi = V(\langle\mathcal{O}^2\rangle - \langle\mathcal{O}\rangle^2)$$

$\Rightarrow \chi_{max} = \chi(\beta_c)$ $\quad \Rightarrow T_c$



lattice coupling β, viz.T

Lee, Yang: only pseudo-critical on finite V !

Order of transition:

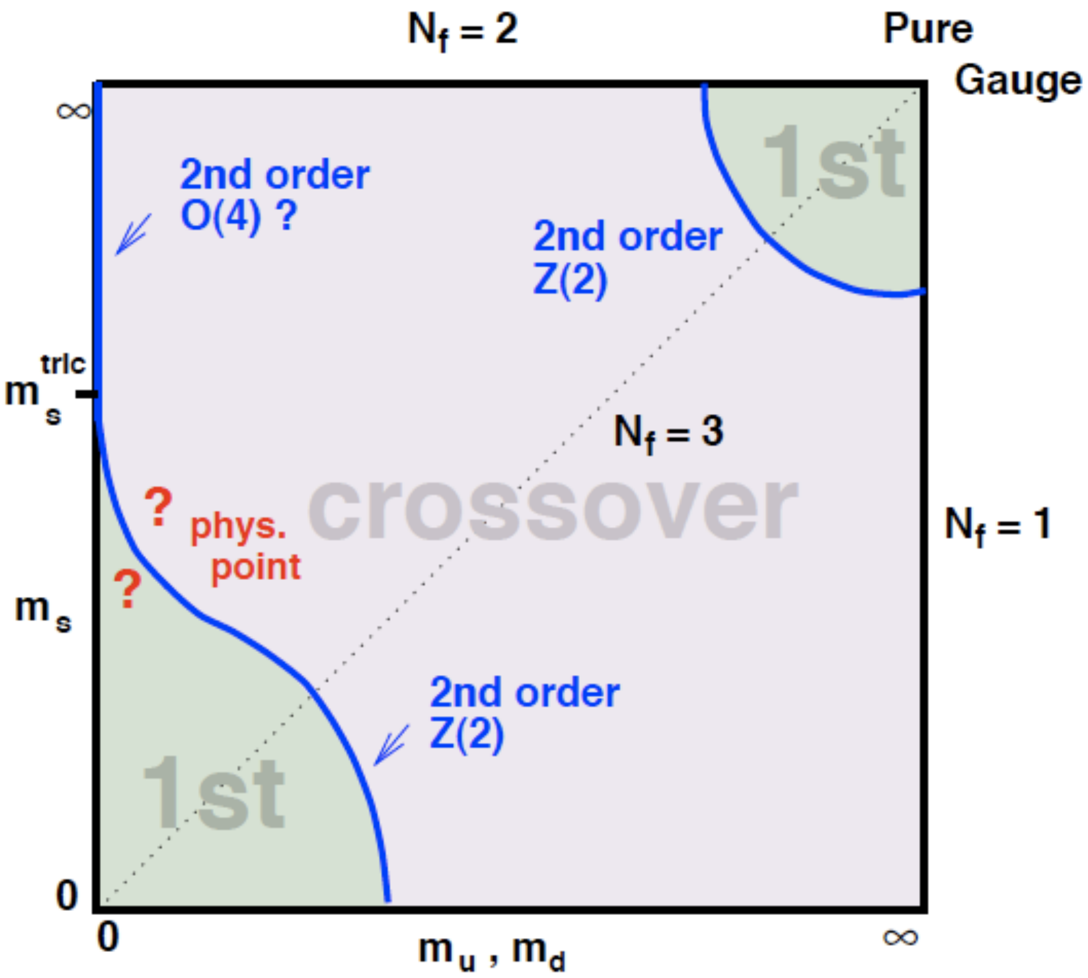
finite volume scaling

$\chi_{max} \sim V^\sigma$

$\sigma = 1$	1st order
$\sigma < 1$	2nd order
$\sigma = 0$	crossover

The order of the p.t., arbitrary quark masses $\mu = 0$

deconfinement p.t.:
breaking of global $Z(3)$



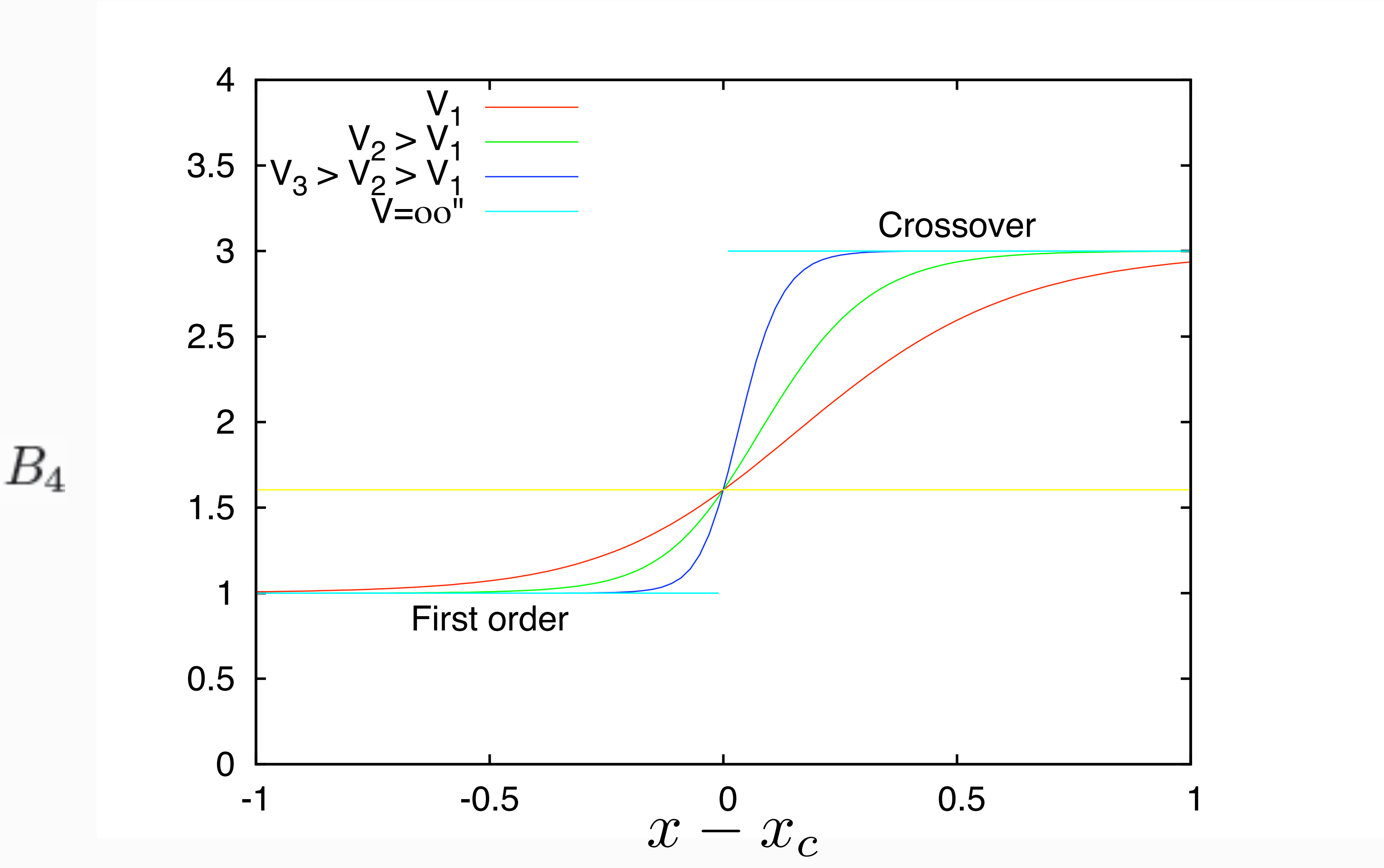
chiral p.t.
restoration of global

$$SU(2)_L \times SU(2)_R \times \underset{\substack{\uparrow \\ \text{anomalous}}}{U(1)_A}$$

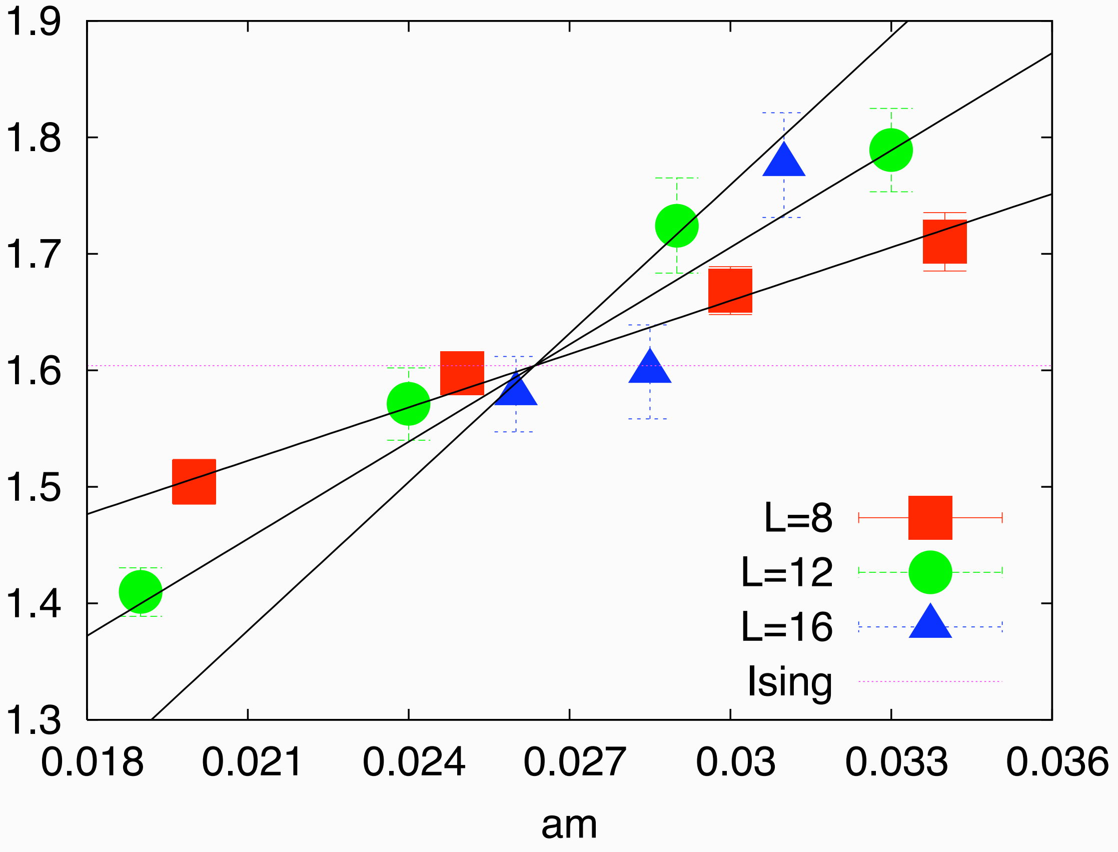
How to identify the order of the phase transition

$$B_4(\bar{\psi}\psi) \equiv \frac{\langle(\delta\bar{\psi}\psi)^4\rangle}{\langle(\delta\bar{\psi}\psi)^2\rangle^2} \xrightarrow{V\to\infty} \begin{cases} 1.604 & \text{3d Ising} \\ 1 & \text{first}-\text{order} \\ 3 & \text{crossover} \end{cases}$$

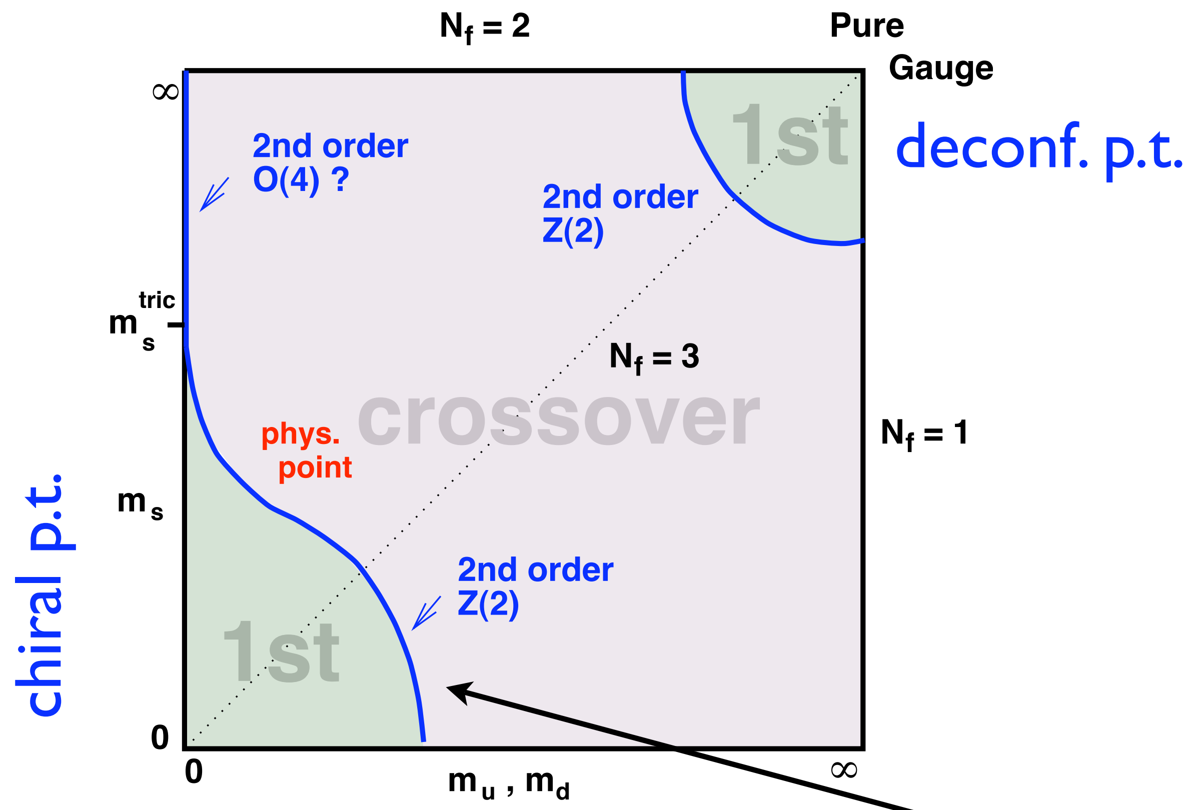
$\mu = 0:$ $$B_4(m,L) = 1.604 + bL^{1/\nu}(m - m_0^c), \quad \nu = 0.63$$

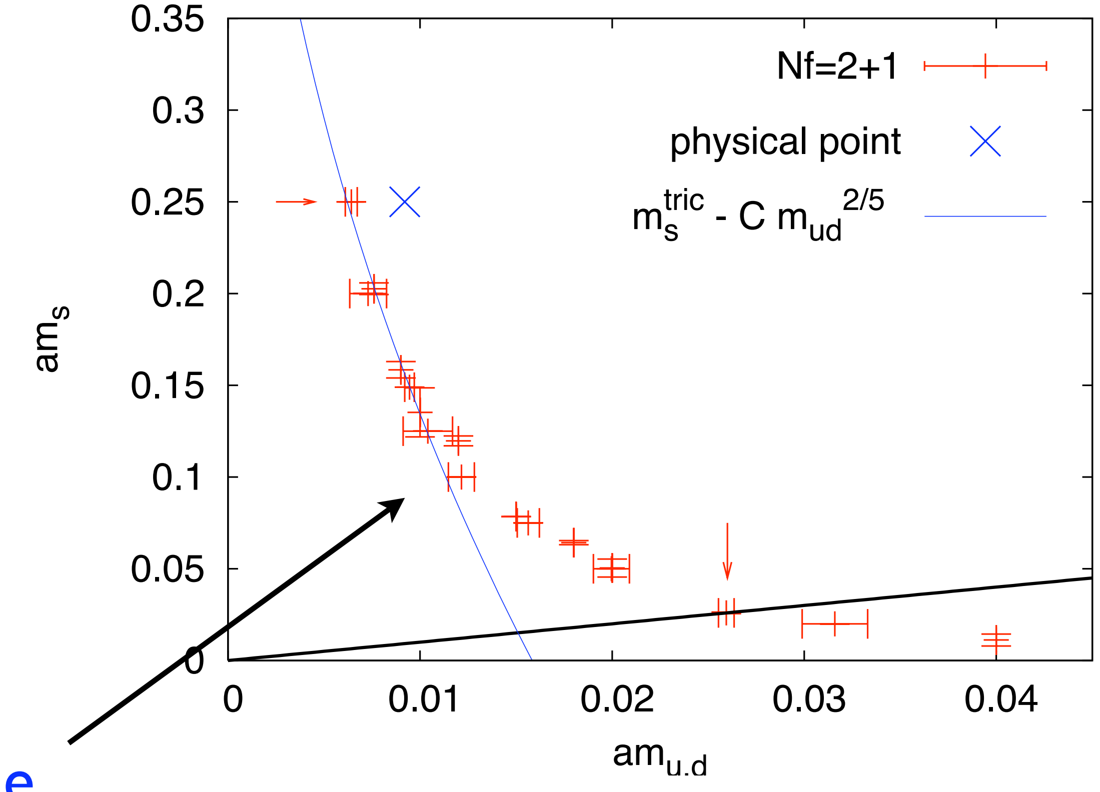


parameter along phase boundary, $T = T_c(x)$



Hard part: order of p.t., arbitrary quark masses $\mu = 0$





chiral critical line

- physical point: crossover in the continuum — Aoki et al 06
- chiral critical line on $N_t = 4, a \sim 0.3$ fm — de Forcrand, O.P. 07
- consistent with tri-critical point at $m_{u,d} = 0, m_s^{\rm tric} \sim 2.8T$
- But: $N_f = 2$ chiral O(4) vs. 1st still open — Di Giacomo et al 05, Kogut, Sinclair 07
  $U_A(1)$ anomaly! — Chandrasekharan, Mehta 07

The ‘sign problem’ is a phase problem

$$Z = \int DU \, [\det M(\mu)]^f \mathrm{e}^{-S_g[U]}$$

importance sampling requires
positive weights

Dirac operator: $\not{D}(\mu)^\dagger = \gamma_5 \not{D}(-\mu^*)\gamma_5$

$\Rightarrow \det(M)$ complex for SU(3), $\mu \neq 0$

$\Rightarrow$real positive for SU(2), $\mu = i\mu_i$

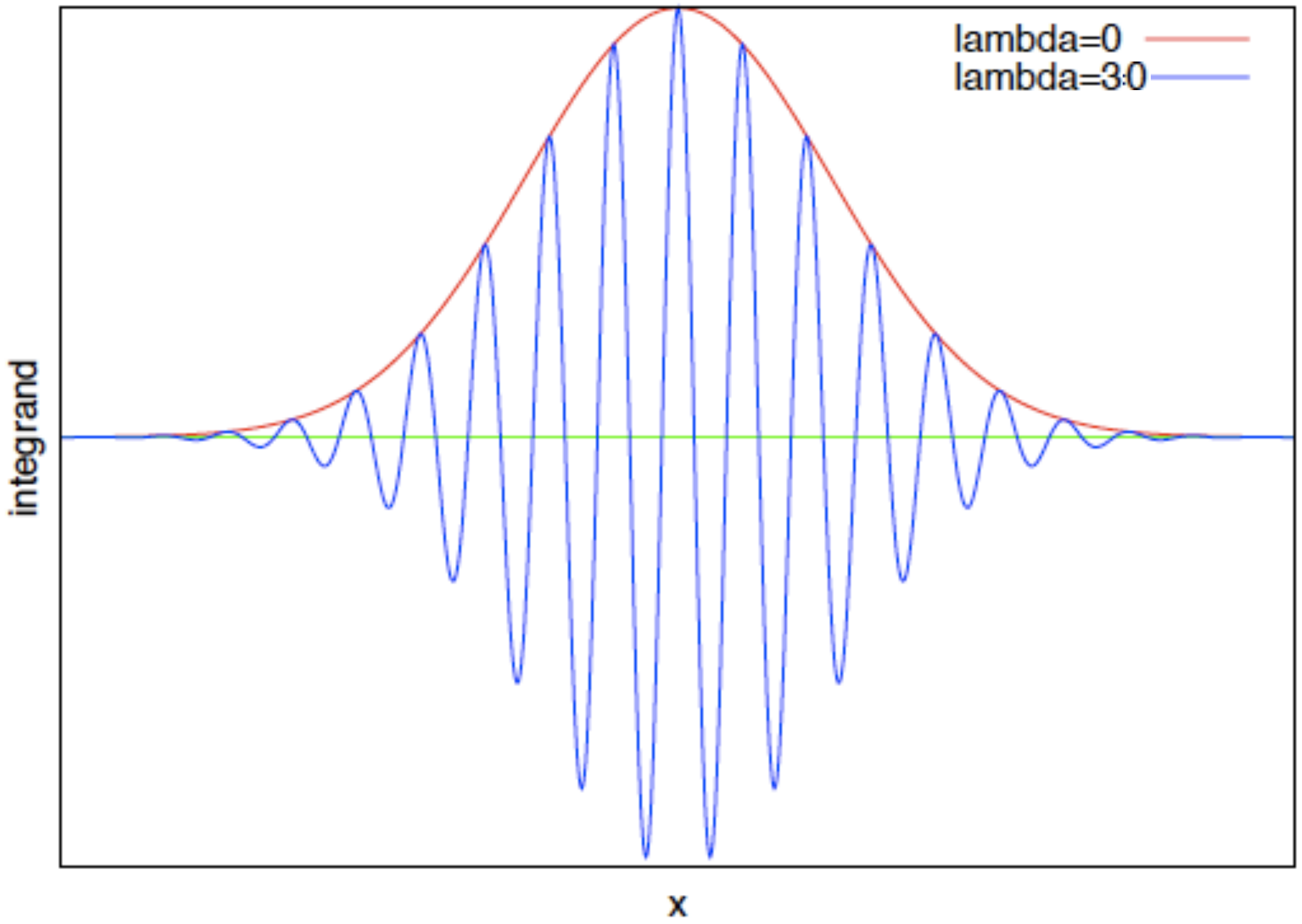
$\Rightarrow$real positive for $\mu_u = -\mu_d$

N.B.: all expectation values real, imaginary parts cancel,
but importance sampling config. by config. impossible!

