Neutrino Physics

What exactly is a "Neutrino" and How Does it Interact?

(and what good is it for studying QCD?)

CTEQ SS10
Lauterbad, Germany

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Objectives of this Lecture

- Birth of Neutrino Physics
- Growing Pains - the puzzles come much more rapidly than the solutions
- Vocabulary of Neutrino Physics
- Where do we stand today with the question "What is a neutrino?" - the current challenges

If some time can be found at this evening's recitation

- How do neutrinos (in a particular state) interact with matter and contribute to QCD studies

(Thanks for slides/figures to B. Kayser and K. McFarland)

Neutrinos Are Everywhere!

Neutrinos outnumber ordinary matter particles in the Universe (electrons, protons, neutrons) by a huge factor (10^8 or so).

- Depending on their masses they may account for a fraction (% or two?) of the "dark matter"
- Neutrinos are important for stellar dynamics: ~ 6.6×10^{10} $cm^{-2}s^{-1}$ stream through the Earth from the sun. Neutrinos also govern Supernovae dynamics, and hence heavy element production.
- Neutrinos carry most (~99%) of the energy from a Supernova explosion
- large numbers formed at the time of the big bang are still whizzing around the Universe ("relic neutrinos"). ~400 / cm^3 of space.
- **To understand the nature of the Universe in which we live we must understand the properties of the neutrino.**

A bit of history… 1930 - Wolfgang Pauli

Dear Radioactive Ladies and Gentlemen….

Dear Radioactive Ladies and Gentlemen,

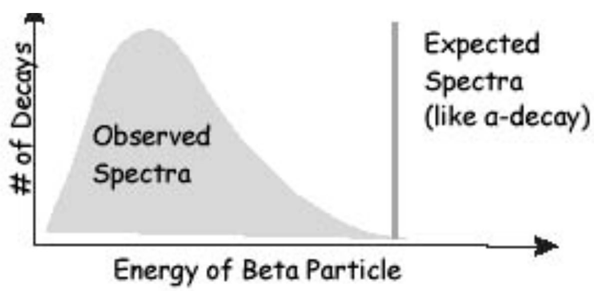
As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant... ....

Unfortunately, I cannot appear in Tubingen personally since I am indispensable here in Zurich because of a ball on the night of 6/7 December. With my best regards to you, and also to Mr Back.

Your humble servant,

W. Pauli

Within a year Pauli was under analysis with C. Jung



N. Bohr suggested energy not conserved in β decays
L. Meitner proposed β^- loses energy through secondary interactions in nulceus yielding gamma rays

First Calculation of Neutrino Cross Sections

using the "Fermi" theory from 1932

Bethe-Peierls (1934): calculation of first cross-section for inverse beta reaction using Fermi's theory for:

yields: $\bar{\nu}_e + p \rightarrow n + e^+$ or $\nu_e + n \rightarrow p + e^-$

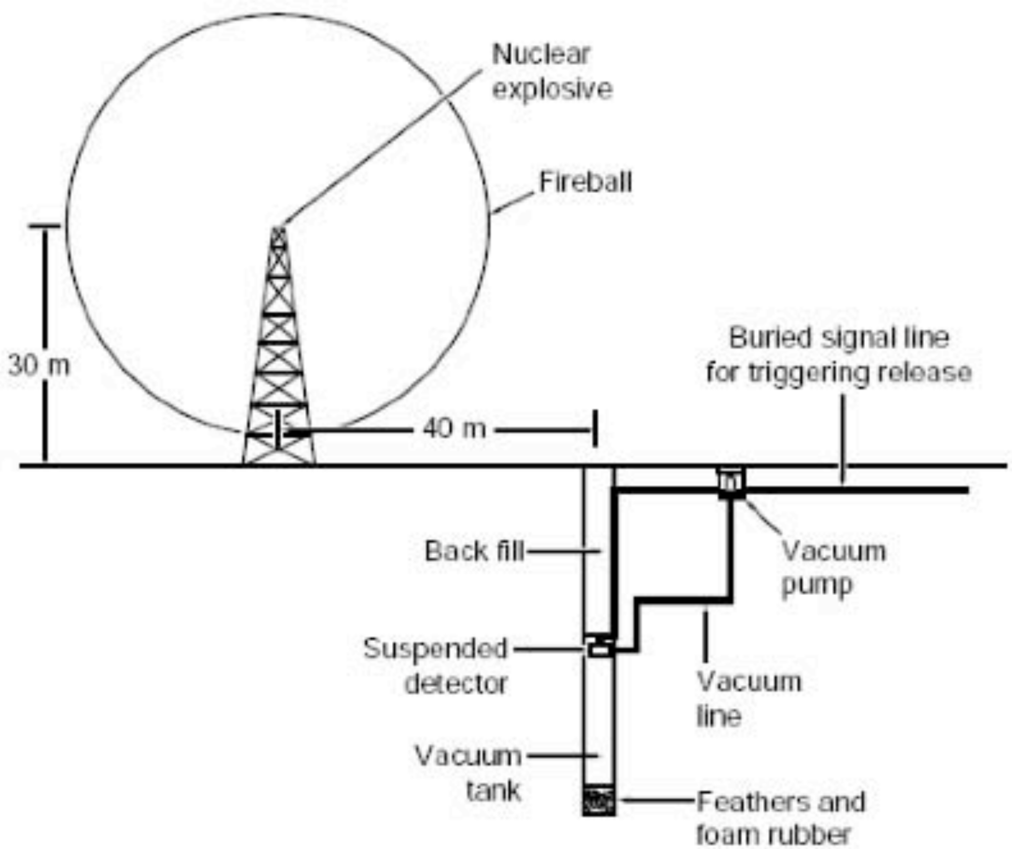
$$\sigma \approx 10^{-44}\ cm^2 \quad \text{for} \quad E(\bar{\nu}) = 2\ MeV$$

This means that the mean free path of a neutrino in water is:

$$\lambda = \frac{1}{n\sigma} \approx 1.5 \times 10^{21}\ cm \approx 1600 \quad light-years$$

Experimentalists groaned - need a very intense source of ν's to detect inverse Beta decay

Project Poltergeist from 1950's



I. Explode bomb
II.At same time let detector fall in vacuum tank
III. Detect neutrinos
IV. Collect Nobel prize

OK – but repeatability is a bit of a problem

They Finally Found the Right Source - Experimental Detection of the Neutrino

In nuclear reactors fission of $_{92}U^{235}$ produces chain of beta reactions

1

$$(A,Z) \rightarrow (A,Z+1) + e^- + \bar{\nu}_e \rightarrow (A,Z+2) + e^- + \bar{\nu}_e \rightarrow \ldots$$

$$N_{\bar{\nu}} \approx 5.6 \times 10^{20}\ s^{-1}\ in\ 4\pi$$

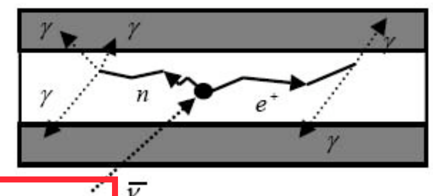
$$\bar{\nu}_e + p \rightarrow n + e^+ \quad \text{or} \quad \nu_e + n \rightarrow p + e^-$$

26 YEARS LATER!!

Reines and Cowan detect in 1953 (Hanford) (discovery confirmed 1956 in Savannah River)

1) Detection of two back-to-back γ's from prompt signal e+e-->γγ at t=0.

2) Neutron thermalization: neutron capture in Cd, emission of late γ's



σ = (11 ± 2.6) x 10^{-44} cm^2 (within 5% of expected)

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2 Existence of "second" neutrino ν_μ established in 1962 by Schwartz, Lederman and Steinberger at Brookhaven National Laboratory

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3 First direct evidence for the third (and last?) neutrino - ν_τ - by the DONUT collaboration at Fermilab in 2000 **70 years after the Pauli hypothesis.**

Power of the Neutrino

$$\bar{\nu}_e + p \rightarrow n + e^+ \quad \text{or} \quad \nu_e + n \rightarrow p + e^-$$

- Neutrinos are picky and "taste" only specific flavors of quarks.
  - Neutrinos interact with d, s, $\bar{u}$ and $\bar{c}$
  - Antineutrinos interact with u, c, $\bar{d}$ and $\bar{s}$
- Neutrinos have the power to change the flavor of the quark with which they interact.
- Chirality/ Iso-spin arguments dictate the selection criteria (more later)

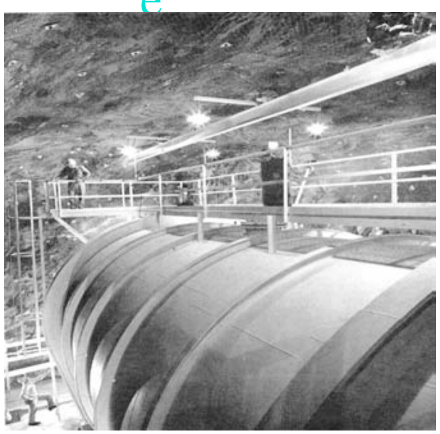
Where the Puzzles Start…Solar Neutrinos

10^{12} solar ν's/sec pass through your brain

Nuclear reactions in the core of the sun produce ν_e and only ν_e.

In 1968, Ray Davis' Homestake experiment measured the higher-E part of the ν_e flux ϕ_{ν_e} that arrives at earth using a huge tank of "cleaning fluid" and $\nu_e + {}^{37}Cl \longrightarrow {}^{37}Ar + e^-$

Theorists, especially John Bahcall, calculated the produced ν_e solar flux vs. E and predicted that Davis should see **36** Ar **atoms** per **month**.



$$\frac{\phi_{\nu_e}\text{(Homestake)}}{\phi_{\nu_e}\text{(Theory)}} = \mathbf{0.34 \pm 0.06}$$

What was going on?

The Possible Solutions:

The theory was wrong.

The experiment was wrong.

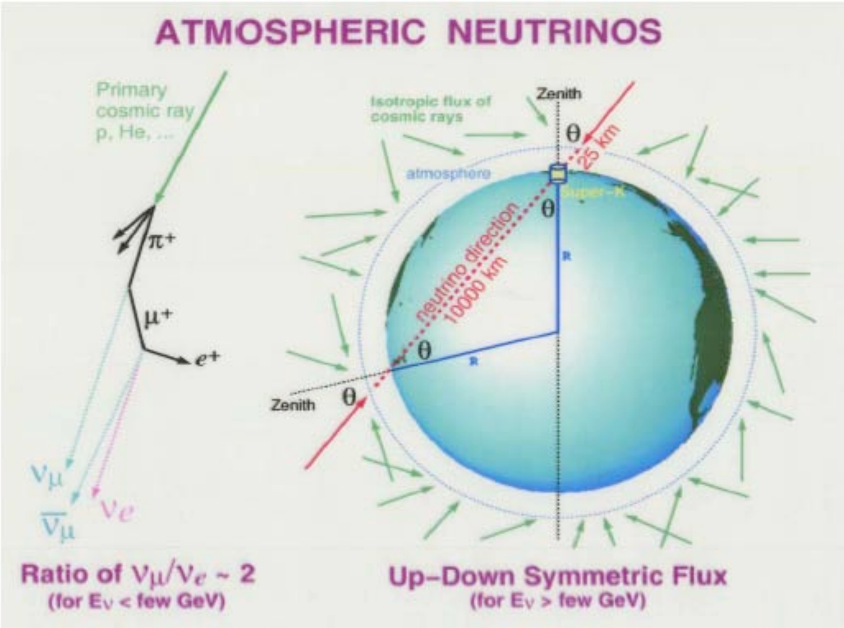
Both were wrong.

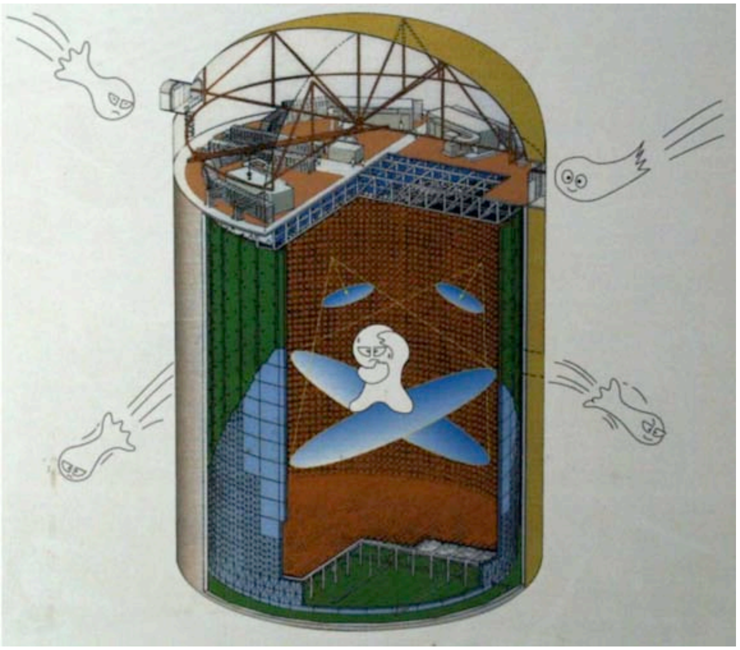
The most radical - NEITHER was wrong.

2/3 of the solar ν_e flux “disappears” on the way to earth

(changes into something that the Homestake experiment could not see).

Next Puzzle - Atmospheric Neutrinos





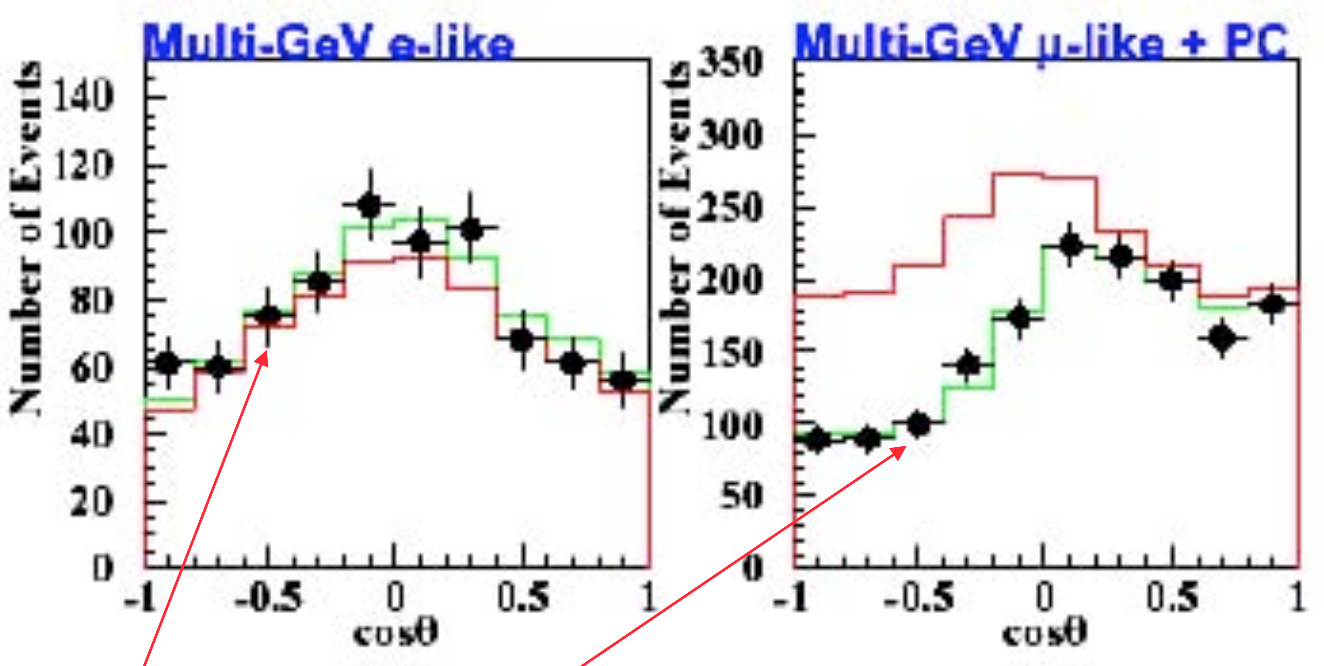
2 GeV cosmic rays hit the earth isotropically, and we expect:

$$\Rightarrow \frac{\phi_{\nu_\mu}(\mathrm{Up})}{\phi_{\nu_\mu}(\mathrm{Down})} \approx 1.0$$

However, Super-Kamiokande (50 kT water) found for $E_\nu > 1.3$ GeV

$$\frac{\phi_{\nu_\mu}(\mathrm{Up})}{\phi_{\nu_\mu}(\mathrm{Down})} = 0.54 \pm 0.04 \, .$$

Resolution of the Atmospheric Neutrino Anomaly



Upward-going muon neutrinos depleted, while upward-going electron neutrinos slightly higher than expected

VERY suggestive of Neutrino Oscillations

Green curve in above figures

Resolution of Solar Neutrino Puzzle:

Neutrinos Change Flavor Between the Sun and the Earth

Sudbury Neutrino Observatory (SNO) measures (high E part):

Smiling John

$$\nu_{sol}\, d \rightarrow e\, p\, p \;\Rightarrow\; \phi_{\nu_e}$$

$$\nu_{sol}\, d \rightarrow \nu\, n\, p \;\Rightarrow\; \phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} \quad \text{Total } \nu_{sol} \text{ flux}$$

$$\frac{\phi_{\nu_e}}{\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}} = 0.340 \pm 0.023\ (\text{stat}) \pm 0.030\ (\text{syst})$$

Total Flux of Neutrinos

SNO: $\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} = (4.94 \pm 0.21 \pm 0.36) \times 10^6/\text{cm}^2\text{sec}$

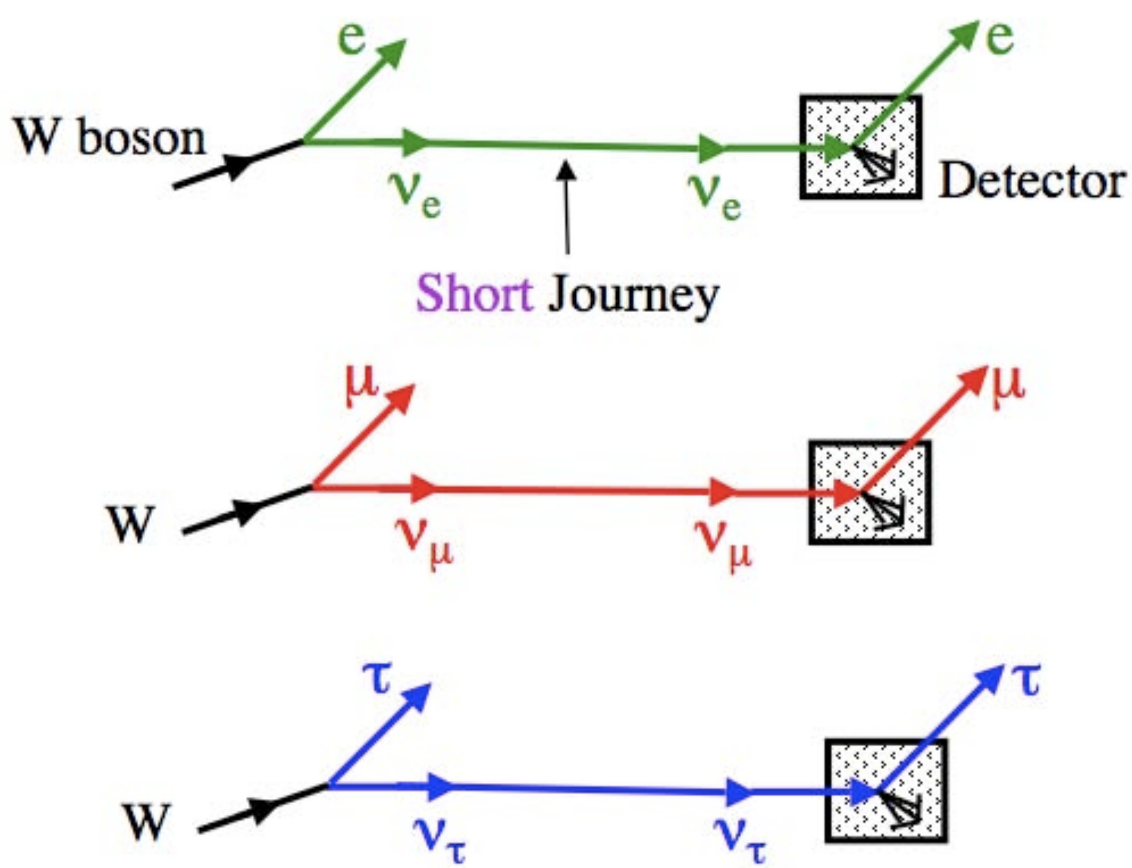
Theory: $\phi_{total} = (5.69 \pm 0.91) \times 10^6/\text{cm}^2\text{sec}$

BOTH RAY DAVIS AND JOHN BAHCALL WERE RIGHT

Oscillation Hypothesis confirmed by KamLAND Reactor Results

What are Neutrino Oscillations ?
Flavor States

- Neutrinos come in (at least) three flavors. Each of the flavors are associated with a charged lepton flavor.



