Photons
in
Large Momentum Transfer Processes

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Outline

1. Introduction
    - Why photons?
    - Motivation for their study
2. Theory overview and comparison to selected data
    - Direct Photon Production
    - Jet Photoproduction
3. Theoretical improvements
    - Resummation techniques
    - Some applications
4. Conclusions
5. Appendix: Notation, kinematics, and other useful information

Why Photons?

- Well understood electromagnetic interaction
- Well defined probe of strong interaction dynamics
- Classic examples
  - Deep inelastic scattering
  - Lepton pair production
  - $e^+e^- \to$ hadrons
- Direct photons, photoproduction, and two photon processes continue this history

Reasons to study photon production

- Gluon PDF is rather indirectly constrained
  - Momentum sum rule
  - Q^2 dependence of PDFs in DIS via DGLAP equations
  - Jet production at colliders via $gg \to gg$ and $gq \to gq$, but $qq \to qq$ dominates at high values of p_T
- QCD Compton process $gq \to \gamma q$ appears at lowest order only with $q\overline{q} \to \gamma g$ so looks to provide an ideal way to constrain the gluon PDF
- Intrinsic interest in seeing if QCD properly describes the photon production mechanisms - offers a new way of looking at QCD dynamics
- Photons are essential for certain search strategies at the LHC, *e.g.*, $H \to 2\gamma$, so photon production must be understood in order to control the backgrounds for such searches

Direct Photon Data

- Initial measurements were done at fixed target energies: $\sqrt{s} = 20 - 40$ GeV
  - With p_T in the 3-10 GeV range this extended the measurements to high values of $x_T = 2p_T/\sqrt{s}$ which is a measure of the parton x being probed in the process
  - Ideal for constraining the gluon PDF, but problems in describing the data surfaced
- Data became available from colliders
  - pp at the ISR with $\sqrt{s} = 44 - 62$ GeV
  - $\overline{p}p$ at CERN and Fermilab with $\sqrt{s} = 540 - 1960$ GeV
  - Now also have pp at RHIC with $\sqrt{s} = 200$ GeV
- Wide kinematic range covered

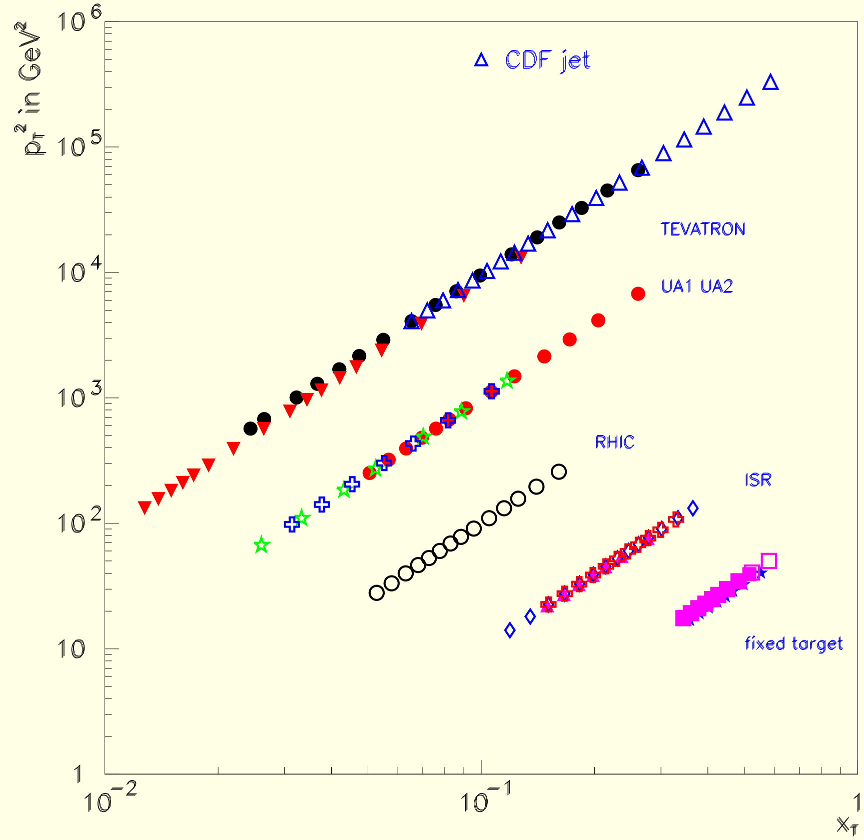
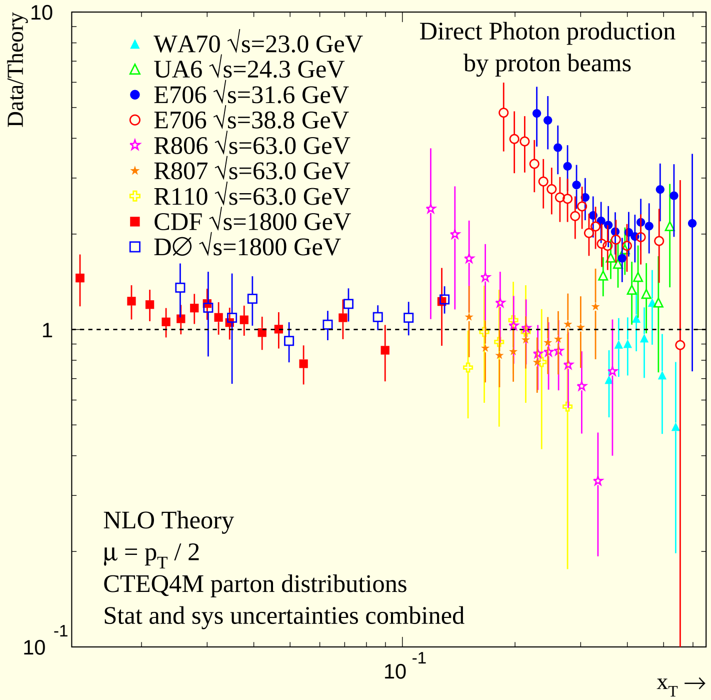


Figure compares kinematic coverage for various direct photon experiments to that for jets at the Tevatron (Aurenche et al, hep-ph/0602133)

- Data Sources
  - Data review by W. Vogelsang and M.R. Whalley, J. Phys. **G23**, Suppl. 7A, A1-A69 (1997)
  - Online database at http://durpdg.dur.ac.uk/HEPDATA
- Usual direct photon talk emphasizes problems describing p_T distributions



- Scatter in data/theory plot perhaps suggests that there is something wrong with direct photon theory!

But direct photon theory *is* QCD ...

- My goal is to examine both the theory and the data to see what works and where improvements are needed

Key predictions of QCD concerning direct photon production include

- Event structure

    1. Where are the photons produced?
    2. How are they correlated with jets?

- Energy dependence of the cross section

- Absolute cross section

Key element underlying each of these is the set of parton-level subprocesses involved.
Can one measure the properties of the subprocesses?

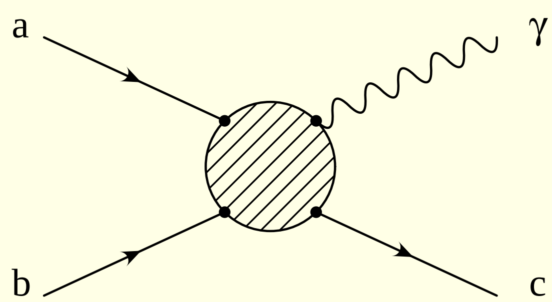
Yes!

Plan

- Review basic theoretical ingredients
- Compare theory and data using observables which directly test the underlying production mechanisms
  - γ-jet mass distribution
  - γ-jet angular distributions
  - Event structure - correlations with produced hadrons
- Compare/contrast with jet photoproduction - which uses the same subprocesses
- Finally - examine the predictions for the γ p_T, rapidity, and center-of-mass energy dependence and compare to other examples of single particle inclusive production.

Theory Overview

- Lowest Order: $\mathcal{O}(\alpha\alpha_s)$
  1. $qg \to \gamma g$ QCD Compton
  2. $q\overline{q} \to \gamma g$ annihilation
- The single photon invariant cross section is given by a convolution with the beam and target parton distribution functions



$$
\begin{aligned}
d\sigma(AB \to \gamma + X) \quad &= \quad G_{a/A}(x_a, \mu_F)\, dx_a\, G_{b/B}(x_b, \mu_F)\, dx_b \\
&\quad\quad \frac{1}{2\hat{s}} \overline{\sum_{ab}} |M(ab \to \gamma c)|^2 d^2PS
\end{aligned}
$$

- d^2PS denotes two-body phase space and μ_F is the factorization scale

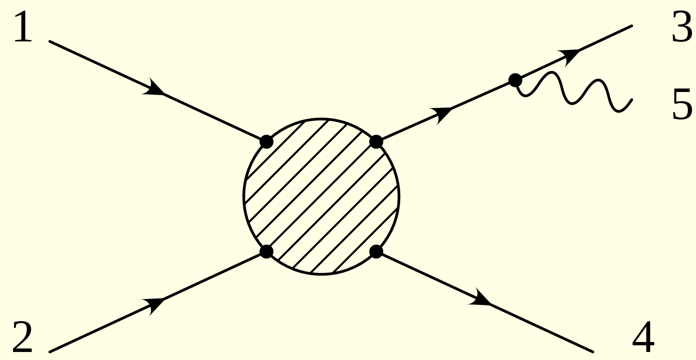
- See the appendix for more details about variables and four-vectors
- Also see the Handbook of Perturbative QCD on the CTEQ web site http://www.cteq.org. The appendix has additional information on how to calculate cross sections for hadronic processes starting at the parton level.

Next-to-Leading Order: $\mathcal{O}(\alpha\alpha_s^2)$

1. one-loop virtual contributions
2. $q\overline{q} \to \gamma g g$
3. $gq \to \gamma q g$
4. $qq' \to \gamma q q'$ plus related subprocesses

- In the next order one sees a new configuration wherein the photon is no longer isolated. Instead, it may be radiated off a high-p_T quark produced in the hard scattering process.

- Consider the subprocess $q(1)q(2) \to q(3)q(4)\gamma(5)$
- Examine the region where $s_{35} = (p_3 - p_5)^2 \approx 0$



$$\overline{\sum}|M(qq \to qq\gamma)|^2 \approx \frac{\alpha}{2\pi}\, P_{\gamma q}(z)\, \frac{1}{s_{35}} \overline{\sum}|M(qq \to qq)|^2$$

- An internal quark line is going on-shell signalling long distance physics effects
- Gives rise to a collinear singularity
- Can factorize the singularity by introducing a *photon fragmentation function*

Photon Fragmentation

- Photon is accompanied by jet fragments on the *same* side
- Factorize the singularity and include it in the bare photon fragmentation function
- Sum large logs with modified Altarelli-Parisi equations

$$
\begin{aligned}
Q^2 \frac{dD_{\gamma/q}(x,Q^2)}{dQ^2} &= \frac{\alpha}{2\pi} P_{\gamma q} + \frac{\alpha_s}{2\pi} \left[D_{\gamma/q} \otimes P_{qq} + D_{\gamma/g} \otimes P_{gq} \right] \\
Q^2 \frac{dD_{\gamma/g}(x,Q^2)}{dQ^2} &= \frac{\alpha_s}{2\pi} \left[\sum_q D_{\gamma/q} \otimes P_{qg} + D_{\gamma/g} \otimes P_{gg} \right]
\end{aligned}
$$

- As with hadron pdfs and fragmentation functions, can't perturbatively calculate the fragmentation functions, but the scale dependence is perturbatively calculable
- Note the $P_{\gamma q}$ splitting function - represents the pointlike coupling of the photon to the quark in $q \to \gamma q$

Fragmentation Component

- The situation has become more complex
- Expect to see two classes of events
    1. Direct (or pointlike) - no hadrons accompanying the photon
    2. Fragmentation (or bremsstrahlung) - photon is a fragment of a high-p_T jet. Part of the fragmentation function is perturbatively calculable.
- Expect (1) to dominate at high-p_T since the energy is not shared with accompanying hadrons.
- The $P_{\gamma q}$ splitting function gives rise to the leading high Q^2 behavior going as $\alpha \log(Q^2/\Lambda^2) \sim \frac{\alpha}{\alpha_s}$ (see the Appendix for a derivation)

So, to our list of contributions add those involving photon fragmentation functions

- $\mathcal{O}(\alpha\alpha_s) : \frac{d\sigma}{d\hat{t}}(ab \to cd) \otimes D_{\gamma/c}$
- $\mathcal{O}(\alpha\alpha_s^2) : \frac{d\sigma}{d\hat{t}}(ab \to cde) \otimes D_{\gamma/c}$

Some Comments

- Photons can be produced as fragments of jets, as is also the case for particles
- Photon production therefore involves all of the subprocesses relevant for jet or particle production
- In addition, one also has the pointlike production processes

Photon production is *more* complicated than jet production, not *less*

Next-to-leading-order Calculations

- Have to integrate over unobserved partons. There are regions of phase space where partons can become parallel to each other (collinear) or soft. Both regions are singular.
- Usually use dimensional regularization to regulate the divergences

Two types of programs exist

1. Phase space integrations done symbolically so expressions for the integrated parton-level subprocess cross sections exist. Integrations over the parton momentum fractions x_a, x_b, and z_c done numerically. This approach is suitable for the single photon inclusive cross section.
2. All integrations done via Monte Carlo
    - Phase space slicing method
    - Subtraction method

With Monte Carlo programs one can examine correlations between the photon and other partons in the final state.

Short summary of two-body kinematics

The following relations are useful when trying to understand the regions of the parton variables which are important for specific observables. More information is available in the appendix.

Consider a photon and a jet produced with approximately balancing p_T and (pseudo)rapidities η_γ and η_{jet} in the hadron-hadron center-of-mass system.

γ-jet invariant mass M: $M^2 = x_a x_b s$ γ-jet rapidity Y: $Y = \frac{1}{2}\ln\frac{x_a}{x_b}$

Scattering angle in the γ-jet rest frame: $\cos\theta^* = \tanh\left(\frac{\eta_\gamma - \eta_{jet}}{2}\right)$

Parton momentum fractions

$$
\begin{aligned}
x_a &= x_T(e^{\eta_\gamma} + e^{\eta_{jet}})/2 = Me^{Y}/\sqrt{s} \\
x_b &= x_T(e^{-\eta_\gamma} + e^{-\eta_{jet}})/2 = Me^{-Y}/\sqrt{s}
\end{aligned}
$$

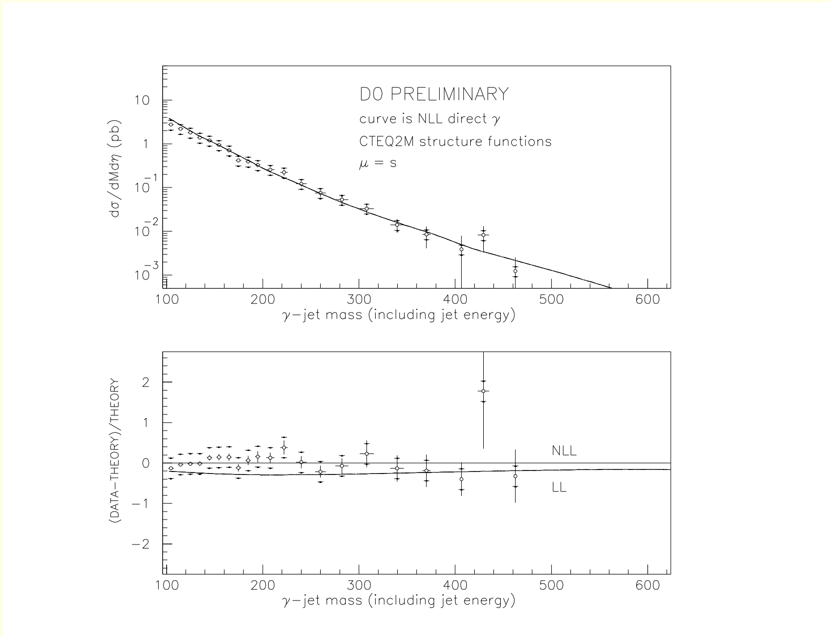
Note: for $Y \approx 0$ one is sensitive to $x_a \approx x_b \approx M/\sqrt{s}$. For $\eta_\gamma \approx \eta_{jet} \approx 0$ one has $x_a \approx x_b \approx x_T$, but this is only a guide, since the cross sections will involve some integrations.