Same problem in many condensed matter systems!

1 dim. illustration

- Example: $Z(\lambda) = \int dx \exp(-x^2 + \mathbf{i}\lambda\mathbf{x})$



- $Z(\lambda)/Z(0) = \exp(-\lambda^2/4)$: exponential cancellations

$\rightarrow$ truncating deep in the tail at $x \sim \lambda$ gives $\mathcal{O}(100\%)$ error Splittorff

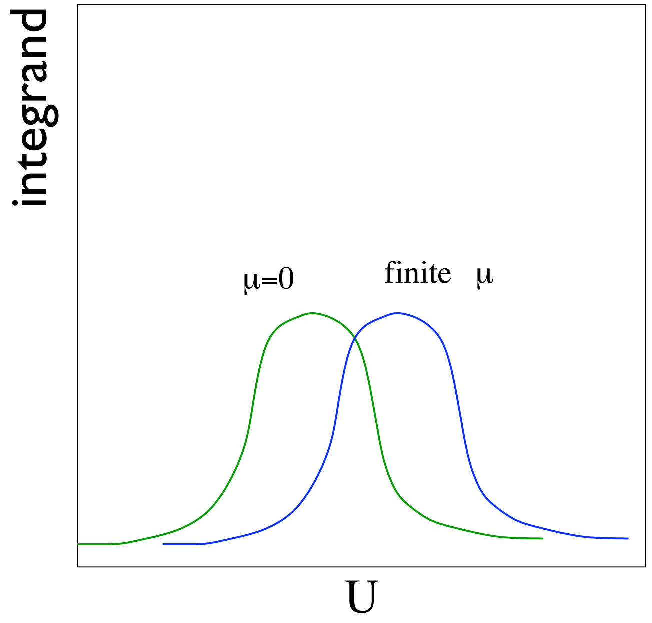
"Every x is important" $\leftrightarrow$ How to sample?

Finite density: methods to evade the sign problem

- **Reweighting:** $Z = \int DU \, \det M(0) \frac{\det M(\mu)}{\det M(0)} \, e^{-S_g}$

  $\uparrow$ use for MC $\quad \uparrow$ calculate

  ~exp(V) statistics needed,
  overlap problem



- Taylor expansion:

$$\langle O\rangle(\mu) = \langle O\rangle(0) + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

coeffs. one by one,
convergence?

- Imaginary $\mu = i\mu_i$: no sign problem, fit by polynomial, then analytically continue

$$\langle O\rangle(\mu_i) = \sum_{k=0}^{N} c_k \left(\frac{\mu_i}{\pi T}\right)^{2k} , \qquad \mu_i \to -i\mu$$

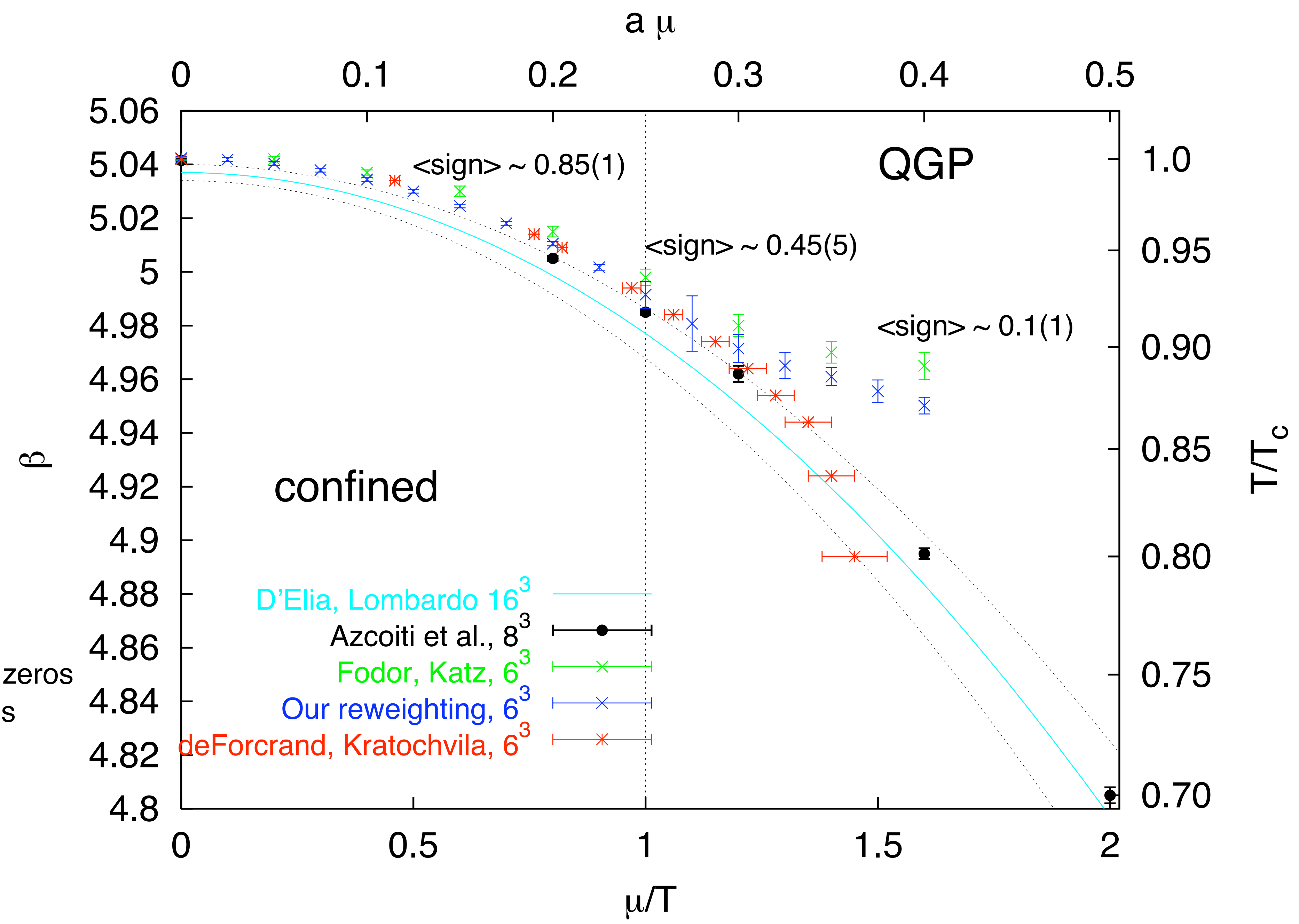
requires convergence
for anal. continuation

All require $\mu/T < 1$!

The good news: comparing $T_c(\mu)$

de Forcrand, Kratochvila 05

$N_t = 4, N_f = 4$; same actions (unimproved staggered), same mass



imaginary μ

2 param. imag. μ

dble reweighting, LY zeros

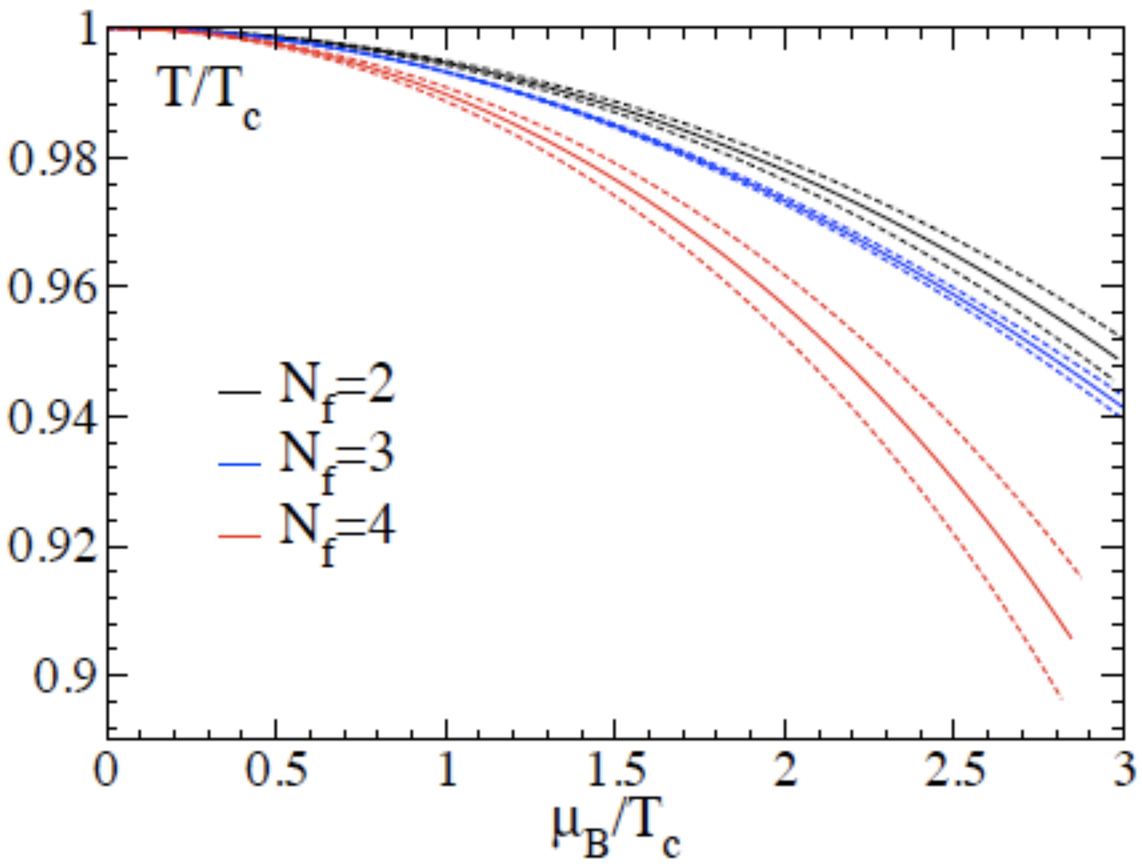
Same, susceptibilities

canonical

Agreement for $\mu/T \lesssim 1$

The (pseudo-) critical temperature

de Forcrand, O.P. 03; d'Elia Lombardo 03



$$\frac{T_c(\mu)}{T_c(\mu=0)} = 1 - c(N_f, m_q)(\frac{\mu}{\pi T})^2 + ..$$

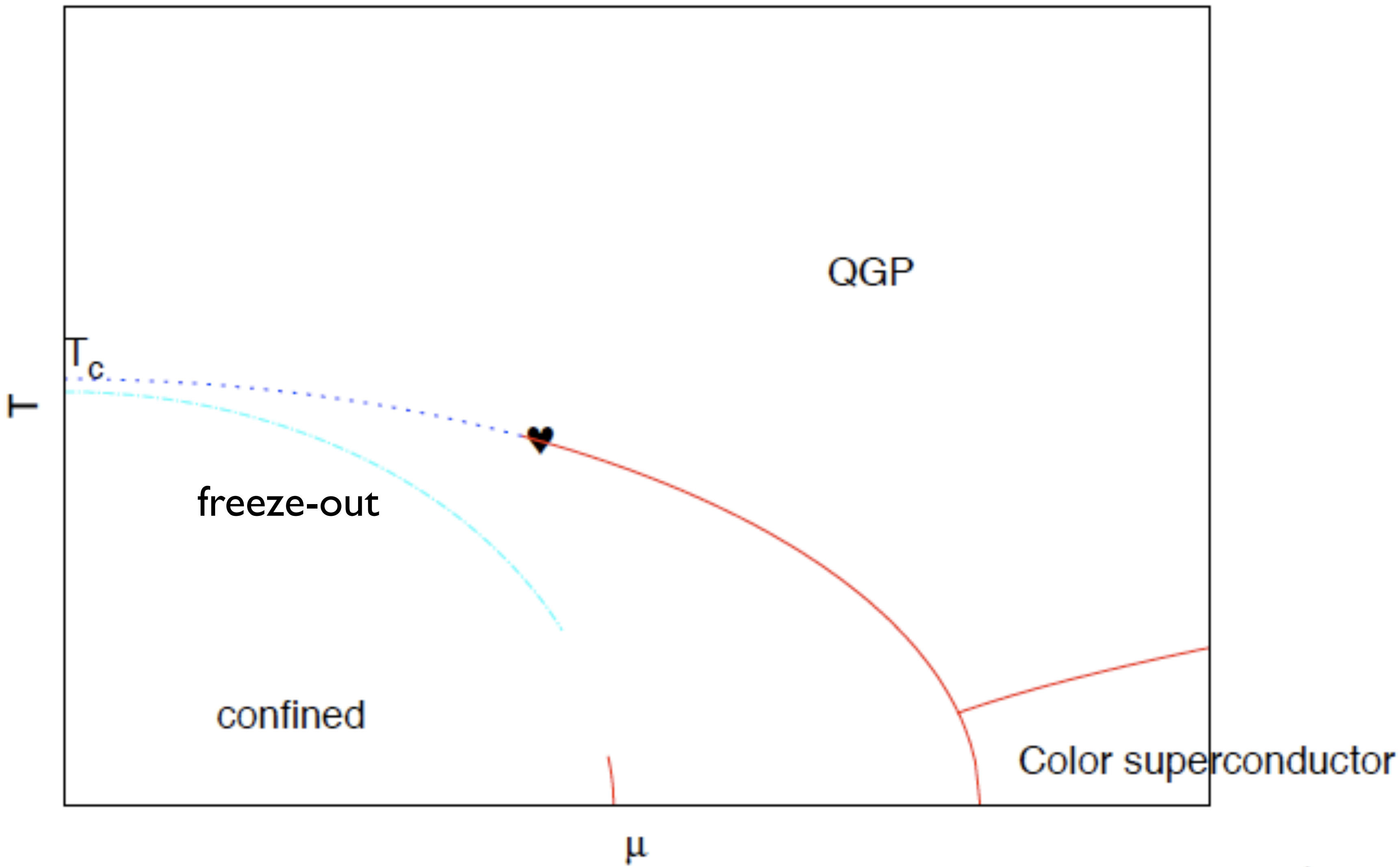
$c \approx 0.500(34), 0.602(9), 0.93(10)$

for light $N_f = 2, 3, 4$

cf. Toublan:($c \propto N_f/N_c$)

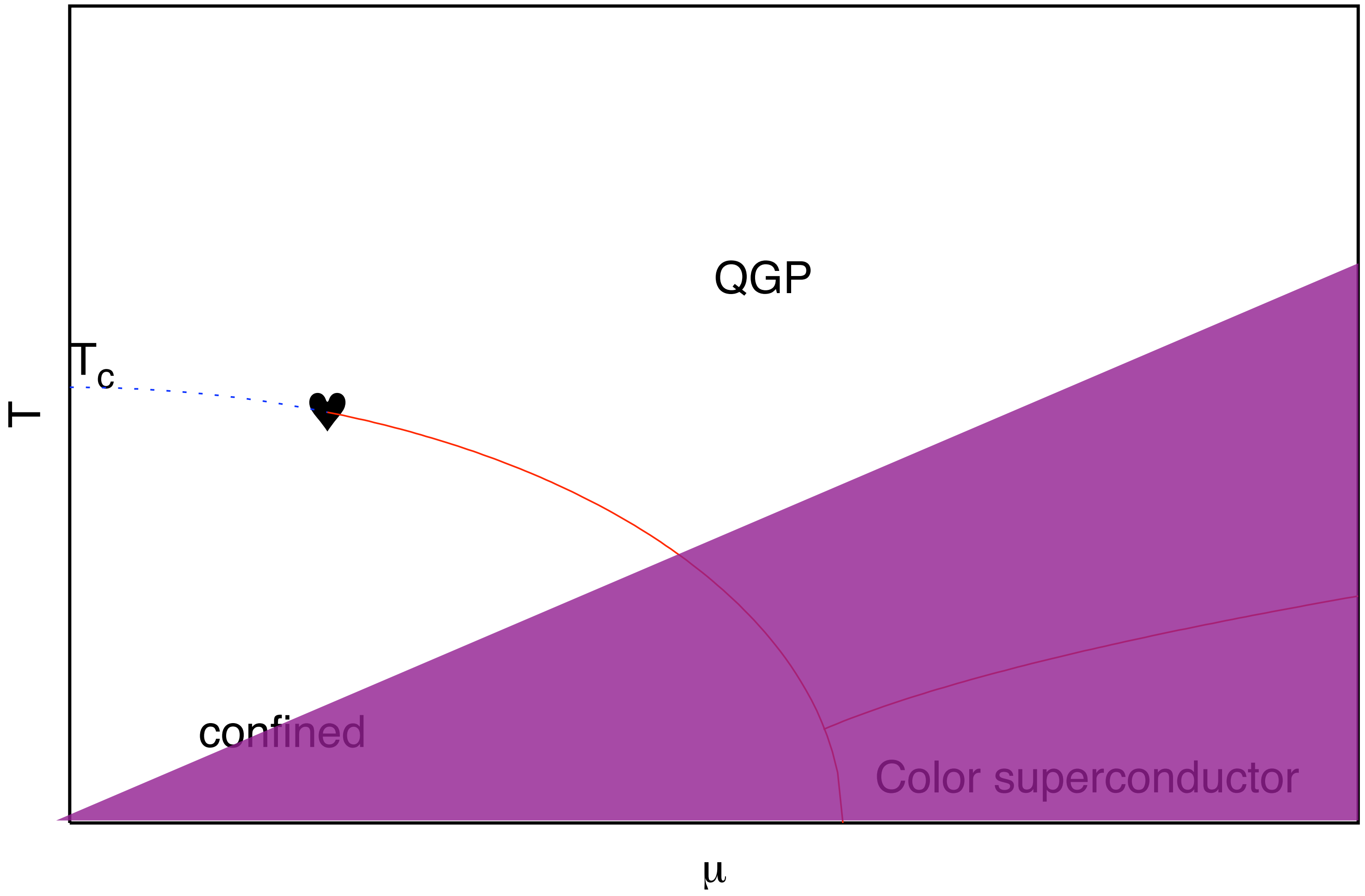
- very flat, but not yet physical masses, coarse lattices
- indications that curvature does not grow towards continuum   de Forcrand, O.P. 07
- extrapolation to physical masses and continuum is feasible!   Budapest-Wuppertal 08