Massive neutrinos…?

The neutrino of flavor α, ν_α, is the one created in W decay together with ℓ_α,

and the one that, when it creates a charged lepton, creates ℓ_α.

But if neutrinos have masses, and leptons mix, then during a ***long*** journey, a neutrino born as ν_α can evolve into something ***different*** that can create a charged lepton ℓ_β of a different flavor from the ℓ_α with which ν_α was born.

Neutrino Mass and Leptonic mixing

There is some spectrum of 3 or more neutrino mass eigenstates ν_i:

Mass (ν_i) $\equiv$ m_i

$\ell_e \equiv e$, $\ell_\mu \equiv \mu$, $\ell_\tau \equiv \tau$

When $W^+ \rightarrow \ell_\alpha^+ + \nu_\alpha$,

e, μ, or τ

the produced neutrino state $|\nu_\alpha\rangle$ is

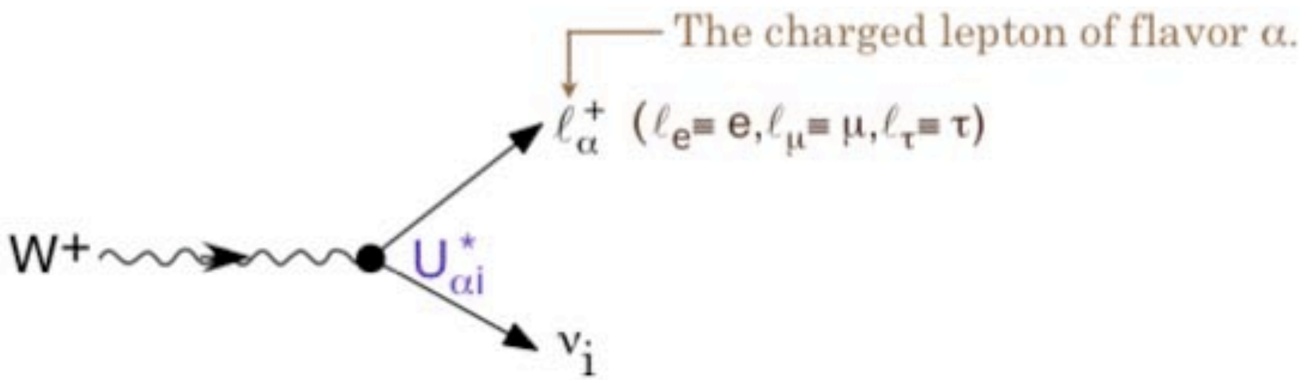
$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle \ .$$

Neutrino of flavor α

Neutrino of definite mass m_i

Leptonic Mixing Matrix

Another way to look at W decay



A given $\ell_\alpha{}^+$ can be accompanied by *any* ν_i.

$$\mathrm{Amp}(W^+ \rightarrow \ell_\alpha{}^+ + \nu_i) = U^*{}_{\alpha i}$$

The neutrino state $|\nu_\alpha>$ produced together with $\ell_\alpha{}^+$

is $|\nu_\alpha> = \sum_i U^*{}_{\alpha i} |\nu_i>$.

Mass $\longleftrightarrow$ Flavor

Just as each neutrino of definite flavor ν_α is a superposition of mass eigenstates ν_i, so each mass eigenstate is a superposition of flavors .

From $|\nu_\alpha> = \Sigma_i U^*_{\alpha i} |\nu_i>$ and the unitarity of U,

$$|\nu_i> = \Sigma_\alpha U_{\alpha i} | \nu_\alpha> .$$

The flavor-α fraction of ν_i is —

$$|<\nu_\alpha|\nu_i>|^2 = |U_{\alpha i}|^2 .$$

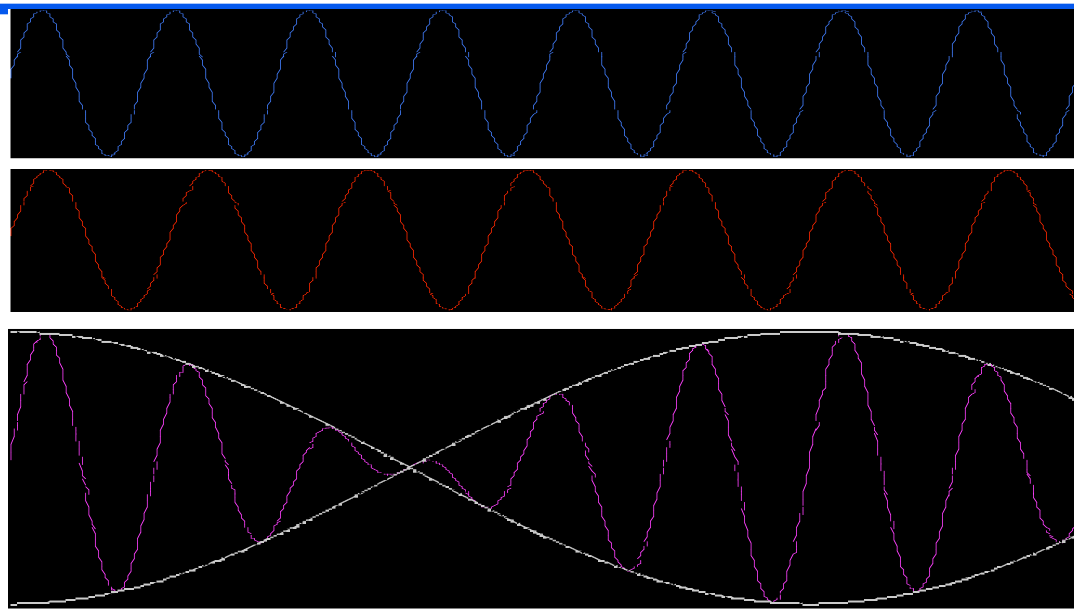
Propagation

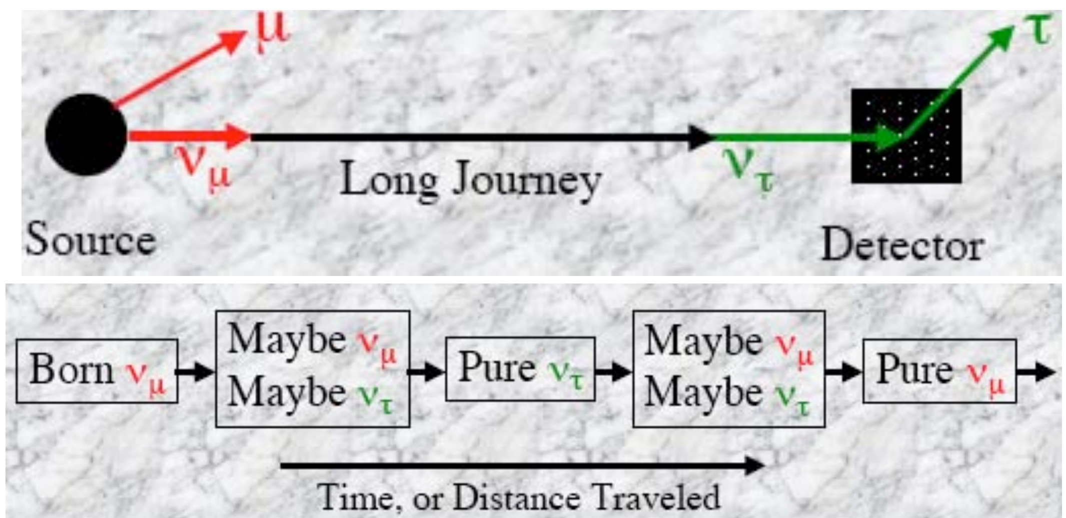
- The U_{lm} are known as the leptonic mixing matrix U.
- The flavor state ν_α is a superposition of several mass states with differing masses which cause them to propagate differently yielding neutrino oscillations.
- The amplitude for the transformation $\nu_\alpha \dashrightarrow \nu_{\alpha'}$ is:

$$A(\nu_l \rightarrow \nu_{l'}) = \sum A(\nu_l \text{ is } \nu_m) A(\nu_m \text{ propagates}) A(\nu_m \text{ is } \nu_{l'})$$

$$A(\nu_m \text{ propagates}) = \exp\left(-i \frac{M_m^2}{2} \frac{L}{E}\right)$$

Oscillating between two different types of ν





2-Flavor Oscillation

- As an example, if there are only two flavors involved in the oscillations then the U matrix takes on the following form and the probability (square of the amplitude) can be expressed as:

$$U = \begin{pmatrix} \cos\theta & e^{i\delta}\sin\theta \\ -e^{-i\delta}\sin\theta & \cos\theta \end{pmatrix} \quad \text{and}$$

$$P(\nu_1 \to \nu_{1'}) = \boxed{\sin^2 2\theta} \sin^2\left[1.27\Delta m^2 (eV^2)\frac{L(km)}{E(GeV)}\right]$$

$$\text{with} \quad \boxed{\Delta m^2 \equiv M_2^2 - M_1^2}$$

- Life is more complicated with 3 flavors, but the principle is the same and we get bonus of possible CP violations as in the quark sector $P(\nu_\mu \to \nu_e) \neq P(\bar{\nu}_\mu \to \bar{\nu}_e)$.

- The components of U now involve θ_{13} , θ_{23} ,θ_{12} and δ and the probabilities involve Δm_{13} , Δm_{23} and Δm_{12} .

Basic 3-flavor Oscillation Phenomenology

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad \Rightarrow U = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}$$

$c_{ij} = \cos\theta_{ij}$ $s_{ij} = \sin\theta_{ij}$

"Solar" "Atmospheric CP Violation "????"

$$P_{\nu_e\nu_\mu(\bar{\nu}_e\bar{\nu}_\mu)}(x) = s_{23}^2 \sin^2 2\theta_{13} \sin^2\left[\frac{\Delta m_{23}^2}{4E}x\right]$$

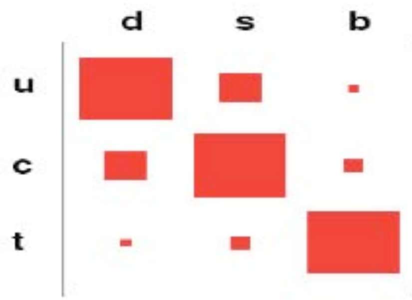
$$P_{\nu_e\nu_\tau(\bar{\nu}_e\bar{\nu}_\tau)}(x) = c_{23}^2 \sin^2 2\theta_{13} \sin^2\left[\frac{\Delta m_{23}^2}{4E}x\right]$$

$$P_{\nu_\mu\nu_\tau(\bar{\nu}_\mu\bar{\nu}_\tau)}(x) = c_{13}^4 \sin^2 2\theta_{23} \sin^2\left[\frac{\Delta m_{23}^2}{4E}x\right]$$

$$\left|\Delta m_{12}^2\right| << \left|\Delta m_{23}^2\right| , \left|\Delta m_{13}^2\right| \approx \left|\Delta m_{23}^2\right|$$

The Neutrino Mixing matrix is quite different than the standard quark mixing matrix - why?

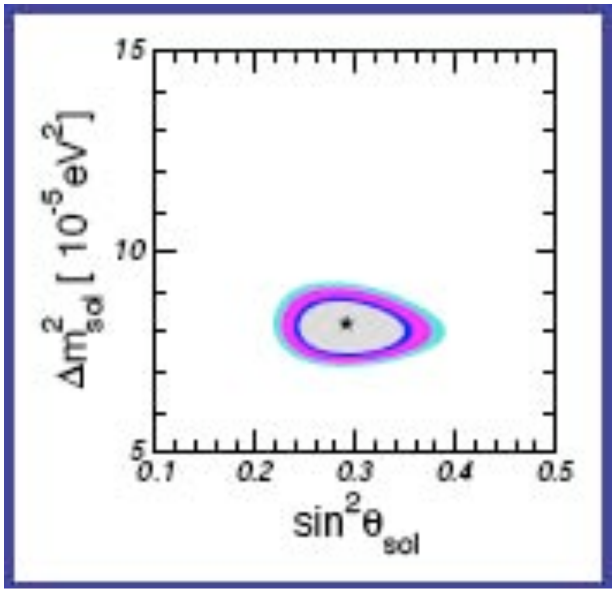
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

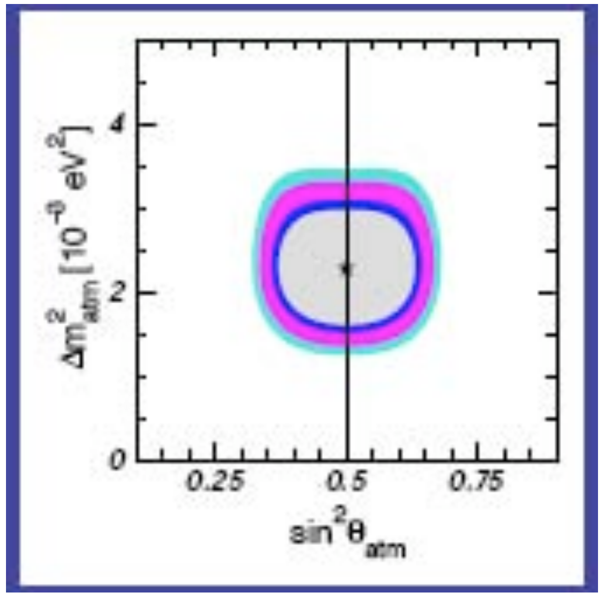


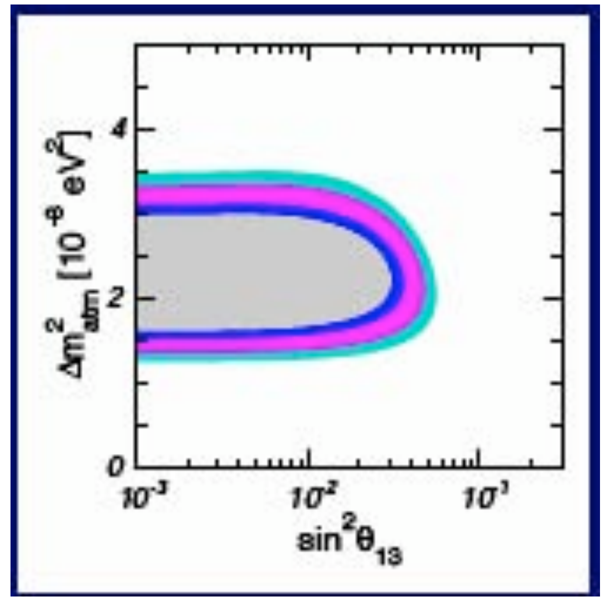
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & >0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$

How are experimental neutrino oscillation results presented?







"Solar"

$\Delta m_{12} = (7.9 \pm 0.3) \times 10^{-5}$ eV^2

$\sin^2\Theta_{12} = (0.31 \pm .03)$

Solar + KamLAND

"Atmospheric

$\Delta m_{23} = (2.2\,^{+.37}_{-.27}) \times 10^{-3}$ eV^2

$\sin^2\Theta_{23} = (0.50 \pm .06)$

SuperK + K2K

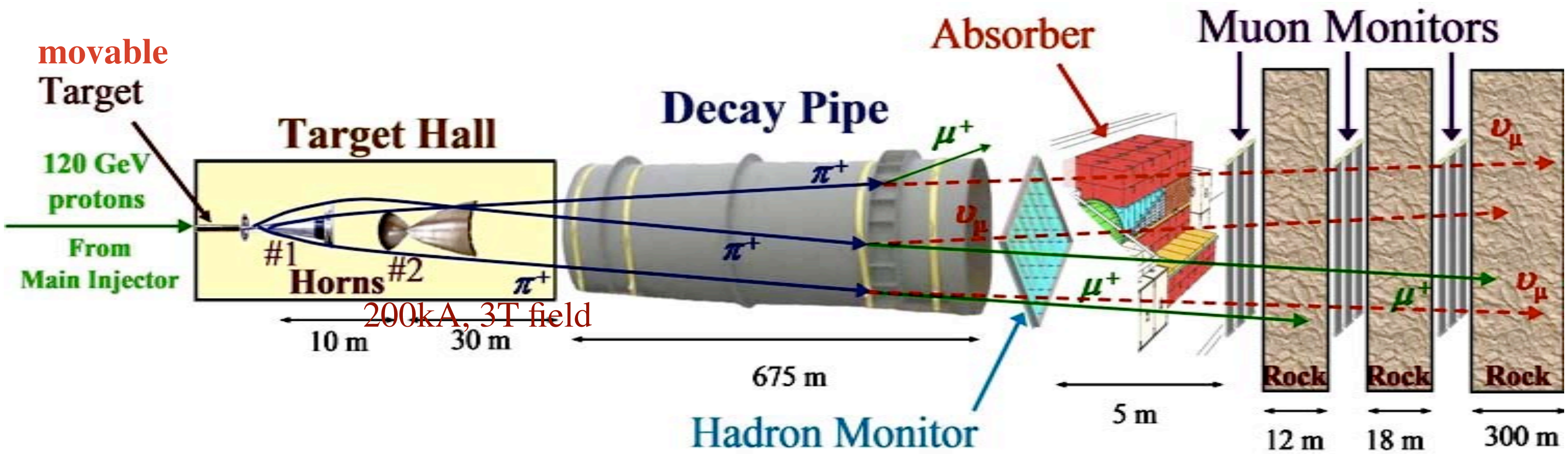
$\nu_e \longleftrightarrow \nu_{\mu/\tau}$ Osc.