Comparison to Data

- Want to first examine data which yield information on the underlying parton subprocesses
- Need γ-jet observables
  - γ-jet invariant mass
  - γ-jet angular distributions
- Start with Tevatron data
  - Higher energies enable jet identification and reconstruction as compared to fixed target data
  - Essentially the only direct photon data with identified jets

γ-jet invariant mass distribution

- Preliminary data from DØ Run I



- Comparison suggests that the photons are being produced at the expected level, but doesn't, by itself, have much to say about the subprocesses
- Note that the relevant region of x in the pdfs is $x \approx M/\sqrt{s}$

γ-jet angular distributions

- QCD Compton and annihilation subprocess both behave as

$$\frac{d\sigma}{d\hat{t}} \sim (1 - \cos(\theta^*))^{-1} \text{ as } \cos(\theta^*) \to 1$$

- Other parton-parton scattering subprocesses ($qq \to qq, qg \to qg, gg \to gg$, etc.) behave as

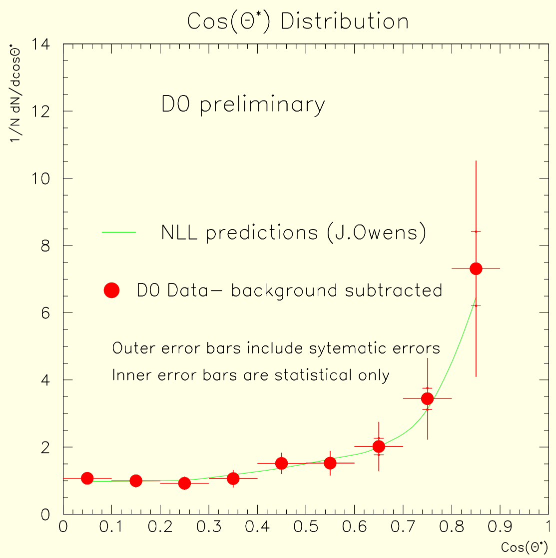
$$(1 - \cos(\theta^*))^{-2}$$

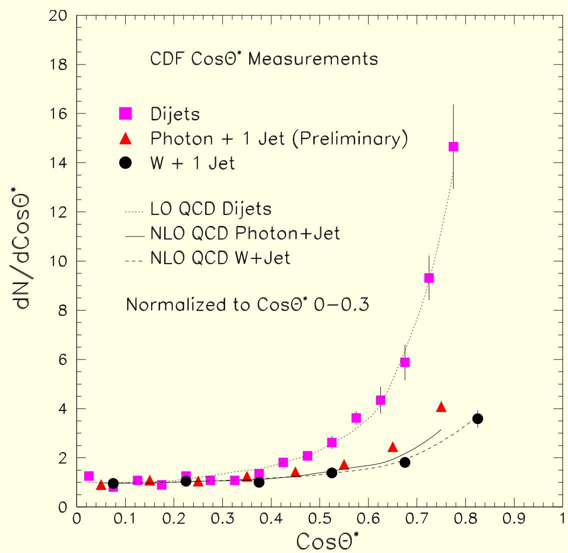
- This means that the γ-jet angular distribution should be flatter than that observed in jet-jet final states.

- See the appendix for a derivation of these relations

Direct Measurement of the γ-jet angular distribution

- Measuring both η_γ and η_{jet} allows one to reconstruct $\cos\theta^* = \tanh\left(\frac{\eta_\gamma - \eta_{jet}}{2}\right)$
- Both DØ and CDF have measured the γ-jet angular distribution

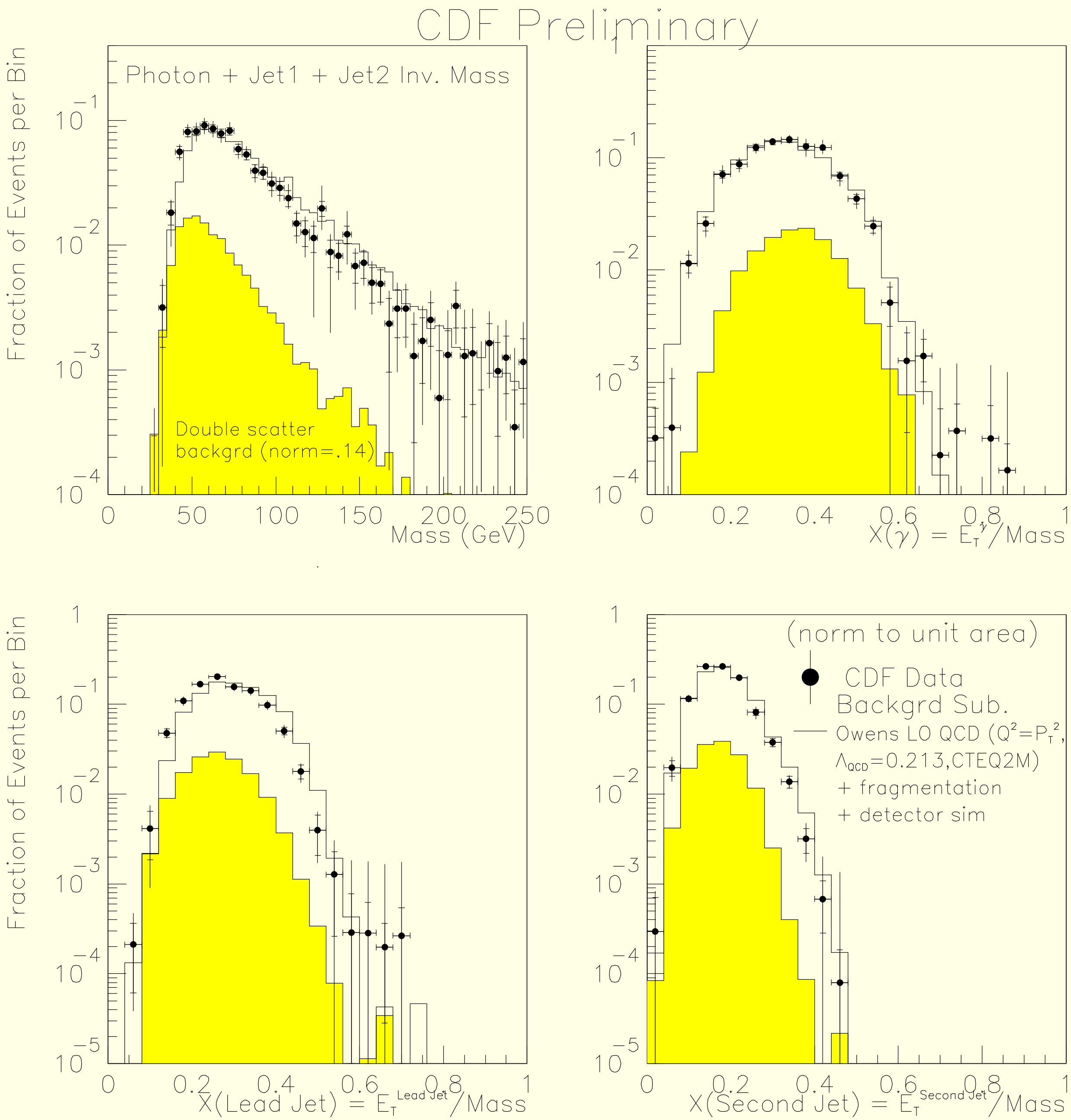




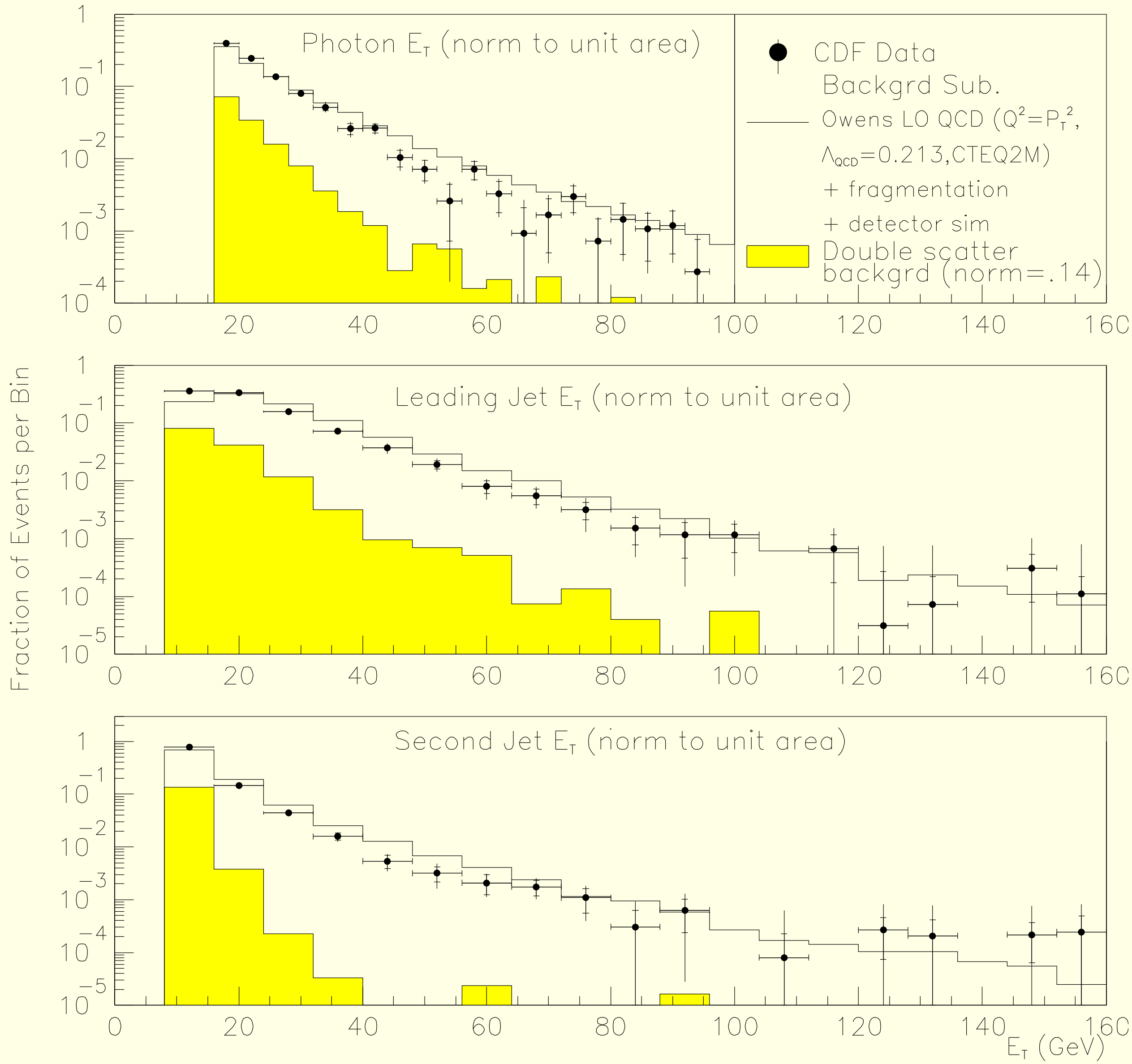
- Both experiments observe a shape consistent with expectations
- Direct photon production is dominated by subprocesses which yield a flatter angular distribution than is observed for dijet production

Three-body Final States

- The next-to-leading-order calculations involve the addition of γ+2 parton final states
- Should be able to examine correlations between the γ and the two final state jets in order to test the underlying subprocesses
- Such measurements have been done by CDF - Abe at al., Phys. Rev. **D57**, 67(1998)
- Caveat - these comparisons are to tree-level predictions based on $2 \rightarrow 3$ subprocesses, so they are not full next-to-leading-order predictions.

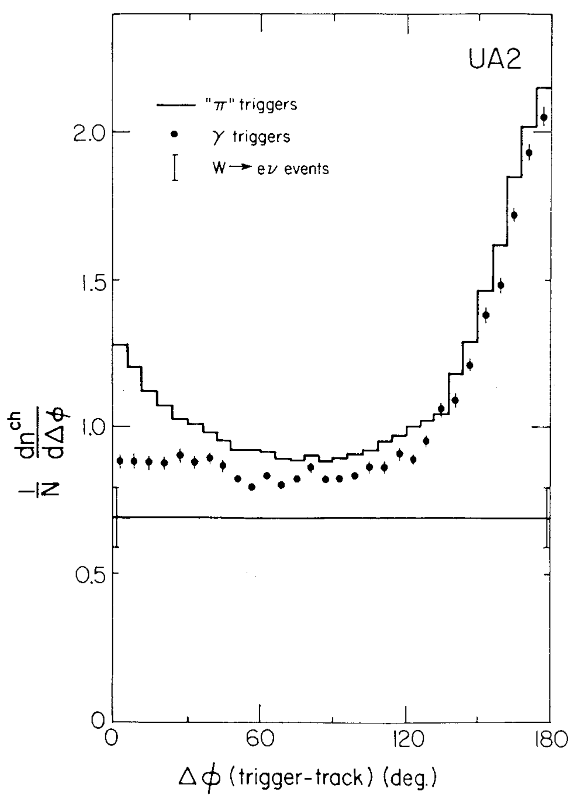

CDF Preliminary
Photon + Jet1 + Jet2 Inv. Mass
Fraction of Events per Bin
Double scatter backgrd (norm=.14)
Mass (GeV)
X(γ) = E_T^γ/Mass
X(Lead Jet) = E_T^Lead Jet/Mass
X(Second Jet) = E_T^Second Jet/Mass
(norm to unit area)
CDF Data Backgrd Sub.
Owens LO QCD (Q²=P_T²,
Λ_QCD=0.213,CTEQ2M)
+ fragmentation
+ detector sim

CDF Preliminary



Event Structure

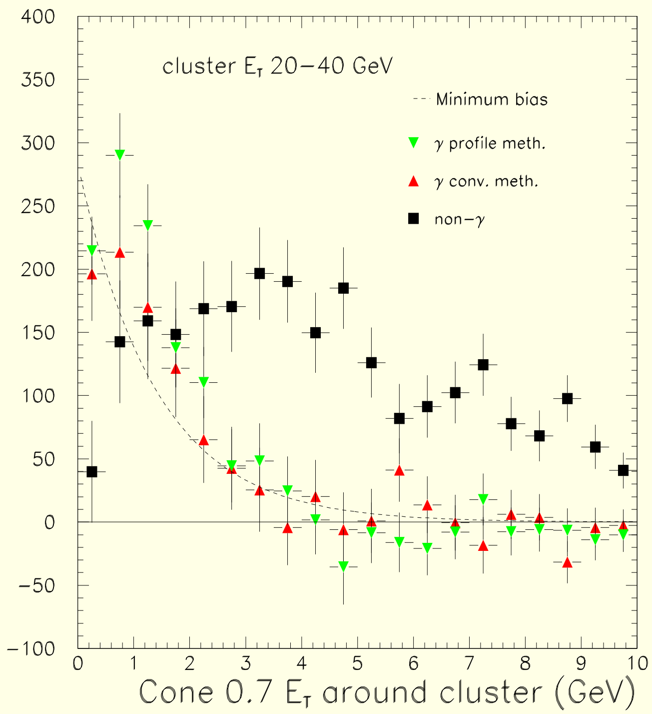
- Lowest order Compton and annihilation subprocesses correspond to an isolated photon recoiling against a jet
- Fragmentation contributions add a component where the photon is accompanied by the hadronic fragments of the parent jet
- Expect to see fewer hadrons on the photon side of the event than would be the case with a hadronic trigger
- Results from UA-2 (R. Ansari et al., Z. Phys. **C41**, 395(1988)) show the reduced number of same side hadrons for photon triggers
- Note that the amount of hadronic activity for photon triggers is larger than that for W production, while being smaller than that for hadronic triggers
- Consistent with a superposition of direct and fragmentation components



UA2
"π" triggers
γ triggers
W→ eν events
$\frac{1}{N}\frac{dn^{ch}}{d\Delta\phi}$
2.0
1.5
1.0
0.5
0
0
60
120
180
Δφ (trigger-track) (deg.)

