Recent developments in inflationary cosmology

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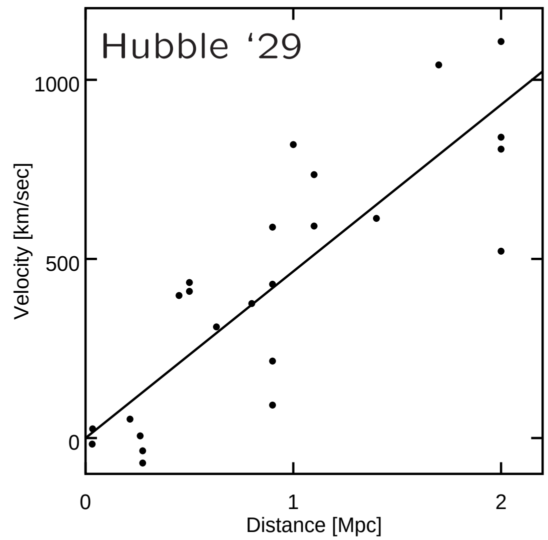
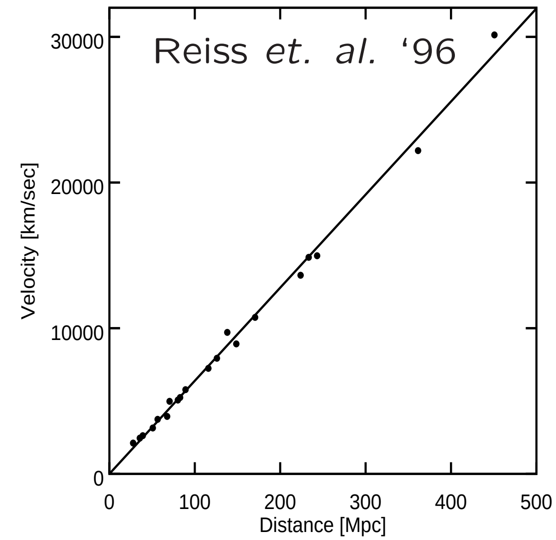
Overview

- Motivation and description of inflation
- CMB Observable for testing inflation - power spectrum, nongaussianity, and polarization
- Basics of Nongaussianity and Polarization in the CMB
- Present status after WMAP7 and future prospects from Planck
- Models of inflation

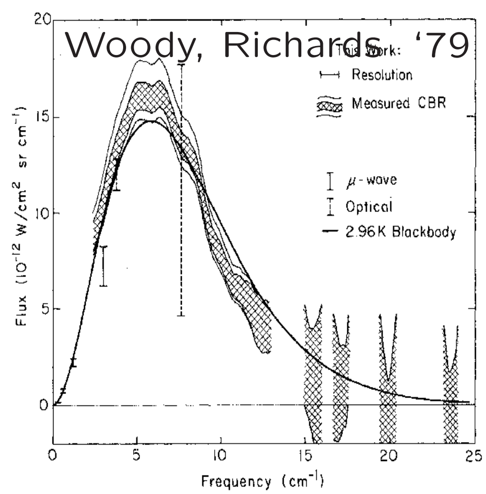
Observational success of Big Bang model

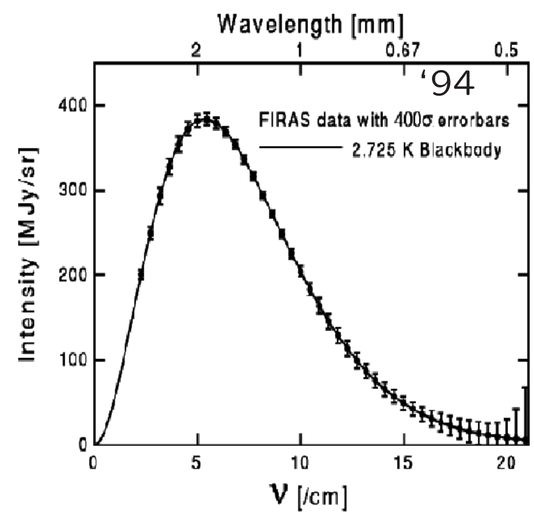
- Hubble law of expanding universe





- Cosmic microwave background radiation (CMB) - blackbody

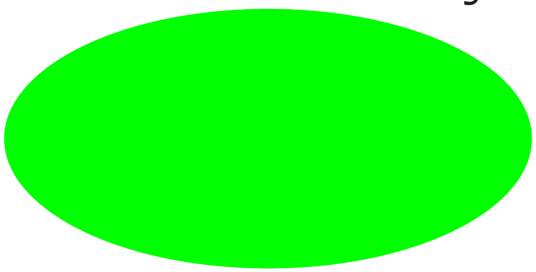


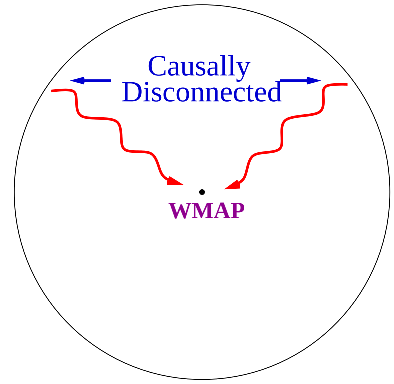


- Nucleosynthesis - explains abundance and uniformity of hydrogen, helium, deuterium and lithium

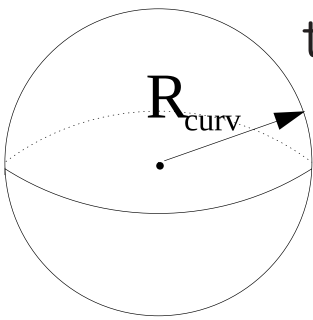
Shortcomings of Standard Cosmology

- Horizon problem - universe homogeneous within 10^{5+} causally disconnected regions





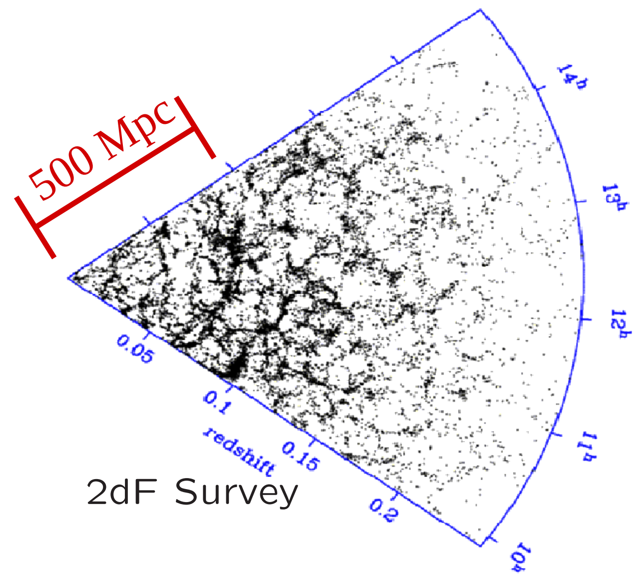
- Flatness problem - present universe close to flat, requires highly tuned initial conditions on initial energy density



$$R_{curv}^2 = H_0^{-2}/|\Omega - 1|$$

$$\Omega \equiv \frac{\rho}{\rho_c};\ \frac{\rho-\rho_c}{\rho} \lesssim 10^{-50}$$

- Formation of structure problem - nonrandom correlations at scales larger than the BB model can explain

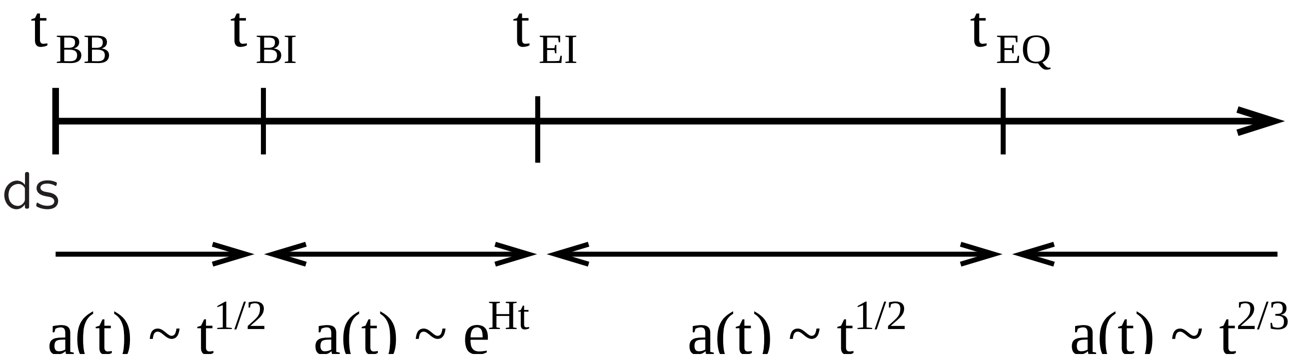


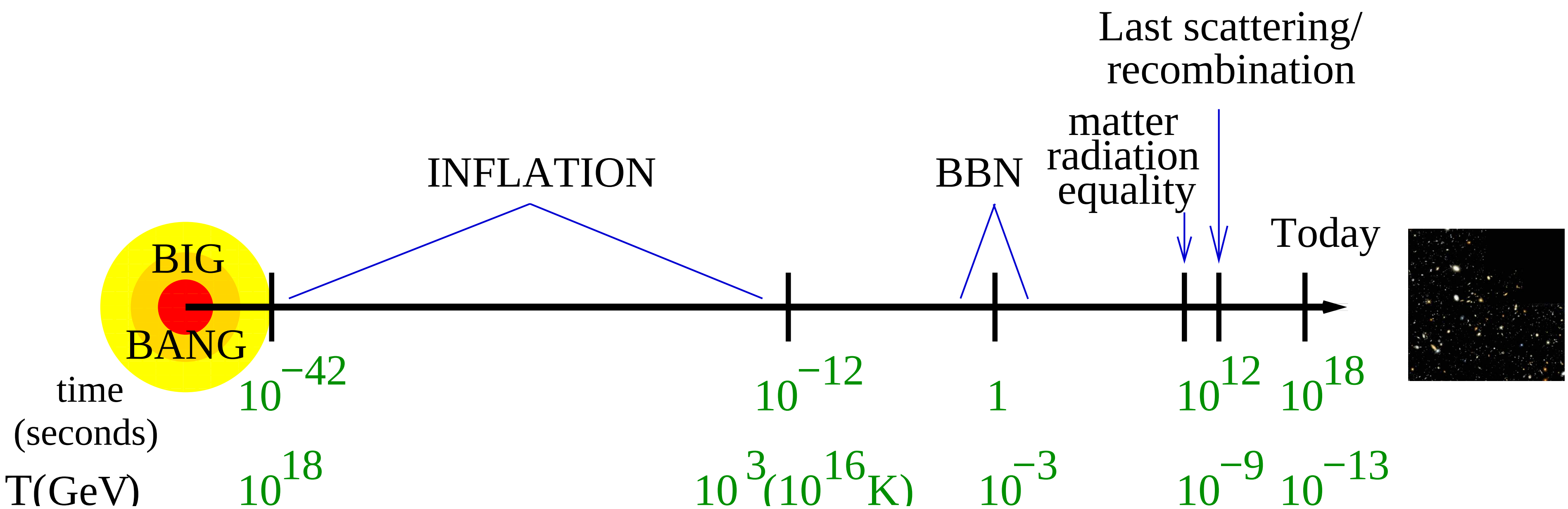
(Also: unwanted relics, breakdown classical ideal fluid approximation)