Comparison with freeze-out curve so far



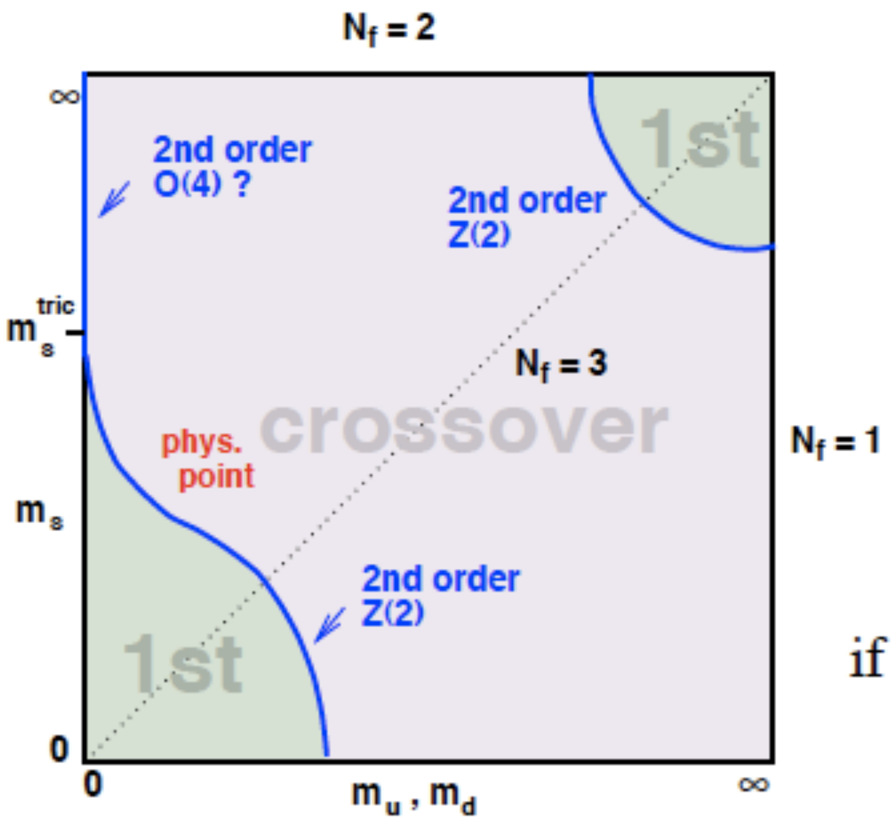
$T_c(\mu)$ considerably flatter than **freeze-out** curve (factor ~ 3 in $\left.\frac{d^2 T_c}{d\mu^2}\right|_{\mu=0}$)

The calculable region of the phase diagram

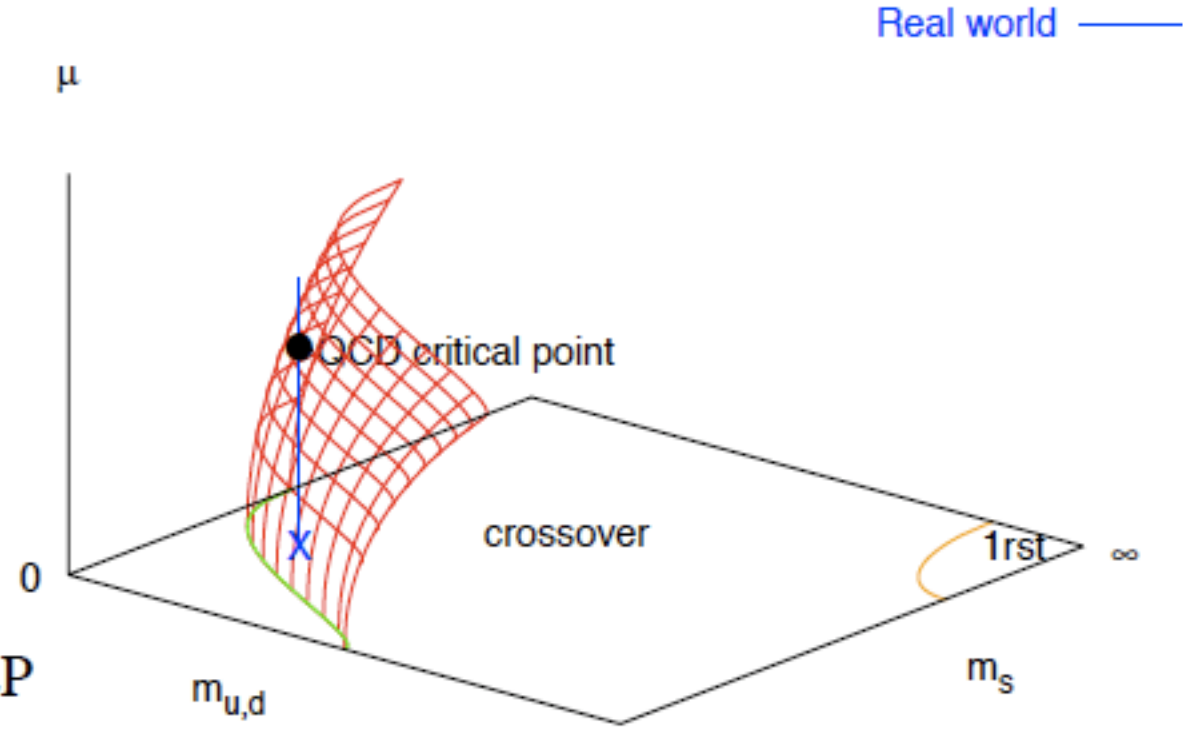


- Upper region: equation of state, screening masses, quark number susceptibilities etc. under control
- Here: phase diagram itself, most difficult!

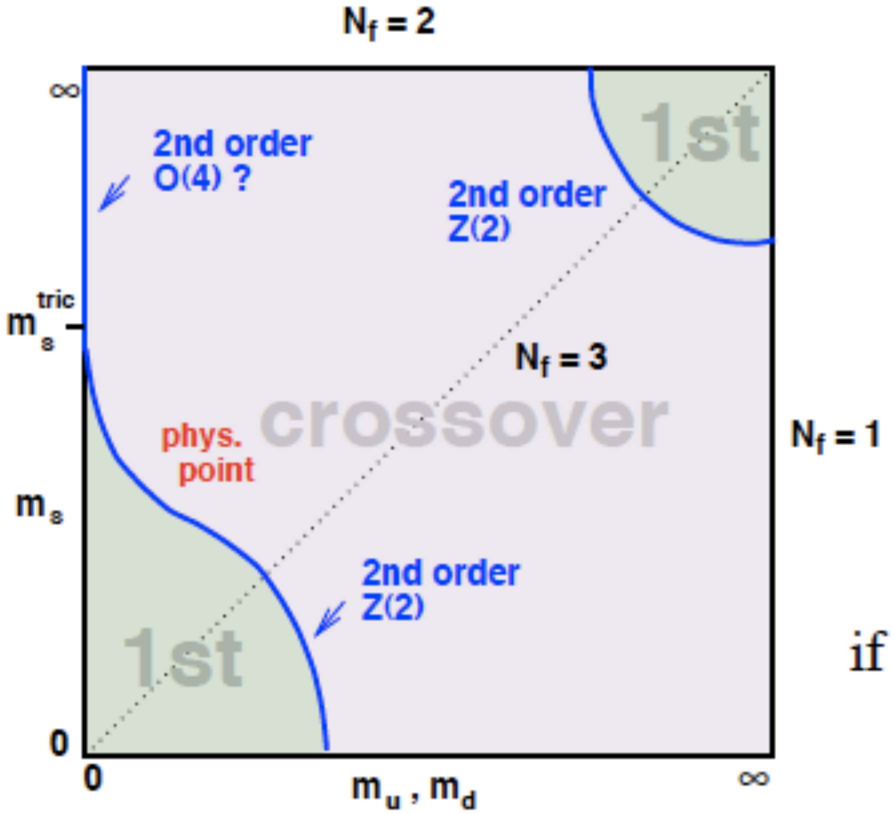
Much harder: is there a QCD critical point?



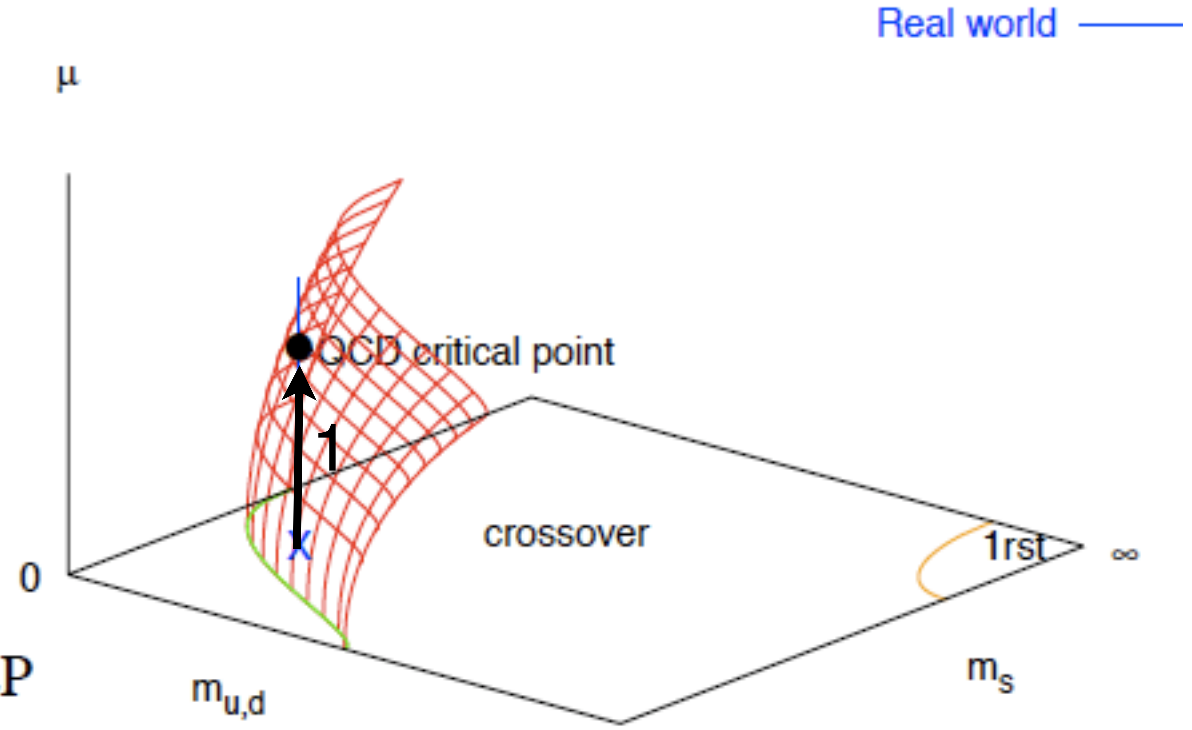
$\underbrace{\longrightarrow}_{\text{if } \textit{chiral} \text{ CEP}}$



Much harder: is there a QCD critical point?



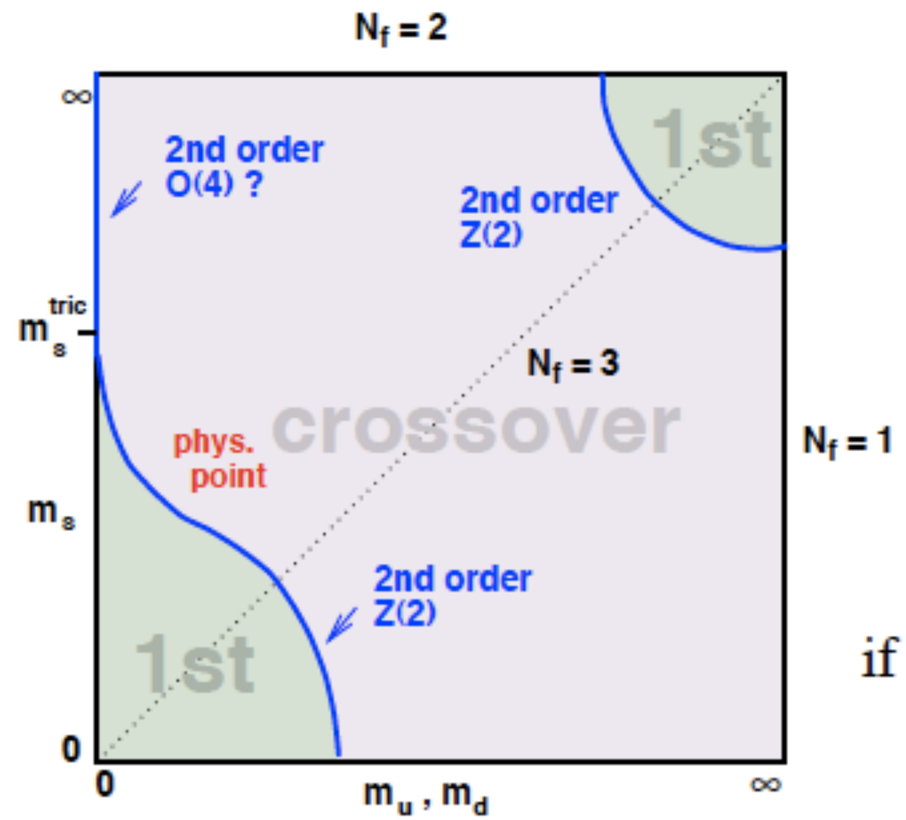
$$\underbrace{\longrightarrow}_{\text{if } chiral \text{ CEP}}$$



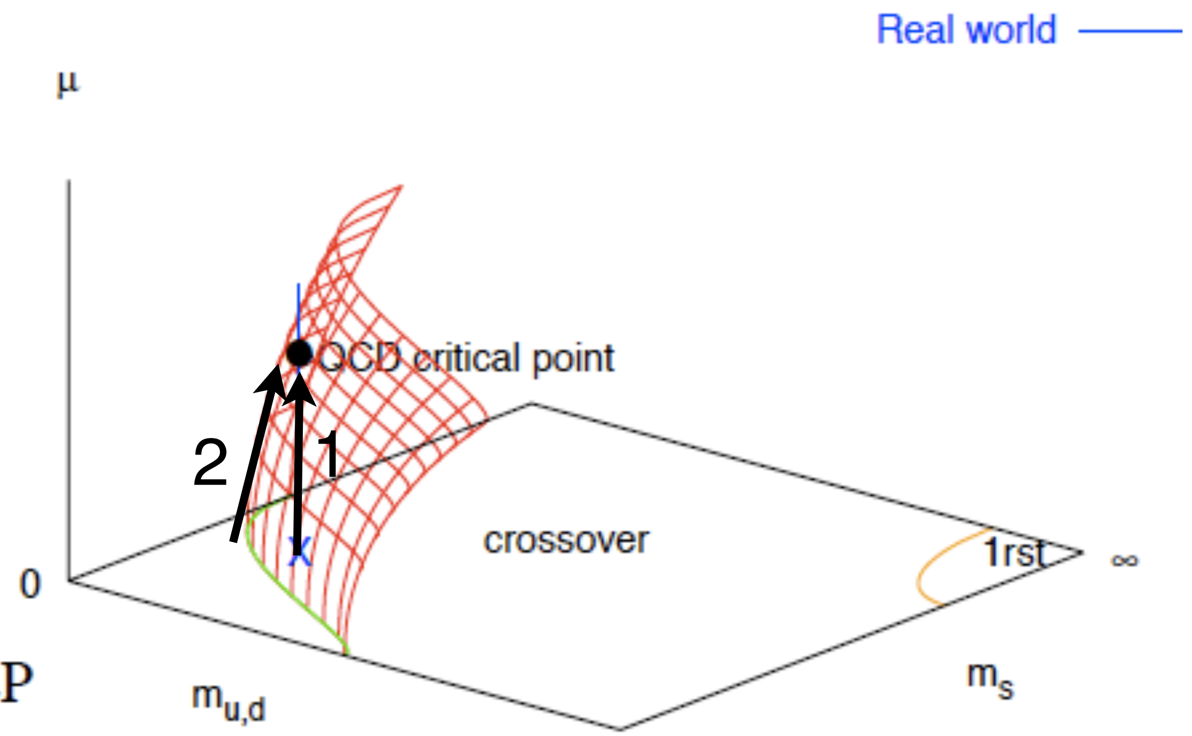
Two strategies:

1 follow vertical line: $m = m_{\rm phys}$, turn on μ

Much harder: is there a QCD critical point?



$\underbrace{\longrightarrow}_{\text{if } chiral \text{ CEP}}$



Two strategies:

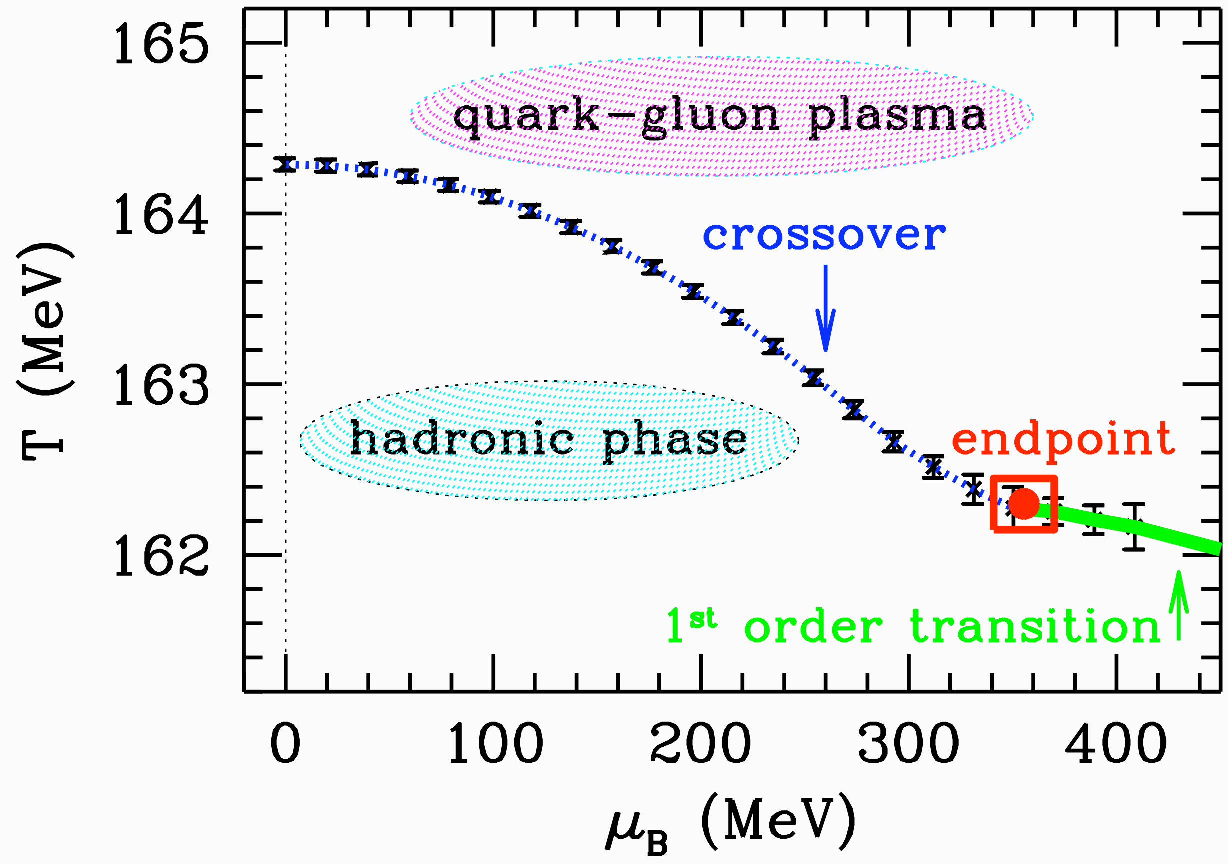
1 follow vertical line: $m = m_{\rm phys}$, turn on μ