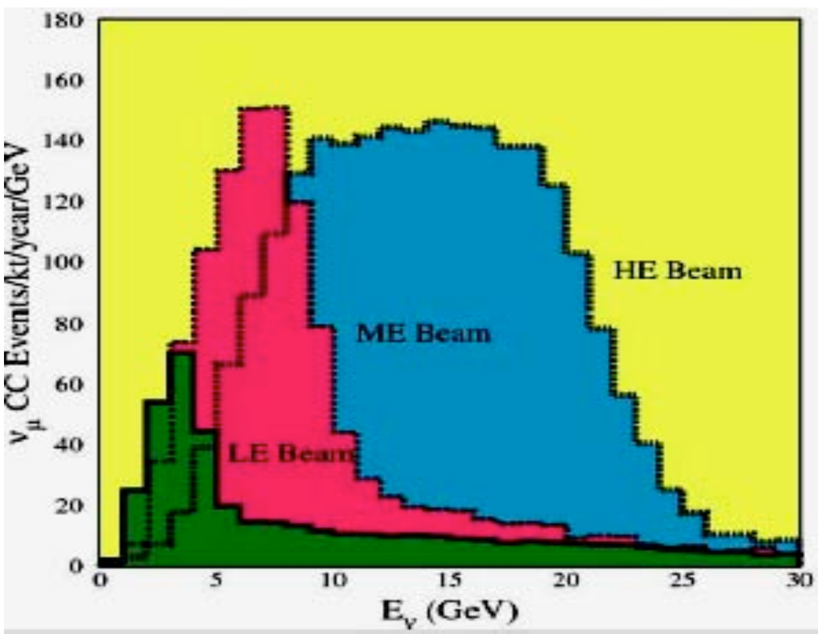
$\Delta m_{13} \approx \Delta m_{23}$

$\sin^2\Theta_{13} < 0.046\ (3\sigma)$

Chooz

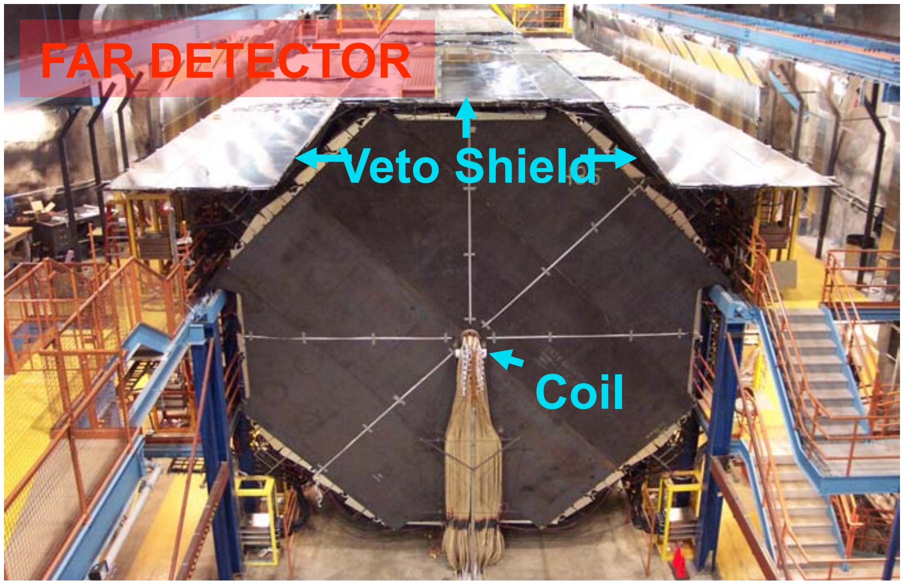
Speaking of experiments… how do we measure these parameters?





Beam	Target z position-cm	FD Events per 1e20 pot
LE-10	-10	390
ME	-100	1500
HE	-350	3410

The MINOS Experiment
Two Neutrino Detectors 735 km apart



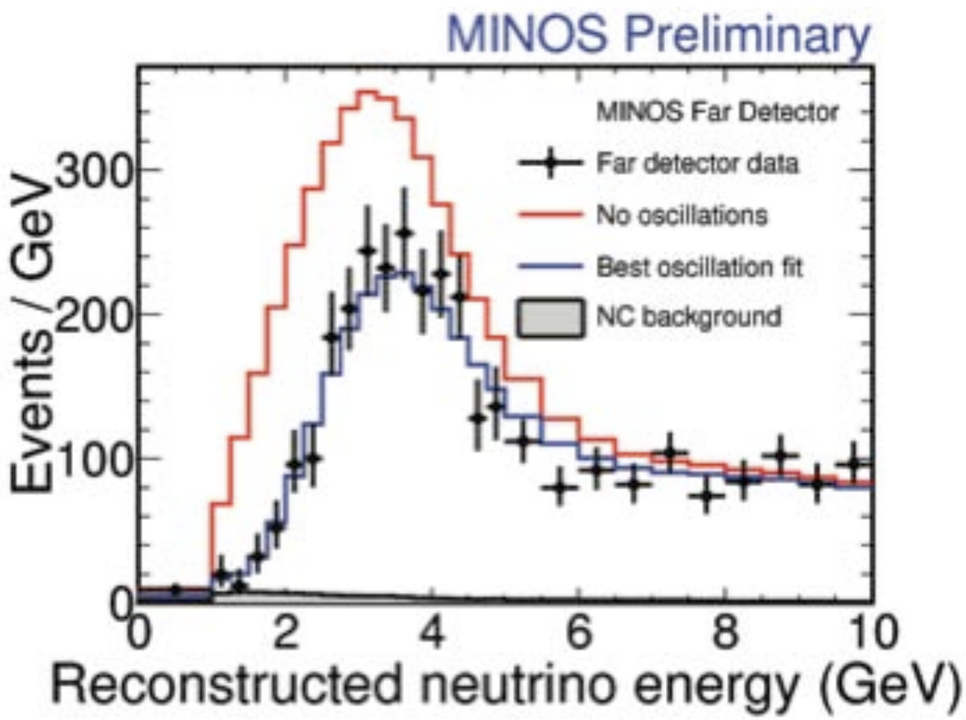
2.54 cm thick magnetized (1.2T) steel plates
4.1x1cm scintillator strips:orthogonal U,V planes

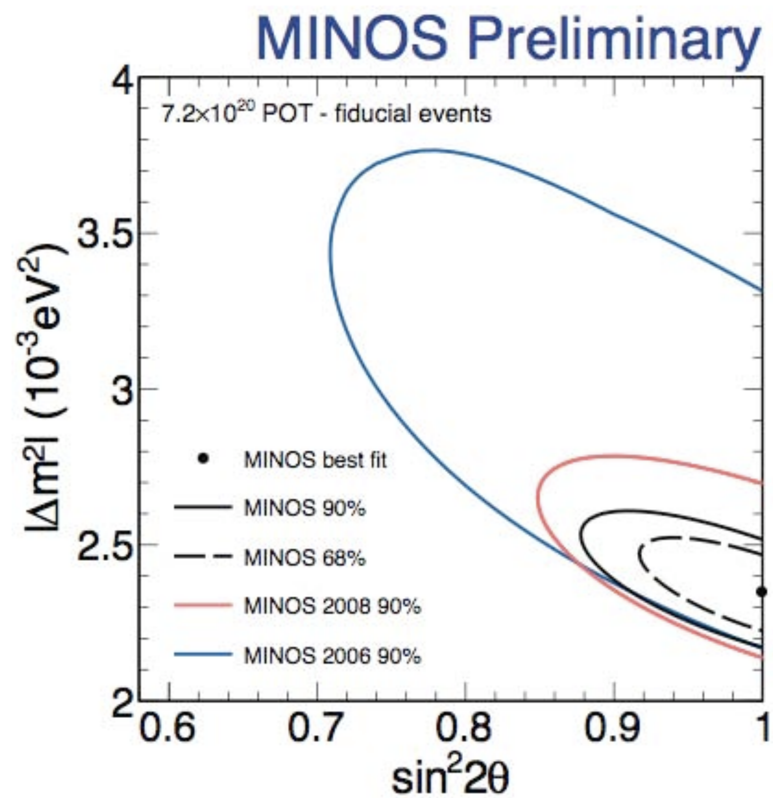
	Far Det	Near Det
Mass(kt)	5.4	1
Size(m^3)	8x8	3.8x4.8
Steel/Scint. Planes	484/484	282/152

MINOS Best-Fit

7.2 x 10^{20} POT

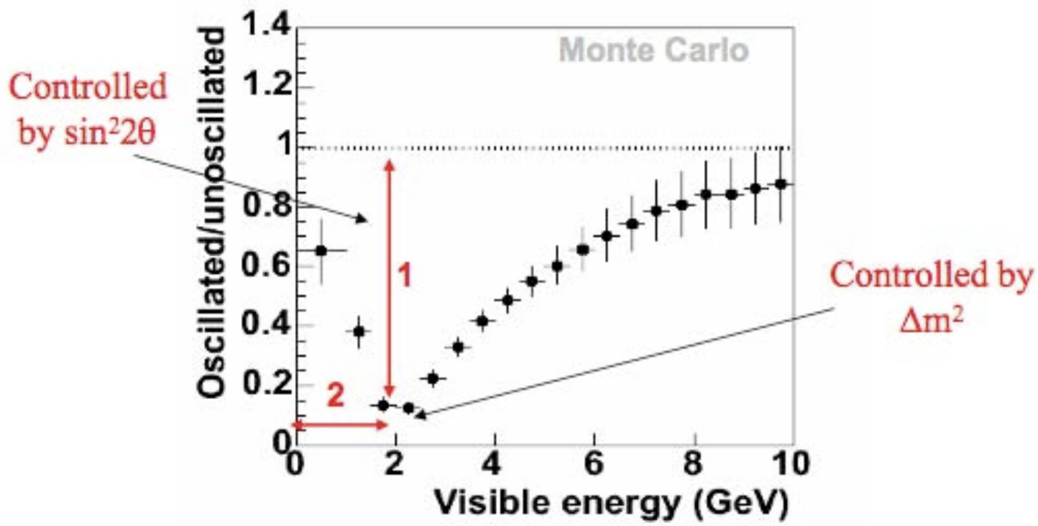
- Observe **1986** events in FD expect **2451** with no oscillations

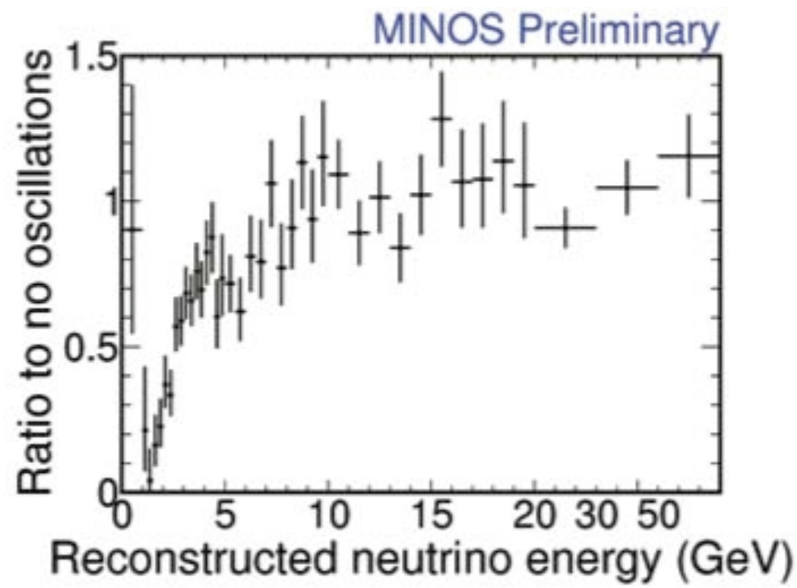


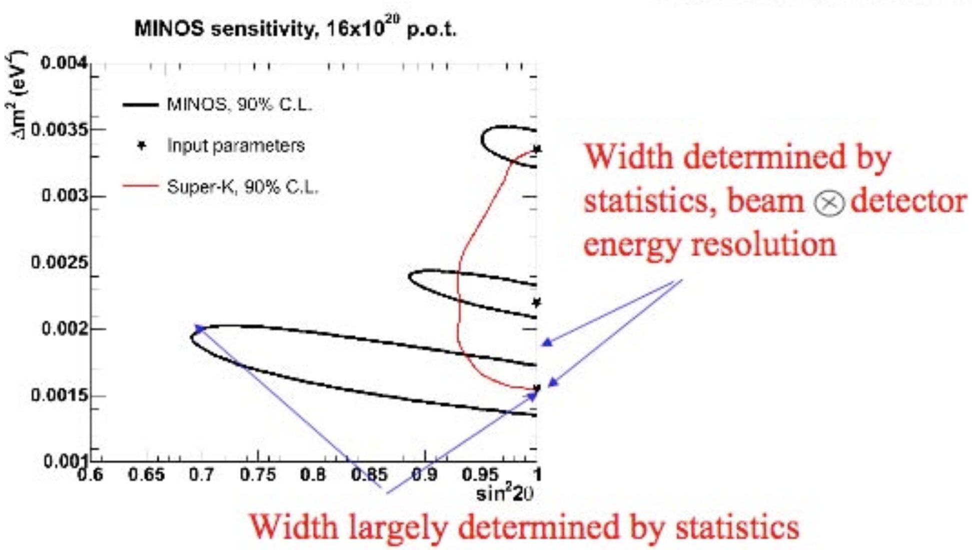


- $\Delta m^2 = 2.35^{+0.11}_{-0.08}$ x 10^{-3} eV^2 (68% CL), $\sin^2(2\theta) > 0.91$ (90% CL)

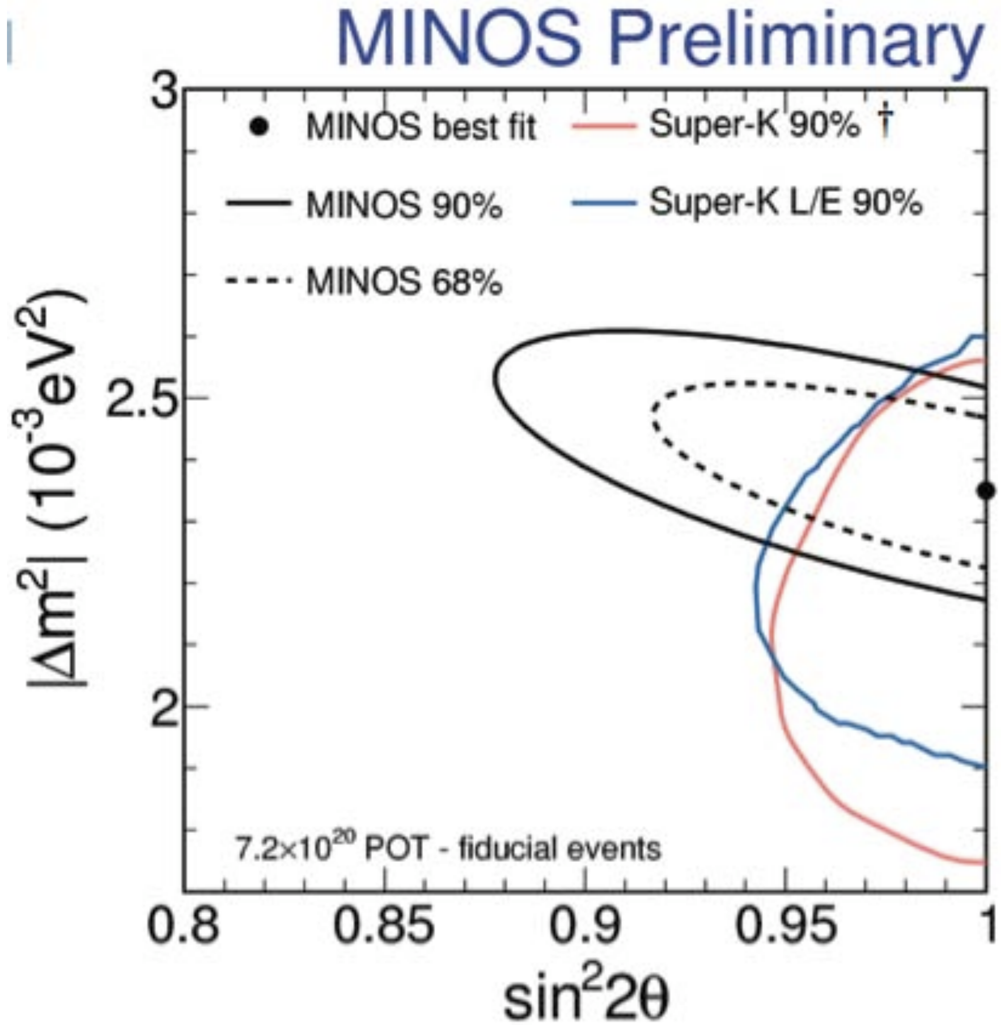
How to interpret oscillation results







Latest MINOS Results compared to SK



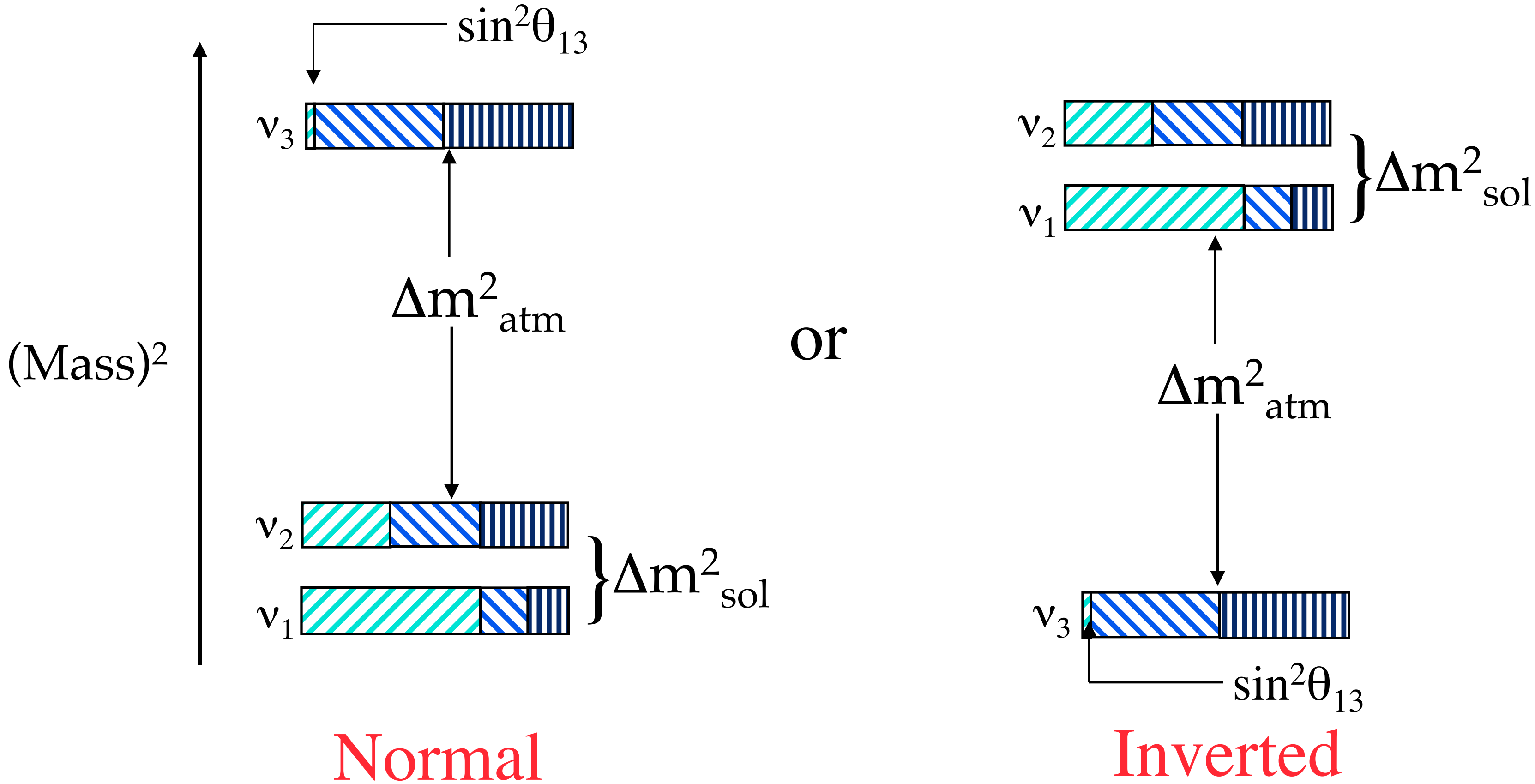
†Super-Kamiokande Collaboration (preliminary)

$$\left|\Delta m^2\right| = 2.35^{+0.11}_{-0.08} \times 10^{-3}\,\mathrm{eV}^2$$

$$\sin^2(2\theta) > 0.91\ (90\%\ \mathrm{C.L.})$$

- Contour includes effects of dominant systematic uncertainties
  - normalization
  - NC background
  - shower energy
  - track energy

