Vector Bosons
in
Large Momentum Transfer Processes

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Outline

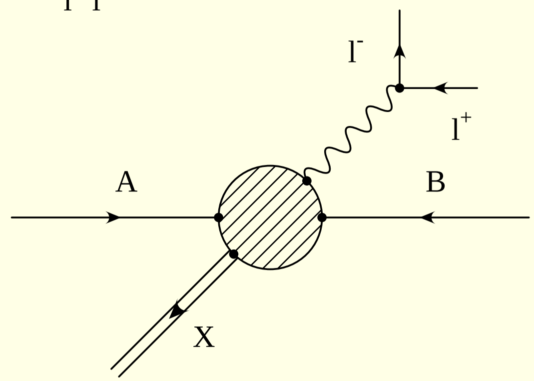
1. Introduction
   - History, motivation, and early ideas
   - Relation to photon production
2. Simple calculation
   - Kinematics, matrix element, and phase space
   - Features and shortcomings
3. Higher Order Calculations
   - p_T distributions and k_T resummation
   - NLO results, NNLO results
   - Rapidity distributions and threshold resummation

Lepton Pair Production

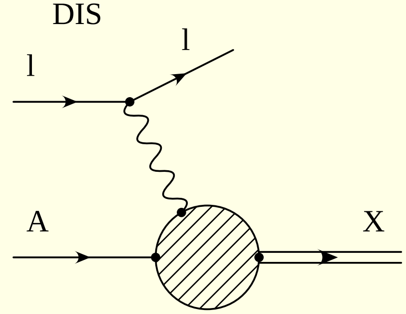
S.D. Drell and T.-M. Yan, Phys. Rev. Lett. **25**, 316 (1970)

- Electromagnetic probe of a hadron-hadron process
- compare to
  - DIS: E-M probe of a single hadron process
  - e^+e^-: E-M probe of hadron production
- Simple description in terms of the (then new) parton model
- Mass of the pair could be varied to insure that the parton momentum fractions were neither too small nor too large (avoid problems with x near 0 or 1)

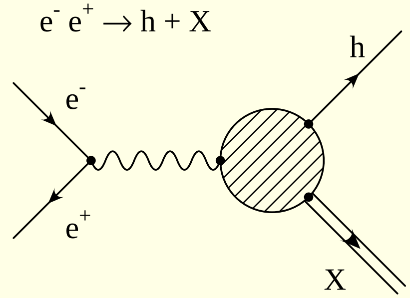
$l^+ l^-$



DIS

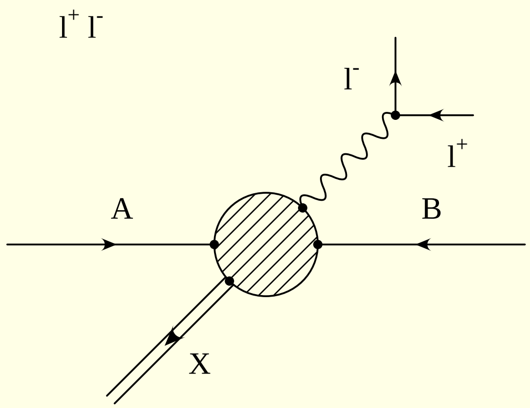


$e^- e^+ \to h + X$



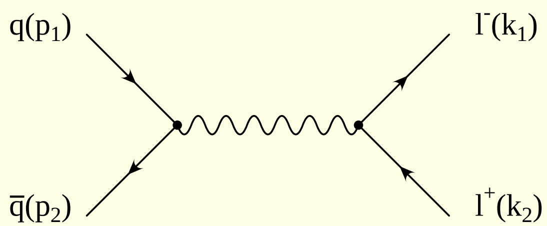
- Structure of the parton model calculation preserved in the presence of QCD corrections
- First example of a calculable hadron-hadron process in the context of the parton model
- Process is of historical interest (2 Nobel prizes)
- Pedagogical importance - one of the early calculations of higher order QCD corrections
- Important for precision Standard Model measurements
- Important roles in searches for new physics

Basic Idea



- Producing a virtual photon with mass Q and $Q^2 > 0$
- Our task is to figure out what is in the shaded circle in the figure
- Simplest possibility: $q\overline{q} \to l^+ l^-$
- Represents purely E-M process in the context of the parton model (treating the quarks as free)
- Simple, testable prediction for the angular distribution of the lepton pair

Born Term



Lorentz invariant variables

$$\begin{aligned}\hat{s} &= (p_1 + p_2)^2 = (k_1 + k_2)^2 \\ \hat{t} &= (p_1 - k_1)^2 = (p_2 - k_2)^2 \\ \hat{u} &= (p_1 - k_2)^2 = (p_2 - k_1)^2\end{aligned}$$

Matrix element

$$M = e_q \frac{e^2}{\hat{s}} \overline{u}(k_1)\gamma_\mu v(k_2)\overline{v}(p_2)\gamma^\mu u(p_1)$$

Spin/color averaged matrix element squared

$$\overline{\sum}|M|^2 = \frac{e_q^2 e^4}{\hat{s}^2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)3\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\mathrm{Tr}\left[\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu\right]\mathrm{Tr}\left[\not{k}_2\gamma_\nu\not{k}_1\gamma_\mu\right]$$

$$= \frac{4}{3}\frac{e_q^2 e^4}{\hat{s}^2}\left[p_1^\nu p_2^\mu + p_1^\mu p_2^\nu - g^{\mu\nu} p_1\cdot p_2\right]\left[k_{2\nu}k_{1\mu} + k_{2\mu}k_{1\nu} - g_{\mu\nu}k_1\cdot k_2\right]$$

Red factors are for the spin average, blue factors are for the color average. Forming the indicated dot products yields

$$\begin{aligned}\overline{\sum}|M|^2 &= \frac{4}{3}\frac{e_q^2 e^4}{\hat{s}^2}\left[2p_1\cdot k_2 p_2\cdot k_1 + 2p_1\cdot k_1 p_2\cdot k_2\right]\\ &= \frac{2}{3}\frac{e_q^2 e^4}{\hat{s}^2}\left[\hat{t}^2 + \hat{u}^2\right]\end{aligned}$$

Center of mass frame: the 4-vectors are

$$p_1 = \frac{\sqrt{\hat{s}}}{2}(1,0,0,1) \quad p_2 = \frac{\sqrt{\hat{s}}}{2}(1,0,0,-1)$$

$$k_1 = \frac{\sqrt{\hat{s}}}{2}(1,\sin\theta,0,\cos\theta) \quad k_2 = \frac{\sqrt{\hat{s}}}{2}(1,-\sin\theta,0,-\cos\theta)$$

yielding the Lorentz scalars

$$\hat{t} = -\frac{\hat{s}}{2}(1-\cos\theta) \text{ and } \hat{u} = -\frac{\hat{s}}{2}(1+\cos\theta)$$

with $\hat{t}^2 + \hat{u}^2 = \frac{\hat{s}^2}{2}(1+\cos^2\theta)$

Inserting these relations into our result yields:

$$\overline{\sum}|M|^2 = \frac{e_q^2 e^4}{3}(1+\cos^2\theta)$$

To make use of this result we need to convert it to a cross section. For this we need the two-body Lorentz invariant phase space factor:

$$
\begin{aligned}
PS^{(2)} &= \frac{d^3k_1}{(2\pi)^3 2E_1}\frac{d^3k_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_1+p_2-k_1-k_2) \\
&= \frac{d^3k}{16\pi^2 E_1 E_2}\delta(\sqrt{\hat{s}}-E_1-E_2).
\end{aligned}
$$

In the center-of-momentum frame we have $k = |\vec{k}_1| = |\vec{k}_2|$ so that in this frame we can write

$$
\begin{aligned}
d(E_1+E_2) &= d\sqrt{\hat{s}}kdk\left(\frac{1}{E_1}+\frac{1}{E_2}\right) \\
&= kdk\frac{E_1+E_2}{E_1E_2}.
\end{aligned}
$$

with $k = \sqrt{\hat{s}}/2$.

This allows the phase space factor to be written as

$$\begin{aligned} PS^{(2)} &= \frac{k^2 dk d\Omega}{16\pi^2 E_1 E_2} \delta(\sqrt{\hat{s}} - E_1 - E_2) \\ &= \frac{k d\sqrt{\hat{s}} d\Omega}{16\pi^2 \sqrt{\hat{s}}} \delta(\sqrt{\hat{s}} - E_1 - E_2) \\ &= \frac{d\Omega}{32\pi^2} \\ &= \frac{d\cos(\theta)}{16\pi} \end{aligned}$$

To get a cross section we multiply the phase space factor times the spin and color averaged squared matrix element and multiply that by a flux factor of $1/2\hat{s}$ yielding

$$\begin{aligned}\sigma(q\bar{q} \to l^+l^-) &= \frac{1}{2\hat{s}} \int_{-1}^{1} \frac{d\cos(\theta)}{16\pi} \frac{e_q^2 e^4}{3} (1 + \cos^2(\theta)) \\ &= \frac{e_q^2}{3} \frac{(4\pi\alpha)^2}{16\pi} \frac{1}{2\hat{s}} \frac{8}{3}\end{aligned}$$

where the fine structure constant $\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$.
The final result for the parton-level cross section is

$$\sigma(q\bar{q} \to l^+l^-) = \frac{4\pi\alpha^2}{9\hat{s}} e_q^2 \equiv \sigma_0.$$

Hadronic Cross Section

Introduce the parton distribution functions: let $q_A(x)dx$ denote the probability of finding a parton of flavor q in hadron A with a momentum fraction lying between x and $x + dx$. Convolute the parton-level cross section σ_0 with the appropriate quark and antiquark parton distribution functions:

$$\sigma(AB \to l^+ l^- + X) = \sum_q \int dx_a dx_b \, \sigma_0 \left[q(x_a) \overline{q}(x_b) + a \leftrightarrow b \right]$$

Note: remember to symmetrize with respect to the beam and target particles. This corresponds to $\hat{t} \leftrightarrow \hat{u}$ here, so σ_0 is unchanged.

Differential Distributions

The total cross section involves a convolution with products of parton distributions. In order to test the theory or to learn more about the parton distributions it has proven to be convenient to undo one or both of the integrations by looking a differential distributions. If we ignore external hadronic masses, we can relate the hadronic and partonic center of mass energies as follows:

$$s = (p_A + p_B)^2 = 2 p_A \cdot p_B = \frac{2 p_1 \cdot p_2}{x_a x_b} = \frac{\hat{s}}{x_a x_b}$$

where it has been assumed that $p_1 = x_a p_A$ and $p_2 = x_b p_B$.

Lepton pair mass distribution

For $q\overline{q} \rightarrow l^+ l^-$ the invariant mass of the lepton pair is just $Q^2 = \hat{s}$. Thus,

$$\frac{d\sigma}{dQ^2} \quad = \quad \sum_q \int dx_a dx_b \, H_q(x_a, x_b) \, \sigma_0 \, \delta(Q^2 - \hat{s})$$

Here the sum over the products of parton distributions is denoted by the function $H_q(x_a, x_b)$. Next, evaluate the δ function as follows:

$$\int dx_a dx_b \delta(Q^2 - x_a x_b s) = \int \frac{dx_a}{x_a s} \delta(x_b - Q^2/x_a s).$$

Thus,

$$\frac{d\sigma}{dQ^2} = \sum_q \int_{x_{amin}}^1 \frac{dx_a}{x_a s} H_q(x_a, \frac{Q^2}{x_a s}) \frac{4\pi\alpha^2}{9Q^2} e_q^2$$

with $x_{amin} = Q^2/s$.

Longitudinal Momentum Distributions

Define a longitudinal scaling variable

$$x_F = p_l/p_{lmax} \approx 2p_l/\sqrt{s}$$

where p_l is the longitudinal momentum in the hadron-hadron cms. For the $q\overline{q} \rightarrow l^+l^-$ subprocess, we are interested in the longitudinal momentum of the lepton pair. We have

$$p_1 = x_a \tfrac{\sqrt{s}}{2}(1,0,0,1) \qquad p_2 = x_b \tfrac{\sqrt{s}}{2}(1,0,0,-1)$$

$$E = \tfrac{\sqrt{s}}{2}(x_a + x_b) \qquad p_l = \tfrac{\sqrt{s}}{2}(x_a - x_b)$$

which yields $x_F = x_a - x_b$.

One can use this to define a double differential cross section

$$\frac{d\sigma}{dQ^2 dx_F} = \frac{4\pi\alpha^2}{9Q^4} \sum_q e_q^2 \int_\tau^1 \frac{dx_a}{x_a}\ \tau H_q(x_a, \frac{\tau}{x_a})\delta(x_F - x_a + \frac{\tau}{x_a})$$

The δ function constraint can be solved for x_a yielding

$$x_a = \frac{1}{2}\left(x_F + \sqrt{x_F^2 + 4\tau}\right).$$

Using $x_b = \tau/x_a$ one derives

$$x_b = \frac{1}{2}\left(-x_F + \sqrt{x_F^2 + 4\tau}\right).$$

The Jacobian factor from the δ function introduces a factor of $x_a/(x_a + x_b)$, so the final result can be written as

$$\frac{d\sigma}{dQ^2 dx_F} = \frac{4\pi\alpha^2}{9Q^4}\frac{1}{\sqrt{x_F^2 + 4\tau}}\tau \sum_q e_q^2 H_q(x_a, \frac{\tau}{x_a}).$$

Rapidity

The rapidity variable is defined as

$$y = \frac{1}{2} \ln \frac{E + p_l}{E - p_l} = \frac{1}{2} \ln \frac{x_a}{x_b}$$

which yields

$$x_a = \sqrt{\tau} e^y \text{ and } x_b = \sqrt{\tau} e^{-y}.$$

Changing variables from (Q^2, x_F) to (y, τ) is done using

$$dQ^2 dx_F = dy d\tau \; s \sqrt{x_F^2 + 4\tau}$$

which gives

$$\frac{d\sigma}{dy d\tau} = \frac{4\pi\alpha^2}{9s} \sum_q \frac{e_q^2}{\tau} H_q(x_a, x_b)$$

with $x_{a,b}$ given above.