2 follow critical surface: $m = m_{\rm crit}(\mu)$

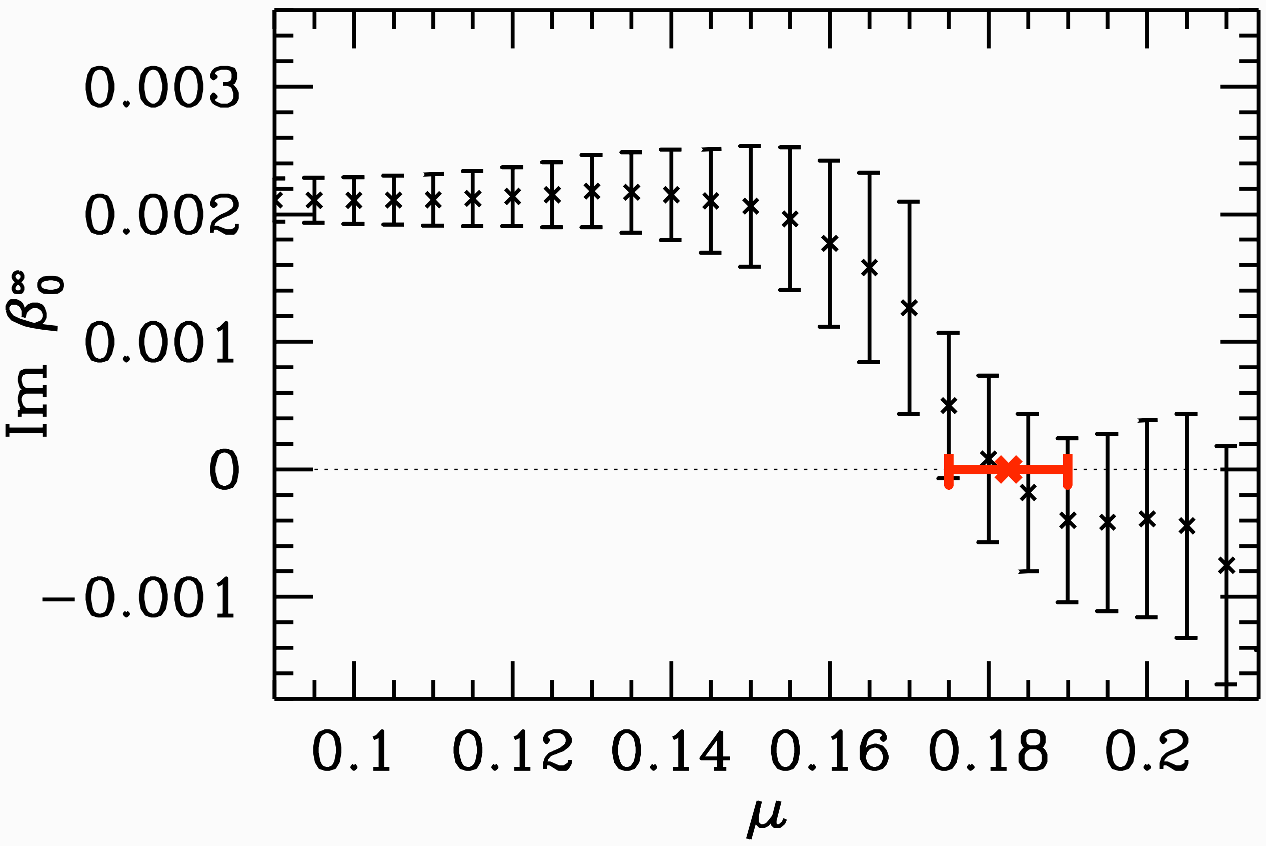
Approach 1a: CEP from reweighting

Fodor, Katz 04

$N_t = 4, N_f = 2+1$ physical quark masses, unimproved staggered fermions



Lee-Yang zero:

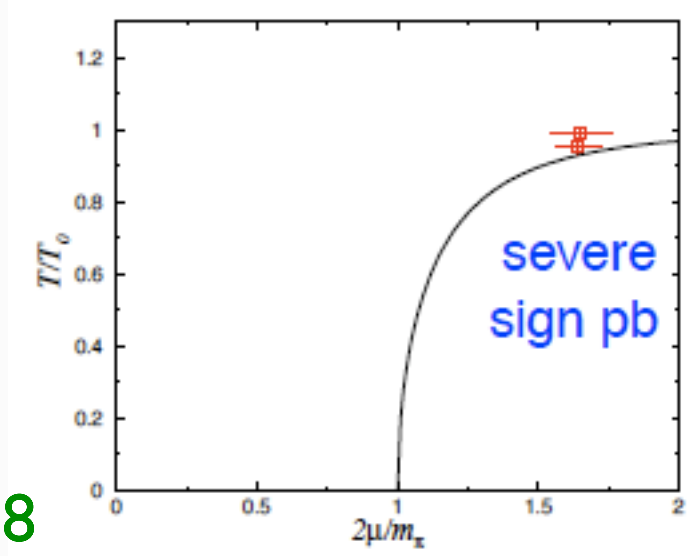


$$(\mu_E^q, T_E) = \ (120(13), 162(2)) \text{ MeV}$$

abrupt change: physics or problem of the method?

(entire curve generated from one point!)

Splittorf 05, Stephanov 08



severe sign pb

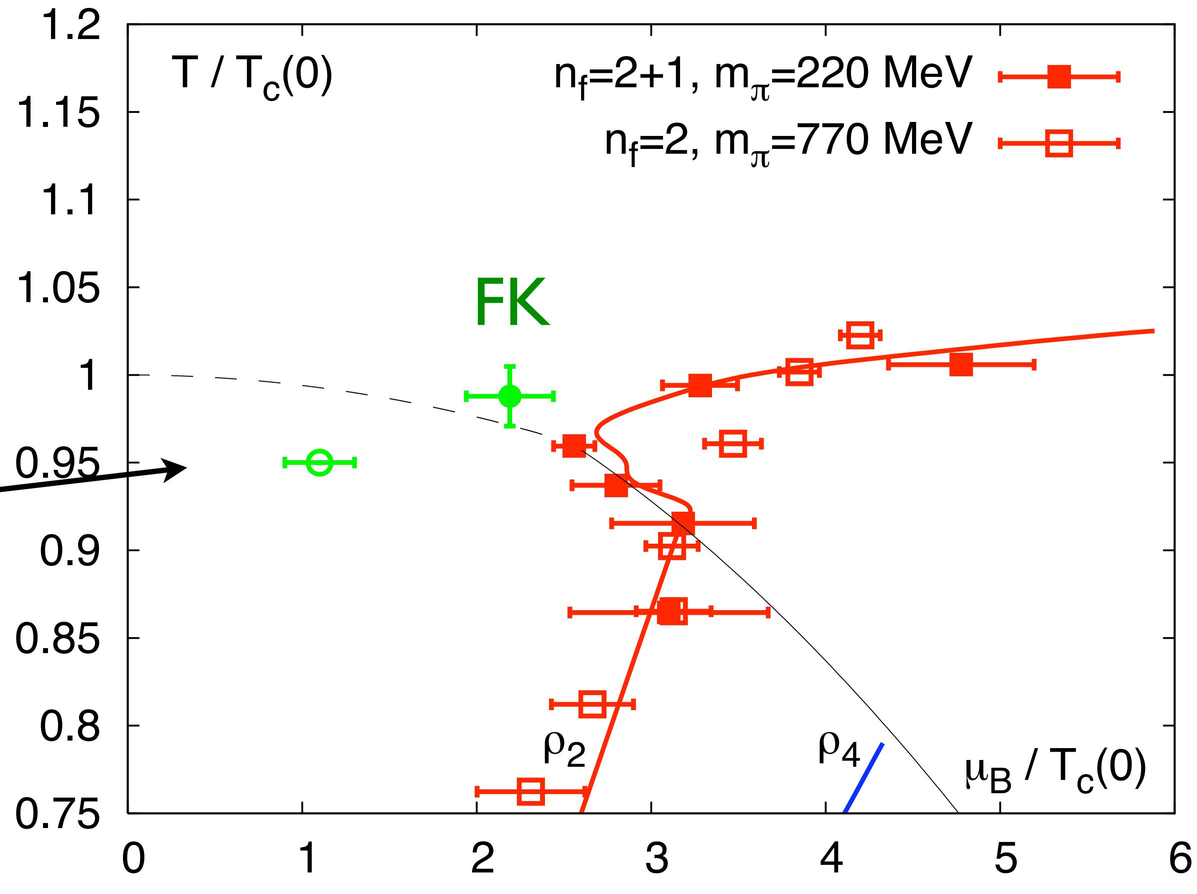
Approach 1b: CEP from Taylor expansion

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu}{T}\right)^{2n}$$

Nearest singularity=radius of convergence $\frac{\mu_E}{T_E} = \lim_{n\to\infty} \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}, \quad \lim_{n\to\infty} \left|\frac{c_0}{c_{2n}}\right|^{\frac{1}{2n}}$

Different definitions agree only for $n \to \infty$ not n=1,2,3

CEP may not be nearest singularity, control of systematics?



Bielefeld-Swansea-RBC

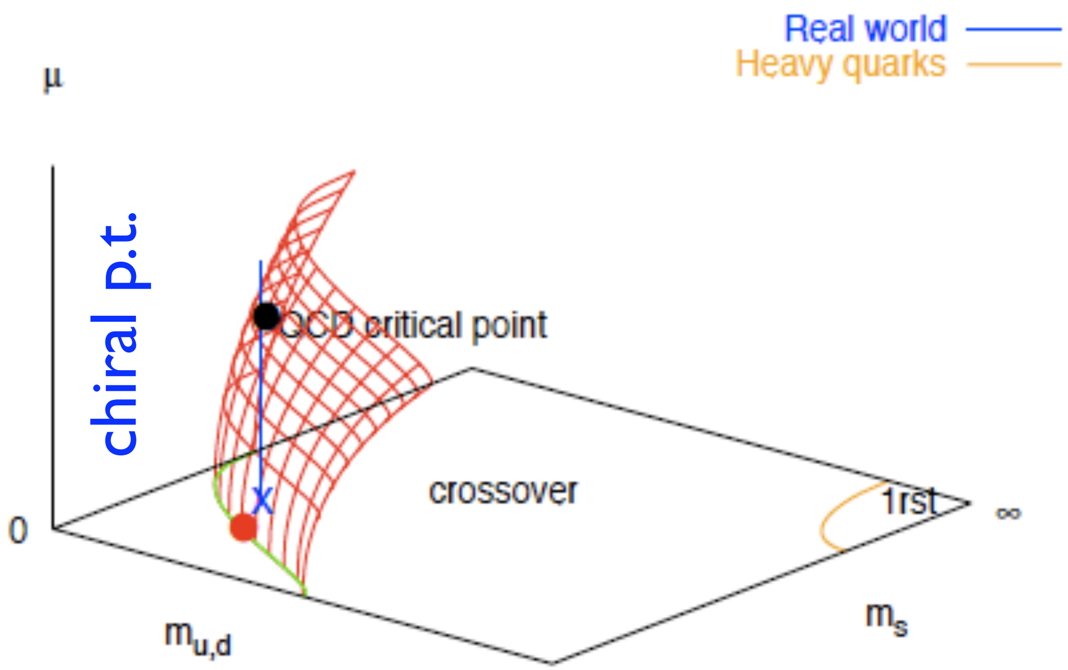
improved staggered

$N_t = 4$

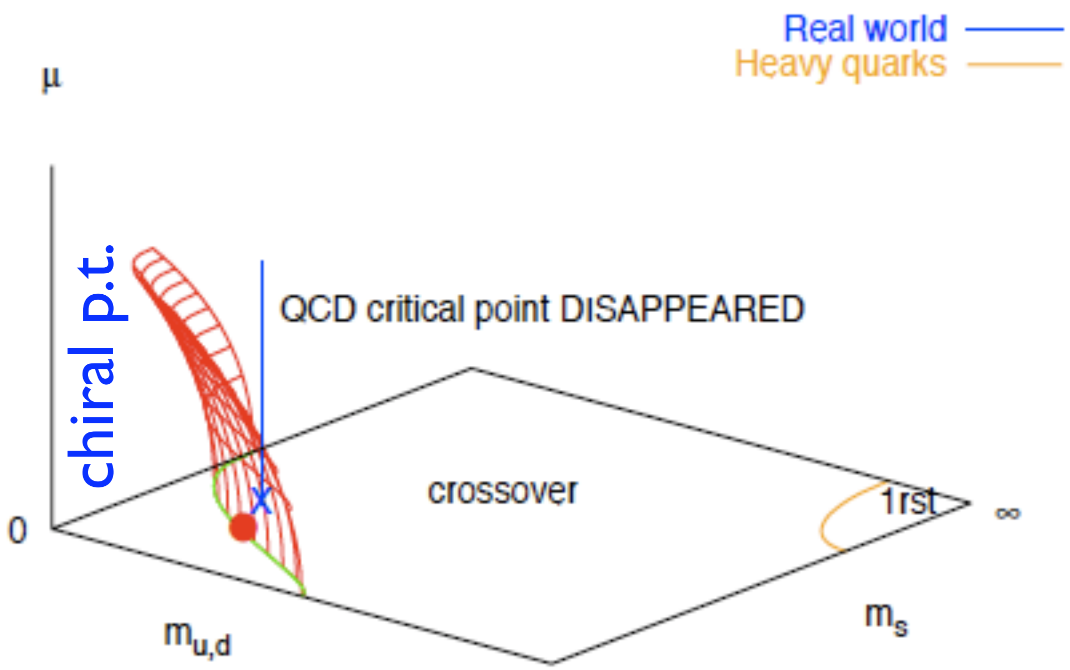
Gavai, Gupta

$N_f = 2$

Approach 2: follow chiral critical line → surface



$c_1 > 0$



$c_1 < 0$

$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

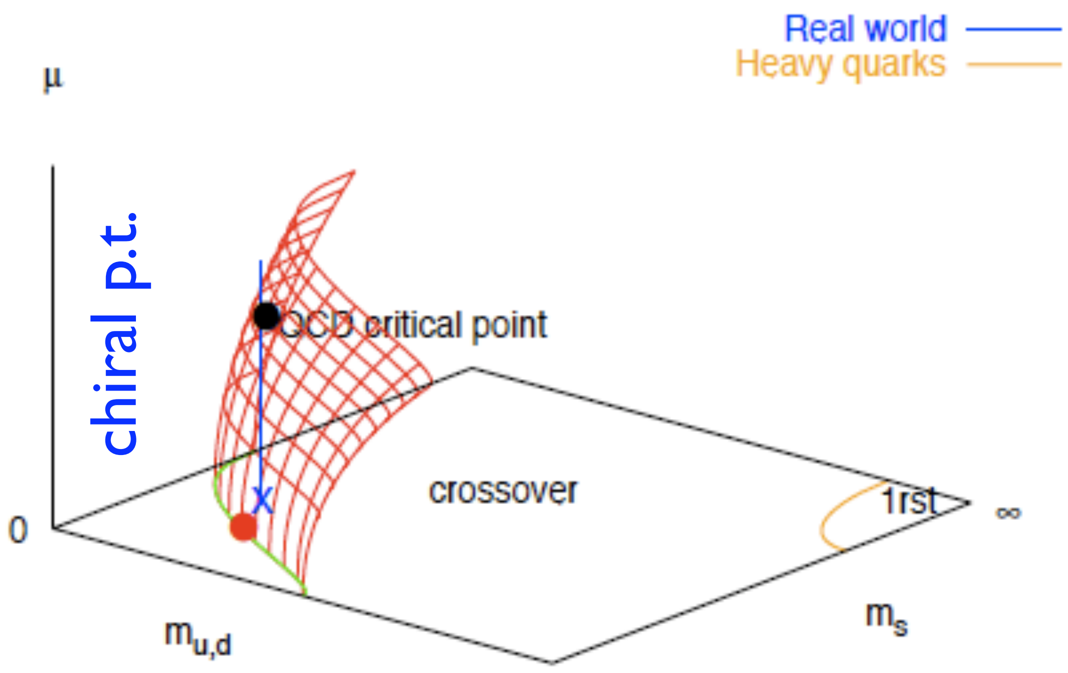
1. Tune quark mass(es) to $m_c(0)$: 2nd order transition at $\mu = 0$, $T = T_c$
   known universality class: 3d Ising

2. Measure derivatives $\frac{d^k m_c}{d\mu^{2k}}|_{\mu=0}$:

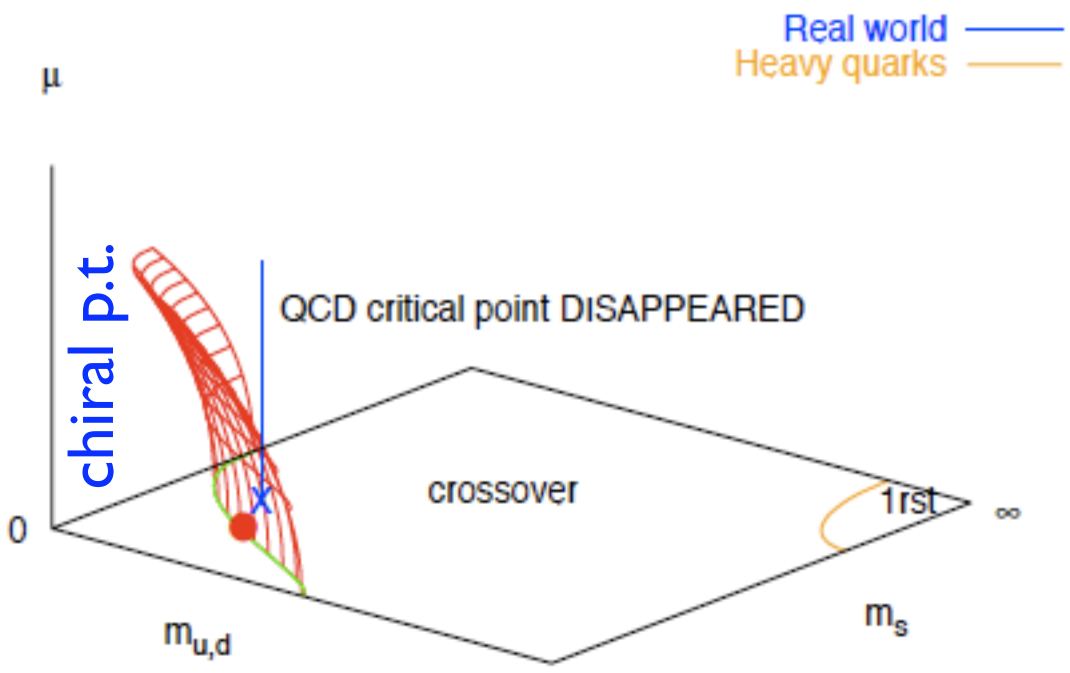
   Turn on imaginary μ and measure $\frac{m_c(\mu)}{m_c(0)}$

de Forcrand, O.P. 08,09

Approach 2: imaginary rather than imagined (?) CEP



$c_1 > 0$



$c_1 < 0$

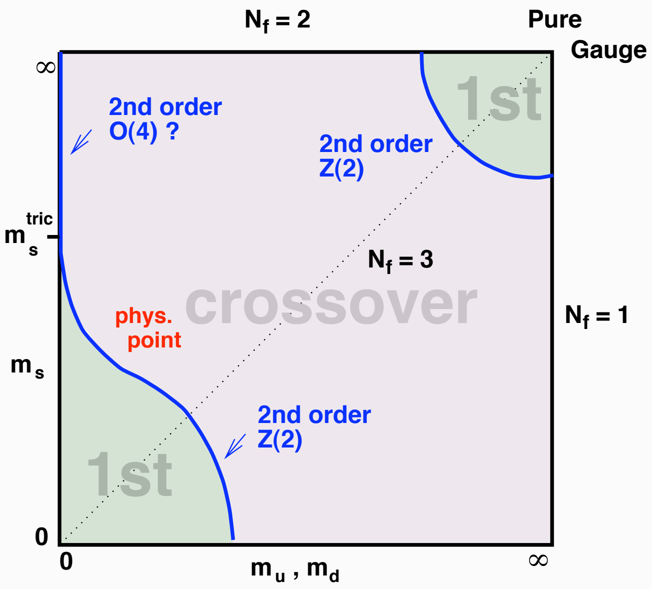
$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