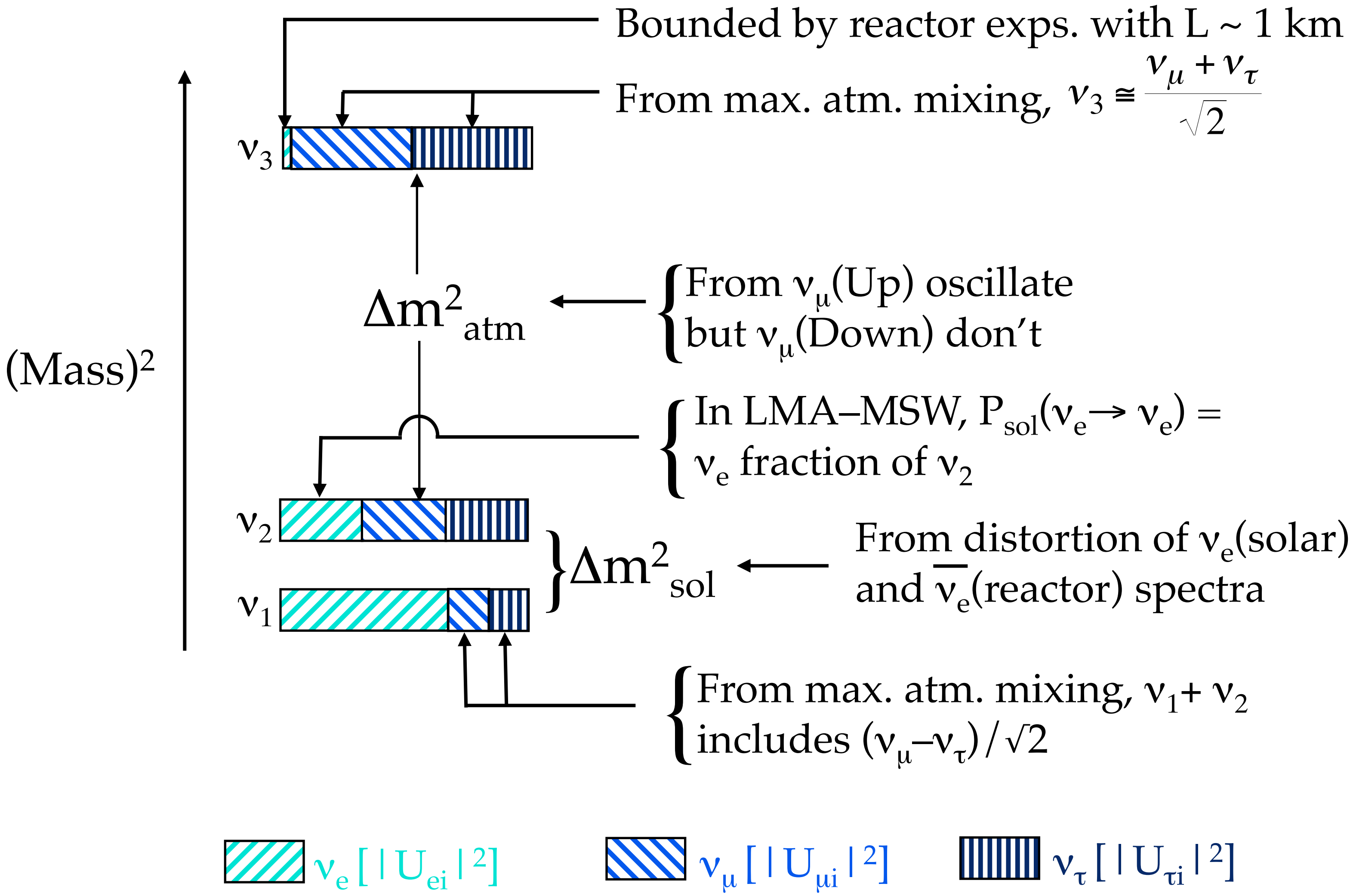
A representation of our knowledge …



$\Delta m^2_{sol} = \sim 8 \times 10^{-5}\ eV^2$, $\Delta m^2_{atm} = \sim 2.5 \times 10^{-3}\ eV^2$

$\nu_e\ [\,|U_{ei}|^2]$ $\nu_\mu [\,|U_{\mu i}|^2]$ $\nu_\tau\ [\,|U_{\tau i}|^2]$

Where Does This Come From?



A Global Fit to Neutrino Data

Dominated by

parameter	best fit	2σ	3σ	
Δm^2_{21} $[10^{-5}\text{eV}^2]$	$7.65^{+0.23}_{-0.20}$	7.25–8.11	7.05–8.34	KamLAND
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3}\text{eV}^2]$	$2.40^{+0.12}_{-0.11}$	2.18–2.64	2.07–2.75	MINOS
$\sin^2\theta_{12}$	$0.304^{+0.022}_{-0.016}$	0.27–0.35	0.25–0.37	SNO
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.39–0.63	0.36–0.67	SuperK
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$	≤ 0.040	≤ 0.056	Chooz

arXiv:0808.2016

How are we doing?

- We are doing pretty well…. right?
- Yes, we seem to have a pretty good experimental grasp with neutrino oscillations.
- However…. we now have enough data to look at accelerator-based oscillations with antineutrinos.

CP

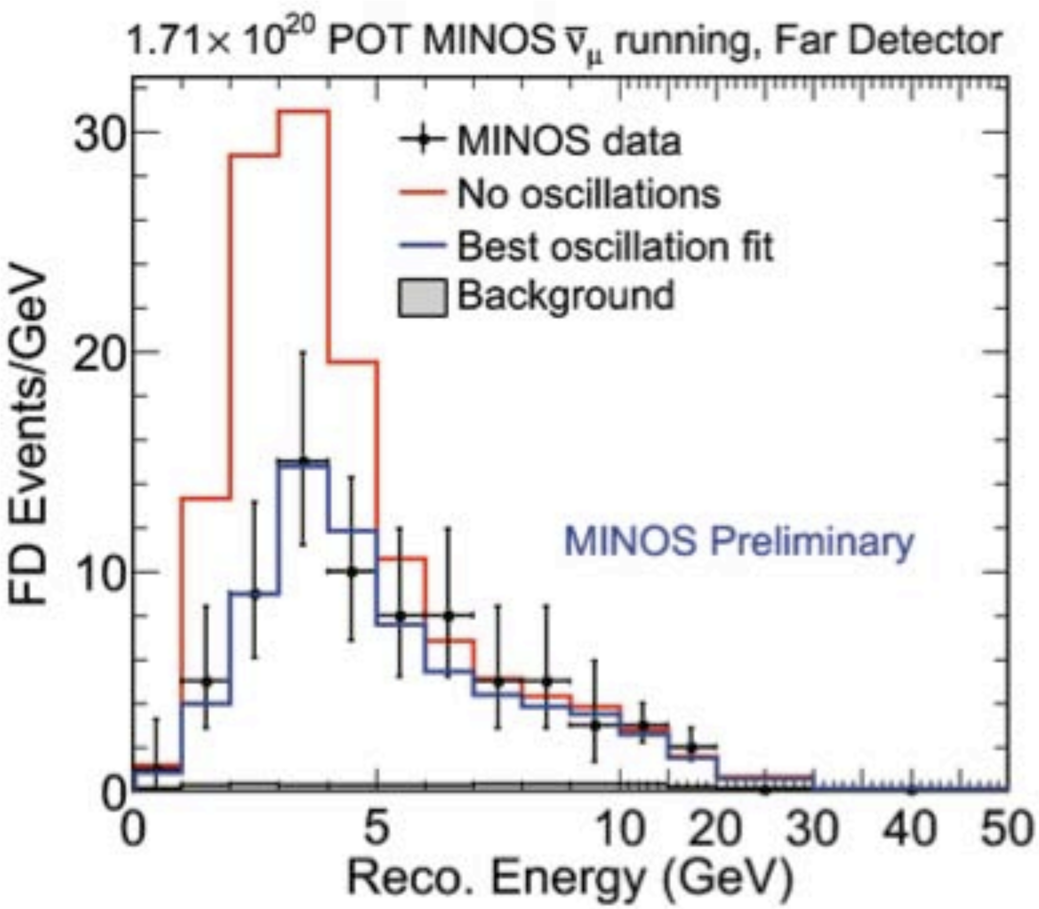
$$\nu_\mu \rightarrow \nu_e \quad \Longleftrightarrow \quad \bar{\nu}_\mu \rightarrow \bar{\nu}_e$$

T $\Updownarrow$ CPT across diagonals $\Updownarrow$ T

$$\nu_e \rightarrow \nu_\mu \quad \Longleftrightarrow \quad \bar{\nu}_e \rightarrow \bar{\nu}_\mu$$

CP

First MINOS Antineutrino Results

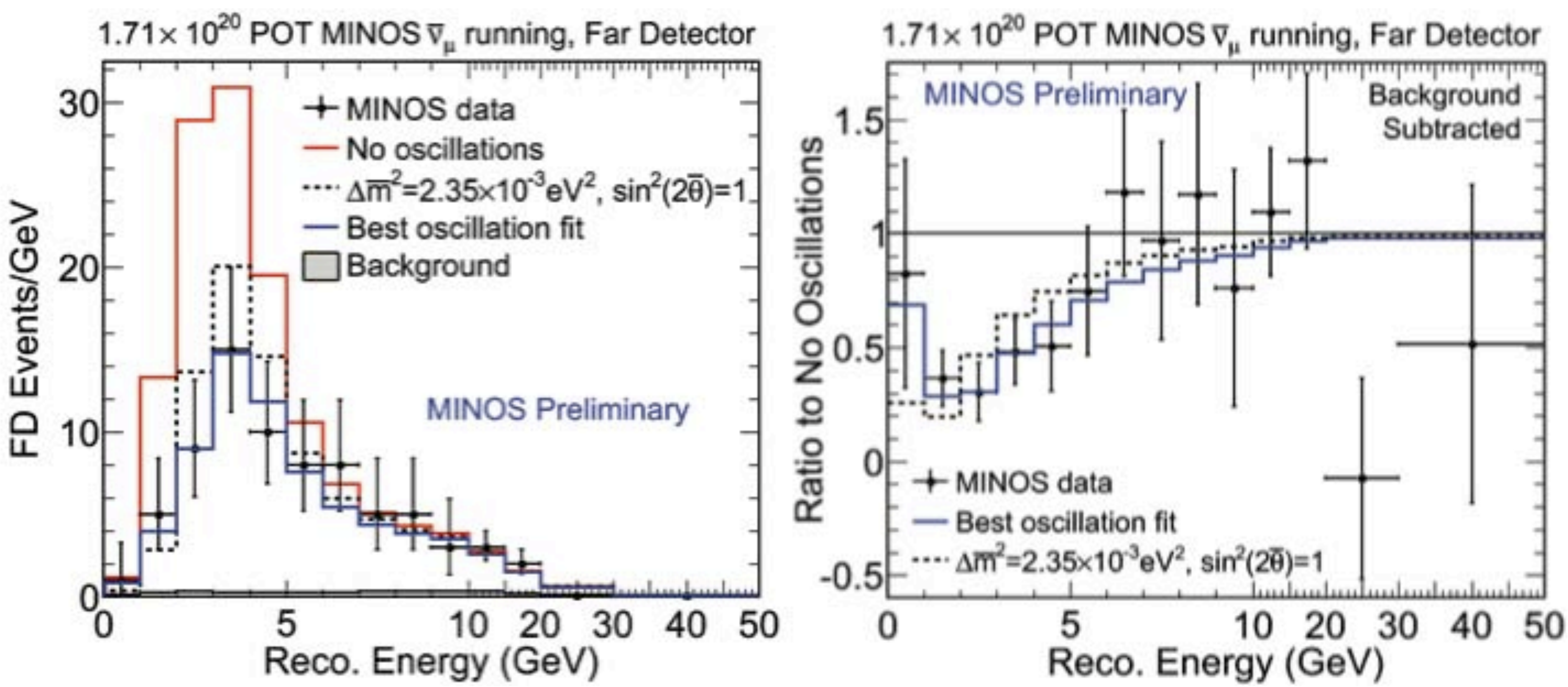


- No oscillation Prediction: **155**
- Observe: **97**
- No oscillations disfavored at 6.3σ

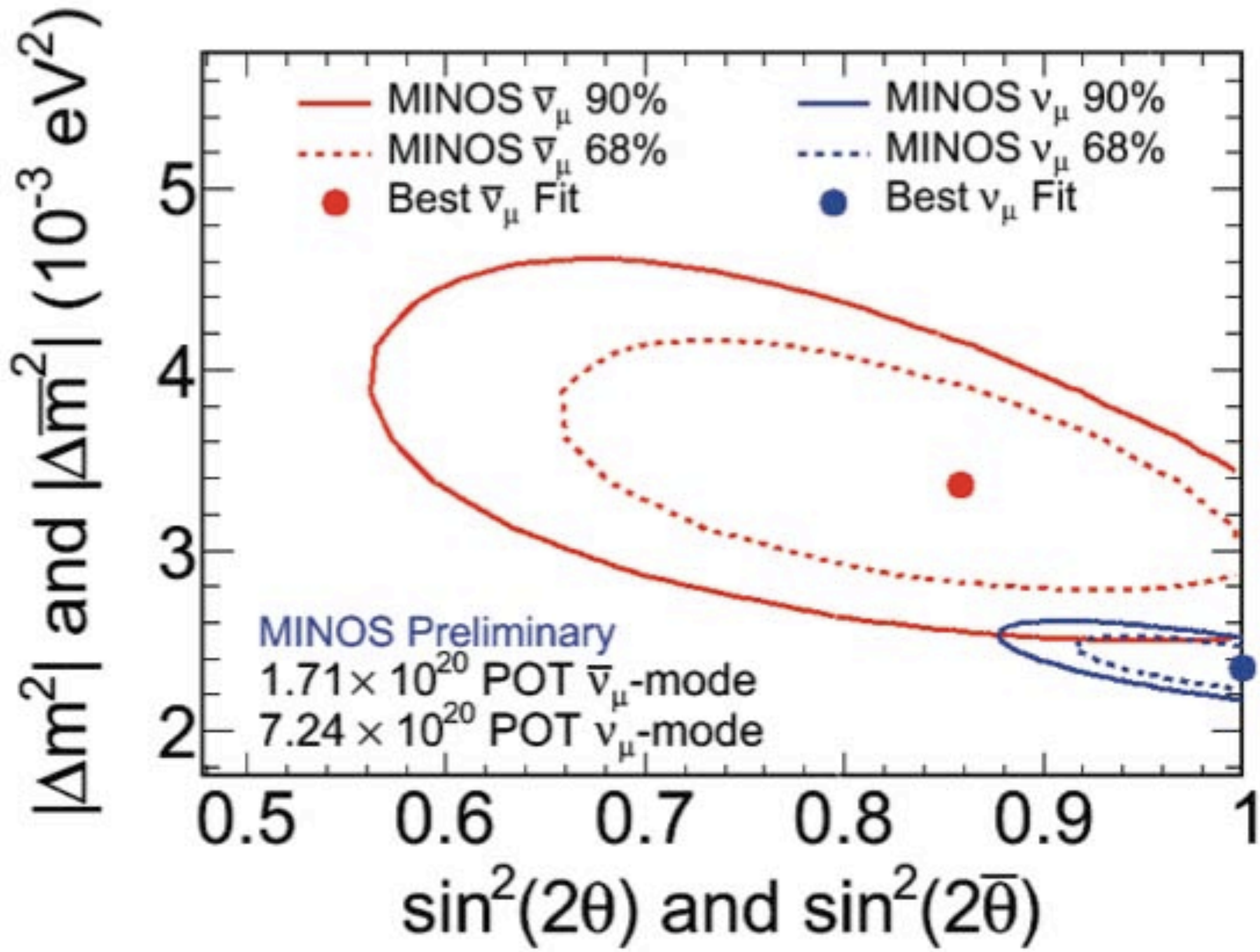
$$\left|\overline{\Delta m^2}\right| = 3.36^{+0.45}_{-0.40} \times 10^{-3}\,\mathrm{eV}^2$$

$$\sin^2(2\bar{\theta}) = 0.86 \pm 0.11$$

Comparison to Neutrinos



Comparison to Neutrinos



Never Become Too Sure of Things with Neutrinos! …and MiniBooNE antineutrino result

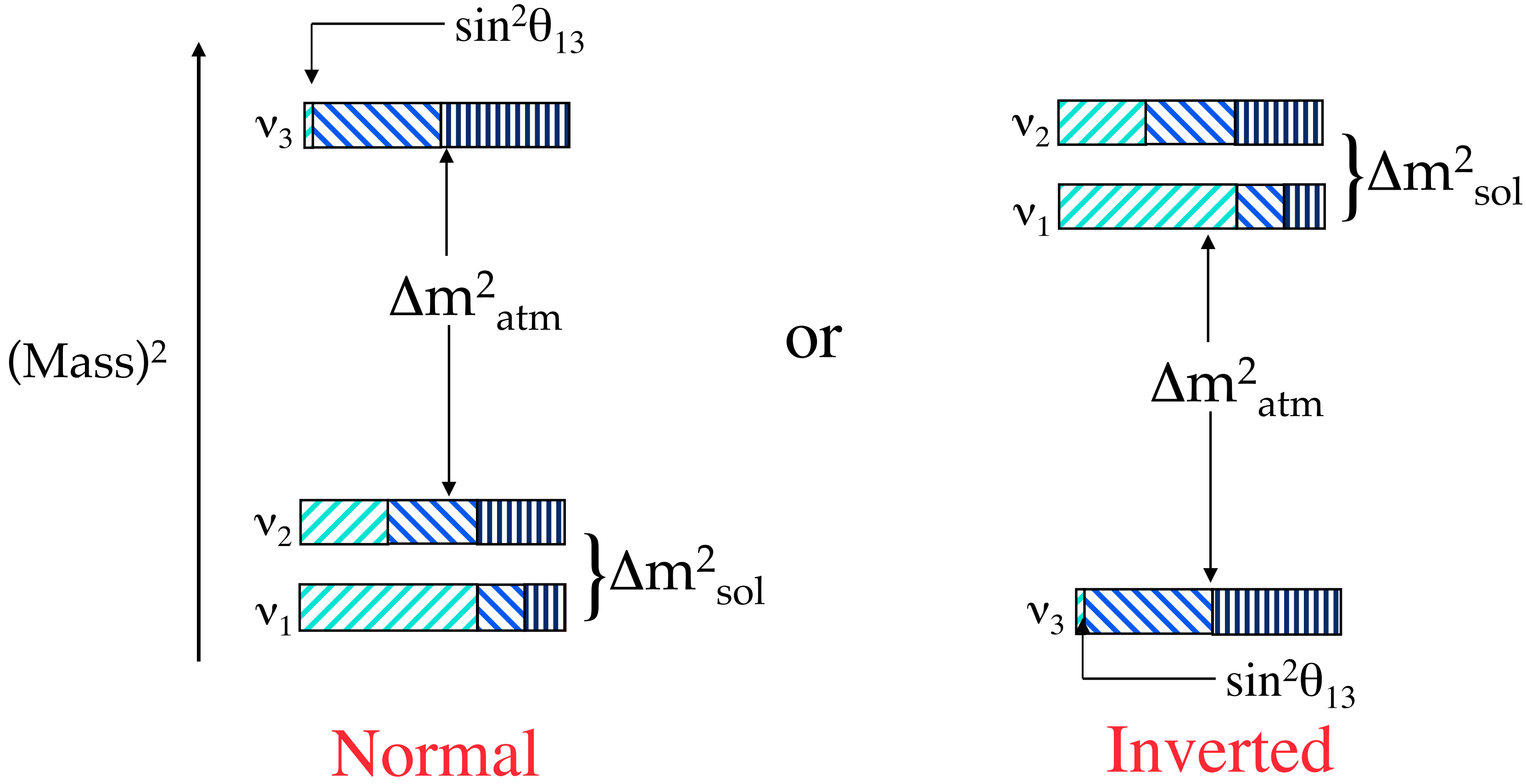
- In addition to the MINOS antineutrino results, MiniBooNE has new antineutrino results and found…
- Antineutrino results NOT consistent with their neutrino results but consistent with an older LSND antineutrino result indicating the need for an apparent antineutrino oscillation with (best fit point):

  $\Delta m^2 = .064 \text{ eV}^2$

  $\sin^2 2\theta = .96$

- … we really didn't need this, thank you. But it points out that the neutrino sector is perhaps even more complex than we thought and needs much more study / statistics.

