PARTON DISTRIBUTIONS
AT THE DAWN OF THE LHC

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CTEQ-MCnet summer school Lauterbad, July 29, 2010

PROLOGUE: THE DISCOVERY OF THE W

LAST TIME WE LOOKED FOR “NEW” PHYSICS AT A HADRON COLLIDER

THEORETICAL PREDICTION...

42 G. Altarelli et al. / Vector boson production

TABLE 2
Values (in nb) of the total cross sections for $W^{\pm}$ and Z^0 production

$\sqrt{s}$ (GeV)	$W^+ + W^-$ GHR	$W^+ + W^-$ DO1	$W^+ + W^-$ DO2	Z^0 GHR	Z^0 DO1	Z^0 DO2	$\frac{\sigma(W^+ + W^-)}{\sigma(Z^0)}$ GHR	$\frac{\sigma(W^+ + W^-)}{\sigma(Z^0)}$ DO1	$\frac{\sigma(W^+ + W^-)}{\sigma(Z^0)}$ DO2
540	4.2	4.3	4.1	1.3	1.3	1.2	3.1	3.4	3.5
700	6.2	6.3	6.1	2.0	1.9	1.8	3.1	3.3	3.4
1000	9.5	9.5	9.6	3.1	3.0	2.9	3.1	3.2	3.3
1300	12.5	12.5	12.9	4.0	3.9	3.9	3.1	3.2	3.3
1600	15.5	15.6	16.5	5.0	4.8	5.0	3.1	3.2	3.3

ALTARELLI, ELLIS, GRECO, MARTINELLI, 1984

...AND EXPERIMENTAL DISCOVERY

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-EP/85-108
11 July 1985

W PRODUCTION PROPERTIES AT THE CERN SPS COLLIDER

UA1 Collaboration, CERN, Geneva, Switzerland

Aachen[1] – Amsterdam (NIKHEF)[2] – Annecy (LAPP)[3] – Birmingham[4] – CERN[5] – Harvard[6] – Helsinki[7] – Kiel[8] – London (Imperial College[9] and Queen Mary College[10]) – Padua[11] – Paris (Coll. de France)[12] – Riverside[13] – Rome[14] – Rutherford Appleton Lab.[15] – Saclay (CEN)[16] – Victoria[17] – Vienna[18] – Wisconsin[19] Collaboration

The corresponding experimental result for the 1984 data at $\sqrt{s}$ = 630 GeV is

$$(\sigma \cdot B)_W = 0.63 \pm 0.05\ (\pm 0.09)\ \text{nb}.$$

This is in agreement with the theoretical expectation [14] of $0.47^{+0.14}_{-0.08}$ nb. We note that the 15%

- AGREEMENT AND UNCERTAINTIES AT 20% CONSIDERED TO BE SATISFACTORY
- RESULTS FROM DIFFERENT PDF SETS DIFFER BY AT LEAST 5%
- NO WAY TO ESTIMATE PDF UNCERTAINTIES

PDFs: THEN AND NOW

PDFs IN 1984

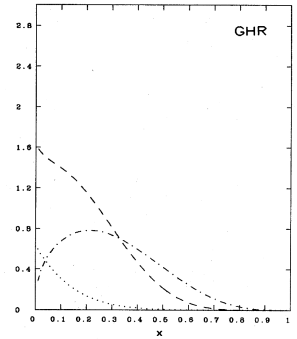


FIG. 25. Parton distributions of Glück, Hoffmann, and Reya (1982), at $Q^2=5$ GeV^2: valence quark distribution $x[u_v(x)+d_v(x)]$ (dotted-dashed line), $xG(x)$ (dashed line), and q_v (dotted line).

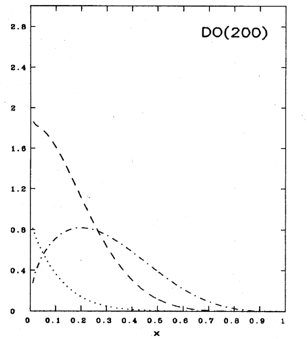


FIG. 27. "Soft-gluon" ($\Lambda=200$ MeV) parton distributions of Duke and Owens (1984) at $Q^2=5$ GeV^2: valence quark distribution $x[u_v(x)+d_v(x)]$ (dotted-dashed line), $xG(x)$ (dashed line), and $q_v(x)$ (dotted line).

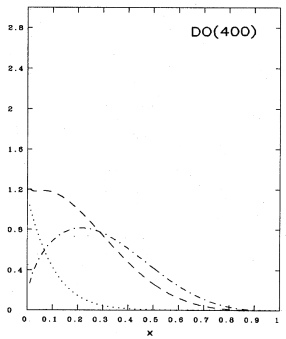
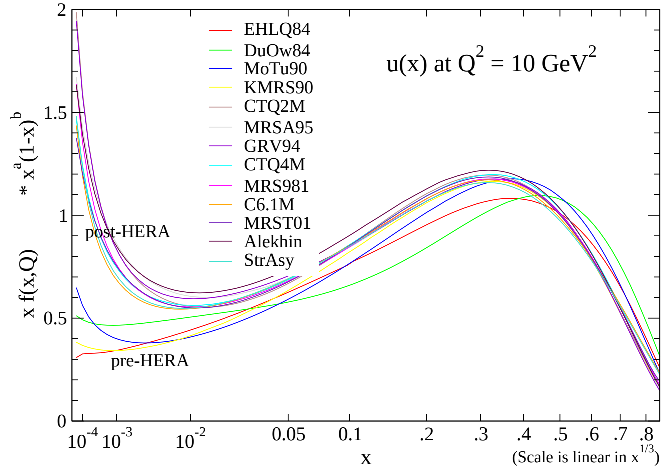


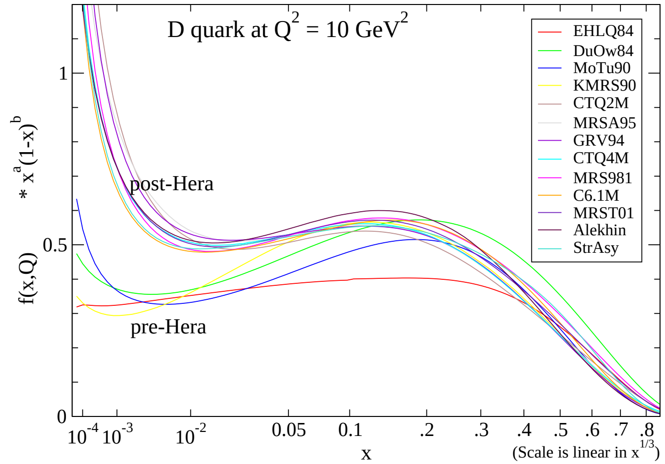
FIG. 26. "Hard-gluon" ($\Lambda=400$ MeV) parton distributions of Duke and Owens (1984) at $Q^2=5$ GeV^2: valence quark distribution $x[u_v(x)+d_v(x)]$ (dotted-dashed line), $xG(x)$ (dashed line), and $q_v(x)$ (dotted line).

Rev. Mod. Phys., Vol. 56, No. 4, October 1984

GHR VS DUKE-OWENS

20 YEARS OF VALENCE PDFs





W.-K. TUNG, 2004

- HADRON COLLIDERS CIRCA 1985 ⇒ QUALITATIVE QCD: DISCOVERY PHYSICS
- DIS AT NMC AND HERA 1995-2005 ⇒ QUANTITATIVE QCD: PRECISION PHYSICS
- HADRON COLLIDERS CIRCA 2010 ⇒ PRECISION QCD ↔ NEW PHYSICS

SUMMARY

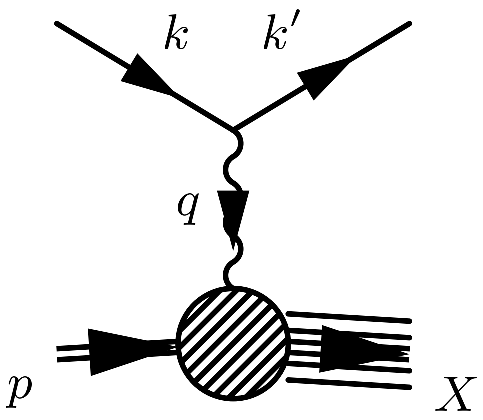
LECTURE I: THE BASICS

- FACTORIZATION
  - PDFs AND PHYSICAL OBSERVABLES
  - EVOLUTION EQUATIONS AND SUM RULES
- STATISTICS
  - HESSIAN APPROACH VS MONTE CARLO APPROACH
  - PARTON PARAMETRIZATION AND UNCERTAINTIES
- DATA
  - FLAVOUR SEPARATION
  - GLUON DETERMINATION

FACTORIZATION

FACTORIZATION I: DEEP-INELASTIC SCATTERING

Structure Functions...



Lepton fractional energy loss: $y = \frac{p\cdot q}{p\cdot k}$;

gauge boson virtuality: $q^2 = -Q^2$

Bjorken x: $x = \frac{Q^2}{2p\cdot q}$

lepton-nucleon CM energy: $s = \frac{Q^2}{xy}$;

virtual boson-nucleon CM energy $W^2 = Q^2\frac{1-x}{x}$;

$$\frac{d^2\sigma^{\lambda_p\lambda_\ell}(x,y,Q^2)}{dxdy} = \frac{G_F^2}{2\pi(1+Q^2/m_W^2)^2}\frac{Q^2}{xy}\left\{\left[-\lambda_\ell\, y\left(1-\frac{y}{2}\right)xF_3(x,Q^2) + (1-y)F_2(x,Q^2)\right.\right.$$

$$\left.\left. +y^2xF_1(x,Q^2)\right] - 2\lambda_p\left[-\lambda_\ell\, y(2-y)xg_1(x,Q^2) - (1-y)g_4(x,Q^2) - y^2xg_5(x,Q^2)\right]\right\}$$

$\lambda_l\ \rightarrow$ lepton helicity; $\lambda_p\ \rightarrow$ proton helicity

$$F_2(x,Q^2) = x\sum_i e_i^2\int_x^1\frac{dy}{y}C\left(\frac{x}{y},\alpha_s(Q^2)\right)f_i(y)$$

	PARITY CONS.	PARITY VIOL.
UNPOL.	$F_1,\ F_2$	F_3
POL.	g_1	$g_4,\ g_5$

FACTORIZATION II: HADRONIC PROCESSES

$$\sigma_X(s, M_X^2) = \sum_{a,b} \int_{x_{\min}}^1 dx_1\, dx_2\, f_{a/h_1}(x_1) f_{b/h_2}(x_2) \hat{\sigma}_{q_a q_b \to X}\left(x_1 x_2 s, M_X^2\right)$$

LEAD. ORD. $= \sigma_0 \sum_{a,b} \int_\tau^1 \frac{dx}{x} f_{a/h_1}(x) f_{b/h_2}(\tau/x) \equiv \sigma_0 \mathcal{L}(\tau)$ $\Rightarrow \mathcal{L}$ PARTON LUMI