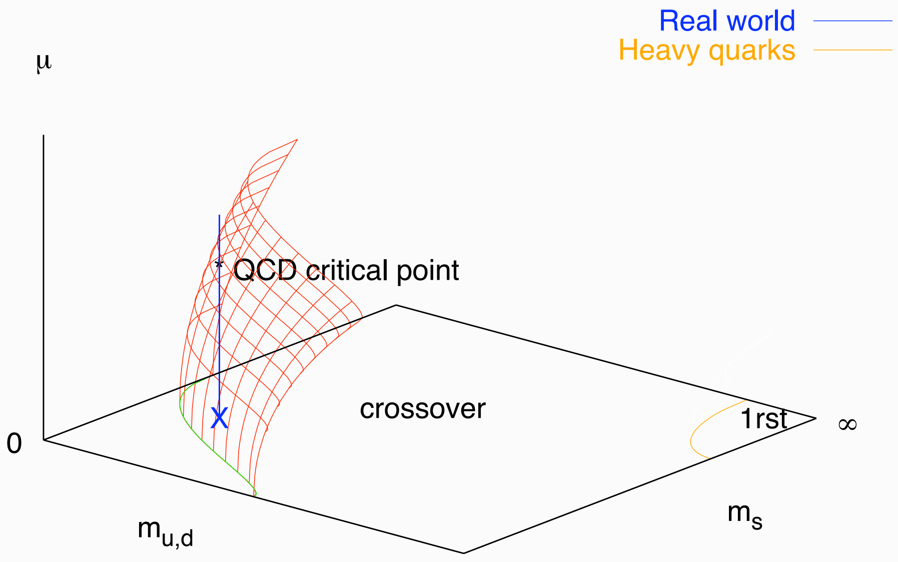
1. Tune quark mass(es) to $m_c(0)$: 2nd order transition at $\mu = 0$, $T = T_c$
   known universality class: 3d Ising
2. Measure derivatives $\frac{d^k m_c}{d\mu^{2k}}|_{\mu=0}$:

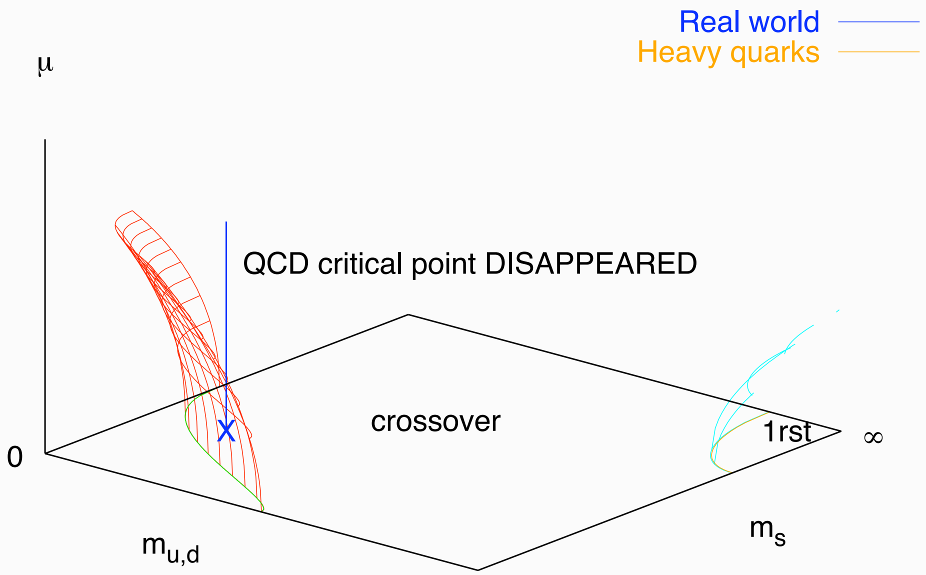
   Turn on imaginary μ and measure $\frac{m_c(\mu)}{m_c(0)}$

de Forcrand, O.P. 08,09

Finite density: chiral critical line ⟶ critical surface



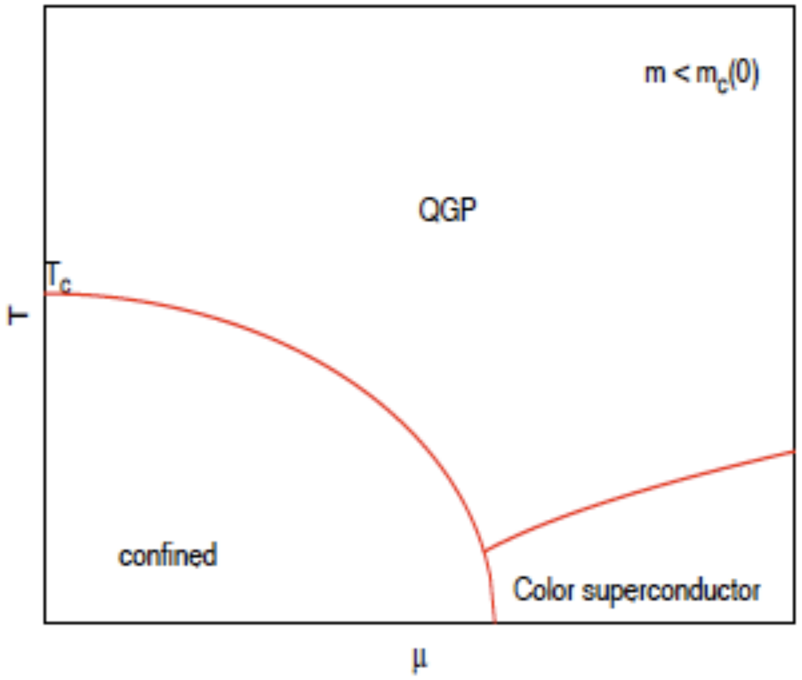


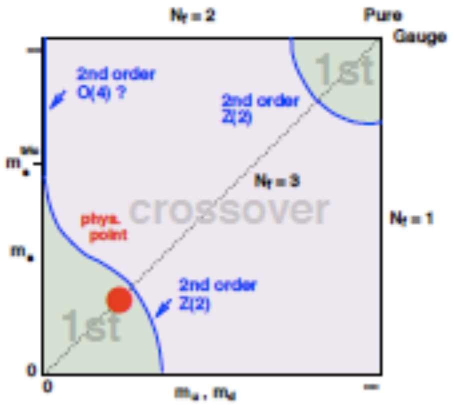


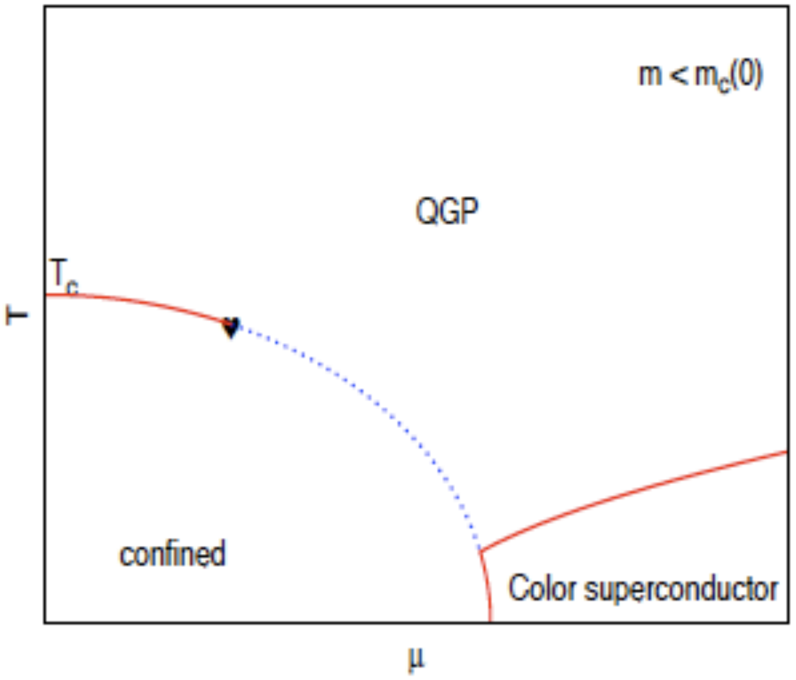
$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

$c_1 > 0$ $c_1 < 0$

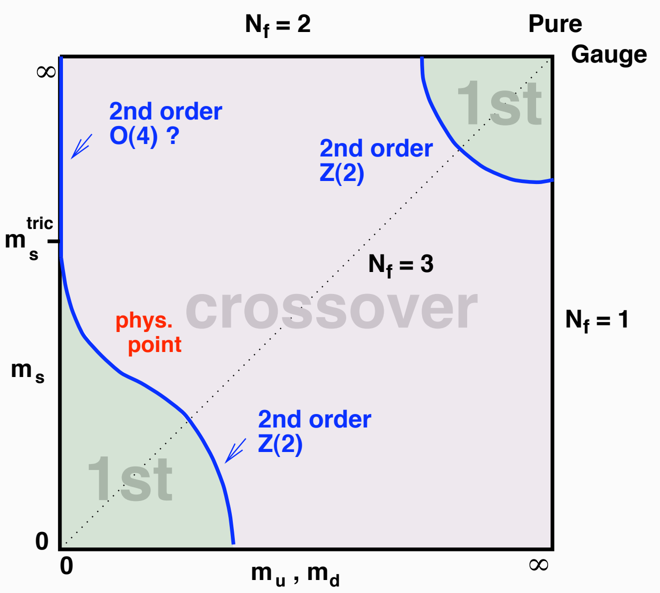
Standard scenario **Exotic scenario**

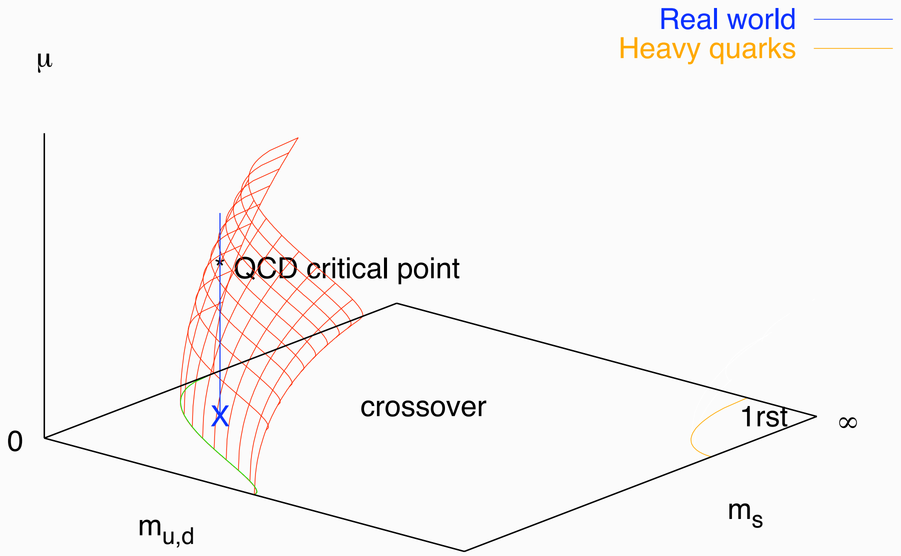


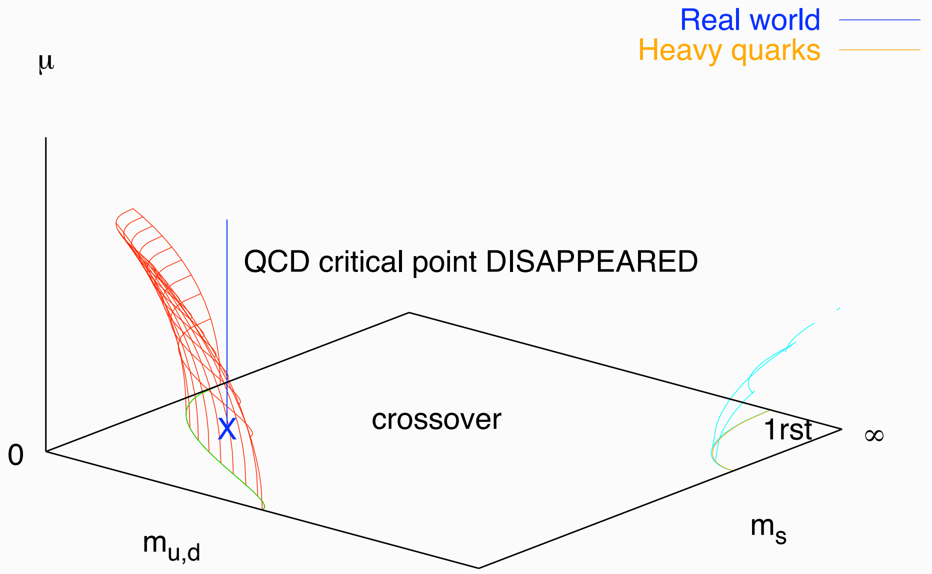




Finite density: chiral critical line ⟶ critical surface





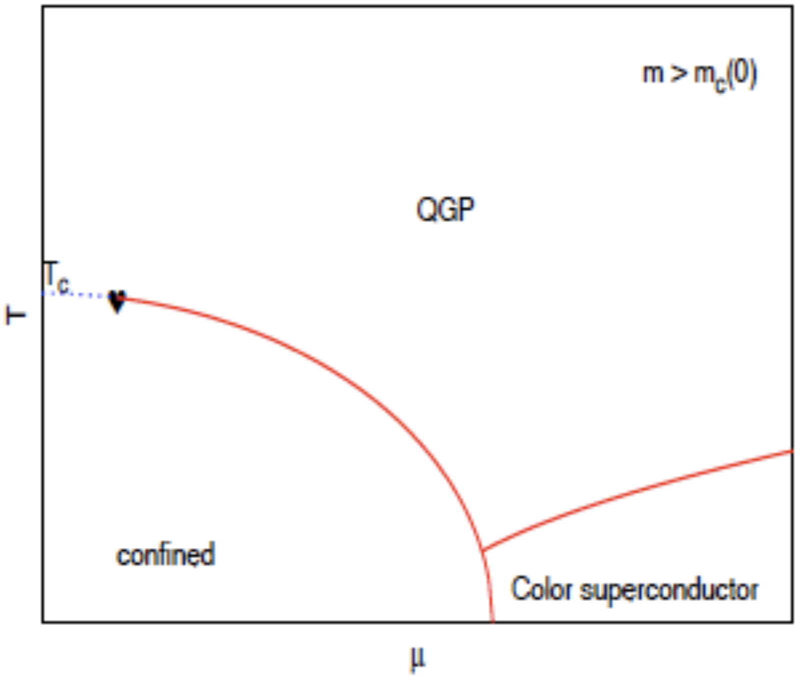


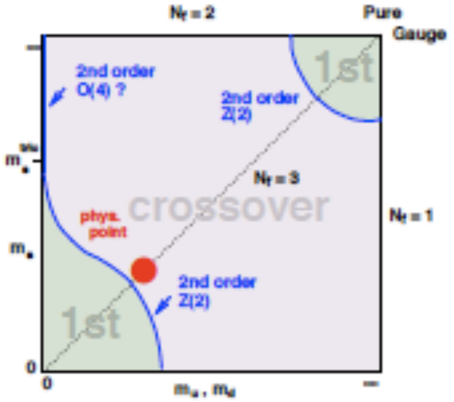
$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

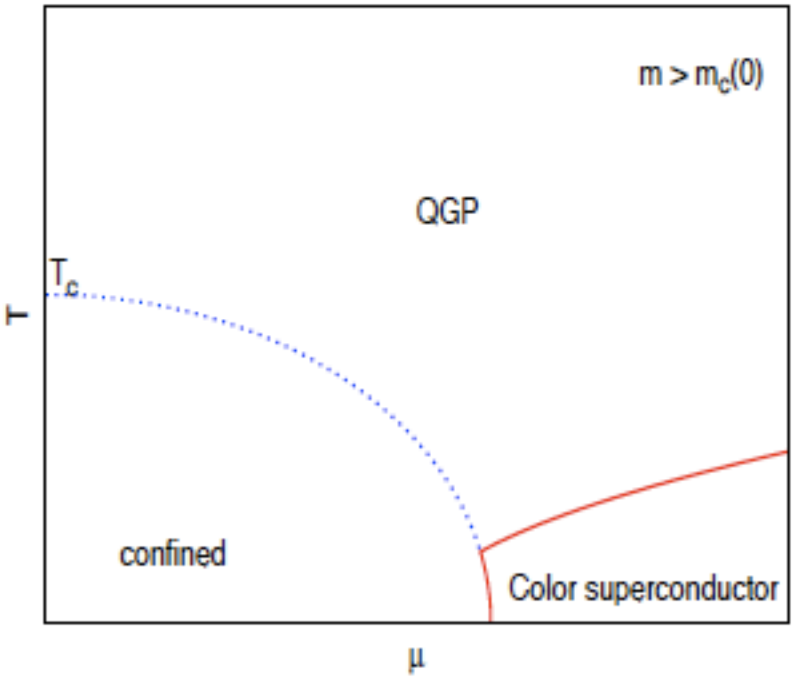
$c_1 > 0$ $c_1 < 0$

Standard scenario

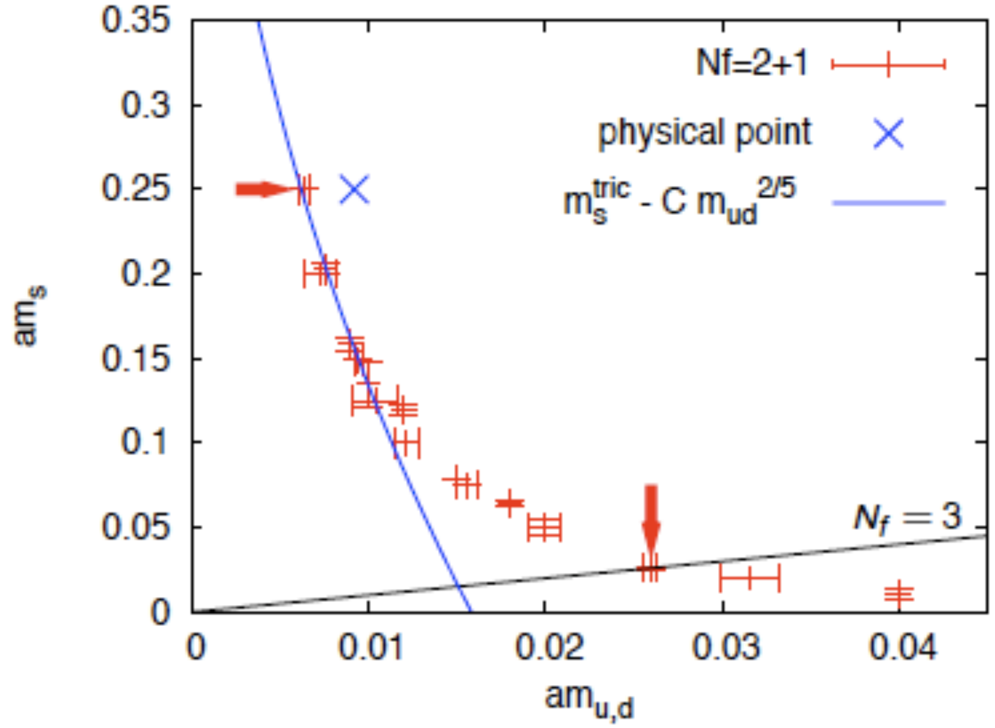
Exotic scenario







Curvature of the chiral critical surface



$N_f = 3$

$N_f = 2+1,\ m_s = m_s^{\text{phys}}$

consistent $8^3 \times 4$ and $12^3 \times 4$, $\sim 5 \times 10^6$ traj.