QCD

To this point we have been reviewing simple parton model results for lepton pair production. Where does QCD enter and how does it modify what we've done so far?

Easy to answer – harder to prove:

- To leading-logarithm accuracy all of the foregoing expressions are correct provided that we use scale dependent parton distributions $q_a(x_a, Q^2)$, etc. when evaluating the function $H_q(x_a, x_b)$! These are obtained as solutions of the DGLAP equations.
- These scale dependent PDFs contain the effects of initial state gluon radiation intergated up to the factorization scale M_f which is typically chosen to be $\mathcal{O}(\alpha_s)$.
- The significance of the mass and longitudinal distributions discussed so far is their easy interpretation in terms of products of PDFs - not quite as direct as for structure fuinctions, but close.

p_T Distribution

- To the order we are working for the hard scattering subprocess , *i.e.*, $\mathcal{O}(\alpha_s^0)$, no transverse momentum is generated via the subprocess itself ($q\overline{q} \rightarrow l^+l^-$).
- Transverse momenta associated with initial state gluon emission have been integrated out in the process of solving the DGLAP evolution equations for the scale dependent PDFs.
- The PDFs retain their dependence on the longitudinal momentum fractions – in the preceeding discussion these have been fixed by specifying Q^2 and either x_F or the rapidity.
- Early parton model predictions treated the lepton pair transverse momentum distributions by attributing a gaussian transverse momentum distribution to the incoming partons ("intrinsic k_T")
  - Data showed $< k_T > \simeq$ 760 MeV per parton at p_{lab} =400 GeV ($\sqrt{s}$ = 27.4 GeV)
  - $< k_T >$ larger than expectations based on hadron size.
  - Data showed a non-gaussian tail.
- Above observations suggested that there was more than just the simple parton model at work here. Turn next to examining higher order QCD corrections to the description obtained thus far.

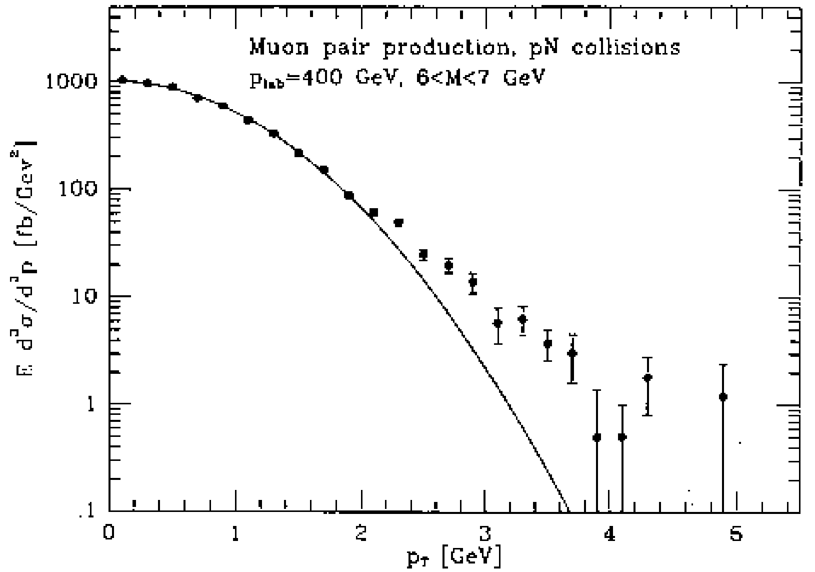
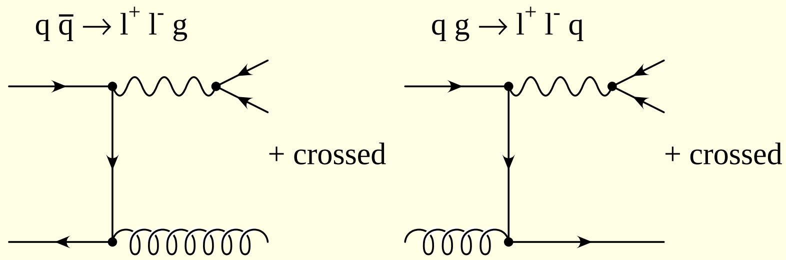


Fig. 9.2. The lepton pair transverse momentum from the CFS collaboration [4]. The curve corresponds to a Gaussian intrinsic k_T distribution for the annihilating partons.

Figure from "QCD and Collider Physics" by Ellis, Stirling, and Webber. Note the gaussian-like behavior in the low-p_T region with a non-gaussian tail appearing at higher values of p_T.

$\mathcal{O}(\alpha_s)$ QCD Contributions



- Lepton pair recoils against a quark or gluon
- Can simplify the calculation by considering the production of a virtual photon of mass Q. Label the momenta by $q(p_1) + \bar{q}(p_2) \longrightarrow \gamma^*(q) + g(k_3)$.

$$
\begin{aligned}
d\sigma &= \frac{1}{2\hat{s}} \overline{\sum} |M(q\bar{q} \to \gamma^* g)|^2 \frac{d^3q}{(2\pi)^3 2E_q} \frac{d^3k_3}{(2\pi)^3 2E_3} \\
&\quad \times (2\pi)^4 \delta(p_1 + p_2 - q - k_3) \\
&= \frac{1}{2\hat{s}} \overline{\sum} |M(q\bar{q} \to \gamma^* g)|^2 \frac{\hat{s} - Q^2}{2\hat{s}} \frac{d\Omega}{16\pi^2}
\end{aligned}
$$

The four-vectors for q and p_1 are given by

$$\begin{aligned} q &= \left[\frac{\hat{s}+Q^2}{2\sqrt{\hat{s}}}, \frac{\hat{s}-Q^2}{2\sqrt{\hat{s}}}\sin(\theta), 0, \frac{\hat{s}-Q^2}{2\sqrt{\hat{s}}}\cos(\theta)\right] \\ p_1 &= \frac{\sqrt{\hat{s}}}{2}(1,0,0,1) \end{aligned}$$

so that

$$\hat{t}, \hat{u} = -\frac{\hat{s}-Q^2}{2}(1 \mp \cos(\theta)) \quad \text{and} \quad d\cos(\theta) = \frac{2}{\hat{s}-Q^2} d\hat{t}$$

Therefore, we have

$$\frac{d\sigma}{d\hat{t}} = \frac{1}{16\pi \hat{s}^2} \overline{\sum} |M(q\overline{q} \to \gamma^* g)|^2$$

Next, consider the decay process $\gamma^*(q) \to l^-(k_1) + l^+(k_2)$. We have

$$M_\mu = e\overline{u}(k_1)\gamma_\mu v(k_2)$$

which yields

$$\begin{aligned}\overline{\sum}|M|^2 &= -\frac{1}{3}e^2\mathrm{Tr}\,[\not{k}_2\gamma_\mu\not{k}_1\gamma^\mu] \\ &= \frac{2}{3}e^2\mathrm{Tr}\,[\not{k}_1\not{k}_2] = \frac{16\pi\alpha Q^2}{3}\end{aligned}$$

Next, consider the full $2 \to 3$ subprocess

$$\begin{aligned}d\sigma &= \frac{1}{2\hat{s}}\overline{\sum}|M(q\overline{q} \to l^+l^-g)|^2\frac{d^3k_1}{(2\pi)^3 2E_1}\frac{d^3k_2}{(2\pi)^3 2E_2}\frac{d^3k_3}{(2\pi)^3 2E_3} \\ &\times(2\pi)^4\delta^4(p_1 + p_2 - k_1 - k_2 - k_3).\end{aligned}$$

Now, insert $d^4q\delta^4(q-k_1-k_2)$ and split the matrix element into production and decay processes.

$$
\begin{aligned}
d\sigma &= \frac{1}{2\hat{s}}\overline{\sum}|M(q\overline{q}\to\gamma^* g)|^2\frac{d^3k_3}{(2\pi)^3 2E_3}d^4q \\
&\quad\times\delta^4(p_1+p_2-q-k_3)\frac{1}{Q^4}\overline{\sum}|M(\gamma^*\to l^+l^-)|^2 \\
&\quad\times\frac{d^3k_1}{(2\pi)^3 2E_1}\frac{d^3k_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(q-k_1-k_2)
\end{aligned}
$$

The factors on the last line are just those for two-body phase space, so they can be rewritten as $\frac{d\Omega}{32\pi^2}$, thereby simplifying the result to

$$
d\sigma = \frac{1}{16\pi\hat{s}Q^4}\overline{\sum}|M(q\overline{q}\to\gamma^* g)|^2\frac{d^3k_3}{(2\pi)^3 2E_3}\frac{16\pi\alpha Q^2}{3}
$$

Next rewrite the k_3 differential as

$$\frac{d^3k_3}{(2\pi)^3 2E_3} = \frac{k_3 dk_3 d\cos(\theta)}{8\pi^2} = \frac{d\hat{t}dQ^2}{16\pi^2\hat{s}} \quad \text{with} \quad k_3 = \frac{\hat{s}-Q^2}{2\sqrt{\hat{s}}}.$$

Therefore, one obtains

$$\frac{d\sigma}{dQ^2 d\hat{t}} = \frac{1}{16\pi^2\hat{s}^2}\frac{\alpha}{3Q^2}\overline{\sum}|M(q\overline{q}\to\gamma^* g)|^2.$$

The end result is that the $2 \to 3$ cross section is proportional to a simpler $2 \to 2$ cross section:

$$\frac{d\sigma}{dQ^2 d\hat{t}}(q\overline{q}\to l^+l^-g) = \frac{\alpha}{3\pi Q^2}\frac{d\sigma}{d\hat{t}}(q\overline{q}\to\gamma^* g).$$

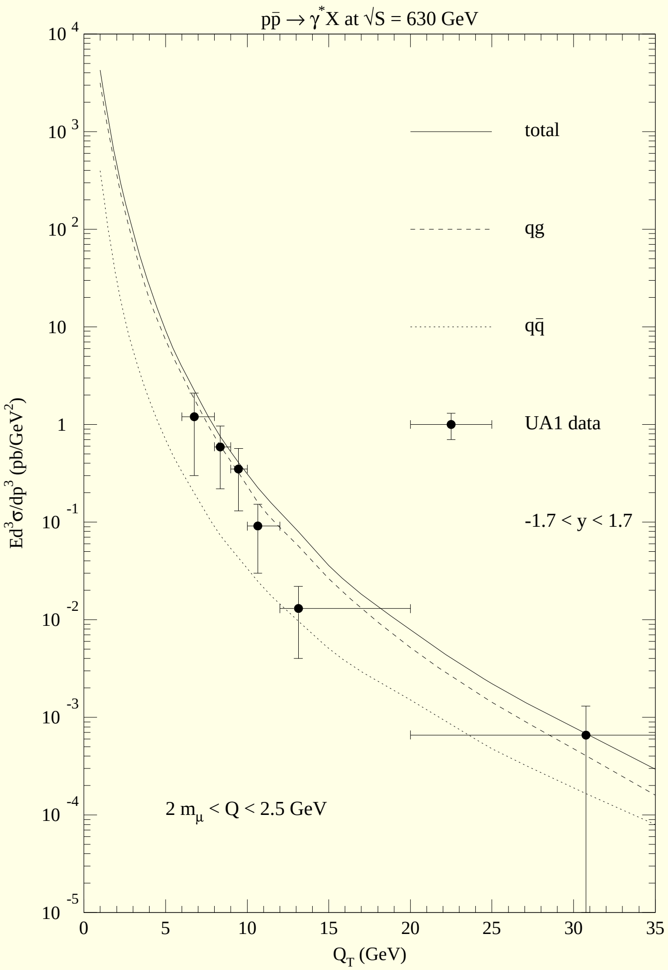
The two-body cross sections are easily calculated in the conventional fashion which, together with the relation just derived, yield the following results:

$$\begin{aligned}
\frac{d\sigma}{dQ^2 d\hat{t}}(q\overline{q} \to l^+ l^- g) &= \frac{\alpha^2 \alpha_s e_q^2}{Q^2 \hat{s}^2} \frac{8}{27} \left[\frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}} + \frac{2Q^2 \hat{s}}{\hat{t}\hat{u}} \right] \\
\frac{d\sigma}{dQ^2 d\hat{t}}(qg \to l^+ l^- q) &= -\frac{\alpha^2 \alpha_s e_q^2}{Q^2 \hat{s}^2} \frac{1}{9} \left[\frac{\hat{t}}{\hat{s}} + \frac{\hat{s}}{\hat{t}} + \frac{2Q^2 \hat{u}}{\hat{s}\hat{t}} \right]
\end{aligned}$$

The next step is to convert these expressions to those for hadronic cross sections.

