THE STANDARD MODEL AND HIGGS PHYSICS

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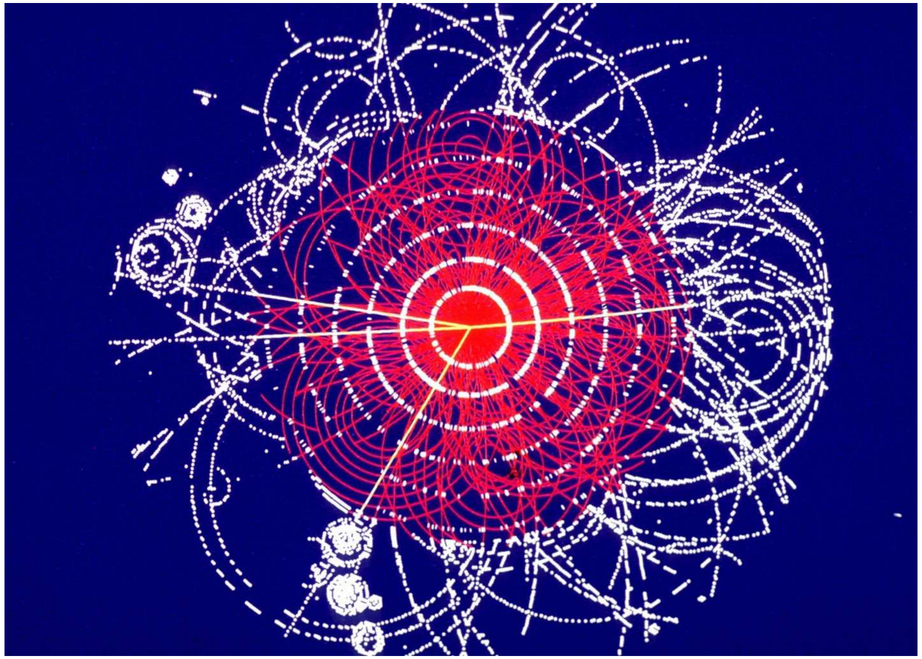
Monday: Standard Model and beyond

- Theoretical introduction
- Constraints on the Higgs
- Supersymmetric Higgs sector

Tuesday: LHC Phenomenology

- Higgs boson decay
- Higgs boson signals at LHC



Field theory description of the SM (and beyond)

- A quick review of non-Abelian gauge theories
  - Yang-Mills theories
  - electroweak interactions
- Spontaneous symmetry breaking and mass generation: the Higgs boson
- Theoretical bounds on the mass of the Higgs boson
- Experimental bounds on the mass of the Higgs boson
- Extension of the Higgs sector: two Higgs-doublet models and the MSSM

Non-Abelian (Yang-Mills) gauge theories

The starting point is a Lagrangian of free or self-interacting fields, that is symmetric under a GLOBAL symmetry

$$\mathcal{L}_\psi(\psi, \partial_\mu\psi) \qquad = i\overline{\psi}\gamma^\mu\partial_\mu\psi - m\,\overline{\psi}\psi$$

where

$$\psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix} = \text{multiplet of a compact Lie group } G$$

The Lagrangian is symmetric under the transformation

$$\psi \to \psi' = U(\theta)\psi \qquad U(\theta) = \exp(igT^a\theta_a) \qquad \text{unitary matrix} \qquad UU^\dagger = U^\dagger U = 1$$

If U is unitary, the T^a are hermitian matrices, called group generators (they "generate" infinitesimal transformation around the unit element of the group)

$$U(\theta) = 1 + igT^a\theta_a + \mathcal{O}\left(\theta^2\right)$$

If U is SU(N) matrix (unitary and det $U = 1$), then there are $N^2 - 1$ traceless, hermitian generators $T^a = \frac{\lambda^a}{2}$

From $\partial_\mu \longrightarrow D_\mu$

We obtain a LOCAL invariant Lagrangian if we make the substitution

$$\mathcal{L}_\psi(\psi, \partial_\mu \psi) \to \mathcal{L}_\psi(\psi, D_\mu \psi) \qquad D_\mu = \partial_\mu - igA^a_\mu(x)T^a \equiv \partial_\mu - igA_\mu(x)$$

with the transformation properties

$$\begin{aligned} \psi(x) &\longrightarrow U(x)\psi(x) \qquad \text{with} \qquad U(x) = U(\theta(x)) = \exp(igT^a\theta_a(x)) \\ D_\mu\psi(x) &\longrightarrow U(x)D_\mu\psi(x) = U(x)D_\mu U^{-1}(x)U(x)\psi(x) \end{aligned}$$

i.e. the covariant derivative must transform as

$$D_\mu \to U(x)D_\mu U^{-1}(x) \qquad \text{implying} \qquad A^a_\mu \to A^a_\mu + \partial_\mu\theta^a(x) + gf^{abc}A^b_\mu\theta^c + \cdots$$

We can build the kinetic term for the A^a_μ fields from

$$F_{\mu\nu} = F^a_{\mu\nu}T^a = \frac{i}{g}[D_\mu, D_\nu] \qquad \text{with} \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$$

which transforms homogeneously under a local gauge transformation

$$F_{\mu\nu} \to UF_{\mu\nu}U^{-1} \qquad \Longrightarrow \qquad F^a_{\mu\nu}F^{\mu\nu}_a \sim \mathrm{tr}F_{\mu\nu}F^{\mu\nu} \to \mathrm{tr}UF_{\mu\nu}U^{-1}\,UF^{\mu\nu}U^{-1} = \mathrm{tr}F_{\mu\nu}F^{\mu\nu}$$

Remarks on Yang-Mills theories

Gauge invariant Yang-Mills (YM) Lagrangian for gauge and matter fields

$$\mathcal{L}_{YM} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \mathcal{L}_\psi(\psi, D_\mu \psi) \qquad \text{with} \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

- Mass terms $A^a_\mu A^{a\mu}$ for the gauge bosons are NOT gauge invariant!
  Gauge bosons of (unbroken) YM theories are massless.
- From the $F^a_{\mu\nu} F^{a\mu\nu}$ term in the Lagrangian, we have cubic and quartic gauge boson self interactions
- gauge invariance combined with renormalizability (absence of higher powers of fields and covariant derivatives in $\mathcal{L}$) determines gauge-boson/matter couplings and gauge-boson self interactions
- if G =SU(3)$_c$ ($N = 3$) and the fermion are in triplets,

$$\psi = \begin{pmatrix} \psi_{\text{red}} \\ \psi_{\text{blue}} \\ \psi_{\text{green}} \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

  we have the QCD Lagrangian with $N^2 - 1 = 8$ gauge bosons = gluons.

Electroweak sector

From experimental facts (charged currents couple only to left-handed fermions, existence of a massless photon and a neutral Z), the gauge group is chosen as $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$.

$$\psi_L \equiv \frac{1}{2}(1-\gamma_5)\psi \qquad \psi_R \equiv \frac{1}{2}(1+\gamma_5)\psi \qquad \psi = \psi_L + \psi_R$$

$$L_L \equiv \frac{1}{2}(1-\gamma_5)\begin{pmatrix} \nu_e \\ e \end{pmatrix} = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \qquad \nu_{eR} \equiv \frac{1}{2}(1+\gamma_5)\nu_e \qquad e_R \equiv \frac{1}{2}(1+\gamma_5)e$$

- $\mathrm{SU}(2)_L$: weak isospin group. Three generators $\Longrightarrow$ three gauge bosons: W^1, W^2 and W^3.
  Generators for doublets are $T^a = \sigma^a/2$, where σ^a are the 3 Pauli matrices
  For gauge singlets (e_R, ν_R): $T^a \equiv 0$. All satisfy $\left[T^a, T^b\right] = i\epsilon^{abc}T^c$.
  The gauge coupling will be indicated with g.
- $\mathrm{U}(1)_Y$: weak hypercharge Y. One gauge boson B with gauge coupling g'.
  One generator (charge) $Y(\psi)$, whose value depends on the fermion field

W^3 and B carry identical quantum numbers ($T_3 = 0$, $Y = 0$) $\Longrightarrow$ they will combine to produce two neutral gauge bosons: Z and γ.

EW gauge-boson sector of the SM

Gauge invariance and renormalizability completely determine the kinetic terms for the gauge bosons

$$\mathcal{L}_{YM} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu}$$

$$\begin{aligned} B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu \\ W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\,\epsilon^{abc}\, W_{b,\mu}\, W_{c,\nu} \end{aligned}$$

The gauge symmetry does NOT allow any explicit mass terms for $W^\pm$ and Z, i.e. forbidden are terms like

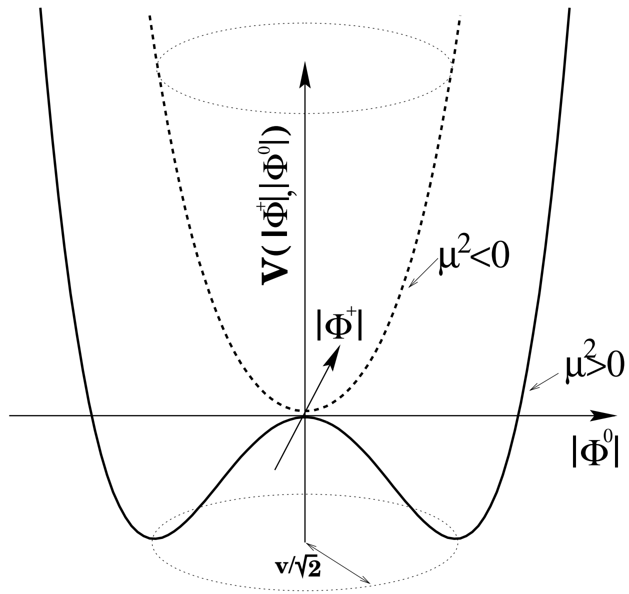
$$\mathcal{L}_{Mass} = \frac{1}{2} m_W^2 W^a_\mu W_a^\mu$$

Spontaneous symmetry breaking

Experimentally, the weak bosons are massive. We give mass to the gauge bosons through the Higgs mechanism: generate mass terms from the kinetic energy term of a scalar doublet field Φ that undergoes spontaneous symmetry breaking.

Introduce a complex scalar doublet

$$
\begin{aligned}
\Phi &= \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad\qquad Y_\Phi = \frac{1}{2} \\
\mathcal{L}_{\mathrm{Higgs}} &= (D_\mu \Phi)^\dagger (D^\mu \Phi) - V\left(\Phi^\dagger \Phi\right) \\
D^\mu &= \partial^\mu - igW_i^\mu \frac{\sigma^i}{2} - ig' Y_\Phi B^\mu \\
V\left(\Phi^\dagger \Phi\right) &= V_0 - \mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2, \qquad\qquad \mu^2, \lambda > 0
\end{aligned}
$$



Notice the "wrong" mass sign. Minimum of potential when Φ has vacuum expectation value

$$
< |\Phi| > \; = \; \frac{v}{\sqrt{2}} \; = \; \frac{|\mu|}{\sqrt{2\lambda}}
$$

Expanding Φ around the minimum

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}\left[v + H(x) + i\chi(x)\right] \end{pmatrix} = \frac{1}{\sqrt{2}} \exp\left[\frac{i\sigma_i\theta^i(x)}{v}\right] \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

We can rotate away the fields $\theta^i(x)$ by an $\mathrm{SU}(2)_L$ gauge transformation

$$\Phi(x) \to \Phi'(x) = U(x)\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

where $U(x) = \exp\left[-\frac{i\sigma_i\theta^i(x)}{v}\right]$.

This gauge choice, called unitary gauge, is equivalent to absorbing the Goldstone modes $\theta^i(x)$.

The vacuum state can be chosen to correspond to the vacuum expectation value

$$\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Notice that only a scalar field can have a vacuum expectation value.The VEV of a fermion or vector field would break Lorentz invariance.

Consequences for the scalar field H

The scalar potential

$$V\left(\Phi^{\dagger}\Phi\right) = \lambda\left(\Phi^{\dagger}\Phi - \frac{v^2}{2}\right)^2$$

expanded around the vacuum state

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

becomes

$$V = \frac{\lambda}{4}\left(2vH + H^2\right)^2 = \frac{1}{2}(2\lambda v^2)H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4$$

Consequences:

- the scalar field H gets a mass which is given by the quartic coupling λ

$$m_H^2 = 2\lambda v^2$$

- there is a term of cubic and quartic self-coupling.

Higgs kinetic terms and coupling to W, Z

$$
\begin{aligned}
D^\mu \Phi &= \left(\partial^\mu - igW_i^\mu \frac{\sigma^i}{2} - ig'\frac{1}{2}B^\mu\right)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \\
&= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2\sqrt{2}}\left[g\begin{pmatrix} W_3^\mu & W_1^\mu - iW_2^\mu \\ W_1^\mu + iW_2^\mu & -W_3^\mu \end{pmatrix} + g'B^\mu\right]\begin{pmatrix} 0 \\ v + H \end{pmatrix} \\
&= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2}(v+H)\begin{pmatrix} g\left(W_1^\mu - iW_2^\mu\right) \\ -gW_3^\mu + g'B^\mu \end{pmatrix}\right] \\
&= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2}\left(1 + \frac{H}{v}\right)\begin{pmatrix} gv \;\; W^{\mu+} \\ -\sqrt{(g^2+g'^2)/2}\;\; v \;\; Z^\mu \end{pmatrix}
\end{aligned}
$$

$$
(D^\mu\Phi)^\dagger D_\mu\Phi = \frac{1}{2}\partial^\mu H \partial_\mu H + \left[\left(\frac{gv}{2}\right)^2 W^{\mu+}W_\mu^- + \frac{1}{2}\frac{\left(g^2+g'^2\right)v^2}{4} Z^\mu Z_\mu\right]\left(1 + \frac{H}{v}\right)^2
$$

Consequences

- The W and Z gauge bosons have acquired masses

$$m_W^2 = \frac{g^2 v^2}{4} \qquad\qquad m_Z^2 = \frac{\left(g^2 + g'^2\right) v^2}{4} = \frac{m_W^2}{\cos^2\theta_W}$$

From the measured value of the Fermi constant G_F

$$\frac{G_F}{\sqrt{2}} = \left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{m_W^2} \qquad \Longrightarrow \qquad v = \sqrt{\frac{1}{\sqrt{2}G_F}} \approx 246.22 \text{ GeV}$$

- the photon stays massless
- HWW and HZZ couplings from $2H/v$ term (and $HHWW$ and $HHZZ$ couplings from H^2/v^2 term)

$$\mathcal{L}_{HVV} = \frac{2m_W^2}{v} W_\mu^+ W^{-\mu} H + \frac{m_Z^2}{v} Z^\mu Z_\mu H \equiv g m_W W_\mu^+ W^{-\mu} H + \frac{1}{2}\frac{g m_Z}{\cos\theta_W} Z^\mu Z_\mu H$$

Higgs coupling proportional to mass

- tree-level HVV (V = vector boson) coupling requires VEV! e.g. $g m_W = g^2 v/2$
  Normal scalar couplings give $\Phi^\dagger\Phi V$ or $\Phi^\dagger\Phi VV$ couplings only.

