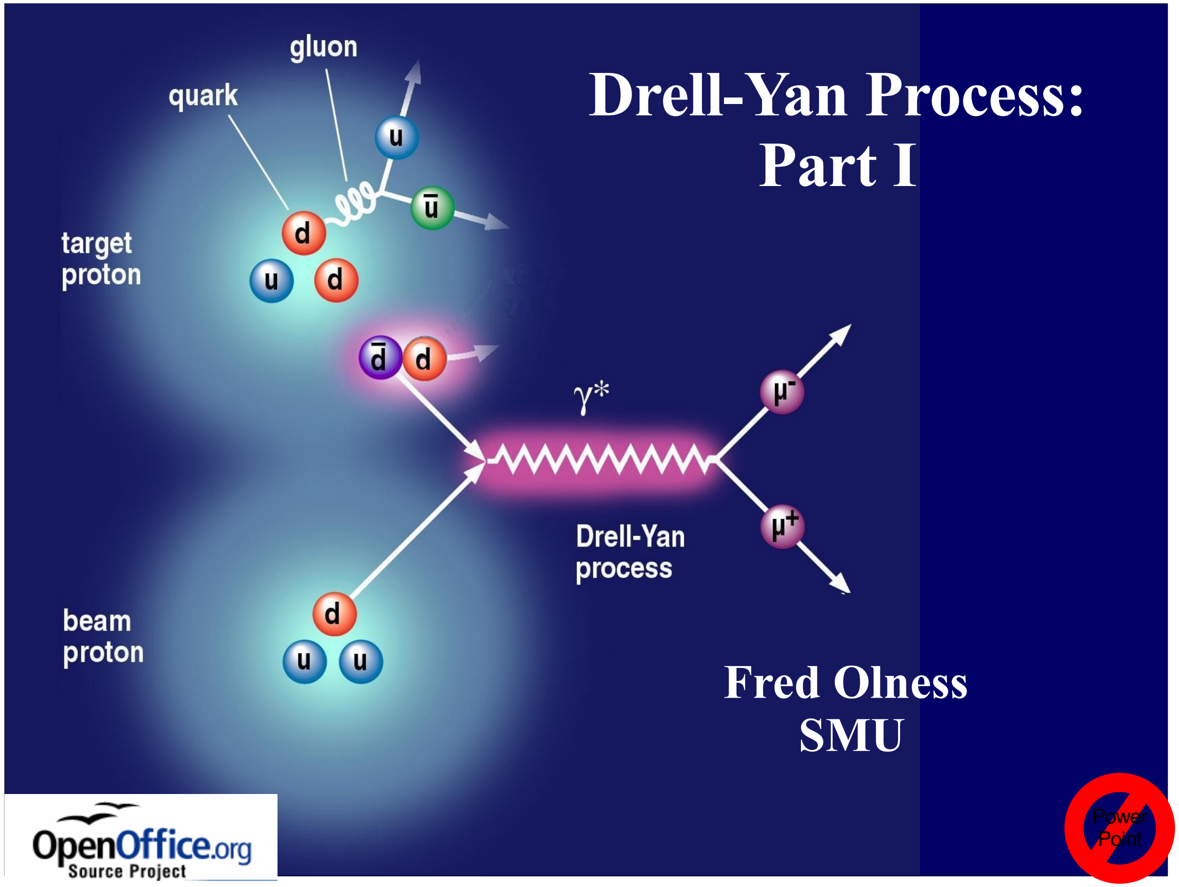
Calculation of: $q + \bar{q} \rightarrow e^{+} + e^{-}$

Taken from:

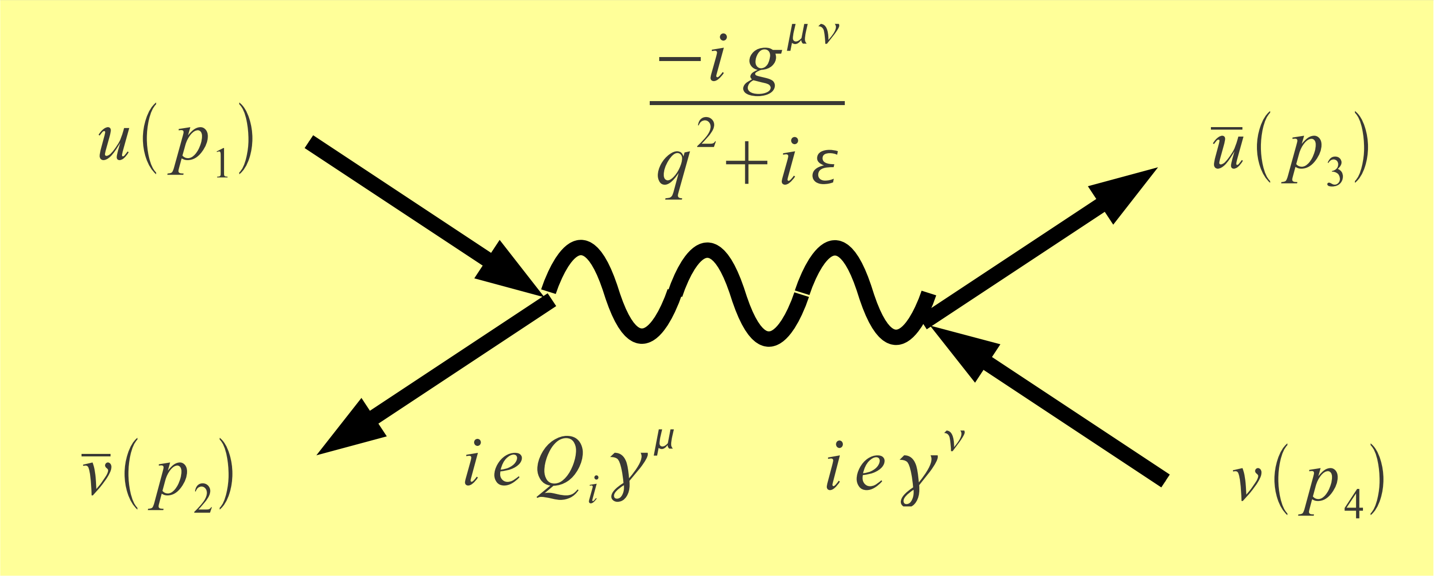


Original at:

http://www.physics.smu.edu/~olness/cteq2003/

Linked from: cteq.org

Let's compute the Born process: $q+\bar{q}\to e^{+}+e^{-}$



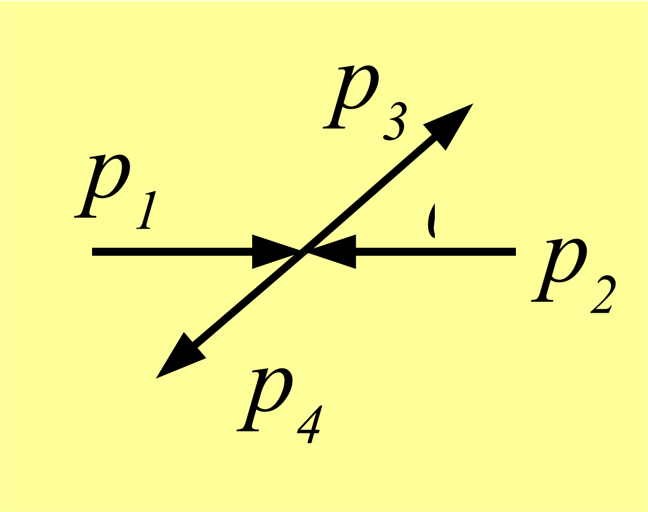
Gathering factors and contracting $g^{\mu\nu}$, we obtain:

$$-i\,M = i\,Q_i\,\frac{e^2}{q^2}\,\{\bar{v}(p_2)\gamma^{\mu}u(p_1)\}\;\{\bar{u}(p_3)\gamma_{\mu}v(p_4)\}$$

Squaring, and averaging over spin and color, ...

$$\overline{|M|^2} = \left(\frac{1}{2}\right)^2 3\left(\frac{1}{3}\right)^2 Q_i^2\,\frac{e^4}{q^4}\,Tr\left[\not{p}_2\gamma^{\mu}\not{p}_1\gamma^{\nu}\right]\;Tr\left[\not{p}_3\gamma_{\mu}\not{p}_4\gamma_{\nu}\right]$$