Figure 12

Similar result seen in preliminary CDF data



- E_T inside a cone of radius 0.7 about the photon looks just like minimum bias case - and not at all like that for hadronic events
- Difficult to identify fragmentation component due to background from beam fragments

Recap

- Thus far we have seen that
  - Photons are produced with the expected γ-jet angular distribution which is flatter than that for dijets
  - Photon-jet rapidity correlations are as expected
  - Photon+2jet events appear to behave as expected
  - Photons appear to be accompanied by fewer hadrons than for purely hadronic triggers, although more than for W events
- All of these items suggest that the basic mechanism for producing high-p_T photons is as expected from QCD
- Is there any other way to probe the dynamics of photon-parton interactions that tests these same mechanisms?

Yes! Jet Photoproduction

Jet Photoproduction

- The same subprocesses are involved, but with the photon crossed to the initial state

$$qg \to \gamma q \quad \Rightarrow \quad \gamma q \to qg \;\; \text{etc.}$$

- Role of photon fragmentation is now in the initial state where it is treated using a photon pdf

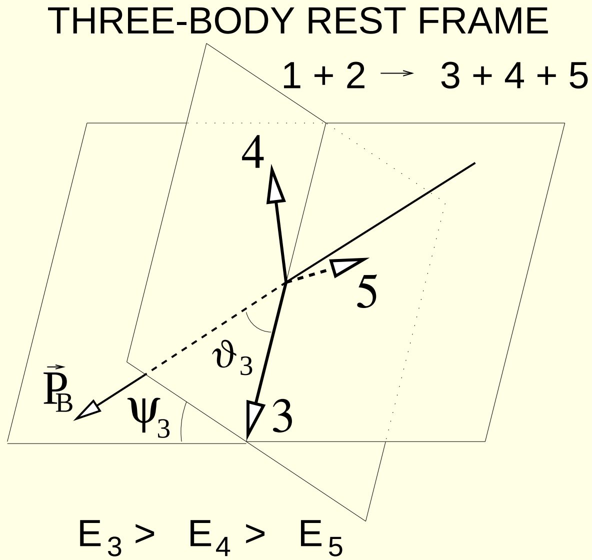
- Details may be found in my CTEQ 2001 summer school lecture at www.cteq.org

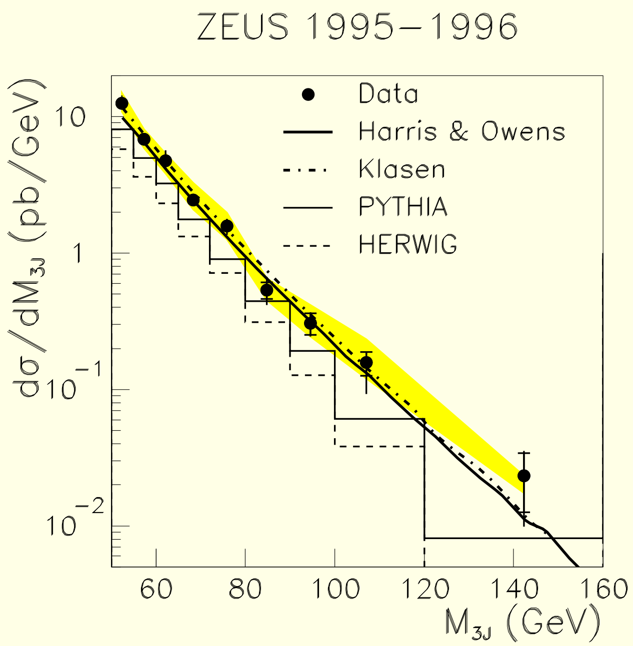
Examples

- Consider several different processes in order of increasing complexity concerning the interactions of the photons
- Photoproduction of three-jet final states
  - Calculation based on $\mathcal{O}(\alpha\alpha_s^2)$ subprocesses
    1. $\gamma q \to qgg$
    2. $\gamma g \to q\overline{q}g$
    3. $\gamma q \to qq'\overline{q}'$
  - Consider three identified high-p_T jets with high three-jet mass
  - Only looking at the pointlike interaction of the photon - no resolved contribution
  - Calculation at this level is lowest order (LO) and also leading-log (LL)
  - Compare to data from the ZEUS Collaboration (hep-ex/9810046)

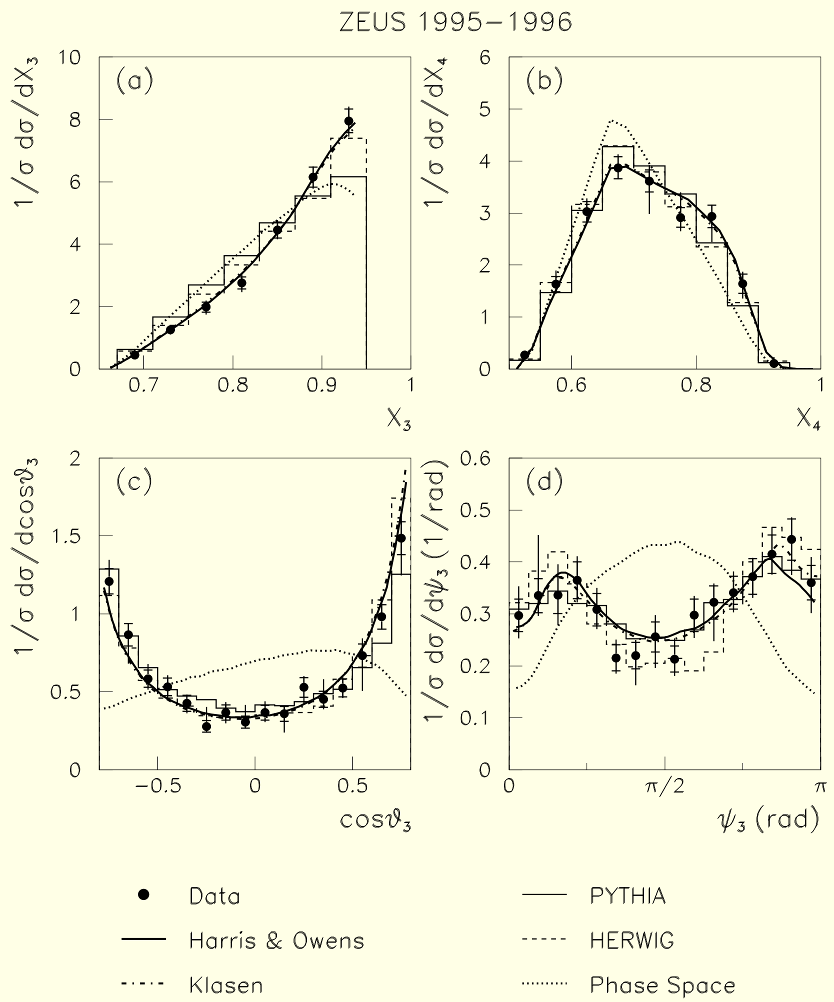
Examine the following

- three-jet mass M_{3j}
- energy fractions: jets ordered in energy with $E_3 > E_4 > E_5$ and energy fractions defined as $x_i = 2E_i/M_{3j}$
- jet angular distributions





- Energy fractions are ordered with $E_3 > E_4 > E_5$
- $\cos\theta_3, \phi_3$ describe the jet orientations



Observe good agreement between matrix element calculations and the data and also between shower Monte Carlo calculations and the data

Single jet and Dijet Photoproduction

- As noted earlier, there will be two components
  - direct or pointlike - photon contributes directly to the hard scattering
  - resolved - both photon and proton parton distributions convoluted with parton-parton scattering subprocesses
  - at $\mathcal{O}(\alpha\alpha_s^2)$ and beyond, specific diagrams can contribute to both. Need an experimental definition to distinguish between the two classes. For example the ZEUS Collaboration has used the following for dijet final states:

$$x_\gamma^{obs} = (E_T^{jet_1} e^{-\eta_{jet_1}} + E_T^{jet_2} e^{-\eta_{jet_2}})/2E_\gamma$$

  - positive η corresponds to the direction of the proton
  - consider two samples with $x_\gamma^{obs} > .75$ (mostly direct) and $x_\gamma^{obs} < .75$ (mostly resolved)

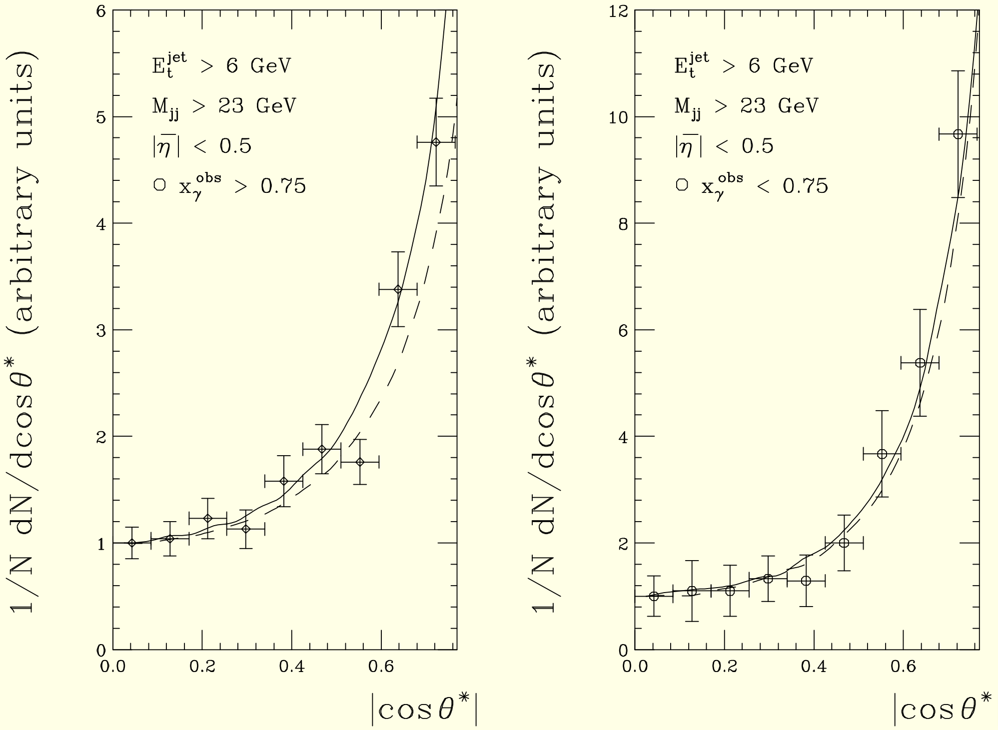
Dijet angular distribution

- $\gamma q \to qg$ and $\gamma g \to q\overline{q}$ are fermion exchange processes which have an angular distribution going as $1/(1-\cos\theta)$ in the dijet center of mass system

- typical boson exchange processes in the resolved part $qq \to qq, qg \to qg, gg \to gg$ etc., behave as $1/(1-\cos\theta)^2$ which is much steeper

Compare to data from the ZEUS Collaboration (M. Derrick *et al.*, Phys./ Lett. B**384**, 401 (1996).)

Curves from Harris and Owens, Phys. Rev. D**56**, 4007 (1997). Dashed lines are LO and solid are NLO.



Note the different scales on these normalized distributions - the dominantly resolved sample has a much steeper distribution

Aside

Why is it possible to separate the direct and resolved components in jet photoproduction, yet it is difficult to isolate the fragmentation or bremsstrahlung component in direct photon production?

- In jet photoproduction one has a sample with two identified jets in the final state. These can be used to reconstruct x_γ and x_p directly.
- Higher order effects and smearing due to beam fragments entering the jet cones smear out the direct component peak at $x_\gamma = 1$, but one can still define two samples of events which are dominantly resolved or dominantly direct.

- In contrast, for direct photon production
  - Some of the fragmentation contribution is removed by isolation cuts used in the trigger (see later)
  - For fragmentation events, one must reconstruct x_a, x_b, and the fragmentation variable z_c. There is not enough information.
- As shown earlier, direct photon events have accompanying hadronic energy that is indistinguishable from the minimum bias case.
- This means that one can not use the accompanying hadronic energy as a flag for fragmentation events.

Lessons Learned

- Photoproduction data support the view that photons interact with partons as expected in QCD
- Two-component picture of the dynamics
  - Direct (point-like) and resolved components in jet photoproduction - resolved piece uses a photon parton distribution to resum large logs coming from configurations where partons are produced collinear with the incoming photon
  - Direct (point-like) and fragmentation components in direct photon production - fragmentation piece uses a photon fragmentation function to resum large logs coming from configurations where the photon is a fragment of a jet and there are partons which are produced collinear with the outgoing photon.
- So, to this point it seems that the theory of direct photon production is supported by the data.

Then, what is all the fuss about?