Relation to direct photon production

- The production of high-p_T photons discussed in the previous lecture is calculable in perturbative QCD since the photon's transverse momentum provides the required large scale
- For lepton pair production the large scale is provided by the lepton pair mass
- But what if we studied the production of lepton pairs with low mass but large p_T?
- The preceeding derivation shows that the subprocesses would be nearly the same as those for direct photon production as long as $Q^2 << \hat{s}, \hat{t}, \hat{u}$
- One could integrate over some region of low Q^2 and have a result that would address the same physics as direct photon production - especially the gluon PDF
- See Beger, Gordon, and Klasen, hep-ph/9803387, Phys. Rev. D58(1998)074012
- See also Berger, Qiu, and Zhang, hep-ph/0107309, Phys. Rev. D65(2002)034006



Comparison to UA-1 data. The idea works and has the potential to address the same issues as direct photon production.

Now, back to our calculation. The next step is to convolute the subprocesses with the appropriate parton distributions:

$$\frac{d\sigma}{dQ^2} = \sum_{ab} \int dx_a dx_b \; G_{a/A}(x_a, Q^2) G_{b/B}(x_b, Q^2) \; \frac{d\sigma_{ab}}{dQ^2 d\hat{t}} \; d\hat{t}$$

Next, use the relations $p_T^2 = \frac{\hat{t}\hat{u}}{\hat{s}}$ and $dp_T^2 = \frac{d\hat{t}}{\hat{s}}|\hat{u} - \hat{t}|$
to get

$$\frac{d\sigma}{dQ^2 dp_T^2} = \sum_{ab} \int dx_a dx_b \; G_{a/A}(x_a, Q^2) G_{b/B}(x_b, Q^2) \frac{d\sigma_{ab}}{dQ^2 d\hat{t}} \frac{\hat{s}}{|\hat{u} - \hat{t}|}$$

This expression is not really that useful since normally one observes the lepton pair only in a restricted range of rapidity. What is really needed is a triple differential cross section

$$\frac{d\sigma}{dQ^2 dy dp_T^2}$$

Hadron-hadron rest frame

$$\begin{aligned} q &= \left[\sqrt{Q^2+p_T^2}\cosh y, p_T, 0, \sqrt{Q^2+p_T^2}\sinh y\right] \\ t &= (p_A-q)^2 = Q^2 - \sqrt{s}\sqrt{Q^2+p_T^2}\ e^{-y} \\ u &= (p_B-q)^2 = Q^2 - \sqrt{s}\sqrt{Q^2+p_T^2}\ e^{y} \end{aligned}$$

Now,

$$\begin{aligned} \hat{t} &= (p_1-q)^2 \Rightarrow \hat{t} - Q^2 = x_a(t-Q^2) \\ \hat{u} &= (p_2-q)^2 \Rightarrow \hat{u} - Q^2 = x_b(u-Q^2) \end{aligned}$$

and $\hat{s}+\hat{t}+\hat{u} = Q^2$, so $x_a x_b s + x_a(t-Q^2) + x_b(u-Q^2) + Q^2 = 0$

This last relation can be rewritten as

$$x_b = -\frac{Q^2 + x_a(t - Q^2)}{x_a s + (u - Q^2)}.$$

Next, let

$$\begin{aligned} x_1 &= -(u - Q^2)/s = \sqrt{Q^2 + p_T^2}\; e^{y}/\sqrt{s} \\ x_2 &= -(t - Q^2)/s = \sqrt{Q^2 + p_T^2}\; e^{-y}/\sqrt{s}. \end{aligned}$$

Then, $x_b = \frac{x_a x_2 - \tau}{x_a - x_1}$ with Q^2, y, p_T^2 fixed.

Using the previously defined variables, one can get

$$\hat{t} = Q^2 - x_a\sqrt{s}\sqrt{Q^2 + p_T^2}\ e^{-y}$$

$$x_b = \frac{x_a\sqrt{Q^2 + p_T^2}\ e^{-y}/\sqrt{s} - \tau}{x_a - \sqrt{Q^2 + p_T^2}\ e^{y}/\sqrt{s}}.$$

One can use these equations to show that

$$dx_b\, d\hat{t} = dy\, dp_T^2\, \frac{x_a x_b}{x_a - x_1}.$$

Therefore,

$$\frac{d\sigma}{dQ^2 dy dp_T^2} = \sum_{ab} \int_{x_{amin}}^{1} dx_a \frac{x_a x_b}{x_a - x_1} G_{a/A}(x_a, Q^2) G_{b/B}(x_b, Q^2) \frac{d\sigma_{ab}}{dQ^2 d\hat{t}}$$

where x_{amin} is determined by $x_b = 1$, yielding $x_{amin} = \frac{x_1 - \tau}{1 - x_2}$

Consider the $q\bar{q} \rightarrow l^+l^-g$ subprocess.

$$\frac{d\sigma}{dQ^2 dy dp_T^2} = \frac{\alpha^2\alpha_s}{Q^2}\frac{8}{27}\int_{x_{a min}}^1 dx_a \frac{x_a x_b}{x_a - x_1}\sum_q H_q(x_a, x_b, Q^2)$$
$$\times \frac{1}{\hat{s}^2}\ \frac{\hat{t}^2 + \hat{u}^2 + 2Q^2\hat{s}}{\hat{t}\hat{u}}$$

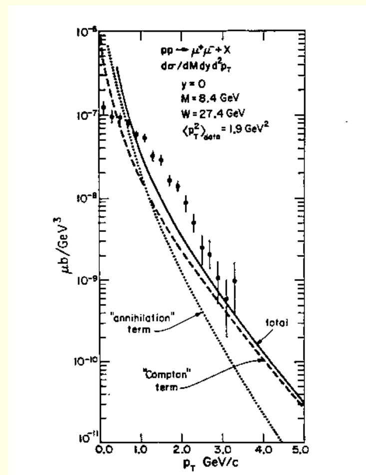
Using variables defined previously, as well as $x_T = 2p_T/\sqrt{s}$, this expression can be rewritten as

$$\frac{d\sigma}{dQ^2 dy dp_T^2} = \frac{\alpha^2\alpha_s}{sQ^2}\frac{8}{27}\frac{1}{p_T^2}\int_{x_{a min}}^1 dx_a \frac{x_a x_b}{x_a - x_1}\sum_q H_q(x_a, x_b, Q^2)$$
$$\times \left[1 - \frac{x_T^2}{2x_a x_b} + \left(\frac{\tau}{x_a x_b}\right)^2\right]$$

Comments

1. The annihilation term gives a p_T^{-2} tail to the p_T distribution (this falls off more slowly than a gaussian)
2. The Compton contribution ($qg \to l^+ l^- q$) is slightly more complicated (must include qg and gq) but similar. This actually dominates at high-p_T for pp or pN collisions.

Lesson: the tail of the p_T distribution can be calculated in QCD.



From "Applications of Perturbative QCD" by R.D. Field

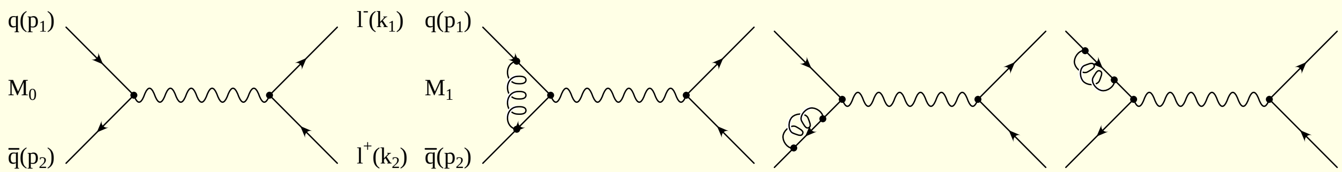
If we can calculate at least the tail of the p_T distribution, then one would think that we could integrate over p_T and get an $\mathcal{O}(\alpha_s)$ contribution to the total cross section. What happens if we integrate the preceeding expression over all p_T?

1. There is a divergence as $p_T \longrightarrow 0$
2. As $p_T \longrightarrow 0, x_1 \longrightarrow x_1^0 = \sqrt{\tau}e^y$. At the same time $x_{amin} \longrightarrow x_1^0$. Therefore, the $1/(x_a - x_1)$ term contributes to the divergence at the low end of the integration range.

$$
\begin{aligned}
-\ln(x_{amin} - x_1) &= -\ln(\frac{x_1 - \tau}{1 - x_2} - x_1) = -\ln\frac{x_1 x_2 - \tau}{1 - x_2} \\
x_1 x_2 &= \frac{Q^2 + p_T^2}{s} = \tau + p_T^2/s
\end{aligned}
$$

Therefore, the cross section diverges as $\frac{\ln(s/p_T^2)}{p_T^2}$ as $p_T^2 \longrightarrow 0$.

Okay, so how do you calculate the $\mathcal{O}(\alpha_s)$ contribution to the cross



Both M_0 and M_1 have the same final state, so one calculates

$$|M_0 + M_1|^2 = |M_0|^2 + 2\text{Re } M_0^* M_1 + ...$$

Add these to the $q\overline{q} \to \gamma^* g$ and $qg \to \gamma^* q$ contributions.

- 1-loop graphs are divergent
- α_s tree graphs are divergent as $p_T \to 0$

Need a regularization scheme: choose dimensional regularization wherein divergences are converted to poles in $\epsilon = \frac{4-n}{2}$ dimensions.

Consider $\frac{d\sigma}{dQ^2}$. This is technically somewhat simpler than $\frac{d\sigma}{dQ^2dy}$. Schematically, the $\mathcal{O}(\alpha_s)$ contributions take the following form:

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma^0}{dQ^2} + \frac{\alpha_s}{2\pi}\left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon} + C\right) \qquad q\bar{q} \to l^+l^-$$
$$\frac{d\sigma}{dQ^2} = \frac{\alpha_s}{2\pi}\left(-\frac{A}{\epsilon^2} + \frac{B'}{\epsilon} + C'\right) \qquad q\bar{q} \to l^+l^-g$$
$$\frac{d\sigma}{dQ^2} = \frac{\alpha_s}{2\pi}\left(\frac{B''}{\epsilon} + C''\right) \qquad qg \to l^+l^-q$$

- $\frac{1}{\epsilon^2}$ terms come from regions in phase space with both soft and collinear divergences.
- $\frac{1}{\epsilon}$ terms come from soft or collinear divergences.
- Soft divergences from the tree graphs cancel infrared divergences from the loop graphs.
- Remaining collinear divergences are related to the PDFs

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma^0}{dQ^2} + \frac{\alpha_s}{2\pi}\left(\frac{B + B' + B''}{\epsilon} + C + C' + C''\right)$$

Detailed result

$$
\begin{aligned}
\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{9Q^2} \int \frac{dx_a}{x_a} \int \frac{dx_b}{x_b} \Big\{ & H_q(x_a, x_b, Q^2) \\
& \times \left[\delta(1-z) + \frac{\alpha_s(\mu^2)}{2\pi} \left(2P_{qq}(z) \left(-\frac{1}{\bar{\epsilon}} + \ln(\frac{Q^2}{\mu^2}) \right) + D_q(z) \right) \right] \\
+ \quad & \frac{\alpha_s(\mu^2)}{2\pi} \left[G_{q/A}(x_a, Q^2) G_{g/B}(x_b, Q^2) + q \leftrightarrow g \right] \\
& \times \left[P_{qg}(z) \left(-\frac{1}{\bar{\epsilon}} + \ln(\frac{Q^2}{\mu^2}) \right) + D_g(z) \right] \Big\}
\end{aligned}
$$

where

$$
\frac{1}{\bar{\epsilon}} = \frac{1}{\epsilon} + \ln 4\pi - \gamma_E \quad \text{and} \quad z = \tau / x_a x_b .
$$

- The ϵ^{-2} soft-collinear poles have cancelled, but some ϵ^{-1} terms remain.
- These are the collinear poles associated with initial state radiation

In the $\overline{\mathrm{MS}}$ scheme we know how to define universal scale dependent parton distributions $G_i(x, \mu^2)$ in terms of the bare distributions $G_i(x)$:

$$G_q(x, \mu^2) = G_q(x) +$$
$$\frac{\alpha_s(\mu^2)}{2\pi}\left(-\frac{1}{\overline{\epsilon}}\right)\int_x^1 \frac{d\xi}{\xi}\left[P_{qq}\left(\frac{x}{\xi}\right)G_q(\xi) + P_{qg}\left(\frac{x}{\xi}\right)G_g(\xi)\right]$$

To see how this helps simplify the previous equation for $d\sigma/dQ^2$, we have to do a bit more work. Consider

$$\int \frac{dx_a}{x_a}\int \frac{dx_b}{x_b} G_q(x_a, \mu^2) G_{\overline{q}}(x_b, \mu^2)\delta(1-z) = \int \frac{dx_a}{x_a}\int \frac{dx_b}{x_b} G_q(x_a) G_{\overline{q}}(x_b)\delta(1-$$
$$- \quad \frac{1}{\overline{\epsilon}}\frac{\alpha_s(\mu^2)}{2\pi}\int \frac{dx_a}{x_a}\int \frac{dx_b}{x_b} G_q(x_a)\int \frac{d\xi}{\xi}\left[P_{qq}\left(\frac{x_b}{\xi}\right)G_{\overline{q}}(\xi) + P_{qg}\left(\frac{x_b}{\xi}\right)G_g(\xi)\right]\delta(1-$$
$$+ \quad (a \leftrightarrow b) + \mathcal{O}(\alpha_s^2)$$

where $z = \tau/x_a x_b$. Use the $\delta(1-\tau/x_a x_b)$ in the above equation to do the x_b integral.