- Scaling variable $\tau = \frac{M_X^2}{s}$

EXAMPLE: DRELL-YAN $\sigma_X \to M^2 \frac{d\sigma}{dM^2}$; $\sigma_0 = \frac{4}{9}\pi\alpha\frac{1}{s}$

FACTORIZATION II: HADRONIC PROCESSES

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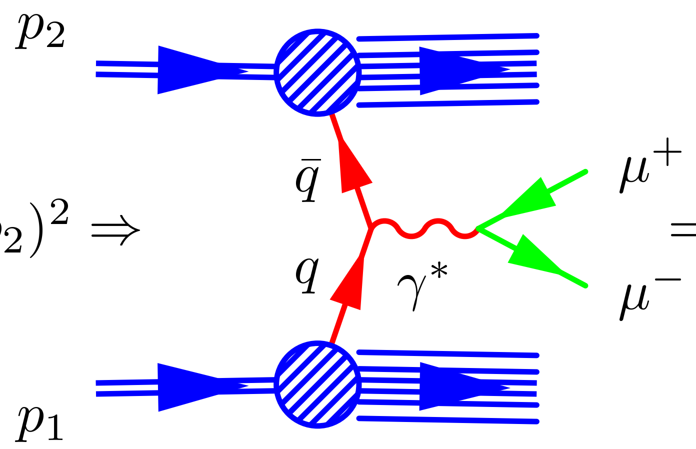
$s=(p_1+p_2)^2 \Rightarrow$



$\Rightarrow M_X^2$

LEADING ORDER

DRELL-YAN

- Hadronic c.m. energy: $s=(p_1+p_2)^2$
- Momentum fractions $x_{1,2} = \sqrt{\frac{\hat s}{s}} \exp \pm y$; Lead. Ord. $\hat s = M^2$
- Partonic c.m. energy: $\hat s = x_1 x_2 s$
- Invariant mass of final state X (dilepton, Higgs,...): $M_W^2 \Rightarrow$ scale of process
- Scaling variable $\tau = \frac{M_X^2}{s}$

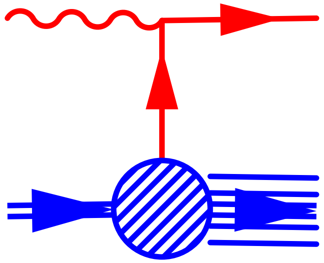
- $\hat\sigma_{q_a q_b\to X} = \sigma_0 C\left(x, \alpha_s(M_H^2)\right)$; $\quad C\left(x,\alpha_s(M_H^2)\right) = \delta(1-x) + O(\alpha_s)$
- $\sigma_X(s,M^2) = \sigma_0 \sum_{a,b}\int_{x_{\min}}^1 dx_1\, dx_2\, f_{a/h_1}(x_1) f_{b/h_2}(x_2)\delta(x_1x_2x-\tau) C\left(x,\alpha_s(M_H^2)\right)$
  $= \sigma_0 \sum_{a,b}\int_{x_2}^1 \frac{dx_1}{x_1}\int_\tau^1 \frac{dx_2}{x_2} f_{a/h_1}(x_1) f_{b/h_2}(x_2) C\left(\frac{\tau}{x_1x_2}, \alpha_s(M_H^2)\right)$

EXAMPLE: DRELL-YAN $\sigma_X \to M^2 \frac{d\sigma}{dM^2}$; $\sigma_0 = \frac{4}{9}\pi\alpha\frac{1}{s}$

Leading Parton Content

(up to $O[\alpha_s]$ corrections)

DEEP-INELASTIC SCATTERING



NC $\quad F_1{}^{\gamma} = \sum_i e_i^2 \, (q_i + \bar{q}_i)$

NC $\quad F_1{}^{Z,\,\mathrm{int.}} = \sum_i B_i \, (q_i + \bar{q}_i)$

NC $\quad F_3{}^{Z,\,\mathrm{int.}} = \sum_i D_i \, (q_i + \bar{q}_i)$

CC $\quad F_1^{W^+} = \bar{u} + d + s + \bar{c}$

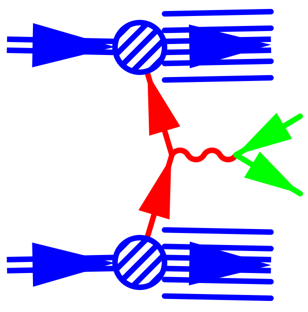
CC $\quad -F_3^{W^+}/2 = \bar{u} - d - s + \bar{c}$

ℓ	e	V	A
u,c,t	$+2/3$	$(+1/2 - 4/3\sin^2\theta_W)$	$+1/2$
d,s,b	-1/3	$(-1/2 + 2/3\sin^2\theta_W)$	-1/2
ν	0	$+1/2$	$+1/2$
e,μ, τ	-1	$(-1/2 + 2\sin^2\theta_W)$	-1/2

$B_q(Q^2) = -2e_q V_\ell V_q P_Z + (V_\ell^2 + A_\ell^2)(V_q^2 + A_q^2)P_Z^2$; $D_q(Q^2) = -2e_q A_\ell A_q P_Z + 4V_\ell A_\ell V_q A_q P_Z^2$; $P_Z = Q^2/(Q^2 + M_Z^2)$

$$W^+ \to W^- \;\Rightarrow\; u \leftrightarrow d,\; c \leftrightarrow s;\; p \to n \;\Rightarrow\; u \leftrightarrow d$$

Drell-Yan



$$L^{ij}(x_1, x_2) \equiv q_i(x_1, M^2)\bar{q}_j(x_2, M^2)$$

$\gamma \qquad \dfrac{d\sigma}{dM^2 dy}(M^2, y) = \dfrac{4\pi\alpha^2}{9M^2 s} \sum_i e_i^2 L^{ii}(x_1, x_2)$

$W \qquad \dfrac{d\sigma}{dy} = \dfrac{\pi G_F M_V^2 \sqrt{2}}{3s} \sum_{i,j} |V_{ij}^{\mathrm{CKM}}| L^{ij}(x_1, x_2)$

$Z \qquad \dfrac{d\sigma}{dy} = \dfrac{\pi G_F M_V^2 \sqrt{2}}{3s} \sum_i \left(V_i^2 + A_i^2\right) L^{ij}(x_1, x_2)$

$V_{ij}^{\mathrm{CKM}} \to$ CKM MATRIX ($i = u,\; c\, t,\; j = d,\; s\, b$), $V_{ij}^{\mathrm{CKM}} = 1 + O(\lambda)$; $\lambda = \sin\theta_C \approx 0.22$

PERTURBATIVE EVOLUTION

THEORY SUMMARY

- DEFINE MELLIN MOMENTS OF PARTON DISTRIBUTIONS
  $f(N,Q^2) \equiv \int_0^1 dx\, x^{N-1} f_2(x,Q^2)$
  NOTE LARGE/SMALL $x \Leftrightarrow$ LARGE/SMALL N
- DEFINE LOGARITHMIC SCALE $t = \ln \frac{Q^2}{\Lambda^2}$: EVOLUTION GIVEN BY ORDINARY DIFFERENTIAL EQUATIONS (RG EQUATIONS)
- ANOMALOUS DIMENSIONS RELATED TO DGLAP SPLITTING FUNCTIONS
  $\gamma(N,\alpha_s(t)) \equiv \int_0^1 dx\, x^{N-1} P(x,\alpha_s(t))$

$$\frac{d}{dt}\Delta q_{NS}(N,Q^2) = \frac{\alpha_s(t)}{2\pi}\gamma_{qq}^{NS}(N,\alpha_s(t))\Delta q_{NS}(N,Q^2),$$

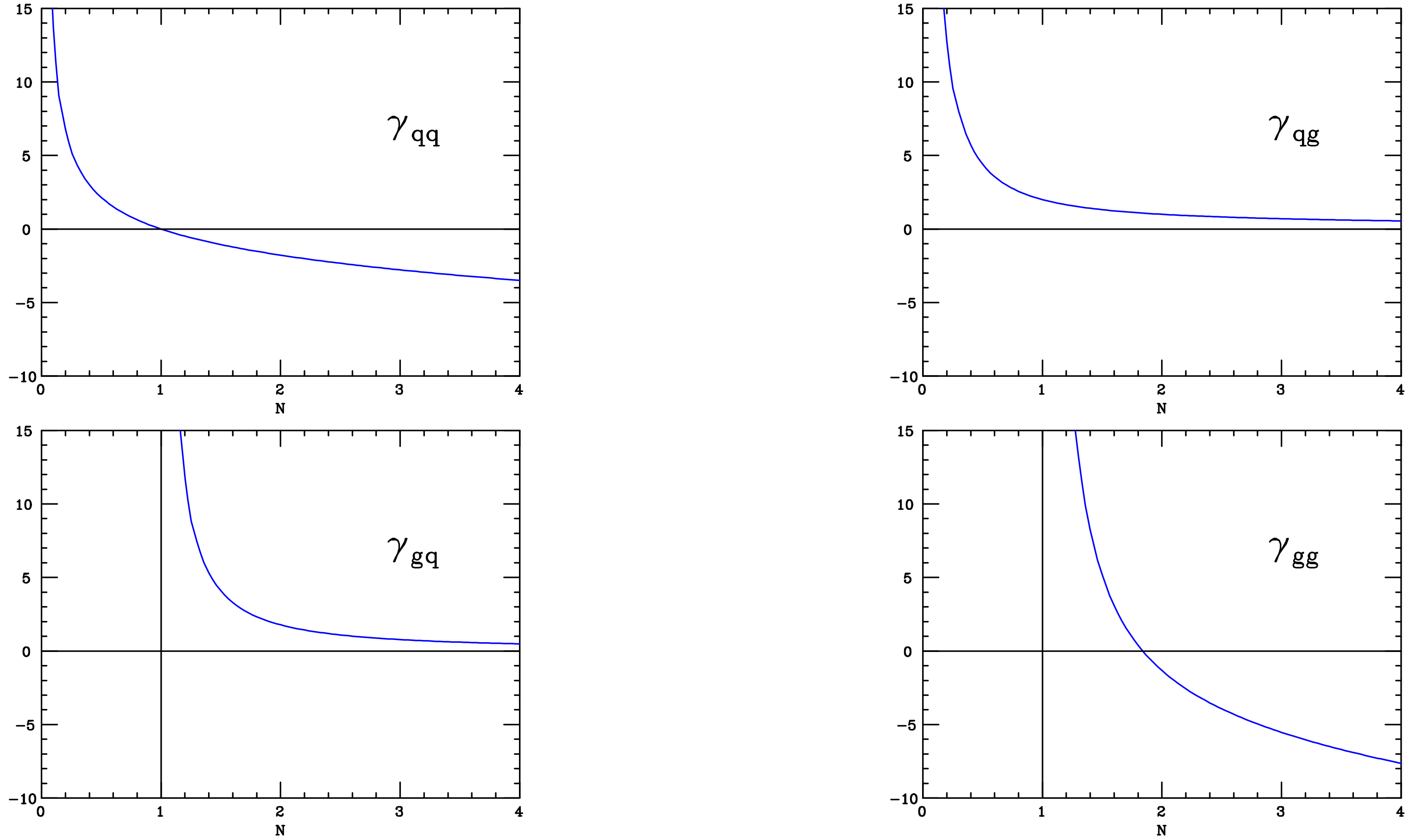
$$\frac{d}{dt}\begin{pmatrix}\Delta\Sigma(N,Q^2)\\ \Delta g(N,Q^2)\end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\begin{pmatrix}\gamma_{qq}^S(N,\alpha_s(t)) & 2n_f\gamma_{qg}^S(N,\alpha_s(t))\\ \gamma_{gq}^S(N,\alpha_s(t)) & \gamma_{gg}^S(N,\alpha_s(t))\end{pmatrix}\otimes\begin{pmatrix}\Delta\Sigma(N,Q^2)\\ \Delta g(N,Q^2)\end{pmatrix},$$