What is cosmic inflation

Short time interval
when universe expands
at accelerated rate

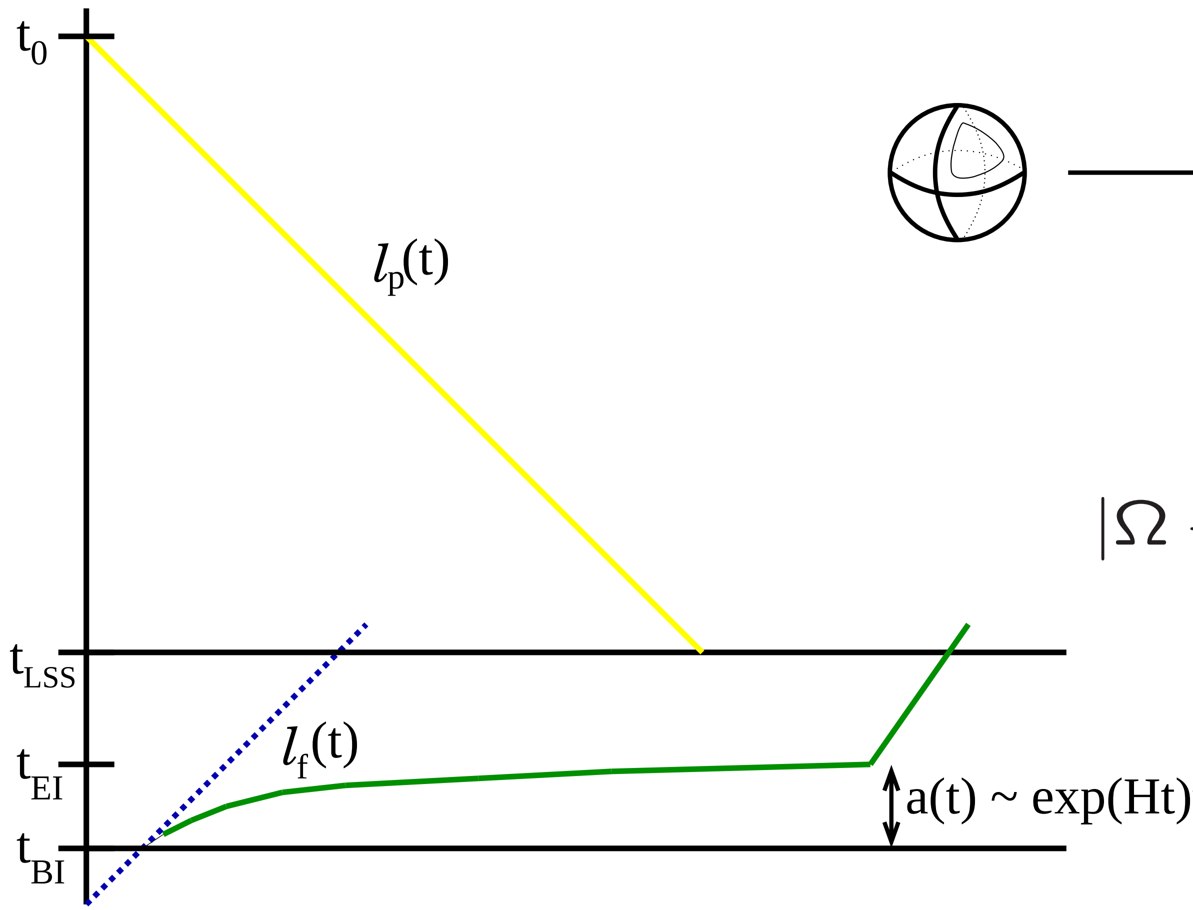
$a(t) \sim \exp(Ht)$



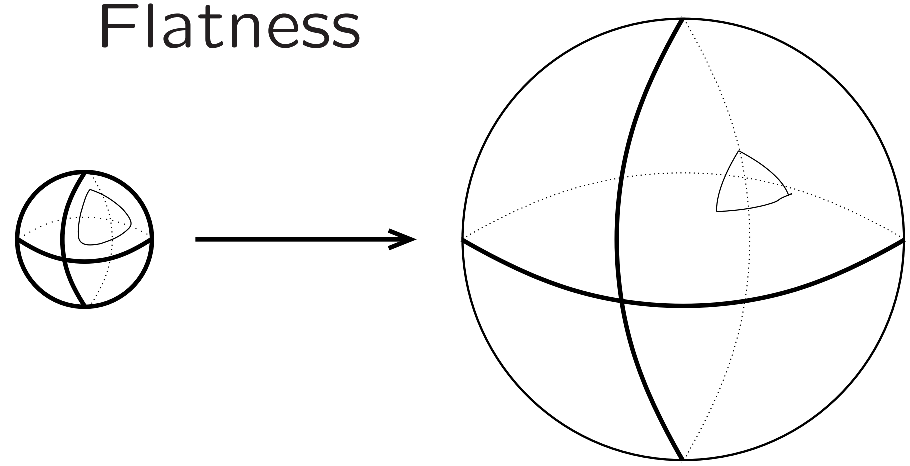


How inflation works

Horizon etc...

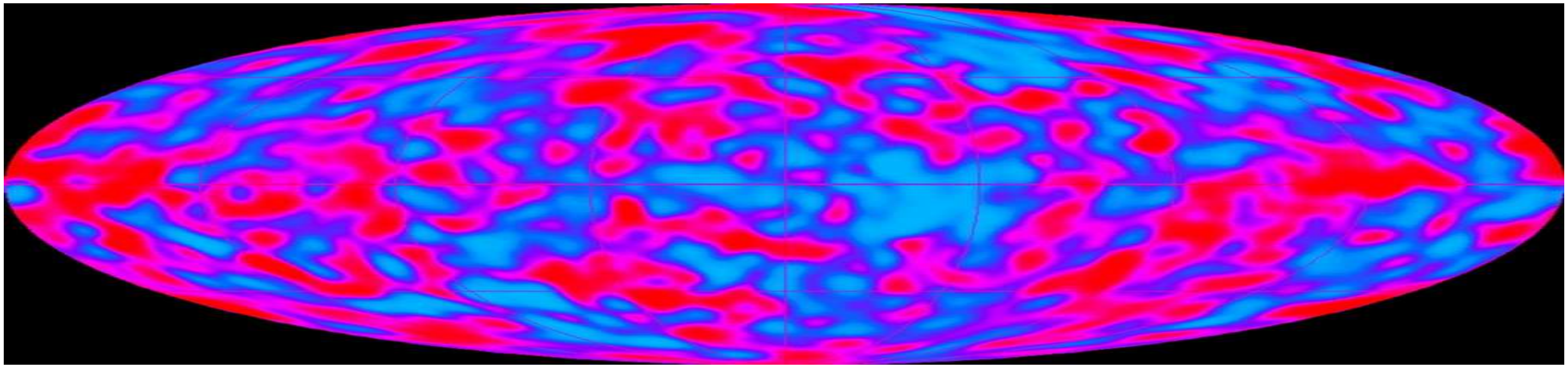


Flatness



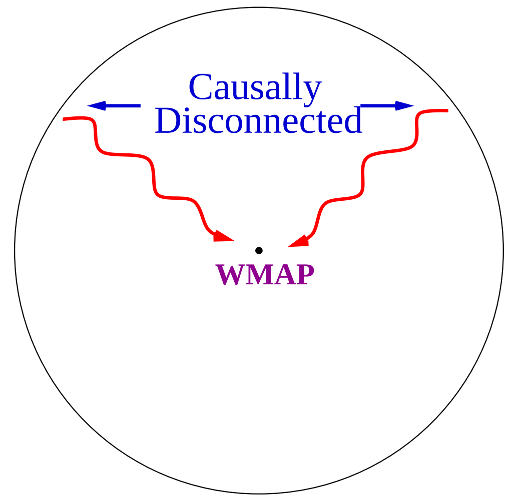
$$|\Omega - 1| \sim \exp(-2Ht)$$

Inflation

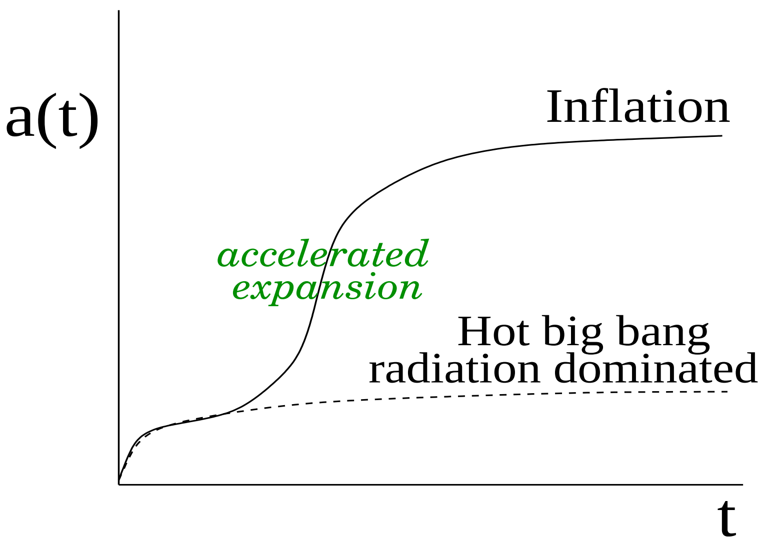


The puzzle of cosmological initial conditions:

$\frac{\Delta T}{T} < 10^{-5}$ in CMB without causal contact



The inflation solution (1981+)



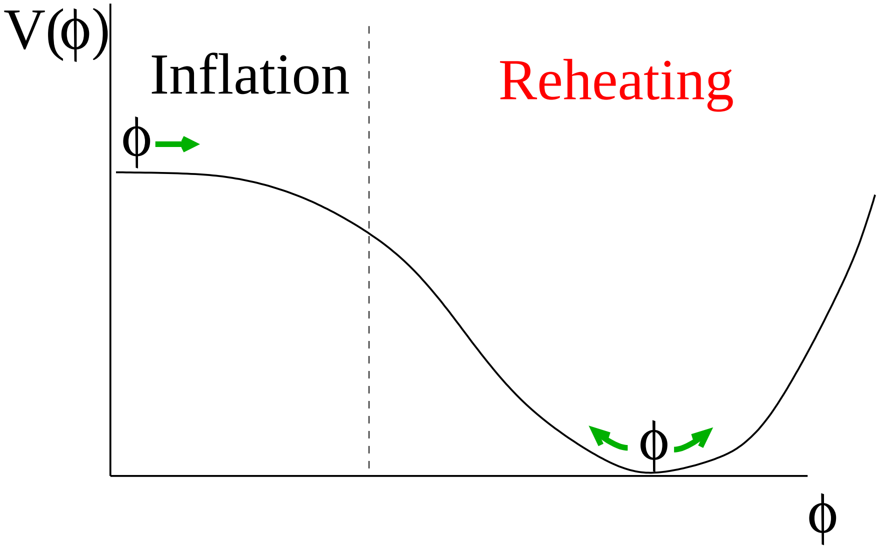
Accelerated expansion $\ddot{a}(t) > 0$

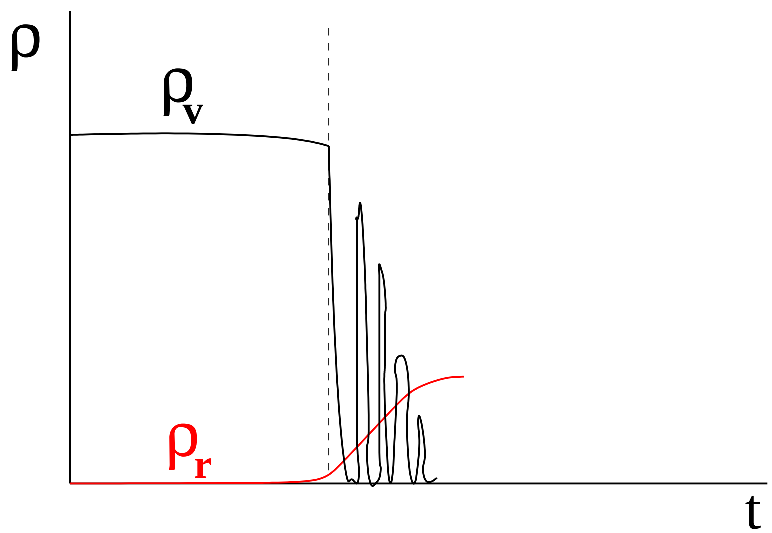
implies $\rho + 3p < 0$

How to get from high energy physics?

Basic inflation picture

Guth, PRD 23, '81; Linde, PLB 108, '82





$$\rho = \frac{\dot{\phi}}{2} + \mathrm{V}(\phi) + \frac{(\nabla\phi)^2}{2R^2}$$

$$p = \frac{\dot{\phi}}{2} - \mathrm{V}(\phi) - \frac{(\nabla\phi)^2}{6R^2}$$

Evolution equation for inflaton ϕ

$$\ddot{\phi} + 3\mathrm{H}\dot{\phi} + \mathrm{V}'(\phi) = 0$$

Just Choose $\mathrm{V}(\phi)$

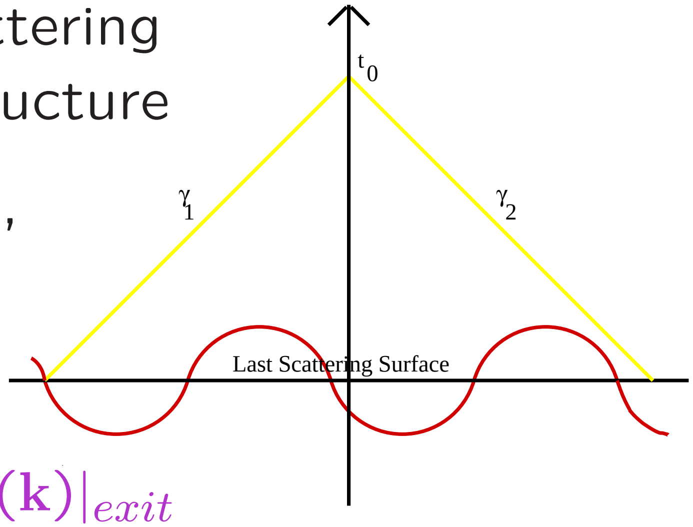
Potential energy dominated

$3\mathrm{H}\dot{\phi} \gg \ddot{\phi}$, "slow-roll"

Nature of the fluctuations

fluctuations during inflation leave "ripples" in early universe

- imprinted on CMB at Last Scattering
- amplified by gravity to make structure



Arise from quantum fluctuations of inflaton field,

(Guth, Pi '82; Hawking '82; Starobinsky '82, Bardeen *et. al.* '83)

$$\delta\phi = \frac{H}{2\pi}$$

Lead to curvature perturbations $\mathcal{R}(\mathbf{k}) = -\frac{H}{\dot{\phi}}\delta\phi(\mathbf{k})|_{exit}$

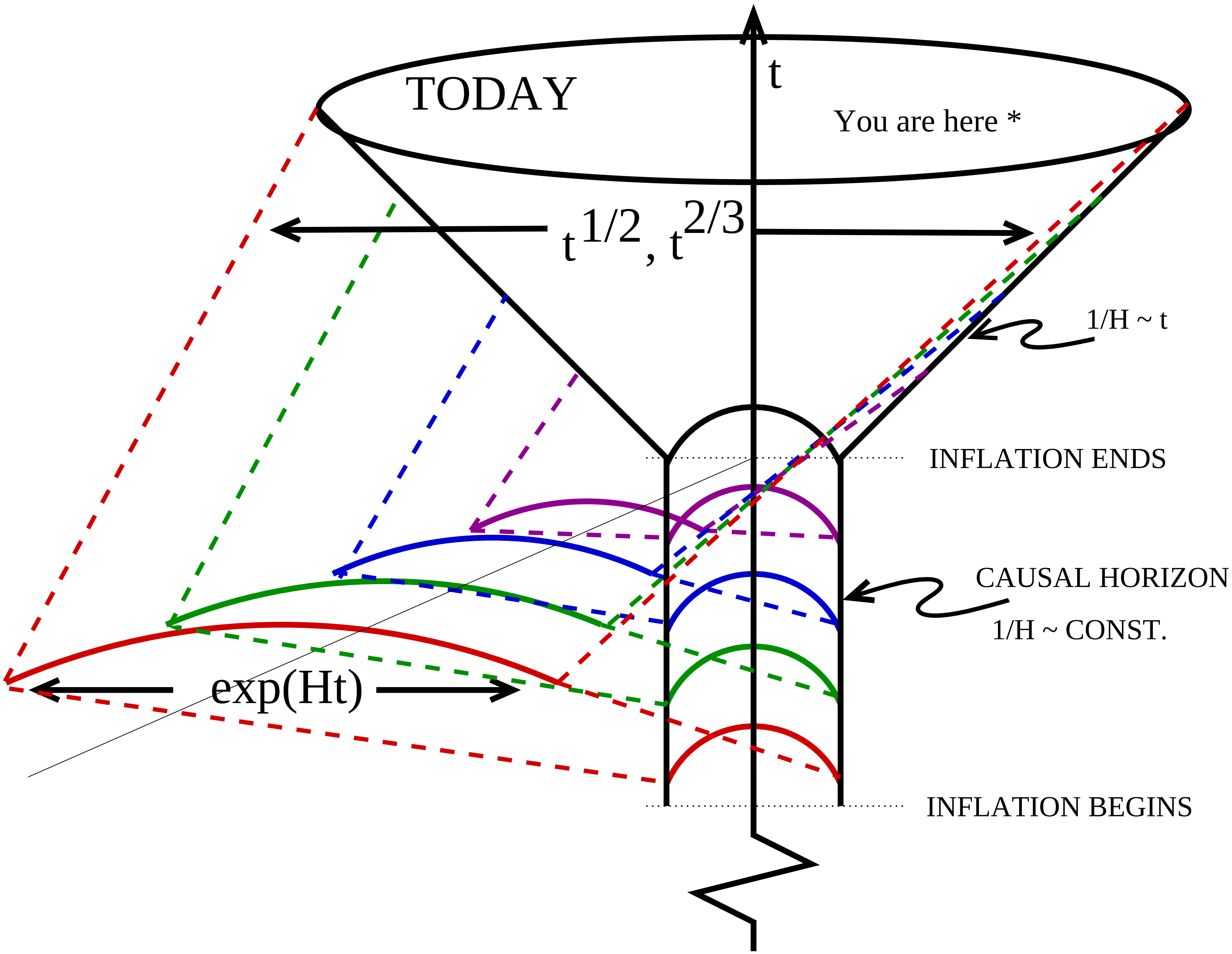
$\mathcal{R}(\mathbf{k})$ constant from epoch when perturbation exits horizon $(exit)$ during inflation to re-enters the horizon after inflation

At largest scales since last scattering $\mathcal{R} \propto$ gravitational potential Φ

Sachs-Wolfe effect: $\frac{\delta T}{T} = \frac{1}{3}\Phi$

Relation between theoretical parameters in inflation model and observational properties of CMB.