Since $x_b = \tau/x_a$, the $\frac{1}{\bar{\epsilon}}$ line can be rewritten as

$$-\frac{1}{\bar{\epsilon}}\frac{\alpha_s(\mu^2)}{2\pi}\int\frac{dx_a}{x_a}G_q(x_a)\int_{\tau/x_a}^{1}\frac{d\xi}{\xi}\left[P_{qq}\left(\frac{\tau}{x_a\xi}\right)G_{\overline{q}}(\xi)+P_{qg}\left(\frac{\tau}{x_a\xi}\right)G_g(\xi)\right]$$

Relabel $\xi \to x_b$ to get

$$-\frac{1}{\bar{\epsilon}}\frac{\alpha_s(\mu^2)}{2\pi}\int\frac{dx_a}{x_a}\int\frac{dx_b}{x_b}G_q(x_a)\left[G_{\overline{q}}(x_b)P_{qq}(z)+G_g(x_b)P_{qg}(z)\right]$$

These are of the same form as the $1/\bar{\epsilon}$ terms in $d\sigma/dQ^2$. Replace

$$G_q(x) \to G_q(x,\mu^2) + \frac{1}{\bar{\epsilon}}\cdots$$

Then, the $1/\bar{\epsilon}$ terms cancel, leaving a finite expression for $d\sigma/dQ^2$. This is a demonstration of the factorization theorem at work. The collinear singularities associated with the PDFs are universal. Once the PDFs are defined using a factorization scheme, the cross sections are finite.

Using these results, the expression for $d\sigma/dQ^2$ can be simplified. In the $\overline{\text{MS}}$ scheme choosing $\mu^2 = Q^2$ we get:

$$
\begin{aligned}
\frac{d\sigma}{dQ^2} &= \frac{4\pi\alpha^2}{9Q^2 s}\sum_q \int \frac{dx_a}{x_a}\int \frac{dx_b}{x_b} \\
&\times \left\{ H_q(x_a, x_b, Q^2)\left[\delta(1-z) + \frac{\alpha_s(Q^2)}{2\pi} D_q(z)\right]\right. \\
&+ \left[(G_q(x_a, Q^2) + G_{\overline{q}}(x_a, Q^2))G_g(x_b, Q^2) + (a \leftrightarrow b)\right] \\
&\times \left. \frac{\alpha_s(Q^2)}{2\pi} D_g(z)\right\}
\end{aligned}
$$

with $z = \tau/x_a x_b$.

- The $\delta(1-z)$ term reproduces the lowest order contribution. $D_q(z)$ and $D_g(z)$ give the finite $\mathcal{O}(\alpha_s)$ corrections. Note the presence of the scale dependent PDFs and the running coupling $\alpha_S(Q^2)$.
- The earlier results on scaling will be modified due to the Q^2 dependence present in these results.

Factorization Schemes

The definition of the the scale dependent PDFs will affect the form of the D_q and D_g functions.

- $\overline{\text{MS}}$: the $1/\bar{\epsilon}$ terms given previously which come from the collinear singularities associated with the initial state radiation are combined with the bare PDFs to give the Q^2 dependent PDFs as has been shown above.
- DIS: additional finite terms are included along with the $1/\bar{\epsilon}$ parts so that the expression for $F_2(x, Q^2)$ in deep inelastic scattering retains its lowest order form when higher order terms are included.

In the $\overline{\text{MS}}$ scheme we have

$$\begin{aligned} D_q(z) &= C_F\left[4(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - 2\frac{1+z^2}{1-z}\ln z \right. \\ &\quad \left. +\delta(1-z)\left(\frac{2\pi^2}{3}-8\right)\right] \\ D_g(z) &= T_R\left[(z^2+(1-z)^2)\ln\left(\frac{(1-z)^2}{z}\right)+\frac{1}{2}+3z-\frac{7}{2}z^2\right] \end{aligned}$$

Note: the gluon spin average is $\frac{1}{2(1-\epsilon)}$ in $n-$dimensions.
In the DIS scheme we get

$$
\begin{aligned}
D_q(z) &= C_F\left[2(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + \frac{3}{(1-z)_+} - 6 - 4z\right. \\
&\quad \left. +\delta(1-z)\left(1+\frac{4\pi^2}{3}\right)\right] \\
D_g(z) &= T_R\left[(z^2+(1-z)^2)\ln(1-z) + \frac{3}{2} - 5z + \frac{9}{2}z^2\right]
\end{aligned}
$$

Comments on the $\mathcal{O}(\alpha_s)$ corrections

1. $\delta(1-z)\frac{\alpha_s}{2\pi}C_F\left(1+\frac{4\pi^2}{3}\right)$ in DIS

   - Part of the "π^2" term comes from the change from $Q^2 < 0$ for DIS to $Q^2 > 0$ for l^+l^- (spacelike virtual photon $\rightarrow$ timelike virtual photon)
   - Phase space contains $(Q^2/\mu^2)^\epsilon \rightarrow (-Q^2/\mu^2)^\epsilon$
   - $(-1)^\epsilon = e^{\epsilon \ln(-1)} = 1 - \epsilon^2\pi^2/2 + \ldots$
   - Multiplied by $(-2/\epsilon^2 + \ldots)$ which gives a finite contribution proportional to π^2.

2. Phase space for DIS and l^+l^- production differ. $z \rightarrow 1$ corresponds to $\hat{s} = Q^2$ so the soft gluon singularities are at $z = 1$. There is a mismatch between the two phase spaces away from $z = 1$ so that the "+" regulator terms are different for the two cases.

These two items help to explain the size of the α_s corrections in l^+l^- relative to DIS.

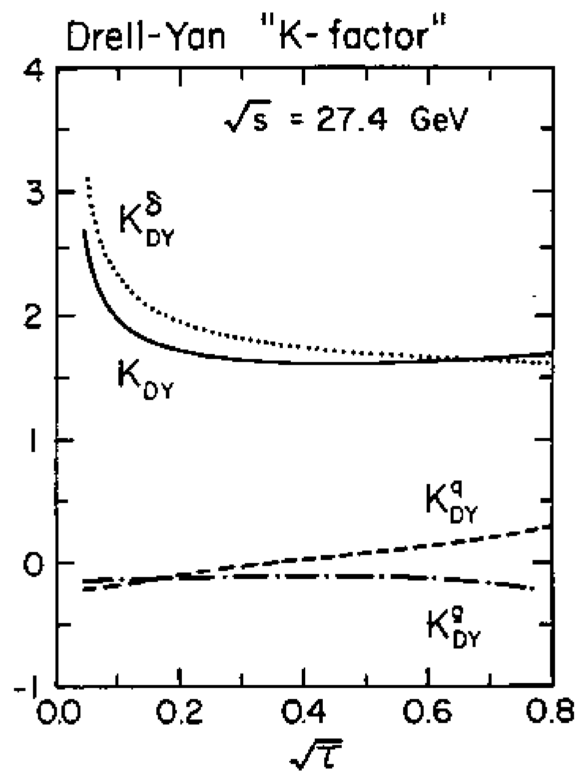


Figure 5.6 Drell-Yan "K-factor", K_{DY}, computed with $\Lambda = 200$ MeV at $\sqrt{s} =$ 27.4 GeV plotted versus $\sqrt{\tau}$. The quark, gluon, and δ-function contributions are shown separately with $K_{DY} = K^{\delta}_{DY} + K^{q}_{DY} + K^{g}_{DY}$.

From "Applications of Perturbative QCD" by R.D. Field

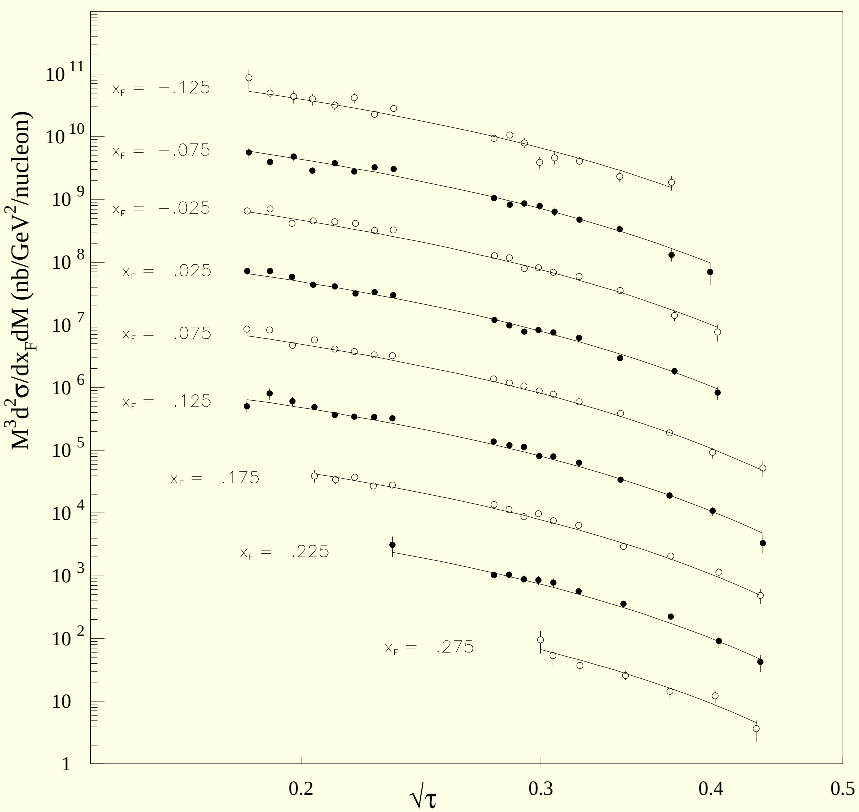
Comparison to data

- Lepton pair production data used extensively in global fits for parton distribution functions
- Sensitive to antiquark distributions in pp, pN collisions

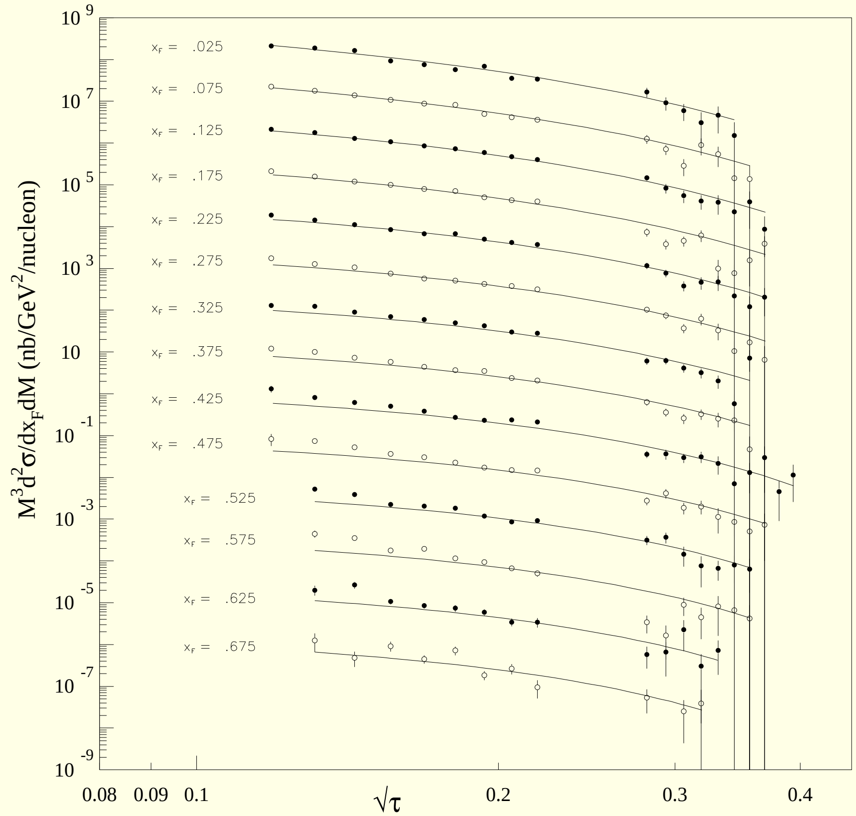
$$\sigma \sim \sum_q e_q^2 [q_a(x_a)\overline{q}_b(x_b) + a \leftrightarrow b]$$

- Corrections through $\mathcal{O}(\alpha_s^2)$ available (although most fits use only through $\mathcal{O}(\alpha_s)$)
- Excellent fits to $d\sigma/dQ^2dy$ and related distributions
- E-866 data using pp and pd instrumental in constraining the $\overline{d}/\overline{u}$ ratio

