

CTEQ-MCnet school on
QCD Analysis and Phenomenology
and the Physics and Techniques of Event Generators

LECTURE 4

Introduction to the Parton Model and Perturbative QCD

Fred Olness (SMU)

Lauterbad (Black Forest), Germany

26 July - 4 August 2010

We already studies

DIS

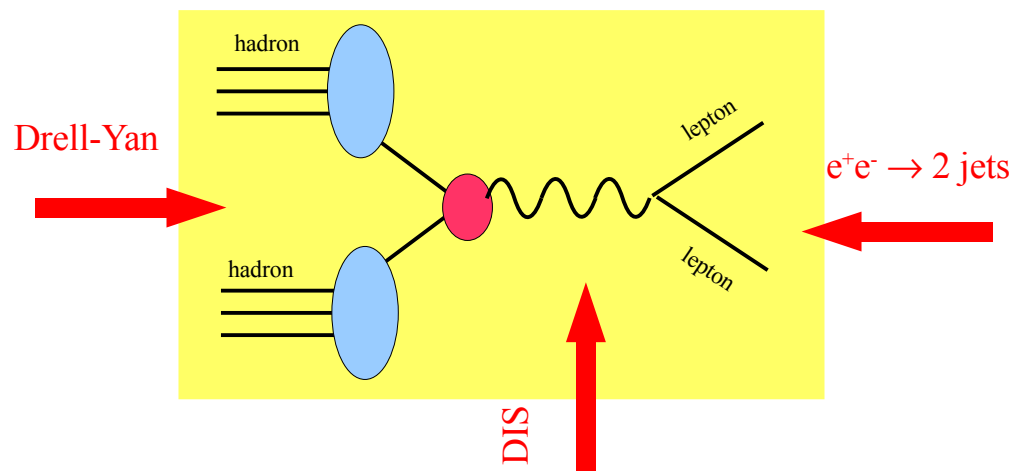
Now we consider

Drell-Yan Process

e^+e^-

Important for Tevatron and LHC

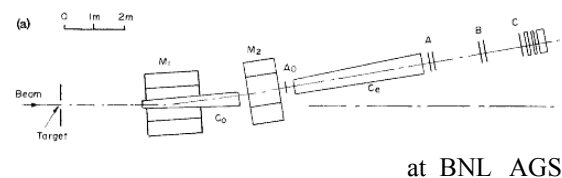
What is the Explanation



Drell-Yan and e^+e^- have an interesting historical relation

A Drell-Yan Example: Discovery of J/Psi

The Process: $p + Be \rightarrow e^+ e^- X$



VOLUME 33, NUMBER 23

PHYSICAL REVIEW LETTERS

2 DECE

Experimental Observation of a Heavy Particle J/ψ

J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen, J. Leong, T. McCorriston, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, and Sau Lan 'Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology Cambridge, Massachusetts 02139

and

Y. Y. Lee

Brookhaven National Laboratory, Upton, New York 11973

(Received 12 November 1974)

We report the observation of a heavy particle J , with mass $m = 3.1$ GeV and width approximately zero. The observation was made from the reaction $p + Be \rightarrow e^+ + e^- + X$ by measuring the e^+e^- mass spectrum with a precise pair spectrometer at the Brookhaven National Laboratory's 30-GeV alternating-gradient synchrotron.

This experiment is part of a large program to daily with a thin Al foil. The beam spot

very narrow width
 \Rightarrow long lifetime

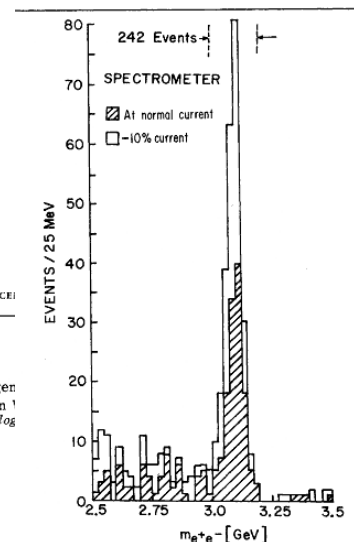


FIG. 2. Mass spectrum showing the existence of J . Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.

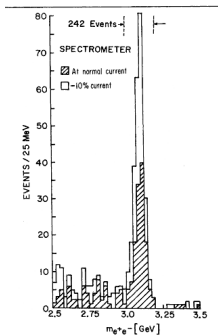
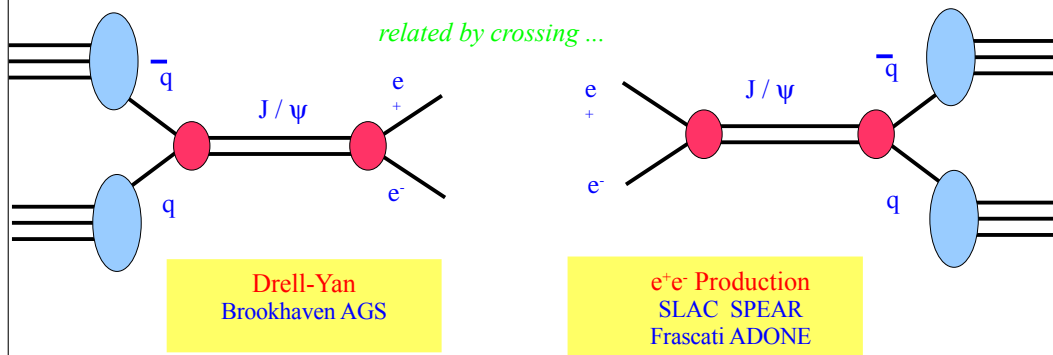
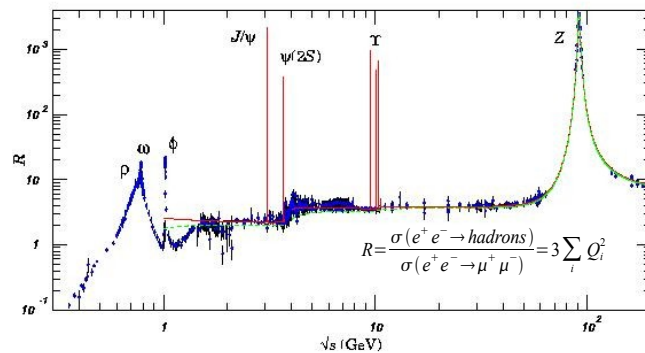


FIG. 2. Mass spectrum showing the existence of J/ψ . Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer current. The run at reduced current was taken two months later than the normal run.