How does $\Delta m^2 = .064\ eV^2$ fit in????
Do we need additional "sterile" neutrinos?



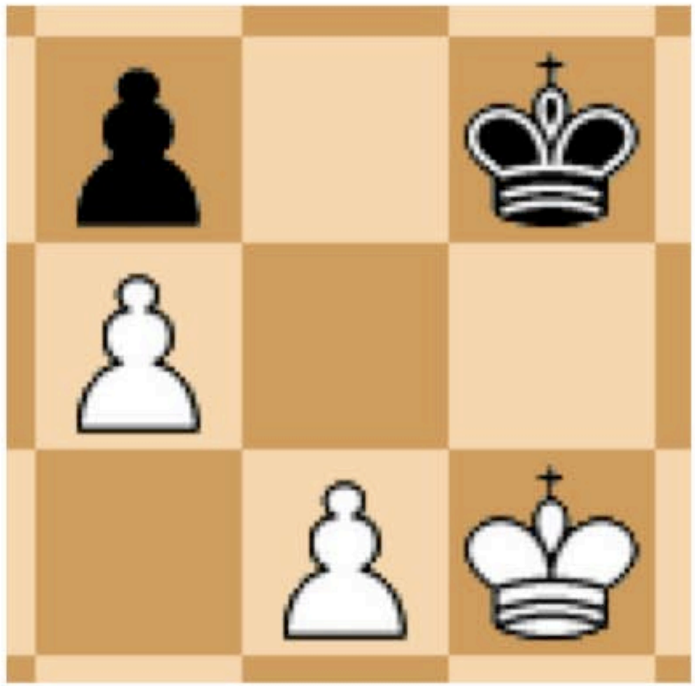
$\Delta m^2_{sol} = \sim 8 \times 10^{-5}\ eV^2$, $\Delta m^2_{atm} = \sim 2.5 \times 10^{-3}\ eV^2$

$\nu_e\ [\,|U_{ei}|^2]$ $\nu_\mu\ [\,|U_{\mu i}|^2]$ $\nu_\tau\ [\,|U_{\tau i}|^2]$

What's going on, where are we in our quest to understand "the neutrino"?

Think of a game of chess….

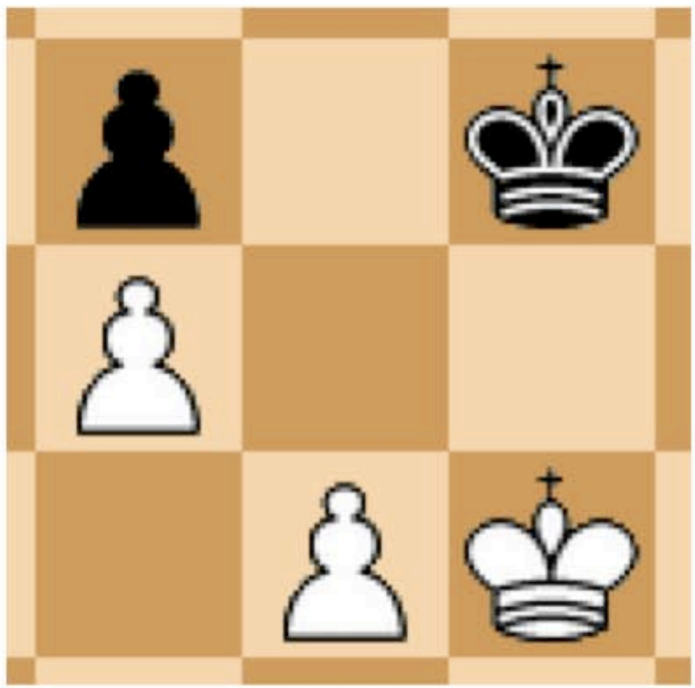
Given this end game:



What’s going on?

A view of the complexity of the situation from Stephen Parke

Given this end game:



Deduce the rules of chess!!!

Neutrino Oscillations:
Current Challenges: Where are we going from here?

- The dominant oscillation parameters will be known reasonably well from solar/ reactor ν and from SuperK, K2K, MINOS, CNGS **FOR NEUTRINOS**
  - Increase precision on the "Solar" and "Atmospheric" parameters - is θ_{23} exactly 45°??
- The physics issues to be investigated are clearly delineated:
  1. Need measurement of missing oscillation probability ($\theta_{13} = \theta_{\mu e}$)
  2. Need determination of **mass hierarchy** (sign of Δm_{13})
  3. WHAT ABOUT ANTINEUTRINOS?
  4. Search for **CP violation** in neutrino sector
  5. Measurement of **CP violation parameters - phase** δ
  6. Testing **CPT** with high precision

All can be accomplished with the $\nu_\mu \Rightarrow \nu_e$ transition or the relative height of the 1^{st} and 2^{nd} oscillation maxima in ν and ν

- **NOνA experiment to measure the sub-dominant $\nu_\mu \Rightarrow \nu_e$.**
- **Fermilab → DUSEL experiment to measure 1^{st} and 2^{nd} oscillation maxima.**

$P(\nu_\mu \rightarrow \nu_e)$ on one slide (3 generations)

$$P(\nu_\mu \rightarrow \nu_e) = P_1 + P_2 + P_3 + P_4$$

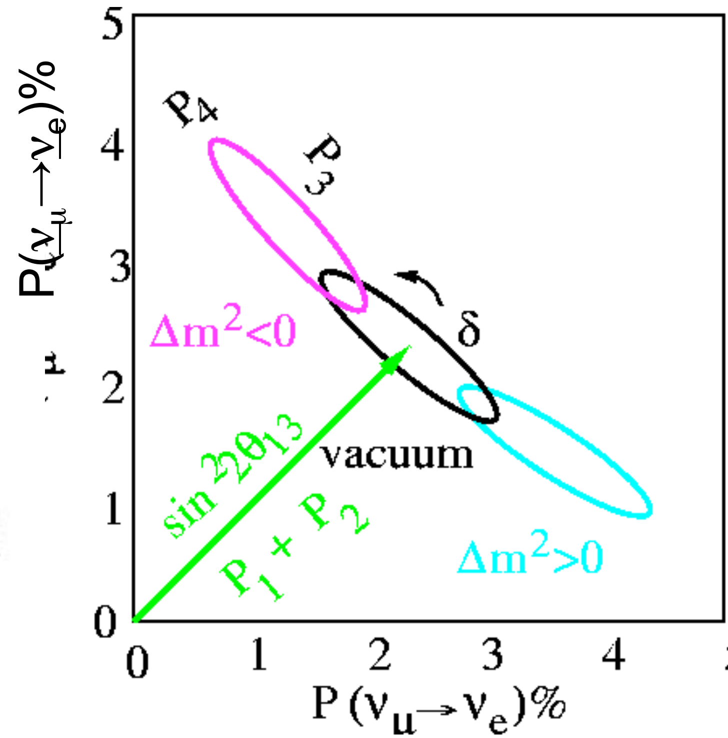
$$P_1 = \sin^2\theta_{23} \sin^2 2\theta_{13} \left(\frac{\Delta_{13}}{B_\pm}\right)^2 \sin^2 \frac{B_\pm L}{2}$$ Atmospheric

$$P_2 = \cos^2\theta_{23} \sin^2 2\theta_{12} \left(\frac{\Delta_{12}}{A}\right)^2 \sin^2 \frac{AL}{2}$$ Solar

$$P_3 = J \cos\delta \left(\frac{\Delta_{12}}{A}\right)\left(\frac{\Delta_{13}}{B_\pm}\right) \cos\frac{\Delta_{13} L}{2} \sin\frac{AL}{2} \sin\frac{B_\pm L}{2}$$

Atmospheric-solar interference

$$P_4 = \mp J \sin\delta \left(\frac{\Delta_{12}}{A}\right)\left(\frac{\Delta_{13}}{B_\pm}\right) \sin\frac{\Delta_{13} L}{2} \sin\frac{AL}{2} \sin\frac{B_\pm L}{2}$$



Minakata & Nunokawa JHEP 2001

$$\Delta_{ij} = \frac{\Delta m^2_{ij}}{2E_\nu}$$
$$A = \sqrt{2} G_F n_e$$

$$B_\pm = |A \pm \Delta_{13}|$$
$$J = \cos\theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23}$$

The $\pm$ is ν or $\overline{\nu}$

Fine, even though we are not entirely sure what a neutrino “IS”

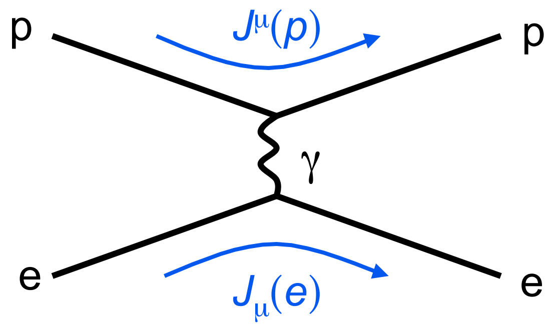
We can ask how do we use the flavor states to study QCD?

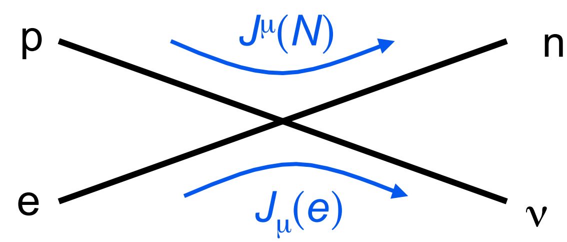
The “Weak Interaction”

Fermi Theory - Current-Current Interaction

1934 Paper rejected by *Nature* because it contains speculations too remote from reality to be of interest to the reader!!

Developed by Fermi in 1932 to describe nuclear β-decay inspired by the success of "current-current" description of electromagnetic interactions:





$$M_{em} = \left(e\bar{u}_p\gamma^\mu u_p\right)\left(\frac{-1}{q^2}\right)\left(-e\bar{u}_e\gamma_\mu u_e\right)$$

$$M_{CC} = G\left(\bar{u}_n\gamma^\mu u_p\right)\left(\bar{u}_\nu\gamma_\mu u_e\right)$$

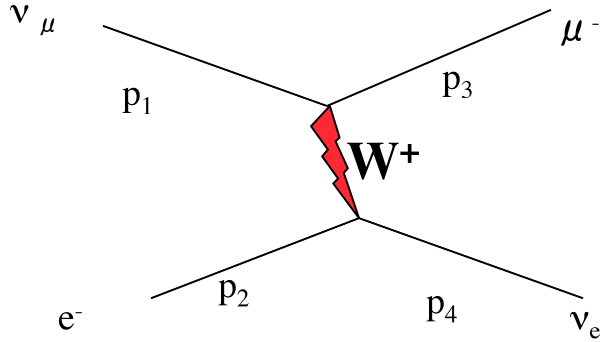
$$\mathcal{H}_{weak} = \frac{G_F}{\sqrt{2}}\left[\bar{l}\gamma_\mu(1-\gamma_5)\nu\right]\left[\bar{f}\gamma^\mu(V-A\gamma_5)f\right] + h.c.$$

Weak interactions are maximally parity violating: $J_\mu \propto \left(\bar{u}_\nu\gamma_\mu(1-\gamma_5)u_e\right)$

Only left-handed fermions, and right-handed anti-fermions, participate in the CC weak interaction!

What is A Weak Interaction?

- An example weak process involving neutrinos, which only feel weak interactions



$$s \equiv (p_1 + p_2)^2$$
$$= (E_\nu + m_e)^2 - (\vec{p}_\nu)^2$$
$$= E_\nu^2 - p_\nu^2 + m_e^2 + 2E_\nu m_e \approx 2E_\nu m_e$$

- For a realistic experiment, the neutrino beam energy is on the order of 100 GeV, so the total center of mass energy is less than 1 GeV
- But W boson rest mass is 80 GeV!!

What is A Weak Interaction - continued?

- Solution… “borrow” energy from the vacuum for a short time.

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

- Numerically, if we have to borrow 80 GeV, $t \sim 8 \times 10^{-27}$s.

$$\therefore t \sim \hbar / \Delta E$$

- Implies the W can travel only 2.5×10^{-18} m, so the weak interaction is very short range.

- Weak interactions are weak because of the massive W and Z bosons exchange $\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$

There are Actually Two Neutrino Weak Interactions

- W exchange gives Charged-Current (CC) events and
  Z exchange gives Neutral-Current (NC) events

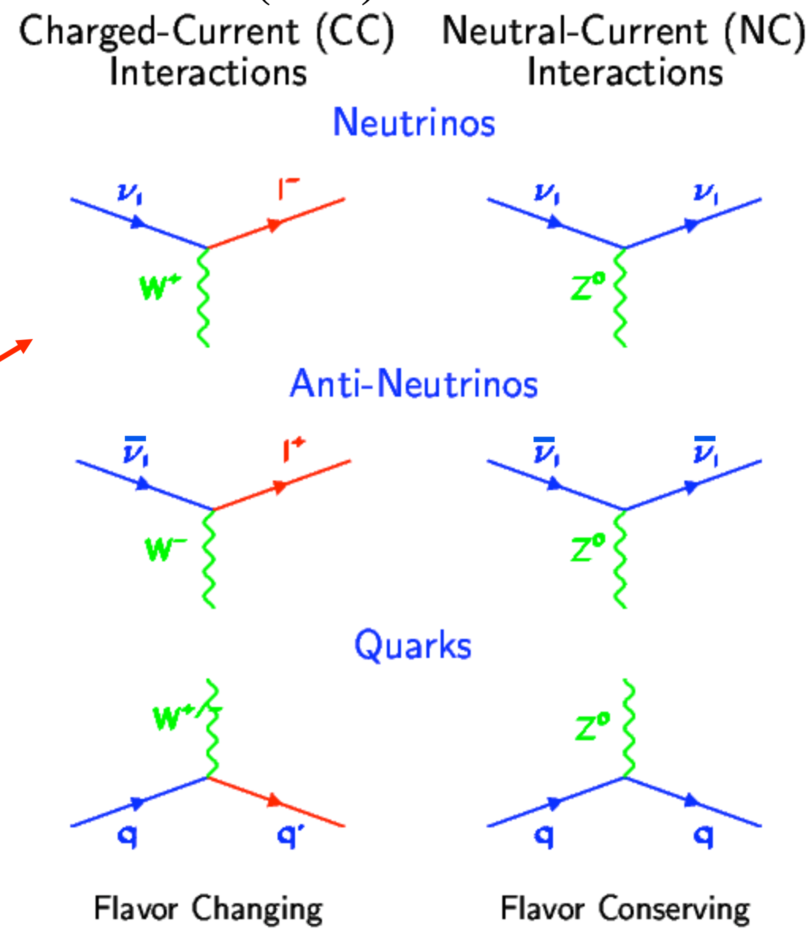
In charged-current events,

Flavor of outgoing lepton tags flavor of neutrino

Charge of outgoing lepton determines if neutrino or antineutrino

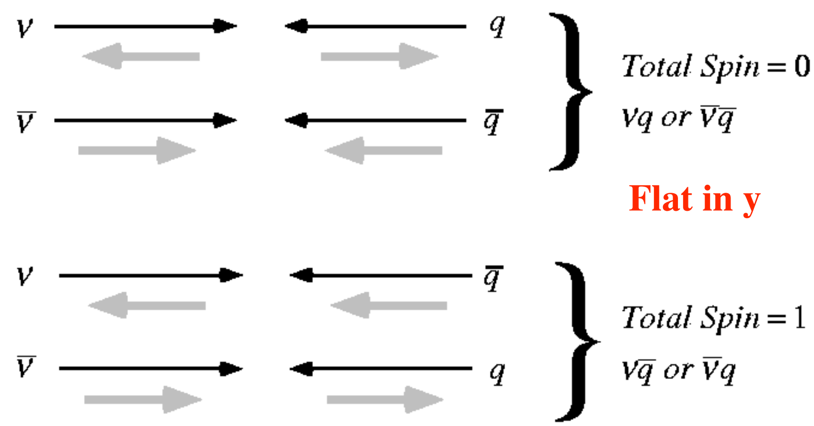
$$l^- \Rightarrow \nu_l$$

$$l^+ \Rightarrow \bar{\nu}_l$$



Chirality in CC ν-quark Scattering

- Total spin determines inelasticity distribution
  - Familiar from neutrino-electron scattering



$\frac{1}{4}(1+\cos\theta^*)^2 = (1-y)^2$

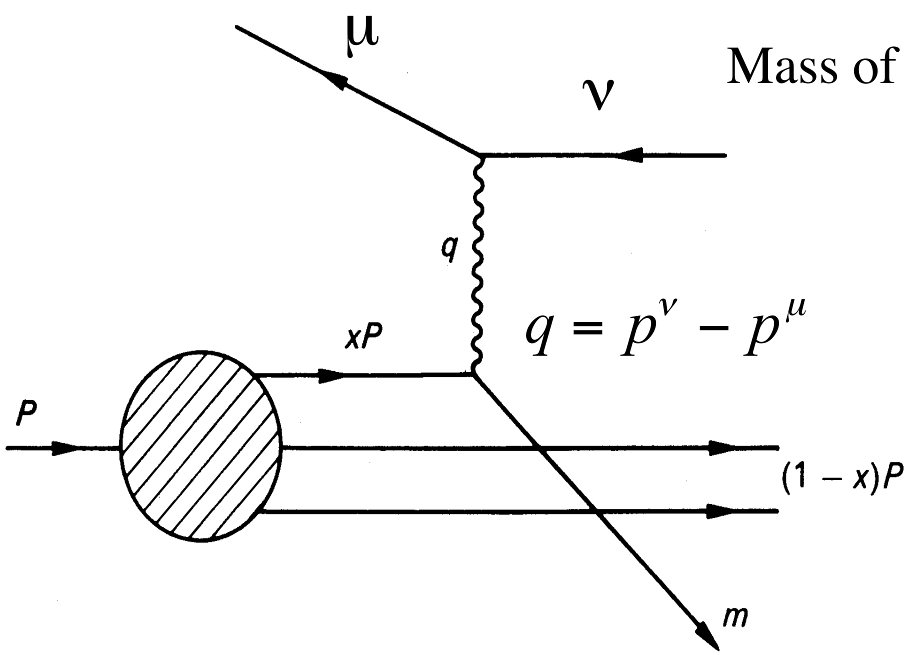
$\int(1-y)^2 dy = 1/3$

$$\frac{d\sigma^{\nu p}}{dxdy} = \frac{G_F^2 s}{\pi}\left(xd(x) + x\bar{u}(x)(1-y)^2\right)$$

$$\frac{d\sigma^{\bar{\nu} p}}{dxdy} = \frac{G_F^2 s}{\pi}\left(x\bar{d}(x) + xu(x)(1-y)^2\right)$$

How does Neutrino Scattering Contribute to Studies of QCD? Parton Interpretation

Mass of target quark $m_q^{\ 2} = x^2 P^2$



$q = p^{\nu} - p^{\mu}$

Mass of final state quark

$$m_{q'}^{\ 2} = (xP + q)^2$$

In "infinite momentum frame", x is momentum of partons inside the nucleon

$$x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2M_T \nu}$$

Neutrino scatters off a point-like parton inside the nucleon. Valid picture at high energies

ν-quark Scattering

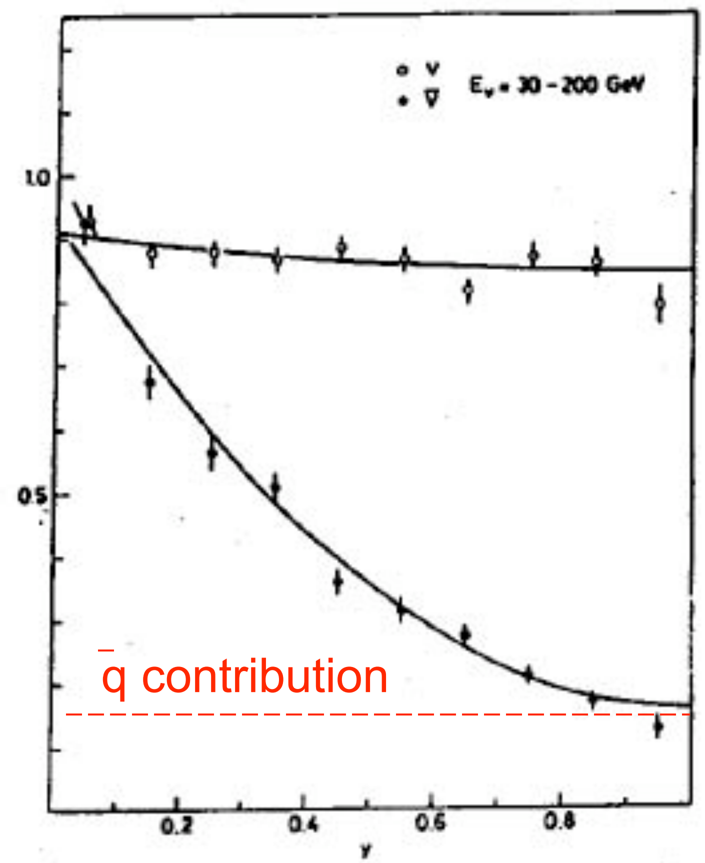
- We know that the helicity combinations (LL,RR = νq, $\bar{\nu}\bar{q}$) are J=0 combinations with flat-y dependence, and LR,RL combinations ($\bar{\nu}$q, ν$\bar{q}$) are J=1 combinations with $(1\text{-}y)^2$ dependence.