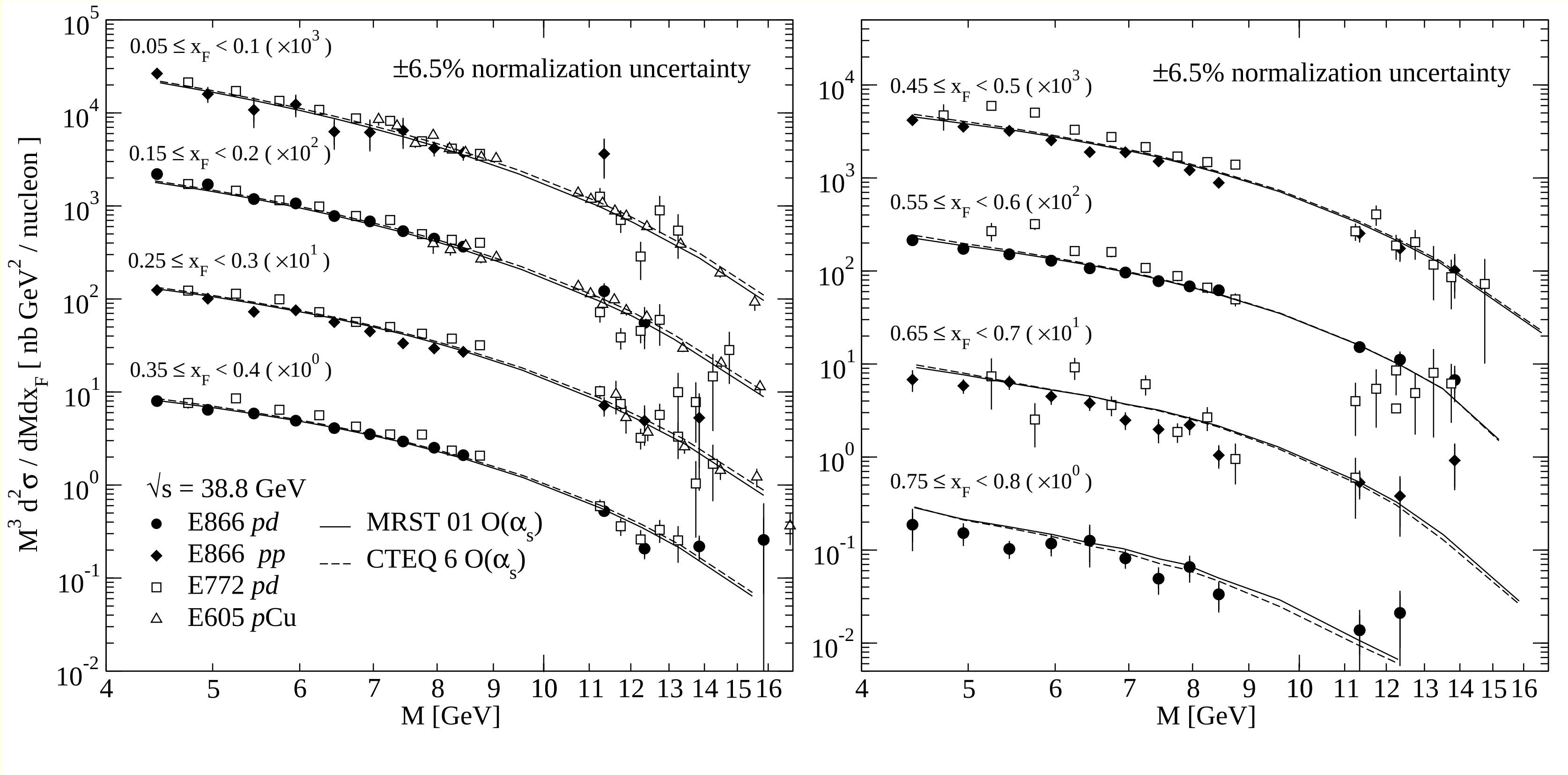
E605 (p Cu $\rightarrow \mu^+\mu^-$ X) p_{LAB} = 800 GeV



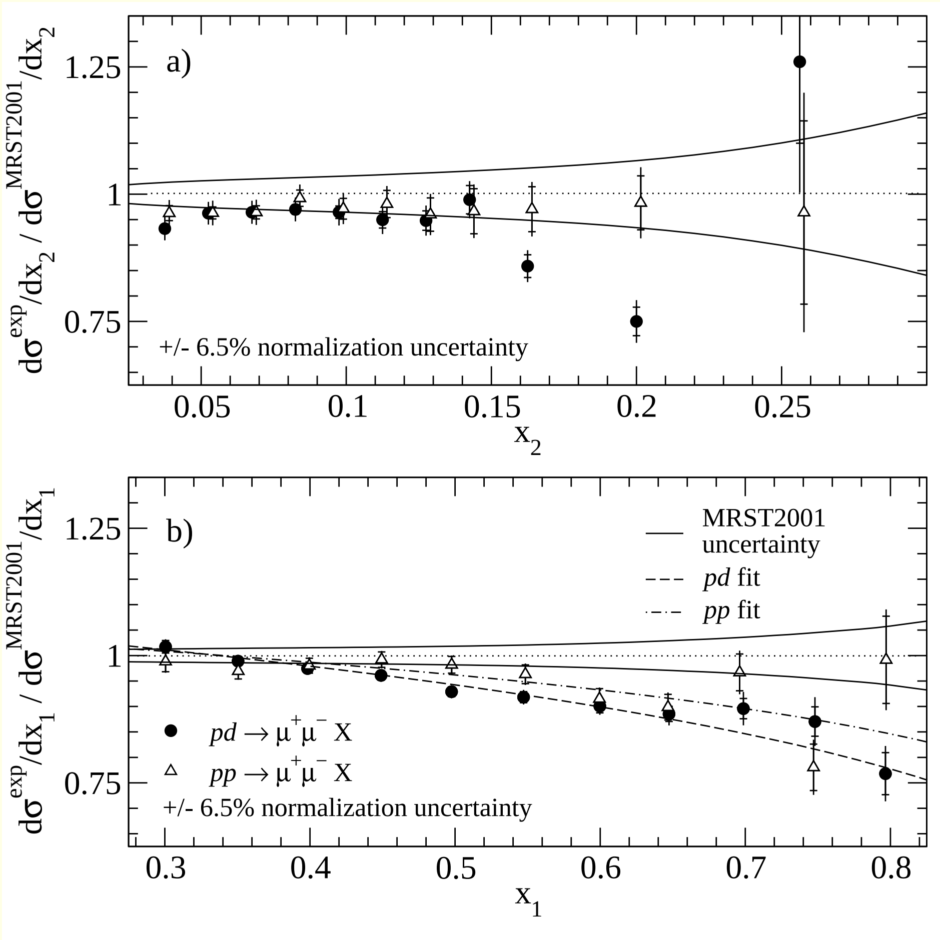
E772 (p d $\rightarrow \mu^+\mu^-$ X) p_{LAB} = 800 GeV



Typical fits to lepton pair production data (from MRST, Martin, Roberts, Stirling, and Thorne, hep-ph/9803445, Eur. Phys. J. C4 (1998) 463)



Results from Fermilab experiment E-866 for *pp* and *pd* interactions



Further results from E-866. These show the impact of having both *pp* and *pd* data for constraining the high-x PDFs

W and Z Production

- Thus far we have seen that fixed target pp and pN lepton pair production experiments provide important information concerning $\overline{u}$ and $\overline{d}$ distributions in the nucleon.
- W and Z production involve subprocesses which are very similar to lepton pair production, e.g., $q\overline{q}' \to W$ and $q\overline{q} \to Z$.
- To date, W/Z production by hadron beams has been studied at high energy $p\overline{p}$ colliders. Since the valence partons in an antiproton are antiquarks, the dominant subprocesses probe the u and d distributions. As we shall see, this places strong constraints on the d/u ratio.
- We shall be working in the narrow width approximation, i.e., the vector bosons will be treated as stable particles of fixed mass. All of the previous lepton pair production results can be easily utilized, providing that we make some changes in the couplings.

Consider $q(p_1)\overline{q}'(p_2) \to W(p)$, for which the matrix element is

$$M = -iV_{qq'}\frac{g}{\sqrt{2}}\epsilon_\alpha \overline{v}(p_2)\gamma^\alpha \frac{1}{2}(1-\gamma_5)u(p_1)$$

where $V_{qq'}$ is the appropriate element of the CKM matrix. The spin/color averaged squared matrix element is given by

$$\begin{aligned}
\overline{\sum}|M|^2 &= |V_{qq'}|^2 \frac{g^2}{96} 2\mathrm{Tr}\left[\not{p}_1(1-\gamma_5)\not{p}_2(1-\gamma_5)\right] \\
&= |V_{qq'}|^2 \frac{g^2}{24}\mathrm{Tr}[\not{p}_1\not{p}_2] = |V_{qq'}|^2 \frac{g^2}{6} p_1 \cdot p_2 \\
&= |V_{qq'}|^2 \frac{G_F M_W^4}{\sqrt{2}}\frac{2}{3}
\end{aligned}$$

where $g^2 = \frac{8G_F M_W^2}{\sqrt{2}}$.

The hadronic cross section σ is given by convoluting the parton level cross section $\hat{\sigma}$ with the appropriate parton distributions:

$$\sigma = \int dx_a dx_b \sum_{qq'} q(x_a)\overline{q}'(x_b)\hat{\sigma}$$

$$\hat{\sigma} = \frac{1}{2\hat{s}}\frac{2}{3}\frac{G_F M_W^4}{\sqrt{2}}|V_{qq'}|^2 \int \frac{d^3p}{(2\pi)^3 2E}(2\pi)^4\delta^4(p-p_1-p_2).$$

The integrand of the phase space integral can be rewritten as

$$2\pi\ d^4p\ \delta^4(p-p_1-p_2)\delta(\hat{s}-M_W^2)$$

which yields

$$\hat{\sigma} = \frac{2\pi}{3}|V_{qq'}|^2\frac{G_F M_W^2}{\sqrt{2}}\delta(\hat{s}-M_W^2).$$

Compare this to our lepton pair production result $\hat{\sigma}_{\gamma^*} = \frac{4\pi^2\alpha}{3}e_q^2\delta(\hat{s}-Q^2)$ which shows that

$$4\pi\alpha e_q^2 \leftrightarrow 2|V_{qq'}|^2\frac{G_F M_W^2}{\sqrt{2}}.$$

Z Production

Here the subprocess is $q(p_1)\overline{q}(p_2) \to Z(p)$, with the matrix element given by

$$M = -ig\epsilon_\alpha \overline{v}(p_2)\gamma^\alpha (g_V + g_A\gamma_5)u(p_1).$$

The partonic cross section is given by

$$\hat{\sigma}_Z = \frac{8\pi}{3}\frac{G_F M_W^2}{\sqrt{2}}(g_V^2 + g_A^2)\delta(\hat{s} - M_Z^2)$$

where

$$g_V^2 + g_A^2 = \frac{1}{8}(1 - 4|e_q|\sin^2\theta_W + 8e_q^2\sin^4\theta_W).$$

Apart from changing the coupling, we can treat W and Z production just like lepton pair production at a fixed value of Q^2.

Rapidity dependence in W production

According to our earlier calculations, the rapidity dependence of W production is given by the x dependence of the PDFs, since the W longitudinal momentum is, in lowest order, given by $\frac{\sqrt{s}}{2}(x_a - x_b)$. Of course, the leptonic decay $W \to l\nu$, by which the W is detected, poses a problem because of the unobserved ν. Nevertheless, the rapidity dependence of the charged lepton gives some useful information. To begin with, consider the rapidity dependence of the W and define an asymmetry $A(y)$ by:

$$A(y) = \frac{\frac{d\sigma}{dy}(W^+) - \frac{d\sigma}{dy}(W^-)}{\frac{d\sigma}{dy}(W^+) + \frac{d\sigma}{dy}(W^-)}.$$

In lowest order

$$\begin{aligned}\frac{d\sigma}{dy}(W^+) &= \frac{2\pi}{3}\frac{G_F}{\sqrt{2}}\sum_{q\overline{q}'}|V_{qq'}|^2[q(x_a)\overline{q}'(x_b) + a \leftrightarrow b] \\ &\approx \frac{2\pi}{3}\frac{G_F}{\sqrt{2}}u(x_a)d(x_b)\end{aligned}$$

for $p\overline{p}$ collisions

Similarly

$$\frac{d\sigma}{dy}(W^-) \approx \frac{2\pi}{3}\frac{G_F}{\sqrt{2}}d(x_a)u(x_b)$$

To this level of approximation we can write the asymmetry A in terms only of the parton distributions.