- EVOLUTION OF SINGLET $\Sigma(x,Q^2) = \sum_{i=1}^{n_f}\left(q_i(x,Q^2)+\bar{q}_i(x,Q^2)\right)$ COUPLED TO GLUON
- ALL "NONSINGLET" QUARK COMBINATIONS $q^{NS}(x,Q^2) = q_i(x,Q^2) - q_j(x,Q^2)$ EVOLVE INDEPENDENTLY
- ANOMALOUS DIMENSIONS COMPUTED IN PERTURBATION THEORY:
  $\gamma_i(N,\alpha_s(t)) = \gamma_i^{(0)}(N) + \alpha_s(t)\gamma_i^{(1)}(N) + \ldots$

PERTURBATIVE EVOLUTION

QUALITATIVE FEATURES

THE LEADING ORDER ANOMALOUS DIMENSIONS



- AS Q^2 INCREASES, PDFS DECREASE AT LARGE x & INCREASE AT SMALL x DUE TO RADIATION
- GLUON SECTOR SINGULAR AT $N = 1 \Rightarrow$ GLUON GROWS MORE AT SMALL x
- $\gamma_{qq}(1) = 0 \Rightarrow$ NUMBER OF QUARKS CONSERVED

SUM RULES

CONSERVED QUANTUM NUMBERS CARRIED BY PARTON DISTRIBUTIONS

- BARYON NUMBER $\int_0^1 dx\,(u^p - \bar{u}^p) = 2 = 2\int_0^1 dx\,\left(d^p - \bar{d}^p\right)$
- MOMENTUM $\int_0^1 dx x\left[\sum_{i=1}^{N_f}\left(q^i(x) + \bar{q}_i(x)\right) + g(x)\right]$=1

FIXED & SCALE–INDEPENDENT

- BARYON NUMBER $\gamma_{qq}(1) - \gamma_{q\bar{q}}(1) = 0$; AT LO $\gamma_{q\bar{q}}(1) = 0$ SO $\gamma_{qq}(1) = 0$
- MOMENTUM $\gamma_{qq}(2) + \gamma_{qg}(2) = 0$, $\gamma_{gq}(2) + \gamma_{gg}(2) = 0$

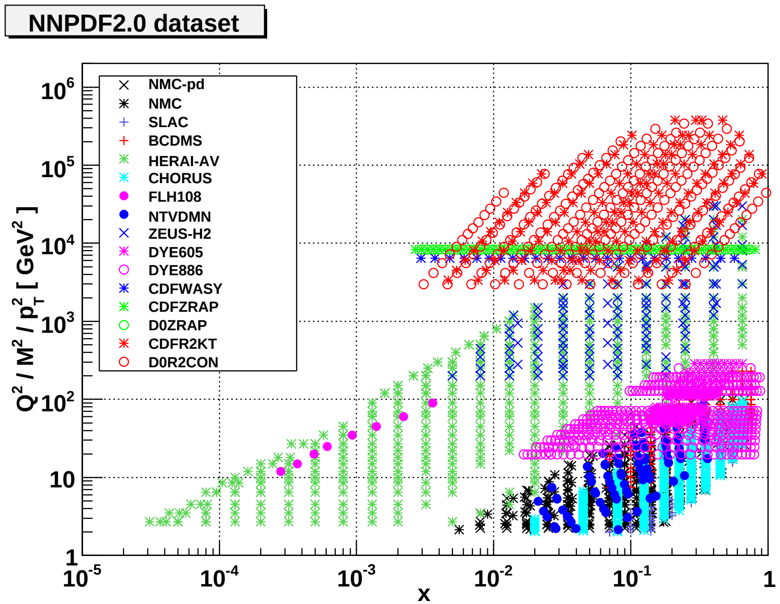
RELATION TO PHYSICAL OBSERVABLES: BARYON NUMBER

- GROSS–LLEWELLYN-SMITH SUM RULE $\frac{1}{2}\int_0^1 dx\,\left(F_3^{\nu p}(x,Q^2) + F_3^{\nu n}(x,Q^2)\right) =$
$= C_{\rm GLS}(Q^2)\int_0^1 dx\,\left[u(x,Q^2) - \overline{u}(x,Q^2) + d(x,Q^2) - \overline{d}(x,Q^2)\right]$
- BJORKEN (UNPOLARIZED) SUM RULE $\frac{1}{2}\int_0^1 dx\,\left(F_1^{\nu p}(x,Q^2) - F_1^{\nu n}(x,Q^2)\right) =$
$= C_{\rm BjU}(Q^2)\int_0^1 dx\,\left[u(x,Q^2) - \overline{u}(x,Q^2) - \left(d(x,Q^2) - \overline{d}(x,Q^2)\right)\right]$

STATISTICS

PARTON FITS

DATA → PARTON DISTRIBUTIONS



ISSUES AND TASKS:

- FROM PHYSICAL OBSERVABLES TO PDFS: SOLVE EVOLUTION EQUATIONS, CONVOLUTE WITH PARTON-LEVEL CROSS-SECTIONS
- DISENTANGLING PDFS: CHOOSE A BASIS OF PDFS ($2N_f$ QUARKS + 1 GLUON) & A SET OF SUITABLE PHYSICAL PROCESSES TO DETERMINE THEM ALL
- PROBABILITY IN THE SPACE OF FUNCTIONS: CHOOSE A STATISTICAL APPROACH (HESSIAN, MONTE CARLO, . . .)
- UNCERTAINTY ON FUNCTIONS: CHOOSE A FUNCTIONAL FORM

THE HESSIAN APPROACH (MSTW, CTEQ):

FUNCTIONAL PARTON FITTING

- CHOOSE A FIXED FUNCTIONAL FORM:
  - MSTW: 20 PARMS.

    $xq(x,Q_0^2) = A(1-x)^\eta(1+\epsilon x^{0.5}+\gamma x)x^\delta$, (5 indep. fns.); $x[\bar{u}-\bar{d}](x,Q_0^2) = A(1-x)^\eta(1+\gamma x+\delta x^2)x^\delta$;

    $x[s-\bar{s}](x,Q_0^2) = A_-(1-x)^{\eta_s}x^{\delta_-}(1-x/x_0)$; $xg(x,Q_0^2) = A_g(1-x)^{\eta_g}(1+\epsilon_g x^{0.5}+\gamma_g x)x^{\delta_g}+A_{g'}(1-x)$
  - CTEQ/TEA: 22→ 26 PARMS.

    $$x\,f(x,Q_0) = a_0\,x^{a_1}\,(1-x)^{a_2}\,\exp\left(a_3 x + a_4 x^2 + a_5\sqrt{x} + a_6 x^{-a_7}\right) \text{ (7 indep. functions)}$$

    a_6, a_7 ONLY USED FOR GLUON
  - BASIS FUNCTIONS: $u_v \equiv u-\bar{u}$, $d_v \equiv d-\bar{d}$, $\bar{u}\pm\bar{d}$ (MSTW) OR $\bar{u}$, $\bar{d}$ (CTEQ), $s^\pm \equiv s\pm\bar{s}$ (CTEQ $\bar{s}+s$ ONLY), g.
- EVOLVE TO DESIRED SCALE & COMPUTE PHYSICAL OBSERVABLES
- DETERMINE BEST-FIT VALUES OF PARAMETERS
- DETERMINE ERROR BY PROPAGATION OF ERROR ON PARMS. ('HESSIAN METHOD') PARM. SCANS ALSO POSSIBLE ('LAGR. MULTIPLIER METHOD')

HESSIAN ERROR ESTIMATES

GENERAL FEATURES

OBSERVABLE X DEPENDING ON PARAMETERS $\vec{z}$: (LINEAR ERROR PROPAGATION)

$$X(\vec{z}) \approx X_0 + z_i \partial_i X(\vec{z}) \qquad \text{ASSUMING MOST LIKELY VALUE AT } \vec{z} = 0$$

VARIANCE: $\sigma_X^2 = \sigma_{ij} \partial_i X \partial_j X,$

$\sigma_{ij} \Rightarrow$ COVARIANCE MATRIX IN PARAMETER SPACE

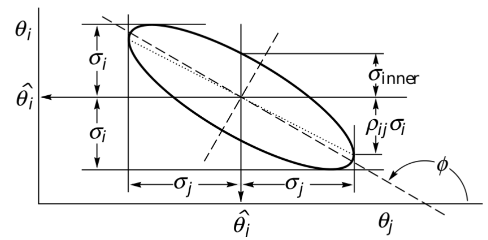
MAXIMUM LIKELIHOOD: COVARIANCE $\Leftrightarrow$ HESSIAN $\sigma_{ij} = \partial_i \partial_j \chi^2$ EVALUATED AT MIN. OF χ^2

DIAGONALIZATION: CHOOSE z_i AS EIGENVECTORS OF σ_{ij} WITH UNIT EIGENVALUES

$$\sigma_X^2 = |\vec{\nabla} X|^2 \text{ (LENGTH OF GRADIENT)}$$

SOME INTERESTING CONSEQUENCES

- THE ONE-σ CONTOUR IN PARAMETER SPACE IS ELLIPSE $\chi^2 = \chi^2_{\text{min}} + 1$
- THE TOTAL UNCERTAINTY IS THE SUM IN QUADRATURE OF UNCERTAINTIES DUE TO EACH PARAMETER (LENGTH OF VECTOR) EVEN WHEN NOT DIAGONALIZING (Lai et al, CTEQ 2010)
- ANY ROTATION (ORTHOGONAL TRANSF.) IN THE SPACE OF PARMS PRESERVES THE GRADIENT $\rightarrow$ CAN DIAGONALIZE A CHOSEN OBSERVABLE WITHOUT SPOILING RESULT (Pumplin 2009)