Perturbations - Hello, goodbye, ... hello again



Power spectra

Power spectrum for curvature (scalar) perturbation,
$P_S(k)\delta^3(\mathbf{k}-\mathbf{k}') = \langle \mathcal{R}(\mathbf{k})\mathcal{R}(\mathbf{k}')\rangle$

Parameterize: $$\frac{k^3 P_S(k)}{2\pi^2} = A_S\left(\frac{k}{k_0}\right)^{n_S(k_0)-1+\frac{1}{2}dn_S/d\ln k}$$

Inflaton will couple with the metric to produce gravity waves (tensor perturbations) proportional to the inflaton energy density

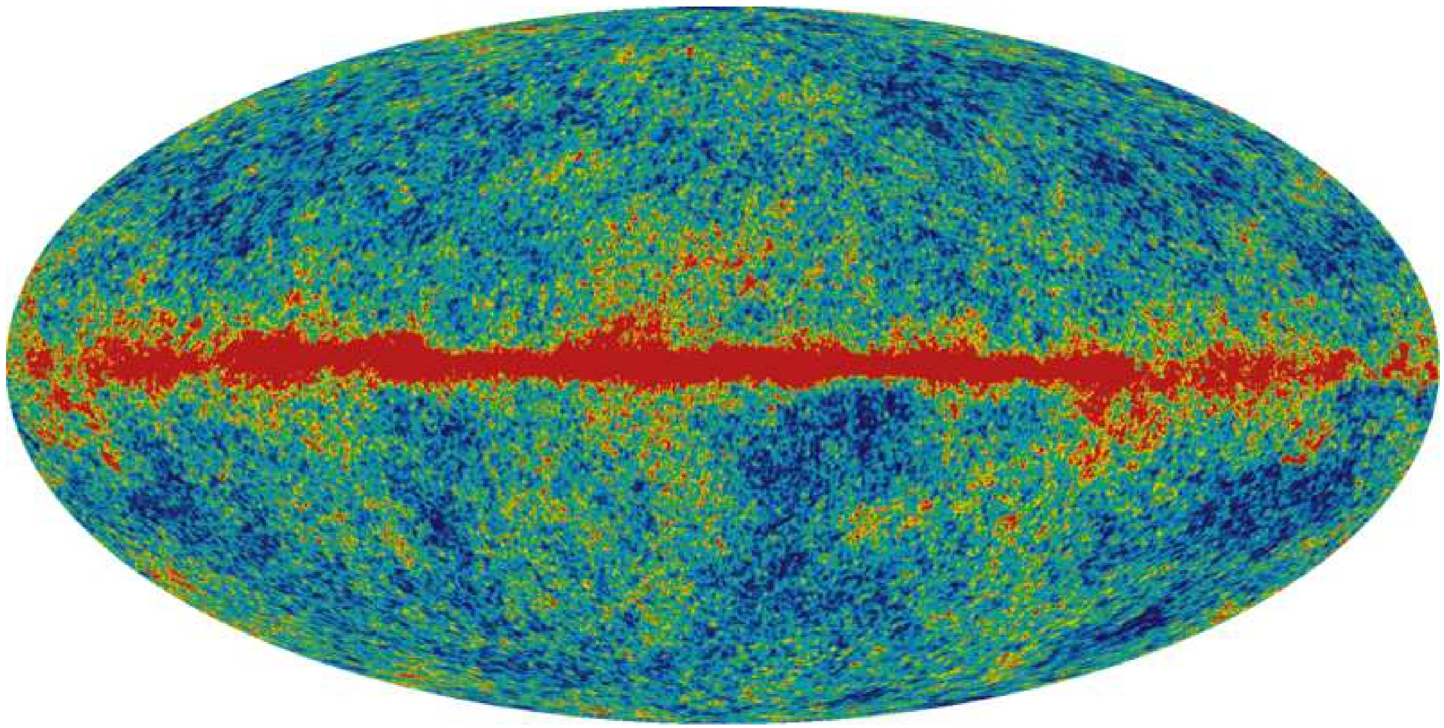
$$P_T(k) = \frac{2}{m_{pl}^2}\left(\frac{H}{2\pi}\right)^2|_{h.e.}$$

Parametrize: $$\frac{k^3 P_T(k)}{2\pi^2} = A_T\left(\frac{k}{k_0}\right)^{n_T}$$

Measurable parameters of inflation:

- Amplitudes A_S, A_T, tensor-scalar index $r \equiv A_T/A_S$
- Index n_S, n_T
- Running of scalar index $dn_S/d\ln k$

CMB observation



From observation obtain:

$$\frac{\delta T}{T}(\theta\phi) = \sum_{\ell,m} a_{\ell,m} Y_{\ell,m}(\theta,\phi)$$

Correlation:

$$C_\ell \delta_{\ell\ell'} \delta_{mm'} = \langle a^*_{\ell m} a_{\ell' m'} \rangle$$

Cosmic variance

In inflationary cosmology the creation of density perturbations is a statistical process

The multipole expansion coefficients $a_{\ell m}$ we observe are just one realization from the ensemble

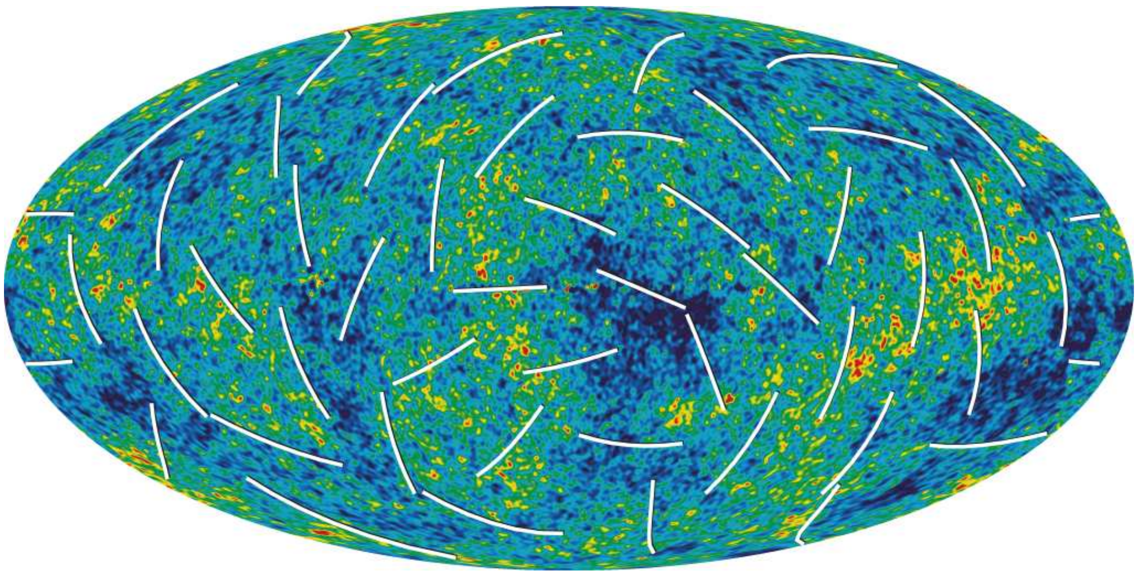
For general inflation models, $a_{\ell,m}$ are (near) Gaussian random variables

$\Rightarrow$ statistical uncertainty in the knowledge of the C_ℓ's

$$\left(\frac{\Delta C_\ell}{C_\ell}\right)_{\text{cosmic variance}} = \sqrt{\frac{2}{2\ell+1}}$$

CV uncertainty most pronounced for low-ℓ (largest length scale)

Polarization in the CMB



For polarized EM wave $\mathbf{E}(t) = \mathrm{Re}[\mathbf{E}e^{i\omega t}]$, Stokes parameters:

$I \equiv \overline{|E_x|}^2 + \overline{|E_y|}^2 \quad Q \equiv \overline{|E_x|}^2 - \overline{|E_y|}^2 \quad U \equiv 2\mathrm{Re}[\overline{E_x^* E_y}]$

I intensity and Q,U plane polarization

For CMB radiation coming in from direction $-\mathbf{n}$

$$Q(-\mathbf{n}) \pm iU(-\mathbf{n}) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} [E_{\ell m} \pm iB_{\ell m}] Y_{\ell m}^{\pm}(\mathbf{n})$$

$Y_{\ell m}^{\pm}$ spin weighted spherical harmonics

Information in CMB polarization

Correlators:

$$\begin{aligned}
\langle a^*_{\ell m} a_{\ell' m'} \rangle &= C_\ell \delta_{\ell\ell'} \delta_{mm'} \\
\langle a^*_{\ell m} E_{\ell' m'} \rangle &= C^{TE}_\ell \delta_{\ell\ell'} \delta_{mm'} \\
\langle E^*_{\ell m} E_{\ell' m'} \rangle &= C^{EE}_\ell \delta_{\ell\ell'} \delta_{mm'} \\
\langle B^*_{\ell m} B_{\ell' m'} \rangle &= C^{BB}_\ell \delta_{\ell\ell'} \delta_{mm'}
\end{aligned}$$

Thomson scattering of photons near last scattering create polarization from initial gradient in photon field

E-mode polarization sensitive to electron density at last scattering, thus useful for measuring optical depth and reionization history - key quantities for breaking degeneracies in spectral index

Expect oscillations in ℓ from T-E correlations

B-mode polarization must be created by tensor perturbations $\Rightarrow$ direct measurement of primordial gravity waves

Nongaussianity in the CMB

Deviation of CMB distribution from Gaussian measured in three-point correlation of Newtonian potential

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3)$$

B called bispectrum

Different models of inflation lead to different bispectrum:

Local model: B peaked for $k_1 \ll k_2, k_3$

$$B^{loc}(k_1,k_2,k_3) = f_{NL}^{loc}\left[P(k_1)P(k_2) + cyc.perm.\right]$$

Equilateral model: B peaked when $k_1 \sim k_2 \sim k_3$

$$B^{eq}(k_1,k_2,k_3) = f_{NL}^{eq}A_S^2\left[-3\frac{1}{k_1^3k_2^3} + 4\frac{1}{k_1^2k_2^2k_3^2} + cyc.perm.\right]$$

Cross product model:

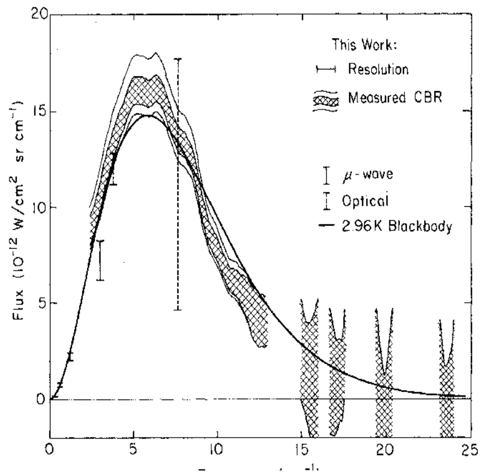
$$B^{cr}(k_1,k_2,k_3) = f_{NL}^{cr}\left[(k_1^{-2}+k_2^{-2})P(k_1)P(k_2)\mathbf{k}_1\cdot\mathbf{k}_2 + cyc.perm.\right]$$