Side Note: From $pp \rightarrow \gamma / Z / W$, we can obtain $pp \rightarrow \gamma / Z / W \rightarrow l^+l^-$

Schematically:

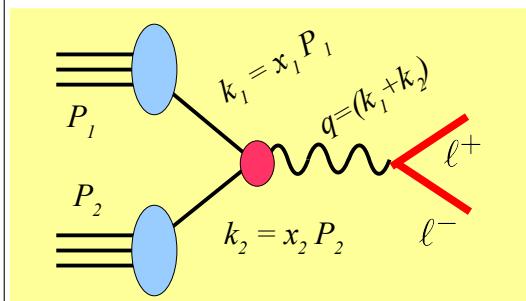
$$d\sigma(q\bar{q} \rightarrow l^+l^-) = d\sigma(q\bar{q} \rightarrow \gamma^*) \times d\sigma(\gamma^* \rightarrow l^+l^-)$$

For example:

$$\frac{d\sigma}{dQ^2 d\hat{t}}(q\bar{q} \rightarrow l^+l^-) = \frac{d\sigma}{d\hat{t}}(q\bar{q} \rightarrow \gamma^*) \times \frac{\alpha}{3\pi Q^2}$$

We'll look at Drell-Yan
Specifically W/Z production

Kinematics in the
hadronic CMS



$$P_1 = \frac{\sqrt{s}}{2} (1, 0, 0, +1) \quad P_1^2 = 0$$

$$P_2 = \frac{\sqrt{s}}{2} (1, 0, 0, -1) \quad P_2^2 = 0$$

$$k_1 = x_1 P_1 \quad k_1^2 = 0$$

$$k_2 = x_2 P_2 \quad k_2^2 = 0$$

$$\frac{d\sigma}{dx_1 dx_2} = \sum_{q, \bar{q}} [q(x_1) \bar{q}(x_2) + q(x_2) \bar{q}(x_1)] \hat{\sigma}$$

Hadronic cross section

Parton distribution functions

Partonic cross section

Trade $\{x_1, x_2\}$ variables for $\{\tau, y\}$

$$x_{1,2} = \sqrt{\tau} e^{\pm y}$$

$$y = \frac{1}{2} \ln \left(\frac{x_1}{x_2} \right)$$

$$\tau = x_1 x_2$$

$$s = (P_1 + P_2)^2 = \frac{\hat{s}}{x_1 x_2} = \frac{\hat{s}}{\tau}$$

Therefore $\tau = x_1 x_2 = \frac{\hat{s}}{s} \equiv \frac{Q^2}{s}$

Fractional energy² between partonic and hadronic system

Using: $dx_1 dx_2 = d\tau dy$

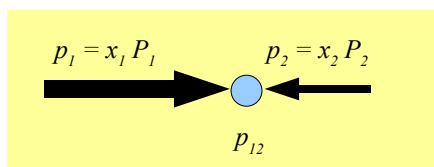
$$\frac{d\sigma}{d\tau dy} = \sum_{q, \bar{q}} [q(x_1) \bar{q}(x_2) + q(x_2) \bar{q}(x_1)] \hat{\sigma}$$

Rapidity & Longitudinal Momentum Distributions

The rapidity is defined as:

$$y = \frac{1}{2} \ln \left(\frac{E_{12} + p_L}{E_{12} - p_L} \right)$$

Partonic CMS has longitudinal momentum w.r.t. the hadron frame



$$p_{12} = (p_1 + p_2) = (E_{12}, 0, 0, p_L)$$

$$E_{12} = \frac{\sqrt{s}}{2} (x_1 + x_2)$$

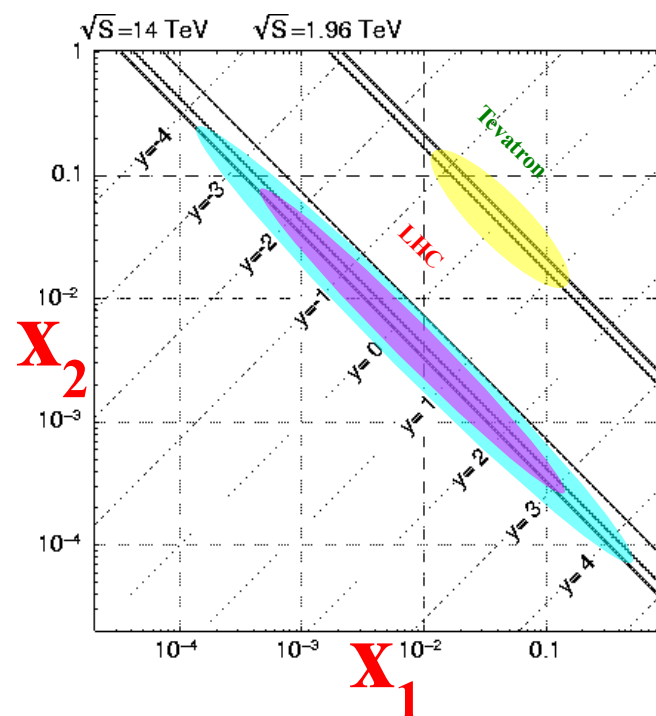
$$p_L = \frac{\sqrt{s}}{2} (x_1 - x_2) \equiv \frac{\sqrt{s}}{2} x_F$$

x_F is a measure of the longitudinal momentum

$$y = \frac{1}{2} \ln \left(\frac{E_{12} + p_L}{E_{12} - p_L} \right) = \frac{1}{2} \ln \left(\frac{x_1}{x_2} \right)$$