HESSIAN ERROR ESTIMATES

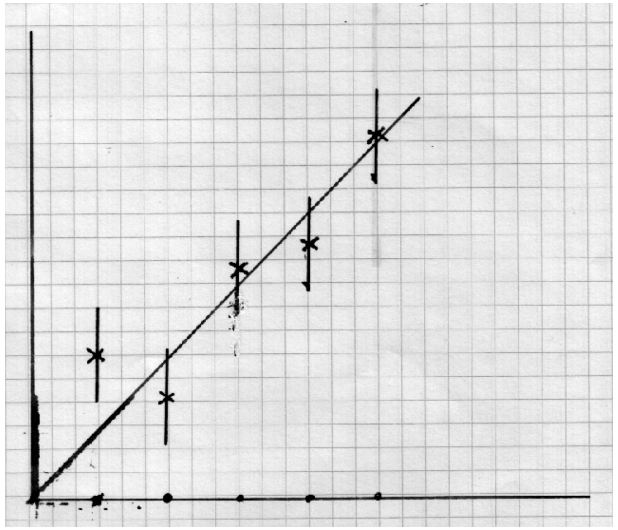
HYPOTESIS TESTING VS. PARAMETER FITTING

"PARADOX"

- THE STANDARD DEVIATION OF χ^2 FOR $N_{\rm dat}$ DATA $\sigma_{\chi^2} = \sqrt{2N_{\rm dat}}$
  HYPOTESIS-TESTING RANGE: COMPARE $\Delta\chi^2 = \chi^2 - \langle\chi^2\rangle$ TO $\sigma^2_{\chi^2}$.
  IF TOO LARGE, SOMETHING WRONG WITH THEORY (OR DATA)
- BUT THE ONE-σ RANGE FOR A PARM. OF THE THEORY IS THE CURVE $\chi^2 - \chi^2_{\rm min} = 1$
  PARAMETER-FITTING RANGE: UNIT DEVIATION FROM THE PARAMETRIC MINIMUM $\chi^2_{\rm min}$

WHY?

- CONSIDER DEVIATIONS Δ_i FROM LINEAR FIT $y = x + k$; DETERMINE INTERCEPT k AS FREE PARAMETER
- IF STANDARD DEVIATION FOR EACH Δ_i IS σ_Δ, THEN AVERAGE SQUARE DEVIATION IN UNITS OF σ_Δ FOR $N_{\rm dat}$ DATA: $\sigma_{\chi^2} = N_{\rm dat}$
- BEST-FIT INCTERCEPT: $k = \langle\Delta_i\rangle$
- UNCERTAINTY ON IT: $\sigma_k = \frac{\sigma_\Delta}{N_{\rm dat}}$
- IF $\Delta k = \sigma_k$, THEN $\Delta\chi^2 = 1$



MONTE CARLO ERROR ESTIMATES

EXACT ERROR PROPAGATION

OBSERVABLE X DEPENDS ON PARAMETERS $\vec{z}$

VARIANCE: $\sigma_X^2 = \langle X^2 \rangle - \langle X \rangle^2$

AVERAGES: $\langle X \rangle = \int d^d z X(\vec{z}) P(\vec{z})$, WITH

$P(\vec{z}) \Rightarrow$ PROBABILITY DISTN. OF PARAMETER VALUES

& INTEGRAL PERFORMED BY MONTE CARLO SAMPLING

HOW MANY REPLICAS DOES ONE NEED?

MONTE CARLO ERROR ESTIMATES

EXACT ERROR PROPAGATION

OBSERVABLE X DEPENDS ON PARAMETERS $\vec{z}$

VARIANCE: $\sigma_X^2 = \langle X^2 \rangle - \langle X \rangle^2$

AVERAGES: $\langle X \rangle = \int d^d z X(\vec{z}) P(\vec{z})$, WITH

$P(\vec{z}) \Rightarrow$ PROBABILITY DISTN. OF PARAMETER VALUES

& INTEGRAL PERFORMED BY MONTE CARLO SAMPLING

HOW MANY REPLICAS DOES ONE NEED? THREE BINS PER PARM $\Rightarrow 3^d$ BINS

FOR 23 PARMS., NEED $> 10^{11}$ REPLICAS

MONTE CARLO ERROR ESTIMATES

EXACT ERROR PROPAGATION

OBSERVABLE X DEPENDS ON PARAMETERS $\vec{z}$
VARIANCE: $\sigma_X^2 = \langle X^2 \rangle - \langle X \rangle^2$
AVERAGES: $\langle X \rangle = \int d^d z X(\vec{z}) P(\vec{z})$, WITH

$P(\vec{z}) \Rightarrow$ PROBABILITY DISTN. OF PARAMETER VALUES
& INTEGRAL PERFORMED BY MONTE CARLO SAMPLING

HOW MANY REPLICAS DOES ONE NEED? THREE BINS PER PARM $\Rightarrow 3^d$ BINS
FOR 23 PARMS., NEED $> 10^{11}$ REPLICAS

DATA SPACE

DIAGONALIZATION: CHOOSE PARM z_1 ALONG $\vec{\nabla} X$
ALL OTHER PARMS $\Rightarrow$ FLAT DIRECTIONS
AVERAGES: $\langle X \rangle = \int dz_1 X(\vec{z}) P(z_1)$
HOW MANY REPLICAS DOES ONE NEED?

MONTE CARLO ERROR ESTIMATES

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AVERAGES: $\langle X \rangle = \int dz_1 X(\vec{z}) P(z_1)$
HOW MANY REPLICAS DOES ONE NEED? ONE–DIMENSIONAL AVERAGE OF n REPLICAS
CONVERGES TO TRUE AVERAGE WITH STANDARD DEV. $\frac{\sigma}{\sqrt{n}}$
10 REPLICAS ENOUGH FOR $\frac{\sigma}{3}$ ACCURACY

MONTE CARLO ERROR ESTIMATES

EXACT ERROR PROPAGATION

OBSERVABLE X DEPENDS ON PARAMETERS $\vec{z}$
VARIANCE: $\sigma_X^2 = \langle X^2 \rangle - \langle X \rangle^2$
AVERAGES: $\langle X \rangle = \int d^d z X(\vec{z}) P(\vec{z})$, WITH

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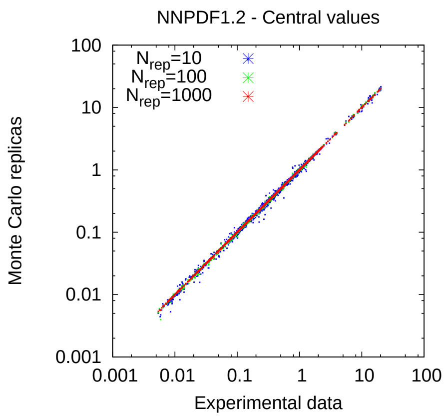
HOW MANY REPLICAS DOES ONE NEED? ONE–DIMENSIONAL AVERAGE OF n REPLICAS CONVERGES TO TRUE AVERAGE WITH STANDARD DEV. $\frac{\sigma}{\sqrt{n}}$
10 REPLICAS ENOUGH FOR $\frac{\sigma}{3}$ ACCURACY
Q: HOW IS IT DONE IN PRACTICE?
A: CHOOSE REPLICAS OF THE DATA, DISTRIBUTED AS THE DATA

THE NNPDF APPROACH:
THE NEURAL MONTE CARLO

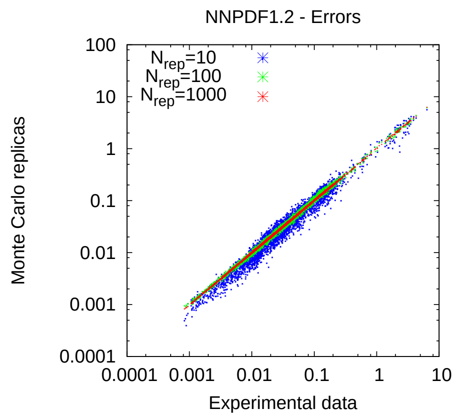
BASIC IDEA: MONTE CARLO SAMPLING OF THE PROBABILITY MEASURE IN THE (FUNCTION) SPACE OF PDFs

- START FROM MONTE CARLO SAMPLING OF DATA SPACE
- SPACE OF FUNCTIONS HUGE
  5 BINS FOR 10 PTS$\times$ 7 FCTNS $\rightarrow 5^{70} \sim 10^{49}$ BINS
- IMPORTANCE SAMPLING: DATA TELL US WHICH BINS ARE POPULATED