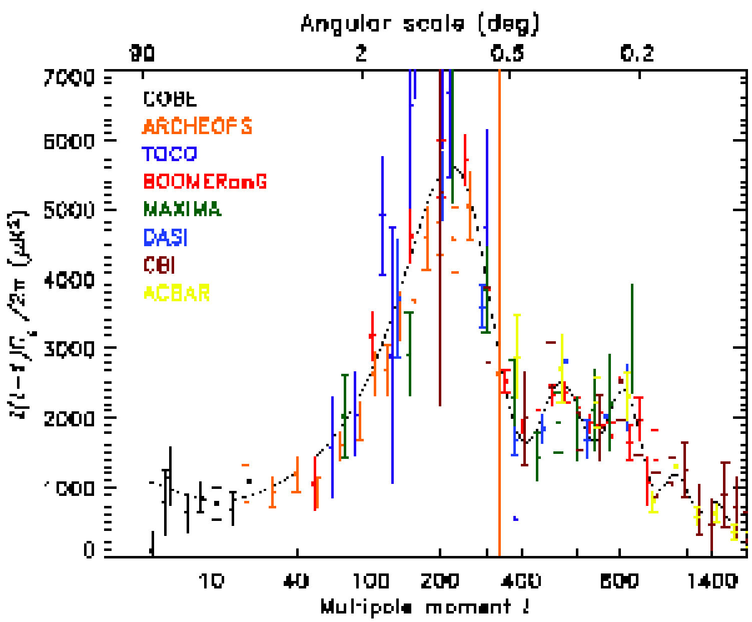
$f_{NL} \sim 100 \Rightarrow$ primordial perturbations Gaussian to 0.1% level

Cosmology - the new black(body)

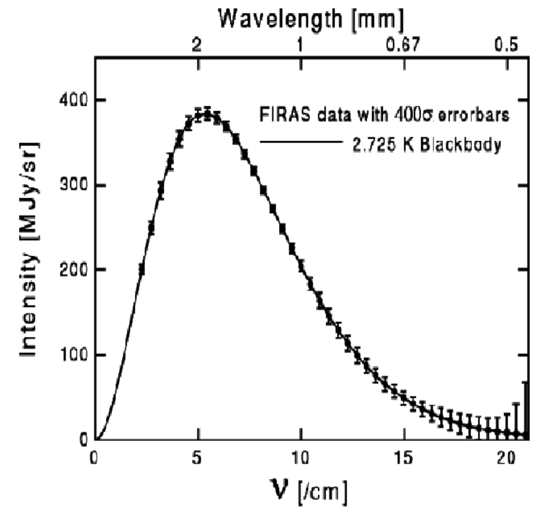
Progress in CMB measurement heralded age of precision cosmology

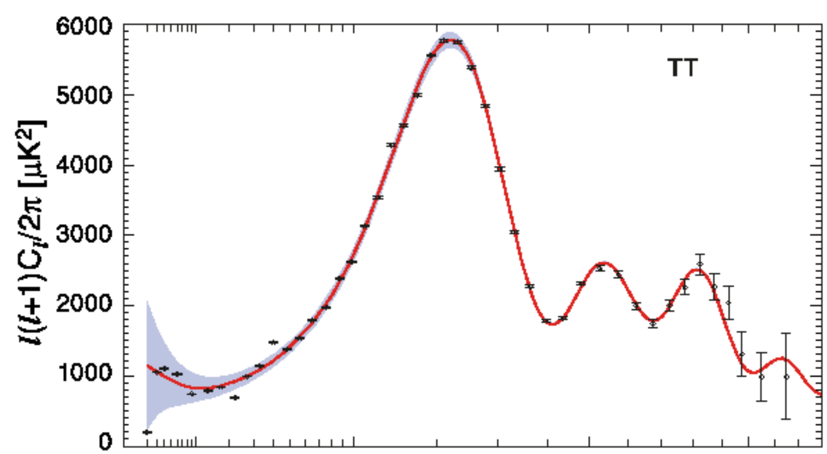
THEN





NOW

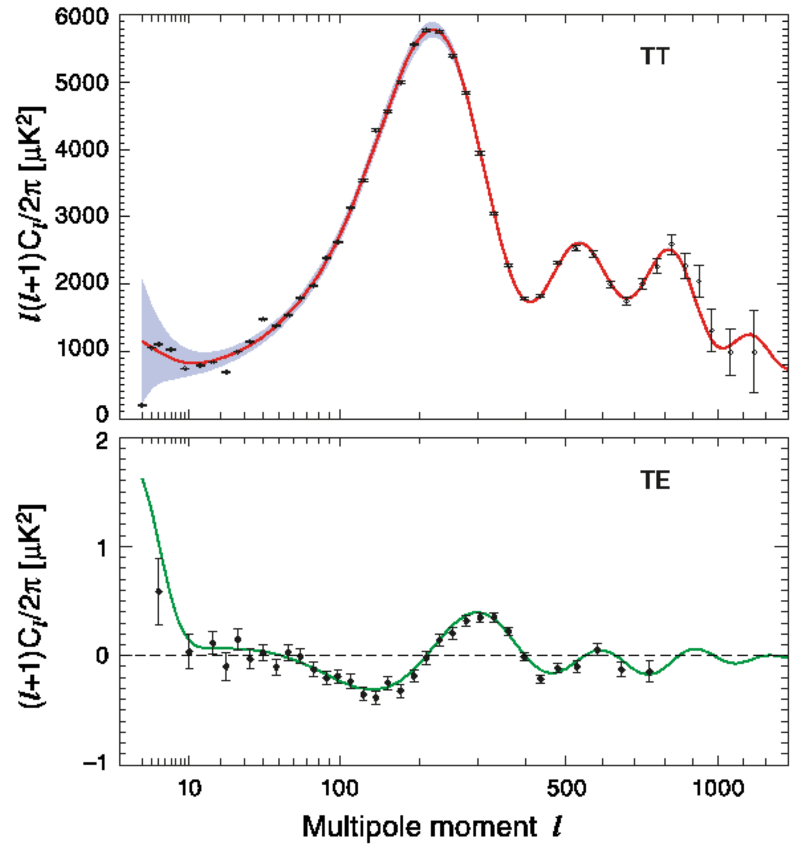




WMAP 7 year

Summary of WMAP data

WMAP 7 year data and angular power spectra:



Scalar amplitude: $A_S = (2.43 \pm 0.11)^{-9}$

Scalar spectral index: $n_S = 0.963 \pm 0.014$

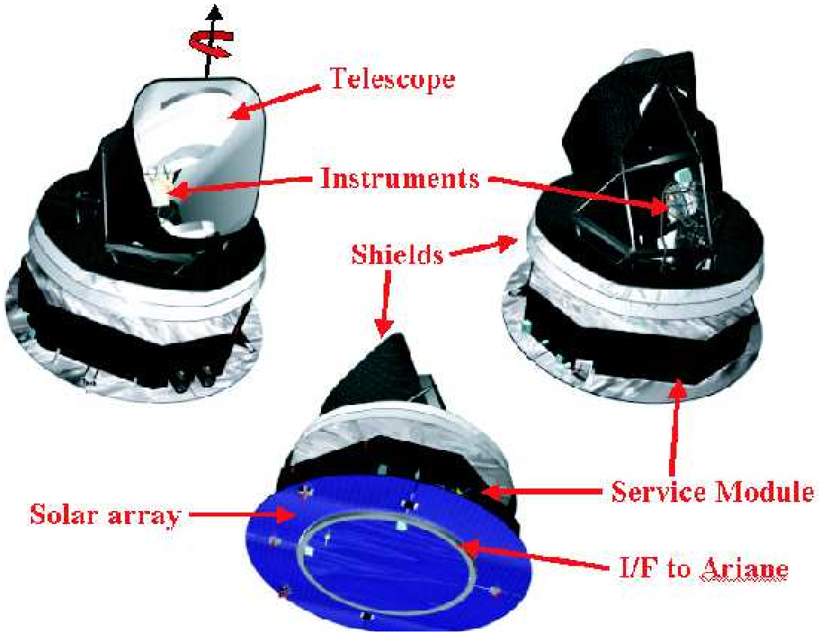
Running of spectral index:
$dn_S/d\ln k = -0.034 \pm 0.026$

Tensor to scalar ratio: $r < 0.36(95\%CL)$

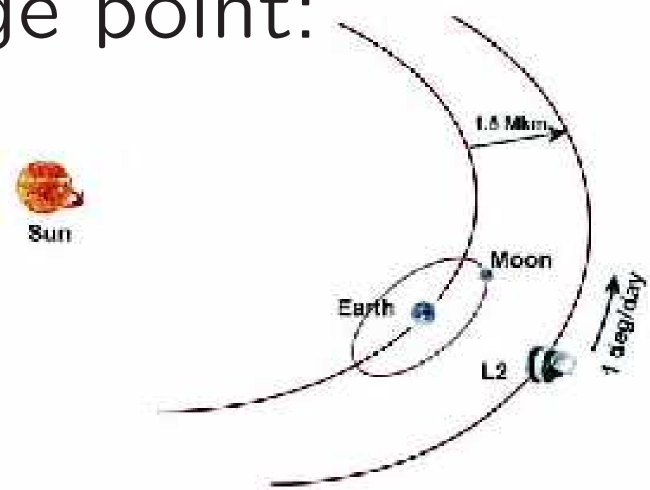
Nongaussianty: $f_{NL}^{loc} = 32 \pm 21$ (68% CL)
Consistent with Gaussian within 95% CL

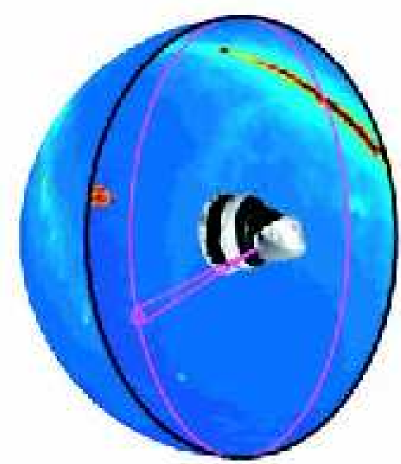
Yadav and Wandelt, 2008 found in WMAP 3 year data
$27 < f_{NL}^{loc} < 147$ $(95\%\ CL)$ with rejection of $f_{NL}^{loc} = 0$ at 2.8σ