Let's work out some parton level kinematics



$$p_1 = \frac{\sqrt{\hat{s}}}{2}\left(1,0,0,+1\right)$$
$$p_2 = \frac{\sqrt{\hat{s}}}{2}\left(1,0,0,-1\right)$$
$$p_3 = \frac{\sqrt{\hat{s}}}{2}\left(1,+\sin(\theta),0,+\cos(\theta)\right)$$
$$p_4 = \frac{\sqrt{\hat{s}}}{2}\left(1,-\sin(\theta),0,-\cos(\theta)\right)$$

$$p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0$$

Defining the Mandelstam variables ...

$$\hat{s} = (p_1 + p_2)^2 = (p_3 + p_4)^2$$
$$\hat{t} = (p_1 - p_3)^2 = (p_2 - p_4)^2$$
$$\hat{u} = (p_1 - p_4)^2 = (p_2 - p_3)^2$$

$$\hat{t} = -\frac{\hat{s}}{2}\left(1-\cos(\theta)\right)$$
$$\hat{u} = -\frac{\hat{s}}{2}\left(1+\cos(\theta)\right)$$

We'll now compute the matrix element M

Manipulating the traces, we find ...

$$Tr\left[\not{p}_2 \gamma^\mu \not{p}_1 \gamma^\nu\right] \; Tr\left[\not{p}_3 \gamma_\mu \not{p}_4 \gamma_\nu\right]$$
$$= 4\left[p_1^\mu p_2^\nu + p_2^\mu p_1^\nu - g^{\mu\nu}(p_1 \cdot p_2)\right] \times 4\left[p_3^\mu p_4^\nu + p_4^\mu p_3^\nu - g^{\mu\nu}(p_3 \cdot p_4)\right]$$
$$= 2^5 \left[(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)\right]$$
$$= 2^3 \left[\hat{t}^2 + \hat{u}^2\right]$$

Where we have used:

$$p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0$$

$$\hat{s} = 2(p_1 \cdot p_2) = 2(p_3 \cdot p_4)$$
$$\hat{t} = 2(p_1 \cdot p_3) = 2(p_2 \cdot p_4)$$
$$\hat{u} = 2(p_1 \cdot p_4) = 2(p_2 \cdot p_3)$$

Putting all the pieces together, we have:

$$\overline{|M|^2} = Q_i^2 \, \alpha^2 \, \frac{2^5 \pi^2}{3} \left(\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}\right)$$

with

$$q^2 = (p_1 + p_2)^2 = \hat{s}$$
$$\alpha = \frac{e^2}{4\pi}$$

... and put it together to find the cross section

$$d\hat{\sigma} \simeq \frac{1}{2\hat{s}} \overline{|M|^2} \, d\Gamma$$

In the partonic CMS system

$$d\Gamma = \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) = \frac{d\cos(\theta)}{16\pi}$$

Recall,

$$\hat{t} = \frac{-\hat{s}}{2}\left(1 - \cos(\theta)\right) \quad and \quad \hat{u} = \frac{-\hat{s}}{2}\left(1 + \cos(\theta)\right)$$

so, the differential cross section is ...

$$\frac{d\hat{\sigma}}{d\cos(\theta)} = Q_i^2 \alpha^2 \frac{\pi}{6} \frac{1}{\hat{s}} \left(1 + \cos^2(\theta)\right)$$

and the total cross section is ...

$$\hat{\sigma} = Q_i^2 \alpha^2 \frac{\pi}{6} \frac{1}{\hat{s}} \int_{-1}^{1} d\cos(\theta) \left(1 + \cos^2(\theta)\right) = \frac{4\pi\alpha^2}{9\hat{s}} Q_i^2 \equiv \hat{\sigma}_0$$

Homework: Part 1

#1) Show:

$$\frac{d^3 p}{(2\pi)^3 2E} = \frac{d^4 p}{(2\pi)^4} (2\pi)\, \delta^+(p^2 - m^2)$$

This relation is often useful as the RHS is manifestly Lorentz invariant

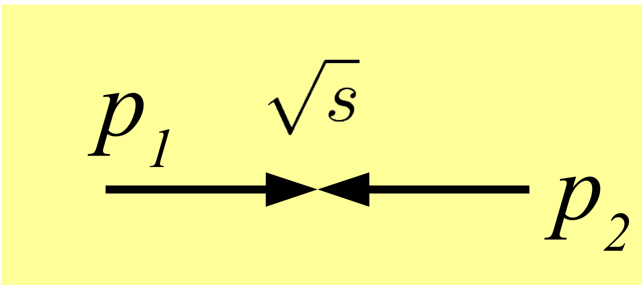
#2) Show that the 2-body phase space can be expressed as:

$$d\Gamma = \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) = \frac{d\cos(\theta)}{16\pi}$$