Single Photon Inclusive Cross Section

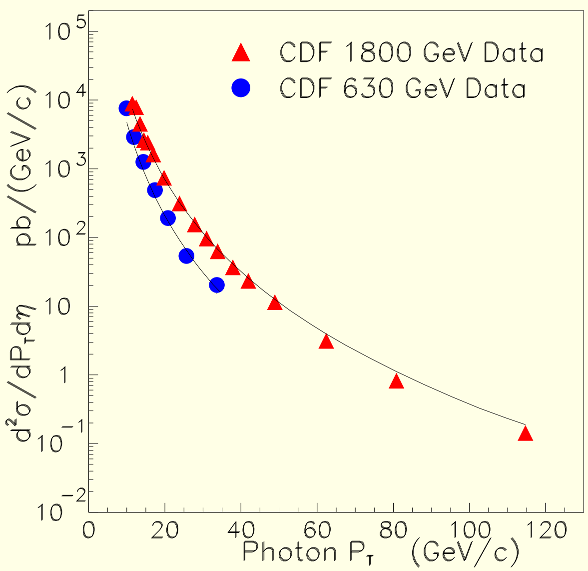
- Integrate over all partons, leaving only the photon as being observed
- In some sense this is the hardest measurement to interpret since if there is a disagreement between theory and experiment one has few clues as to the origin - all other details have been integrated out
- On the other hand, the inclusive nature of the observable makes the calculation easier - you don't have to model the distributions that are integrated over. Rather, you only have to get the integral over the distribution correct.
- Can measure - and calculate - the photon p_T and η distributions
- p_T spectrum is steeply falling so that it is especially susceptible to resolution smearing effects on the experimental side and approximations made in the theoretical calculations

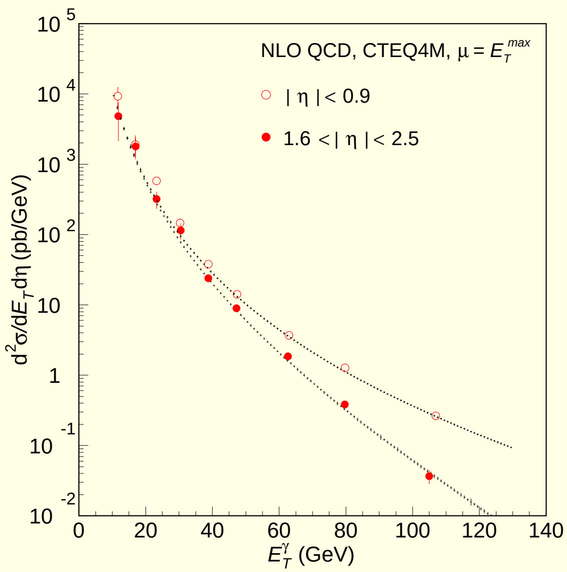
Calculations use the same techniques as described previously

- Analytic calculations result in faster running programs, since many of the integrations have already been done and only the convolutions with pdfs and fragmentation functions need to be done.
- Monte Carlo based programs are able to investigate the effects of the photon isolation cuts to be discussed below, although approximation techniques exist so that the analytic programs can also invoke these cuts.

Tevatron data

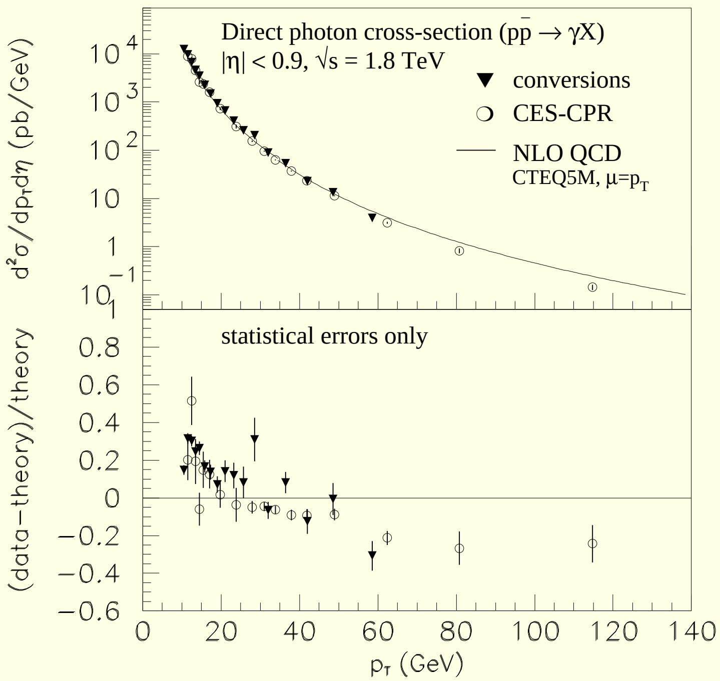
CDF: $\sqrt{s} = 1800, 630$ GeV, $|\eta| < 0.9$
DØ: $\sqrt{s} = 1800, 630$ GeV, $|\eta| < 0.9, 1.6 < |\eta| < 2.5$

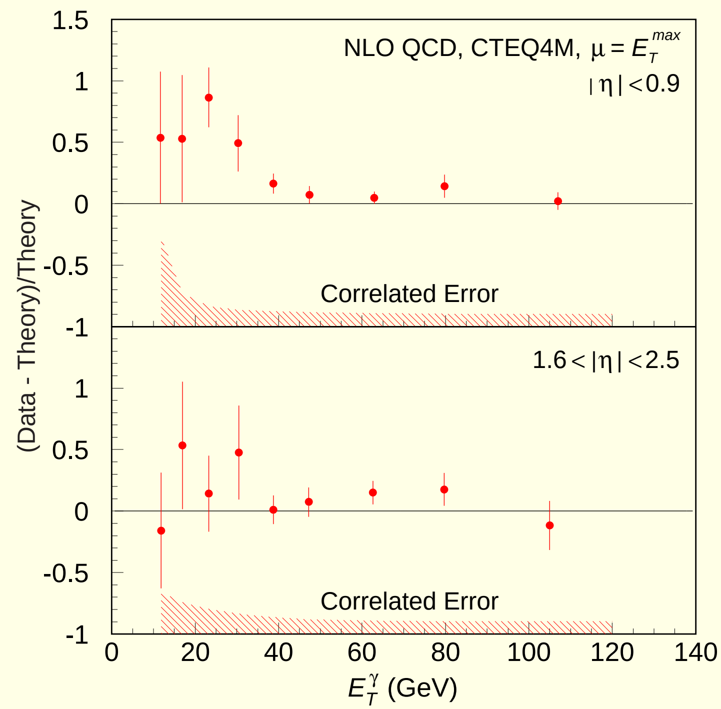




- Agreement looks good when plotted on a logarithmic scale
- Confirms expectation that the QCD description of direct photon production is correct
- But what if we look closer . . . ?

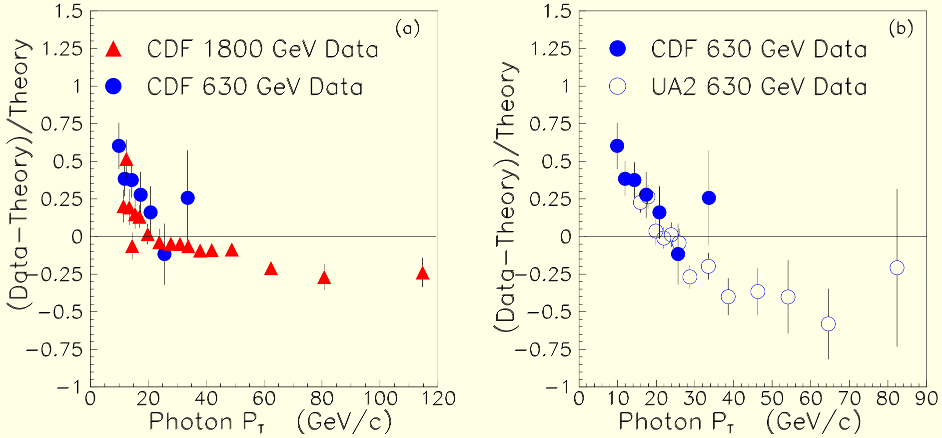
Look on a linear scale . . .



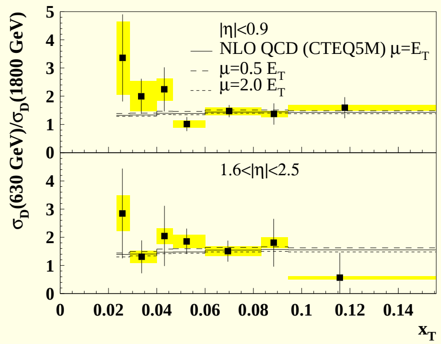


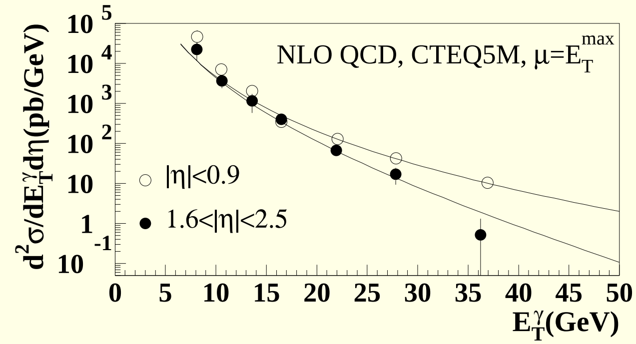
Both CDF and DØ see an excess at the low p_T end in the central region

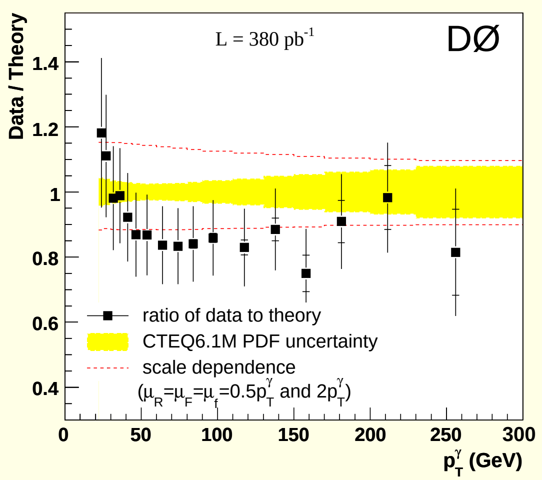
- Problem seen by CDF at both 1800 and 630 GeV
- Excess occurs at low p_T, not at fixed x_T, so the solution can not be a simple adjustment of the pdfs
- Effect also seen by UA-2



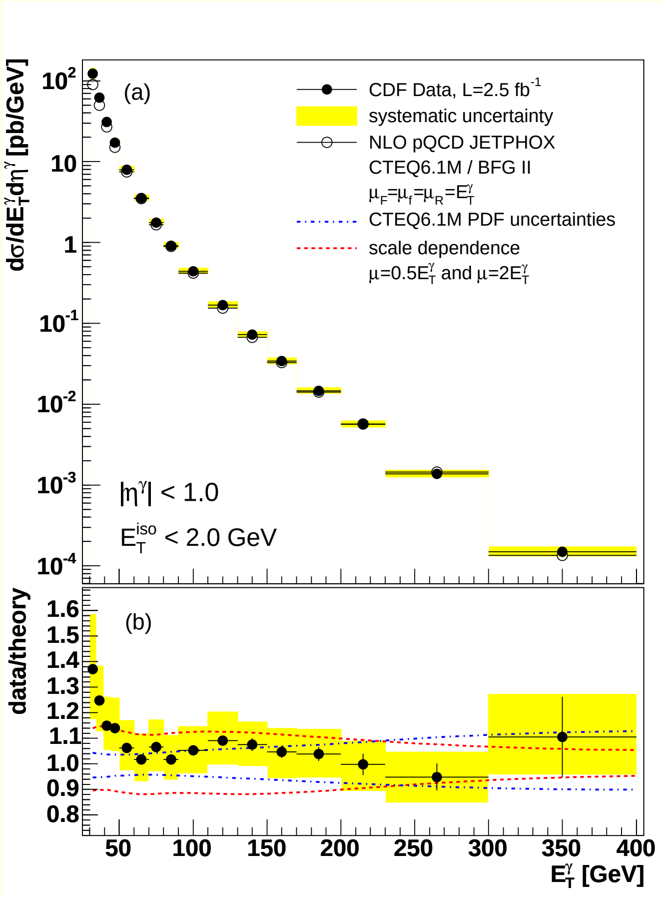
Also seen by DØ at 630 and 1960 GeV







Latest Run II CDF data shows the same effect



Effect appears below about 50 GeV transverse momentum - agreement is good above that

Isolation Cuts

- So, what can explain the disparity between shapes of the experimental and theoretical distributions?
- One item to consider is that these are not truly inclusive measurements. Rather, they are measurements of inclusive cross sections for *isolated* photons.
- Algorithms are different for each experiment (UA-2, CDF, DØ) but all limit the allowed hadronic energy in the vicinity of the electromagnetic trigger

$\Rightarrow$ fragmentation component is reduced

- Expect fragmentation to be most important at the low p_T end of the distribution
- Fragmentation provides only a portion of the available energy to the photon. The pointlike or direct subprocesses are more efficient and so dominate at high values of p_T
- Errors on modeling the effects of the isolation cuts could especially affect the low p_T end of the spectrum

Specific Example - CDF cone algorithm

- Require that there be less than 1 GeV of hadronic transverse energy in a cone of radius
$$\mathcal{R} = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} < 0.4$$
about the direction of the photon.

Theoretical modeling (phase space slicing method)

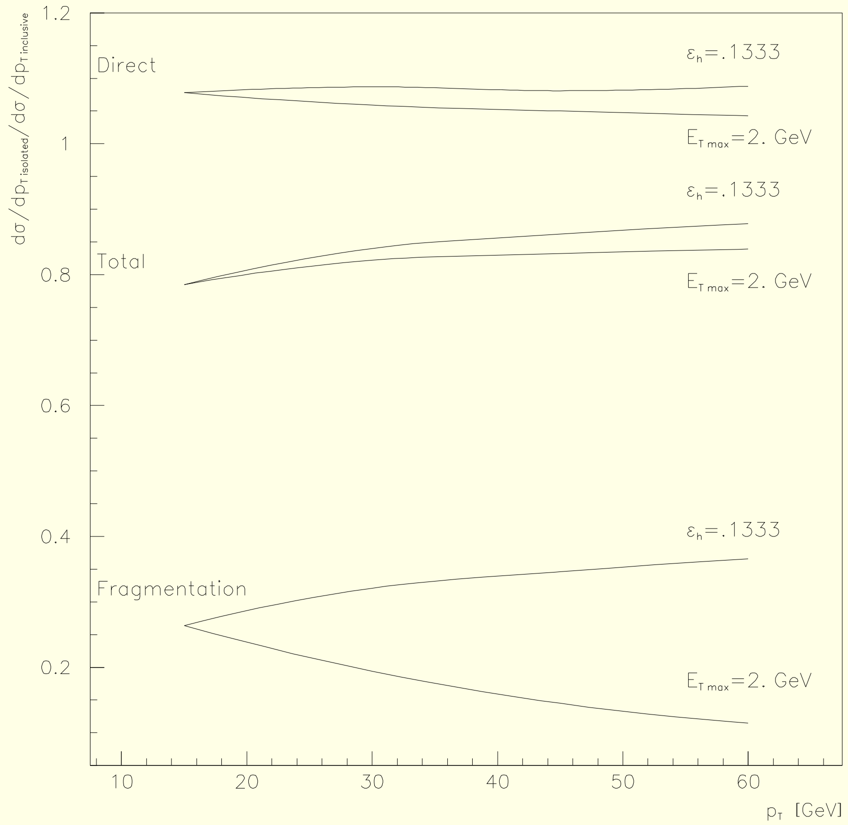
- Must treat the two- and three-body contributions separately
- For the $2 \rightarrow 3$ pointlike subprocesses, one can explicitly enforce the isolation condition on an event-by-event basis in the Monte Carlo at the parton level.

