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Introduction to Monte Carlo Event Generators

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1. (yesterday) Introduction and Overview; Monte Carlo Techniques

2. (yesterday) Matrix Elements; Parton Showers I

3. (today) Parton Showers II; Matching Issues

4. (today) Multiple Parton–Parton Interactions

5. (tomorrow) Hadronization and Decays; Generator Status

Event Physics Overview

Repetition: from the "simple" to the "complex", or from "calculable" at large virtualities to "modelled" at small

Matrix elements (ME):

1) Hard subprocess: $|\mathcal{M}|^2$, Breit-Wigners, parton densities.



Parton Showers (PS):

3) Final-state parton showers.



2) Resonance decays: includes correlations.



4) Initial-state parton showers.



5) Multiple parton–parton interactions.



6) Beam remnants, with colour connections.



5) + 6) = Underlying Event

7) Hadronization



8) Ordinary decays: hadronic, τ , charm, ...



Parton Distribution Functions

Hadrons are composite, with time-dependent structure:



 $f_i(x, Q^2)$ = number density of partons *i* at momentum fraction *x* and probing scale Q^2 .

Linguistics (example): $F_2(x,Q^2) = \sum_i e_i^2 x f_i(x,Q^2)$ structure function parton distributions Absolute normalization at small Q_0^2 unknown. Resolution dependence by DGLAP:

$$\frac{\mathrm{d}f_b(x,Q^2)}{\mathrm{d}(\ln Q^2)} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z} f_a(x',Q^2) \frac{\alpha_s}{2\pi} P_{a\to bc} \left(z = \frac{x}{x'}\right)$$



Initial-State Shower Basics

- Parton cascades in p are continuously born and recombined.
- Structure at Q is resolved at a time $t \sim 1/Q$ before collision.
- A hard scattering at Q^2 probes fluctuations up to that scale.
- A hard scattering inhibits full recombination of the cascade.



Event generation could be addressed by forwards evolution: pick a complete partonic set at low Q_0 and evolve, see what happens. Inefficient:

have to evolve and check for *all* potential collisions, but 99.9...% inert
 impossible to steer the production e.g. of a narrow resonance (Higgs)

Backwards evolution

Backwards evolution is viable and ~equivalent alternative: start at hard interaction and trace what happened "before"



Monte Carlo approach, based on *conditional probability*: recast

$$\frac{\mathrm{d}f_b(x,Q^2)}{\mathrm{d}t} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z} f_a(x',Q^2) \frac{\alpha_s}{2\pi} P_{a\to bc}(z)$$
with $t = \ln(Q^2/\Lambda^2)$ and $z = x/x'$ to
$$\mathrm{d}\mathcal{P}_b = \frac{\mathrm{d}f_b}{f_b} = |\mathrm{d}t| \sum_a \int \mathrm{d}z \frac{x'f_a(x',t)}{xf_b(x,t)} \frac{\alpha_s}{2\pi} P_{a\to bc}(z)$$
then solve for *de*creasing *t*, i.e. backwards in time, starting at high Q^2 and moving towards lower, with Sudakov form factor $\exp(-\int \mathrm{d}\mathcal{P}_b)$

Ladder representation combines whole event:



cf. previously:



One possible
Monte Carlo order:
1) Hard scattering
2) Initial-state shower from center outwards
3) Final-state showers

Coherence in spacelike showers



• kinematics only: $Q_3^2 > z_1Q_1^2, Q_5^2 > z_3Q_3^2, \dots$ i.e. Q_i^2 need not even be ordered

- coherence of leading collinear singularities: $Q_5^2 > Q_3^2 > Q_1^2$, i.e. Q^2 ordered
- coherence of leading soft singularities (more messy):
 - $\begin{array}{ll} E_{3}\theta_{4} > E_{1}\theta_{2}, \text{ i.e. } z_{1}\theta_{4} > \theta_{2} \\ z \ll 1: & E_{1}\theta_{2} \approx p_{\perp 2}^{2} \approx Q_{3}^{2}, E_{3}\theta_{4} \approx p_{\perp 4}^{2} \approx Q_{5}^{2} \\ & \text{ i.e. reduces to } Q^{2} \text{ ordering as above} \\ z \approx 1: & \theta_{4} > \theta_{2}, \text{ i.e. angular ordering of soft gluons} \\ \implies \text{ reduced phase space} \end{array}$