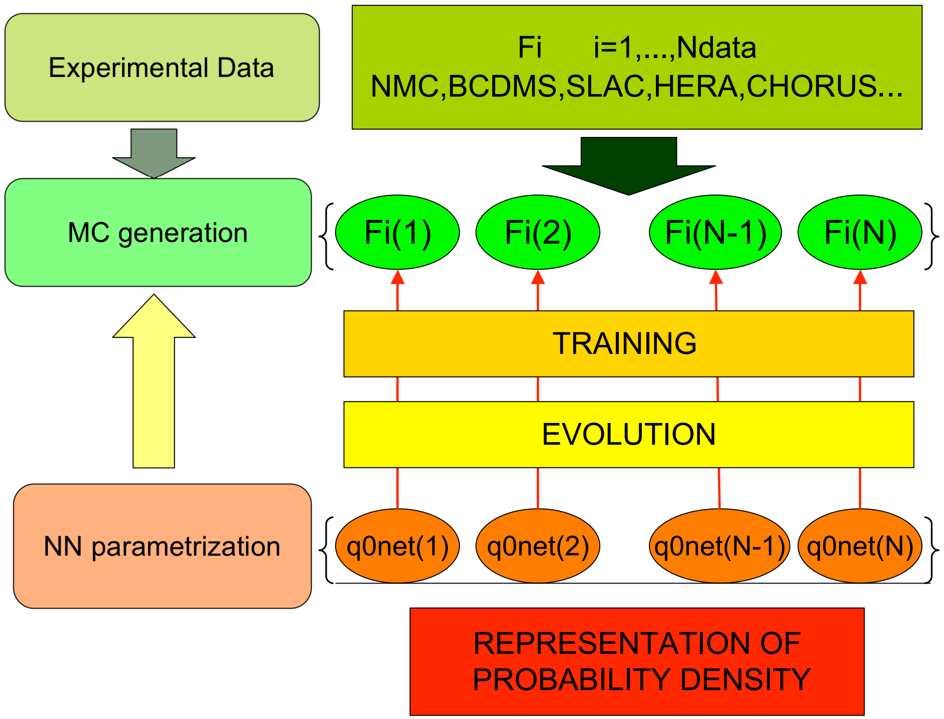
replica averages vs. central values



replica standard dev. vs. uncertainties

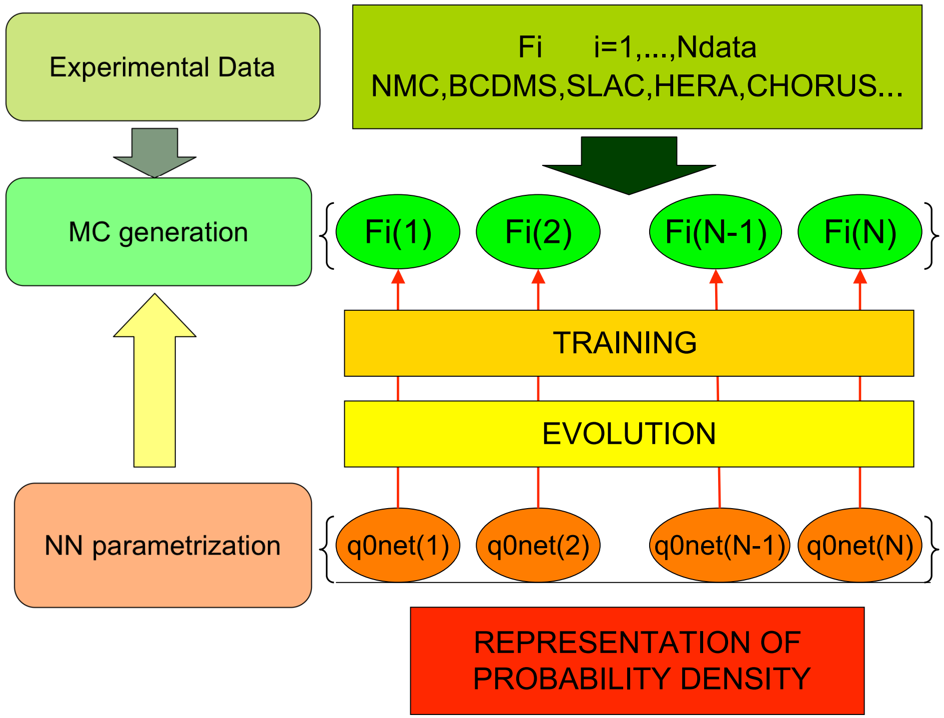


10 REPLICAS ENOUGH FOR CENTRAL VALS, 100 FOR UNCERTAINTIES, 1000 FOR CORRELNS



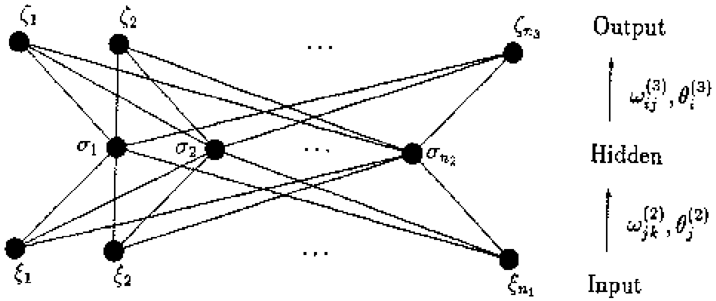
DATA MONTE CARLO ⇒ PDF MONTE CARLO
NEURAL NETWORK PARM+ CROSS-VALIDATION METHOD

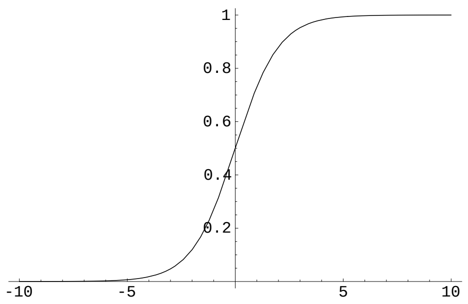
- EACH PDF↔ NEURAL NETWORK PARAMETRIZED BY 37 PARAMETERS
- NNPDF1.2: $37 \otimes 7 = 259$ PARMS (RECALL MSTW, CTEQ → 20 FREE PARAMETERS) "INFINITE" NUMBER OF PARAMETERS⇒ CAN REPRESENT ANY FUNCTION
- COMPLEX SHAPES (LARGE NO.OF PARAMETERS) REQUIRE LONGER FITTING
- FIT STOPS WHEN QUALITY OF FIT TO RANDOMLY SELECTED "VALIDATION" DATA (NOT FITTED) STOPS IMPROVING
- CAN OBTAIN A FIT WITH χ^2 LOWER THAN BEST FIT ("OVERLEARNING")



NEURAL NETWORKS

A NONLINEAR FUNCTIONAL FORM





MULTILAYER FEED-FORWARD NETWORKS

- Each neuron receives input from neurons in preceding layer and feeds output to neurons in subsequent layer
- Activation determined by weights and thresholds
  $$\xi_i = g\left(\sum_j \omega_{ij}\xi_j - \theta_i\right)$$
- Sigmoid activation function
  $$g(x) = \frac{1}{1+e^{-\beta x}}$$

EXAMPLE: A 1-2-1 NN

$$f(x) = \frac{1}{1+e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1+e^{\theta_1^{(2)} - x\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1+e^{\theta_2^{(2)} - x\omega_{21}^{(1)}}}}}$$

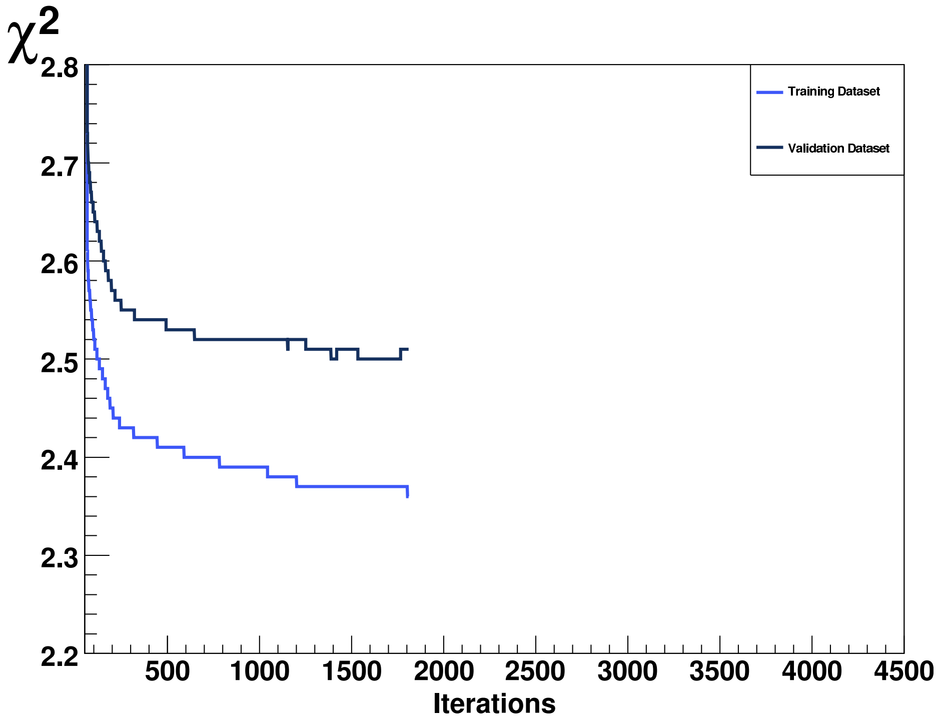
THANKS TO NONLINEAR BEHAVIOUR,
ANY FUNCTION CAN BE REPRESENTED BY A SUFFICIENTLY BIG NEURAL NETWORK

CROSS-VALIDATION
HOW TO DETERMINE THE OPTIMAL FIT

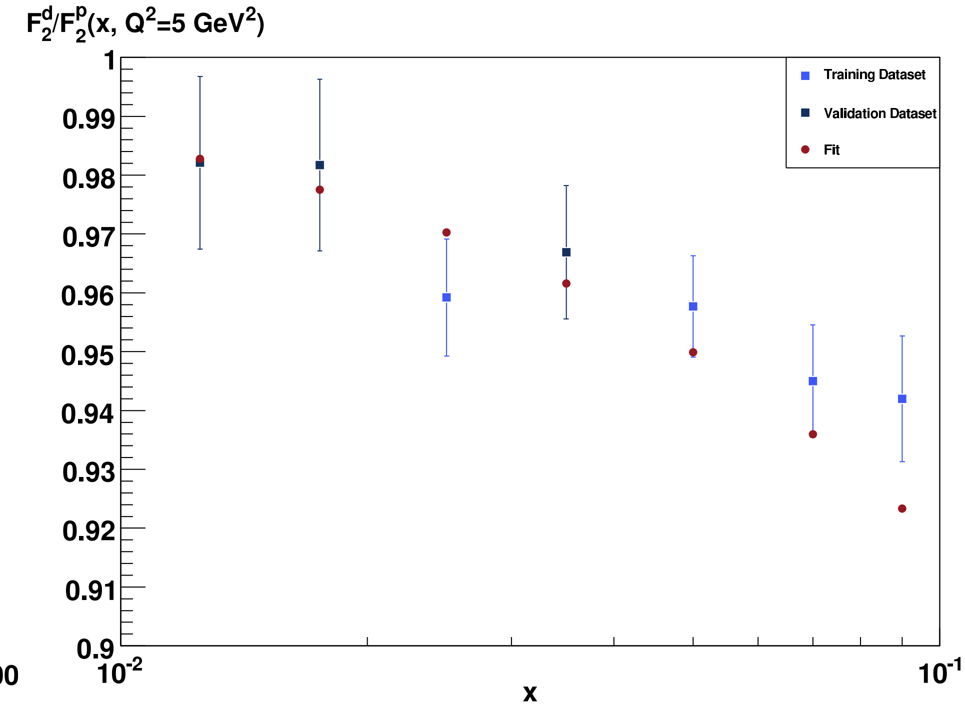
- REPLICAS ARE FITTED TO A DATA SUBSET
- A DIFFERENT SUBSET OF DATA USE FOR EACH REPLICA
- OPTIMAL FIT WHEN FIT TO VALIDATION (CONTROL) DATA STOPS IMPROVING
-