Gauging the symmetry: fermion Lagrangian

Following the gauge recipe (for one generation of leptons, quarks work the same way)

$$\mathcal{L}_\psi = i\,\bar{L}_L \,\not{D}\, L_L + i\,\bar{\nu}_{eR}\,\not{D}\,\nu_{eR} + i\,\bar{e}_R\,\not{D}\,e_R$$

where

$$D^\mu = \partial^\mu - igW_i^\mu\,T^i - ig'Y_\psi\,B^\mu \qquad T^i = \frac{\sigma^i}{2} \quad \text{or} \quad T^i = 0, \qquad i = 1,2,3$$

$$\mathcal{L}_\psi \equiv \mathcal{L}_{kin} + \mathcal{L}_{CC} + \mathcal{L}_{NC}$$

$$\begin{aligned}
\mathcal{L}_{kin} &= i\,\bar{L}_L\,\not{\partial}\,L_L + i\,\bar{\nu}_{eR}\,\not{\partial}\,\nu_{eR} + i\,\bar{e}_R\,\not{\partial}\,e_R \\
\mathcal{L}_{CC} &= g\,W_\mu^1\,\bar{L}_L\,\gamma^\mu\,\frac{\sigma_1}{2}\,L_L + g\,W_\mu^2\,\bar{L}_L\,\gamma^\mu\,\frac{\sigma_2}{2}\,L_L = \frac{g}{\sqrt{2}}\,W_\mu^+\,\bar{\nu}_L\,\gamma^\mu\,e_L + \frac{g}{\sqrt{2}}\,W_\mu^-\,\bar{e}_L\,\gamma^\mu\,\nu_L \\
\mathcal{L}_{NC} &= \frac{g}{2}\,W_\mu^3\,[\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} - \bar{e}_L\,\gamma^\mu\,e_L] + g'\,B_\mu\Big[Y_L\,(\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} + \bar{e}_L\,\gamma^\mu\,e_L) \\
&\quad + Y_{\nu_{eR}}\,\bar{\nu}_{eR}\,\gamma^\mu\,\nu_{eR} + Y_{e_R}\,\bar{e}_R\,\gamma^\mu\,e_R\Big]
\end{aligned}$$

with

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$$

Fermion couplings fixed by renormalizability and gauge quantum numbers

				$SU(3)$	$SU(2)$	$U(1)_Y$
$Q_L^i =$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{6}$
$u_R^i =$	u_R	c_R	t_R	3	1	$\frac{2}{3}$
$d_R^i =$	d_R	s_R	b_R	3	1	$-\frac{1}{3}$
$L_L^i =$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	$-\frac{1}{2}$
$e_R^i =$	e_R	μ_R	τ_R	1	1	-1
$\nu_R^i =$	ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	1	1	0

Weak mixing angle

W_μ^3 and B_μ mix to produce two orthogonal mass eigenstates

$$\begin{aligned}\text{massive partner}: \quad g\,W_\mu^3 - g'\,B_\mu &= \sqrt{g^2+g'^2}\,Z_\mu = \sqrt{g^2+g'^2}\left(W_\mu^3\cos\theta_W - B_\mu\sin\theta_W\right)\\ \text{orthogonal, massless}: \quad g'\,W_\mu^3 + g\,B_\mu &= \sqrt{g^2+g'^2}\,A_\mu = \sqrt{g^2+g'^2}\left(W_\mu^3\sin\theta_W + B_\mu\cos\theta_W\right)\end{aligned}$$

with mixing angle fixed by $\cos\theta_W = \dfrac{g}{\sqrt{g^2+g'^2}}$ $\quad\sin\theta_W = \dfrac{g'}{\sqrt{g^2+g'^2}}$

Write the NC Lagrangian in terms of these mass eigenstates

$$\mathcal{L}_{NC} = \bar{\psi}\gamma_\mu\left(gT_3W_3^\mu + g'YB^\mu\right)\psi = \bar{\psi}\gamma_\mu\left(\frac{1}{\sqrt{g^2+g'^2}}(g^2T_3 - g'^2Y)Z^\mu + \frac{gg'}{\sqrt{g^2+g'^2}}(T_3+Y)A^\mu\right)\psi$$

Must identify positron charge, e, as

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} = g\sin\theta_W = g'\cos\theta_W$$

and the charge of a particle, as a multiple of the positron charge, is given by the Gell-Mann–Nishijima formula: $Q = T_3 + Y$

The neutral current

It is customary to write the Z coupling to fermions in terms of the electric charge Q and the third component of isospin ($T_3 = \pm 1/2$ for left-chiral fermions, 0 for right-chiral fermions)

$$\mathcal{L}_{NC} = \overline{\psi}\gamma_\mu \left(\frac{1}{\sqrt{g^2+g'^2}}(g^2 T_3 - g'^2 Y) Z^\mu + \frac{gg'}{\sqrt{g^2+g'^2}}(T_3+Y)A^\mu \right)\psi = e\overline{\psi}\gamma_\mu Q \psi A^\mu + \overline{\psi}\gamma_\mu Q_Z \psi Z^\mu$$

Q_Z is given by

$$Q_Z = \frac{1}{\sqrt{g^2+g'^2}}(g^2 T_3 - g'^2(Q - T_3)) = \frac{e}{\cos\theta_W \sin\theta_W}\left(T_3 - Q\sin^2\theta_W\right)$$

This procedure works for leptons and also for the quarks (see more later)

$$Q_L^i = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \qquad \begin{array}{l} u_R^i = u_R, c_R, t_R \\ d_R^i = d_R, s_R, b_R \end{array}$$

Fermion mass generation

A direct mass term is not invariant under $\mathrm{SU}(2)_L$ or $\mathrm{U}(1)_Y$ gauge transformation

$$m_f \overline{\psi}\psi = m_f \left(\overline{\psi}_R \psi_L + \overline{\psi}_L \psi_R\right)$$

Generate fermion masses through Yukawa-type interaction terms

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} &= -\Gamma_d \overline{Q}_L \Phi d_R - \Gamma_d \overline{d}_R \Phi^\dagger Q_L \\ &\quad -\Gamma_u \overline{Q}_L \Phi_c u_R + \text{h.c.} \\ &\quad -\Gamma_e \overline{L}_L \Phi e_R + \text{h.c.} \\ &\quad -\Gamma_\nu \overline{L}_L \Phi_c \nu_R + \text{h.c.} \end{aligned}$$

$$\Phi_c = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

where Q, L are left-handed doublet fields and d_R, u_R, e_R, ν_R are right-handed $SU(2)$ -singlet fields.

Notice: neutrino masses can be implemented via Γ_ν term. Since $m_\nu \approx 0$ we neglect it in the following.

Fermion masses for three generations

Generate fermion masses for three generations of quarks and leptons by generalizing

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} \quad &= \quad -\Gamma_d^{ij}\,\overline{Q}_L'^{\,i}\,\Phi d_R'^{\,j} - \Gamma_d^{ij*}\,\overline{d}_R'^{\,i}\,\Phi^\dagger Q_L'^{\,j} \\
&\quad -\Gamma_u^{ij}\,\overline{Q}_L'^{\,i}\,\Phi_c u_R'^{\,j} + \text{h.c.} \\
&\quad -\Gamma_e^{ij}\overline{L}_L^i \Phi e_R^j + \text{h.c.}
\end{aligned}$$

$$\Phi_c = i\sigma_2\Phi^* = \frac{1}{\sqrt{2}}\begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

where Q', u' and d' are quark fields that are generic linear combination of the mass eigenstates u and d and Γ_u, Γ_d and Γ_e are 3×3 complex matrices in generation space, spanned by the indices i and j.

$\mathcal{L}_{\text{Yukawa}}$ is Lorentz invariant, gauge invariant and renormalizable, and therefore it can (actually it must) be included in the Lagrangian.

Expanding around the vacuum state

In the unitary gauge we have

$$\begin{aligned}
\overline{Q}_L^{\prime\, i}\, \Phi\, d_R^{\prime\, j} &= \left(\overline{u}_L^{\prime\, i}\;\; \overline{d}_L^{\prime\, i}\right) \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} d_R^{\prime\, j} = \frac{v+H}{\sqrt{2}}\, \overline{d}_L^{\prime\, i}\, d_R^{\prime\, j} \\
\overline{Q}_L^{\prime\, i}\, \Phi_c\, u_R^{\prime\, j} &= \left(\overline{u}_L^{\prime\, i}\;\; \overline{d}_L^{\prime\, i}\right) \begin{pmatrix} \frac{v+H}{\sqrt{2}} \\ 0 \end{pmatrix} u_R^{\prime\, j} = \frac{v+H}{\sqrt{2}}\, \overline{u}_L^{\prime\, i}\, u_R^{\prime\, j}
\end{aligned}$$

and we obtain

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\Gamma_d^{ij}\, \frac{v+H}{\sqrt{2}}\, \overline{d}_L^{\prime\, i}\, d_R^{\prime\, j} - \Gamma_u^{ij}\, \frac{v+H}{\sqrt{2}}\, \overline{u}_L^{\prime\, i}\, u_R^{\prime\, j} - \Gamma_e^{ij}\, \frac{v+H}{\sqrt{2}}\, \overline{e}_L^{i}\, e_R^{j} + \text{h.c.} \\
&= -\left[M_u^{ij}\, \overline{u}_L^{\prime\, i}\, u_R^{\prime\, j} + M_d^{ij}\, \overline{d}_L^{\prime\, i}\, d_R^{\prime\, j} + M_e^{ij}\, \overline{e}_L^{i}\, e_R^{j} + \text{h.c.} \right] \left(1 + \frac{H}{v}\right)
\end{aligned}$$

with mass matrices $\quad M^{ij} = \Gamma^{ij} \frac{v}{\sqrt{2}}$

Diagonalizing M_f

It is always possible to diagonalize M_f^{ij} ($f = u, d, e$) with a bi-unitary transformation ($U^f_{L/R}$ must be unitary in order to preserve the form of the kinetic terms in the Lagrangian)

$$
\begin{aligned}
f'_{Li} &= \left(U_L^f\right)_{ij} f_{Lj} \\
f'_{Ri} &= \left(U_R^f\right)_{ij} f_{Rj}
\end{aligned}
$$

with U_L^f and U_R^f chosen such that

$$\left(U_L^f\right)^\dagger M_f U_R^f = \text{diagonal}$$

For example:

$$(U_L^u)^\dagger M_u U_R^u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \qquad \left(U_L^d\right)^\dagger M_d U_R^d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

Mass terms

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\sum_{f,i,j} M_f^{ij}\, \overline{f}_L'^{\,i}\, f_R'^{\,j} \left(1 + \frac{H}{v}\right) + \text{h.c.} \\
&= -\sum_{f,i,j} \overline{f}_L^{\,i} \left[\left(U_L^f\right)^\dagger M_f U_R^f\right]_{ij} f_R^j \left(1 + \frac{H}{v}\right) + \text{h.c.} \\
&= -\sum_f m_f \left(\overline{f}_L f_R + \overline{f}_R f_L\right) \left(1 + \frac{H}{v}\right)
\end{aligned}$$

We succeed in producing fermion masses and we got a fermion-antifermion-Higgs coupling proportional to the fermion mass.

The Higgs Yukawa couplings are flavor diagonal: no flavor changing Higgs interactions.

Mass diagonalization and charged current interaction

The charged current interaction is given by

$$\frac{e}{\sqrt{2}\sin\theta_W}\,\overline{u}'^{\,i}_L\, \not{W}^+\, d'^{\,i}_L + \text{h.c.}$$

After the mass diagonalization described previously, this term becomes

$$\frac{e}{\sqrt{2}\sin\theta_W}\,\overline{u}^{i}_L \left[(U^u_L)^\dagger\, U^d_L\right]_{ij} \not{W}^+ d^j_L + \text{h.c.}$$

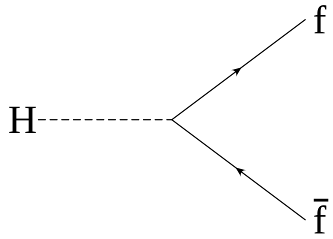
and we define the Cabibbo-Kobayashi-Maskawa matrix V_{CKM}

$$V_{CKM} = (U^u_L)^\dagger\, U^d_L$$

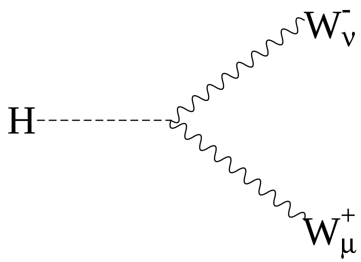
- V_{CKM} is not diagonal and then it mixes the flavors of the different quarks.

- It is a unitary matrix and the values of its entries must be determined from experiments.

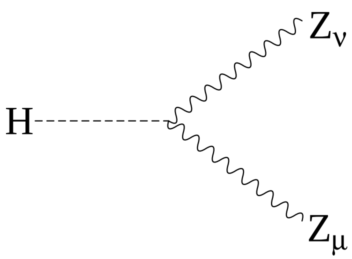
Feynman rules for Higgs couplings



$$-i\frac{m_f}{v}$$



$$ig\, m_W\, g_{\mu\nu}$$



$$i\, g\, \frac{1}{\cos\theta_W}\, m_Z\, g_{\mu\nu}$$

Within the Standard Model, since almost all masses have been measured, the Higgs couplings are almost completely known. The only free parameter (not yet measured) is the Higgs mass

$$m_H^2 = 2\lambda v^2$$

Constraints on the Higgs Boson Mass

We had found that the Higgs boson mass is related to the value of the quartic Higgs coupling λ:

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \lambda \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^2$$

leads to

$$m_H^2 = 2\lambda v^2$$

So far we have measured neither m_H nor $\lambda \Longrightarrow$ no direct experimental information

This raises several questions

- Can we get rid of the Higgs by setting $m_H = \infty$ and $\lambda = \infty$? Can we eliminate the Higgs from the SM?
- Does consistency of the SM as a renormalizable field theory provide constraints?
- Is there indirect information on m_H, e.g. from precision observables and loop effects?