Kinematics for W / Z / Higgs Production

12



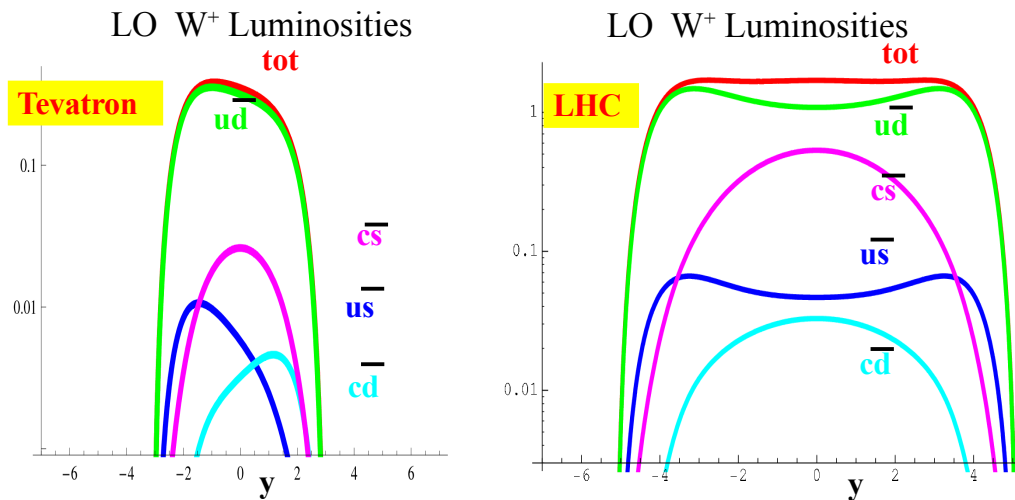
$$y = \frac{1}{2} \ln \left(\frac{x_1}{x_2} \right)$$

$$\tau = x_1 x_2$$

$$x_{1,2} = \sqrt{\tau} e^{\pm y}$$

Z
W

H
Z
W



$$d\sigma = \int dx_1 \int dx_2 \int d\tau \left[q(x_1) \bar{q}(x_2) + q(x_2) \bar{q}(x_1) \right] \hat{\sigma} \delta\left(\tau - \frac{M^2}{S}\right)$$

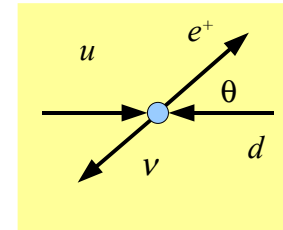
$$\frac{d\sigma}{d\tau} = \frac{dL}{d\tau} \hat{\sigma}(\tau)$$

$$\frac{dL}{d\tau} = f \otimes f$$

The CMS of the parton-parton system is moving longitudinally relative to the hadron-hadron system

How do we measure the W-boson mass?

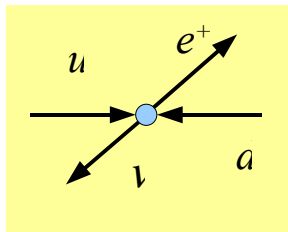
$$u + \bar{d} \rightarrow W^+ \rightarrow e^+ \nu$$



Can't measure W directly
Can't measure ν directly
Can't measure longitudinal momentum

We can measure the P_T of the lepton

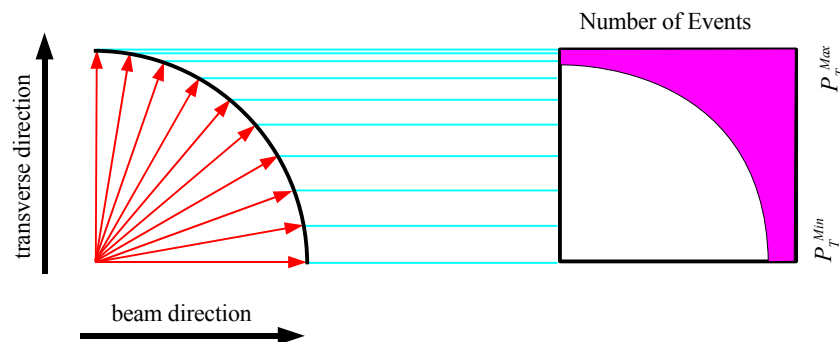
The Jacobian Peak



Suppose lepton distribution is uniform in θ

The dependence is actually $(1+\cos\theta)^2$, but we'll worry about that later

What is the distribution in P_T ?



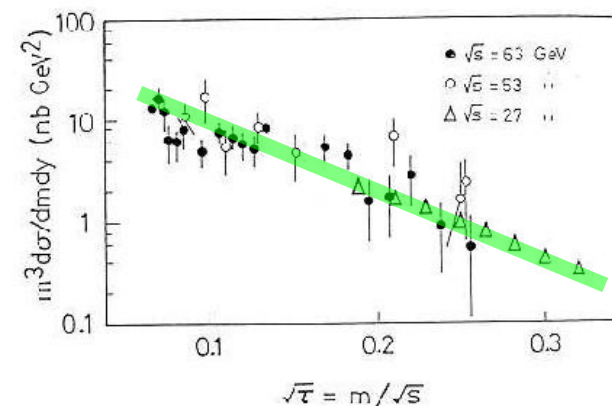
We find a peak at $P_T^{max} \approx M_W/2$

Drell-Yan Cross Section and the Scaling Form

$$\text{Using: } \hat{\sigma}_0 = \frac{4\pi\alpha^2}{9\hat{s}} Q_i^2 \quad \text{and} \quad \delta(Q^2 - \hat{s}) = \frac{1}{sx_1} \delta\left(x_2 - \frac{\tau}{x_1}\right)$$

we can write the cross section in the scaling form:

$$Q^4 \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{9} \sum_{q, \bar{q}} Q_i^2 \int_{\tau}^1 \frac{dx_1}{x_1} \tau \left[q(x_1) \bar{q}(\tau/x_1) + \bar{q}(x_1) q(\tau/x_1) \right]$$



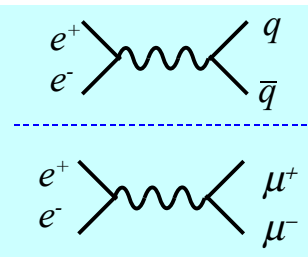
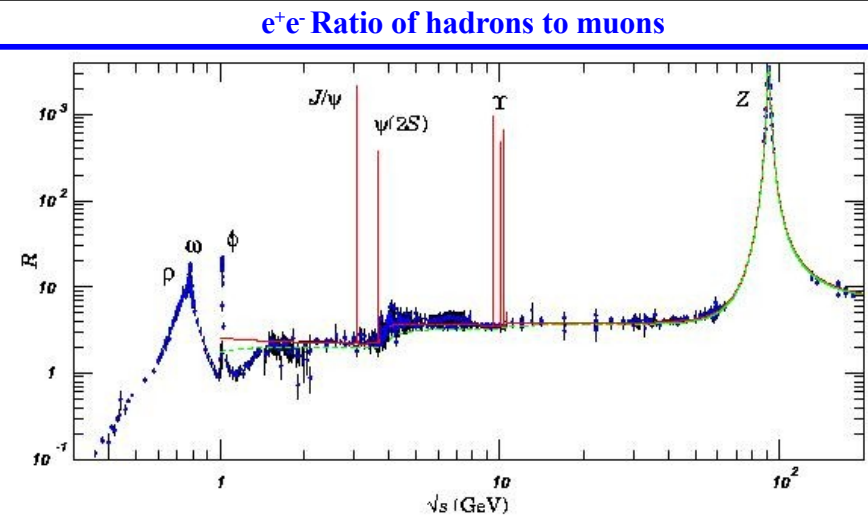
Notice the RHS is a function of only τ , not Q .