- For the two-body fragmentation component there is no dependence on $\mathcal{R}$ since the fragmentation functions are inclusive quantities.
  - Work in the collinear approximation (all emitted partons or photons are collinear with the parent parton)
  - parent parton transverse momentum is p_{Tpart}
  - photon transverse momentum is $p_{T\gamma} = zp_{Tpart}$
  - hadronic E_T is $(1-z)p_{Tpart} = (1-z)p_{t\gamma}/z$.
- Requiring that the hadronic E_T is less than E_{Tcut} results in

$$z > \frac{1}{1 + E_{Tcut}/p_{T\gamma}}.$$

- One can also enforce a similar isolation condition on the $2 \rightarrow 3$ fragmentation component

Study of the behavior of the isolated photon cross section by Catani, Fontannaz, Guillet, and Pilon, hep-ph/0204023



Ratio of isolated to inclusive for the total and each component

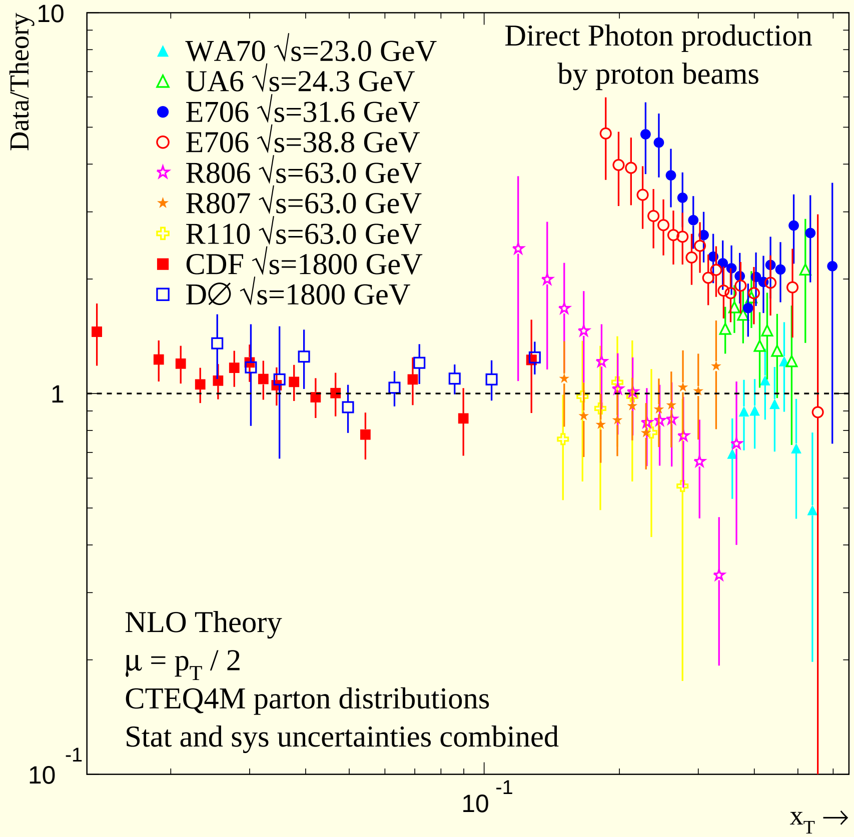
Consider the "Total" curve for $E_{Tmax} = 2$ GeV. The cone radius is $\mathcal{R} = .7$ (similar to $E_{Tmax} = 1$ GeV in a cone of radius .4)

- $\sim 22\%$ reduction due to isolation at p_T=15 GeV
- Expected effect of the isolation slowly decreases as p_T increases
- Even if the isolation was *totally removed*, the cross section would only increase by 28 %.

Strongly suggests that the modelling of the isolation cuts is not responsible for the discrepancy between theory and experiment

Fixed Target and Lower Energy Collider Data

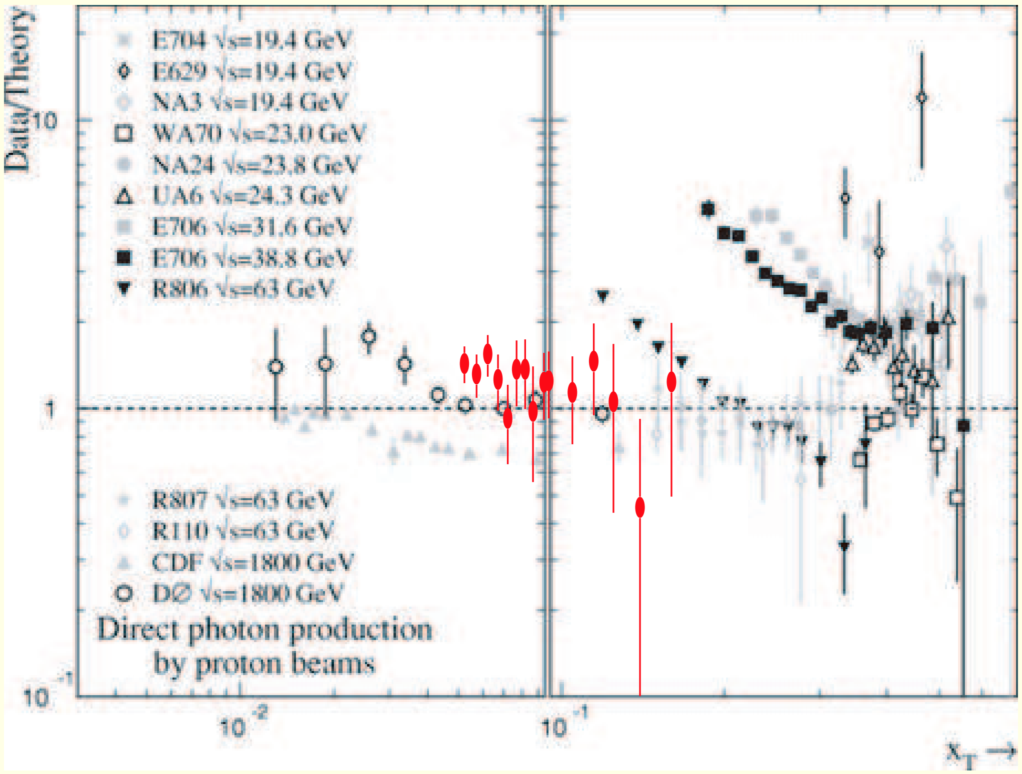
It is now time to consider the situation for the inclusive cross section at lower energies



Comments

1. Data/Theory plotted versus x_T
2. Data plotted at same x_T but different $\sqrt{s}$ correspond to different p_T's
3. E706 higher than theory, UA6 somewhat above theory, WA70 and theory agree
   - Likely that there is some experimental inconsistency here since the range in $\sqrt{s}$ is relatively small and it would take a significant modification of the theory to explain all three sets simultaneously.
4. See some shape disagreements among the ISR experiments
5. Plotted on this scale, the previously noted deviations of the theory from the CDF and DØ data look pretty darned small!

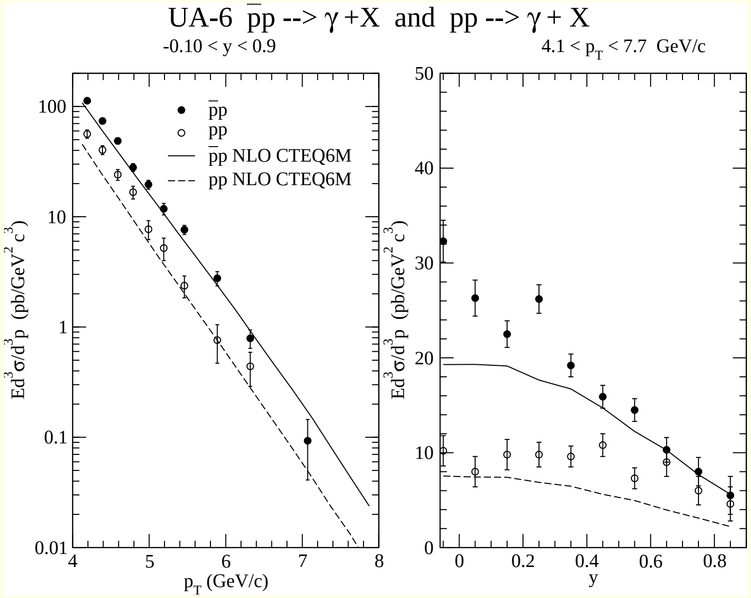
- Similar plot showing PHENIX data (nucl-ex/0504013)



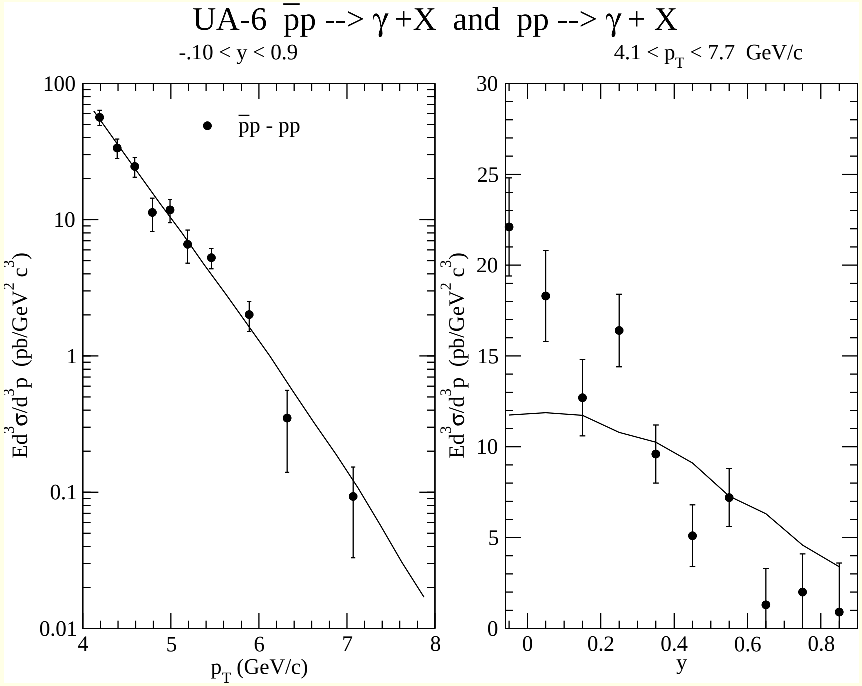
- Agreement is comparable to that for other collider experiments and better than for the fixed target regime

Example - UA-6

- Measured both pp and $\bar{p}p$ at $\sqrt{s} = 24.3$ GeV/c
- Initial state gluon and gluon fragmentation contributions cancel in the $\bar{p}p - pp$ difference



- Theory below the data at the lower end of the p_T range
- Rapidity theory curves are flatter than the data



- Cross section difference cancels contributions from gluons
- p_T difference is well described
- Rapidity theory curve is somewhat flatter than the data
- So, the situation is mixed - the $\bar{p}p$ and pp curves are individually below the data, the p_T difference is well described, while the rapidity difference curve is a bit too flat

So what is going on?

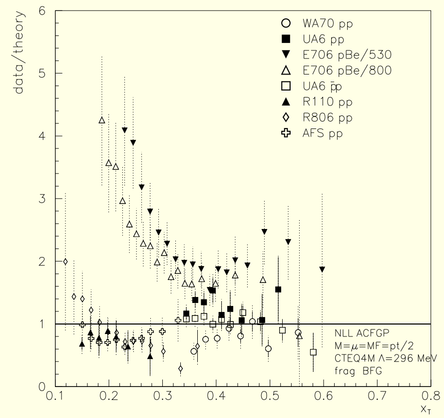
- Theory and data have different shapes for $\sqrt{s} = 630$, 1800 GeV with the theory being flatter than the data
- Some of the lower energy experiments show this same behavior to an even larger degree - others do not
- Critical review of the situation for the lower energy experiments: Aurenche, Fontannaz, Guillet, Kniehl, Pilon, and Werlen, hep-ph/9811382

Is this behavior seen for any other processes?

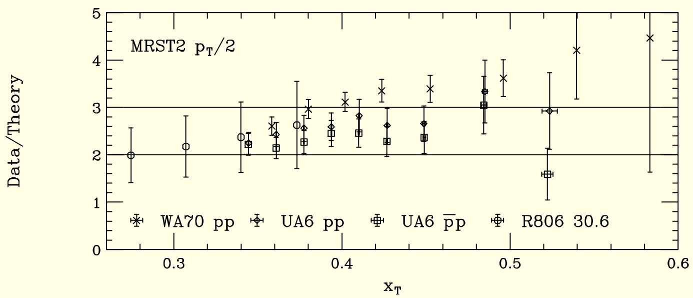
Yes - Inclusive single hadron production!

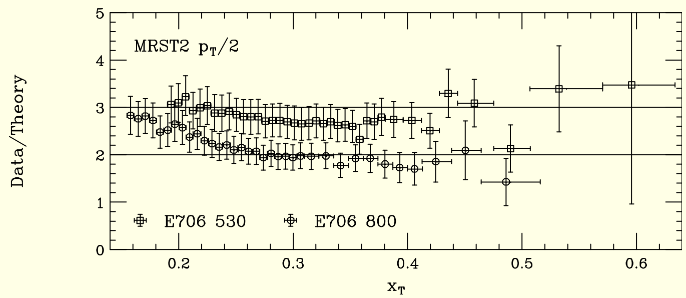
- Situation reviewed by Aurenche, Fontannaz, Guillet, Kniehl, and Werlen, hep-ph/9910252

Example plot for direct photon production



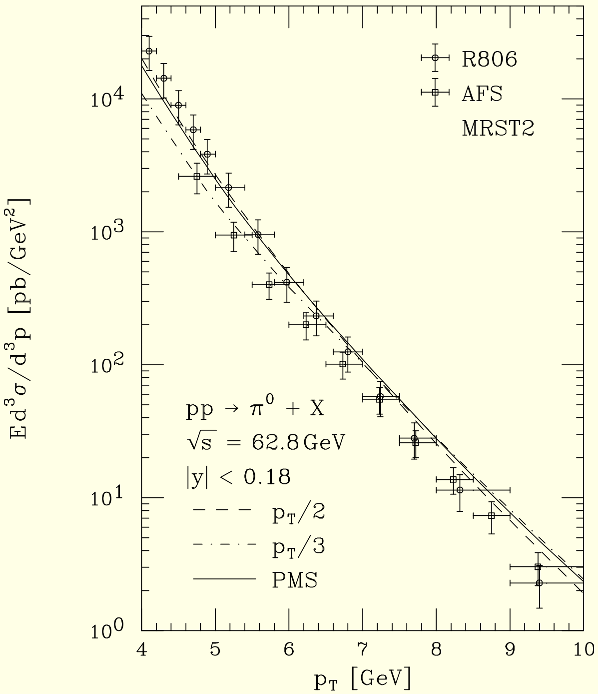
And two for π^0 production

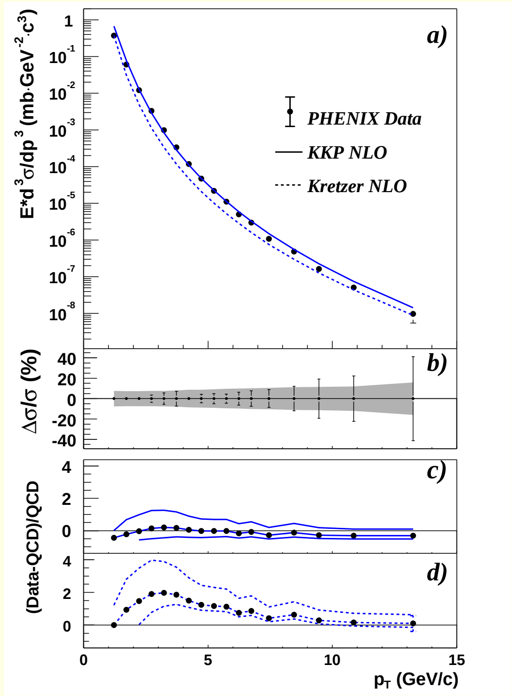


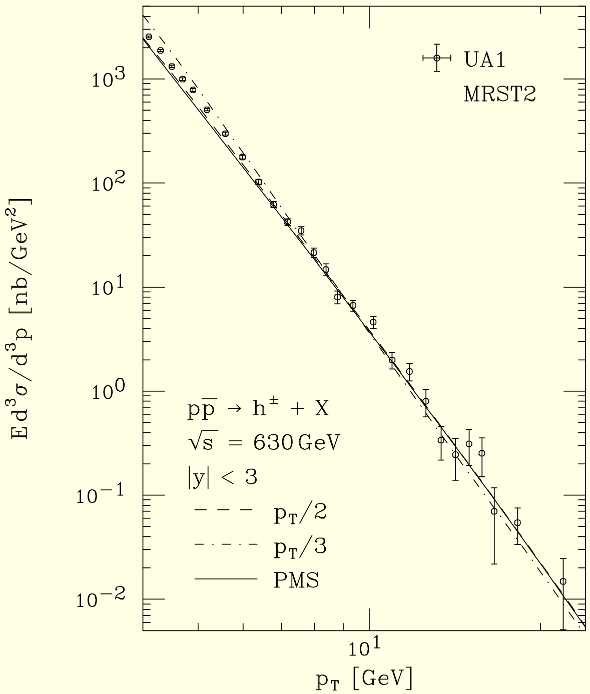


See similar excesses of data over theory

But a strange thing happens as one goes up in energy. . .