Evolution procedures



DGLAP: Dokshitzer–Gribov–Lipatov–Altarelli–Parisi evolution towards larger Q^2 and (implicitly) towards smaller xBFKL: Balitsky–Fadin–Kuraev–Lipatov evolution towards smaller x (with small, unordered Q^2) CCFM: Ciafaloni–Catani–Fiorani–Marchesini interpolation of DGLAP and BFKL GLR: Gribov–Levin–Ryskin nonlinear equation in dense-packing (saturation) region, where partons recombine, not only branch

Initial-State Shower Comparison

Two(?) CCFM Generators: (SMALLX (Marchesini, Webber)) CASCADE (Jung, Salam) LDC (Gustafson, Lönnblad): reformulated initial/final rad. \implies eliminate non-Sudakov



Test 1) forward (= p direction) jet activity at HERA





but also explained by DGLAP with leading order pair creation + flavour excitation (\approx unordered chains) + gluon splitting (final-state radiation)

CCFM requires off-shell ME's + unintegrated parton densities

$$F(x,Q^2) = \int^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \mathcal{F}(x,k_{\perp}^2) + (\text{suppressed with } k_{\perp}^2 > Q^2)$$

so not ready for prime time in pp

Initial- vs. final-state showers

Both controlled by same evolution equations

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_{\rm S}}{2\pi} \frac{\mathrm{d}Q^2}{Q^2} P_{a\to bc}(z) \,\mathrm{d}z \,\cdot\, (\mathrm{Sudakov})$$

but



decreasing E, m^2, θ both daughters $m^2 \ge 0$ physics relatively simple \Rightarrow "minor" variations: Q^2 , shower vs. dipole, ... Initial-state showers: Q^2 spacelike ($\approx -m^2$) E_0, Q_0^2 E_2, m_2^2 E_1, Q_1^2

decreasing E, increasing Q^2 , θ one daughter $m^2 \ge 0$, one $m^2 < 0$ physics more complicated \Rightarrow more formalisms: DGLAP, BFKL, CCFM, GLR, ...

Combining FSR with ISR



Separate processing of ISR and FSR misses interference (\sim colour dipoles)



ISR+FSR add coherently in regions of colour flow

in "normal" shower by azimuthal anisotropies

automatic in dipole (by proper boosts)



FIG. 13. Observed η_3 distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.



FIG. 15. Observed α distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.



Sherpa dipoles



Evolution of timelike sidebranch cascades can reduce p_{\perp} :



Matrix Elements vs. Parton Showers

- ME : Matrix Elements
 - + systematic expansion in α_{s} ('*exact*')
 - + powerful for multiparton Born level
 - + flexible phase space cuts
 - loop calculations very tough
 - negative cross section in collinear regions
 - \Rightarrow unpredictive jet/event structure
 - no easy match to hadronization
- **PS** : Parton Showers
 - approximate, to LL (or NLL)
 - $\begin{array}{ll} & \text{main topology not predetermined} \\ \Rightarrow \text{ inefficient for exclusive states} \end{array}$
 - + process-generic \Rightarrow simple multiparton
 - + Sudakov form factors/resummation
 ⇒ sensible jet/event structure
 - + easy to match to hadronization





(T. Plehn, D. Rainwater, P. Skands)

Matrix Elements and Parton Showers

Recall complementary strengths:

- ME's good for well separated jets
- PS's good for structure inside jets

Marriage desirable! But how?

Problems: • gaps in coverage?

- doublecounting of radiati
- doublecounting of radiation?
- Sudakov?
- NLO consistency?