$$\begin{aligned} A &\approx \frac{u(x_a)d(x_b) - d(x_a)u(x_b)}{u(x_a)d(x_b) + d(x_a)u(x_b)} \\ &= \frac{R_{du}(x_b) - R_{du}(x_a)}{R_{du}(x_b) + R_{du}(x_a)} \end{aligned}$$

where

$$R_{du}(x) = \frac{d(x)}{u(x)}.$$

Now,

$$x_{\substack{a\\b}} = \frac{M_W}{\sqrt{s}} e^{\pm y} \approx x_0(1 \pm y)$$

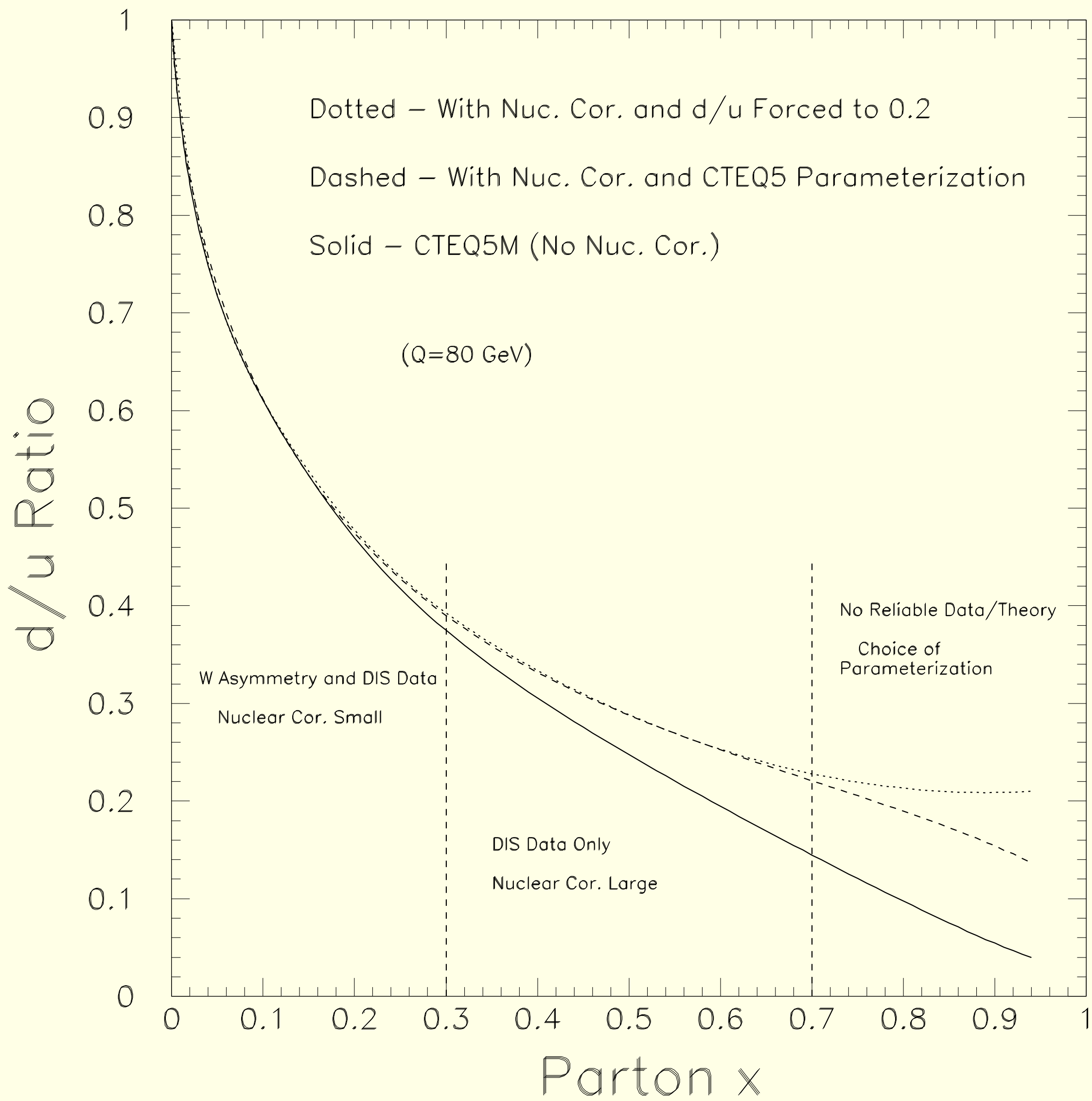
for small y. Here $x_0 = M_W/\sqrt{s}$. Then,

$$R_{du}(x_{\substack{a\\b}}) \approx R_{du}(x_0) \pm y x_0 R'_{du}(x_0).$$

Therefore, in this approximation we obtain

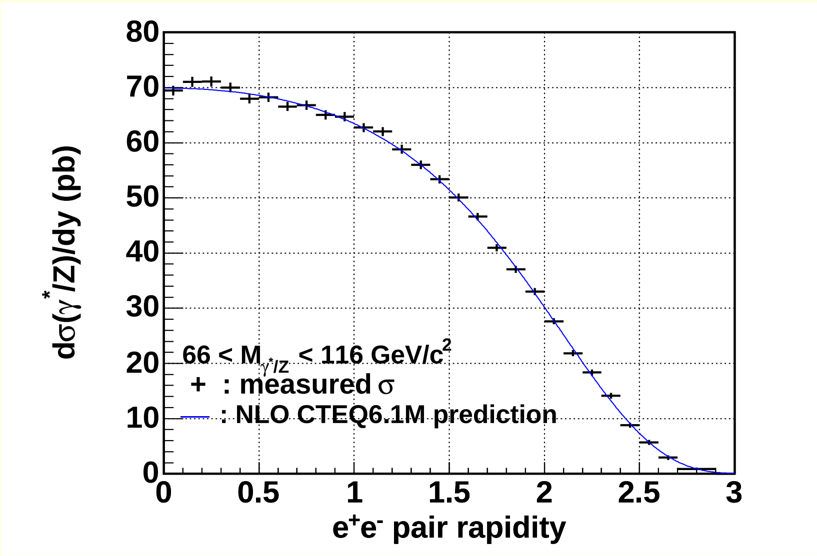
$$A(y) \approx -x_0 y \frac{R'_{du}(x_0)}{R_{du}(x_0)}.$$

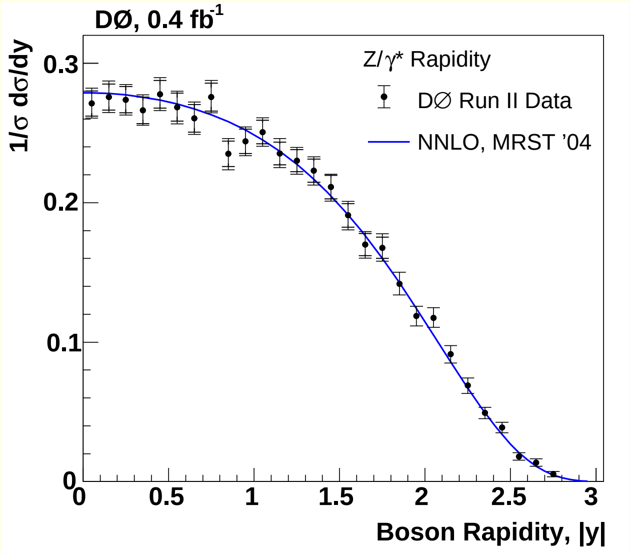
So, A gives us information about the slope of the d/u ratio. We'll see shortly that we can do the same for the charged lepton asymmetry. This yields valuable constraints in the low- to moderate-x range as shown below (from S. Kuhlmann et al., hep-ph/9912283, Phys. Let. B76 (2000) 291:



Dotted – With Nuc. Cor. and d/u Forced to 0.2
Dashed – With Nuc. Cor. and CTEQ5 Parameterization
Solid – CTEQ5M (No Nuc. Cor.)
(Q=80 GeV)
W Asymmetry and DIS Data
Nuclear Cor. Small
DIS Data Only
Nuclear Cor. Large
No Reliable Data/Theory
Choice of
Parameterization
d/u Ratio
Parton x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

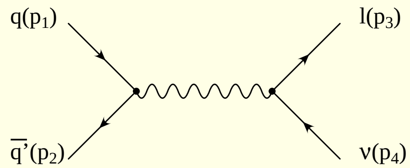
Z Rapidity Distribution





Good agreement betwen theory and data from CDF and DØ

As mentioned above, the presence of the ν in the leptonic decay of the W complicates matters somewhat. Let's look at the distribution of the charged lepton in W production. We'll work in lowest order, but the same ideas apply in higher order. The subprocess is $q(p_1)\overline{q}'(p_2) \to l(p_3)\nu(p_4)$.



The matrix element is

$$M = V_{qq'}\frac{g^2}{2}\overline{v}(p_2)\gamma_\mu\frac{1}{2}(1-\gamma_5)u(p_1)\frac{1}{\hat{s}-M_W^2+iM_W\Gamma_W}\overline{u}(p_3)\gamma^\mu\frac{1}{2}(1-\gamma_5)v(p_4).$$

The squared matrix element is given by

$$\overline{\sum}|M|^2 = \frac{1}{4}\frac{1}{3}|V_{qq'}|^2\frac{g^4}{4}\frac{1}{(\hat{s}-M_W^2)^2+M_W^2\Gamma_W^2}$$
$$\times \quad \frac{1}{4}\mathrm{Tr}[\not{p}_1\gamma_\nu(1-\gamma_5)\not{p}_2\gamma_\mu(1-\gamma_5)]\frac{1}{4}\mathrm{Tr}[\not{p}_4\gamma^\nu(1-\gamma_5)\not{p}_3\gamma^\mu(1-\gamma_5)]$$

To simplify the traces, use the following relations:

$$\begin{aligned} \mathrm{Tr}[\not{p}_1\gamma_\nu\not{p}_2\gamma_\mu] &= 4[p_{1\nu}p_{2\mu} + p_{1\mu}p_{2\nu} - g_{\mu\nu}p_1\cdot p_2] \\ \mathrm{Tr}[\not{p}_1\gamma_\nu\not{p}_2\gamma_\mu\gamma_5] &= -4i\epsilon_{\alpha\nu\beta\mu}p_1^\alpha p_2^\beta \\ \epsilon^{\alpha\beta\mu\nu}\epsilon_{\alpha\beta\sigma\tau} &= -2(g^\mu_\sigma g^\nu_\tau - g^\mu_\tau g^\nu_\sigma) \end{aligned}$$

and note that ϵ contracted with a symmetric function gives zero. The result is

$$\overline{\sum}|M|^2 = \frac{1}{3}|V_{qq'}|^2\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^2 \frac{4\hat{s}^2(1+\cos\theta)^2}{(\hat{s}-M_W^2)^2 + M_W^2\Gamma_W^2}.$$

Next, note that phase space gives $d\cos\theta/16\pi$ and there is a flux factor of $1/2\hat{s}$. Thus,

$$\frac{d\hat{\sigma}}{d\cos\theta} = \frac{1}{24\pi}|V_{qq'}|^2\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^2 \frac{4\hat{s}^2(1+\cos\theta)^2}{(\hat{s}-M_W^2)^2 + M_W^2\Gamma_W^2}.$$

Comment: If the width $\Gamma \ll M$ one can use the narrow width approximation wherein

$$\int_{-\infty}^{\infty} \frac{ds}{(s-M^2)^2 + M^2\Gamma^2} = \frac{\pi}{M\Gamma}$$

(let $\frac{s-M^2}{M\Gamma} = \tan\theta$. Then the integral is elementary.) Thus,

$$\frac{1}{(s-M^2)^2 + M^2\Gamma^2} \approx \frac{\pi}{M\Gamma}\delta(s-M^2) \text{ for } \Gamma \ll M.$$

We are interested in the hadron-hadron cm rapidity y. This is related to the parton-parton cm rapidity $\hat{y}$ by

$$y = \hat{y} + \frac{1}{2}\ln\frac{x_a}{x_b}.$$

Furthermore, $\hat{y} = \ln\cot\theta/2 = \frac{1}{2}\ln\frac{1+\cos\theta}{1-\cos\theta}$. Then,

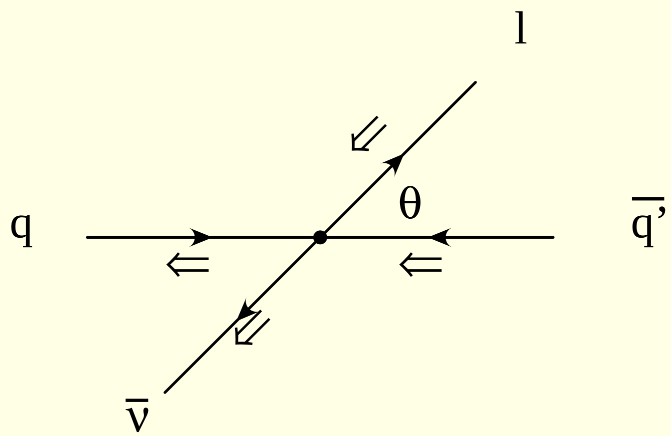
$$\frac{d\hat{y}}{d\cos\theta} = \frac{1}{\sin^2\theta} \text{and } \frac{d\hat{\sigma}}{d\hat{y}} = \frac{d\hat{\sigma}}{d\cos\theta}\sin^2\theta.$$

We therefore obtain

$$\frac{d\sigma}{dy} = \sum_q \int dx_a dx_b \frac{d\hat{\sigma}}{d\cos\theta}\sin^2\theta[q(x_a)\overline{q}'(x_b) + a \leftrightarrow b].$$

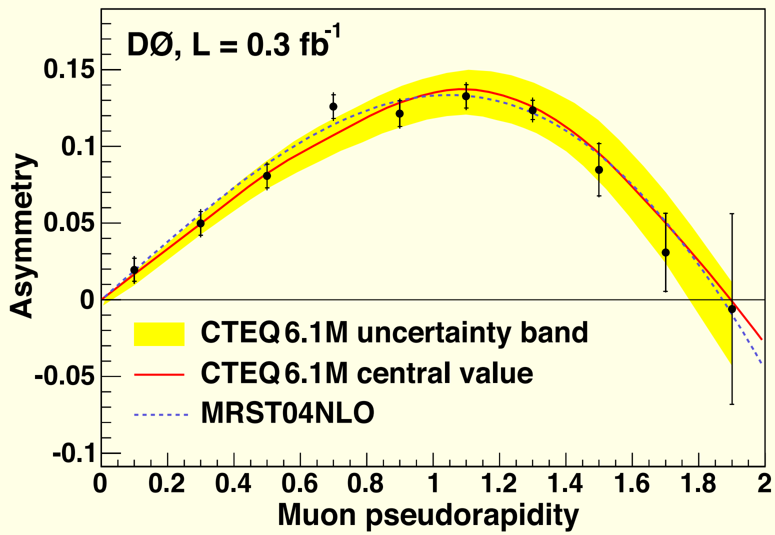
Note: given x_a, x_b, y, and s one gets $\hat{s}$ and $\hat{y}$ and then uses $\sin\theta = \frac{1}{\cosh\hat{y}}$. From here one can get the lepton p_T if a cut on p_T is desired.
This type of calculation can be extended to higher orders. Thus, one can calculate the rapidity dependence of the charged lepton and, therefore, the lepton rapidity asymmetry. This, in turn can be used to constrain PDFs in global fits.