The perturbative unitarity bound

A very severe constraint on the Higgs boson mass comes from unitarity of the scattering amplitude.

$$\text{unitarity} \Longleftrightarrow \text{QM probability} < 1$$

Scattering probability bounded from above!

Considering the elastic scattering of longitudinally polarized Z bosons

$$Z_L Z_L \to Z_L Z_L$$

$$\mathcal{M} = -\frac{m_H^2}{v^2}\left[\frac{s}{s-m_H^2} + \frac{t}{t-m_H^2} + \frac{u}{u-m_H^2}\right] \qquad \text{in the } s \gg m_Z^2 \text{ limit}$$

where s, t and u are the usual Mandelstam variables.

The perturbative unitary bound on the $J = 0$ partial wave amplitude takes the form

$$s \gg m_H^2 : \qquad |\mathcal{M}_0|^2 = \left[\frac{3}{16\pi}\frac{m_H^2}{v^2}\right]^2 < 1 \qquad \Longrightarrow \qquad m_H < \sqrt{\frac{16\pi}{3}}v \approx 1\ \text{TeV}$$

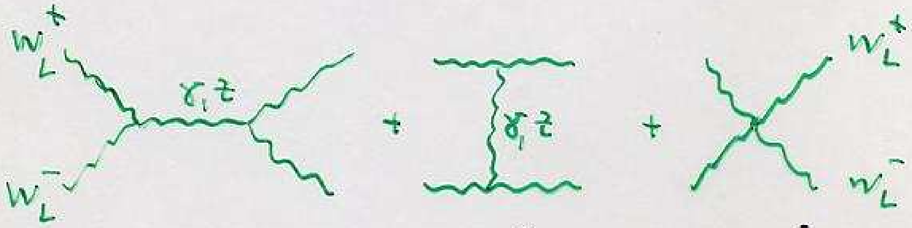
WW scattering and unitarity

Consider longitudinal W's

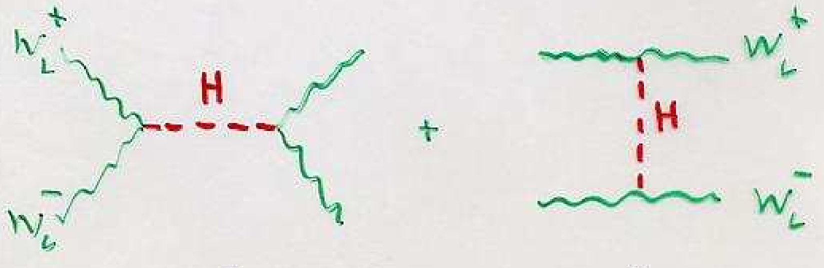
$$W_L^+ W_L^- \to W_L^+ W_L^-$$

Polarisation vector

$$\varepsilon_L^\mu = \frac{p^\mu}{m_W} + O\left(\frac{m_W}{E}\right) \sim \frac{\sqrt{s}}{m_W}$$



$$\mathcal{M} \sim \underbrace{\frac{s^2}{m_W^4} \qquad \frac{s^2}{m_W^4} \qquad \frac{s^2}{m_W^4}}_{s/m_W^2}$$



$$\frac{s}{m_W^2} \qquad \frac{s}{m_W^2}$$

sum $\sim s^0$ for $s \gg m_H^2$

Unitarity of WW scattering

Partial wave amplitudes are bounded by a constant

$\Longrightarrow \quad \mathcal{M} \sim \frac{s}{m_W^2}$ violates unitarity at sufficiently high energy

Without the Higgs contribution, the $J = 0$ partial wave violates unitarity for $\sqrt{s} > 1.2$ TeV

Destructive interference between Higgs exchange amplitudes and gauge boson scattering amplitudes works for $s > m_H^2$ only

$$\Longrightarrow \quad m_H \lesssim 1 \text{ TeV}$$

or new physics at the TeV scale

or both

Running of λ

The one-loop renormalization group equation (RGE) for $\lambda(\mu)$ is

$$\frac{d\lambda(\mu)}{d\log\mu^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + \frac{3}{8}g^4 + \frac{3}{16}\left(g^2+g'^2\right)^2 - 3h_t^4 - 3\lambda g^2 - \frac{3}{2}\lambda\left(g^2+g'^2\right) + 6\lambda h_t^2\right]$$

where

$$m_t = \frac{h_t v}{\sqrt{2}} \qquad \text{and} \qquad m_H^2 = 2\lambda v^2$$

This equation must be solved together with the one-loop RGEs for the gauge and Yukawa couplings, which, in the Standard Model, are given by

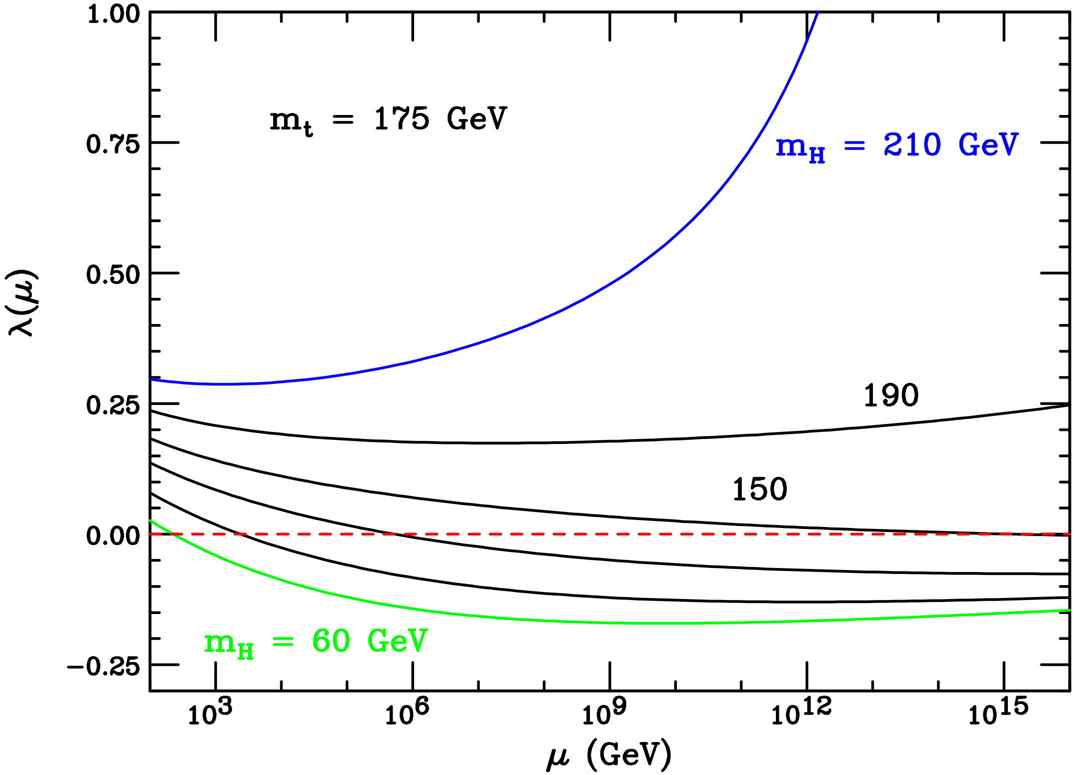
$$\begin{aligned}
\frac{dg(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2}\left(-\frac{19}{6}g^3\right)\\
\frac{dg'(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2}\frac{41}{6}g'^3\\
\frac{dg_s(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2}\left(-7g_s^3\right) = \frac{1}{32\pi^2}\left(-(11-\frac{2}{3}n_f)g_s^3\right)\\
\frac{dh_t(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2}\left[\frac{9}{2}h_t^3 - \left(8g_s^2 + \frac{9}{4}g^2 + \frac{17}{12}g'^2\right)h_t\right]
\end{aligned}$$

here g_s is the strong interaction coupling constant, and the $\overline{\text{MS}}$ scheme is adopted.

Solutions for $\lambda(\mu)$

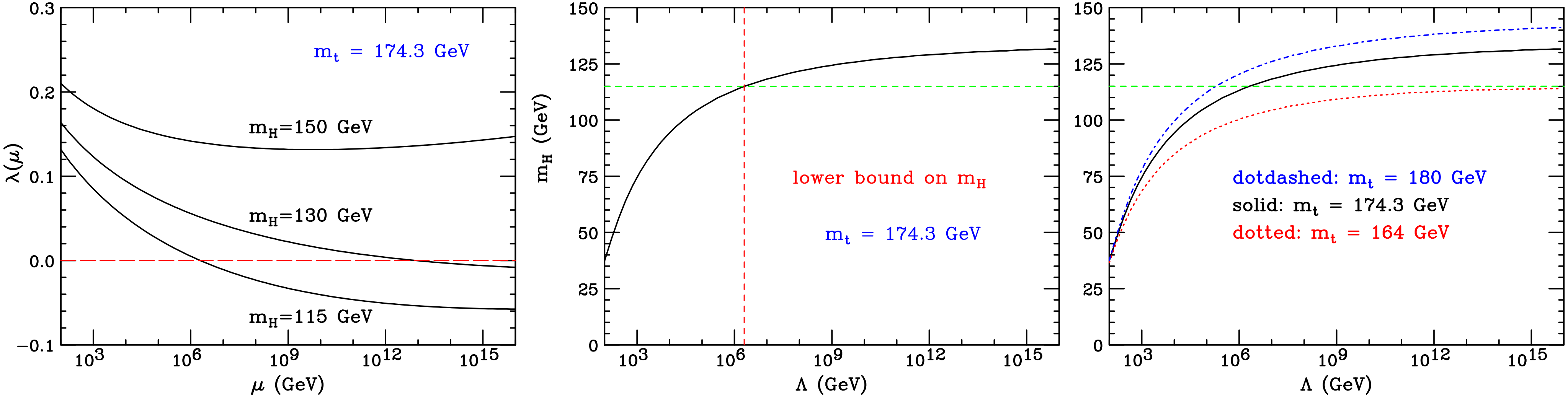
Solving this system of coupled equations with the initial condition

$$\lambda\left(m_H\right) = \frac{m_H^2}{2v^2}$$



Lower bound for m_H: vacuum stability

It can be shown that the requirement that the Higgs potential be bounded from below, even after the inclusion of radiative corrections, is fulfilled if $\lambda(\mu)$ stays positive, at least up to a certain scale $\mu \approx \Lambda$, the maximum energy scale at which the theory can be considered reliable.

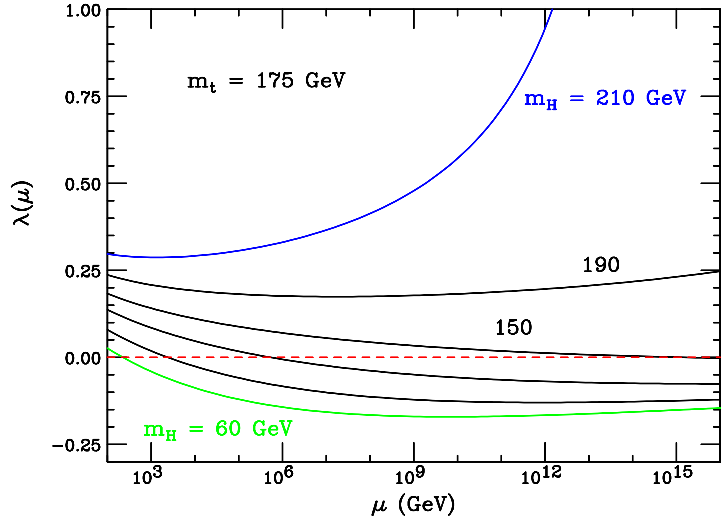


✗ This limit is extremely sensitive to the top-quark mass.

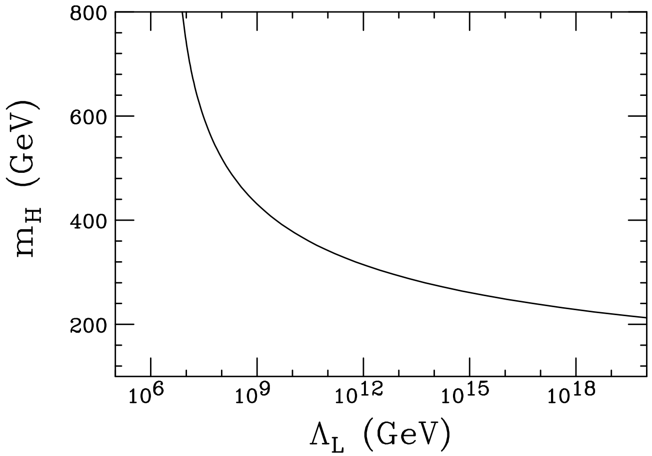
✓ The stability lower bound can be relaxed by allowing metastability

Upper bound for m_H: triviality bound

For large values of the Higgs boson mass, the coupling $\lambda(\mu)$ grows with increasing μ, and eventually leaves the perturbative domain ($\lambda \lesssim 1$): the solution has a singularity in μ , known as the Landau pole.
For the theory to make sense up to a scale Λ, we must ask $\lambda(\mu) \lesssim 1$ (or something similar), for $\mu \leq \Lambda$.
Neglecting gauge and Yukawa coupling, we have



$$\lambda(\mu^2) = \frac{\lambda(m_H^2)}{1 - \frac{3}{4\pi^2}\lambda(m_H^2)\log\frac{\mu^2}{m_H^2}} \qquad \text{singular when} \qquad \mu^2 \approx \Lambda_L^2 \equiv m_H^2 \exp\left[\frac{4\pi^2}{3\lambda(m_H^2)}\right]$$

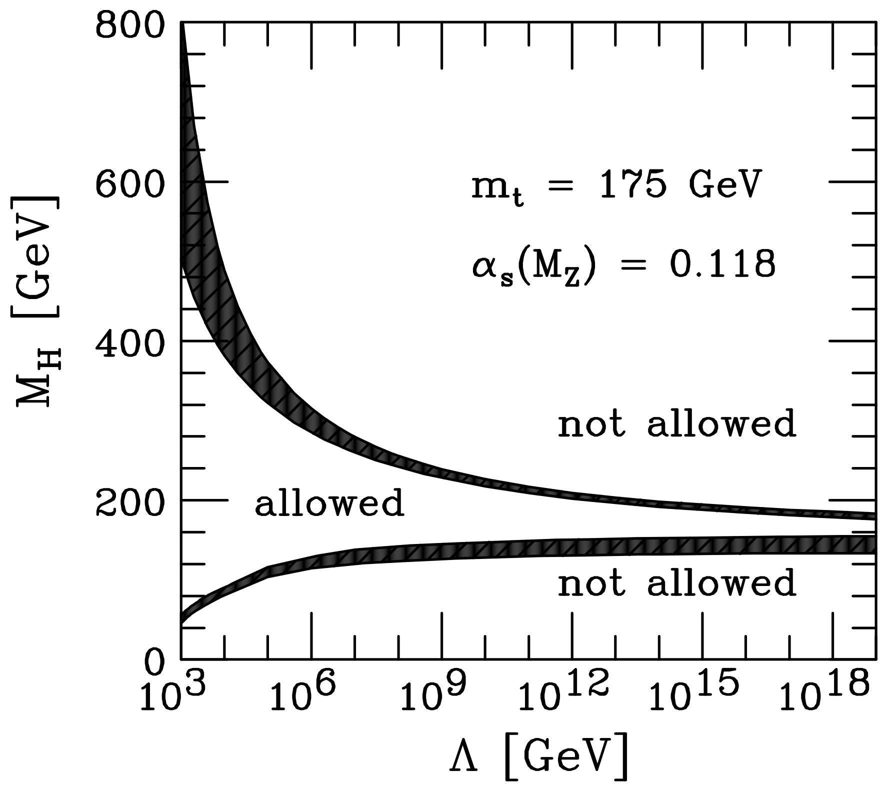


For any value of $\lambda\left(m_H^2\right)$ the theory has an upper scale Λ of validity.
$\Lambda \rightarrow \infty$ for pure scalar theory possible only if $\lambda(\mu) \equiv 0$, i.e. no scalar self-coupling $\Longrightarrow$ free or trivial theory

Higgs boson mass bounds

Renormalization group constraints on the Higgs boson mass, $m_H = \sqrt{2\lambda}v$

Riesselmann, hep-ph/9711456



Notice the small window $130\ \mathrm{GeV} \lesssim m_H \lesssim 180\ \mathrm{GeV}$, where the theory is valid up to the Planck scale $M_{\mathrm{Planck}} = (\hbar c/G_{\mathrm{Newton}})^{1/2} \approx 1.22 \times 10^{19}$ GeV.