- From weak-isospin we see that neutrinos scatter from T_3=-1/2, anti-nu from T_3=+1/2

$$\frac{d\sigma^{\nu p}}{dxdy} = \frac{G^2 s}{\pi}\left(xd(x) + xs(x) + x\bar{u}(x)(1-y)^2\right)$$

$$\frac{d\sigma^{\bar{\nu} p}}{dxdy} = \frac{G^2 s}{\pi}\left(x\bar{d}(x) + x\bar{s}(x) + xu(x)(1-y)^2\right)$$

(ignoring c, b,t quarks., c quark mass)



Neutrino Deep-inelastic Scattering

- Deep inelastic neutrino-nucleon scattering reactions have large q^2 ($q^2 >> m_N^2, E_\nu >> m_N$): $\nu_l(p) + N \rightarrow l^-(p') + X$
- Quark-parton model valid due to asymptotic freedom of QCD, which makes quarks behave as free point-like particles.
- Infinite momentum frame: a parton takes a fraction x $(0<x<1)$, of momentum when struck by a neutrino. Final quark state:

$$(xp_N + q)^2 = m_q^{\,2} \Rightarrow x \approx -\frac{q^2}{2p_N \cdot q} \quad if\ q^2 >> m_q^{\,2}$$

- Variables in DIS:

$$s = (p + p_N)^2 \approx 2ME_\nu = 2ME$$

$$Q^2 = -q^2 = -(p + p')^2 = 4EE' \sin^2\frac{\theta}{2}$$

$$W^2 = E_X^{\,2} - p_X^{\,2} = -Q^2 + 2M\nu + M^2$$

$$\nu = \frac{q \cdot p_N}{M} = E - E'$$

Bjorken Variables ($0<x<1$, $0<y<1$):

$$x = \frac{-q^2}{2q \cdot p_N} = \frac{Q^2}{2M\nu}$$

$$y = \frac{q \cdot p_N}{p \cdot p_N} = \frac{\nu}{E} = \frac{Q^2}{2MEx}$$

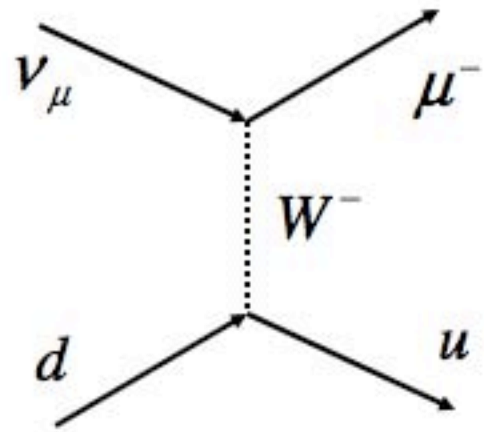
Neutrino Deep-inelastic Scattering

- Scattering off proton:

$$\frac{d\sigma_{CC}(\nu_\mu p)}{dxdy} = \frac{G_F{}^2 ME}{\pi} 2x\left\{[d(x)+s(x)]+[\bar{u}(x)+\bar{c}(x)](1-y)^2\right\}$$

$$\frac{d\sigma_{CC}(\bar{\nu}_\mu p)}{dxdy} = \frac{G_F{}^2 ME}{\pi} 2x\left\{[u(x)+c(x)](1-y)^2+[\bar{d}(x)+\bar{s}(x)]\right\}$$

- Structure functions:



$$F_2^{\nu p}(x) = 2x[d(x)+\bar{u}(x)+s(x)+\bar{c}(x)]$$

$$xF_3^{\nu p}(x) = 2x[d(x)-\bar{u}(x)+s(x)-\bar{c}(x)]$$

$$F_2^{\bar{\nu} p}(x) = 2x[u(x)+c(x)+\bar{d}(x)+\bar{s}(x)]$$

$$xF_3^{\bar{\nu} p}(x) = 2x[u(x)+c(x)-\bar{d}(x)-\bar{s}(x)]$$

- Neutron (isospin symmetry):

$$F_2^{\nu n}(x) = 2x[u(x)+\bar{d}(x)+s(x)+\bar{c}(x)]$$

$$xF_3^{\nu n}(x) = 2x[u(x)-\bar{d}(x)+s(x)-\bar{c}(x)]$$

Total DIS Cross Sections

- Scattering off isoscalar target (equal number neutrons and protons):

$$q \equiv u + d + s + c \qquad \bar{q} \equiv \bar{u} + \bar{d} + \bar{s} + \bar{c}$$

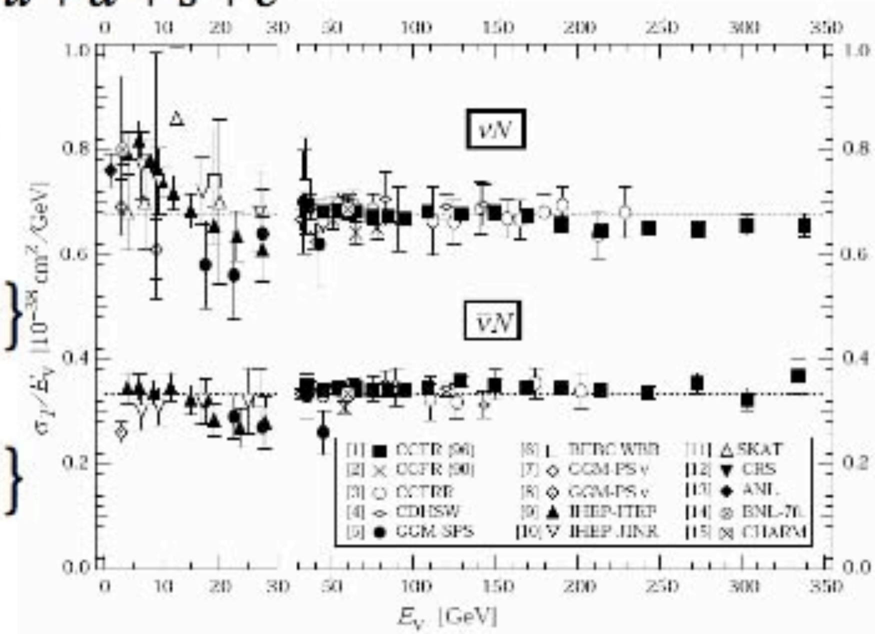
$$F_2^{\nu N}(x) = x\left[q(x) + \bar{q}(x)\right]$$

$$xF_3^{\nu N}(x) = x\left[q(x) - \bar{q}(x) + 2\left(s(x) - c(x)\right)\right]$$

$$xF_3^{\bar{\nu} N}(x) = x\left[q(x) - \bar{q}(x) - 2\left(s(x) - c(x)\right)\right]$$

$$\frac{d\sigma_{CC}(\nu_\mu N)}{dxdy} = \frac{G_F^{\,2} ME}{\pi} x\left\{q(x) + \bar{q}(x)\,(1-y)^2\right\}$$

$$\frac{d\sigma_{CC}(\bar{\nu}_\mu N)}{dxdy} = \frac{G_F^{\,2} ME}{\pi} x\left\{q(x)(1-y)^2 + \bar{q}(x)\right\}$$



- Total cross-section:

$$\sigma_{CC}(\nu_\mu N) = \frac{G_F^{\,2} s}{2\pi}\left[\langle Q\rangle + \frac{1}{3}\langle \bar{Q}\rangle\right] = (0.677 \pm 0.014)\times 10^{-38}\, cm^2 / GeV \times E(GeV)$$

$$\sigma_{CC}(\bar{\nu}_\mu N) = \frac{G_F^{\,2} s}{2\pi}\left[\frac{1}{3}\langle Q\rangle + \langle \bar{Q}\rangle\right] = (0.334 \pm 0.008)\times 10^{-38}\, cm^2 / GeV \times E(GeV)$$

Quark and Anti-quark Densities from ν DIS

- Quark content of nucleons from CC cross-sections
- Define:

$$U = \int_0^1 xu(x)dx\, ,\, etc.$$

- Experimental values from y distribution of cross-sections yields:

$$\frac{\overline{Q}}{Q+\overline{Q}} = 0.15 \pm 0.03 \qquad \frac{S}{Q+\overline{Q}} = 0.00 \pm 0.03 \qquad \frac{\overline{Q}+S}{Q+\overline{Q}} = 0.16 \pm 0.01$$

- If $r \equiv \frac{\sigma_{CC}(\overline{\nu}N)}{\sigma_{CC}(\nu N)} = 0.493 \pm 0.016$ (measured) $\Rightarrow \frac{\overline{Q}}{Q} = \frac{3r-1}{3-r} \approx 0.19$

$$Q_V = Q - \overline{Q} \approx 0.33 \qquad Q_S = \overline{Q}_S = \overline{Q} \approx 0.08$$

$$\int_0^1 F_2^{\nu N}(x)dx = Q + \overline{Q} \approx 0.49$$

- Quarks and antiquarks carry 49% of proton momentum, valence quarks only 33% and sea quarks only 16%.

Latest ν DIS Scattering Results - NuTeV

The NuTeV Experiment at Fermilab the most recent neutrino experiment to investigate QCD:

NuTeV accumulated over 3 million ν/ ν events with $20 \leq E_\nu \leq 400$ GeV.

NuTeV considered 23 systematic uncertainties.

NuTeV agrees with **charge lepton** data for $x < 0.5$.

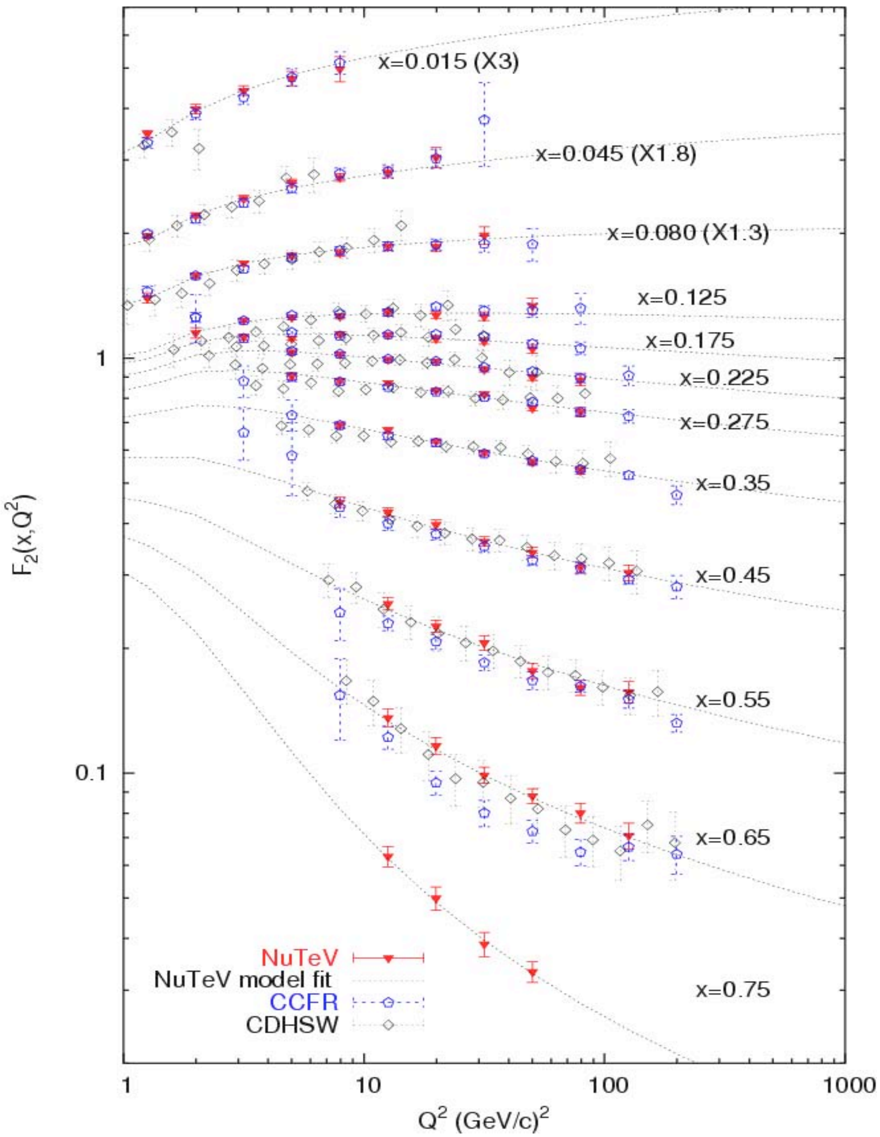
Perhaps smaller nuclear correction at high-x for neutrino scattering.

NuTeV F_2 and xF_3 agrees with **theory** for medium x.

At low x different Q^2 dependence.

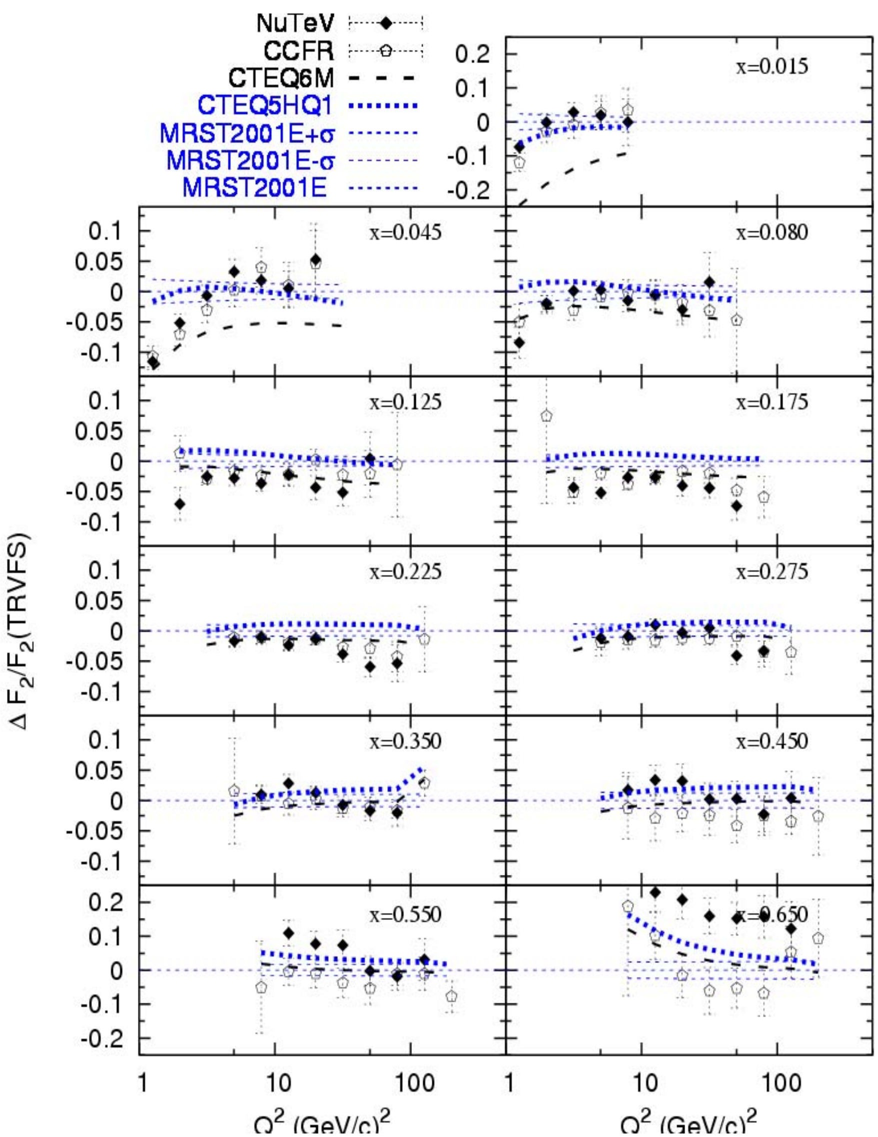
At high x (x>0.6) NuTeV is systematically higher.