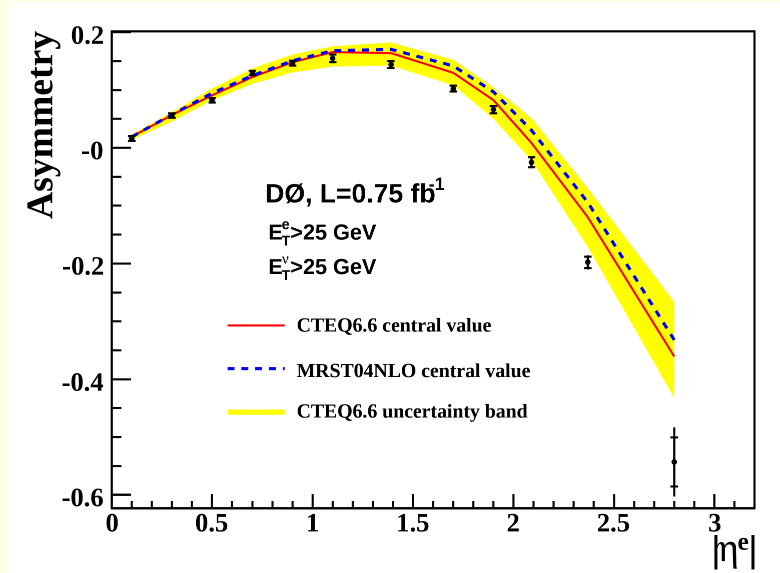
Comment: In the calculation if the $\cos\theta$ dependence we encountered a factor of $(1+\cos\theta)^2$. The source of this is easy to understand. The W couples to left-handed particles and right-handed antiparticles.



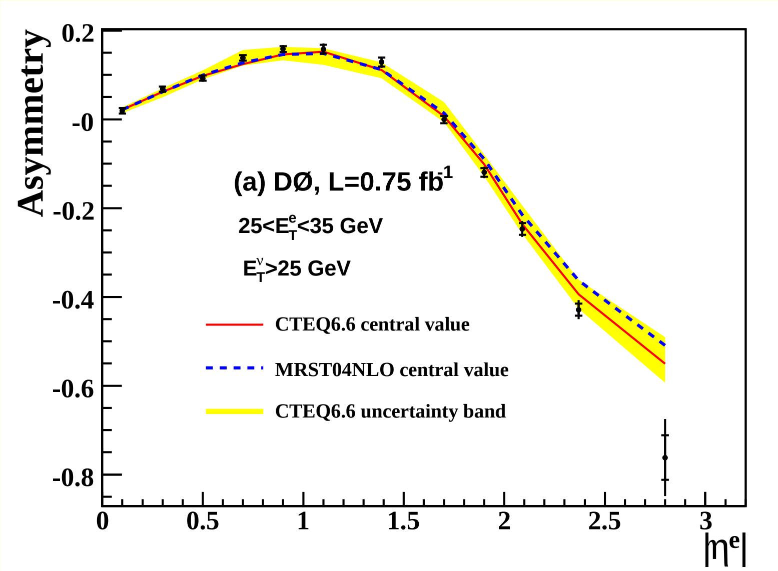
When $\theta \to \pi$ the cross section must vanish since angular momentum would not be conserved. However, $\theta = 0$ is allowed. The $(1+\cos\theta)$ factor ensures this.

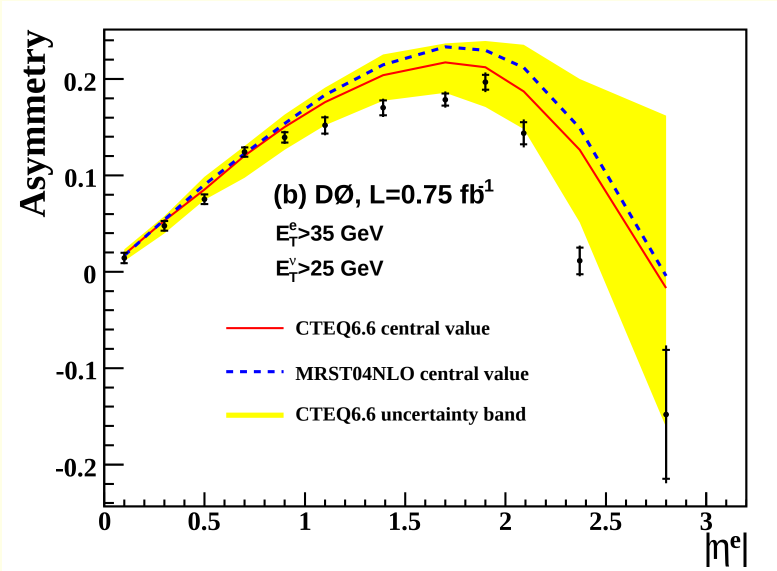
Examples of W-lepton charge asymmetries





- DØ muon and electron charge asymmetries from W production
- The highest rapidity electron data suggest that the theory is too high
- This suggests that the high-x d/u ratio should be increased

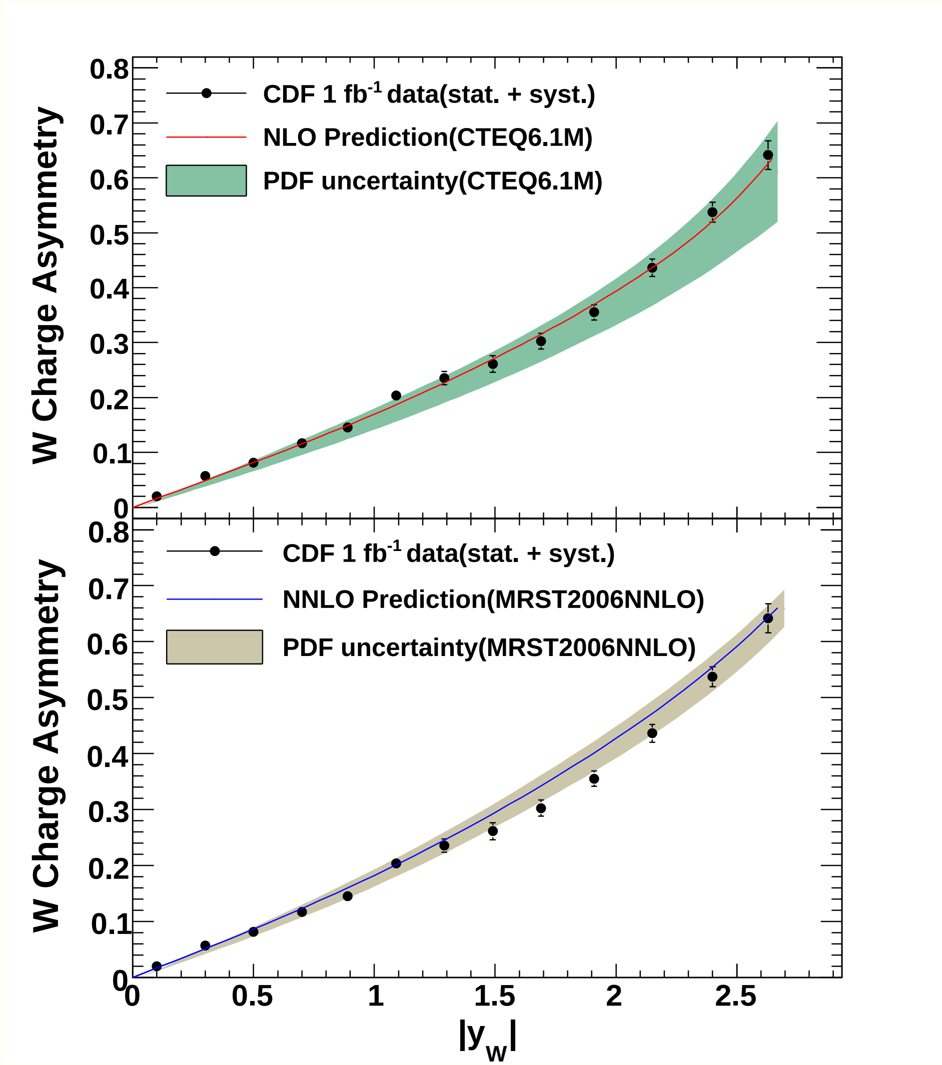




- When the data are binned in $W\,p_T$ the high rapidity discrepancy is worse
- Suggests that there may be a problem accounting for the p_T cuts correctly in the theory calculation

W Asymmetry

- The transverse momentum of the W decay neutrino can be determined via transverse momentum conservation
- This can not be done for the longitudinal momentum, since an unknown amount of momentum will be associated with particles going down (or near) the beam pipe
- This prevents a unique determination of the W rapidity
- CDF has utilized a method that allows the W charge asymmetry to be determined, however.
  - The kinematics of the decay allows two solutions for the neutrino longitudinal momentum
  - Both solutions are retained, each with a model dependent weight associated with it
- Although model dependent, the result allows much closer contact with the W production mechanism and the underlying PDFs



Results in good agreement within errors for both PDF sets shown

Comments on the W and W-lepton asymmetries

- Recent comparisons by the CDF and DØ groups show that their W-lepton asymmetry data agree.
- There is some indication from the W-lepton data that the d/u ratio at high values of x should be increased
- When the data are binned in $W - p_T$ slices the disagreement is enhanced
- The W asymmetry data appear to be in better agreement with current PDFs
- The best other source of information on the large-x d PDF is deep inelastic scattering from a deuterium target.
- Nuclear corrections are needed to in order to use these data properly, although the tendency in the past has been to ignore nuclear corrections in deuterium
- See the discussion by the MSTW group in arXiv:1006.2753[hep-ph] for a phenomenological discussion of the situation

p_T distributions revisited

The p_T distribution for W production is of vital importance for the precision measurement of the W mass. Previously, we saw how to calculate the high-p_T tail of the distribution. Now, we need to reexamine this issue with an eye towards calculating the full distribution. First, consider the $q\bar{q} \rightarrow l^+l^-g$ annihilation subprocess.

$$\begin{aligned}\frac{d\sigma}{dQ^2dydp_T^2} &= \frac{\alpha^2\alpha_s}{sQ^2}\frac{8}{27}\frac{1}{p_T^2}\int_{x_{amin}}^{1}\frac{dx_a}{x_a-x_1}\sum_q H_q(x_a,x_b,\mu^2)e_q^2 \\ &\times \left[1+\frac{\tau^2}{(x_ax_b)^2}-\frac{x_T^2}{2x_ax_b}\right]\end{aligned}$$

where

$$
\begin{aligned}
x_b &= \frac{x_a x_2 - \tau}{x_a - x_1} \\
x_{amin} &= \frac{x_1 - \tau}{1 - x_2} \\
x_1 &= -(u - Q^2)/s = \sqrt{Q^2 + p_T^2}\; e^{y}/\sqrt{s} \\
x_2 &= -(t - Q^2)/s = \sqrt{Q^2 + p_T^2}\; e^{-y}/\sqrt{s}
\end{aligned}
$$

Now, consider the limit as $p_T \to 0$:

$$x_b \to \sqrt{\tau} e^{-y} = x_b^0 \text{ and } x_{amin} \to \sqrt{\tau} e^{y} = x_a^0$$

so

$$(x_{amin} x_b)^2 \sim \tau^2.$$

As $p_T \to 0$ the [ ] term above goes to 2. Near $p_T = 0$ we can integrate the $\frac{1}{x_a - x_1}$ term, approximating the rest of the integrand as $\sim$ constant.