OPTIMAL FITTING

χ^2



FIT TO DATA



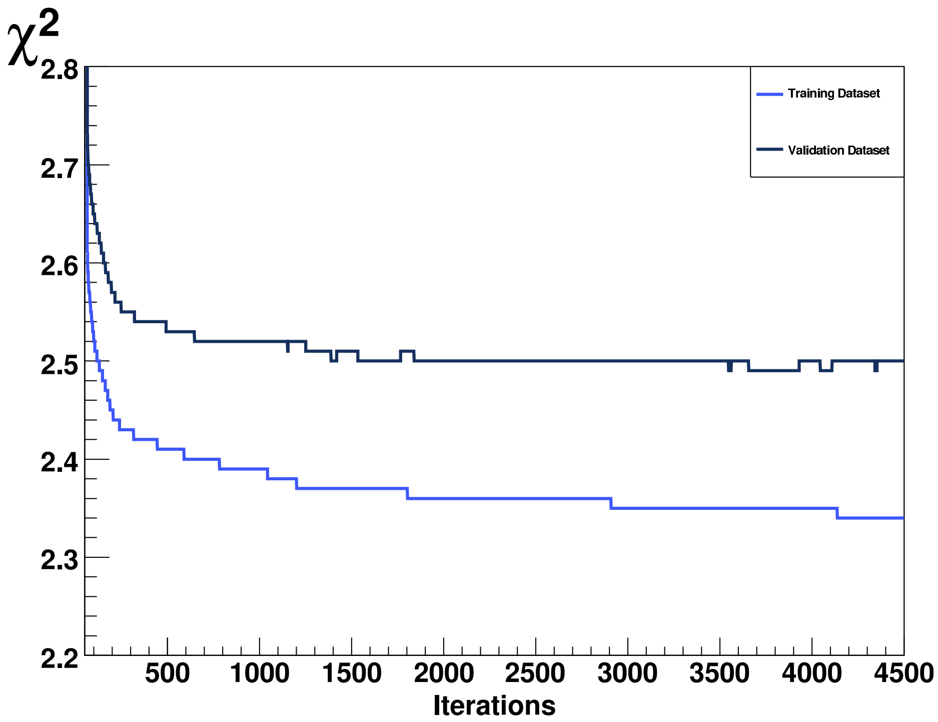
CROSS-VALIDATION

HOW TO DETERMINE THE OPTIMAL FIT

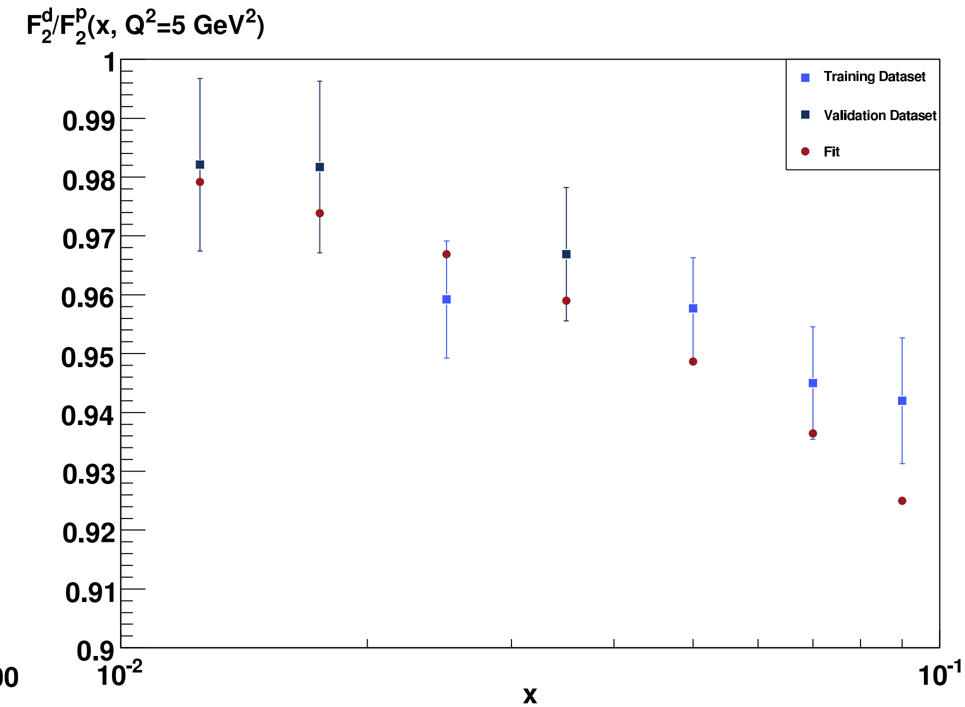
- REPLICAS ARE FITTED TO A DATA SUBSET
- A DIFFERENT SUBSET OF DATA USE FOR EACH REPLICA
- OPTIMAL FIT WHEN FIT TO VALIDATION (CONTROL) DATA STOPS IMPROVING
- THE BEST FIT IS NOT AT THE MINIMUM OF THE χ^2

OVERFITTING

χ^2



FIT TO DATA



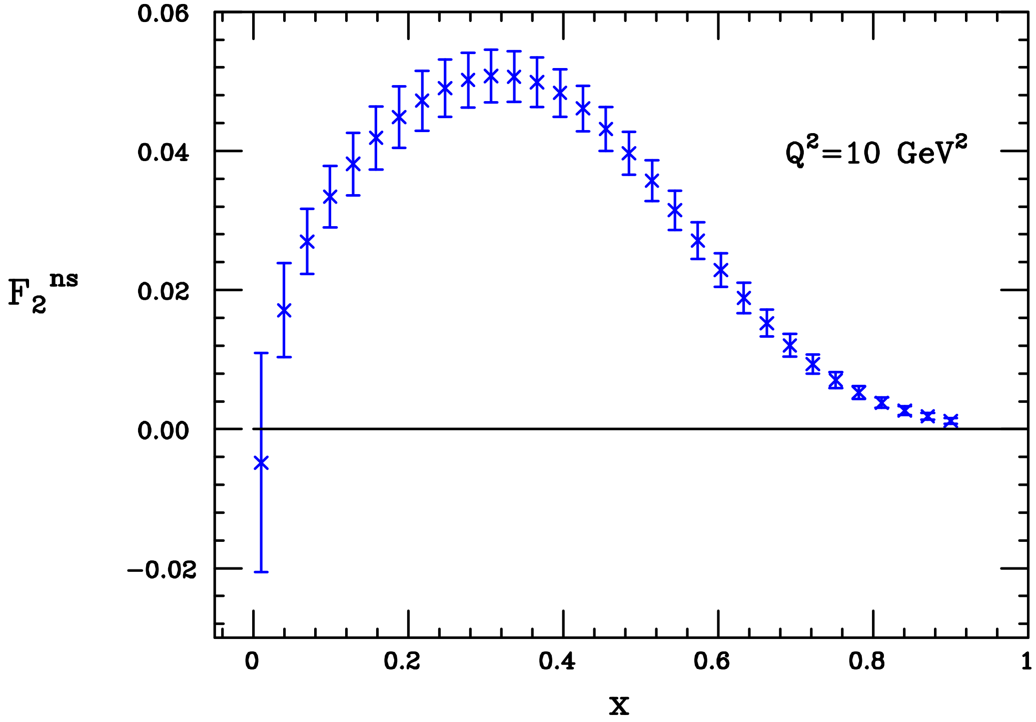
DATA

DISENTANGLING QUARKS: UP VS. DOWN

WITH THE HELP OF ISOSPIN SYMMETRY

$$u^p(x,Q^2) = d^n(x,Q^2); \quad d^p(x,Q^2) = u^n(x,Q^2)$$

$$F_2^p(x,Q2) - F_2^d(x,Q^2) = \frac{1}{3}\left[(u^p + \bar{u}^p) - \left(d^p + \bar{d}^p\right)\right]\left[1 + O(\alpha_s)\right]$$



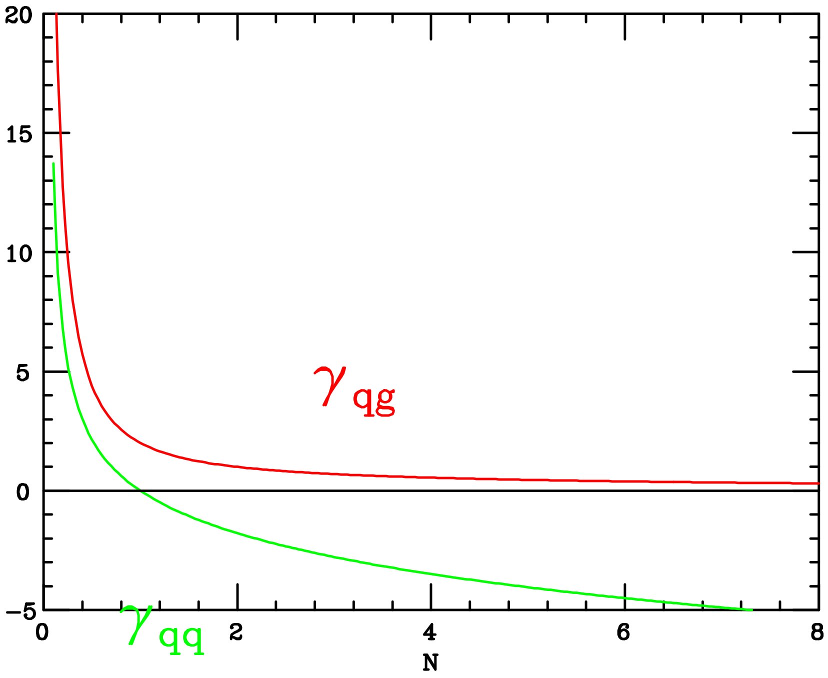
DETERMINING THE GLUON

SMALL x (SMALL N):

SINGLET SCALING VIOLATIONS (HERA)

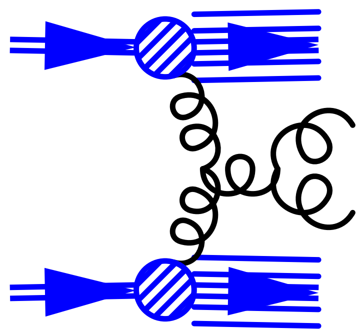
$$\frac{d}{dt} F_2^s(N,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\left[\gamma_{qq}(N)F_2^s + 2\,n_f\,\gamma_{qg}(N)g(N,Q^2)\right] + O(\alpha_s^2)$$

ANOMALOUS DIMENSIONS



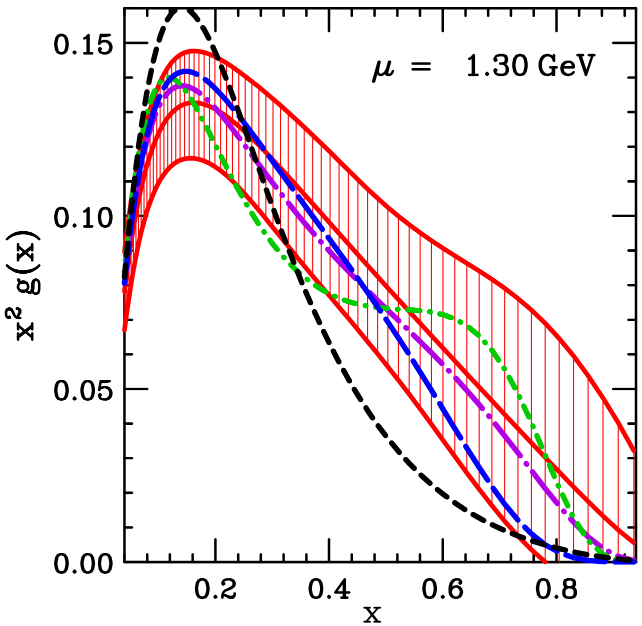
LARGE x:

HIGH p_T JETS (TEVATRON)



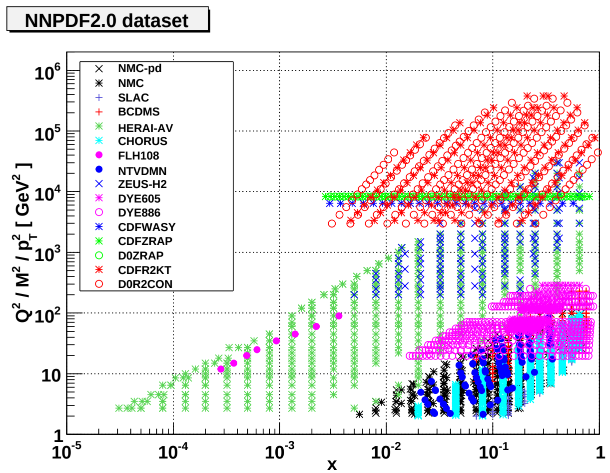
CTEQ6.6 GLUON

(DASHED: NO JET DATA)