NuTeV F_2 Measurement on Iron



- **Isoscalar ν-Fe F_2**
- **NuTeV F_2 is compared with earlier results the line is a fit to NuTeV data**
- **All systematic uncertainties are included**
- **All data sets agree for *0.1<x<0.4*.**
- **At *x>0.4* NuTeV is systematically above earlier results**

Comparison with Theory for F_2



- **Baseline is TRVFS(MRST2001E)**
- **NuTeV and CCFR F_2 are compared to TRVFS(MRST2001E)**

$$\frac{F_2^{NuTeV} - F_2^{TRVFS}}{F_2^{TRVFS}}$$

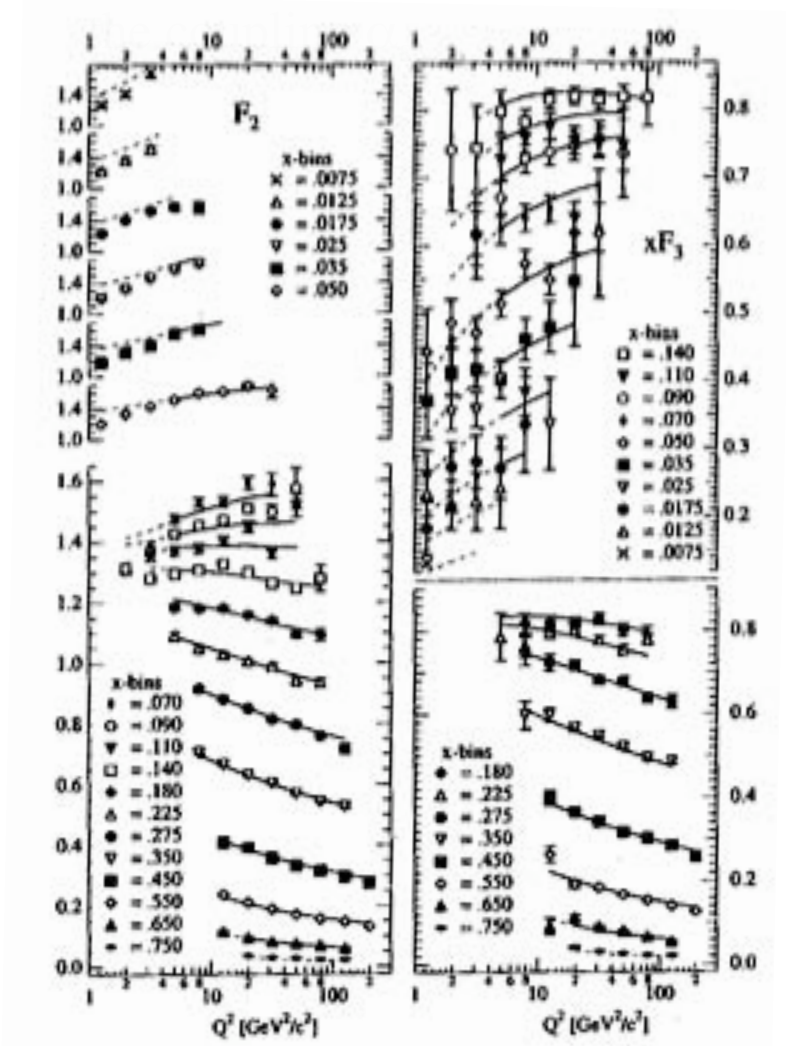
- **Theoretical models shown are:**
  - **- ACOT(CTEQ6M)**
  - **- ACOT(CTEQ5HQ1)**
  - **- TRVFS (MRST2001E)**
- **Theory curves are corrected for:**
  - **- target mass** *(H. Georgi and H. D. Politzer,*
- **NuTeV F_2 agrees with theory for medium x.**
- **At low x different Q^2 dependence.**
- **At high x >0.6) NuTeV is systematically higher.**
- **nuclear effects – parameterization from charge lepton data, assumed to be the same for neutrino scattering ---- WRONG!**

Summary

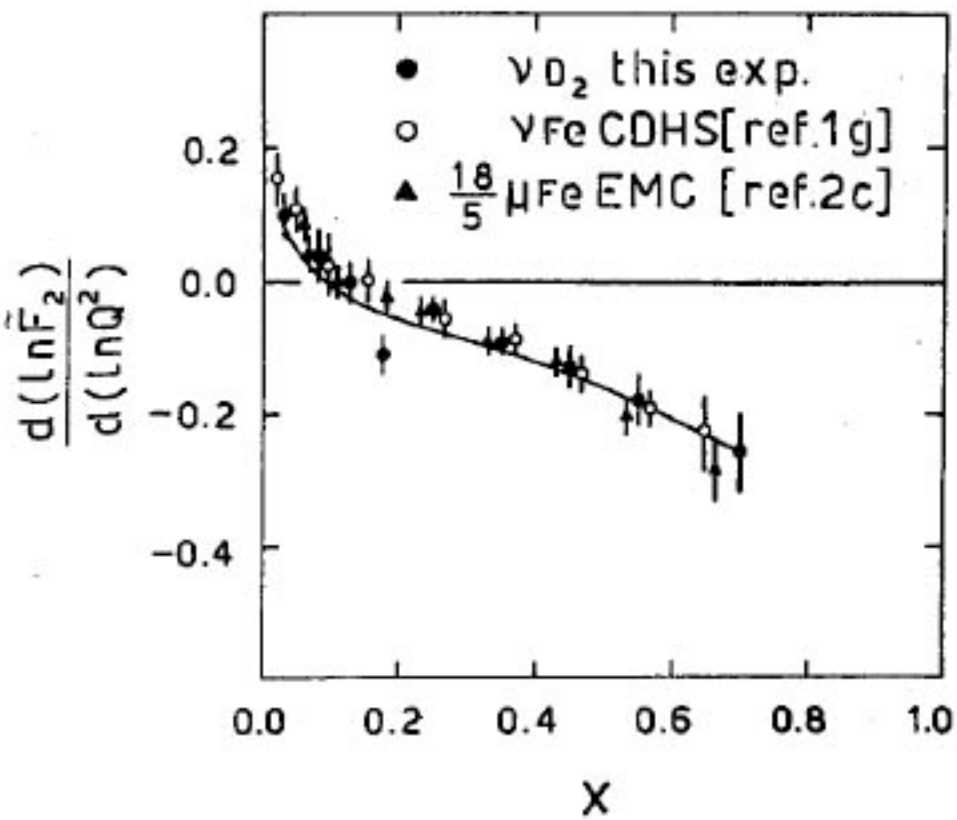
- **Very exciting times in Neutrino Physics**
- Neutrinos not only have surprised us with a small but significant mass but they are demonstrating mixing in a very different manner than quarks… why?
- Are antineutrino oscillations really so different than neutrino oscillations?
- Still many open questions in the neutrino sector? Very crucial but experimentally very difficult questions to answer:
- Neutrinos, with their ability to taste particular quarks can add significantly to our QCD studies if we can only determine how nuclear effects mask their quark level interactions.

QCD and ν scattering

- QCD therefore predicts the Q^2 evolution of the structure functions in terms of



$$\frac{\partial xF_3(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_0^1 P_{qq}(x/y)\,xF_3(y,Q^2)\,dy/y$$



Neutrino Deep-inelastic Scattering

- Neutrino proton CC scattering: $\nu_\mu(p) + p \to \mu^-(p') + X$

  $u(x)dx$ = number of u-quarks in proton between x and x+dx

  $u(x) = u_V(x) + u_S(x) \qquad d(x) = d_V(x) + d_S(x)$

  In the sea: $u_S(x) = \bar{u}(x) \quad d_S(x) = \bar{d}(x)$

  For proton (uud):
  $$\int_0^1 u_V(x)dx = \int_0^1 [u(x) - \bar{u}(x)]dx = 2$$
  $$\int_0^1 d_V(x)dx = \int_0^1 [d(x) - \bar{d}(x)]dx = 1$$

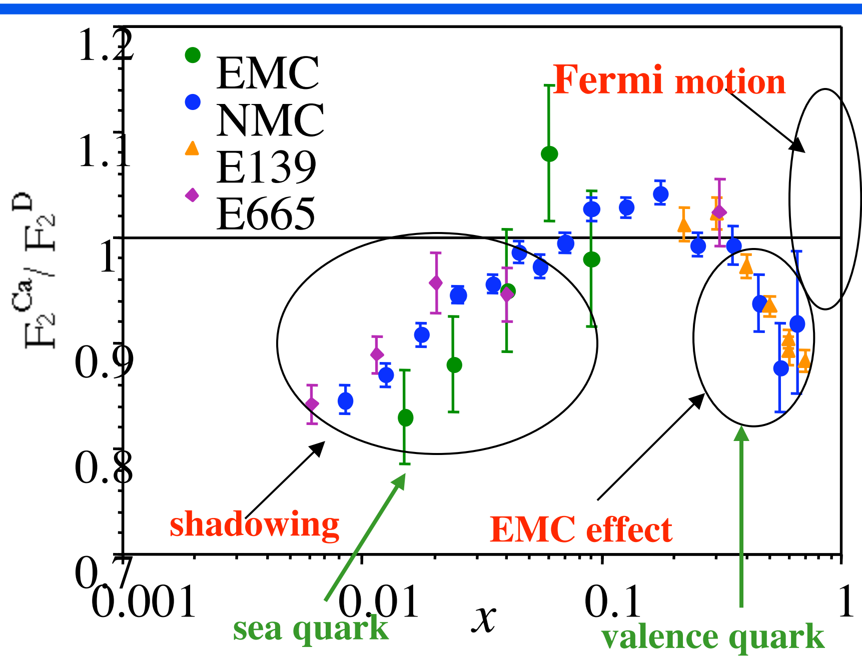
- Scattering off quarks:

$$\frac{d\sigma_{CC}(\nu_\mu q)}{dy} = \frac{d\sigma_{CC}(\bar{\nu}_\mu \bar{q})}{dy} = \frac{2G_F{}^2 m_q E}{\pi} \quad \text{with } y = 1 - \frac{E}{E'} = \frac{1}{2}(1 - \cos\theta)$$

$$\frac{d\sigma_{CC}(\nu_\mu \bar{q})}{dy} = \frac{d\sigma_{CC}(\bar{\nu}_\mu q)}{dy} = \frac{2G_F{}^2 m_q E}{\pi}(1 - y)^2$$

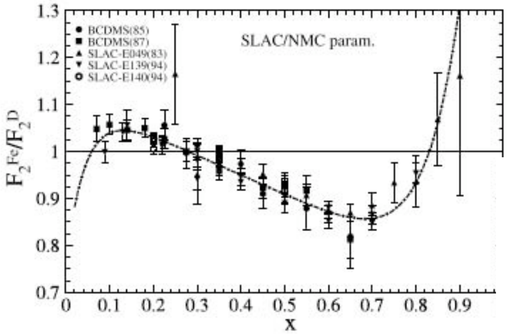
Experimental Studies of Nuclear Effects with Neutrinos:

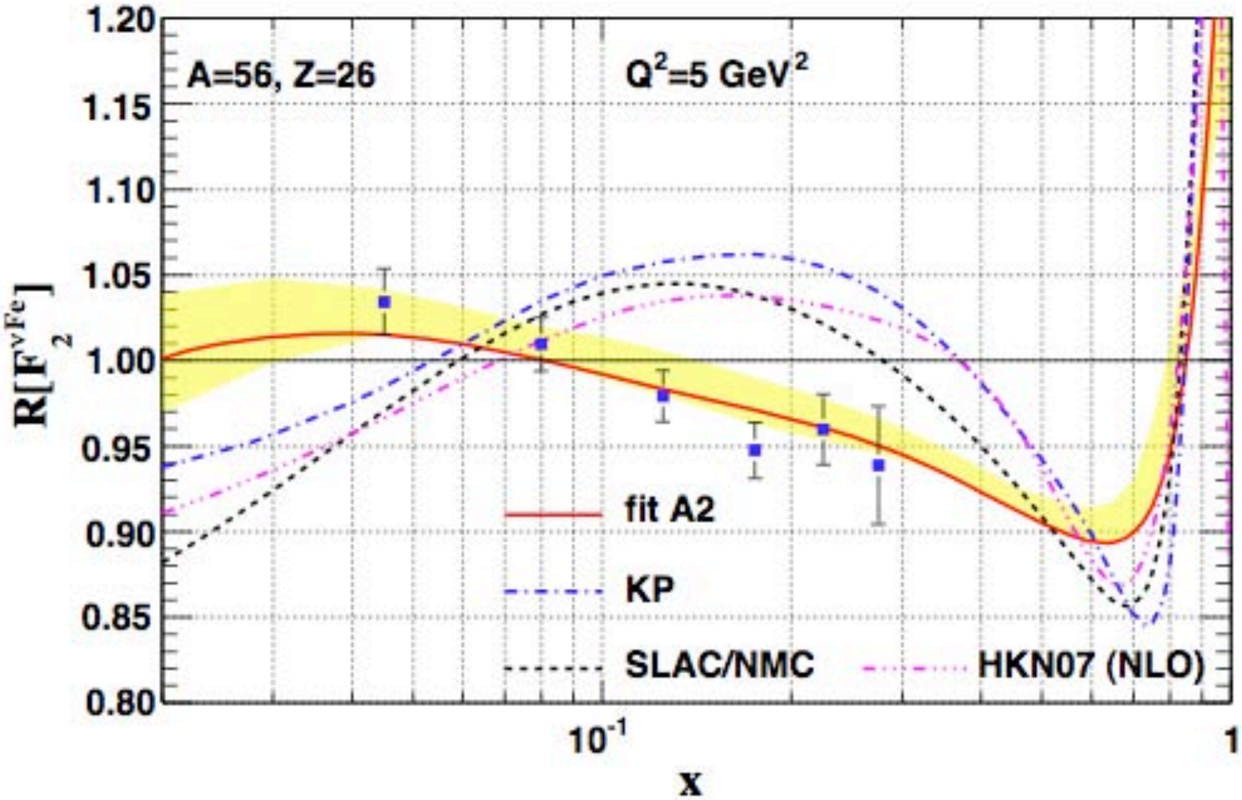
NON-EXISTENT



- F_2 / nucleon changes as a function of A. Measured in μ/e - A, **not in ν − A**
- **Good reason to consider nuclear effects are DIFFERENT in ν - A.**
  - **Presence of axial-vector current.**
  - **Different nuclear effects for valance and sea --> different shadowing for xF_3 compared to F_2.**

F_2 Structure Function Ratios: ν-Iron





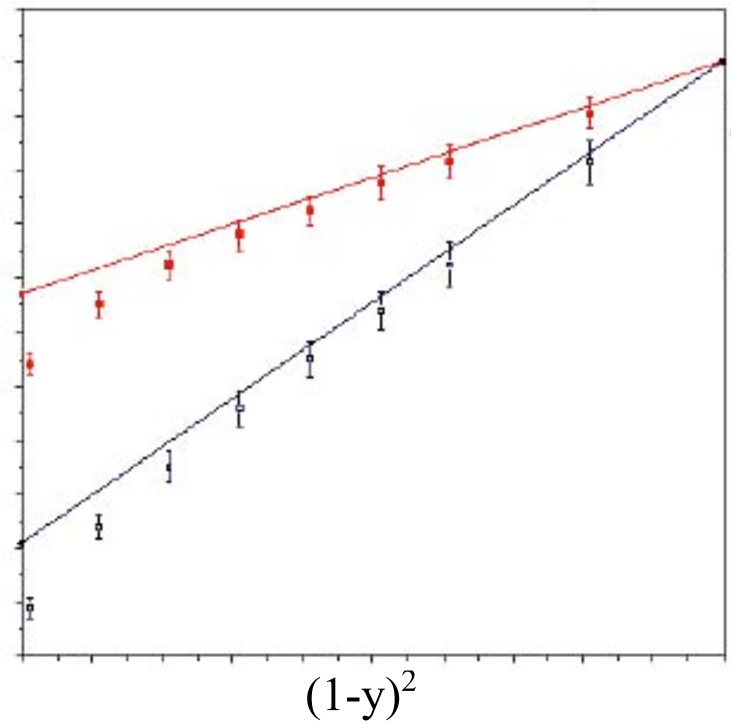
Structure Function Extraction

$$\frac{d\sigma^{\nu A}}{dxdQ^2} = \frac{G_F^2}{2\pi x}\left[\frac{1}{2}\left(F_2^{\nu A}(x,Q^2) + xF_3^{\nu A}(x,Q^2)\right) + \frac{(1-y)^2}{2}\left(F_2^{\nu A}(x,Q^2) - xF_3^{\nu A}(x,Q^2)\right)\right.$$

$$\left. + y^2 F_L \right]$$

$$\frac{d\sigma^{\bar{\nu} A}}{dxdQ^2} = \frac{G_F^2}{2\pi x}\left[\frac{1}{2}\left(F_2^{\bar{\nu} A}(x,Q^2) - xF_3^{\bar{\nu} A}(x,Q^2)\right) + \frac{(1-y)^2}{2}\left(F_2^{\bar{\nu} A}(x,Q^2) + xF_3^{\bar{\nu} A}(x,Q^2)\right)\right]$$

$$\frac{\sigma\left(x,Q^2,(1-y)^2\right)}{G^2/2\pi x}$$



Neutrino
Statistical + 5% systematic

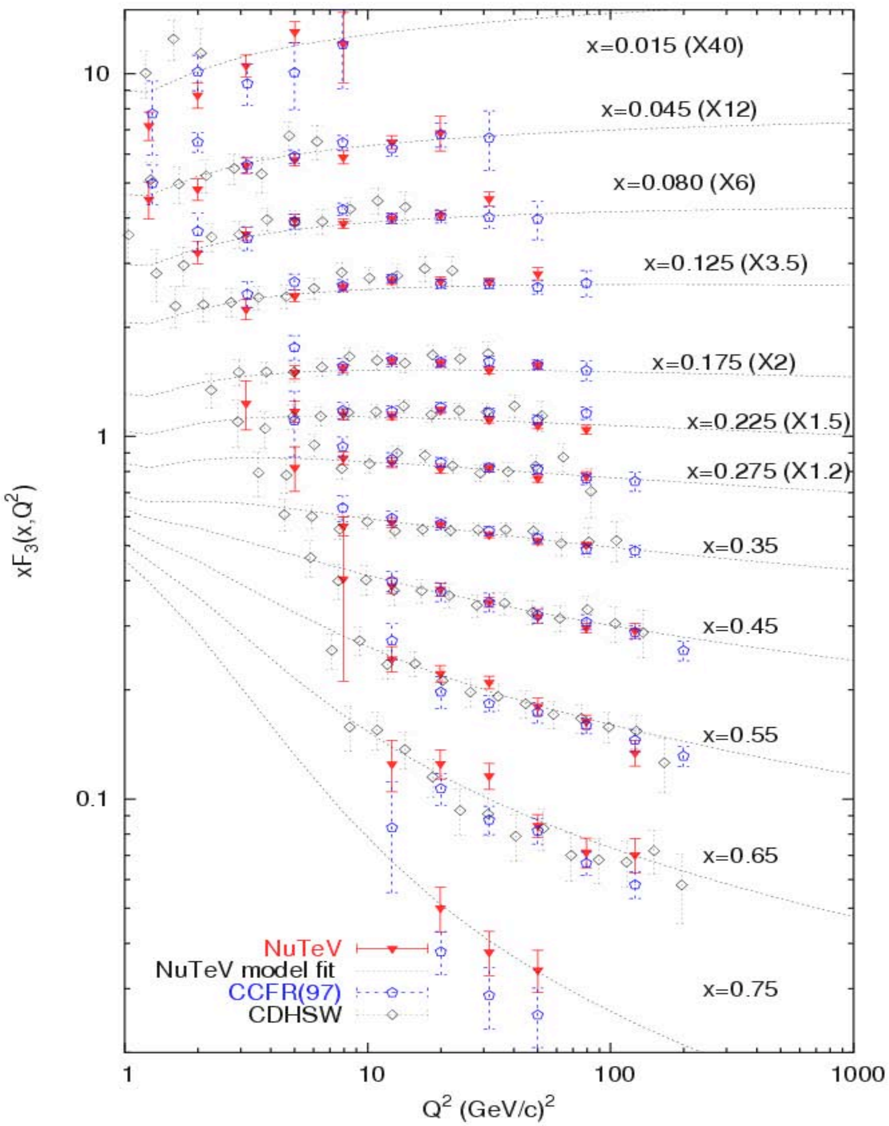
Anti-Neutrino
Statistical only

$R = R_{whitlow}$

X = 0.1 - 0.125
$Q^2 = 2 - 4\ GeV^2$

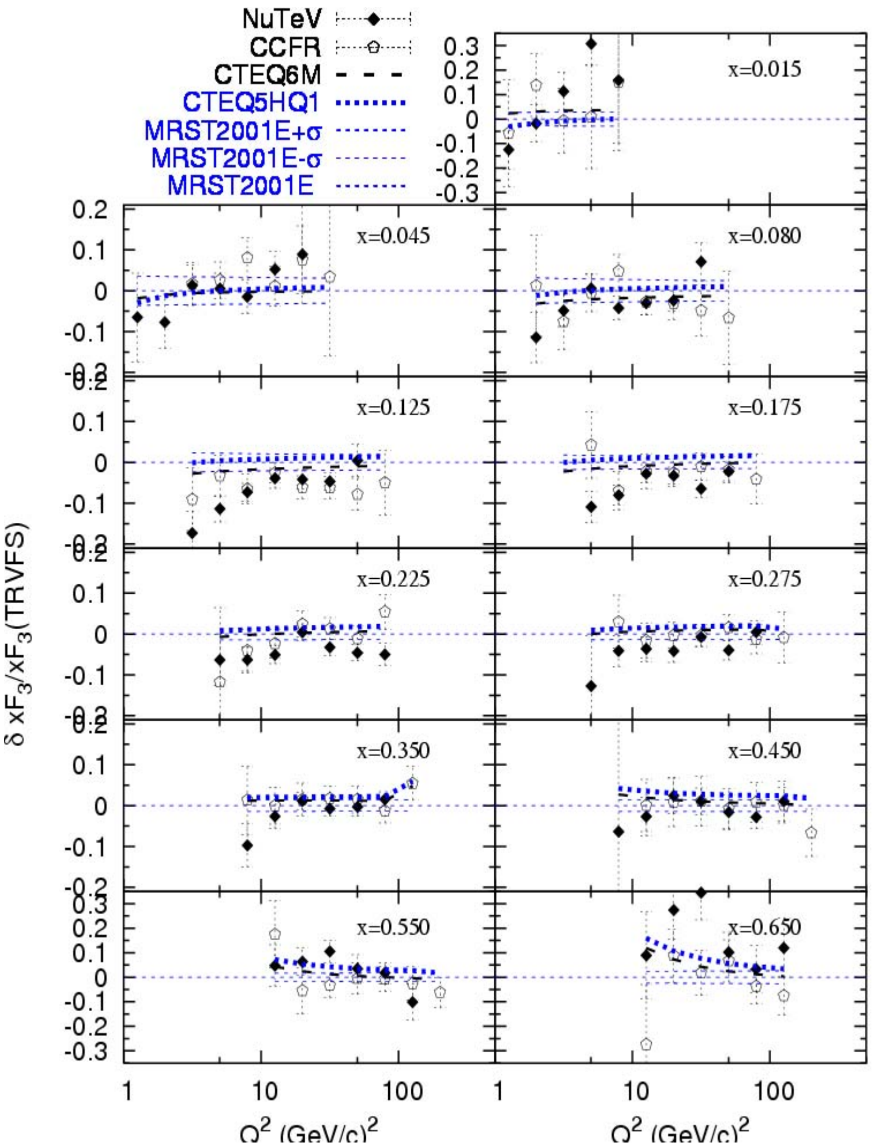
Meant to give an impression only!
Kinematic cuts in (1-y) not shown.