This quantity should lie on a universal scaling curve.

Cf., DIS case, & scattering of point-like constituents

e^+e^- R ratio

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$



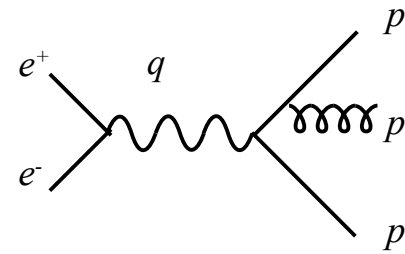
$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3 \sum_i Q_i^2 \left[1 + \frac{\alpha_s}{\pi} \right]$$

3 quark colors

NLO correction

e^+e^-

NLO corrections



Define the energy fractions E_i :

$$x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2p_i \cdot q}{s}$$

Energy Conservation:

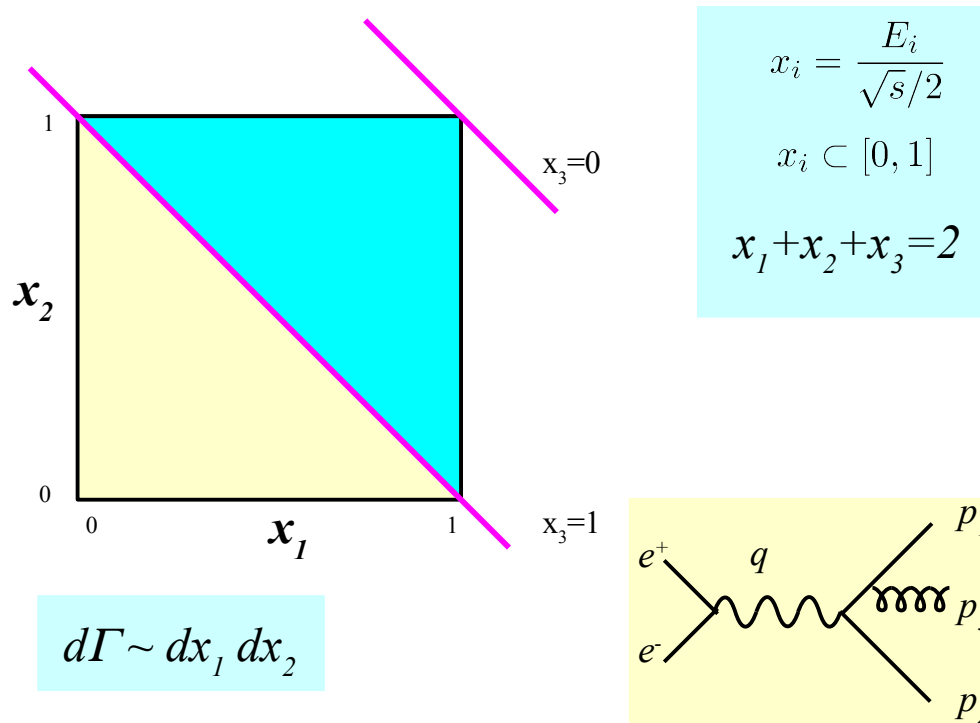
$$\sum_i x_i = 2$$

Range of x :