Much work ongoing \implies no established orthodoxy

Three main areas, in ascending order of complication:

1) Match to lowest-order nontrivial process — merging

2) Combine leading-order multiparton process — vetoed parton showers

3) Match to next-to-leading order process — MC@NLO, POWHEG

Merging

= cover full phase space with smooth transition ME/PS Want to reproduce $W^{ME} = \frac{1}{\sigma(LO)} \frac{d\sigma(LO+g)}{d(phasespace)}$ by shower generation + correction procedure $W^{ME} = W^{PS} \qquad \frac{W^{ME}}{W^{PS}}$

• Exponentiate ME correction by shower Sudakov form factor:

$$W_{\text{actual}}^{\text{PS}}(Q^2) = W^{\text{ME}}(Q^2) \exp\left(-\int_{Q^2}^{Q_{\text{max}}^2} W^{\text{ME}}(Q'^2) dQ'^2\right)$$

• Do not normalize W^{ME} to $\sigma(NLO)$ (error $\mathcal{O}(\alpha_s^2)$ either way)



• Normally several shower histories \Rightarrow \sim equivalent approaches

Final-State Shower Merging

Merging with $\gamma^*/Z^0 \rightarrow q\overline{q}g$ for $m_q = 0$ since long (M. Bengtsson & TS, PLB185 (1987) 435, NPB289 (1987) 810)

For $m_q > 0$ pick $Q_i^2 = m_i^2 - m_{i,\text{onshell}}^2$ as evolution variable since $W^{\text{ME}} = \frac{(\dots)}{Q_1^2 Q_2^2} - \frac{(\dots)}{Q_1^4} - \frac{(\dots)}{Q_2^4}$

Coloured decaying particle also radiates:



Subsequent branchings $q \rightarrow qg$: also matched to ME, with reduced energy of system

PYTHIA performs merging with generic FSR $a \rightarrow bcg$ ME, in SM: $\gamma^*/Z^0/W^{\pm} \rightarrow q\overline{q}, t \rightarrow bW^+, H^0 \rightarrow q\overline{q},$ and MSSM: $t \rightarrow bH^+, Z^0 \rightarrow \tilde{q}\overline{\tilde{q}}, \tilde{q} \rightarrow \tilde{q}'W^+, H^0 \rightarrow \tilde{q}\overline{\tilde{q}}, \tilde{q} \rightarrow \tilde{q}'H^+,$ $\chi \rightarrow q\overline{\tilde{q}}, \chi \rightarrow q\overline{\tilde{q}}, \tilde{q} \rightarrow q\chi, t \rightarrow \tilde{t}\chi, \tilde{g} \rightarrow q\overline{\tilde{q}}, \tilde{q} \rightarrow q\tilde{g}, t \rightarrow \tilde{t}\tilde{g}$

g emission for different colour, spin and parity:

 $R_3^{bl}(y_c)$: mass effects in Higgs decay:



Initial-State Shower Merging



Merging in HERWIG

HERWIG also contains

merging, for

- $\bullet \ Z^0 \to q \overline{q}$
- t \rightarrow bW⁺
- $\bullet \ q \overline{q} \to Z^0$

and some more

Special problem: angular ordering does not cover full phase space; so (1) fill in "dead zone" with ME (2) apply ME correction in allowed region

Important for agreement with data:



Vetoed Parton Showers

S. Catani, F. Krauss, R. Kuhn, B.R. Webber, JHEP 0111 (2001) 063; L. Lönnblad, JHEP0205 (2002) 046;

F. Krauss, JHEP 0208 (2002) 015; S. Mrenna, P. Richardson, JHEP0405 (2004) 040;

M.L. Mangano et al., JHEP0701 (2007) 013

Generic method to combine ME's of several different orders to NLL accuracy; has become a standard tool for many studies

Basic idea:

- consider (differential) cross sections $\sigma_0, \sigma_1, \sigma_2, \sigma_3, \ldots$, corresponding to a lowest-order process (e.g. W or H production), with more jets added to describe more complicated topologies, in each case to the respective leading order
- σ_i , $i \geq 1$, are divergent in soft/collinear limits
- absent virtual corrections would have ensured "detailed balance", i.e. an emission that adds to σ_{i+1} subtracts from σ_i
- such virtual corrections correspond (approximately) to the Sudakov form factors of parton showers
- so use shower routines to provide missing virtual corrections
 rejection of events (especially) in soft/collinear regions
 - \Rightarrow rejection of events (especially) in soft/collinear regions

Veto scheme:

1) Pick hard process, mixing according to $\sigma_0 : \sigma_1 : \sigma_2 : ...,$ above some ME cutoff (e.g. all $p_{\perp i} > p_{\perp 0}$, all $R_{ij} > R_0$), with large fixed α_{s0}