NuTeV xF_3 Measurement on Fe



- **Isoscalar ν-Fe xF_3**
- **NuTeV xF_3 is compared with earlier results the line is a fit to NuTeV data**
- **All systematic uncertainties are included**
- **All data sets agree for *0.1<x<0.4*.**
- **At *x>0.4* NuTeV is systematically above earlier results**

Comparison with Theory for xF_3

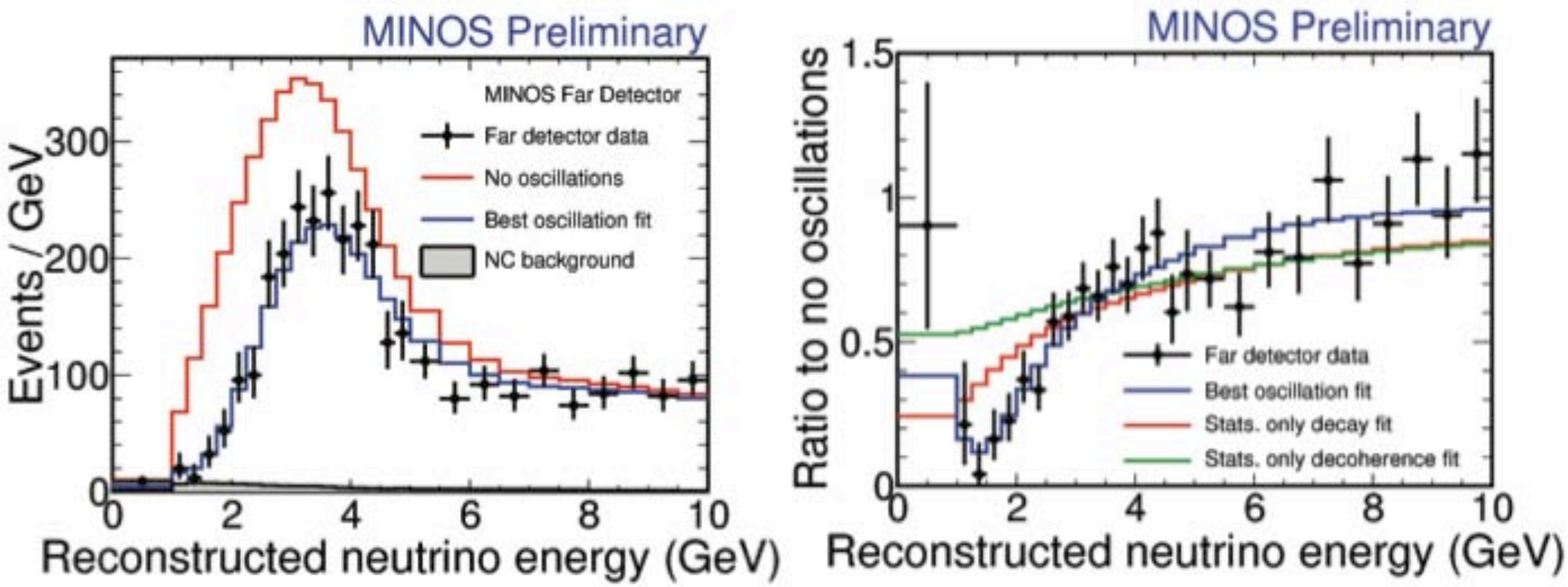


- **Baseline is TRVFS(MRST2001E).**
- **NuTeV and CCFR xF_3 are compared to TRVFS(MRST2001E)**

$$\frac{xF_3^{NuTeV} - xF_3^{TRVFS}}{xF_3^{TRVFS}}$$

- **Theoretical models shown are:**
  - **- ACOT(CTEQ6M)**
  - **- ACOT(CTEQ5HQ1)**
  - **- TRVFS (MRST2001E)**
- **theory curves are corrected for:**
  - **- target mass** *(H. Georgi and H. D. Politzer,*
- **NuTeV xF_3 agrees with theory for medium x.**
- **At low x different Q^2 dependence.**
- **At high x (x>0.6) NuTeV is systematically higher.**
- **nuclear effects – parameterization from charge lepton data, assumed to be the same for neutrino scattering ---- WRONG!**

Are we sure it is oscillations?



- Oscillations fit the data well, 66% of experiments have worse χ^2
- Pure decoherence† disfavored: > **8σ**
- Pure decay‡ disfavored: > **6σ**
  (**7.8σ** if NC events included)

†G.L. Fogli *et al.*, PRD 67:093006 (2003) ‡V. Barger *et al.*,PRL 82:2640 (1999)

P. Vahle, Neutrino 2010

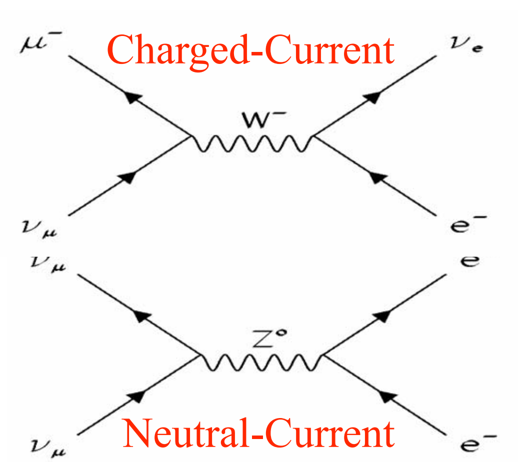
Electroweak Theory

- Standard Model
  - SU(2) ⊗ U(1) gauge theory unifying weak/EM
    ⇒ weak NC follows from EM, Weak CC
  - Measured physical parameters related to mixing parameter for the couplings, $g' = g \tan\theta_W$

Z Couplings	g_L	g_R
ν_e, ν_μ, ν_τ	1/2	0
e, μ, τ	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$
u, c, t	$1/2 - 2/3 \sin^2\theta_W$	$-2/3 \sin^2\theta_W$
d, s, b	$-1/2 + 1/3 \sin^2\theta_W$	$1/3 \sin^2\theta_W$

$$e = g \sin\theta_W,\ G_F = \frac{g^2\sqrt{2}}{8M_W^2},\ \frac{M_W}{M_Z} = \cos\theta_W$$

- Neutrinos are special in SM
  - Right-handed neutrino has **NO** interactions!



Milestones in the History of Neutrino Physics

- 1930 - Pauli postulates the existence of the neutrino
- 1934 - Enrico Fermi develops a comprehensive theory of radioactive decays, including Pauli's hypothetical particle, which Fermi coins the neutrino (Italian: "little neutral one").
- 1959 - Discovery of a particle fitting the expected characteristics of the neutrino is announced by Clyde Cowan and Fred Reines.
- 1962 - Experiment at Brookhaven National Laboratory discovered a second type of neutrino (ν_μ).
- 1968 - The first experiment to detect ν_e produced by the Sun's burning (using a liquid Chlorine target deep underground) reports that less than half the expected neutrinos are observed.
- 1985 - The IMB experiment observes fewer atmospheric ν_μ interactions than expected.
- 1989 - Kamiokande becomes the second experiment to detect ν_e from the Sun finding only about 1/3 the expected rate.
- 1994 - Kamiokande finds that ν_μ traveling the greatest distances from the point of production to the detector exhibit the greatest depletion.
- 1997 - Super-Kamiokande reports a deficit of cosmic-ray ν_μ and solar ν_e, at rates agreeing with earlier experiments.
- 1998 - The Super-Kamiokande collaboration announces evidence of non-zero neutrino mass at the Neutrino '98 conference.
- 2000 - First direct evidence for the ν_τ announced at Fermilab by DONUT collaboration.
- 2004 - K2K Experiment confirms (with limited statistics) Super -Kamiokande discovery .
- 2005 - MINOS starts data-taking to **STUDY** Neutrino Oscillation Phenomena

Neutrino Structure Functions Wonderfully Efficient in Isolating Quark Flavors

Recall Neutrinos have the ability to directly resolve flavor of the nucleon's constituents: ν interacts with d, s, $\bar{u}$, and $\bar{c}$ while $\bar{\nu}$ interacts with u, c, $\bar{d}$ and $\bar{s}$.

Using Leading order expressions:

$$F_2^{\bar{\nu}N}(x,Q^2) = x\left[u + \bar{u} + d + \bar{d} + 2\bar{s} + 2c\right]$$

$$F_2^{\nu N}(x,Q^2) = x\left[u + \bar{u} + d + \bar{d} + 2s + 2\bar{c}\right]$$

$$xF_3^{\bar{\nu}N}(x,Q^2) = x\left[u + d - \bar{u} - \bar{d} - 2\bar{s} + 2c\right]$$

$$xF_3^{\nu N}(x,Q^2) = x\left[u + d - \bar{u} - \bar{d} + 2s - 2\bar{c}\right]$$

Taking combinations of the Structure functions

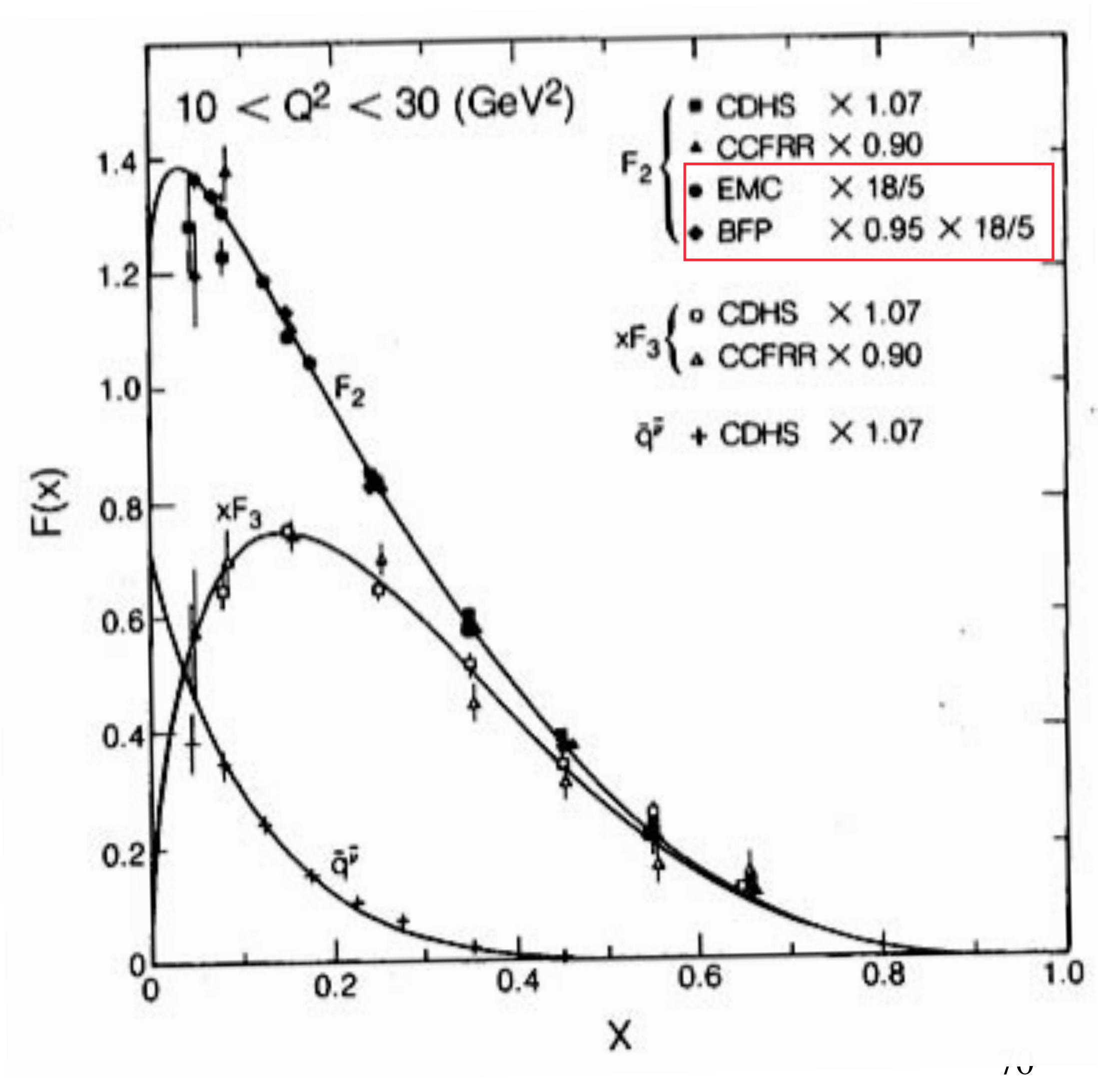
$$F_2^{\nu} - xF_3^{\nu} = 2\left(\bar{u} + \bar{d} + 2\bar{c}\right)$$

$$F_2^{\bar{\nu}} - xF_3^{\bar{\nu}} = 2\left(\bar{u} + \bar{d} + 2\bar{s}\right)$$

$$xF_3^{\nu} - xF_3^{\bar{\nu}} = 2\left[\left(s + \bar{s}\right) - \left(\bar{c} + c\right)\right]$$

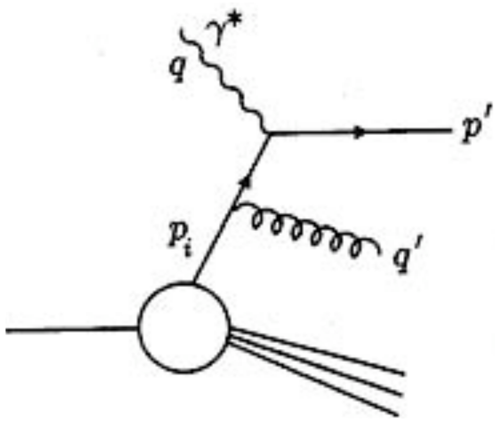
Momentum Distributions and Parton Universality

- It is straightforward to relate the structure functions from charged lepton and neutrino scattering.
- The fact that they are in good agreement justifies earlier claims of parton universality!



QCD and Scaling Violations

- At higher order in QCD the nucleon looks somewhat different



$$\alpha_s(Q^2) = 12\pi/[(33 - 2N_f)ln(Q^2/\Lambda^2)]$$

Calculations of the structure functions in terms of parton distributions now are somewhat more complicated and involve the "splitting functions"

Pqq(x/y) = probability of finding a quark with momentum x within a quark with momentum y

Pgq(x/y) = probability of finding a quark with momentum x within a gluon with momentum y.

$$P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{(1-z)} + 2\delta(1-z)$$

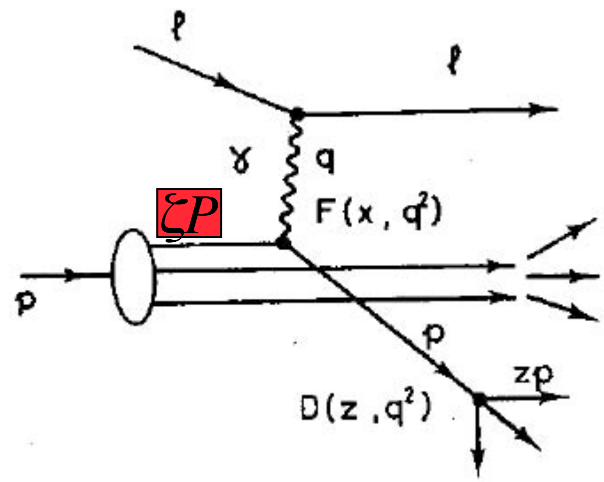
$$P_{gq}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

Heavy Quark Production

- Production of heavy quarks like charm requires a re-examination of the parton kinematics:

$$(q + \zeta p)^2 = m_c^2$$

$$q^2 + 2\zeta p \bullet q + \zeta^2 M^2 = m_c^2$$



$$\zeta \cong \frac{Q^2 + m_c^2}{2M\nu} = \frac{Q^2 + m_c^2}{Q^2 / x}$$

$$\zeta \cong x\left(1 + \frac{m_c^2}{Q^2}\right)$$

"slow rescaling" - The effects of the ~ 1 GeV charm mass are not negligible even at 100 GeV neutrino energy.

Charm identified through decays to μ+, di-muon events allow measurement of:

- CKM matrix elements
- m_c - from threshold behavior
- s and sbar quark distributions

Probability for ν_e Apperance

$P(\nu_\mu \rightarrow \nu_e$ in vacumn$) = P_1 + P_2 + P_3 + P_4$

- $P_1 = \sin^2(\theta_{23}) \sin^2(2\theta_{13}) \sin^2(1.27\ \Delta m_{13}^2\ L/E)$ "Atmospheric"
- $P_2 = \cos^2(\theta_{23}) \sin^2(2\theta_{12}) \sin^2(1.27\ \Delta m_{12}^2\ L/E)$ "Solar"
- $P_3 = J \sin(\delta) \sin(1.27\ \Delta m_{13}^2\ L/E)$
- $P_4 = J \cos(\delta) \cos(1.27\ \Delta m_{13}^2\ L/E)$

} Atmospheric-solar interference

where $J = \cos(\theta_{13}) \sin(2\theta_{12}) \sin(2\theta_{13}) \sin(2\theta_{23}) \sin(1.27\ \Delta m_{13}^2\ L/E) \sin(1.27\ \Delta m_{12}^2\ L/E)$

In matter at oscillation maximum, P_1 will be approximately multiplied by $(1 \pm 2E/E_R)$ and P_3 and P_4 will be approximately multiplied by $(1 \pm E/E_R)$ ($E_R \approx 11$ GeV for the earth's Crust), where the top sign is for neutrinos with normal mass hierarchy and antineutrinos with inverted mass hierarchy.

This is about ±30% effect for NuMI, about ±11% effect for T2K