For a cutoff scale of $\Lambda > 1000$ TeV the Higgs boson should lie in the mass window $110\ \mathrm{GeV} < m_H < 300\ \mathrm{GeV}$

Constraints from precision data

$$
\begin{aligned}
\alpha &= \frac{1}{4\pi}\frac{g^2 g'^2}{g^2+g'^2} = \frac{1}{137.03599976(50)} \\
G_F &= \frac{1}{\sqrt{2}v^2} = 1.16637(1)\times 10^{-5}\ \mathrm{GeV}^{-2} \\
m_Z &= \frac{1}{2}\sqrt{g^2+g'^2}\,v = 91.1875(21)\ \mathrm{GeV}\ ,
\end{aligned}
$$

where the uncertainty is given in parentheses. The value of α is extracted from low-energy experiments, G_F is extracted from the muon lifetime, and m_Z is measured from e^+e^- annihilation near the Z mass.

At tree level, we can express m_W as

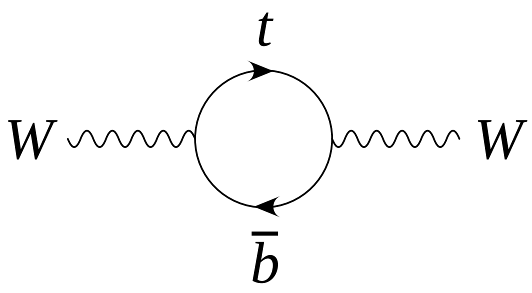
$$
m_W^2 = \frac{1}{\sin^2\theta_W}\frac{\pi\alpha}{\sqrt{2}G_F}
$$

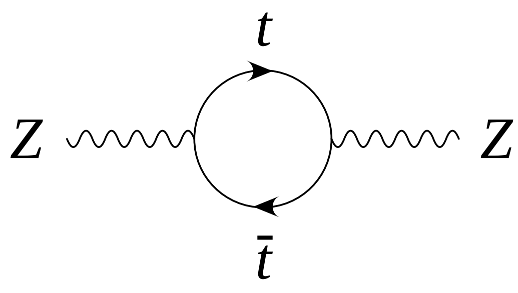
where

$$
\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2}
$$

Clues to the Higgs boson mass

From the sensitivity of electroweak observables to the mass of the top, we are able to measure its mass, even without directly producing it



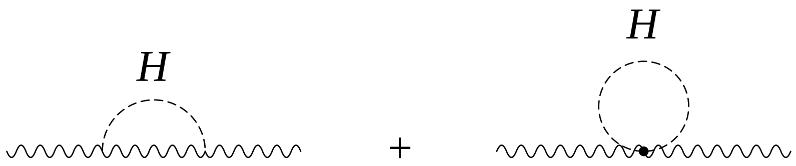


These quantum corrections alter the link between W and Z boson masses

$$m_W^2 = \frac{1}{\sin^2\theta_W\,(1-\Delta\rho)}\,\frac{\pi\alpha}{\sqrt{2}G_F} \qquad\qquad \Delta\rho_{(\mathrm{top})} \approx -\frac{3G_F}{8\pi^2\sqrt{2}}\,\frac{1}{\tan^2\theta_W}\,m_t^2$$

The strong dependence on m_t^2 accounts for the precision of the top-quark mass estimates derived from electroweak observables.

The Higgs boson quantum corrections are typically smaller than the top-quark corrections, and exhibit a more subtle dependence on m_H than the m_t^2 dependence of the top-quark corrections.



$$\Delta\rho_{(\text{Higgs})} = \frac{11 G_F m_Z^2 \cos^2\theta_W}{24\sqrt{2}\pi^2} \log\left(\frac{m_H^2}{m_W^2}\right)$$

Since m_Z has been determined at LEP to 23 ppm, it is interesting to examine the dependence of m_W upon m_t and m_H.

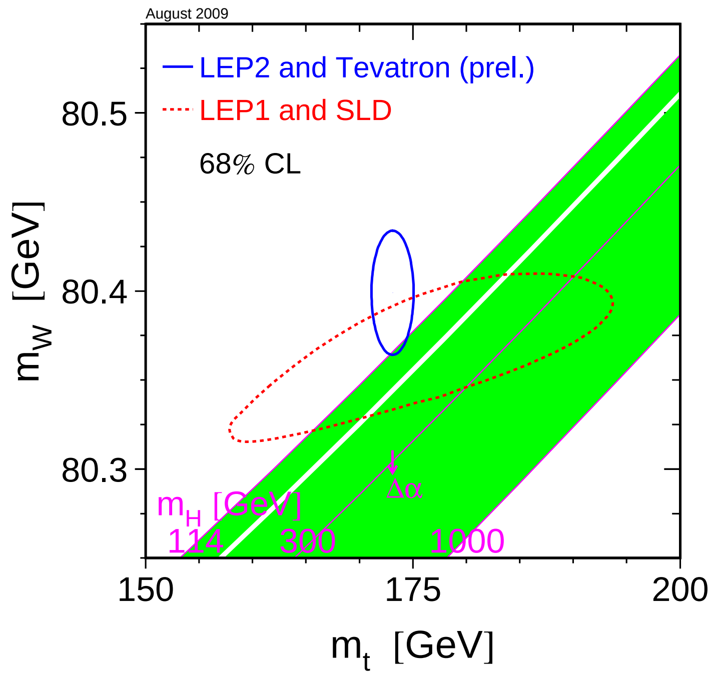
Indirect measurements of m_W and m_t (dashed line)

Direct measurements of m_W and m_t (solid line)

$m_t = 173.1 \pm 1.3$ GeV

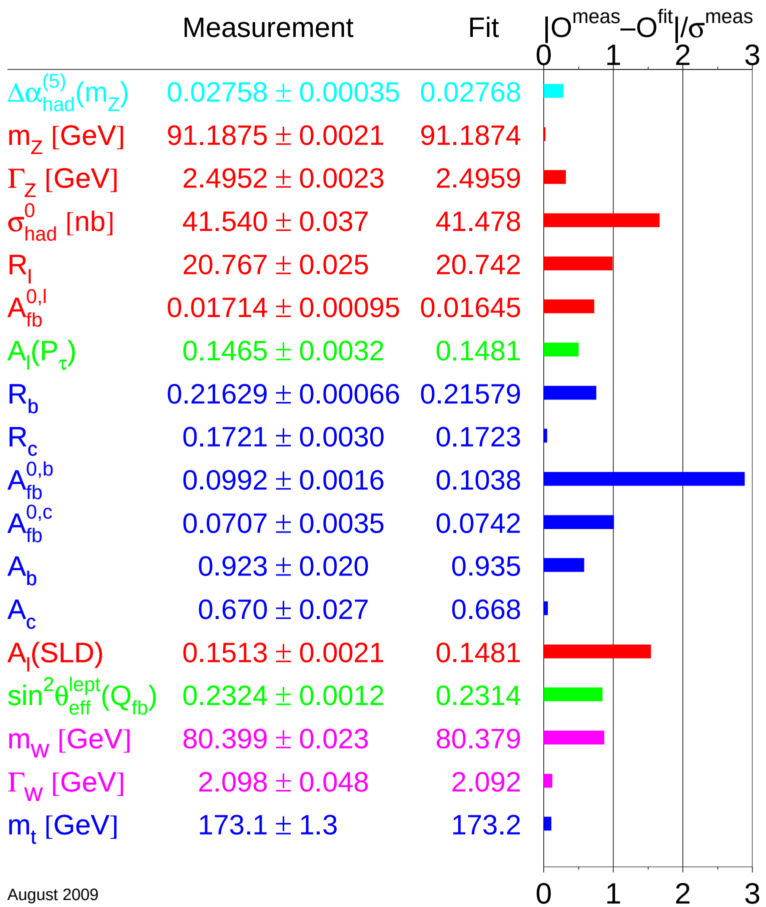
$m_W = 80.399 \pm 0.023$ GeV

both shown as one-standard-deviation regions.



The indirect and direct determinations are in reasonable agreement and both favor a light Higgs boson, within the framework of the SM.

Summary of EW precision data



Better estimates of the SM Higgs boson mass are obtained by combining all available data:

Summary of electroweak precision measurements (status summer 2009) as given on LEP-EWWG page:
http://lepewwg.web.cern.ch/LEPEWWG/

SM Higgs mass fit to EW precision data

$$m_H = 87^{+35}_{-26}\ \text{GeV}$$

Including theory uncertainty

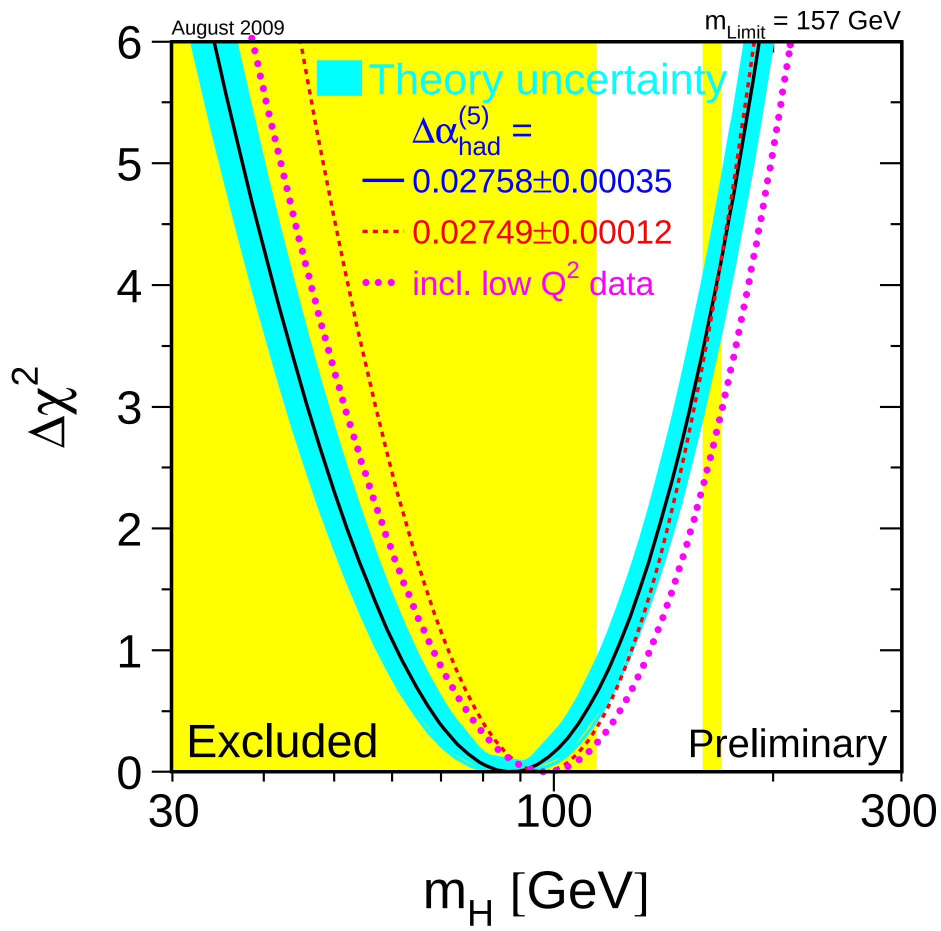
$$m_H < 157\ \text{GeV} \quad (95\%\ \text{CL})$$

Does not include
Direct search limit from LEP

$$m_H > 114\ \text{GeV} \quad (95\%\ \text{CL})$$

Renormalize probability for
$m_H > 114$ GeV to 100%:

$$m_H < 186\ \text{GeV} \quad (95\%\ \text{CL})$$



The MSSM Higgs sector

The SM uses the conjugate field $\Phi_c = i\sigma_2\Phi^*$ to generate down quark and lepton masses. In supersymmetric models this must be an independent field

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} &= -\Gamma_d \bar{Q}_L \Phi_1 d_R - \Gamma_e \bar{L}_L \Phi_1 e_R + \text{h.c.} \\ & \quad -\Gamma_u \bar{Q}_L \Phi_2 u_R + \text{h.c.} \end{aligned}$$

Two complex Higgs doublet fields Φ_1 and Φ_2 receive mass and VEVs v_1, v_2 from generalized Higgs potential. Mass eigenstates constructed out of these 8 real fields are

Neutral sector:

2 CP even Higgs bosons: h and H

1 CP odd Higgs boson: A

1 Goldstone boson: χ_0

Charged sector:

charged Higgs bosons: $H^\pm$

charged Goldstone boson: $\chi^\pm$

Higgs mixing and MSSM parameters

The Higgs potential leads to general mixing of the 2 doublet fields

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}[H^- \ \sin\beta - \chi^- \ \cos\beta] \\ v_1 + [H \ \cos\alpha - h \ \sin\alpha] + i[A \sin\beta + \chi_0 \cos\beta] \end{pmatrix} \to \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H^- \ \sin\beta \\ v_1 + \varphi_1 + iA \sin\beta \end{pmatrix}$$

$$\Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 + [H \ \sin\alpha + h \ \cos\alpha] + i[A \cos\beta - \chi_0 \sin\beta] \\ \sqrt{2}[H^+ \ \cos\beta + \chi^+ \ \sin\beta] \end{pmatrix} \to \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 + \varphi_2 + iA \cos\beta \\ \sqrt{2} H^+ \ \cos\beta \end{pmatrix}$$

The angle β is determined by the VEVs:

$$v_1 = v \ \cos\beta \,, \qquad v_2 = v \ \sin\beta \,, \qquad \Longrightarrow \qquad \frac{v_2}{v_1} = \tan\beta$$

The mixing angle α between the 2 CP even scalars and the masses are determined by

$$\tan\beta \,, \qquad m_A \,, \qquad v = \sqrt{v_1^2 + v_2^2} = 246 \text{ GeV}$$

SUSY Higgs mass relations

Higgs potential in the MSSM produces distinct mass relations at tree level

$$\begin{aligned} m_h^2, m_H^2 &= \frac{1}{2}\left[m_A^2 + m_Z^2 \pm \sqrt{\left(m_A^2 + m_Z^2\right)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta}\right] \\ m_{H^\pm} &= \sqrt{m_A^2 + m_W^2} > m_W \end{aligned}$$

Pseudoscalar mass m_A sets scale for H and $H^\pm$ mass, but h must be light

$$m_h^2 = \frac{2m_A^2 m_Z^2 \cos^2 2\beta}{m_A^2 + m_Z^2 + \sqrt{\left(m_A^2 + m_Z^2\right)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta}} < m_Z^2 \cos^2 2\beta$$

because quartic coupling is proportional to g^2, g'^2

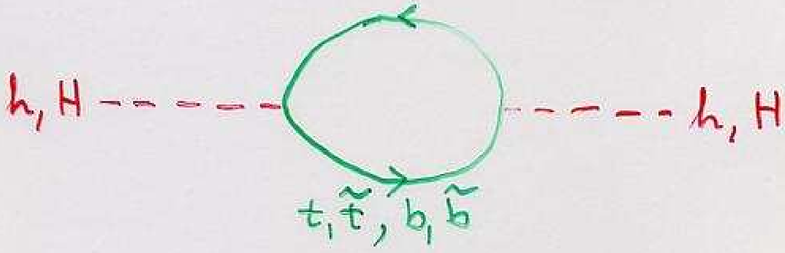
Problem: $m_h < m_Z$ is ruled out by LEP data! $\Longrightarrow$ need to include radiative corrections

Behaviour for $m_A \gg m_Z$:

$$m_H^\pm \approx m_A \approx m_H\,, \qquad\qquad m_h = m_Z|\cos 2\beta|$$

m_h is largest for $\tan\beta \rightarrow 0, \infty$.
Later: h has SM couplings in $m_A \rightarrow \infty$ limit (decoupling limit)

Include radiative corrections



Change h/H mass matrix

$$m^2_{H/h} = m^2_0 + \delta m^2$$

diagonalize $\Rightarrow m^2_h, m^2_H$, mixing angle α

Consider special case: $m_A \gg m_Z, \tan\beta \gg 1$

Lowest order: $m_h = m_Z$ (= upper bound)

h has SM couplings

$\Rightarrow t, \tilde{t}_1, \tilde{t}_2$ loops dominate

strong dependence on stop mixing:

$$X_t = A_t - \mu \cot\beta$$

governs $\tilde{t}_L, \tilde{t}_R \Longleftrightarrow \tilde{t}_1, \tilde{t}_2$ mass eigenstat

Quadratic divergencies cancel at scal

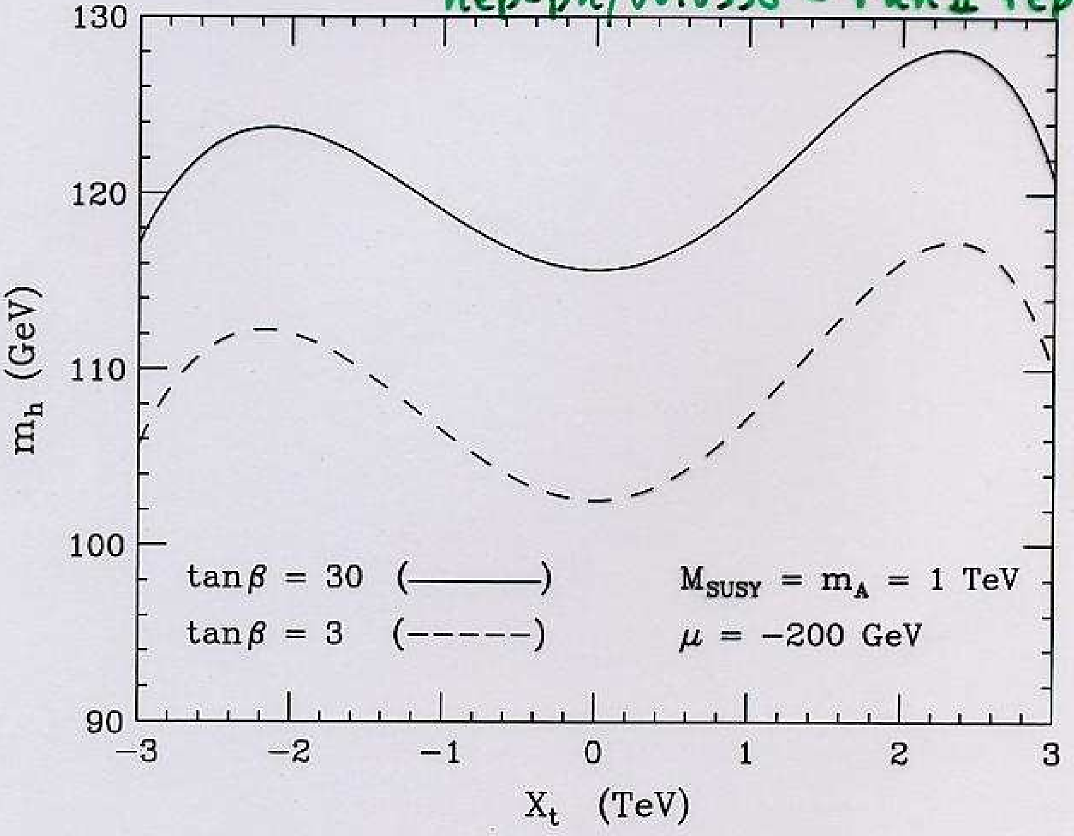
$$M_S^2 = \frac{1}{2}\left(m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2}\right)$$

upper bound (reached for large $\tan\beta$)

$$m_h^2 \leq m_Z^2 + \frac{3\, m_t^4}{2\pi^2 v^2}\left[\ln\frac{M_S^2}{m_t^2} + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12\, M_S^2}\right)\right]$$

include all 1-loop + leading 2-loop corrections:

hep-ph/0010338 = run II report

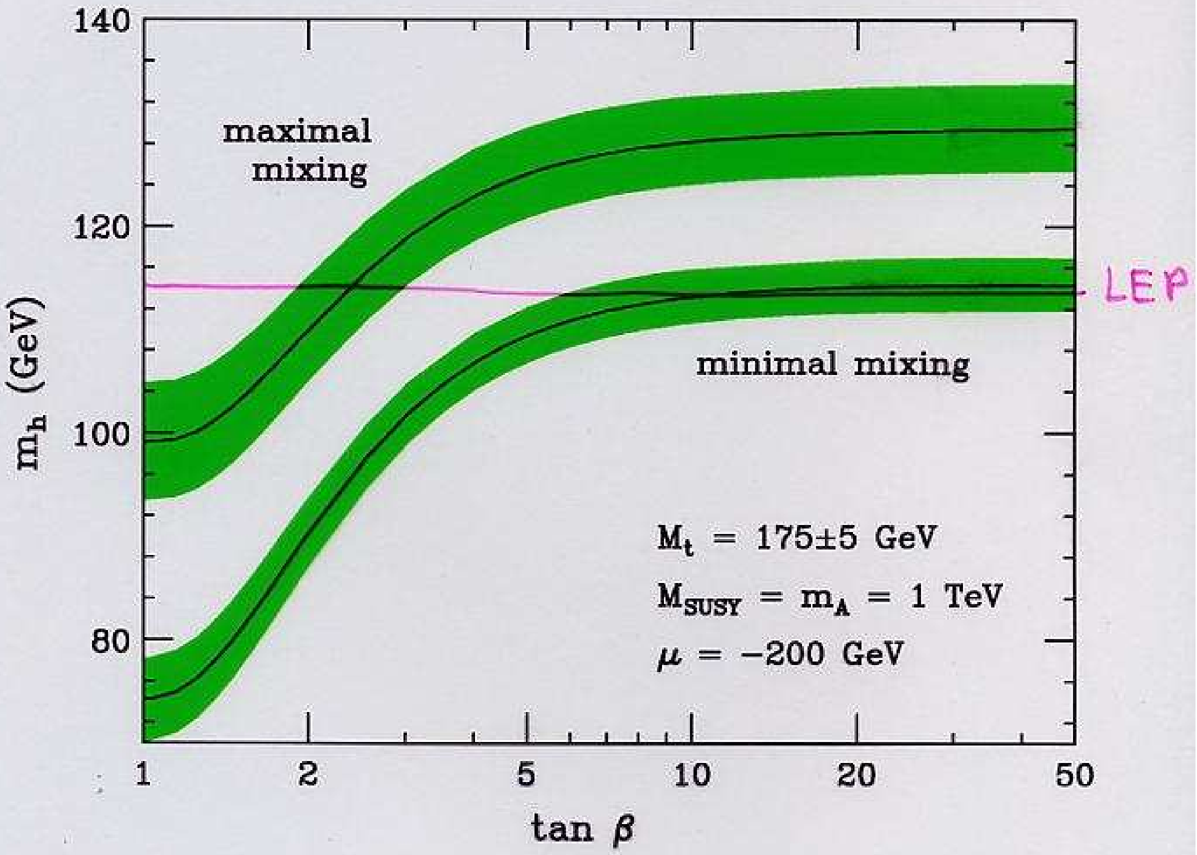


maximal mixing: $X_t = \sqrt{6}\, M_S$

minimal mixing: $X_t = 0$

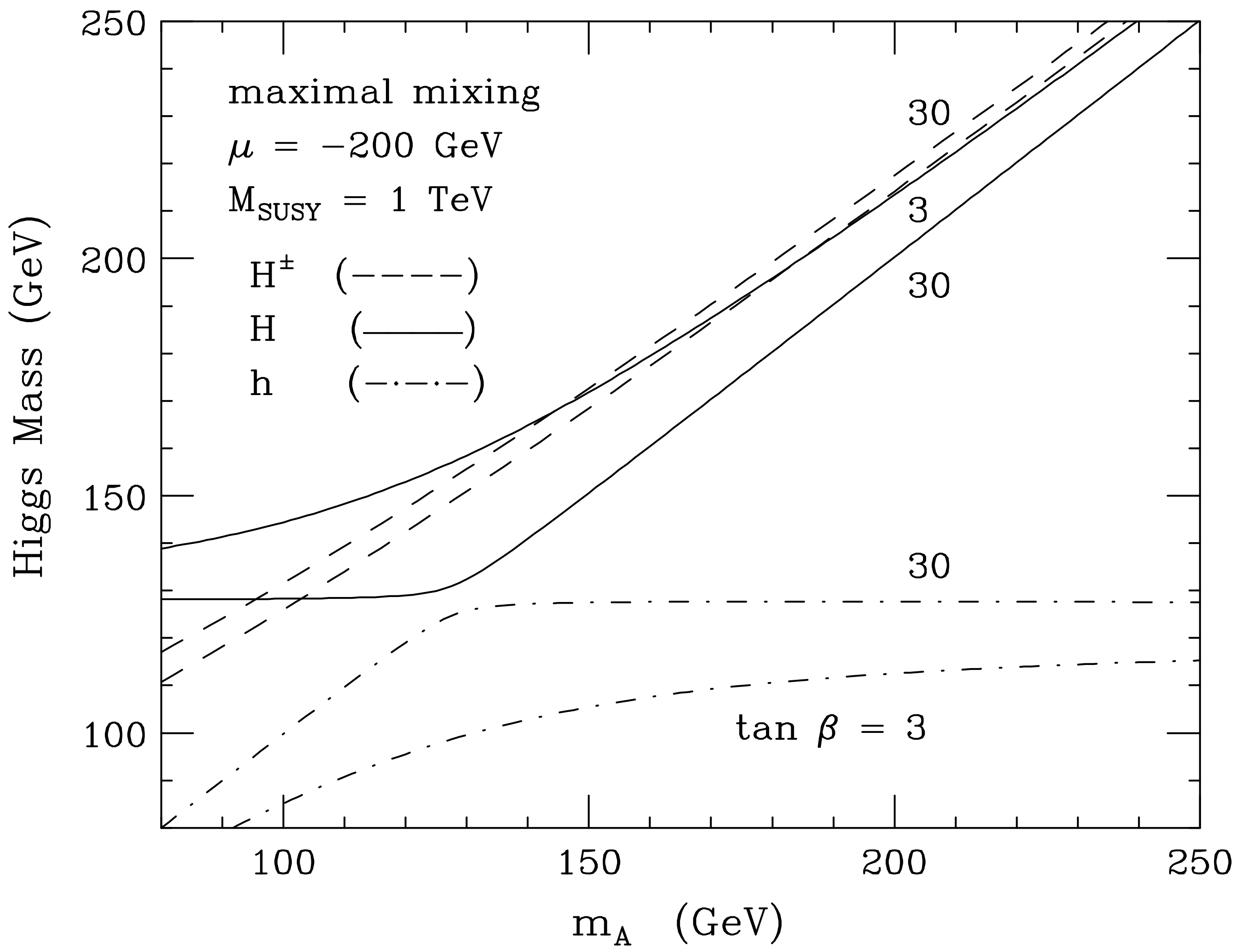
tan β dependence of m_h

green band: $m_t = 170 \ldots\ldots 180$ GeV



LEP limit of $m_h > 114$ GeV for SM Higgs

- rules out $\tan\beta \approx 1$
- favors large stop mixing



Lightest Higgs mass $m_h \lesssim 135$ GeV since quartic coupling is given by gauge couplings,

$$V_{quartic} = (g^2 + g'^2)/8 \left(\Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2\right)^2 + g^2/2 \; \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1$$