Note, we are working with massless partons, and θ is in the partonic CMS frame

Homework : Part 2

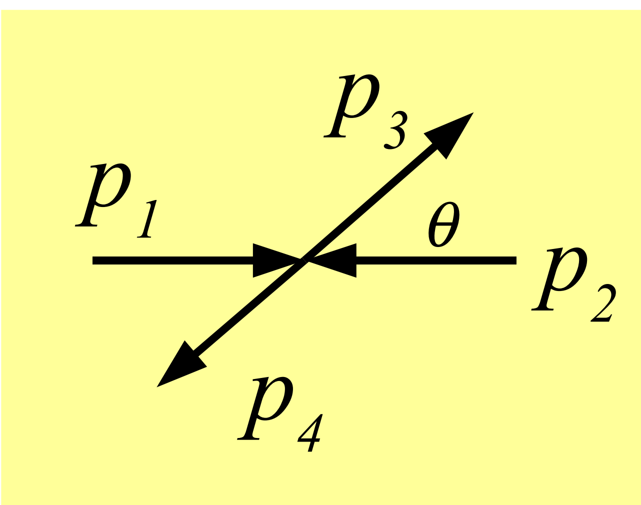
1) Let's work out the general 2→2 kinematics for general masses.



a) Start with the incoming particles.
Show that these can be written in the general form:

$$p_1 = (E_1, 0, 0, +p) \qquad p_1^2 = m_1^2$$
$$p_2 = (E_2, 0, 0, -p) \qquad p_2^2 = m_2^2$$

... with the following definitions:



$$E_{1,2} = \frac{\hat{s} \pm m_1^2 \mp m_2^2}{2\sqrt{\hat{s}}} \qquad p = \frac{\Delta(\hat{s}, m_1^2, m_2^2)}{2\sqrt{\hat{s}}}$$

$$\Delta(a,b,c) = \sqrt{a^2+b^2+c^2-2(ab+bc+ca)}$$

Note that $\Delta(a,b,c)$ is symmetric with respect to its arguments, and involves the only invariants of the initial state: s, m_1^2, m_2^2.

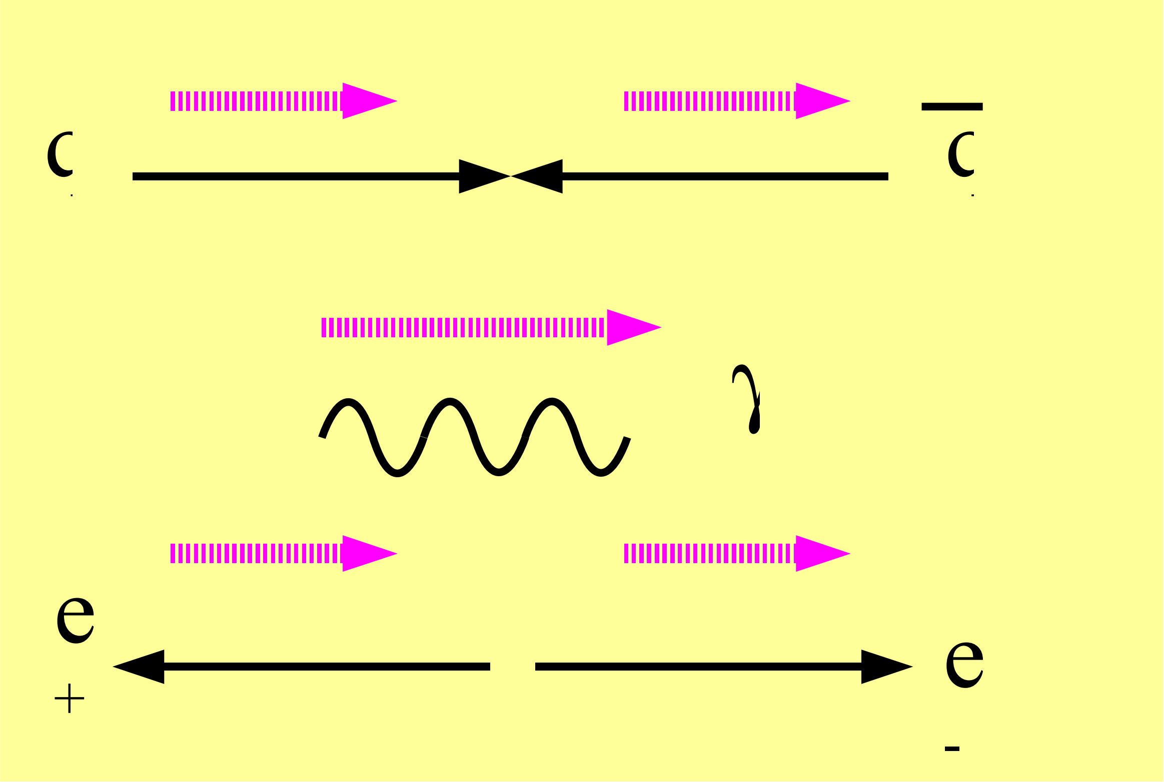
b) Next, compute the general form for the final state particles, p_3 and p_4. Do this by first aligning p_3 and p_4 along the z-axis (as p_1 and p_2 are), and then rotate about the y-axis by angle θ.