Agreement between theory and data gets better at higher energies *and at lower p_T's!*

So where do we stand?

1. Situation with fixed target direct photon production is confused by some disagreement between experiments
   - ▷ See Apanasevich et al., hep-ph/0007191 for a discussion of the systematics of γ/π^0 ratios and consistency between experiments
2. All experiments see an excess of data over theory for single hadron inclusive production at fixed target energies
3. Agreement between theory and data for single hadron production improves with increasing energy and is excellent by $\sqrt{s} = 200$ GeV
4. Likely that we need an *improved method of calculating single particle inclusive cross sections* in the fixed target energy range - one that would improve agreement for both photon and hadron production
5. A reassessment of systematic errors on the existing fixed target photon experiments might also help resolve the discrepancies between data sets

Theoretical Ideas and Scenarios

Start by considering the case of single hadron production where all experiments in the fixed target regime see an excess of data over theory

- The steeply falling spectra in the fixed target region force the fragmentation variable z to be near one. As one goes up in energy the distributions flatten somewhat and $< z >$ decreases
- Fragmentation functions are not well constrained by data at high values of z
- Fragmentation functions behave as $(1 - z)^n$ with $n \approx 2 - 3$.
- As $z \to 1$ large logarithms of $(1 - z)$ should be resummed

Resummation may offer a way of significantly increasing the fixed target predictions ($< z >$ near 1) while not raising the already successful higher energy predictions too much (here $< z > << 1$)

Threshold Resummation

Basic Physics -

- For inclusive calculations singularities from soft real gluon emission cancel against infrared singularities from virtual gluon emission
- Limitations on real gluon emission imposed by phase space constraints can upset this cancellation
- Singular terms still cancel, but there can be large logarithmic remainders
- Classic example is thrust distribution in $e^+e^- \to jets$
- For hadronic reactions with PDFs and FFs the collinear factors actually conspire to enhance the partonic cross sections (See my previous lecture at this school)

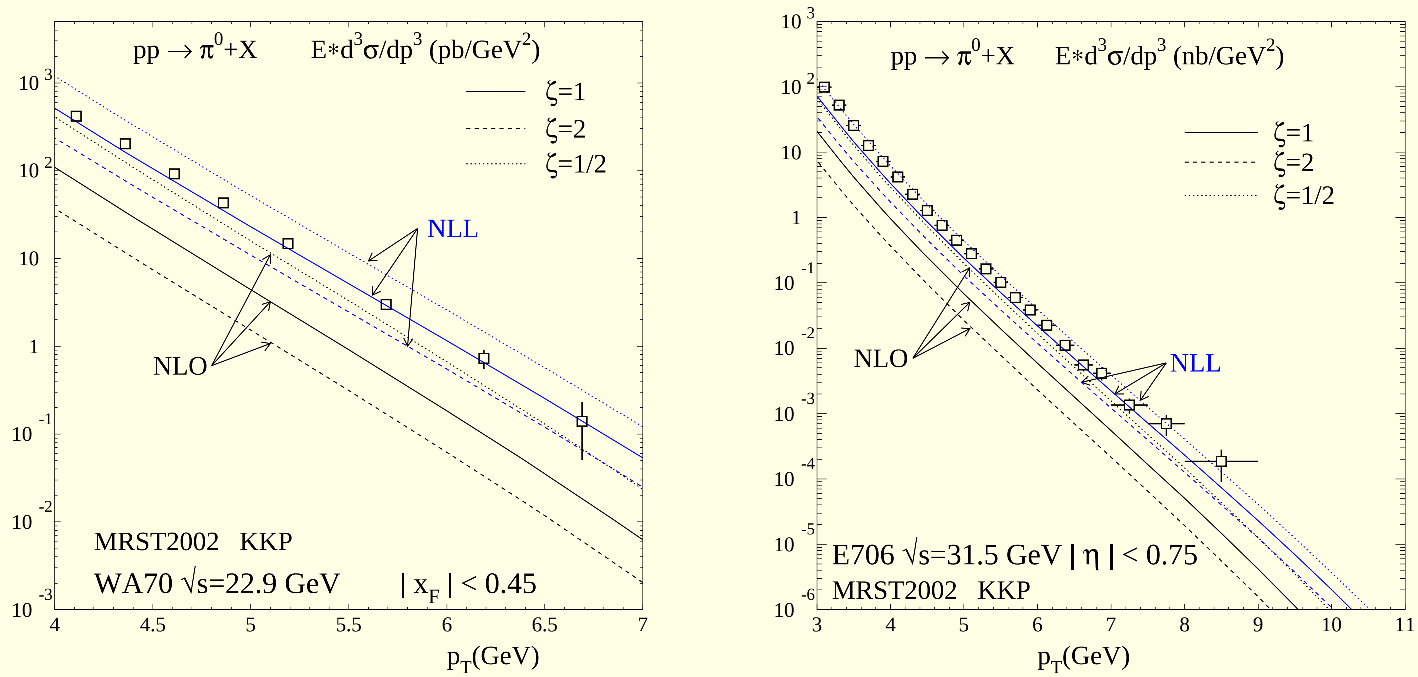
High-p_T particle production

- For typical fixed target energies the PDFs are evaluated at rather large x values and the fragmentation functions are evaluated at large z
- For example, $\sqrt{s} = 30$ GeV and $p_T = 7.5$ GeV gives $x_T = .5$
- Steeply falling PDFs and fragmentation functions constrain real gluon emission when high-p_T is required since it costs a significant amount of the parton-parton center-of-mass energy to emit additional partons beyond the one that is fragmenting into the observed hadron.
- Phase space for gluon emission is limited near kinematic threshold in the parton-parton scattering subprocess for producing the hadron with the observed value of p_T

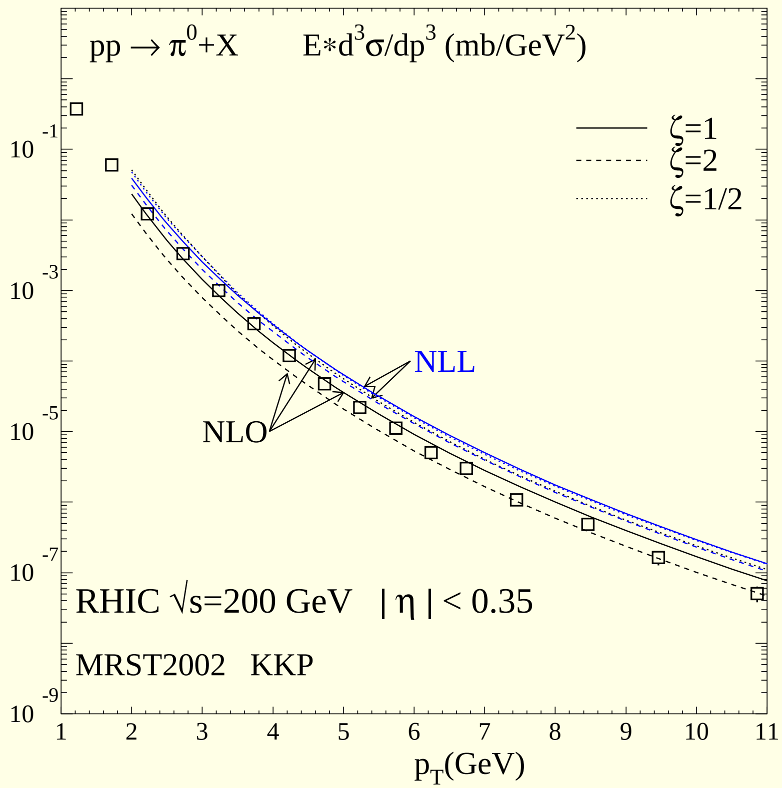
Define $v = 1 + t/s$ and $w = -u/(s+t)$.
Threshold occurs at $w = 1 \;\; (s + t + u = 0)$
Soft gluons emission gives rise to terms in the partonic cross sections which behave like $\alpha_s^m \left(\frac{\ln^n(1-w)}{1-w} \right)_+$ relative to the Born terms which are $\mathcal{O}(\alpha_{\int}^{\in})$ in this case.
Can sum leading logs ($n = 2m - 1$), next-to-leading-logs ($n = 2m - 2$), etc.

Application to the π^0 cross section

- Paper by de Florian and Vogelsang (hep-ph/0501258) applies threshold resummation to π^0 production
- Large values of the fragmentation variable z relevant for fixed target energies leads to large threshold resummation corrections there.
- Enhancement is strongly energy dependent since the relevant values of z decrease as one goes to higher energies at fixed p_T.
- Enhancement is larger than that observed in jet production since the jet cross section doesn't involve fragmentation functions



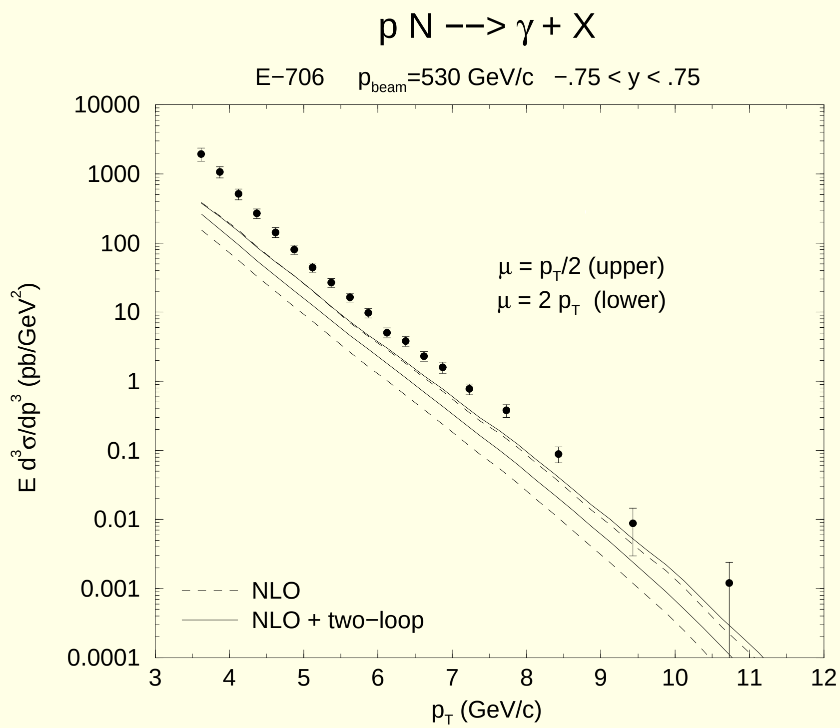
- Blue curves include the resummation corrections properly matched to an existing NLO calculation in order to avoid double counting.
- Note the reduced scale dependence of the resummed results.



- Note reduced enhancement at RHIC energy compared to the previous fixed target results

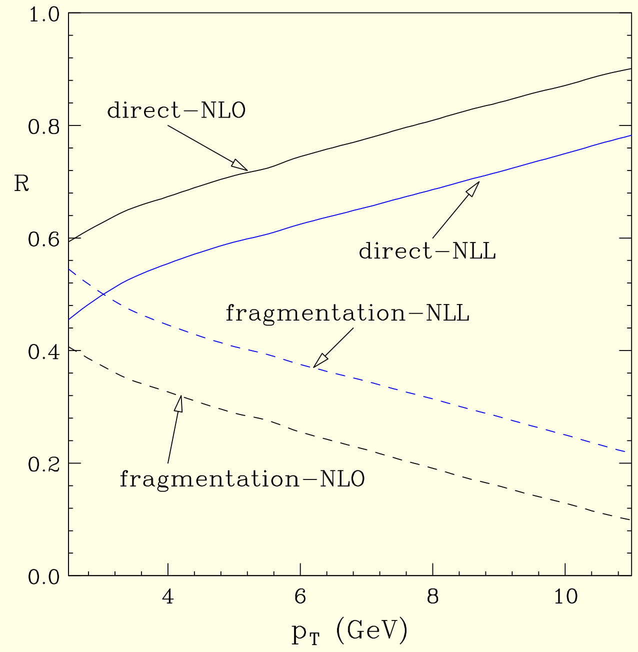
What about direct photons? Can threshold resummation help?