Higgs mixing and MSSM parameters

The Higgs potential leads to general mixing of the 2 doublet fields

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}[H^- \ \sin\beta - \chi^- \ \cos\beta] \\ v_1 + [H \ \cos\alpha - h \ \sin\alpha] + i[A \sin\beta + \chi_0 \cos\beta] \end{pmatrix} \to \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^- \ \sin\beta \\ v_1 + \varphi_1 + iA \sin\beta \end{pmatrix}$$

$$\Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 + [H \ \sin\alpha + h \ \cos\alpha] + i[A \cos\beta - \chi_0 \sin\beta] \\ \sqrt{2}[H^+ \ \cos\beta + \chi^+ \ \sin\beta] \end{pmatrix} \to \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 + \varphi_2 + iA \cos\beta \\ \sqrt{2}H^+ \ \cos\beta \end{pmatrix}$$

The angle β is determined by the VEVs:

$$v_1 = v \ \cos\beta \,, \qquad v_2 = v \ \sin\beta \,, \qquad \Longrightarrow \qquad \frac{v_2}{v_1} = \tan\beta$$

The mixing angle α between the 2 CP even scalars and the masses are determined by

$$\tan\beta \,, \qquad m_A \,, \qquad v = \sqrt{v_1^2 + v_2^2} = 246 \text{ GeV}$$

Coupling to gauge bosons

$$\begin{aligned}\mathcal{L} &= (D^\mu\Phi_1)^\dagger D_\mu\Phi_1 + (D^\mu\Phi_2)^\dagger D_\mu\Phi_2 \\ &= \frac{1}{2}|\partial_\mu\phi_1|^2 + \frac{1}{2}|\partial_\mu\phi_2|^2 + \left(\frac{g_Z^2}{8}Z_\mu Z^\mu + \frac{g^2}{4}W_\mu^+ W^{-\mu}\right)\left[(v_1+\varphi_1)^2 + (v_2+\varphi_2)^2\right] \quad + \quad \ldots\end{aligned}$$

The $v_1^2 + v_2^2 = v^2$ term gives same masses to W, Z as in the SM

$$m_W^2 = \frac{g^2 v^2}{4} \qquad\qquad m_Z^2 = \frac{(g^2+g'^2)\,v^2}{4} = \frac{m_W^2}{\cos^2\theta_W}$$

The couplings to the gauge bosons arise from

$$2v_1\varphi_1 + 2v_2\varphi_2 = 2v\,[\ H\cos(\beta-\alpha) + h\sin(\beta-\alpha)\ ]$$

$\Longrightarrow$ extra coupling factors for hVV and HVV couplings as compared to SM

$$hVV \sim \sin(\beta-\alpha) \qquad\qquad HVV \sim \cos(\beta-\alpha)$$

At tree level

$$\cos^2(\beta-\alpha) = \frac{m_h^2\,(m_Z^2 - m_h^2)}{m_A^2\,(m_H^2 - m_h^2)}$$

$$\longrightarrow \frac{m_Z^4 \sin^2 4\beta}{4\, m_A^4} \quad \text{for large } m_A$$

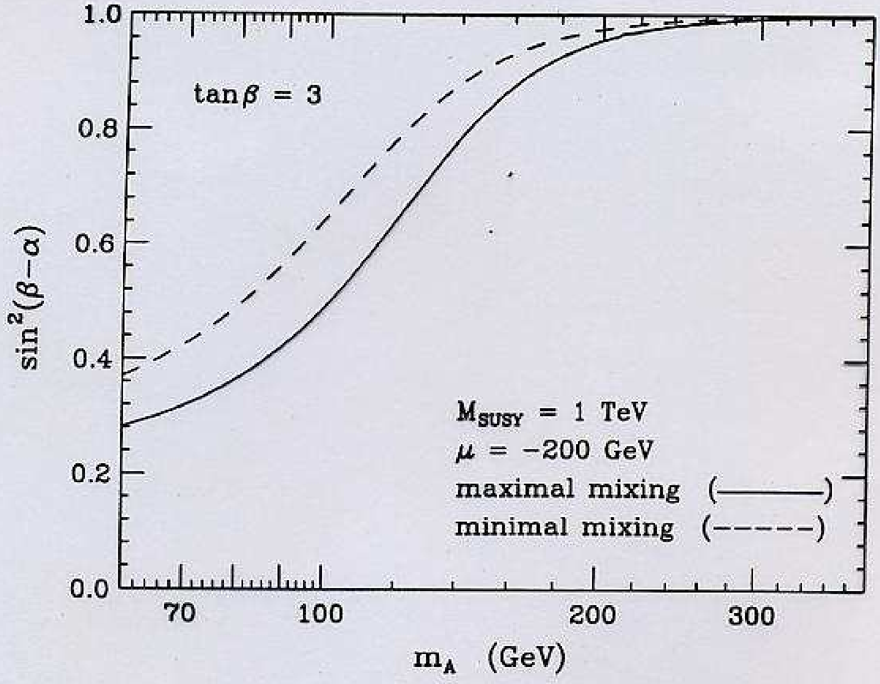
$$\Rightarrow \left.\begin{array}{l} \cos(\beta-\alpha) \to 0 \\ \sin(\beta-\alpha) \to 1 \end{array}\right\} \text{ for large } m_A$$

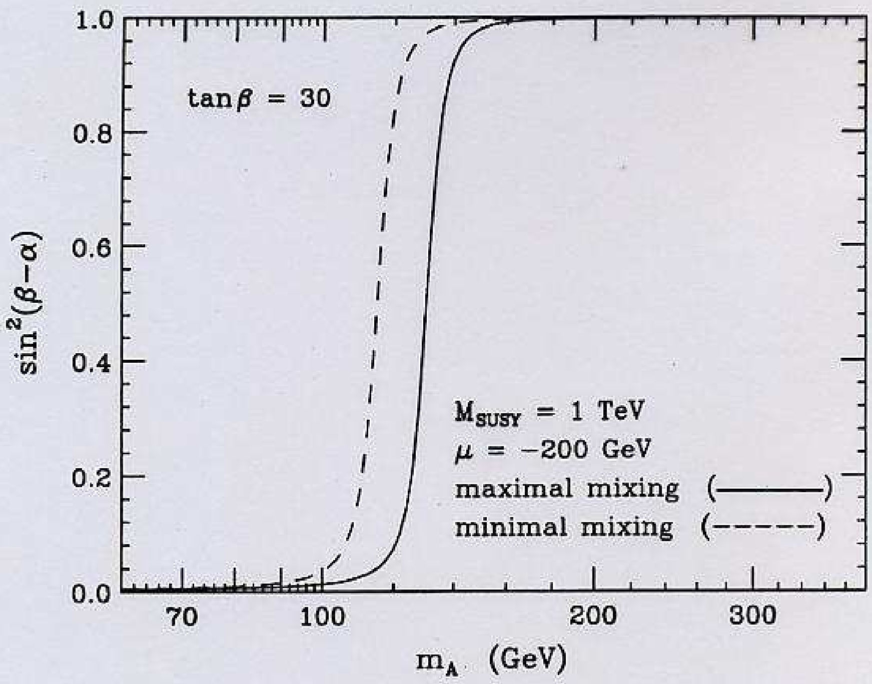
$\left.\begin{array}{l} H \text{ decouples from } W, Z \\ h \text{ couples like SM} \end{array}\right\}$ as $m_A \to \infty$

Radiative corrections: no significant change of tree level result for decoupling.

radiative corrections included

run II report





Coupling to fermions

$$\mathcal{L}_{\text{Yuk.}} = -\Gamma_b \bar{b}_L \Phi_1^0 b_R - \Gamma_t \bar{t}_L \Phi_2^0 u_R + \text{h.c.}$$

$$= -\Gamma_b \bar{b}_L \frac{v_1 + H\cos\alpha - h\sin\alpha + iA\,\sin\beta}{\sqrt{2}} b_R - \Gamma_t \bar{t}_L \frac{v_2 + H\sin\alpha + h\cos\alpha + iA\,\cos\beta}{\sqrt{2}} t_R + \text{h.c.}$$

The v_1, v_2 terms are the fermion masses

$$m_b = \frac{\Gamma_b v_1}{\sqrt{2}} \qquad m_t = \frac{\Gamma_t v_2}{\sqrt{2}} \qquad \Longrightarrow \qquad \frac{\Gamma_b}{\sqrt{2}} = \frac{m_b}{v\cos\beta} \qquad \frac{\Gamma_t}{\sqrt{2}} = \frac{m_t}{v\sin\beta}$$

Expressed in terms of masses the Yukawa Lagrangian is

$$\mathcal{L}_{\text{Yuk.}} = -\frac{m_b}{v}\bar{b}\left(v + H\frac{\cos\alpha}{\cos\beta} - h\frac{\sin\alpha}{\cos\beta} - i\gamma_5 A\,\tan\beta\right)b - \frac{m_t}{v}\bar{t}\left(v + H\frac{\sin\alpha}{\sin\beta} + h\frac{\cos\alpha}{\sin\beta} - i\gamma_5 A\,\cot\beta\right)t$$

$\Longrightarrow$ coupling factors compared to SM hff coupling $-i\, m_f/v$

Decoupling limit for fermions

Consider limit $\quad \sin(\beta-\alpha)\to 1, \quad \cos(\beta-\alpha)\to 0$

- hbb, $h\tau\tau$:

$$-\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\,\cos(\beta-\alpha) \to 1$$

- htt:

$$\frac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) + \frac{\cos(\beta-\alpha)}{\tan\beta} \to 1$$

- Hbb, $H\tau\tau$:

$$\frac{\cos\alpha}{\cos\beta} = \cos(\beta-\alpha) + \tan\beta\,\sin(\beta-\alpha) \to \tan\beta$$

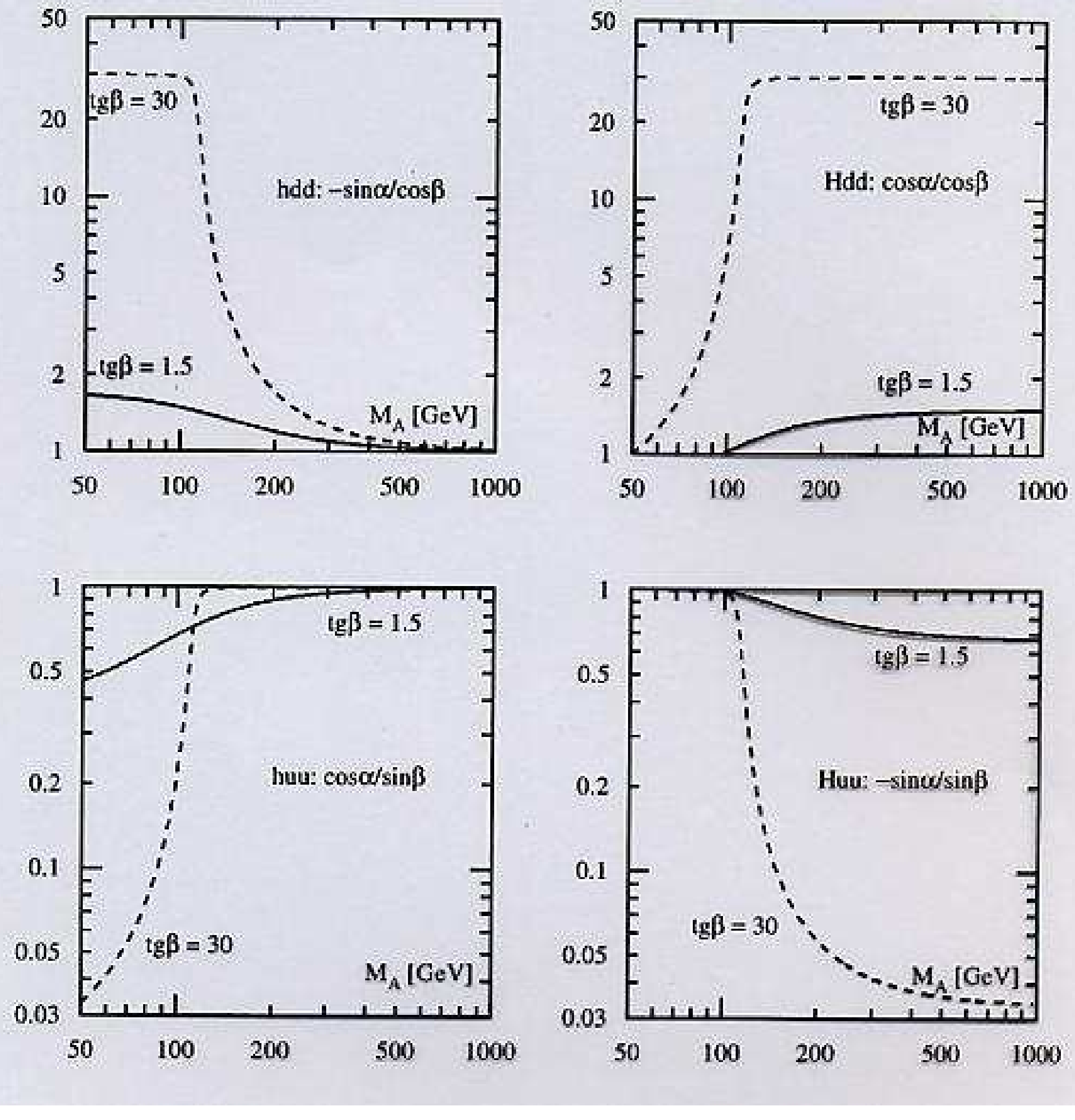
- Htt:

$$\frac{\sin\alpha}{\sin\beta} = \cos(\beta-\alpha) - \frac{\sin(\beta-\alpha)}{\tan\beta} \to \frac{-1}{\tan\beta}$$

In the large m_A regime

- light h couplings to fermions approach SM values
- $h\bar{b}b$ (and $A\bar{b}b$, $H/A\tau\tau$) couplings are enhanced $\sim \tan\beta$ $\Longrightarrow$ large cross sections at LHC

Spira, hep-ph/9705337



Higgs phenonomenology

Importance of decoupling limit in MSSM (large m_A) $\Longrightarrow$ Concentrate on SM case

Higgs couples to fermions and gauge bosons proportional to their mass $\Longrightarrow$
Heavy SM particles are involved in both production and decay processes