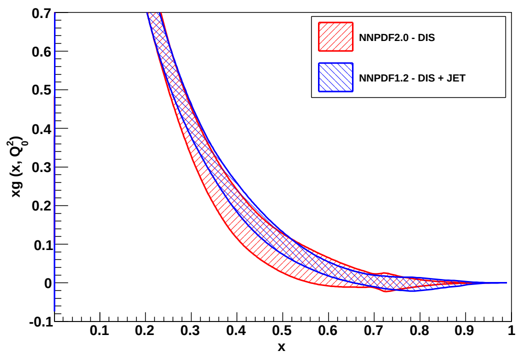
THE IMPACT OF JET DATA(NNPDF2.0)

	DIS	DIS+JET	NNPDF2.0
$\chi^2_{\rm tot}$	1.20	1.18	1.21
NMC-pd	0.85	0.86	0.99
NMC	1.69	1.66	1.69
SLAC	1.37	1.31	1.34
BCDMS	1.26	1.27	1.27
HERAI	1.13	1.13	1.14
CHORUS	1.13	1.11	1.18
FLH108	1.51	1.49	1.49
NTVDMN	0.71	0.75	0.67
ZEUS-H2	1.50	1.49	1.51
CDFR2KT	**0.91**	**0.79**	0.80
D0R2CON	**1.00**	**0.93**	0.93
DYE605	7.32	10.35	0.88
DYE866	2.24	2.59	1.28
CDFWASY	13.06	14.13	1.85
CDFZRAP	3.12	3.31	2.02
D0ZRAP	0.65	0.68	0.47



- HIGH E_T JET DATA WELL REPRODUCED EVEN WHEN NOT FITTED $\Rightarrow$ LARGE x GLUON WELL DETERMINED BY SCALING VIOLATIONS!
- SIGNIFICANT IMPROVEMENT IN LARGE x GLUON ACCURACY

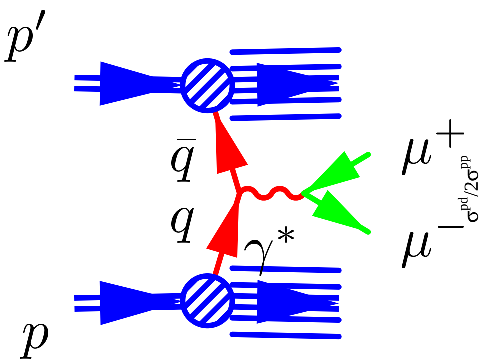
GLUON

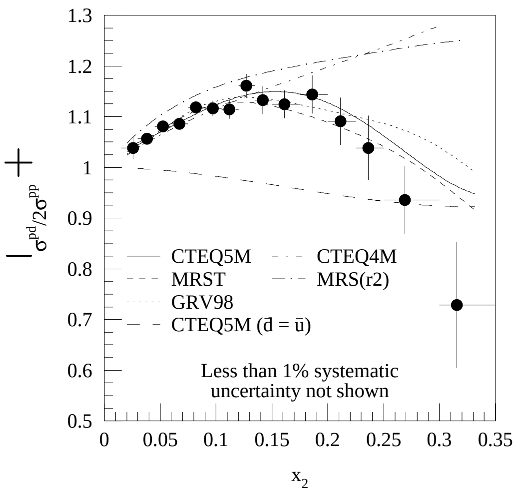


DISENTANGLING QUARKS FROM ANTIQUARKS

γ^* DIS ONLY MEASURES $q+\bar{q}$ COMBINATION!

DRELL-YAN p/d ASYMMETRY



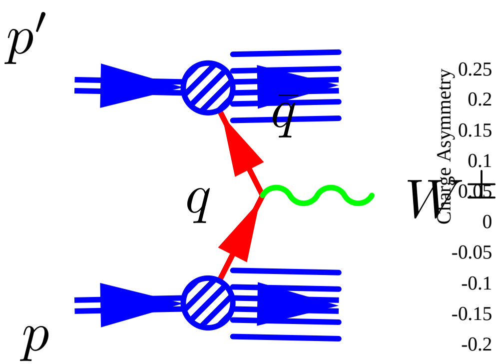


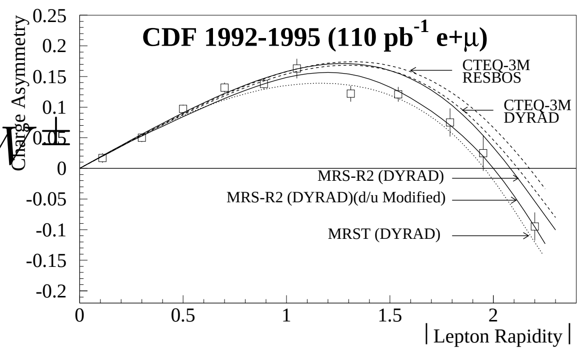
E866 (2001)

LIGHT ANTIQUARK ASYMMETRY

$$\frac{\sigma^{pn}}{\sigma^{pp}} \sim \left.\frac{\frac{4}{9}u^p\bar{d}^p+\frac{1}{9}d^p\bar{u}^p}{\frac{4}{9}u^p\bar{u}^p+\frac{1}{9}d^p\bar{d}^p}\right|_{\text{large } x} \approx \frac{\bar{d}}{\bar{u}}$$

$W^\pm$ ASYMMETRY





CDF (1998)

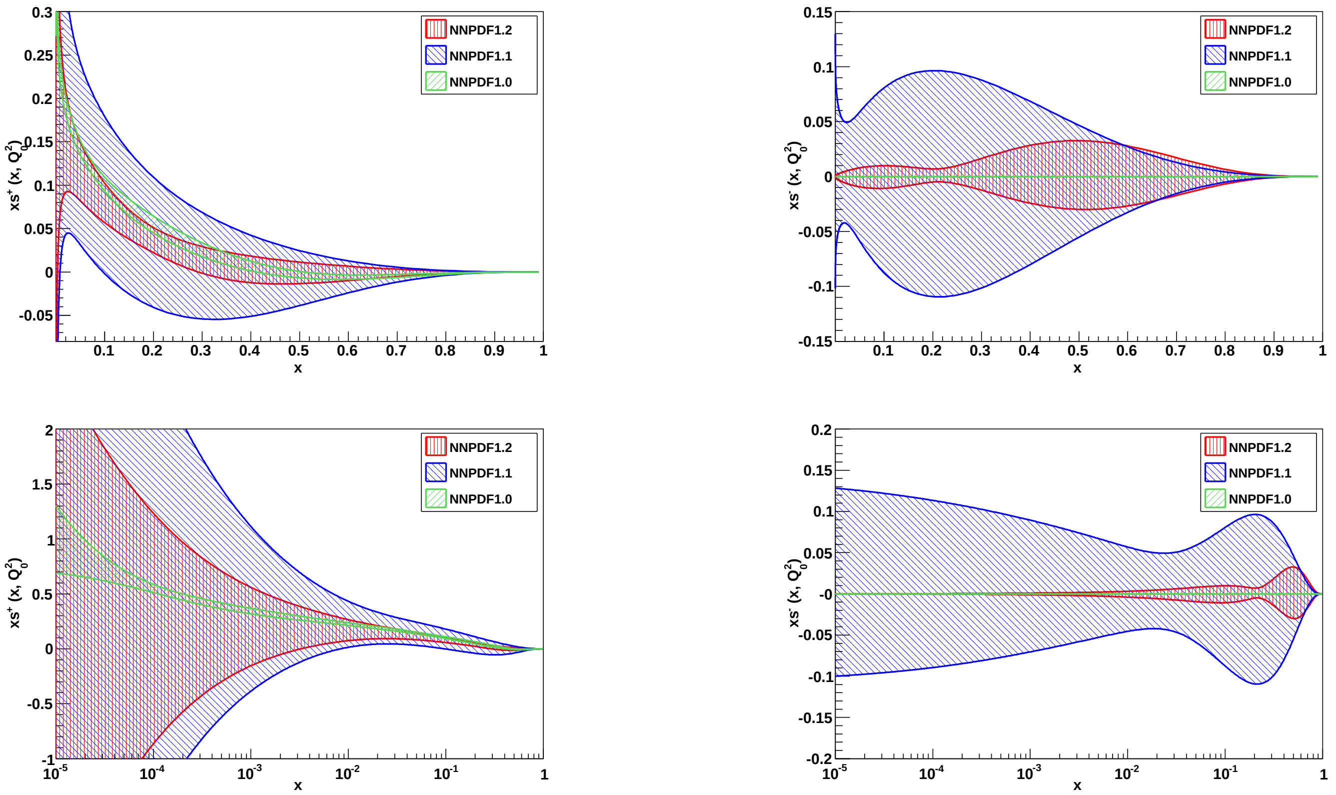
LIGHT QUARK ASYMMETRY

$$\frac{\sigma^{p\bar{p}}_{W^+}}{\sigma^{p\bar{p}}_{W^-}} \sim \frac{u^p\,d^p}{d^p\,u^p} \qquad (q^p=\bar{q}^{\bar{p}})$$

DISENTANGLING STRANGENESS

- STRANGENESS ALMOST UNCONSTRAINED BY INCLUSIVE DIS DATA
  NNPDF1.1: s, $\bar{s}$ (actually $s^{\pm}$) indep. parametrized, no dimuon data
- IN PARTON FITS UP TO 2009 $\to$ STRANGENESS FIXED BY ASSUMPTION
  NNPDF1.0: $s(x, Q_0^2) = \bar{s}(x, Q_0^2)$, $s + \bar{s} = \frac{1}{2}(\bar{u} + \bar{d})$
- IN CURRENT PARTON FITS $\to$ STRANGENESS FIXED BY DIS DIMUON PRODUCTION
  $\nu + s \to c$ & COLLIDER W PRODUCTION
  NNPDF1.2: s, $\bar{s}$ (actually $s^{\pm}$) indep. parametrized, dimuon data

STRANGE PDFS: THE IMPACT OF DIMUON DATA



THE IMPACT OF DRELL-YAN+ W-PROD. DATA(NNPDF2.0)