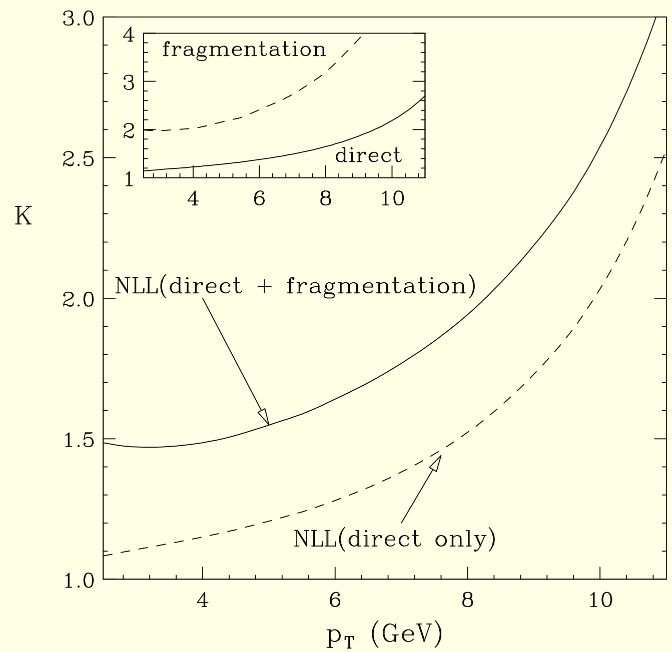
- Example application to the fixed target data - N. Kidonakis and J.F. Owens, Phys. Rev. D61, 094004, 2000; hep-ph/9912388
- Fixed target region dominated by annihilation and Compton subprocesses
- Fragmentation doesn't play as large a role as at higher energies since it costs extra energy to have a photon produced by fragmentation
- No significant enhancement to the annihilation and Compton terms
- Reduced scale dependence observed

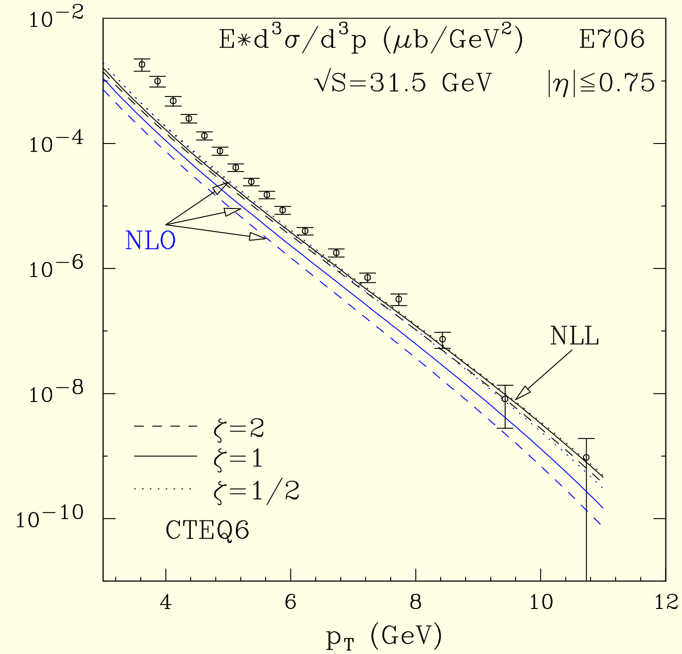

p N −−> γ + X
E−706 p_beam=530 GeV/c −.75 < y < .75
μ = p_T/2 (upper)
μ = 2 p_T (lower)
NLO
NLO + two−loop
E d³σ/dp³ (pb/GeV²)
p_T (GeV/c)
10000
1000
100
10
1
0.1
0.01
0.001
0.0001
3
4
5
6
7
8
9
10
11
12

But...

- The fragmentation contribution is *not* zero at fixed target energies
- Vogelsang and de Florian had previously shown that the fragmentation contribution in *hadro*production was significantly enhanced by threshold resummation
- They subsequently applied the formalism to direct photons in hep-ph/0506150
- Relative contribution of fragmentation versus direct *is* enhanced

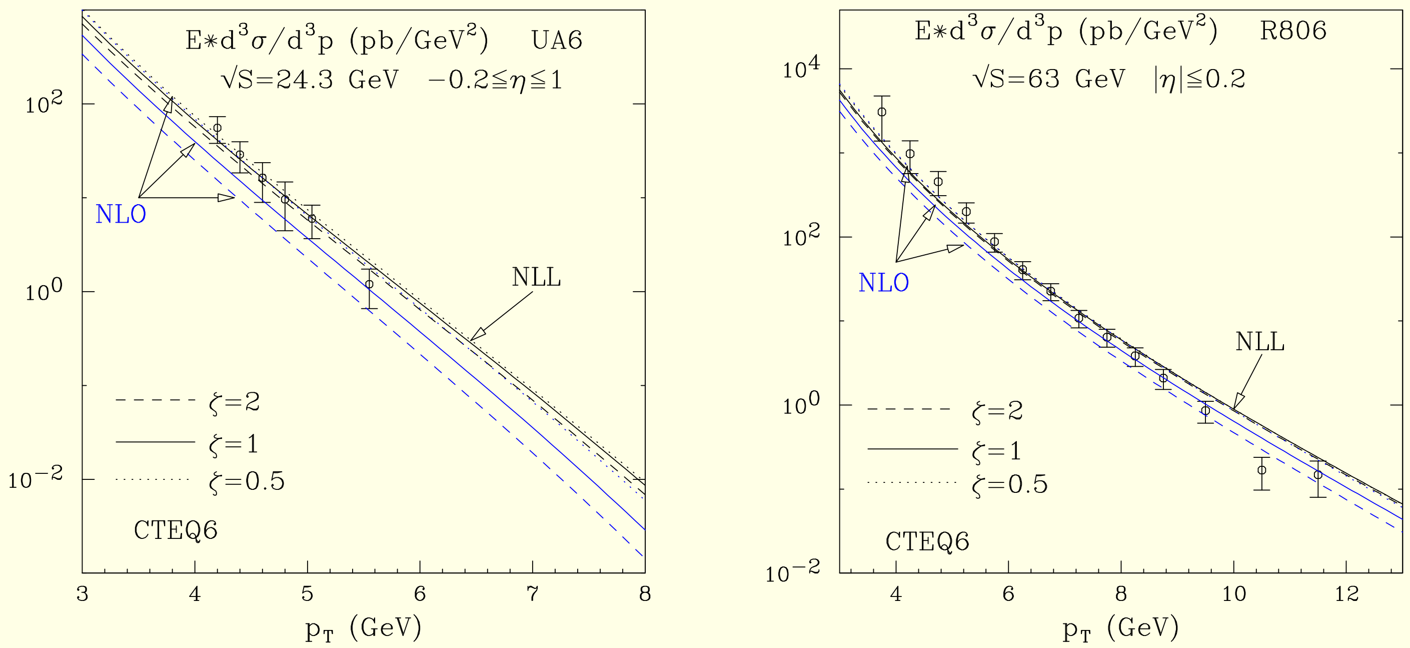






- Resumming the fragmentation component results in a larger increase than if just the direct component is resummed
- Still isn't enough to describe the E-706 results

Resummation *can* result in a good description of the UA-6 *pp* data



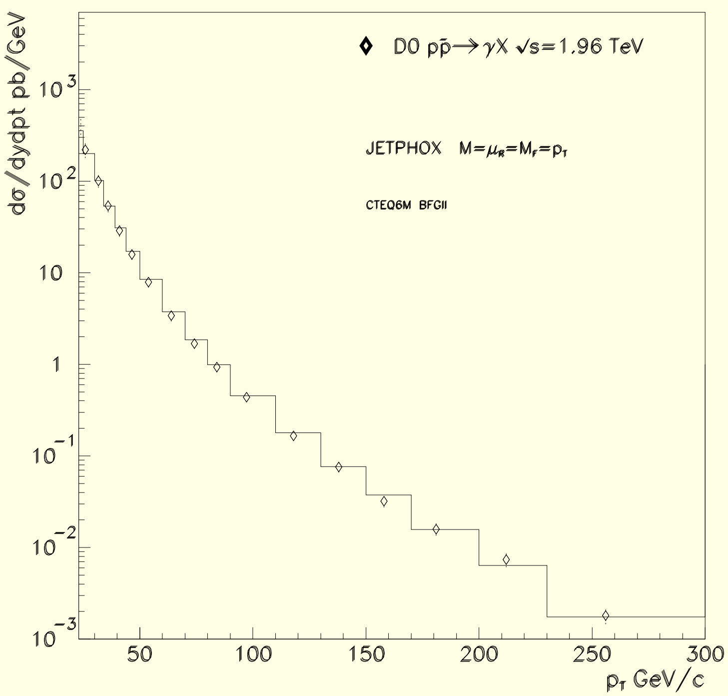
- Fragmentation component largely cancels in the $\overline{p}p - pp$ difference, so the previous good agreement for this is retained
- Enhancement decreases rapidly with energy as in the hadroproduction case so that agreement with higher energy data is retained

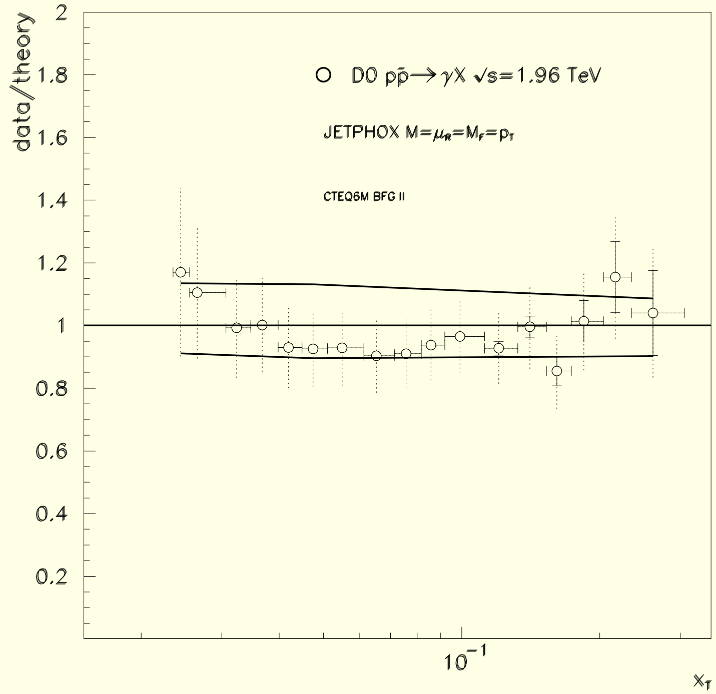
Bottom line on threshold resummation

- Provides reduced scale dependence
- Provides an enhancement in the fixed target regime, but the effect is much smaller at higher energies
- Can improve the agreement with some fixed target experiments without adversely affecting the agreement at higher energies

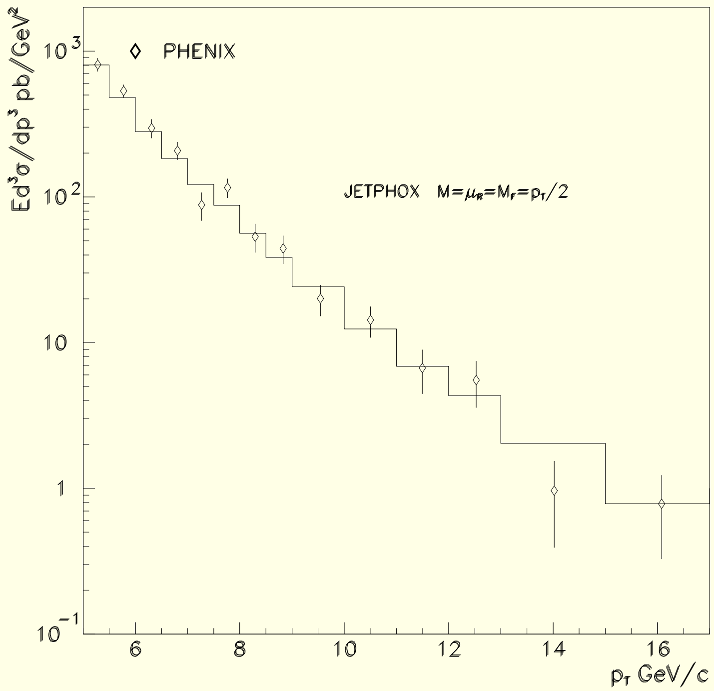
So where do we stand?

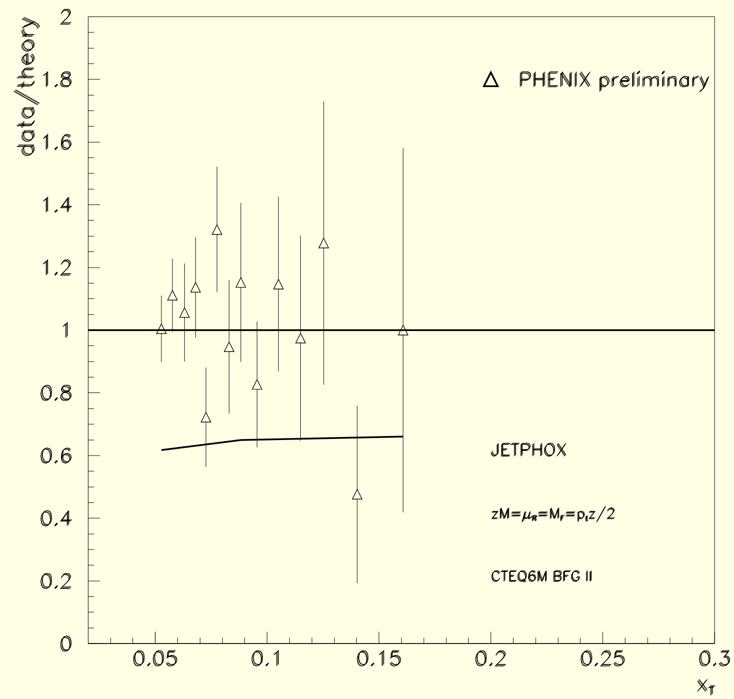
- Situation recently reviewed by Aurenche, Fontannaz, Guillet, Pilon, and Werlen in hep-ph/0602133, Phys. Rev. D73:094007,2006.
- Incorporated new data from DØ and PHENIX (as discussed earlier)
- All calculations done with the JETPHOX package
- CTEQ6M pdfs used along with the Bourhis-Fontannaz-Guillet photon fragmentation functions
- All scales set equal to $p_T/2$ except where noted



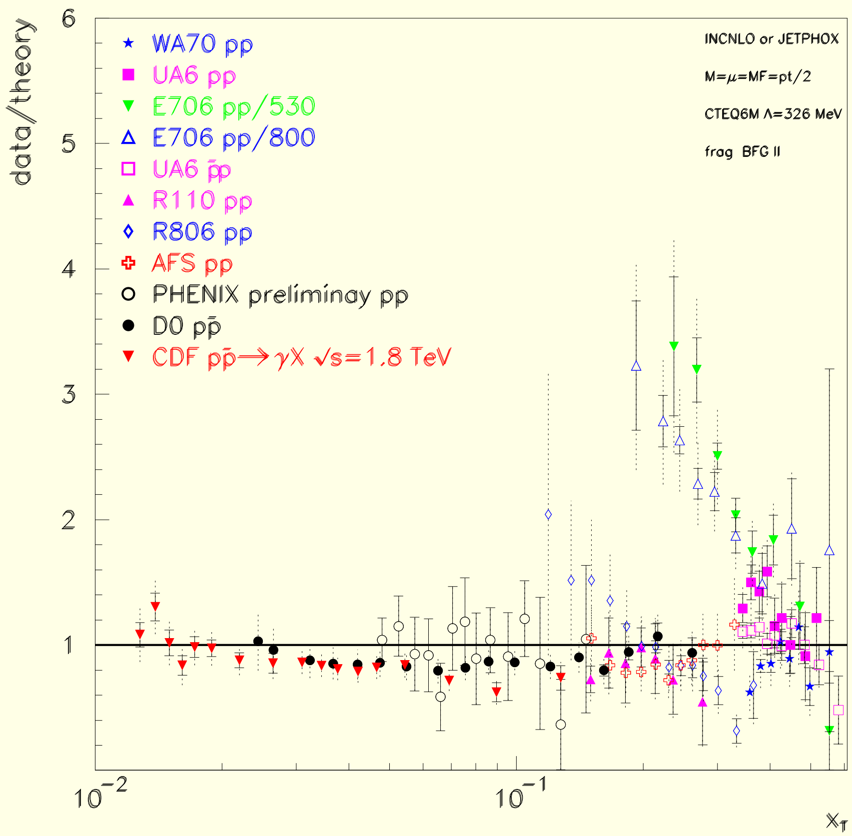


Excellent agreement between NLO QCD and the DØ data (note that these data have p_T greater than 20 GeV and that all scales have been set equal to p_T)