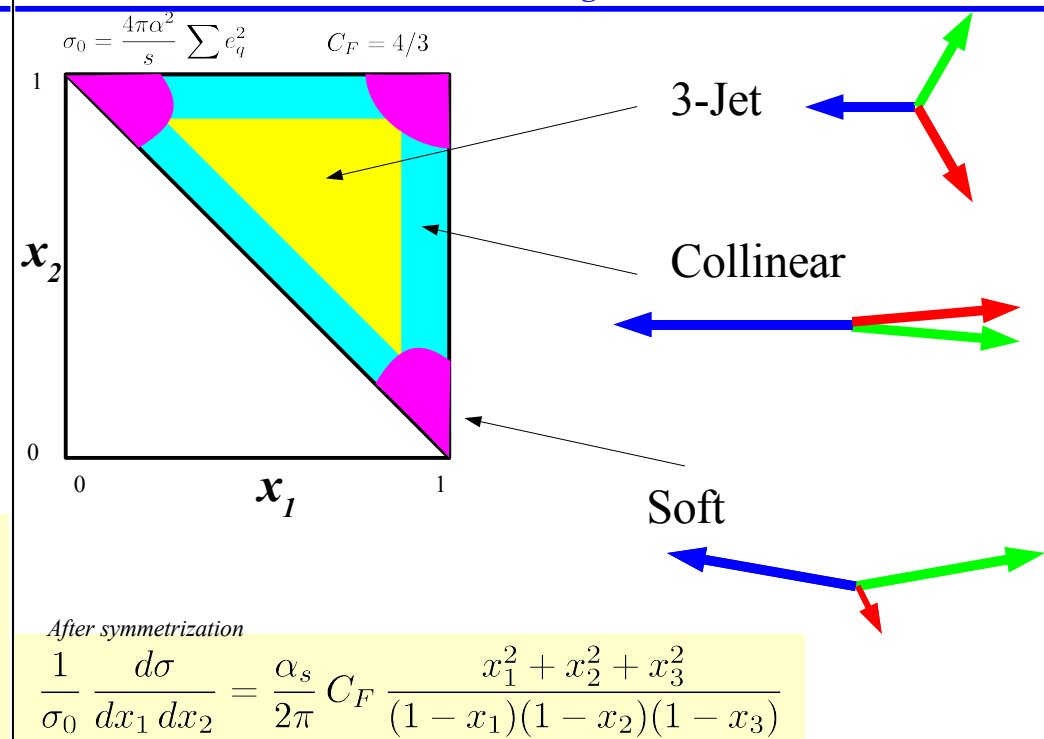
$$x_i \in [0, 1]$$

Exercise: show 3-body phase space is flat in $dx_1 dx_2$

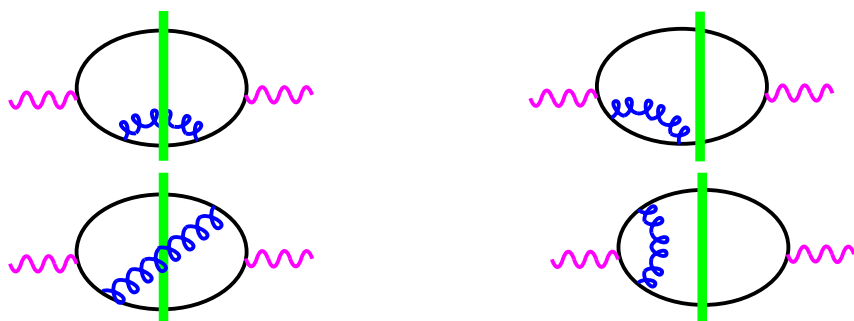
3-Particle Phase Space



3-Particle Configurations



Singularities cancel between 2-particle and 3-particle graphs



$$\sigma_2^{(\epsilon)} = \sigma_0 C_F \frac{\alpha_s}{\pi} (...) \left[+\frac{-1}{\epsilon^2} + \frac{-3}{2\epsilon} + \frac{\pi^2}{2} - 4 \right]$$

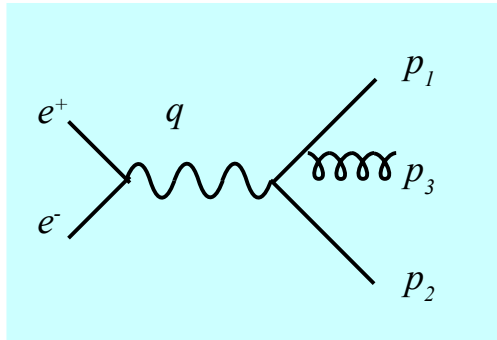
$$\sigma_3^{(\epsilon)} = \sigma_0 C_F \frac{\alpha_s}{\pi} (...) \left[+\frac{+1}{\epsilon^2} + \frac{+3}{2\epsilon} + \frac{-\pi^2}{2} - \frac{19}{4} \right]$$

$$\sigma_2^{(\epsilon)} + \sigma_3^{(\epsilon)} = \sigma_0 C_F \frac{\alpha_s}{\pi} (...) \left[0 + 0 + 0 + -\frac{35}{4} \right]$$

Same result with
gluon mass
regularization

e^+e^-
 Differential
 Cross Sections

Differential Cross Section



What do we do about soft and collinear singularities???

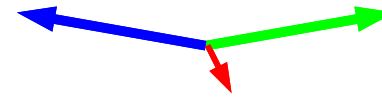
Introduce the concept of “Infrared Safe Observable”

The soft and collinear singularities will cancel
ONLY
if the physical observables are appropriately defined.

Infrared Safe Observables

Observables must satisfy the following requirements:

Soft



if $p_s \rightarrow 0$

$$\mathcal{O}_{n+1}(p_1, \dots, p_n, p_s) \longrightarrow \mathcal{O}_n(p_1, \dots, p_n)$$

Collinear

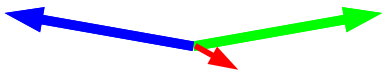


if $p_a \parallel p_b$

$$\mathcal{O}_{n+1}(p_1, \dots, p_a, p_b, \dots, p_n) \longrightarrow \mathcal{O}_n(p_1, \dots, p_a + p_b, \dots, p_n)$$

Infrared Safe Observables

Soft



if $p_s \rightarrow 0$

$$\mathcal{O}_{n+1}(p_1, \dots, p_n, p_s) \longrightarrow \mathcal{O}_n(p_1, \dots, p_n)$$



Collinear



if $p_a \parallel p_b$

$$\mathcal{O}_{n+1}(p_1, \dots, p_a, p_b, \dots, p_n) \longrightarrow \mathcal{O}_n(p_1, \dots, p_a + p_b, \dots, p_n)$$



Examples: Infrared Safe Observables

Infrared Safe Observables:

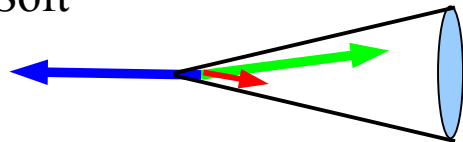
- Event shape distributions
- Jet Cross sections

Un-Safe Infrared Observables:

- Momentum of the hardest particle
 - (affected by collinear splitting)
- 100% isolated particles
 - (affected by soft emissions)
- Particle multiplicity
 - (affected by both soft & collinear emissions)

Infrared Safe Observables: Define Jets

Soft

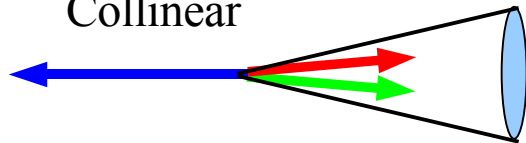


if $p_s \rightarrow 0$

$$\mathcal{O}_{n+1}(p_1, \dots, p_n, p_s) \longrightarrow \mathcal{O}_n(p_1, \dots, p_n)$$



Collinear



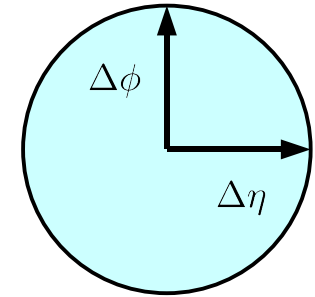
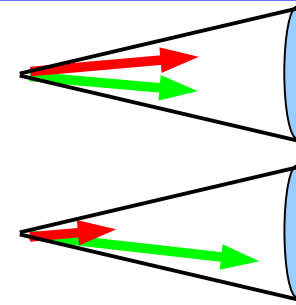
if $p_a \parallel p_b$

$$\mathcal{O}_{n+1}(p_1, \dots, p_a, p_b, \dots, p_n) \longrightarrow \mathcal{O}_n(p_1, \dots, p_a + p_b, \dots, p_n)$$