Keeping the most singular terms, we get

$$\begin{aligned}
\frac{d\sigma}{d\tau dy dp_T^2} &\approx \frac{\alpha^2\alpha_s}{Q^2}\frac{8}{27}\frac{\ln s/p_T^2}{p_T^2} 2\sum_q H_q(x_a^0, x_b^0) \\
&\approx \frac{4\alpha_s}{3\pi}\left(\frac{d\sigma}{d\tau dy}\right)_{Born}\frac{\ln s/p_T^2}{p_T^2}
\end{aligned}$$

Next, following the arguments of Parisi and Petronzio, Nucl. Phys. B154, 427 (1979), as discussed in my earlier resummation lecture, we know that the integral over all p_T^2 is finite, so

$$\int_0^s \frac{d\sigma}{d\tau dy dp_T^2}\, dp_T^2 = \left(\frac{d\sigma}{d\tau dy}\right)_{Born} + \mathcal{O}(\alpha_s)$$

and, using the above results,

$$\begin{aligned}
\int_0^{p_T^2} \frac{d\sigma}{d\tau dy dp_T^2}\, dp_T^2 &= \left(\frac{d\sigma}{d\tau dy}\right)_{Born}\left(1 - \int_{p_T^2}^s \frac{4\alpha_s}{3\pi}\frac{\ln s/p_T^2}{p_T^2}\, dp_T^2\right) \\
&= \left(\frac{d\sigma}{d\tau dy}\right)_{Born}\left[1 - \frac{2\alpha_s}{3\pi}\ln^2 s/p_T^2\right]
\end{aligned}$$

Extended to higher orders, it can be shown that the square bracketed term exponentiates. Hence,

$$\int_0^{p_T^2} \frac{d\sigma}{d\tau dy dp_T^2}\, dp_T^2 = \left(\frac{d\sigma}{d\tau dy}\right)_{Born} \exp(-\frac{2\alpha_s}{3\pi} \ln^2 s/p_T^2).$$

For more details, see Dokshitzer, D'yakanov, and Troyan, Phys. Rep. 58, 271 (1980) and Curci, Greco, and Srivastava, Phys. rev. Lett 43, 834 (1979) and Nucl. Phys. B159, 451 (1979).

Differentiating the above results yields

$$\frac{d\sigma}{d\tau dy dp_T^2} = \left(\frac{d\sigma}{d\tau dy}\right)_{Born} \frac{4\alpha_s}{3\pi} \frac{\ln s/p_T^2}{p_T^2} \exp(-\frac{2\alpha_s}{3\pi} \ln^2 s/p_T^2).$$

The exponential is referred to as a Sudakov form factor. It represents the summation of the leading double-log terms. Notice that the exponential kills the divergence at $p_T = 0$. Physically, this represents the fact that the probability to produce a massive lepton pair with no additional radiation is zero.

- In this approximation the gluon emissions are treated as uncorrelated. If the lepton pair is to have zero p_T, then all the gluons must have zero p_T.
- This suppression ia actually too strong. One can have two or more gluons whose $\vec{p}_T$ adds to zero. Thus, configurations with balancing gluons should be included. However, these are subleading terms, even though they may be dominant at sufficiently small values of p_T (see Parisi and Petronzio, Nucl. Phys. B154, 427 (1979)).
- Resummation techniques exist which include these subleading terms and which give non-zero cross sections at $p_T = 0$ (see Collins, Soper, and Sterman, Nucl. Phys. B250, 199 (1985)).

Exponentiating in impact parameter space is a way of ensuring conservation of transverse momentum which includes the previously mentioned subleading terms.

$$\frac{d\sigma}{d\tau dy dp_T^2} \sim \sum_q \frac{\sigma_0^{q\overline{q}}}{2} \int_0^\infty bdbJ_0(bp_T)exp(-S(b,Q))$$
$$\times \quad [q(x_a, b_0/b)\overline{q}(x_b, b_0/b) + a \leftrightarrow b]$$
$$S(b,Q) = \int_{(b_0/b)^2}^{Q^2} \frac{dq^2}{q^2} \left[A \ln Q^2/q^2 + B\right] \text{ with } x_{\substack{a\\b}} = \sqrt{\tau} e^{\pm y}$$

A and B have perturbative expansions. Additional non-leading contributions can be systematically included.

Note: the above expressions must be supplemented with a prescription for treating the large b region where non-perturbative effects will come into play.

Technical aside

To avoid the large b region one technique is to replace $b \to b^* = \frac{b}{\sqrt{1+(b/b_{max})^2}}$ in $S(b, Q)$ and in the PDFs. In addition, one adds a term S_{np} to S:

$$S(b, Q) \to S(b^*, Q) + S_{np}(b, Q)$$

where S_{np} is often parametrized as

$$S_{np} = b^2 \left[g_1 + g_2 \ln \frac{Q b_{max}}{2} \right].$$

Note: S_{np} parametrizes the large b non-perturbative region. Its form is suggested by requiring it to smoothly tie on to $S(b, Q)$.
Note: The gaussian term in b (g_1) corresponds to a gaussian in p_T since

$$\int_0^\infty b db J_0(b p_T) e^{-\alpha b^2} = \frac{1}{2\alpha} e^{-p_T^2/4\alpha}.$$

Thus, this form for the resummation contains the gaussian smearing ansatz originally used to describe the lepton pair p_T distribution at low values of p_T.

An alternative formalism exists which avoids the use of impact parameter space - see Ellis and Veseli, Nucl. Phys. B511, 649 (1998). The approach follows the original work of Dokshitzer, D'yakanov and Troyan (DDT formula).

$$\frac{d\sigma}{d\tau dy dp_T^2} = \sum_q \sigma_0^{q\overline{q}} \frac{d}{dp_T^2} \{[q(x_a, p_T)\overline{q}(x_b, p_T) + a \leftrightarrow b] \times \exp\left(- \int_{p_T^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A \ln Q^2/\mu^2 + B \right] \right) \Bigg\} .$$

Here, too, A and B have perturbative expansions.

- The physics behind this expression is the same as in the $b-$space form. Adjustments are made so that the leading, next-to-leading, and next-to-next-to-leading logs are the same.
- In this case some adjustment must be made in the small-p_T region, since the PDFs can not be evolved to arbitrarily small p_T.

Comparison to data

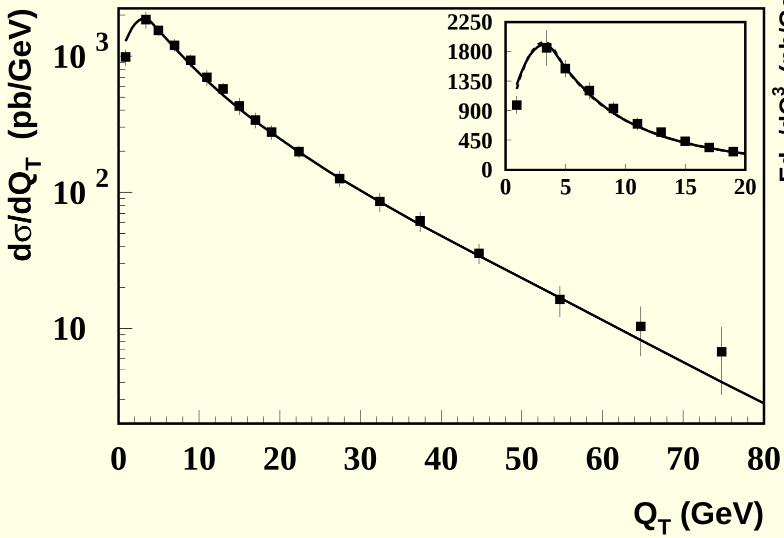
The resummation formalisms discussed for lepton pair production can equally well be applied to W and Z production.
Some references

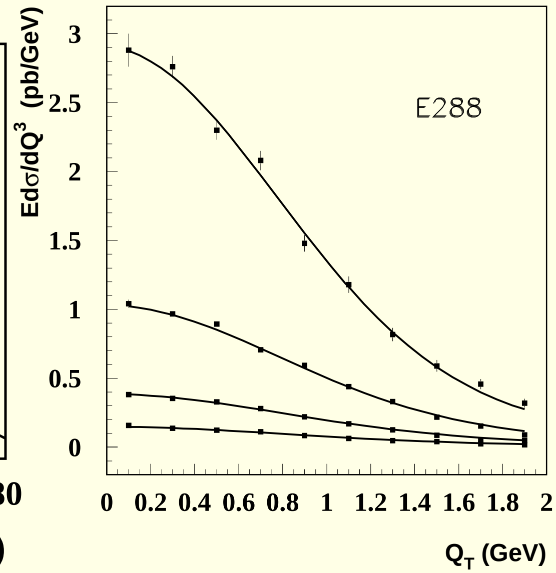
1. Altarelli, Ellis, Greco, and Martinelli, Nucl. Phys. B246, 12 (1984) (first detailed comparison of resummed perturbation theory with vector boson production data.)

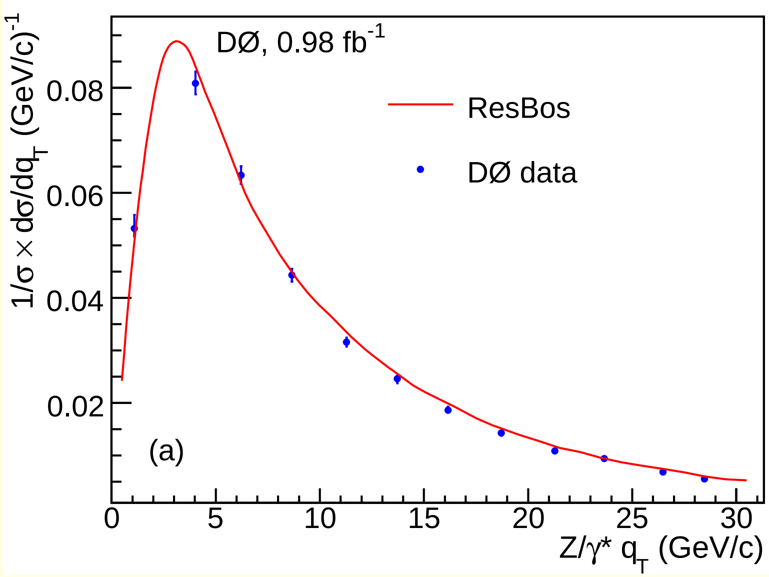
2. Ellis and Veseli, hep-ph/9706526, Nucl. Phys. B511, 649 (1998)

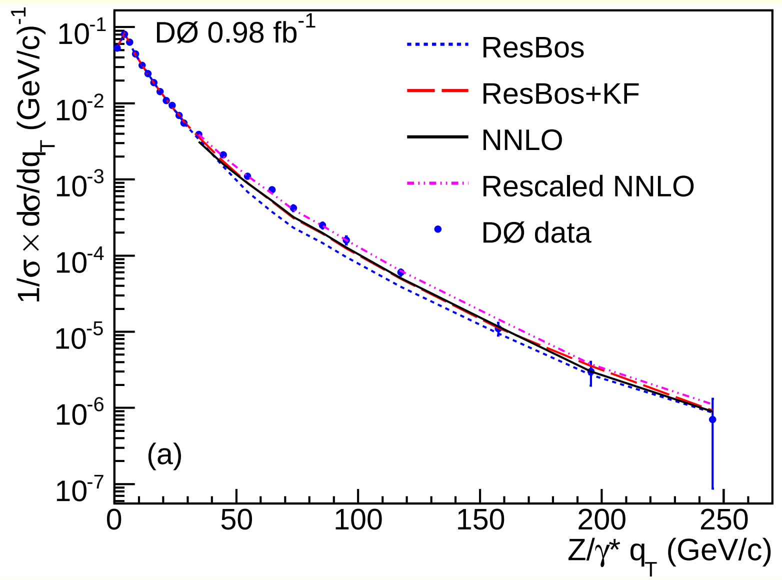
3. Ellis, Ross, and Veseli, hep-ph/9704239, Nucl. Phys. B503, 309 (1997)

4. J. Qiu and X. Zhang, Phys. Rev. D63:114011,2001


dσ/dQ_T (pb/GeV)
Q_T (GeV)
2250
1800
1350
900
450
0
0 5 10 15 20
10^3
10^2
10
0 10 20 30 40 50 60 70 80


Edσ/dQ^3 (pb/GeV)
E288
Q_T (GeV)
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
0 0.5 1 1.5 2 2.5 3


DØ, 0.98 fb^-1
ResBos
DØ data
(a)
1/σ × dσ/dq_T (GeV/c)^-1
Z/γ* q_T (GeV/c)
0.02 0.04 0.06 0.08
0 5 10 15 20 25 30


DØ 0.98 fb^-1
ResBos
ResBos+KF
NNLO
Rescaled NNLO
DØ data
(a)
1/σ × dσ/dq_T (GeV/c)^-1
Z/γ* q_T (GeV/c)
10^-1 10^-2 10^-3 10^-4 10^-5 10^-6 10^-7
0 50 100 150 200 250

Conclusions

- Lepton pair production has a long history of serving as a physics probe of hadronic interactions and new physics
- QCD corrections to the basic parton model picture have been calculated though $\mathcal{O}(\alpha_s^2)$
- Resummation techniques to handle the two-scale problem (p_T, Q) have been developed
- W and Z production serve as sources of information on Standard Model physics and beyond
- Production properties serve to constrain PDFs needed to refine measurements of the W mass
- Technology developed for lepton pair production is directly applicable to W and Z production

References

The following references may prove helpful for further studies of vector boson production. Each reference contains numerous references to the original literature.

- Collider Physics, V. Barger and R. Phillips, published by Addison-Wesley (originally published in 1987; a 1996 edition is available)
- QCD and Collider Physics, R.K. Ellis, W.J. Stirling, and B.R. Webber, published by Cambridge University Press (2003)

In addition to these references, an article based on notes from Jack Smith's 1995 DESY-CTEQ lectures on the Drell-Yan process is available from Björn Pötter at

- www.desy.de/~poetter or
- www.phys.psu.edu/~cteq/schools/summer95/dy.ps.

This writeup covers the calculation of the $\mathcal{O}(\alpha_s)$ corrections using dimensional regularization. Several useful appendices covering calculational techniques in n dimensions are included.