Planck satellite

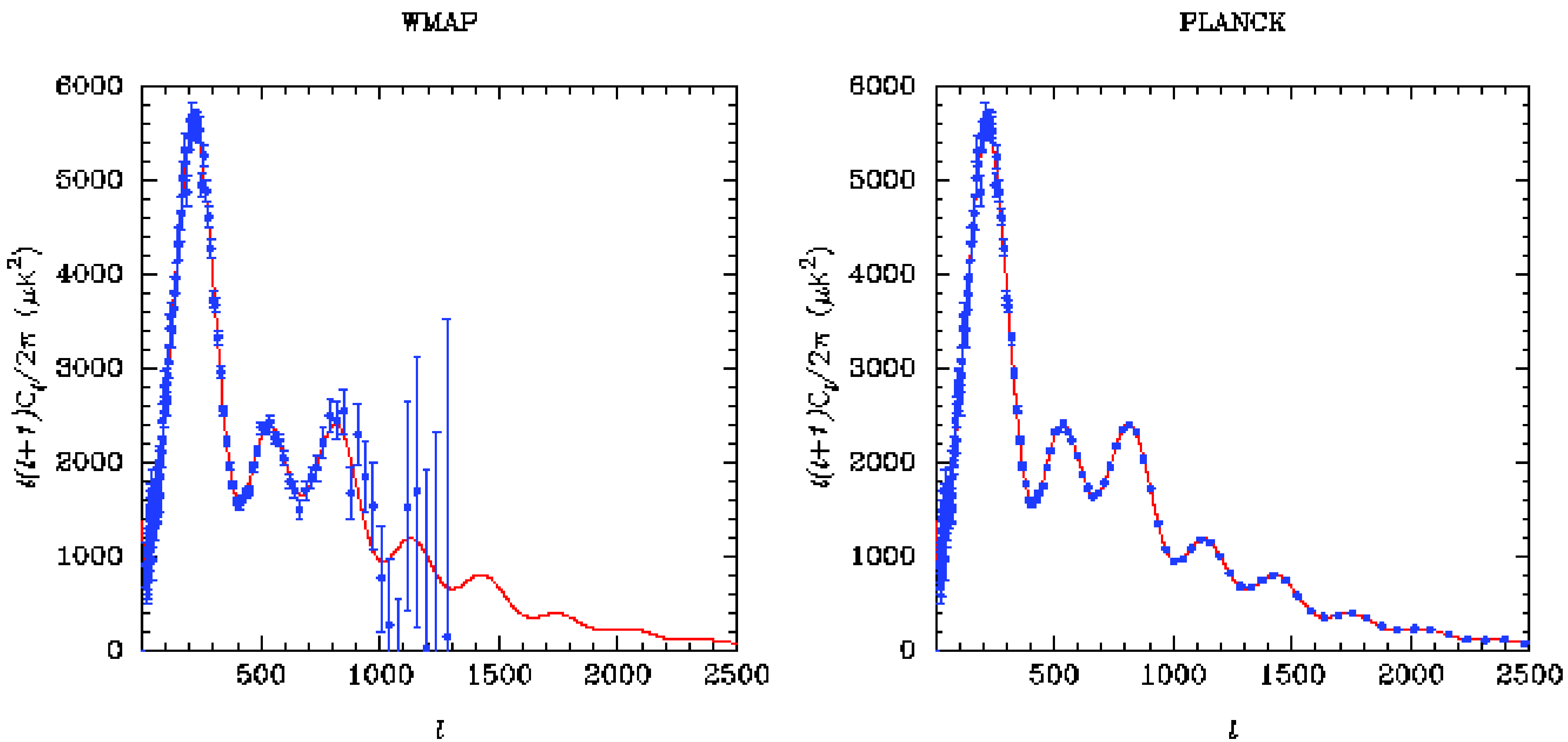


Orbits at L2 Lagrange point:





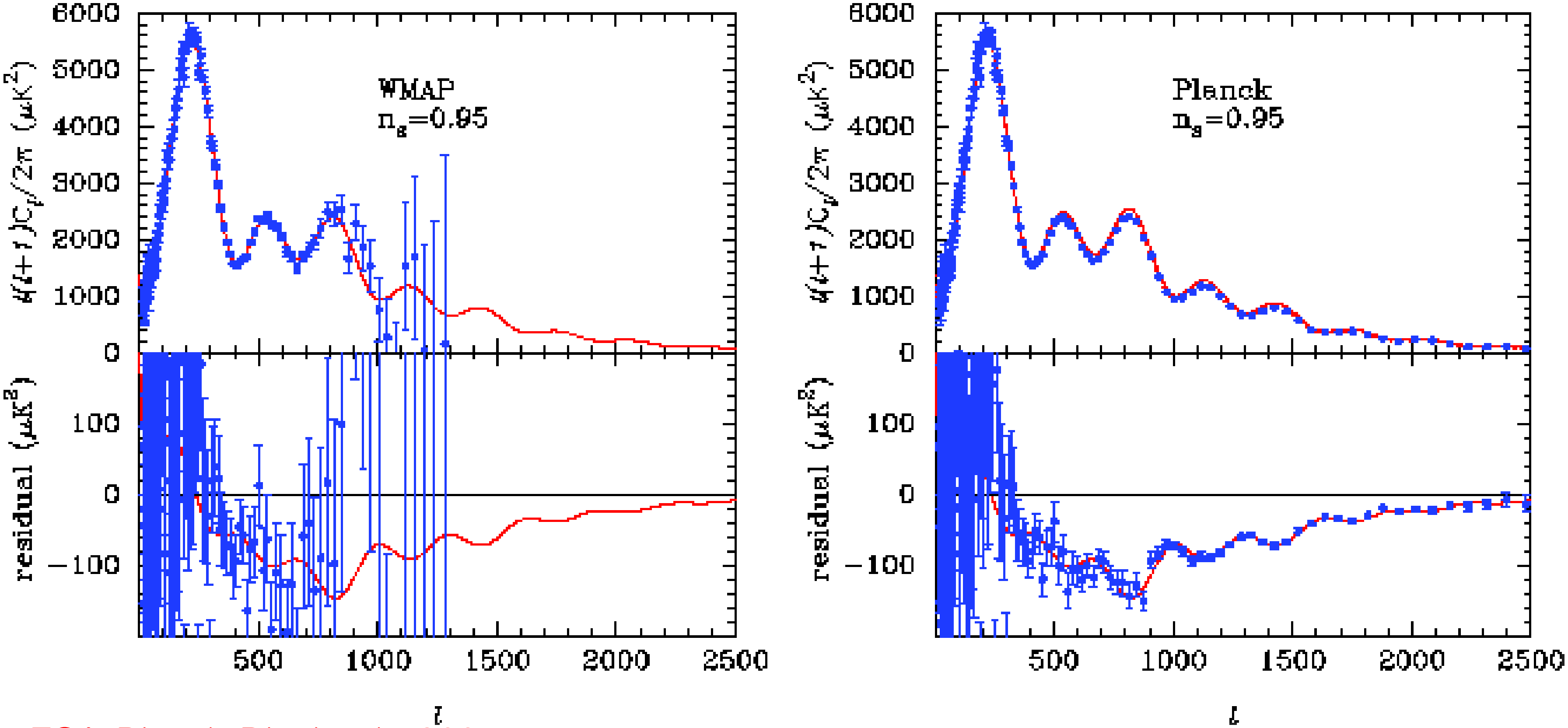
What Planck can achieve



ESA Planck Bluebook, 2005

CMB power spectrum for concordance ΛCMB model (red line) compare WMAP to (projected) Planck data

Distinguishing models

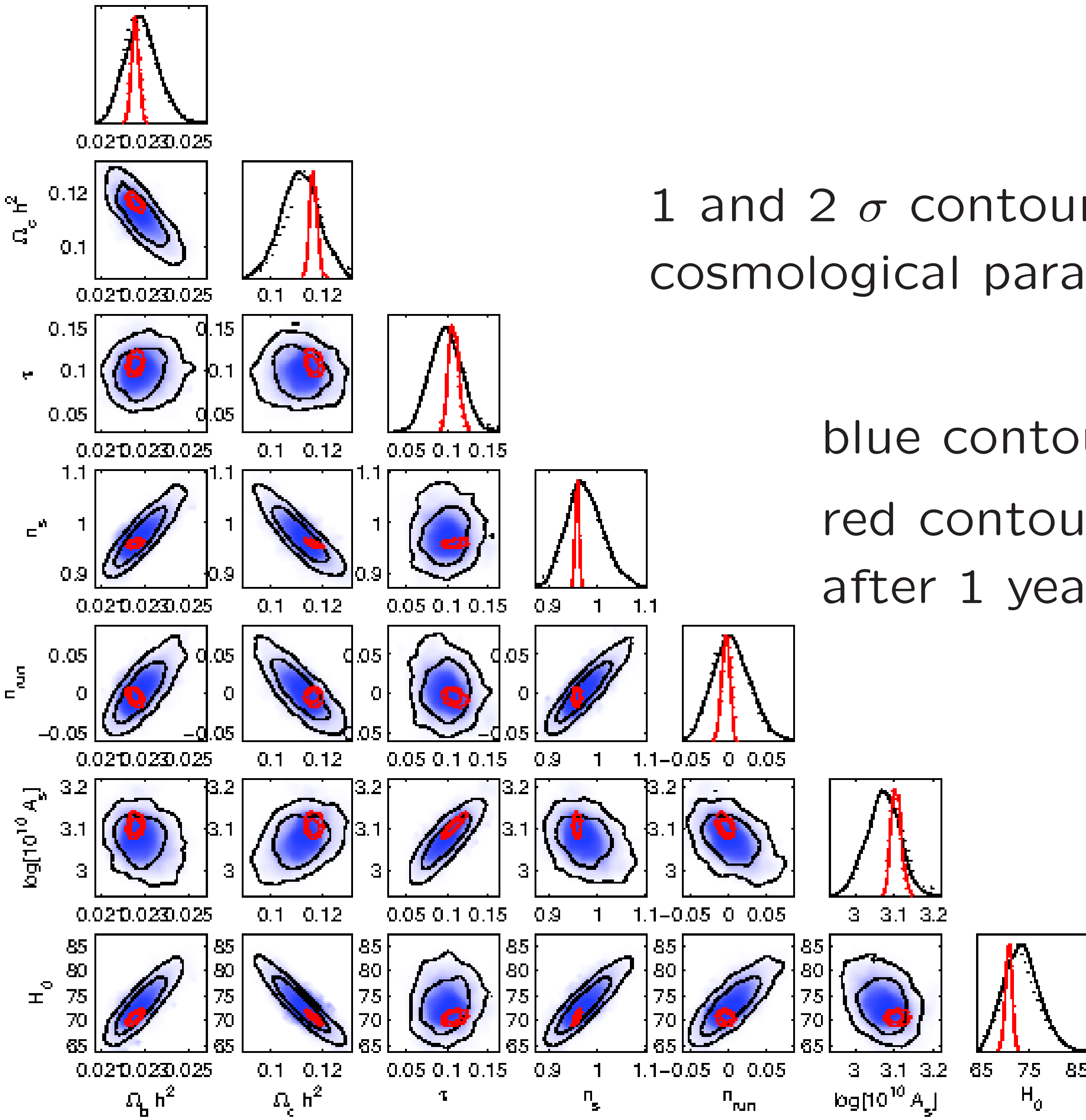


ESA Planck Bluebook, 2005

Solid red lines are concordance ΛCDM model with spectral index $n_S = 0.95$ and 1