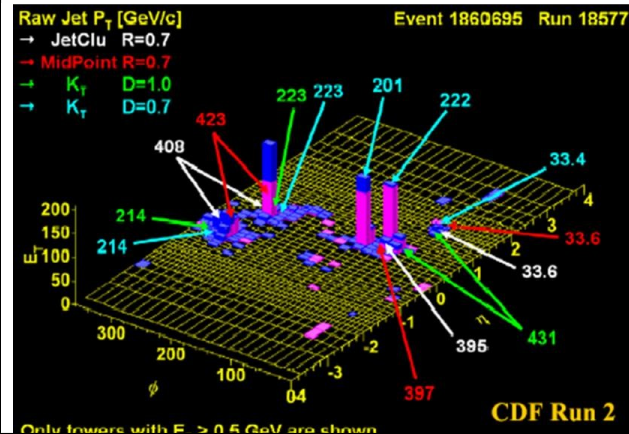
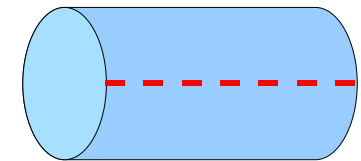
Infrared Safe Observables: Define Jets

Jet Cone



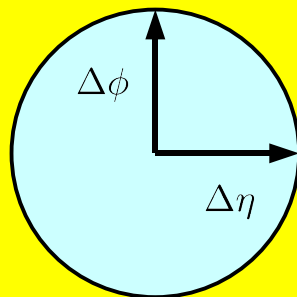
$$R^2 = (\Delta\eta)^2 + (\Delta\phi)^2$$

Let's examine this definition a bit more closely



Pseudo-Rapidity vs. Angle

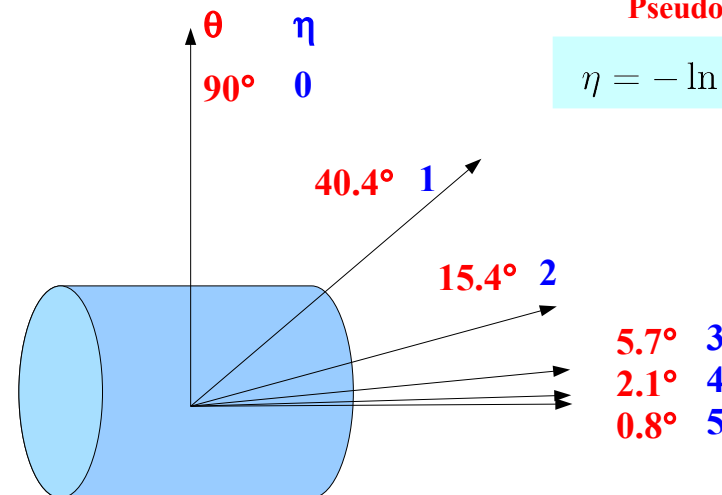
Jet Cone

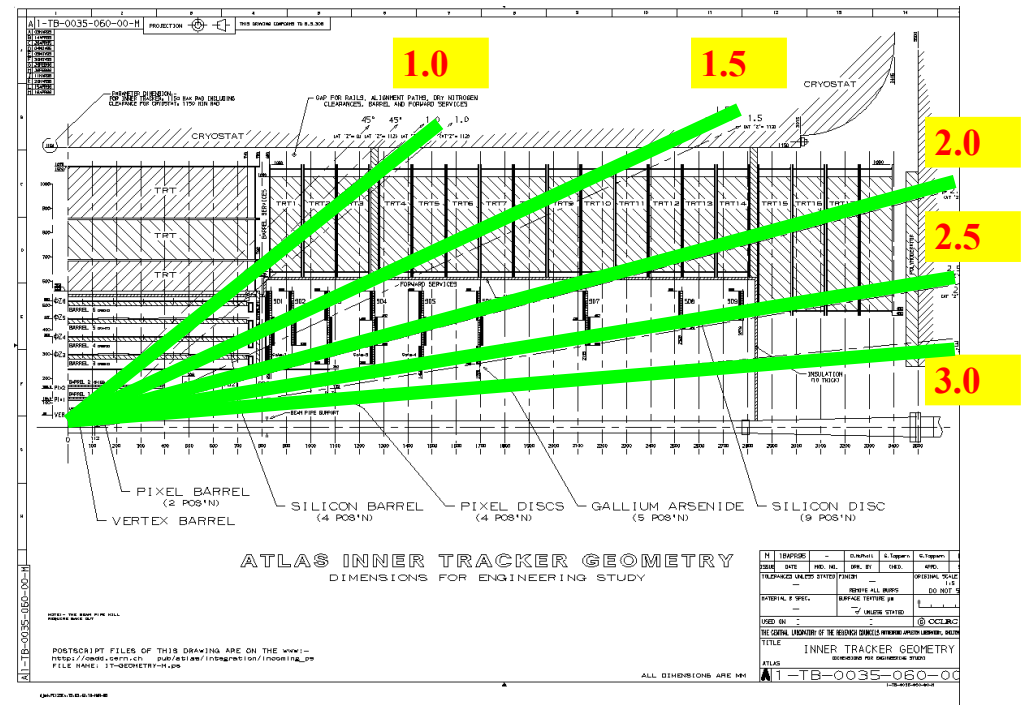
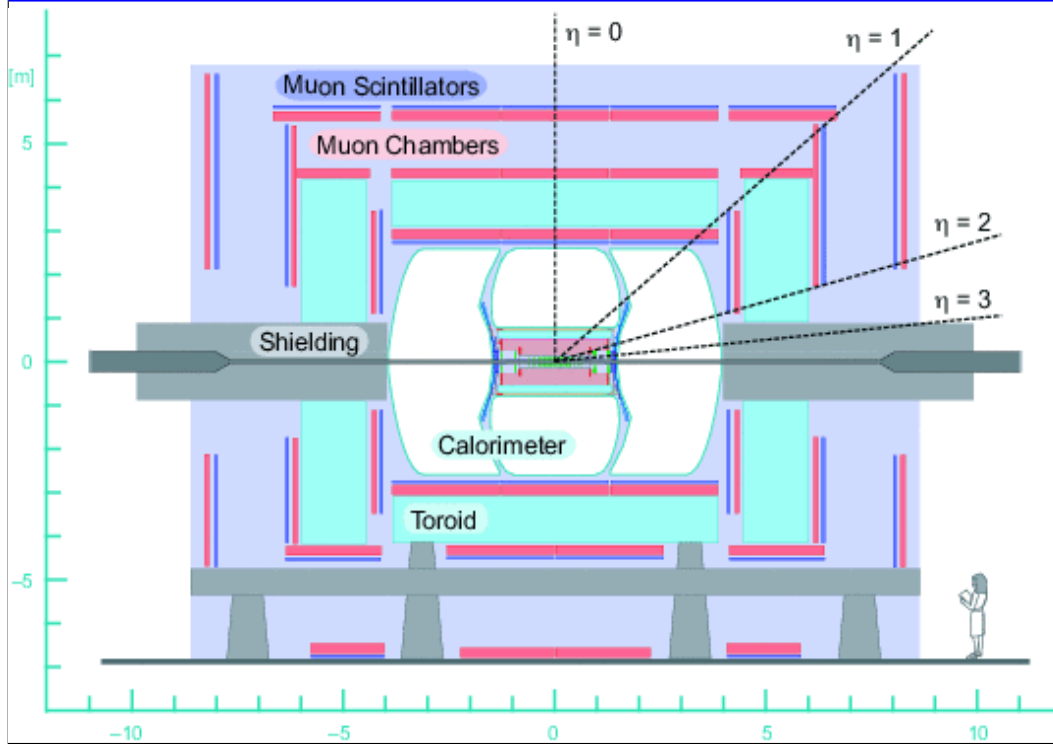