2) Reconstruct imagined shower history (in different ways) 3) Weight $W_{\alpha} = \prod_{\text{branchings}} (\alpha_{s}(k_{\perp i}^{2})/\alpha_{s0}) \Rightarrow \text{accept/reject}$

CKKW-L:

4) Sudakov factor for non-emission on all lines above ME cutoff W_{Sud} = ∏ "propagators" Sudakov(k²_{⊥beg}, k²_{⊥end})
4a) CKKW : use NLL Sudakovs
4b) L: use trial showers
5) W_{Sud} ⇒ accept/reject
6) do shower, vetoing emissions above cutoff MLM:

- 4) do parton showers
- 5) (cone-)cluster showered event
- 6) match partons and jets
- 7) if all partons are matched, and $n_{jet} = n_{parton}$, keep the event, else discard it





0

50

100

150

200

250

 $p_{\perp}(Z)$ [GeV]

IPPP Durham



• Example: All-jets p_T 's in DY-pair production

CDF Data: PRL 100 (2008) 102001

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QA

Introduction	Matrix elements	Parton showers	Merging	Soft physics	Forthcoming attractions

Z^0 +jets at Tevatron: cross sections

CDF data from PRL 100 (2008) 102001 and D0/, arXiv:0808:1296

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Impact of α_{S} - global in SHERPA



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CKKW mix of W + (0, 1, 2, 3, 4) partons, hadronized and clustered to jets:



(S.Mrenna, P. Richardson)



Spread of W + jets rate for different matching schemes + showers, top: Tevatron, bottom: LHC.

ALPGEN: MLM + HERWIG ARIADNE: CKKW-L + ARIADNE HELAC: MLM + PYTHIA MADEVENT: MLM/CKKW + PYTHIA SHERPA: CKKW + SHERPA model varation: α_{s} , cuts, ...

(Alwall, Krauss, Lavesson, Lönnblad, Mangano, Worek, ...)

MC@NLO

Objectives:

- Total rate should be accurate to NLO.
- NLO results are obtained for all observables when (formally) expanded in powers of α_s .
- Hard emissions are treated as in the NLO computations.
- Soft/collinear emissions are treated as in shower MC.
- The matching between hard and soft emissions is smooth.
- The outcome is a set of "normal" events, that can be processed further.

Basic scheme (simplified!):

- 1) Calculate the NLO matrix element corrections to an *n*-body process (using the subtraction approach).
- 2) Calculate analytically (no Sudakov!) how the first shower emission off an *n*-body topology populates (n + 1)-body phase space.
- 3) Subtract the shower expression from the (n + 1) ME to get the "true" (n + 1) events, and consider the rest of σ_{NLO} as *n*-body.
- 4) Add showers to both kinds of events.



MC@NLO in comparison:

- Superior with respect to "total" cross sections.
- Equivalent to merging for event shapes (differences higher order).
- Inferior to CKKW-L for multijet topologies.
- \Rightarrow pick according to current task and availability.

MC@NLO 2.31 [hep-ph/0402116]

IPROC	Process
-1350-IL	$H_1H_2 \to (Z/\gamma^* \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
-1360-IL	$H_1H_2 \to (Z \to)l_{\rm IL}\bar{l}_{\rm IL} + X$
-1370-IL	$H_1H_2 \to (\gamma^* \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
-1460-IL	$H_1H_2 \to (W^+ \to) l_{\rm IL}^+ \nu_{\rm IL} + X$
-1470-IL	$H_1H_2 \to (W^- \to) l_{\rm IL}^- \bar{\nu}_{\rm IL} + X$
-1396	$H_1H_2 \to \gamma^* (\to \sum_i f_i \bar{f}_i) + X$
-1397	$H_1 H_2 \to Z^0 + X$
-1497	$H_1H_2 \to W^+ + X$
-1498	$H_1 H_2 \to W^- + X$
-1600-ID	$H_1 H_2 \to H^0 + X$
-1705	$H_1H_2 \to b\bar{b} + X$
-1706	$H_1H_2 \to t\bar{t} + X$
-2850	$H_1H_2 \to W^+W^- + X$
-2860	$H_1 H_2 \to Z^0 Z^0 + X$
-2870	$H_1H_2 \to W^+Z^0 + X$
-2880	$H_1 H_2 \to W^- Z^0 + X$

(Frixione, Webber)

- Works identically to HERWIG: the very same analysis routines can be used
- Reads shower initial conditions from an event file (as in ME corrections)
- Exploits Les Houches accord for process information and common blocks
- Features a self contained library of PDFs with old and new sets alike
- LHAPDF will also be implemented

Later additions: single top, H^0W^{\pm} , H^0Z^0 , tW, ...