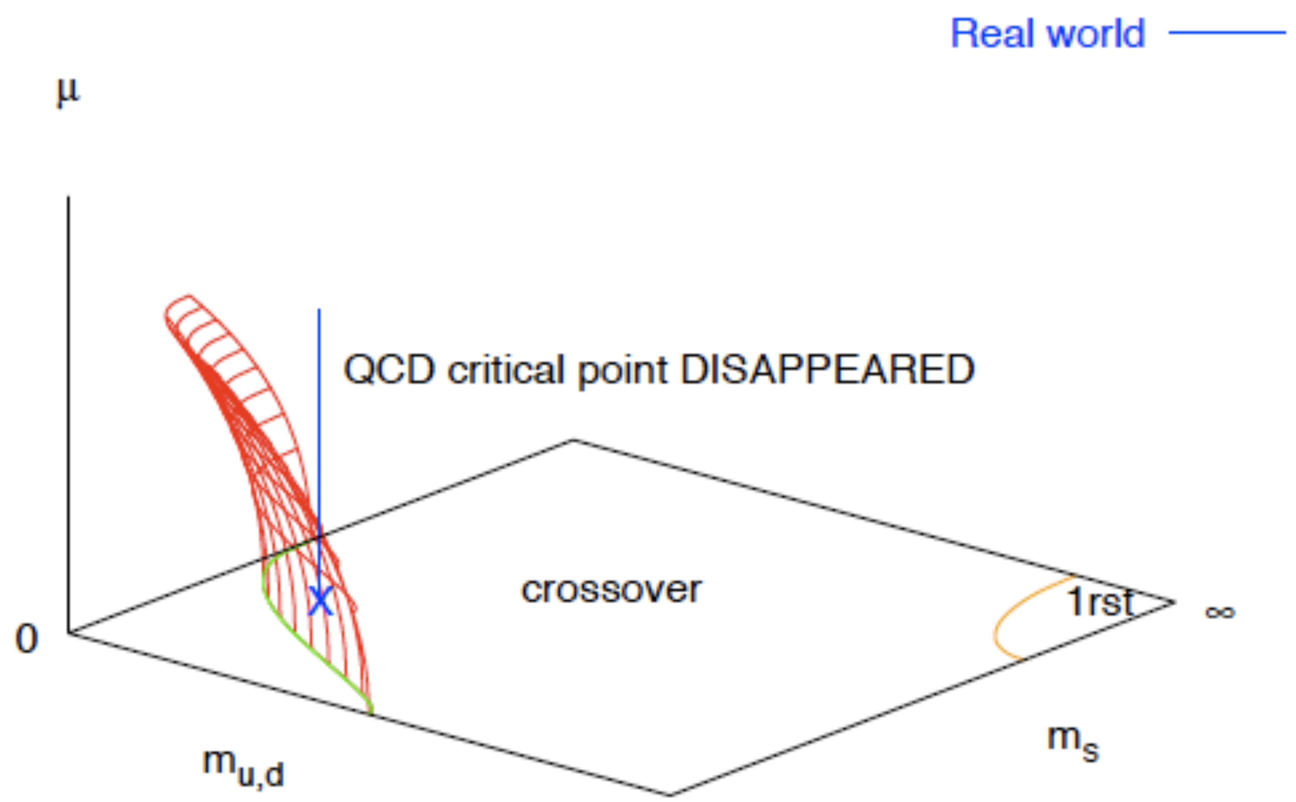
$$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3)\left(\frac{\mu}{\pi T}\right)^2 \underbrace{- 47(20)\left(\frac{\mu}{\pi T}\right)^4}_{\text{8th derivative of P}} - \ldots$$

$16^3 \times 4$, Grid computing, $\sim 10^6$ traj.

$$\frac{m_c^{u,d}(\mu)}{m_c^{u,d}(0)} = 1 - 39(8)\left(\frac{\mu}{\pi T}\right)^2 - \ldots$$

de Forcrand, O.P. 08,09

The chiral critical surface on a coarse lattice:



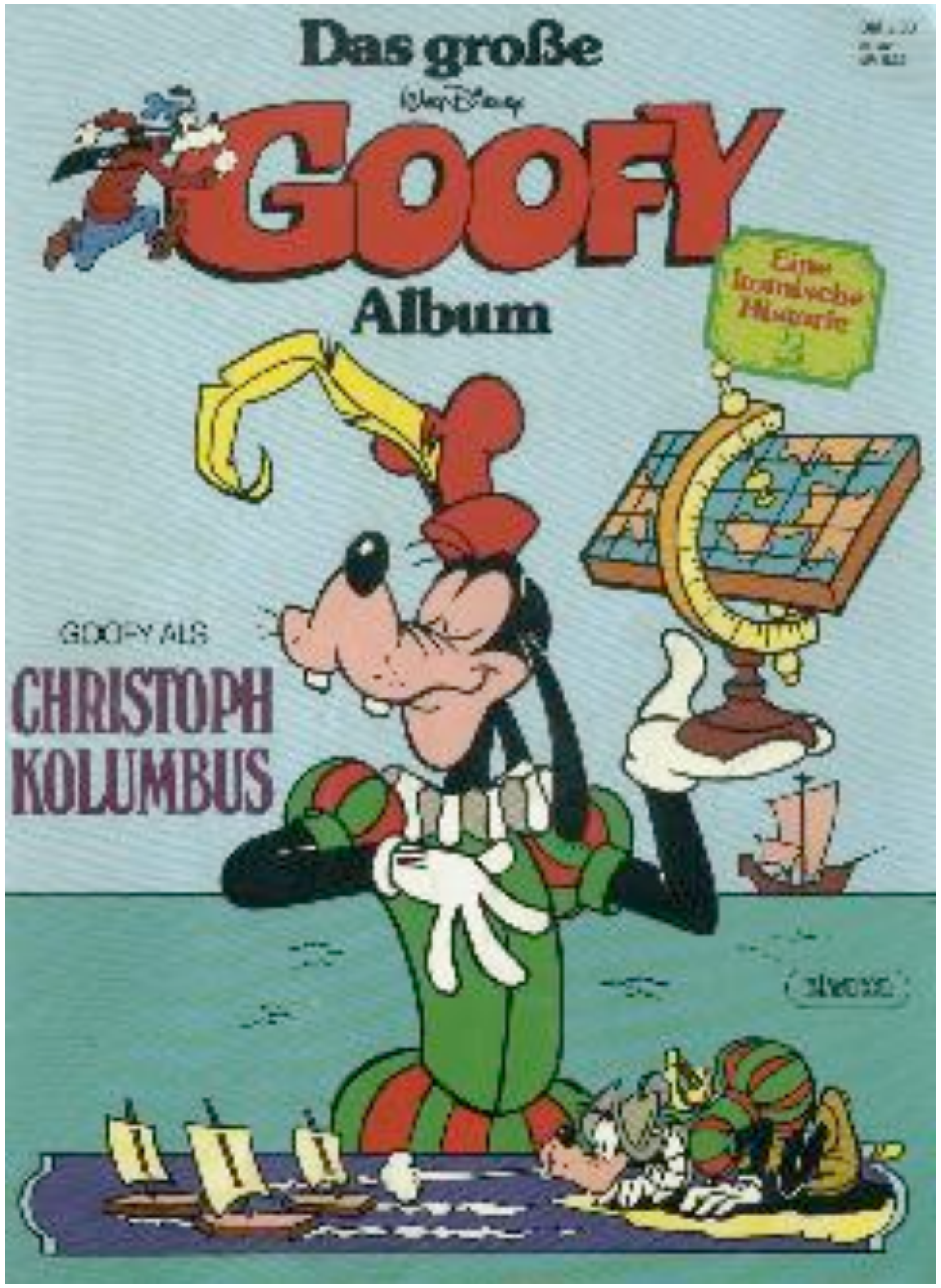
No chiral crit. pt. at small chem. pot., $\frac{\mu}{T} \lesssim o(1)$, for $N_t = 4$ ($a \sim 0.3$ fm)

cf. Ejiri 08 $\rightarrow \left(\frac{\mu}{T}\right)^{\text{CEP}} \sim 2.4$

- Higher order terms? Convergence?
- Cut-off effects?
- In any case: picture for QCD phase diagram not as clear as anticipated.....

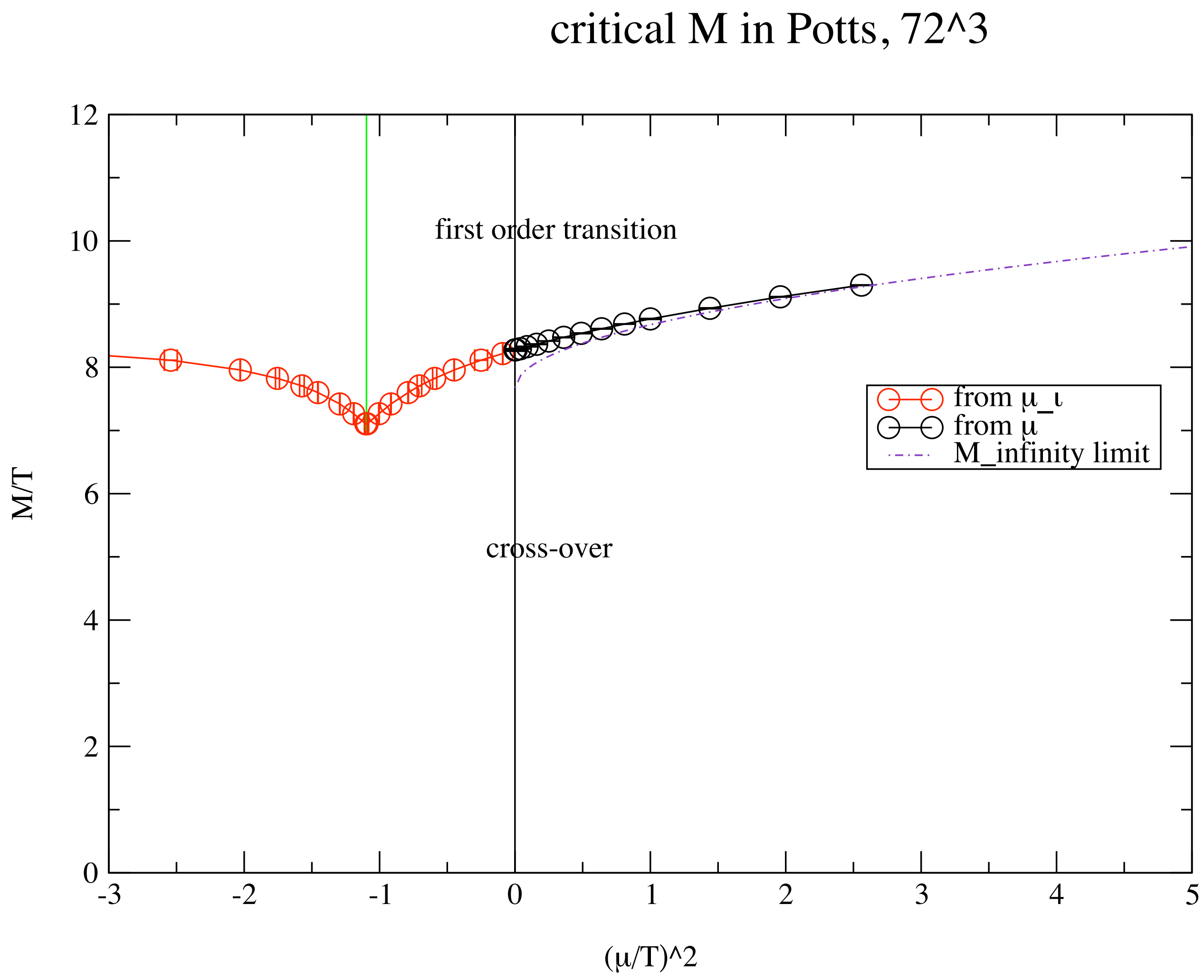
Un-discovering a critical point feels like...

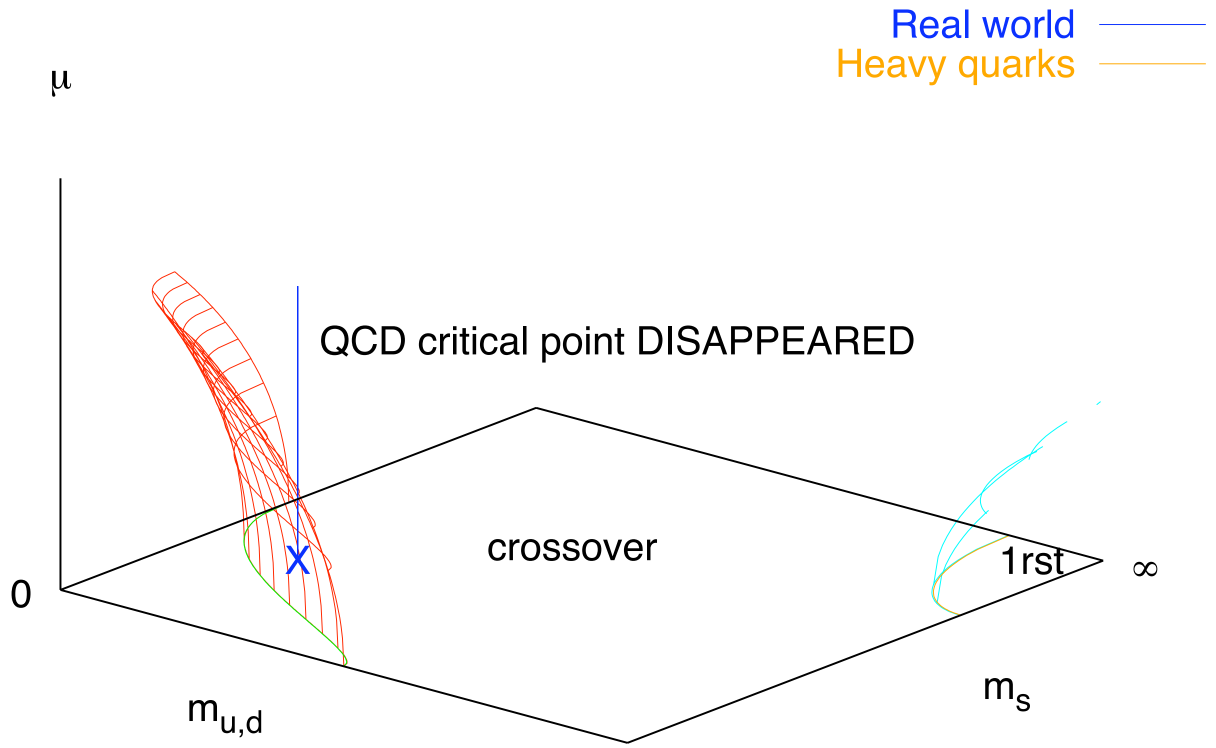
Un-discovering a critical point feels like...



Scenario unusual? ...the same happens for heavy quark masses!

effective heavy quark theory, same universality class: 3-state Potts model





Real μ: first order region shrinking!

de Forcrand, Kim, Takaishi

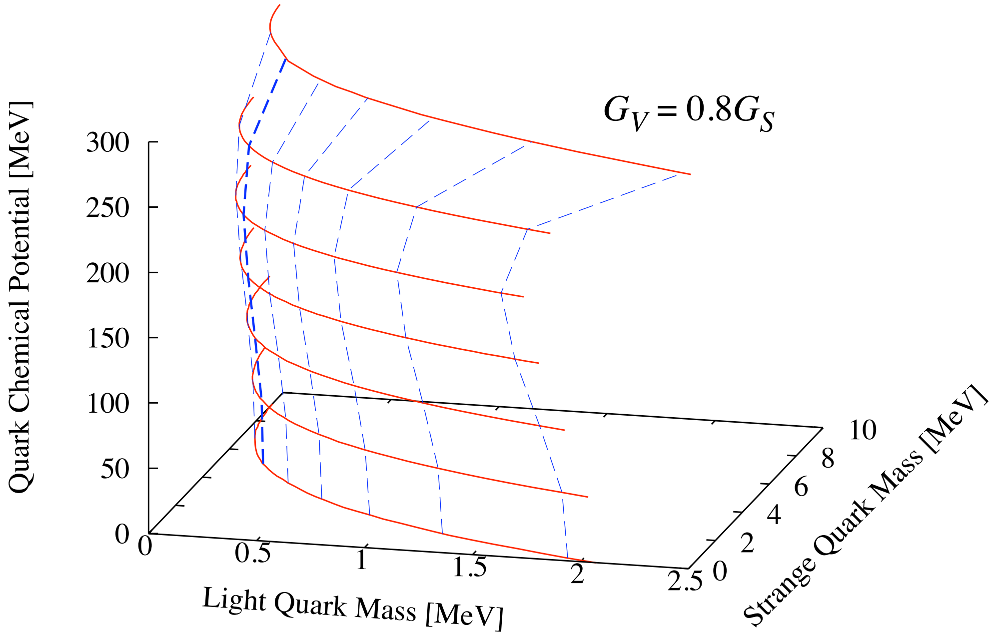
also for finite isospin chemical potential

Kogut, Sinclair

N.B.: non-chiral critical point still possible!