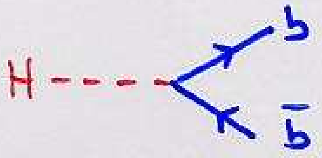
$$W, Z, t, b, \tau$$

Consider

- Higgs decay: total width and decay branching fractions
- Production cross sections at LHC
- Signatures and backgrounds
- Measurement of Higgs couplings

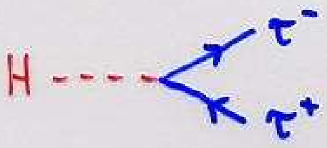
Main Higgs decay channels

$H \to b\bar{b}$



$m_H \lesssim 150$ GeV

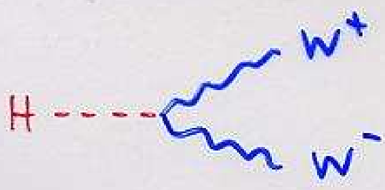
$H \to \tau^+\tau^-$



$m_H \lesssim 140$ GeV

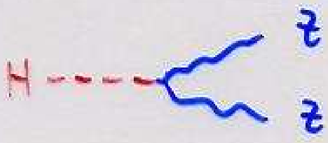
and into gauge bosons

$H \to W^+W^-$



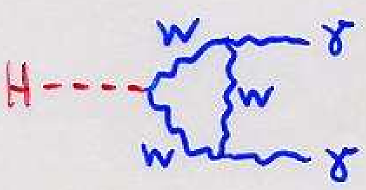
$m_H \gtrsim 120$ GeV

$H \to ZZ$



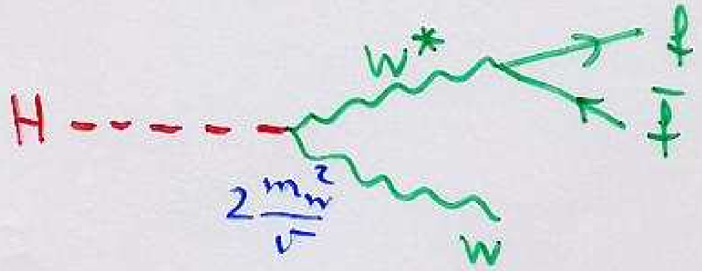
$m_H \gtrsim 120/180$ GeV

$H \to \gamma\gamma$



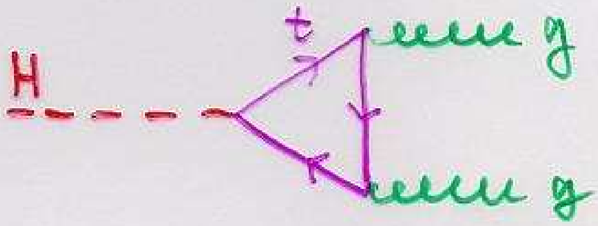
$m_H \lesssim 150$ GeV

For $m_H \gtrsim 110$ GeV : $H \to WW^*$

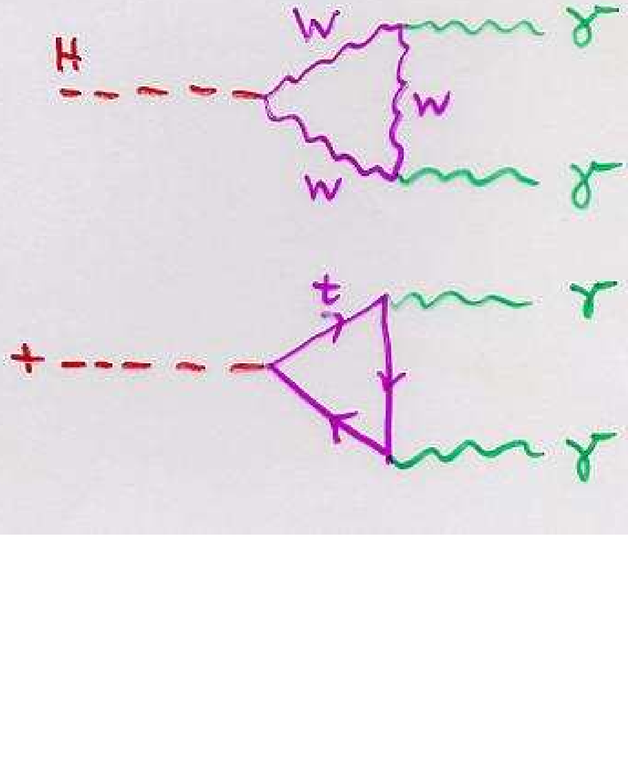


Loop decays

$H \to gg$

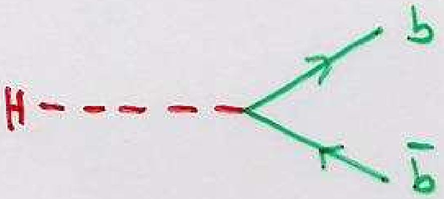


$H \to \gamma\gamma$



Higgs decays

For $m_H \lesssim 135$ GeV, $H \to b\bar{b}$ dominate



$$-i\,\frac{m_b}{v}$$

$$\Gamma(H\to b\bar{b}) = 3\,\frac{m_H}{8\pi}\left(\frac{\overline{m}_b(m_H)}{v}\right)^2 \beta^3\left(1+\frac{17}{3}\frac{\alpha_s}{\pi}+\dots\right.$$

QCD radiative corrections are important

- Use running b mass $\overline{m}_b(m_H)$

$$\overline{m}_b(m_H = 100\text{ GeV}) \approx 2.9\text{ GeV} \approx 0.69\,\overline{m}_b(m_b$$

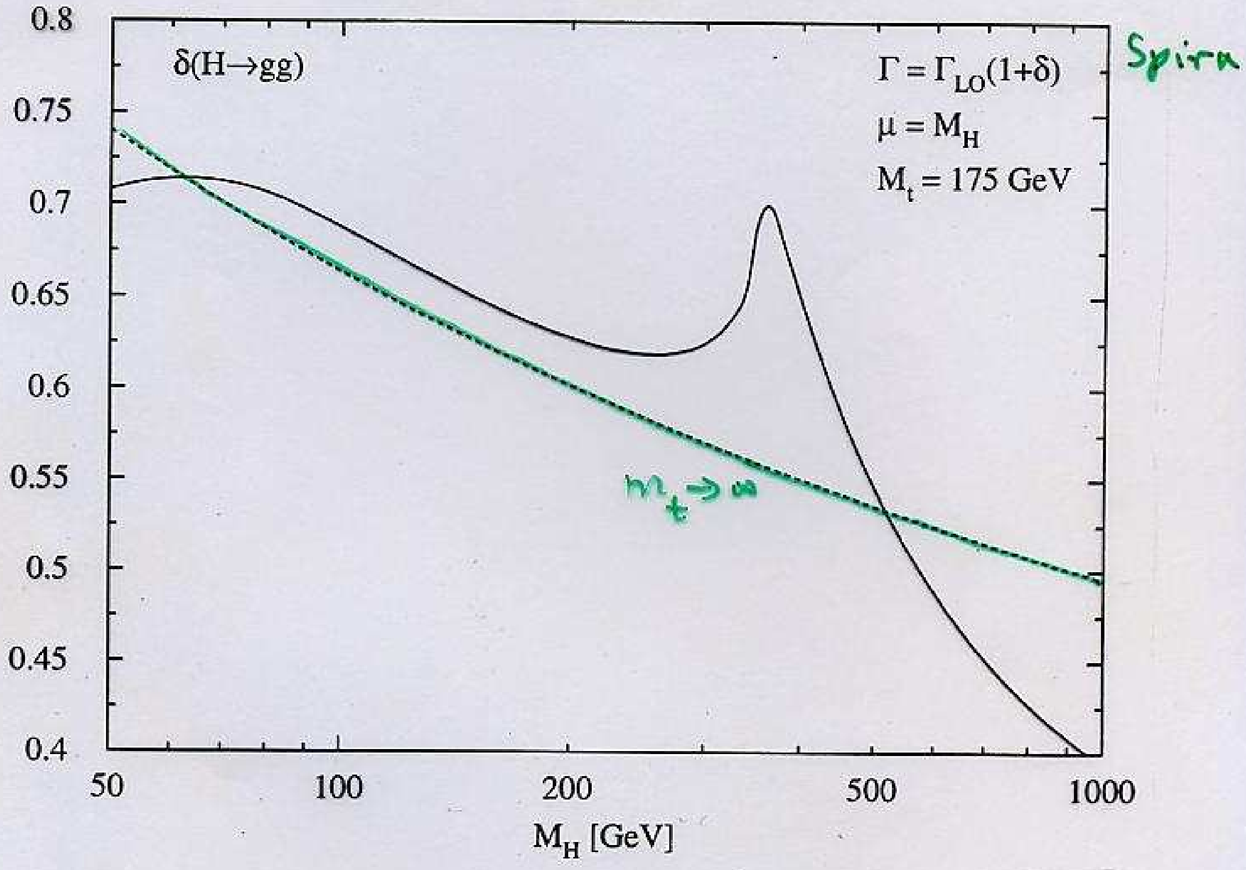
- include 2 loop QCD corrections

f	m_f	$m_f(100\text{ GeV})$
b	4.7 GeV	2.92 GeV
c	1.2 GeV	0.62 GeV
τ	1.8 GeV	1.8 GeV

$\Gamma(H\to c\bar{c}) < \Gamma(H\to\tau\tau$

NLO QCD corrections to $\Gamma(H \to gg)$

$$\Gamma(H \to gg, q\bar{q}g) = \Gamma_{LO}(H \to gg)\,(1+\delta)$$

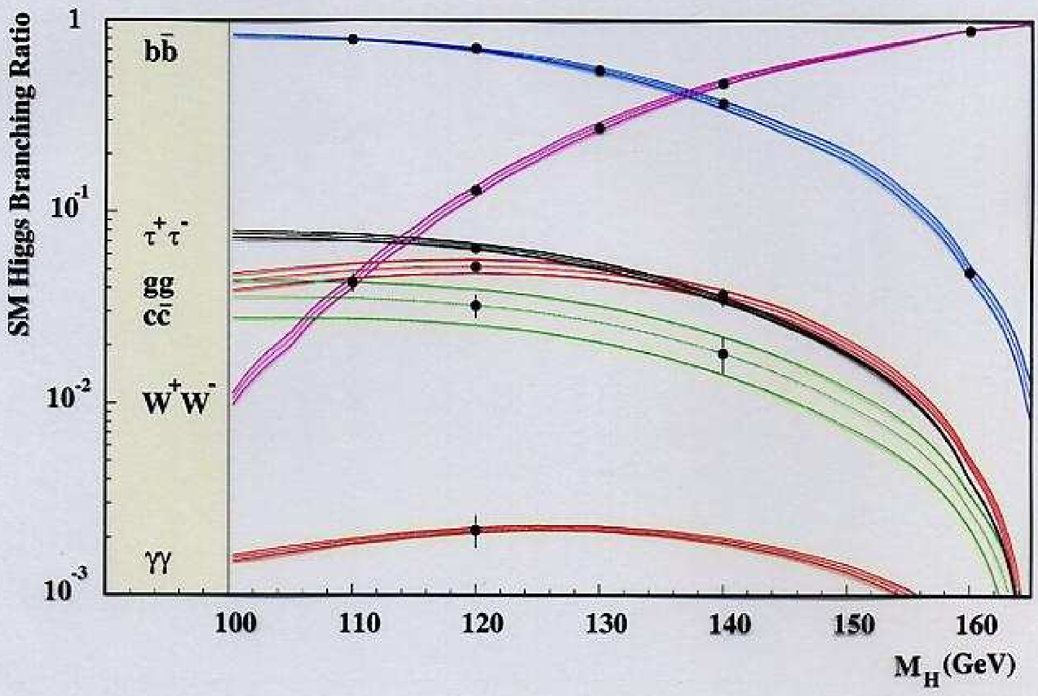


Radiative corrections for various decay modes implemented in HDECAY

Djouadi, Kalinowski, Spira, hep-ph/9704448

Continuously updated for SM & MSSM

Present theoretical accuracy



Example: $M_H = 120$ GeV

Decay mode:	$b\bar{b}$	WW^*	$\tau^+\tau^-$	$c\bar{c}$	gg	$\gamma\gamma$
Theory	1.4%	2.3%	2.3%	23%	5.7%	2.3%

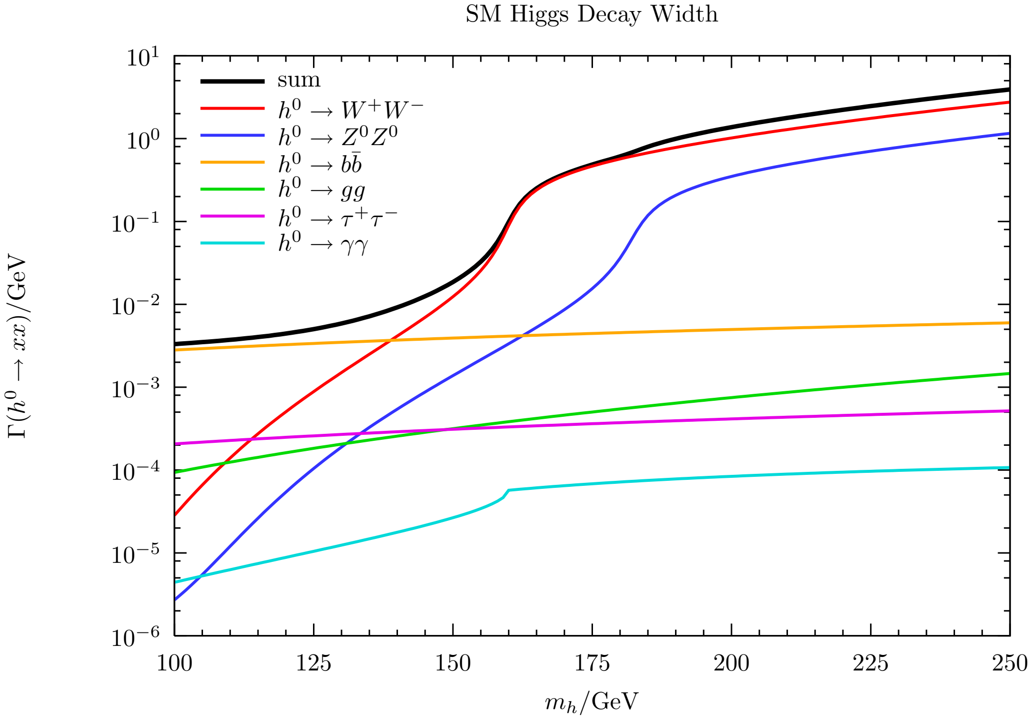
Mainly due to: pole masses m_c and m_b, and $\alpha_s(\mu)$.