WMAP has difficulty distinguishing between the models vs. Planck can distinguish very well

Improving parameter estimation



1 and 2 σ contour regions for cosmological parameters

blue contours WMAP 4

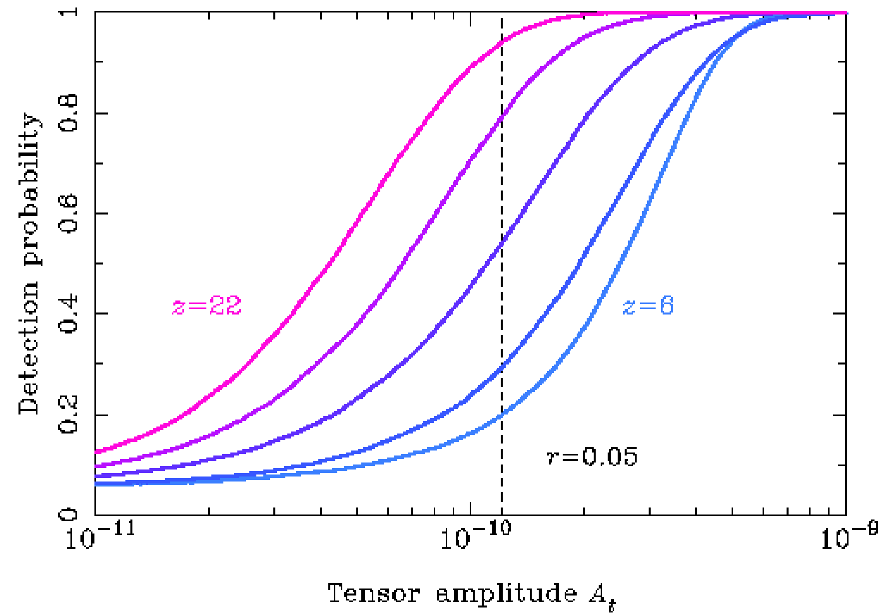
red contours Planck projected after 1 year of observation

ESA Planck Bluebook, 2005

Parameter forecast for Planck

Parameter Forecasts for WMAP and Planck

Parameter	Input Value	June'03	June'03 +2dF	*WMAP*$_4$	*Planck*	*WMAP*$_4$ ACT/SPT
Flat+weak priors						
ω_b	0.2240	0.00095	0.00090	0.00047	0.00017	0.00025
ω_c	0.1180	0.011	0.007	0.0039	0.0016	0.0035
n_S	0.9570	0.026	0.024	0.0125	0.0045	0.0080
τ	0.108	0.059	0.056	0.020	0.005	0.021
+running						
ω_b	0.2240	0.00162	0.00090	0.00047	0.00017	0.00025
ω_c	0.1180	0.0158	0.007	0.0039	0.0016	0.0035
$n_S(k_n)$	0.9570	0.055	0.024	0.0125	0.0045	0.0080
n_{run}	0.0	0.033	0.029	0.025	0.005	0.0092
τ	0.108	0.112	0.074	0.019	0.006	0.0266



ESA Planck Bluebook, 2005

Experiments	f_{NL} (Bispectrum)	f_{NL} (Skewness)
COBE	600	800
WMAP	20	80
Planck	5	70
Ideal	3	60

Bartolo, *et al.*, 2005

Error on scalar index reduced to half a percent

Tensor-scalar ratio detectable down to ~ 0.05 (optimistic)

If no detection of r in Planck $\Rightarrow$ low energy scale $V^{1/4}$ of inflation,

$V^{1/4} = 3.3 \times 10^{16} r^{1/4}$GeV

Models of inflation

Standard class of inflation models:

Large field models $\phi > m_{pl}$ Linde 1983:

power law potentials $V = \lambda\phi^p$

Small field models $\phi < m_{pl}$ Linde '82, Albrecht & Steinhardt '82:

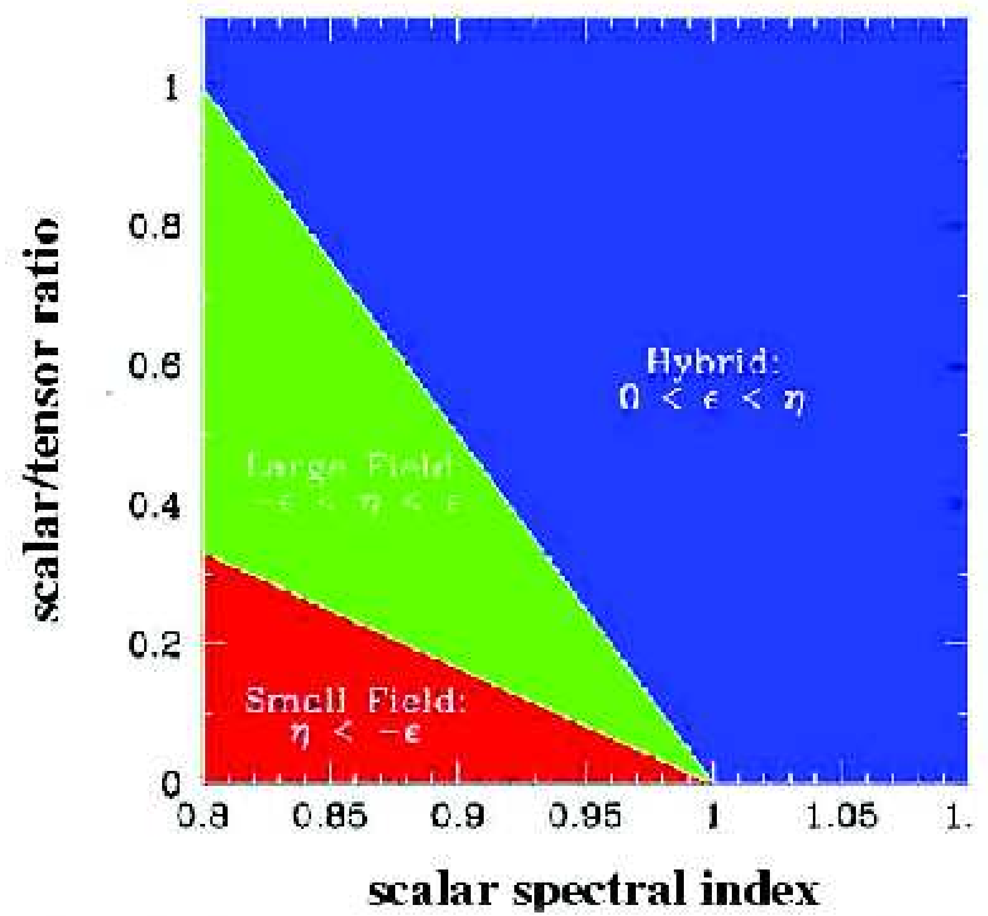
hilltop potentials $V = V_0(1 - \mu\phi^p)$

Hybrid models Linde '91:

Two field potential

$$V = V_0 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}m_\chi\chi^2 + \frac{1}{4}\lambda\chi^4 + \frac{1}{2}g^2\phi^2\chi^2$$

Standard inflation models - predictions



Kinney, *et al.*, 2001

Slow roll parameters:

$$\epsilon \equiv \frac{m_{pl}^2}{16\pi}\left(\frac{V'}{V}\right) \ll 1$$

$$\eta \equiv \frac{m_{pl}^2}{8\pi}\left(\frac{V''}{V}\right) \ll 1$$

$f_{NL} \lesssim 1$ in all models

$p = 4$ ruled out by WMAP data

If $r < 0.1$ is found by Planck, monomial cold inflation models ruled out

If $f_{NL} \gtrsim 5$ is found by Planck all these simplest of inflation models would be ruled out

Warm inflation

AB, PRL 75, 1995

Evolution equation includes dissipation:

$\ddot{\phi} + 3\mathrm{H}\dot{\phi} + \Upsilon\dot{\phi} + \mathrm{V}'(\phi) = 0$

Dissipation term leads to radiation production during inflation,

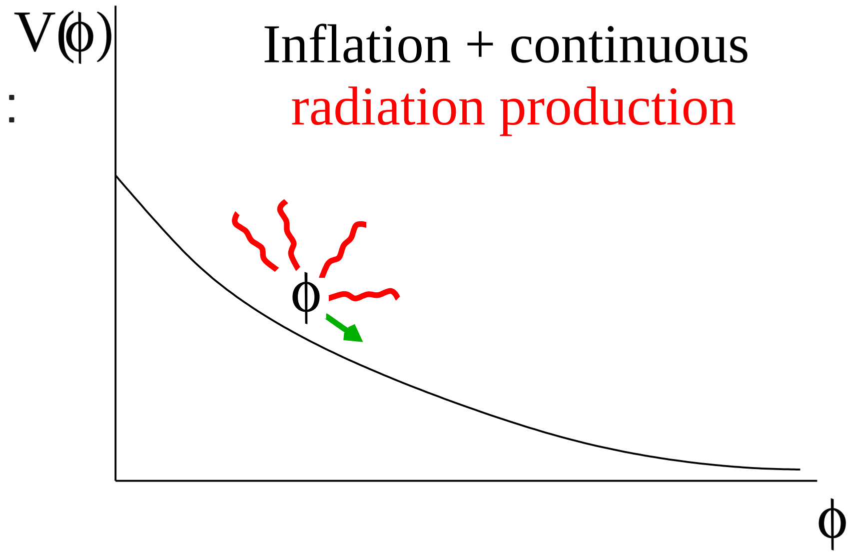
$\dot{\rho}_r = -4H\rho_r + \Upsilon\dot{\phi}^2$