$$R^2 = (\Delta\eta)^2 + (\Delta\phi)^2$$

Pseudo-Rapidity

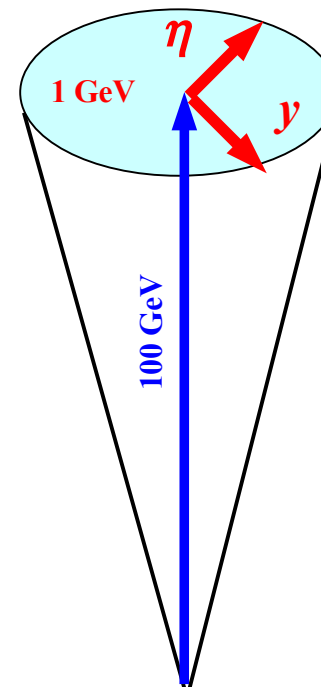
$$\eta = -\ln[\tan(\theta/2)]$$





homework

HOMEWORK: Jet Cone Definition



PROBLEM #2: In a Tevatron detector, consider two particles traveling in the transverse direction:

$$p_1^\mu = \{E, 100, 0, 1\}$$

$$p_2^\mu = \{E, 100, 1, 0\}$$

where the components are expressed in GeV units. E is defined such that the particles are massless.

- Compute E .
- For each particle, compute the pseudorapidity η and azimuthal angle ϕ .
- Explain how the above exercise justifies the correct jet radius definition to be:

$$R = \sqrt{\eta^2 + \phi^2}$$

In particular, why is the above correct and $R = \sqrt{\eta^2 + 2\phi^2}$, for example, incorrect.

HOMEWORK: Light-Cone Coordinates & Boosts

$$P_\mu = \{P_t, P_x, P_y, P_z\}$$

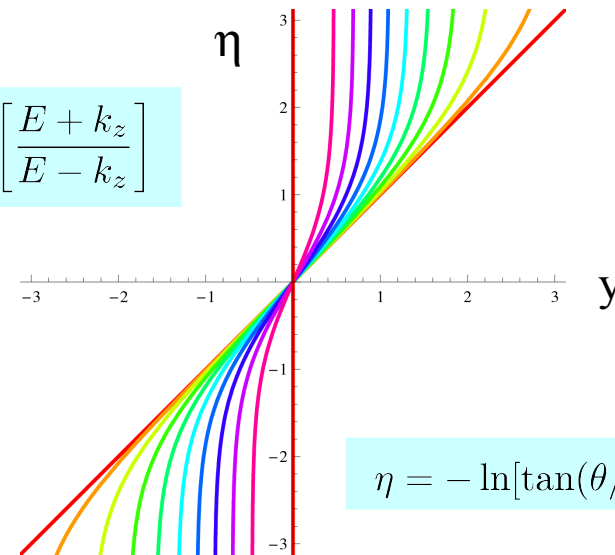
$$P_\mu = \{P_+, \vec{P}_\perp, P_-\} \quad \vec{P}_\perp = \{P_x, P_y\}$$

$$P_\pm = \frac{1}{\sqrt{2}} (P_t \pm P_z)$$

- 1) Compute the metric $g_{\mu\nu}$ in the light-cone frame, and compute $\vec{P}_1 \cdot \vec{P}_2$ in terms of the light-cone components.
- 2) Compute the boost matrix B for a boost along the z -axis, and show the light-cone vector transforms in a particularly simple manner.
- 3) Show that a boost along the z -axis uniformly shifts the rapidity of a vector by a constant amount.

Rapidity vs. Pseudo-Rapidity

$$y = \frac{1}{2} \ln \left[\frac{E + k_z}{E - k_z} \right]$$



$$\frac{M}{E} = \{0, 0.1, 0.2, \dots\}$$

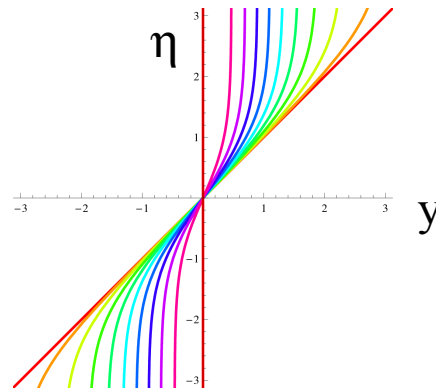
HOMEWORK: Rapidity vs. Pseudo-Rapidity

PROBLEM #1: Consider the rapidity y and the pseudo-rapidity η :

$$y = \frac{1}{2} \ln \left(\frac{E + P_T}{E - P_T} \right)$$

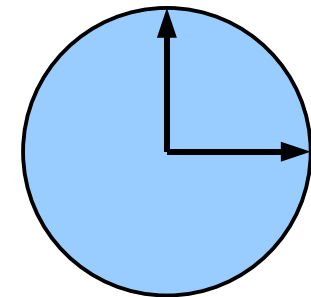
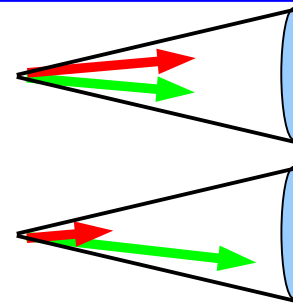
$$\eta = -\ln \left[\tan \left(\frac{\theta}{2} \right) \right]$$

- a) Make a parametric plot of $\{y, \eta\}$ as a function of m/E where m is the mass of the particle.
- b) Show that in the limit $m \rightarrow 0$ that $y \rightarrow \eta$.
- c) Make a table of η for $\theta = [0^\circ, 180^\circ]$ in steps of 5 degrees.
- d) Make a table of θ for $\eta = [0, 10]$ i