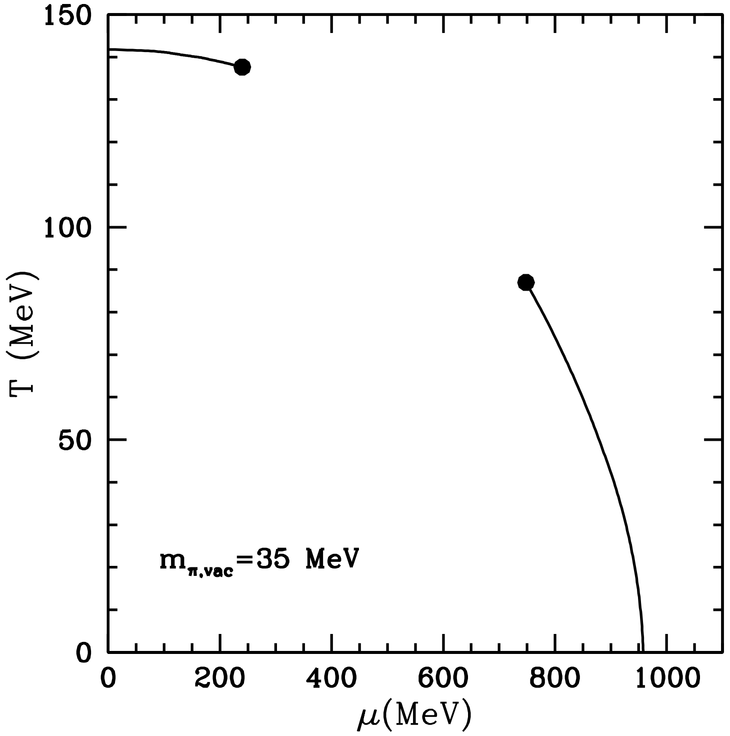
Recent model studies with similar results

K. Fukushima 08



NJL-Polyakov loop model with vector-vector interaction

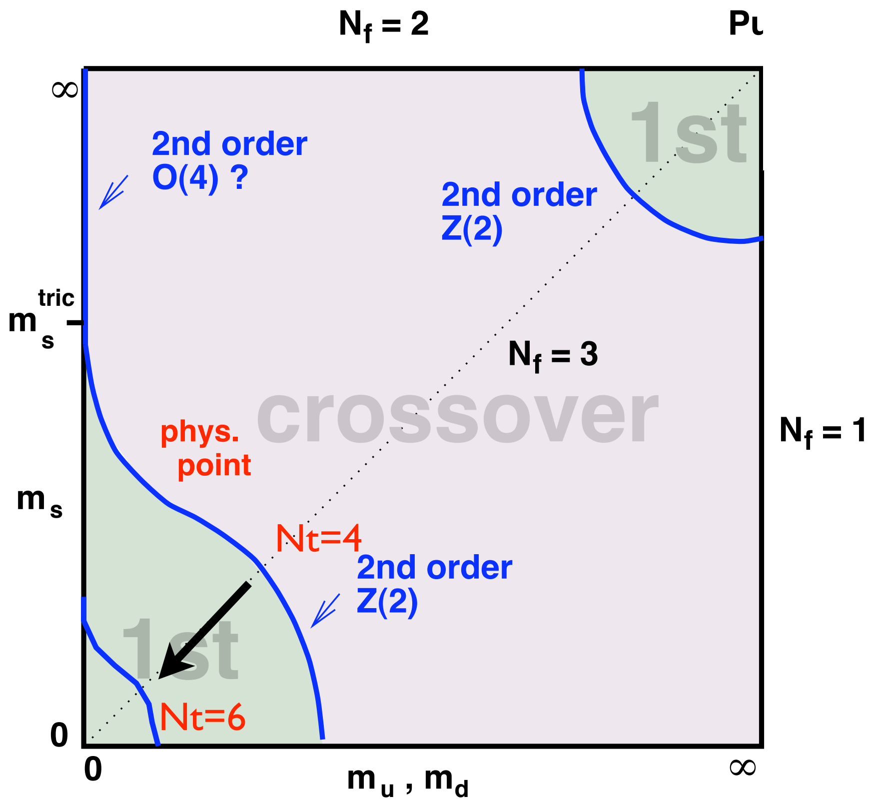
Bowman, Kapusta 08

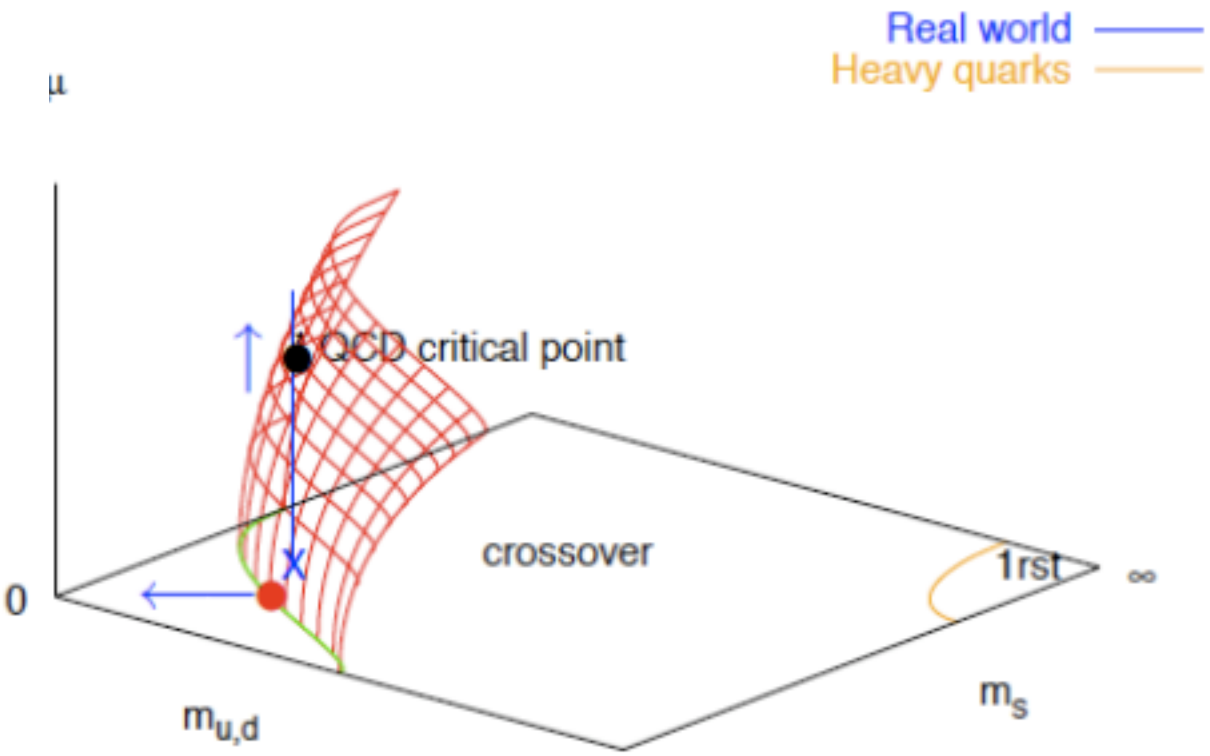


Linear sigma model with quarks

Qualitative behaviour as in exotic lattice scenario!

Towards the continuum: $N_t = 6, a \sim 0.2\ \mathrm{fm}$





$$\frac{m_\pi^c(N_t = 4)}{m_\pi^c(N_t = 6)} \approx 1.77 \qquad N_f = 3$$

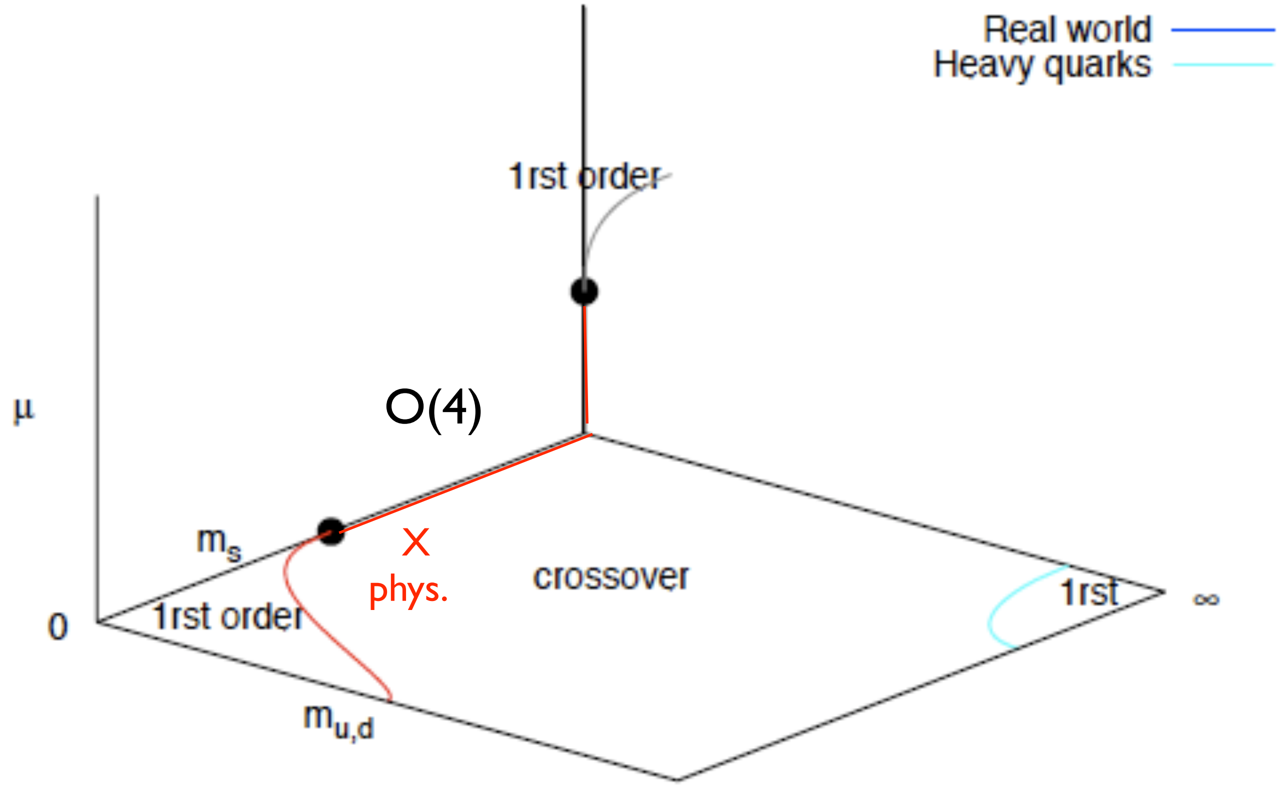
de Forcrand, Kim, O.P. 07
Endrodi et al 07

- Physical point deeper in crossover region as $a \longrightarrow 0$
- Cut-off effects stronger than finite density effects!
- Curvature of crit. surface (so far) consistent with 0; strong quark mass sensitivity!

The interplay of Nf=2 and Nf=2+1

- $O(4)$ transition for 2 massless flavors Pisarski & Wilczek

$\Rightarrow$ tricritical points $(m_{u,d} = 0, m_s = \infty, \mu = \mu^*)$ and $(m_{u,d} = 0, m_s = m_s^*, \mu = 0)$

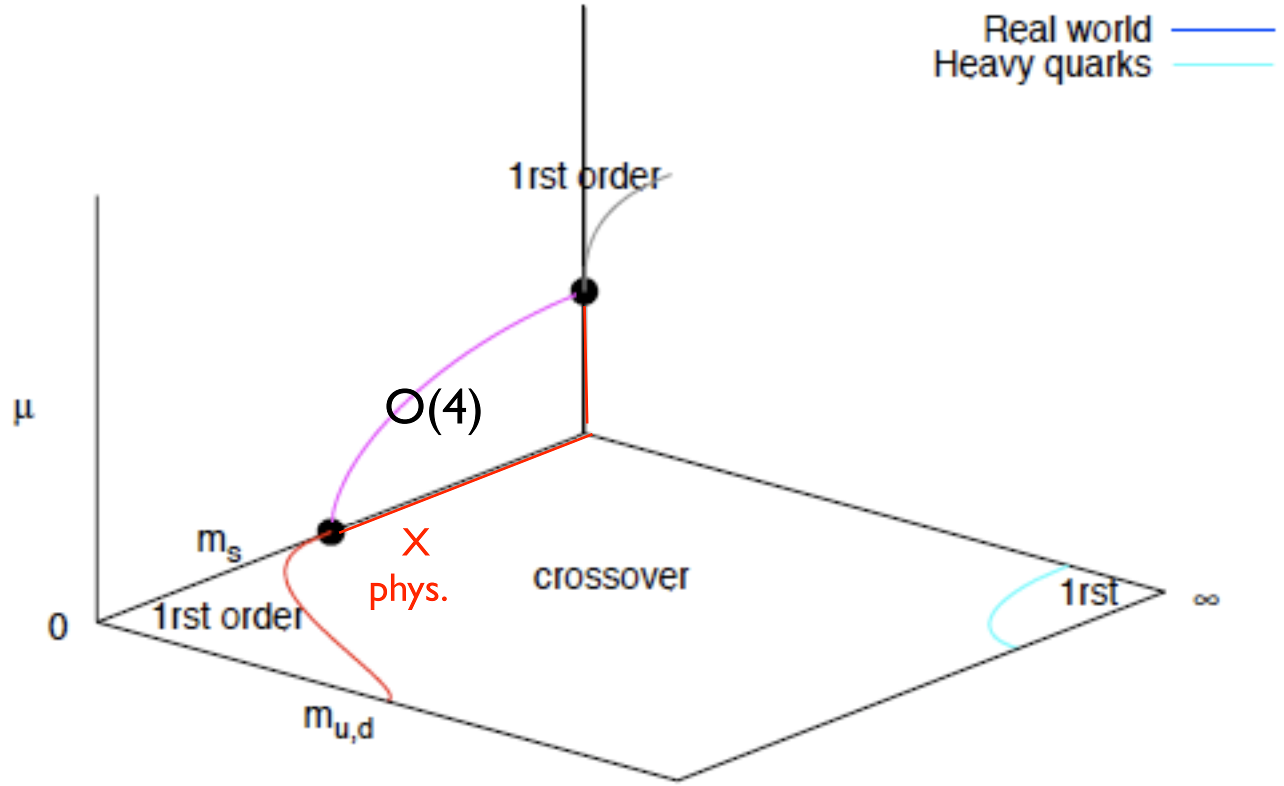


The interplay of Nf=2 and Nf=2+1

- $O(4)$ transition for 2 massless flavors Pisarski & Wilczek

$\Rightarrow$ tricritical points $(m_{u,d} = 0, m_s = \infty, \mu = \mu^*)$ and $(m_{u,d} = 0, m_s = m_s^*, \mu = 0)$

- $N_f = 2$ and $N_f = 2 + 1$ analytically connected

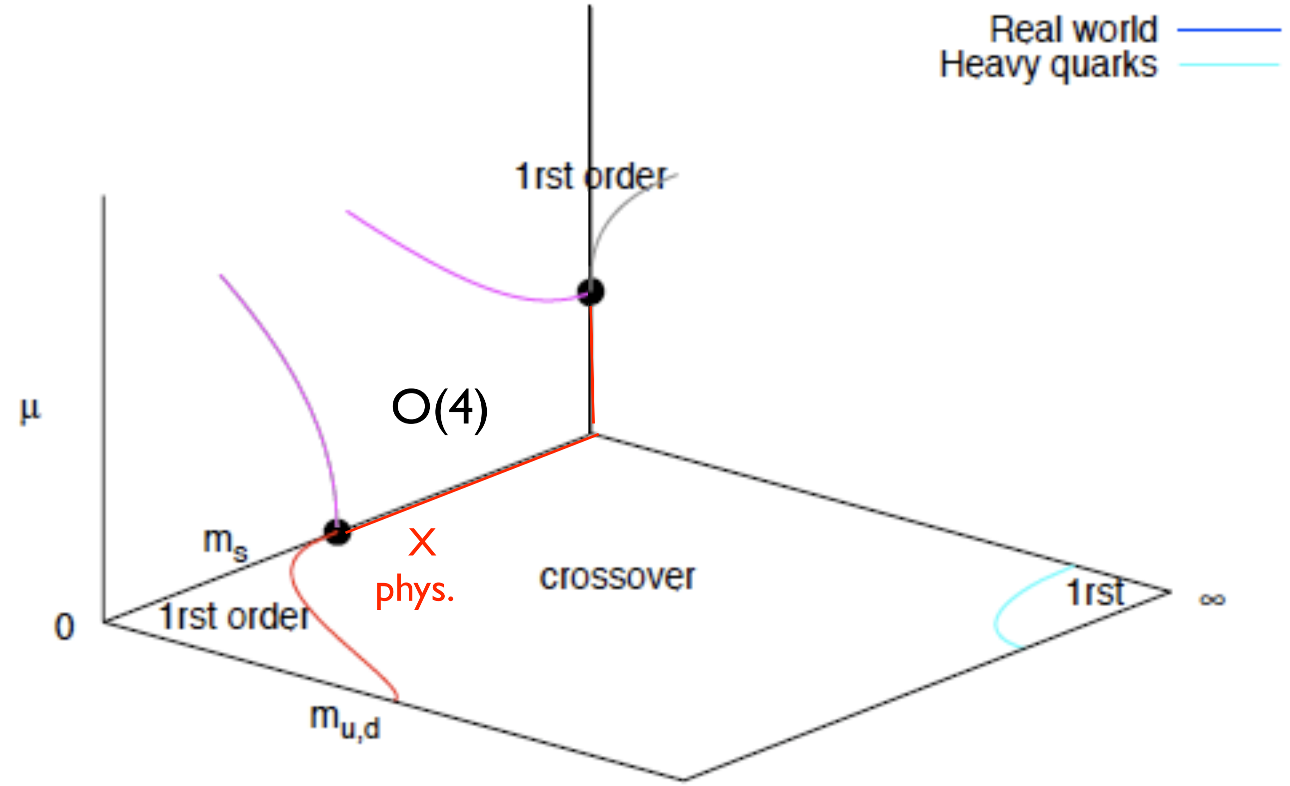


The interplay of Nf=2 and Nf=2+1

- $O(4)$ transition for 2 massless flavors Pisarski & Wilczek

$\Rightarrow$ tricritical points $(m_{u,d} = 0, m_s = \infty, \mu = \mu^*)$ and $(m_{u,d} = 0, m_s = m_s^*, \mu = 0)$

- $N_f = 2$ and $N_f = 2 + 1$ analytically connected



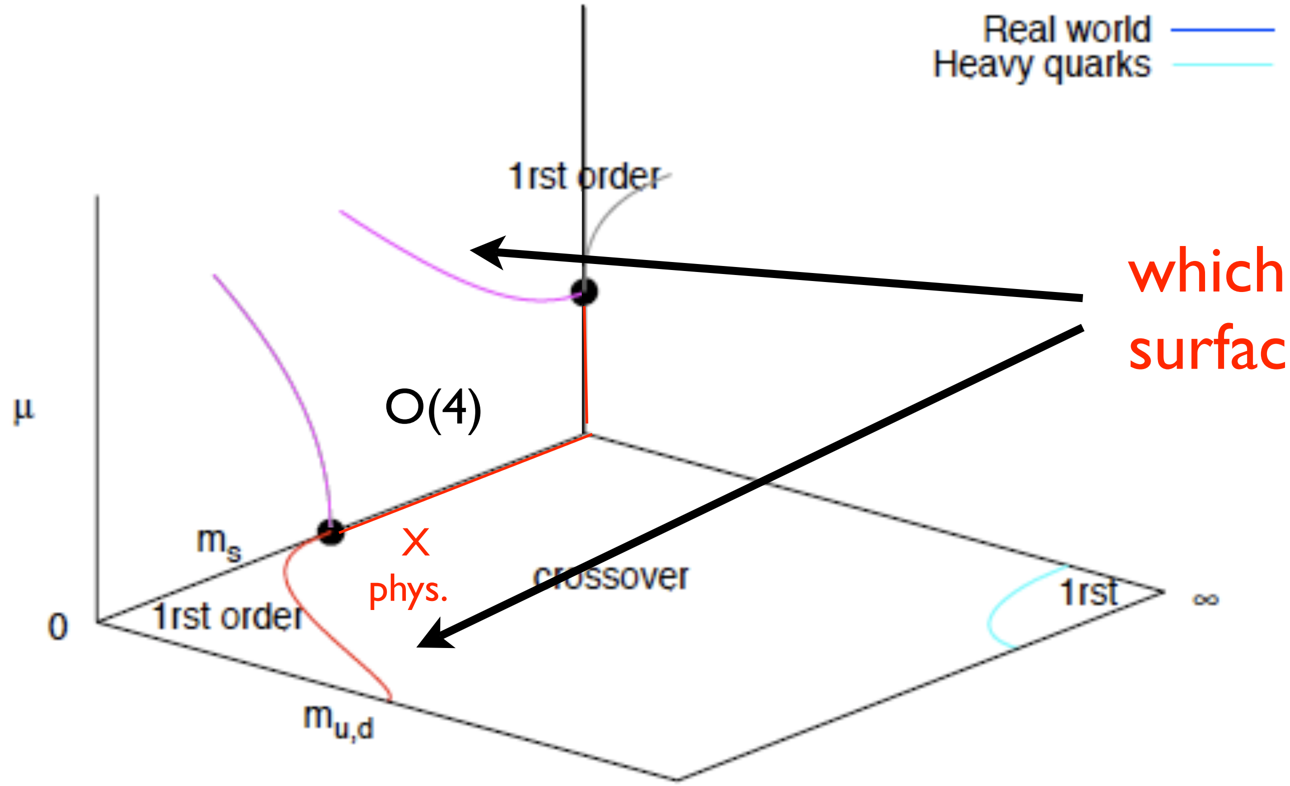
- $N_f = 2$ and $N_f = 2 + 1$ need not be connected

The interplay of Nf=2 and Nf=2+1

- $O(4)$ transition for 2 massless flavors Pisarski & Wilczek

⇒ tricritical points $(m_{u,d}=0, m_s=\infty, \mu=\mu^*)$ and $(m_{u,d}=0, m_s=m_s^*, \mu=0)$

- $N_f=2$ and $N_f=2+1$ analytically connected



which critical surface do we need?