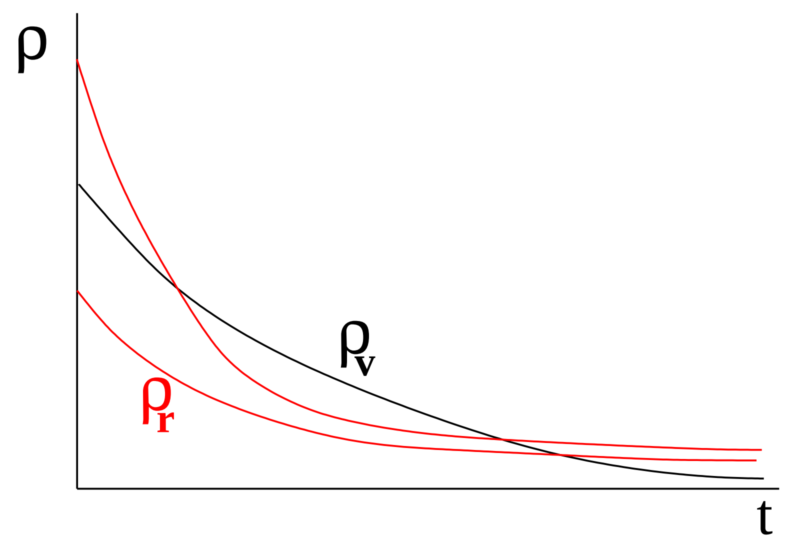
Strong dissipative regime:

$\Upsilon > 3H,\ T > H$

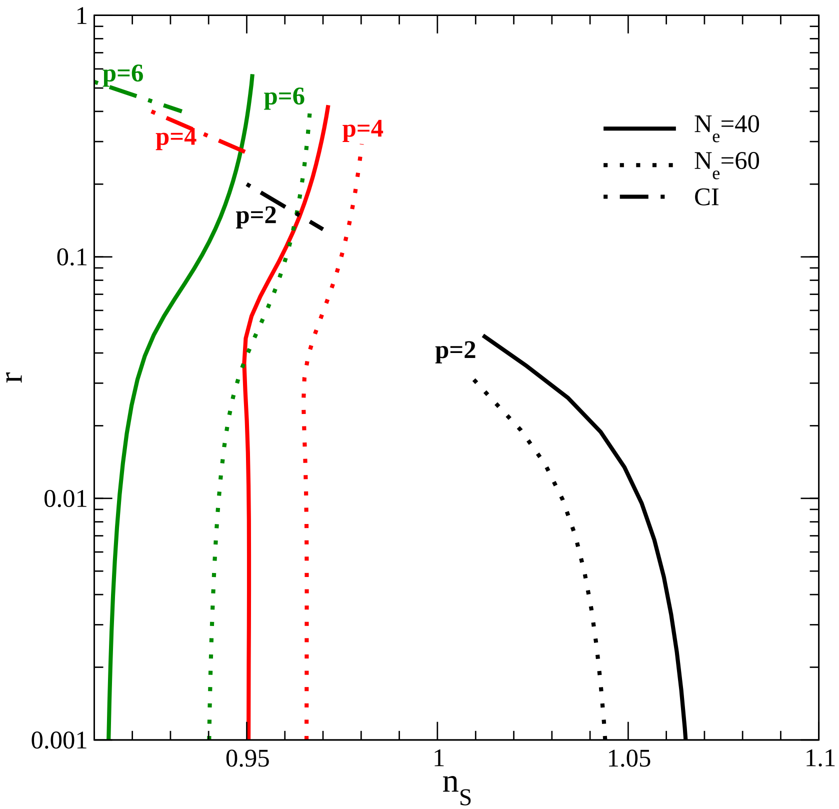
Weak dissipative regime:

$\Upsilon < 3H,\ T > H$





Warm inflation predictions



Bastero-Gil & AB, 2009

Many specific models:

monomial ϕ^p, hilltop $V_0 - \lambda\phi^p$, hybrid, etc....

Predictions for spectral indices range from red to blue

Generally predicts low tensor-scalar ratio $r < 0.1$

Nongaussianity:

Weak dissipative regime: $f_{NL} < 1$ Gupta, *et al.*, 2002

Strong dissipative regime: $f_{NL} \approx -15 \ln \left(1 + \frac{Q}{14}\right) - \frac{5}{2}$ Moss & Xiong, 2007

If Planck tightens bound on $r < 0.1$ and finds $f_{NL} \gtrsim 5$, favorable regime for warm inflation models

Curvaton model

(Mollerach, '90, Lyth & Wands, 2002)

Inflaton field ϕ need not immediately generate curvature perturbations

Isocurvature perturbations in some other scalar field σ, "curvaton", with sub-leading energy density during inflation

$$V(\phi,\sigma) = \frac{1}{2}M^2\phi^2 + \frac{1}{2}m^2\sigma^2$$

During inflation - curvaton close to minima and constant $\sigma \approx \sigma_*$, energy subdominant $m^2\sigma_*^2 \ll M^2\phi^2$

Early stage of inflation sets global mean σ_* in Universe

Perturbations via quantum fluctuations $\delta\sigma \approx H/2\pi$, to be Gaussian $\Rightarrow$ $\sigma_*^2 \gg H^2/4\pi^2$

Advantage: parameter space less constrained for model building

Predictions: lower r than single field models

Can generate nongaussianity, $f_{NL} = -5/(4x)$, $x \equiv \rho_{cv}/\rho$ energy density in curvaton before its decay (small $x \Rightarrow$ large f_{NL})

Model building challenge

- Inflation models must have very flat potentials for slow-roll and satisfying density perturbation constraints

  Rely on Supersymmetry, but this leads to η-problem that SUGRA corrections will typically lead to mass corrections $\sim H$

  Can solve with certain cold inflation models, i.e. D-term inflation, minimal choice of Kahler potential etc... Warm inflation in strong dissipative regime is free of this problem.

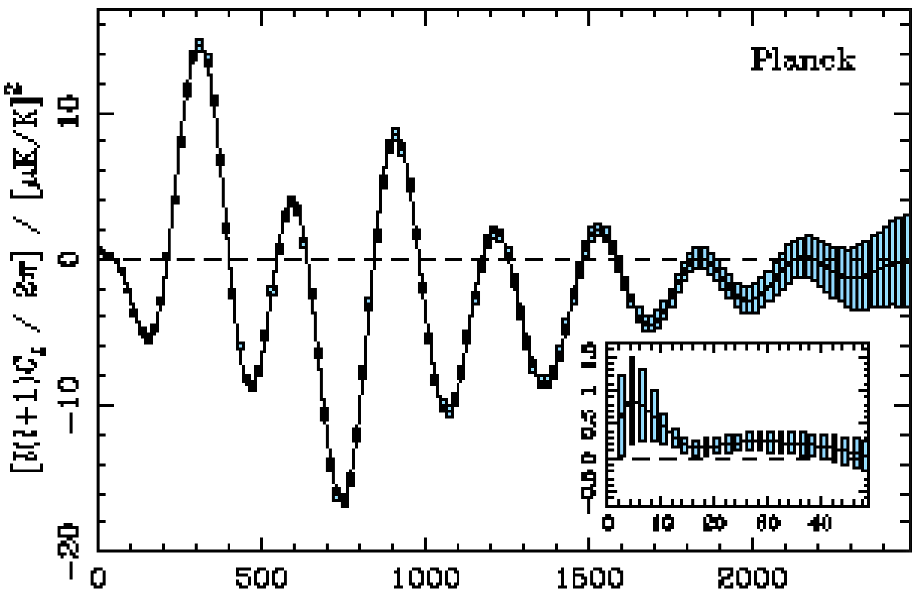
- Simplest inflation models, large field models ϕ^p typically have $\phi > m_{pl}$. Effective theory not well defined in this regime. Warm inflation monomial models alleviate this problem since ϕ amplitude is below m_{pl}.

Summary

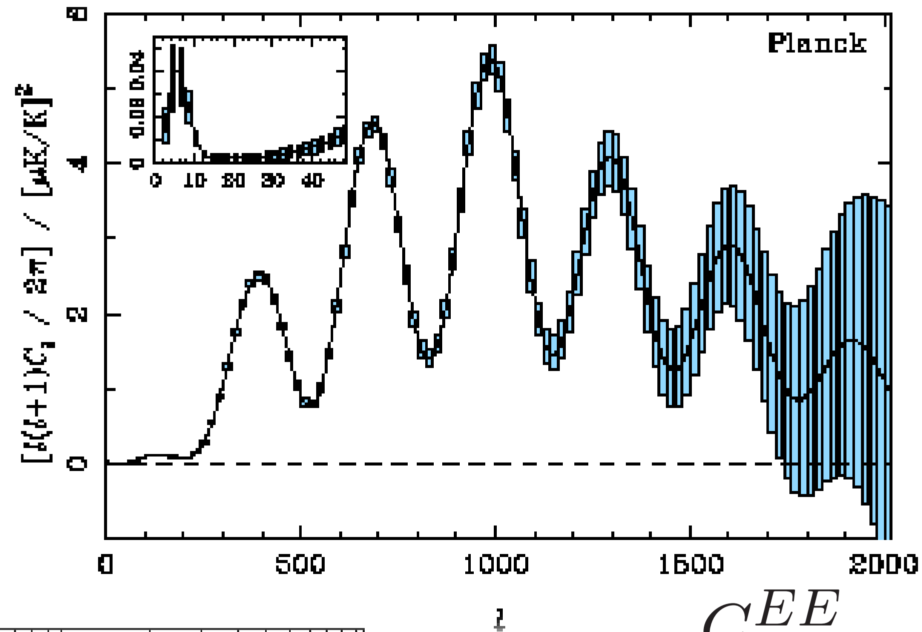
- High precision cosmology data available from CMB
- Benchmark model has scale invariant perturbations, Gaussian distributed with no tensor mode
- CMB data is now at a stage where can look from deviations from the benchmark model
- This can help discriminate inflation models
- Planck will be major leap in this direction, but without either a signal for nongaussianity or tensor modes found, model discrimination and cleanly verifying inflation will be hard to do

A thanks to Thomas Binoth for the talk invitation . . .

Projected Planck polarization spectra



C_ℓ^{TE}



C_ℓ^{EE}

C_ℓ^{BB}