 W^+W^- Observables



These correlations are problematic: the soft and hard emissions are both relevant. MC@NLO does well, resumming large logarithms, and yet handling the large-scale physics correctly

Solid: MC@NLO Dashed: HERWIG $\times \frac{\sigma_{NLO}}{\sigma_{LO}}$ Dotted: NLO

POWHEG

Nason; Frixione, Oleari, Ridolfi (e.g. JHEP **0711** (2007) 070) Better (?) alternative to MC@NLO:

$$d\sigma = \bar{B}(v)d\Phi_v \left[\frac{R(v,r)}{B(v)}\exp\left(-\int_{p_{\perp}}\frac{R(v,r')}{B(v)}d\Phi_r'\right)d\Phi_r\right]$$

where

$$\overline{B}(v) = B(v) + V(v) + \int \mathrm{d}\Phi_r [R(v,r) - C(v,r)] \,.$$

and

 $v, d\Phi_v$ Born-level *n*-body variables and differential phase space $r, d\Phi_r$ extra n + 1-body variables and differential phase space B(v) Born-level cross section

V(v) Virtual corrections

R(v,r) Real-emission cross section

C(v,r) Conterterms for collinear factorization of parton densities.

Basic idea:

- Pick the real emission with largest p_{\perp} according to complete ME's, with NLO normalization.
- Let showers do subsequent evolution downwards from this p_{\perp} scale.

Relative to MC@NLO:

- + no negative weights (except in regions with extreme virtual corrections)
- + clean separation to shower stage
- \pm optimal for p_{\perp} -ordered showers, messy but manageable for others
- \pm different higher-order terms
- as of yet fewer processes than $\ensuremath{\mathsf{MC@NLO}}$



Status of POWHEG

Up to now, the following processes have been implemented in POWHEG:

- $hh \rightarrow ZZ$ (Ridolfi, P.N., 2006)
- $e^+e^- \rightarrow \text{hadrons}$, (Latunde-Dada, Gieseke, Webber, 2006), $e^+e^- \rightarrow t\bar{t}$, including top decays at NLO (Latunde-Dada, 2008),
- $hh \rightarrow Q\bar{Q}$ (Frixione, Ridolfi, P.N., 2007)
- $hh \rightarrow Z/W$ (Alioli, Oleari, Re, P.N., 2008;) (Hamilton,Richardson,Tully, 2008;)
- $hh \rightarrow H$ (gluon fusion) (Alioli, Oleari, Re, P.N., 2008; Herwig++)
- $hh \rightarrow H, hh \rightarrow HZ/W$ (Hamilton, Richardson, Tully, 2009;)
- $hh \rightarrow t + X$ (single top) NEW (Alioli, Oleari, Re, P.N., 2009)
- $hh \rightarrow Z + jet$, Very preliminary (Alioli, Oleari, Re, P.N., 2009)
- The POWHEG BOX, Very preliminary, (Alioli, Oleari, Re, P.N., 2009)

Summary Lecture 3

 Showers bring us *from* few-parton "pencil-jet" topologies to multi-broad-jet states.

• Necessary complement to matrix elements: • * Do not trust off-the-shelf ME for $R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} \lesssim 1 \star$ * Do not trust unmatched PS for $R \gtrsim 1 \star$

• Two main lines of evolution: •

 \star (1) Improve algorithm as such: evolution variables, kinematics, dipoles, NLL, small-x, k_{\perp} factorization, BFKL/CCFM, ... \star

* (2) Improve matching ME-PS: merging,
 vetoed parton showers, MC@NLO, POWHEG *

 $\star \Rightarrow$ active area of development; high profile \star