Infrared Safe Observables: Define Jets

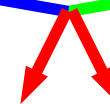
Jet Cone



Problem:
The cone definition is simple,
BUT
it is too simple

$$R^2 = (\Delta\eta)^2 + (\Delta\phi)^2$$

Such configurations can be mis-identified as a 3-jet event

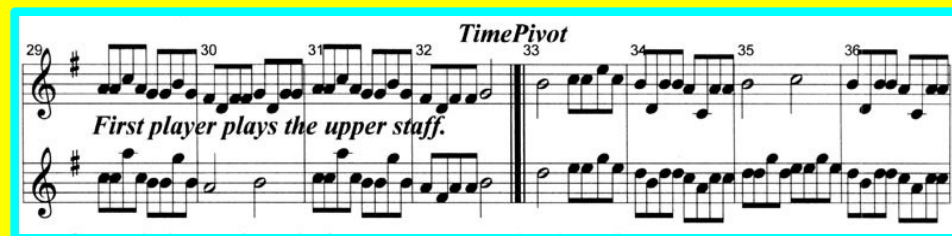


See talk by
Ken Hatakeyama
(Jets)

- Drell-Yan: Tremendous discovery potential
 - Need to compute 2 initial hadrons
- e^+e^- processes:
 - Total Cross Section:
 - Differential Cross Section: singularities
- Infrared Safe Observables
 - Stable under soft and collinear emissions
- Jet definition
 - Cone definition is simple:
 - ... it is TOO simple

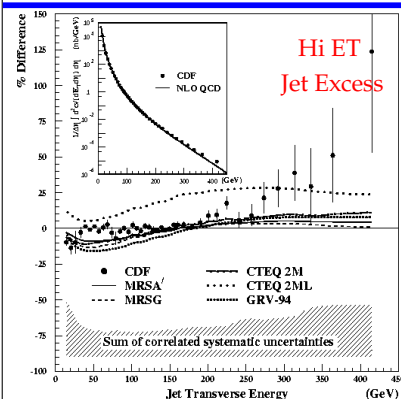
Final Thoughts

$$\begin{aligned}\mathcal{L}_{\text{QCD}} &= \bar{\psi}_i (i\gamma^\mu (D_\mu)_{ij} - m \delta_{ij}) \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} \\ &= \bar{\psi}_i (i\gamma^\mu \partial_\mu - m) \psi_i - g G_\mu^a \bar{\psi}_i \gamma^\mu T_{ij}^a \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}\end{aligned}$$

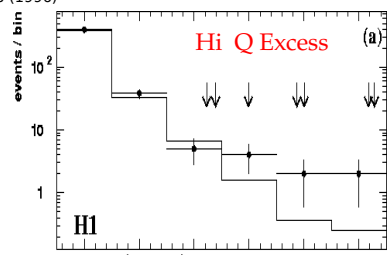


Scaling, Dimensional Analysis, Factorization, Regularization & Renormalization, Infrared Safety ...

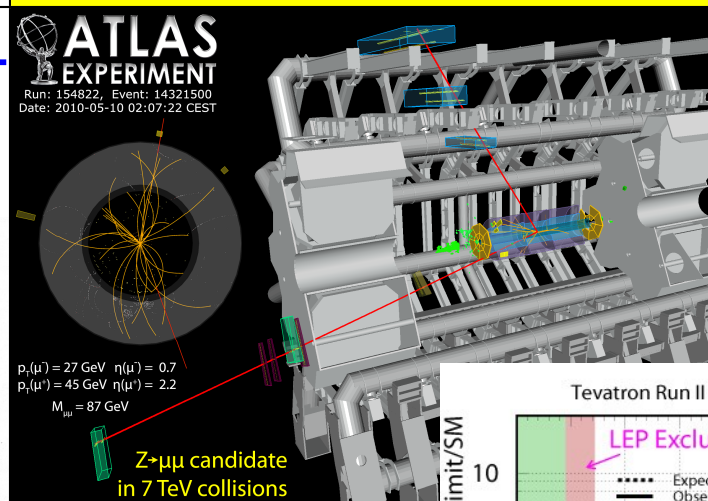
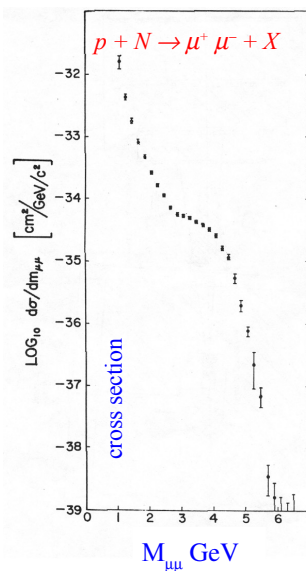
Can you find the Nobel Prize???



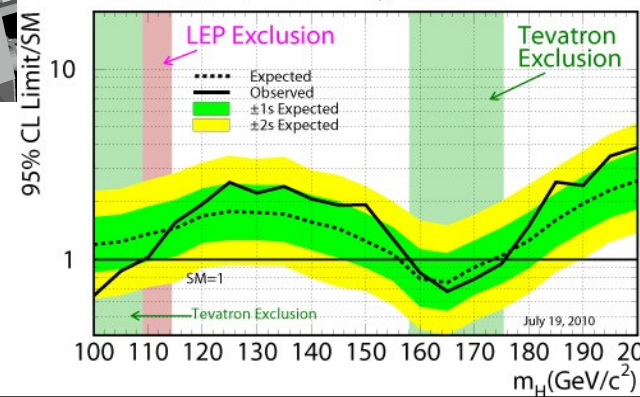
CDF Collaboration, PRL 77, 438 (1996)



H1 Collaboration, ZPC74, 191 (1997) Q_0^2 (GeV²)
ZEUS Collaboration, ZPC74, 207 (1997)



Tevatron Run II Preliminary, $\langle L \rangle = 5.9 \text{ fb}^{-1}$



Thanks to:

Dave Soper, George Sterman, Steve Ellis
for ideas borrowed from previous CTEQ
introductory lecturers

Thanks to Randy Scalise for the help on the
Dimensional Regularization.

Thanks to my friends at Grenoble who
helped with suggestions and corrections.

Thanks to Jeff Owens for help on Drell-Yan
and Resummation.

To the CTEQ and MCnet folks for making
all this possible.



END OF LECTURE 4