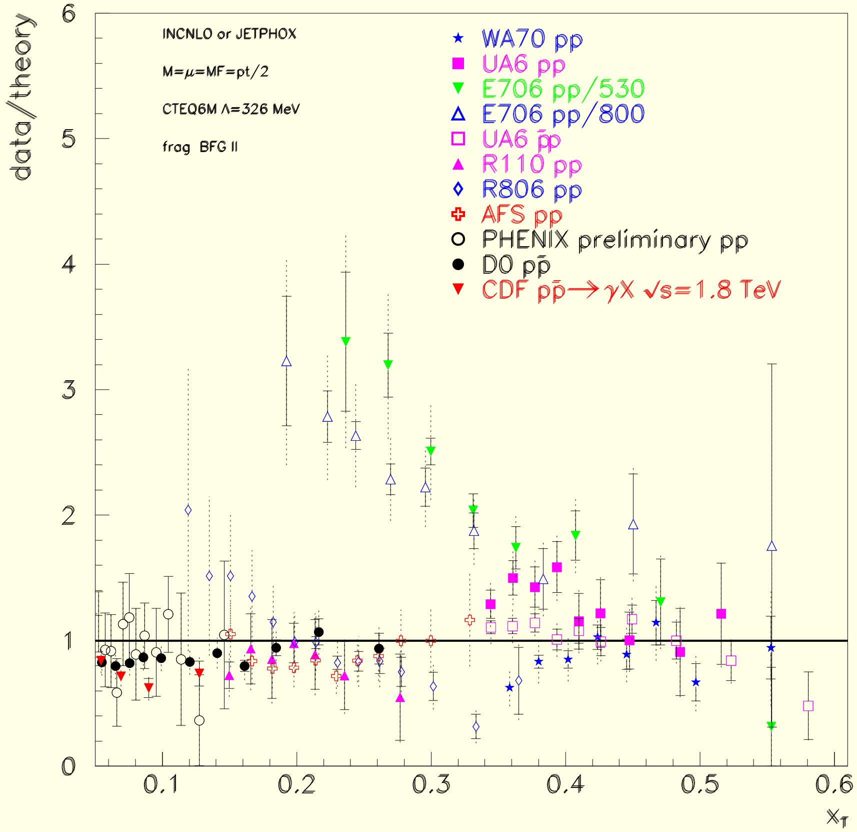




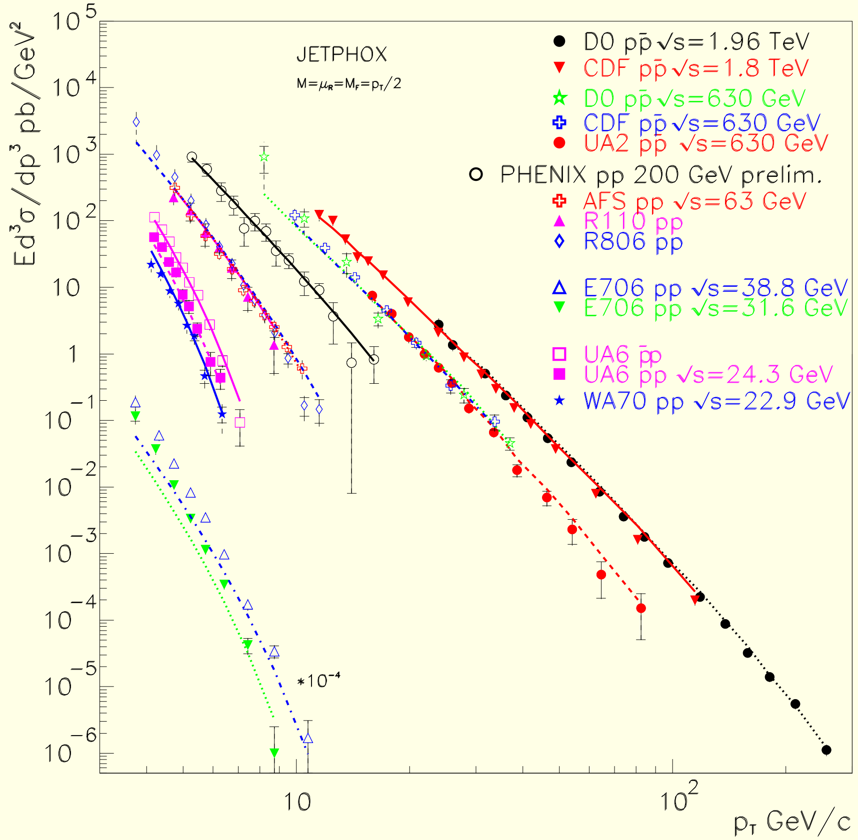
Agreement with the PHENIX data (only statistical errors shown) is also excellent



Data/theory for collider and fixed target data shows a reasonably consistent comparison with some exceptions



Same as the previous figure, except on a linear x_T scale to emphasize the fixed target region at large x_T



This is a summary of the world's data on direct photon production. There is a reasonably consistent picture covering 9 orders of magnitude

Assessment

- A rather consistent picture seems to be emerging
  - For $p_T > 50$ GeV the Tevatron data from DØ and CDF are well described by the theory
  - For the fixed target energy range, threshold resummation applied to both the direct and fragmentation components will help improve the agreement between theory and experiment while preserving the agreement with the collider data
- Both the DØ and CDF results show an excess of data/theory below p_T of 50 GeV or so
- Threshold resummation won't affect this region significantly
- Isolation cuts and the methods used to calculate their effects do not seem to be responsible as the isolation effects are not as large as the excess and the effects are rather smooth in p_T with no sharp onset

- Resummation of small x_T logarithms has been suggested as a possible explanation, but a recent calculation (Diana, Rojo, and Ball, arXiv:1006.4250 [hep-ph]) shows that this is not the case - one must go to much smaller values of x_T to see even a modest effect
- A combination of k_T and threshold resummation by George Sterman and collaborators (called joint resummation) can give some enhancement at small x_T, but this depends on fitting some nonperturbative parameters. The challenge would be to explain the excess without modifying the agreement found elsewhere - see hep-ph/0904234
- This excess remains unexplained

Summary and Conclusions

1. Examining γ-jet observables suggests that high-p_T photons are produced in accordance with the expectations based on QCD
2. There is broad agreement between the theory and most experimental results for the photon p_T distribution
3. Photoproduction observables confirm that QCD matrix elements give a good description of photon interactions there, as well
4. Threshold resummation has been shown to play an important role in hadroproduction at fixed target energies and can offer some improvement for direct photons
    - Enhances the fragmentation contribution more than was previously anticipated in the fixed target regime
    - Effects are reduced at collider energies as the since smaller values of x_T are probed

While there are a few remaining issues, the overall description of the data over nine orders of magnitude is encouraging

Appendix: some miscellaneous and hopefully useful stuff

1. Check out the CTEQ web page at www.cteq.org
   - information on past summer schools, including transparencies of many of the lectures
   - information and links for parton distributions
   - CTEQ List of Challenges in Perturbative QCD
   - CTEQ Pedagogical Page
   - CTEQ Handbook of Perturbative QCD
2. Four-vectors and rapidity
   - rapidity is defined as $y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z}$. For massless particles this reduces to the pseudorapidity which is defined as $\eta = \ln\cot\theta/2$.
   - the four-vector for a massless particle with transverse momentum p_T and rapidity y may be conveniently expressed as

$$p^\mu = (p_T\cosh y, p_T, 0, p_T\sinh y)$$

3. Mandelstam variables

- For a two-body process $p_1 + p_2 \to p_3 + p_4$ the three Mandelstam variables are defined as

$$
\begin{aligned}
s &= (p_1 + p_2)^2 \\
t &= (p_1 - p_3)^2 \\
u &= (p_1 - p_4)^2
\end{aligned}
$$

- For processes with more particles one sometimes encounters variables such as $s_{ij} = (p_i + p_j)^2$ which is just the squared invariant mass of particles i and j and $t_{ij} = (p_i - p_j)^2$ which is the squared four-momentum transfer between particles i and j.

4. Another example: direct photon production $qg \rightarrow \gamma + q$

- four-vectors in the hadron-hadron center-of-mass frame

$$\begin{aligned} p_q &= \frac{\sqrt{s}}{2} x_a (1,0,0,1) \\ p_g &= \frac{\sqrt{s}}{2} x_b (1,0,0-1) \\ p_\gamma &= p_T (\cosh y, 1, 0, \sinh y) \end{aligned}$$

- Substituting these four-vectors into the expressions for the Mandelstam variables above yields

$$\begin{aligned} \hat{s} &= x_a x_b \; s \\ \hat{t} &= -x_a \; p_T \sqrt{s} \; e^{-y} \\ \hat{u} &= -x_b \; p_T \sqrt{s} \; e^{y} \end{aligned}$$

- A ˆ is often used to denote a variable at the parton level.

5. Convolutions

- The symbol $\otimes$ is sometimes used to denote a convolution:

$$
\begin{aligned}
f \otimes g &= \int_0^1 dy \int_0^1 dz\; f(y)\; g(z)\; \delta(x - yz) \\
&= \int_x^1 \frac{dz}{z}\; f(x/z)\; g(z)
\end{aligned}
$$

6. Subprocesses and angular distributions

The two lowest order subprocesses for direct photon production are (in units of $\pi\alpha\alpha_s/\hat{s}^2$)

$$\begin{aligned}\frac{d\sigma}{d\hat{t}}(gq \to \gamma q) &= -\frac{e_q^2}{3}\left[\frac{\hat{u}}{\hat{s}} + \frac{\hat{s}}{\hat{u}}\right] \\ \frac{d\sigma}{d\hat{t}}(q\overline{q} \to \gamma g) &= \frac{8}{9}e_q^2\left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}}\right]\end{aligned}$$

The dominant parton-parton scattering subprocesses for hadroproduction are (in units of $\pi\alpha_s^2/\hat{s}^2$)

$$
\begin{aligned}
\frac{d\sigma}{d\hat{t}}(qq' \to qq') &= \frac{4}{9}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right] \\
\frac{d\sigma}{d\hat{t}}(qg \to qg) &= -\frac{4}{9}\left[\frac{\hat{s}}{\hat{u}}+\frac{\hat{u}}{\hat{s}}\right]+\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} \\
\frac{d\sigma}{d\hat{t}}(gg \to gg) &= \frac{9}{2}\left[3-\frac{\hat{t}\hat{u}}{\hat{s}^2}-\frac{\hat{s}\hat{u}}{\hat{t}^2}-\frac{\hat{s}\hat{t}}{\hat{u}^2}\right]
\end{aligned}
$$

- In the parton-parton center-of-mass frame, one can write

$$\hat{t} = -\frac{\hat{s}}{2}(1-\cos(\theta^*))$$
$$\hat{u} = -\frac{\hat{s}}{2}(1+\cos(\theta^*))$$

- Therefore, as $\cos(\theta^*) \to 1(-1), \hat{t}\,(\hat{u}) \to 0$. Hence, in this limit, the first two subprocesses on the preceding page behave as $(1-|\cos(\theta^*)|)^{-1}$ while the next three behave as $(1-|\cos(\theta^*)|)^{-2}$.

7. Center of mass scattering angle

Start in the parton-parton center of mass frame where one has

$$\begin{aligned} p_1 &= \frac{\sqrt{\hat{s}}}{2}(1,0,0,1) \quad p_2 = \frac{\sqrt{\hat{s}}}{2}(1,0,0-1) \\ p_3 &= \frac{\sqrt{\hat{s}}}{2}(1,\sin\theta^*,0,\cos\theta^*) \quad p_4 = \frac{\sqrt{\hat{s}}}{2}(1,-\sin\theta^*,0,-\cos\theta^*) \end{aligned}$$

from which one can derive

$$\hat{t} = -\frac{\hat{s}}{2}\left(1-\cos(\theta^*)\right) \quad \hat{u} = -\frac{\hat{s}}{2}\left(1+\cos(\theta^*)\right).$$

Next, write the parton four-vectors in the hadron-hadron frame as

$$
\begin{aligned}
p_1 &= \frac{\sqrt{s}}{2} x_a (1,0,0,1) \\
p_2 &= \frac{\sqrt{s}}{2} x_b (1,0,0-1) \\
p_3 &= p_T (\cosh y_3, 1, 0, \sinh y_3) \\
p_4 &= p_T (\cosh y_4, -1, 0, \sinh y_4)
\end{aligned}
$$

which can be used to derive

$$\begin{aligned}\hat{t} &= -\sqrt{s}x_a p_T e^{-y_3} \\ \hat{u} &= -\sqrt{s}x_a p_T e^{-y_4}.\end{aligned}$$

From these two sets of expressions one can obtain

$$\frac{\hat{t}}{\hat{u}} = e^{-(y_3-y_4)} = \frac{1-\cos\theta^*}{1+\cos\theta^*}.$$

It then follows that

$$\cos\theta^* = \tanh\frac{y_3-y_4}{2}.$$

8. Some comments on the asymptotic solution of the evolution equations for parton distributions in a photon

   - Rewrite the evolution equations by taking moments of both sides using the following definitions:

$$\begin{aligned}
M_q^n &= \int_0^1 dx\ x^{n-1}\ G_{q/\gamma}(x) \\
M_g^n &= \int_0^1 dx\ x^{n-1}\ G_{g/\gamma}(x) \\
A_{ij}^n &= \frac{1}{2\pi b}\int_0^1 dx\ x^{n-1}\ P_{ij}(x) \\
a^n &= \frac{\alpha}{2\pi}\int_0^1 dx\ x^{n-1}\ P_{q\gamma} \\
\alpha_s(t) &= \frac{1}{bt}
\end{aligned}$$

     where $t = \ln(Q^2/\Lambda^2)$.

- The evolution equations can now be written as

$$\begin{aligned}\frac{dM_q^n}{dt} &= e_q^2\, a^n + \frac{1}{t}\left[A_{qq}^n\, M_q^n + A_{qg}^n\, M_g^n\right]\\ \frac{dM_g^n}{dt} &= \frac{1}{t}\left[\sum_q A_{gq}^n M_q^n + A_{gg}^n M_g^n\right]\end{aligned}$$

- If each of the moments is proportional to t, the t dependence drops out of the equations and they may be solved algebraically

- The asymptotic solution is

$$
\begin{aligned}
M_q^n &= a^n \left(\frac{e_q^2 - 5/18}{1 - A_{qq}^n} + \frac{5}{18} \frac{1 - A_{gg}^n}{F^n} \right) t \\
M_g^n &= \frac{5f}{9} a^n \frac{A_{gg}^n}{F^n} t \\
F^n &= 1 - A_{qq}^N - A_{gg}^n + A_{qq}^n A_{gg}^n - 2f A_{qg}^n A_{gq}^n
\end{aligned}
$$

where f is the number of flavors

- Note how the moments are each proportional to t
- Compare to the case where $P_{q\gamma} = 0$ where the moments are of the form

$$
M^n(t_0) \left(\frac{t}{t_0} \right)^{A^n}
$$

- Note that one can add any solution of the homogeneous evolution equations to this asymptotic solution