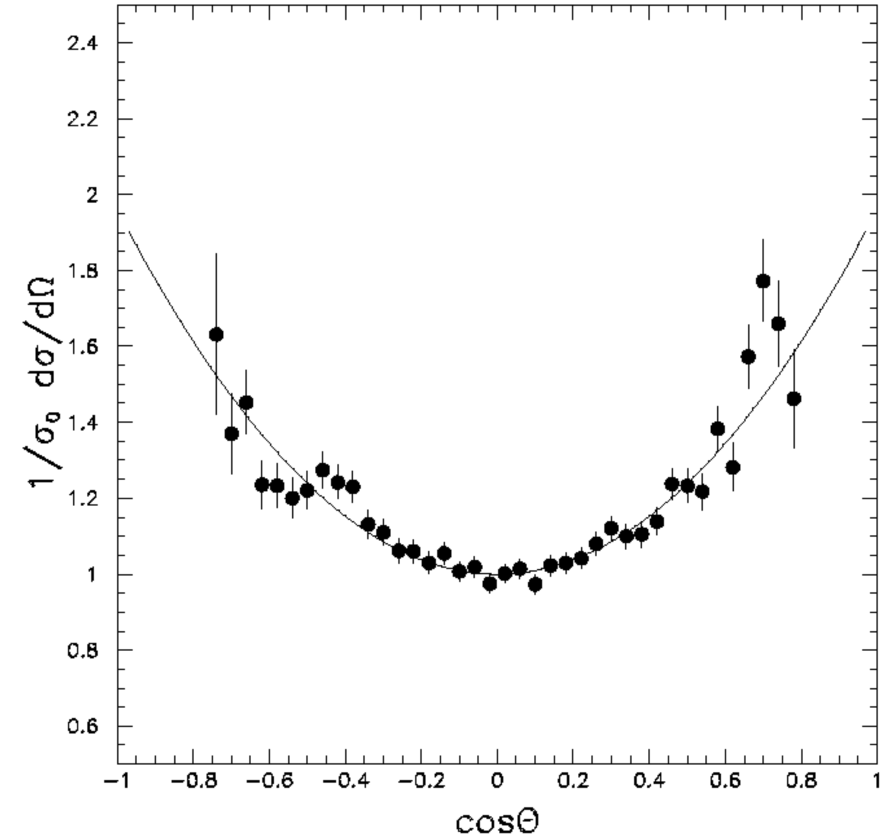
What does the angular dependence tell us?

Observe, the angular dependence: $q+\bar{q}\rightarrow e^{+}+e^{-}$

$$\frac{d\hat{\sigma}}{d\cos(\theta)} = Q_i^2\,\alpha^2\,\frac{\pi}{6}\,\frac{1}{\hat{s}}\left(1+\cos^2(\theta)\right)$$

Characteristic of scattering of spin ½ constitutients by a spin 1 vector





Note, for the photon, the mirror image of the above is also valid; hence the symmetric distribution. The W has V-A couplings, so we'll find: $(1+\cos\theta)^2$