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ZEUS-H2	1.50	1.49	1.51
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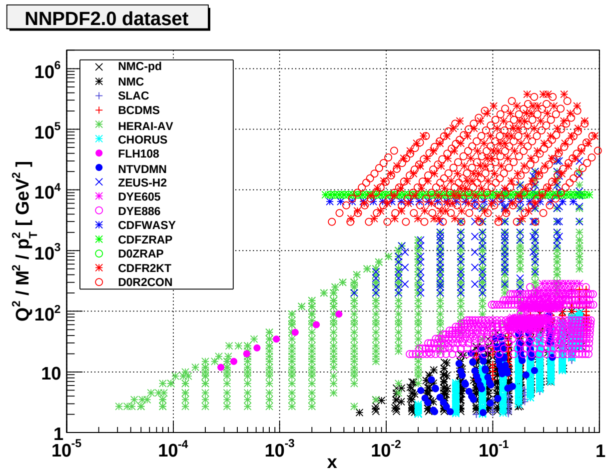
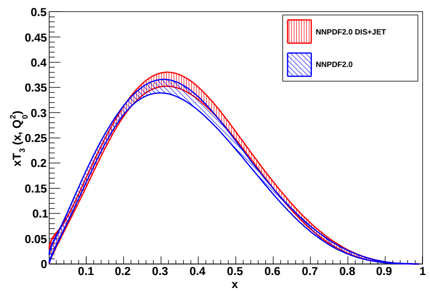
D0R2CON	1.00	0.93	0.93
DYE605	7.32	**10.35**	**0.88**
DYE866	2.24	**2.59**	**1.28**
CDFWASY	13.06	**14.13**	**1.85**
CDFZRAP	3.12	**3.31**	**2.02**
D0ZRAP	0.65	**0.68**	**0.47**

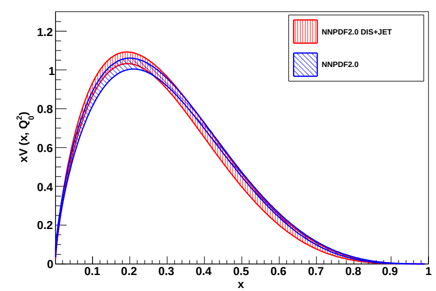


- VERY SUBSTANTIAL IMPROVEMENT IN FIT QUALITY WHEN DATA INCLUDED $\Rightarrow$ SOME PDF COMBINATIONS POORLY DETERMINED WITHOUT THESE DATA
- HUGE IMPROVEMENT IN SEA ASYM $\bar{u} - \bar{d}$ & STRANGENESS $s - \bar{s}$
- SIGNIFICANT IMPROVEMENT IN TOTAL VALENCE $(\sum_i (q_i - \bar{q}_i))$ & ISOTRIPLET $(u + \bar{u} - (d + \bar{d}))$

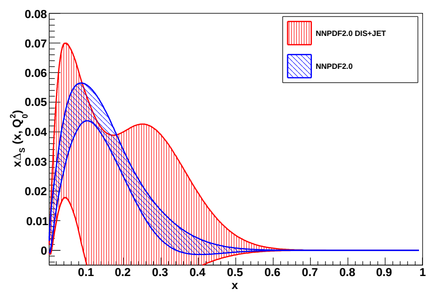
ISOTRIPLET



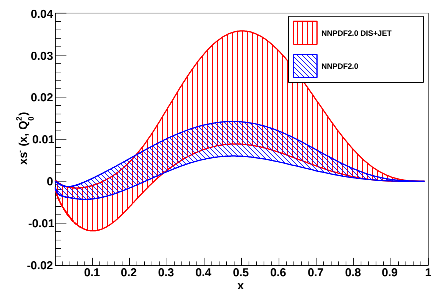
VALENCE



$\bar{d} - \bar{u}$



$s - \bar{s}$

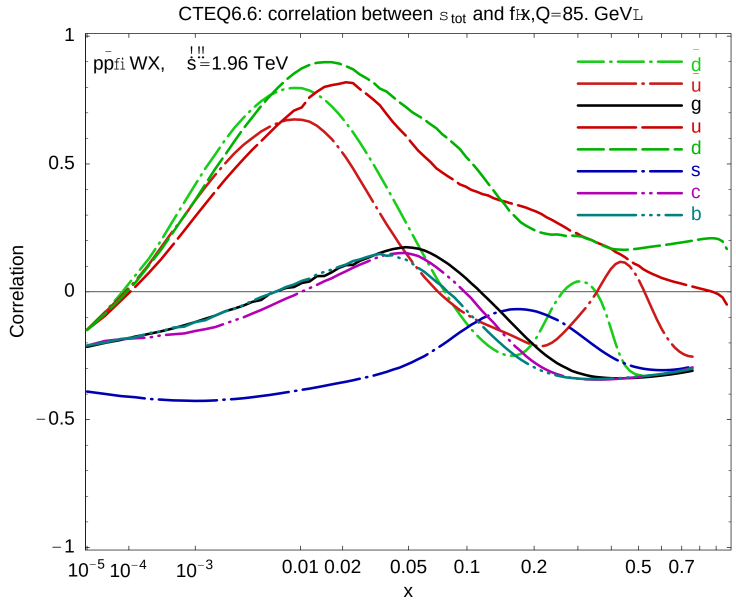


THE IMPACT OF DRELL-YAN+ W-PROD. DATA(CTEQ6.6)

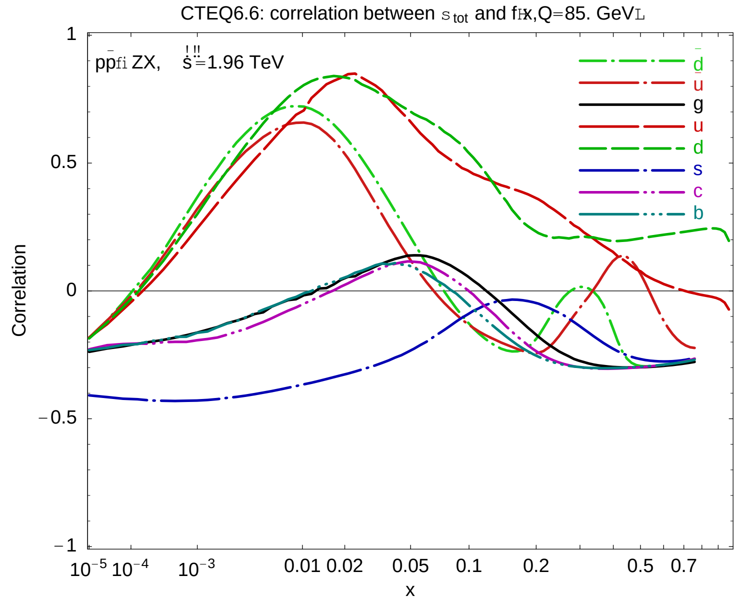
CORRRELATION COEFFICIENT BETWEEN THE CROSS SECTION AND INDIVIIDUAL PDFS

TEVATRON

W

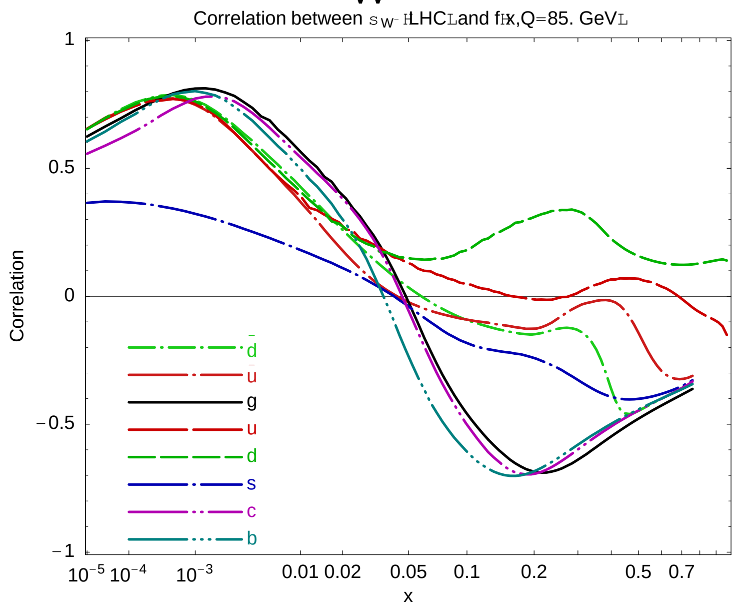


Z

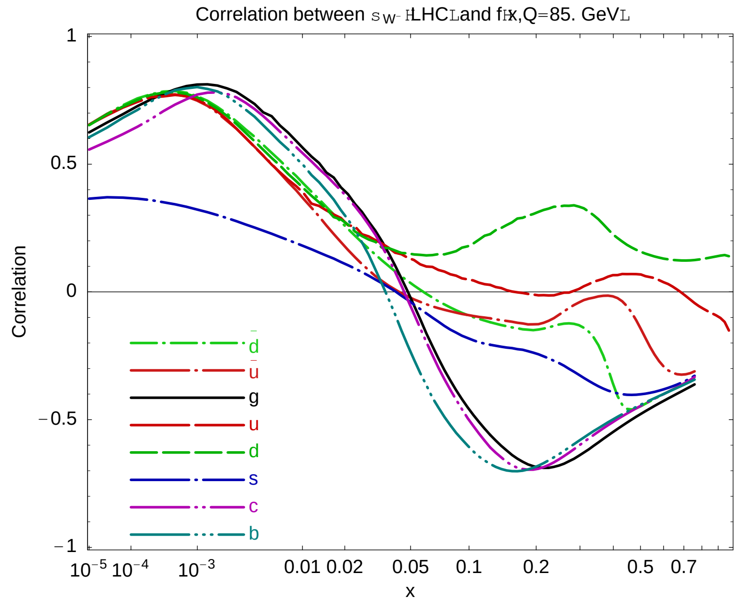


LHC

W



Z



PRESENT

- SMALL x GLUON & SINGLET ⇔ PRECISE HERA DATA
- SMALL x FLAVOUR SEPARATION ⇔ TEVATRON W ASYMMETRY DATA
- MEDIUM x FLAVOUR SEPARATION ⇔ FIXED TARGET p/d DIS DATA, DRELL YAN, NEUTRINO INCLUSIVE
- STRANGENESS ⇔ NEUTRINO DIMUON
- LARGE x GLUON ⇔ TEVATRON JETS

PRESENT

- SMALL x GLUON & SINGLET $\Leftrightarrow$ PRECISE HERA DATA
- SMALL x FLAVOUR SEPARATION $\Leftrightarrow$ TEVATRON W ASYMMETRY DATA
- MEDIUM x FLAVOUR SEPARATION $\Leftrightarrow$ FIXED TARGET p/d DIS DATA, DRELL YAN, NEUTRINO INCLUSIVE
- STRANGENESS $\Leftrightarrow$ NEUTRINO DIMUON
- LARGE x GLUON $\Leftrightarrow$ TEVATRON JETS

FUTURE

- W ASYMMETRY $\Leftrightarrow$ FULL FLAVOUR SEPARATION AT MEDIUM/SMALL x
- HIGH p_T JETS $\Leftrightarrow$ PRECISE GLUON AT INTERMEDIATE x
- HQ PRODUCTION $\Leftrightarrow$ INDIVIDUAL QUARK FLAVOURS & GLUON AT SMALL x
- HIGGS PRODUCTION $\Leftrightarrow$ MEDIUM x GLUON