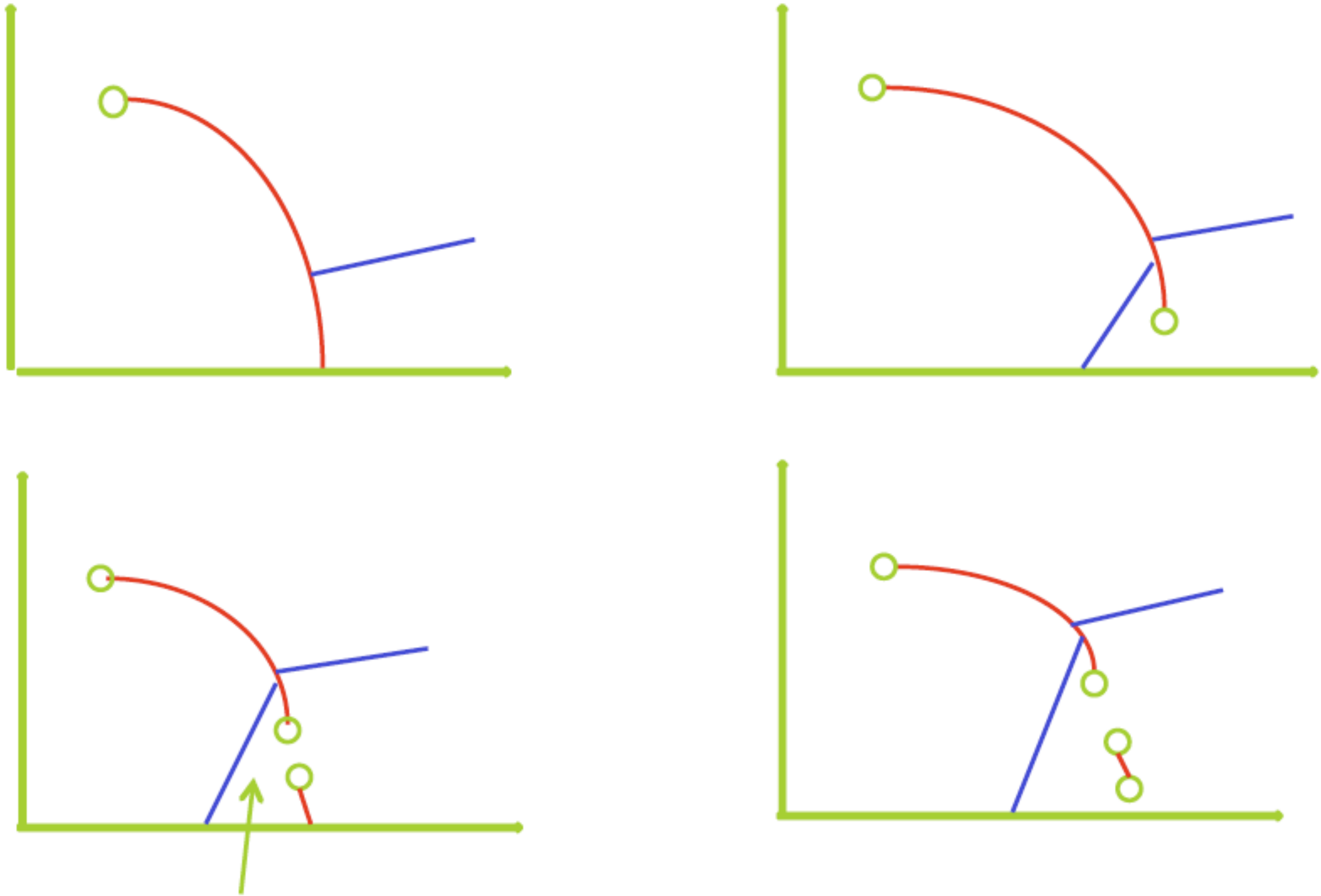
- $N_f=2$ and $N_f=2+1$ need not be connected

More (and non-chiral) critical points?

Good time for models: NJL with vector interactions — Zhang, Kunihiro, Fukushima 09

Ginzburg-Landau approach for quark condensates — Baym et al. 06

...



Conclusions

- Working lattice methods available for $\mu < T$
- $T_c(\mu)$, EoS under control at small density
- On coarse lattices a~0.3 fm no chiral critical point for $\mu < T$
- Large cut-off and quark mass effects
- Uncharted territory: do QCD critical points exist?

Conclusions

- Working lattice methods available for $\mu < T$
- $T_c(\mu)$, EoS under control at small density
- On coarse lattices a~0.3 fm no chiral critical point for $\mu < T$
- Large cut-off and quark mass effects
- Uncharted territory: do QCD critical points exist?

The greatest trick that Santa ever pulled was convincing the world he didn't exist.

The argument: $N_f = 2, \quad \mu, m = 0$

Pisarski, Wilczek 84

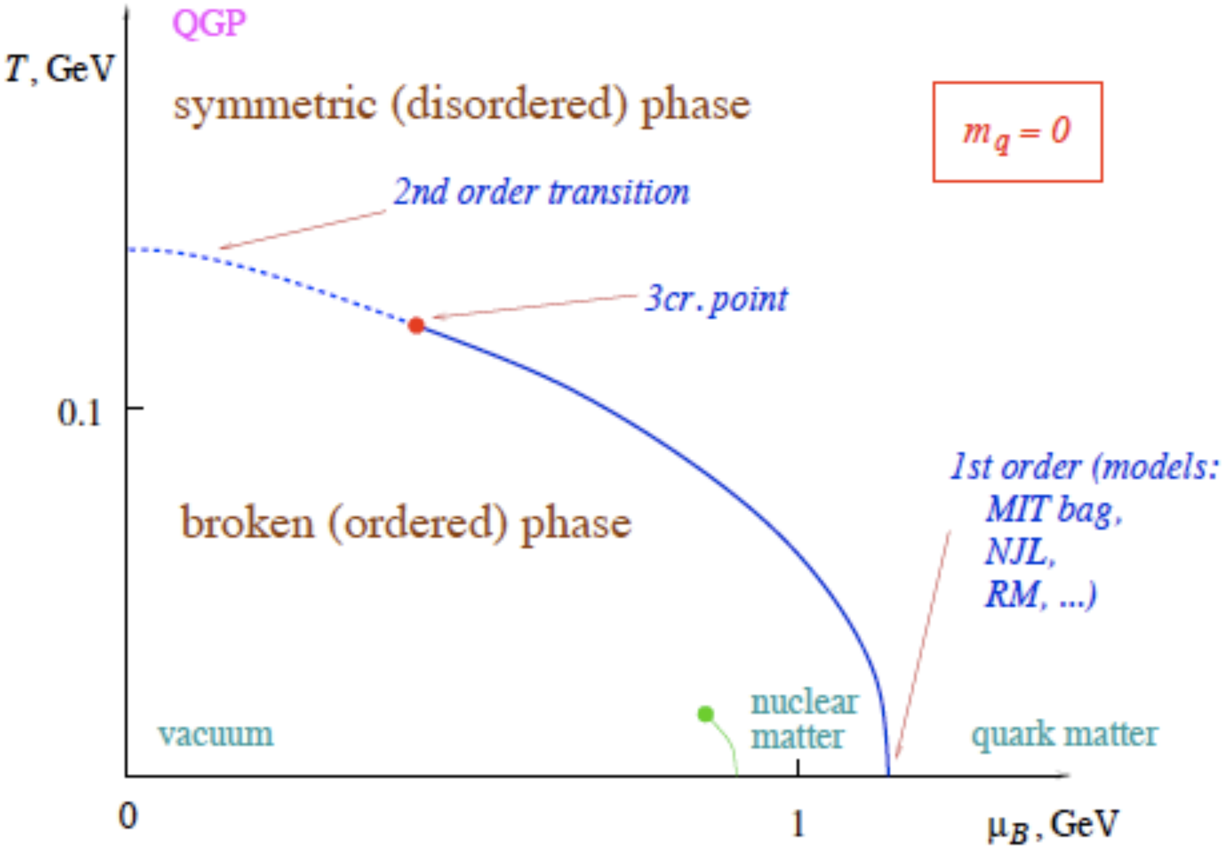
spont. chiral symmetry breaking $SU(2)_V \times SU(2)_A \rightarrow SU(2)_V$; restored at finite T

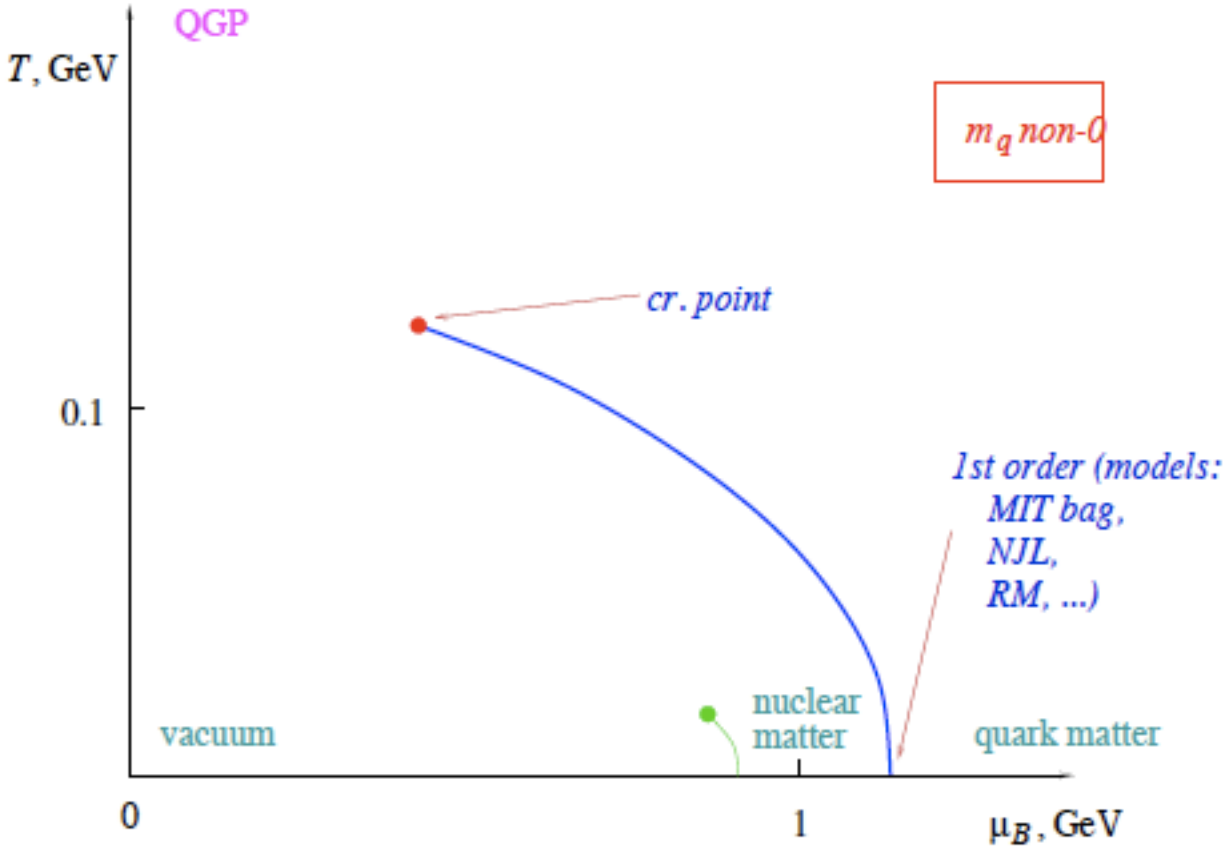
true order parameter $\langle\bar{\psi}\psi\rangle \Rightarrow$ separate phases $\Rightarrow$ non-analytic transition

The assumptions:

Halasz et al. 98; Rajagopal, Shuryak, Stephanov 98

- $N_f = 2$ chiral transition is second order $\Rightarrow O(4)$
- $T = 0$ and $\mu = 0$ transitions are continuously connected



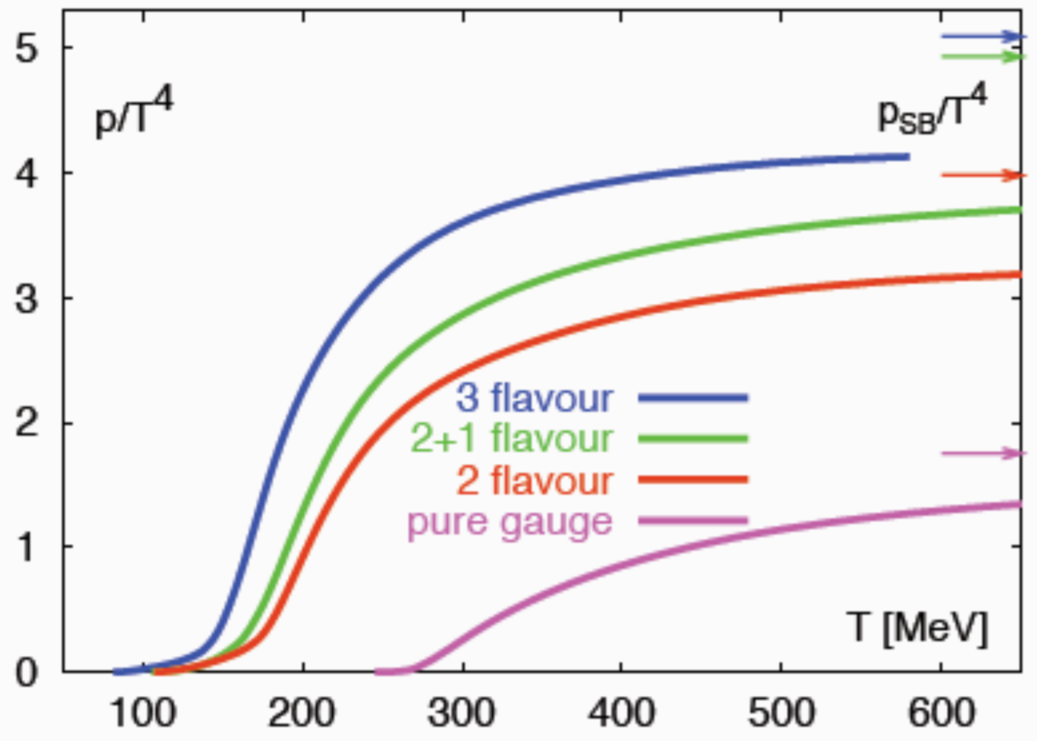


- $N_f = 2 + 1$: adding m_s only shifts location of line and (tri-)critical point

The equation of state

staggered p4-improved, $N_\tau = 4$

Bielefeld



compare with ideal gas:

$$\frac{\epsilon_{SB}}{T^4} = \frac{3p_{SB}}{T^4} = \begin{cases} 3\frac{\pi^2}{30} & , T < T_c \\ (16 + \frac{21}{2}N_f)\frac{\pi^2}{30} & , T > T_c \end{cases}$$

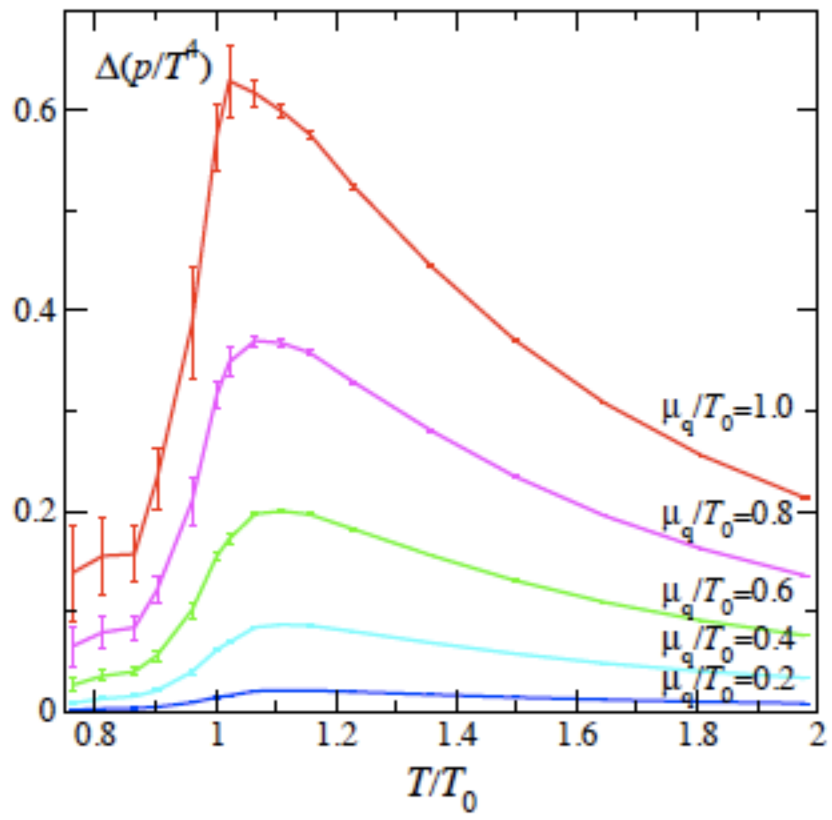
$T > T_c$: more degrees of freedom, but significant interaction!

➡ sQGP or `almost ideal' gas....?

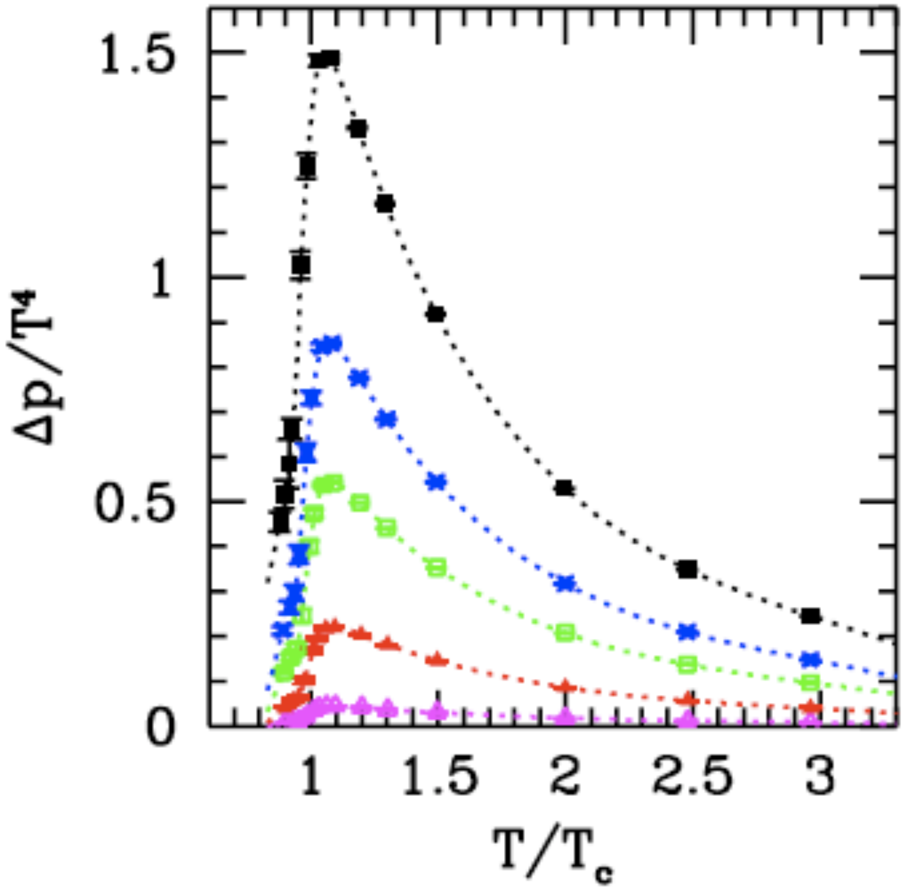
Continuum extrapolation with physical quarks feasible in the near future

Equation of state at finite baryon density

Taylor expansion, second order **Bielefeld-Swansea**



Reweighting **Wuppertal**



The nature of the transition for phys. masses

Aoki et al. 06

...in the staggered approximation...in the continuum...is a crossover!