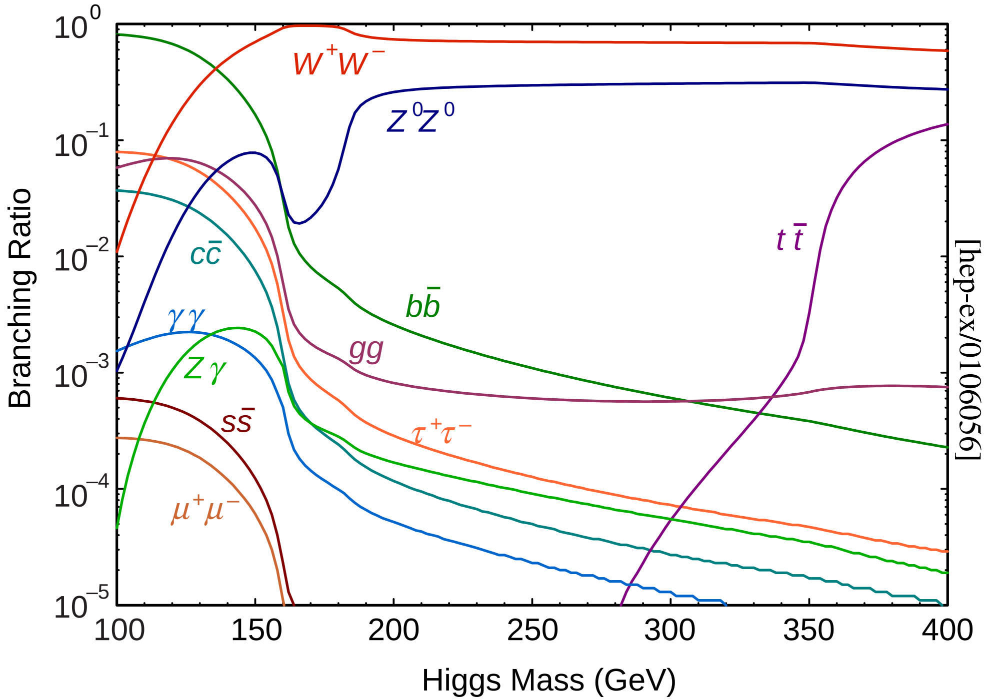
From HDECAY when (Carena et al., hep-ph/0106116)

$$\alpha_s(M_Z) = 0.1185 \pm 0.0020$$
$$m_c(m_c) = 1.23 \pm 0.09 \text{ GeV}$$
$$m_b(m_b) = 4.17 \pm 0.05 \text{ GeV}$$

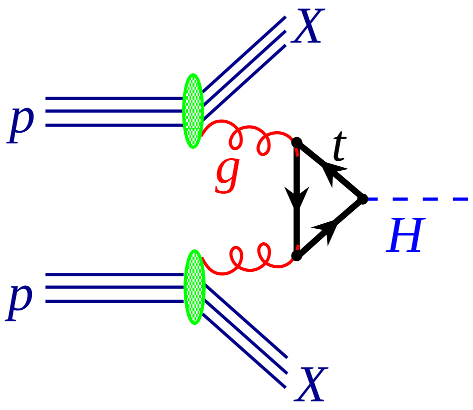
Decay of the SM Higgs

Higgs decay width and branching fractions within the SM

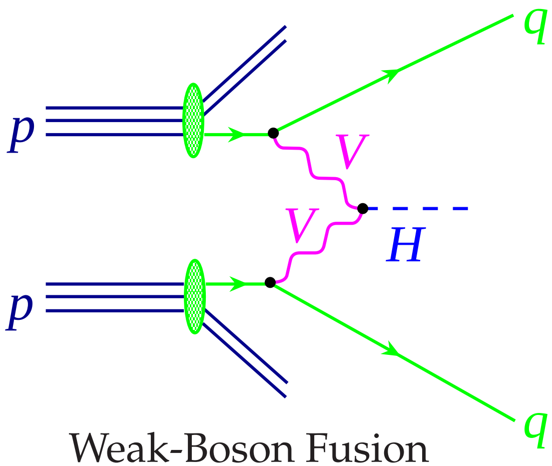




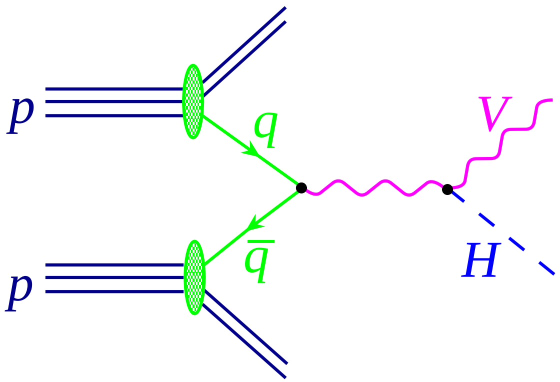
Higgs Production Modes at Hadron Colliders



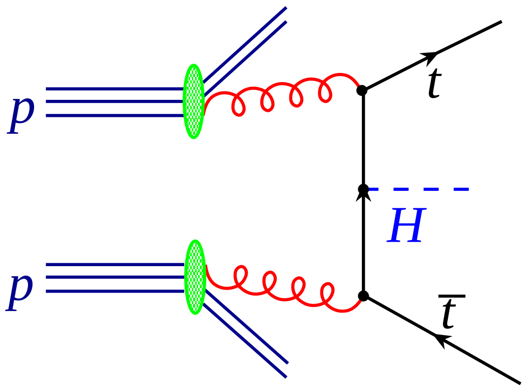
Gluon fusion



Weak-Boson Fusion

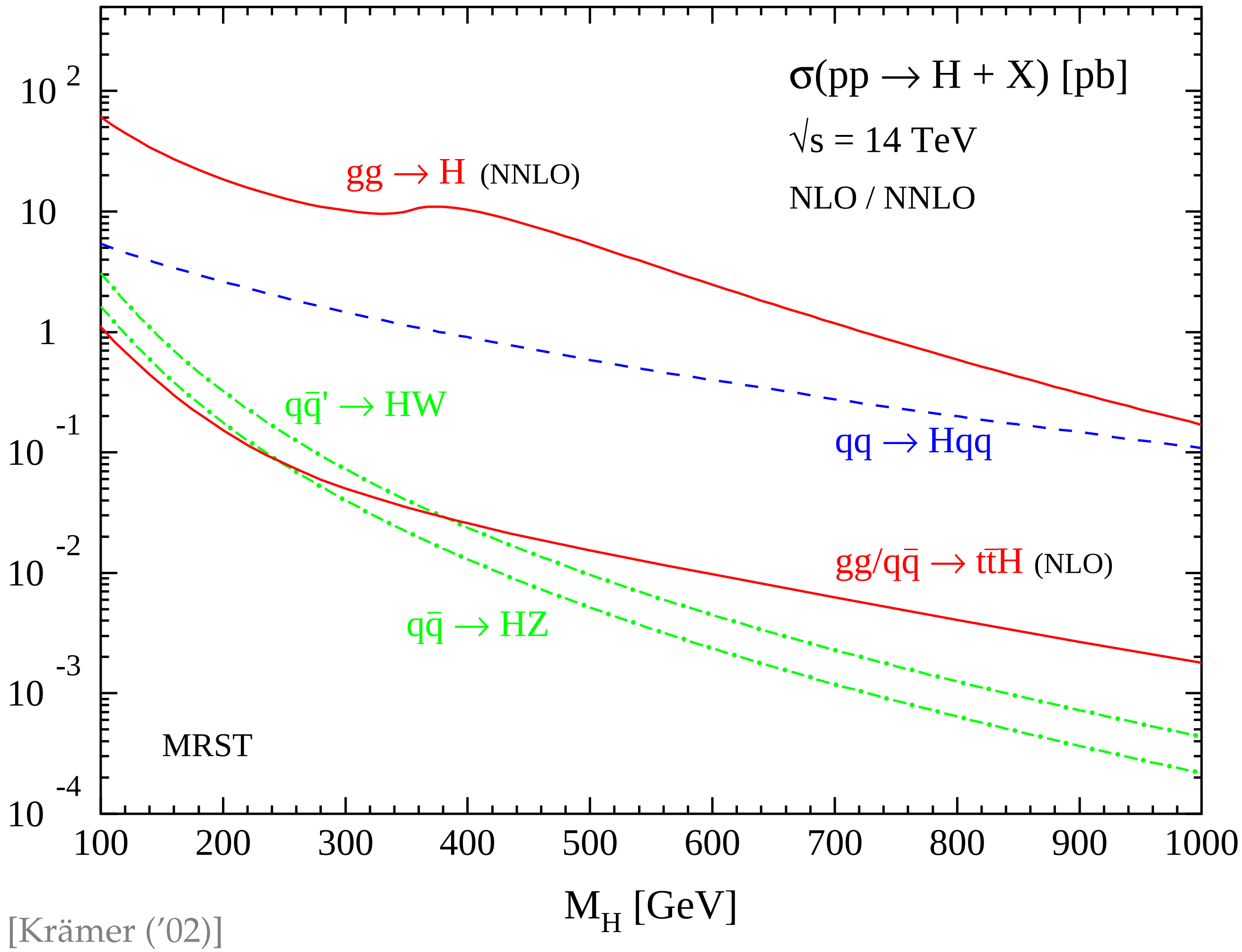


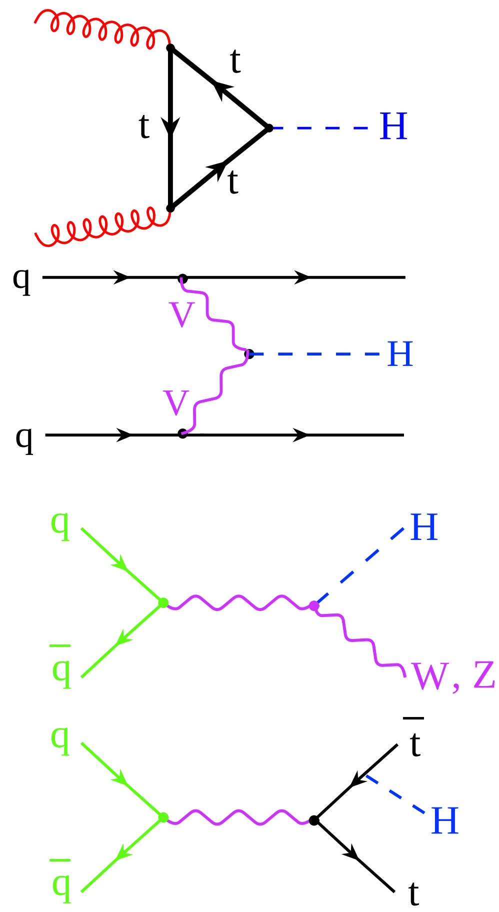
Higgs Strahlung



$t\bar{t}H$

Total SM Higgs cross sections at the LHC



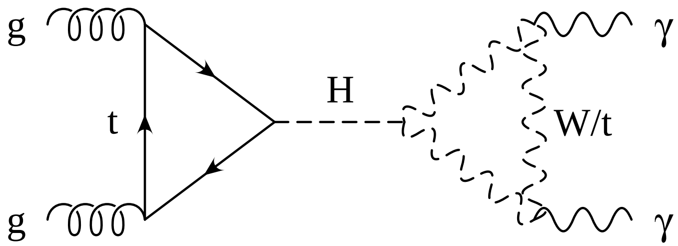


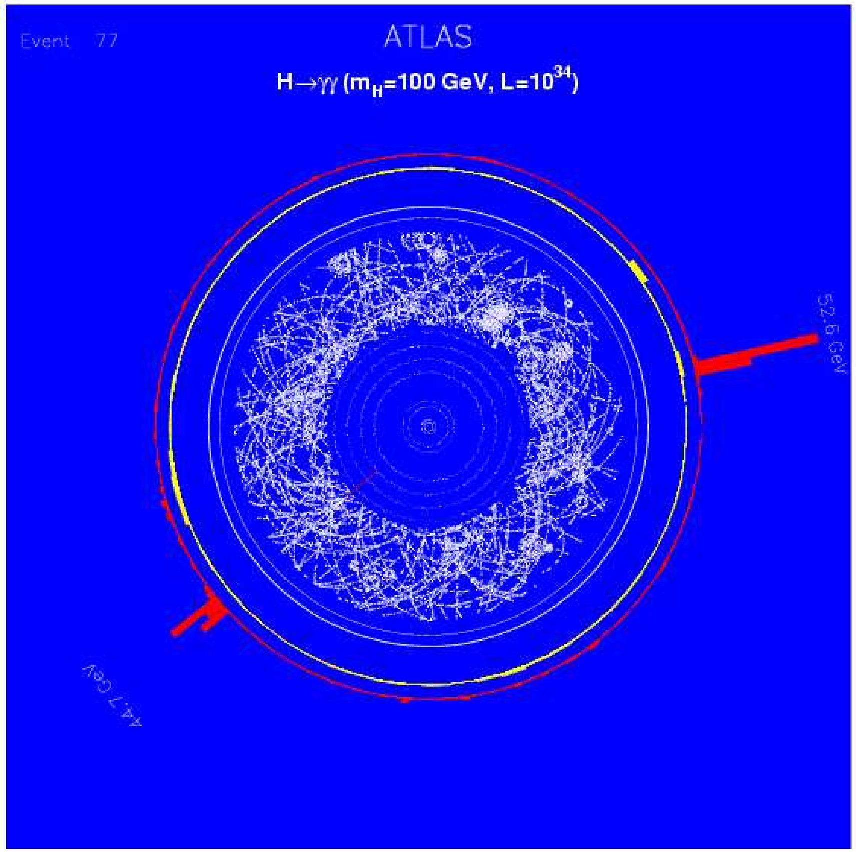
[Krämer ('02)]

Inclusive search channels

- inclusive search for
$$H \to \gamma\gamma$$
invariant-mass peak, for $m_H < 150$ GeV
- inclusive search for
$$H \to ZZ^* \to \ell^+\ell^-\ell^+\ell^-$$
for $m_H \geq 130$ GeV and $m_H \neq 2m_W$.
- inclusive search for
$$H \to W^+W^- \to \ell^+\bar{\nu}\ell^-\nu$$
for $140\ \text{GeV} \leq m_H \leq 200$ GeV

$H \longrightarrow \gamma\gamma$





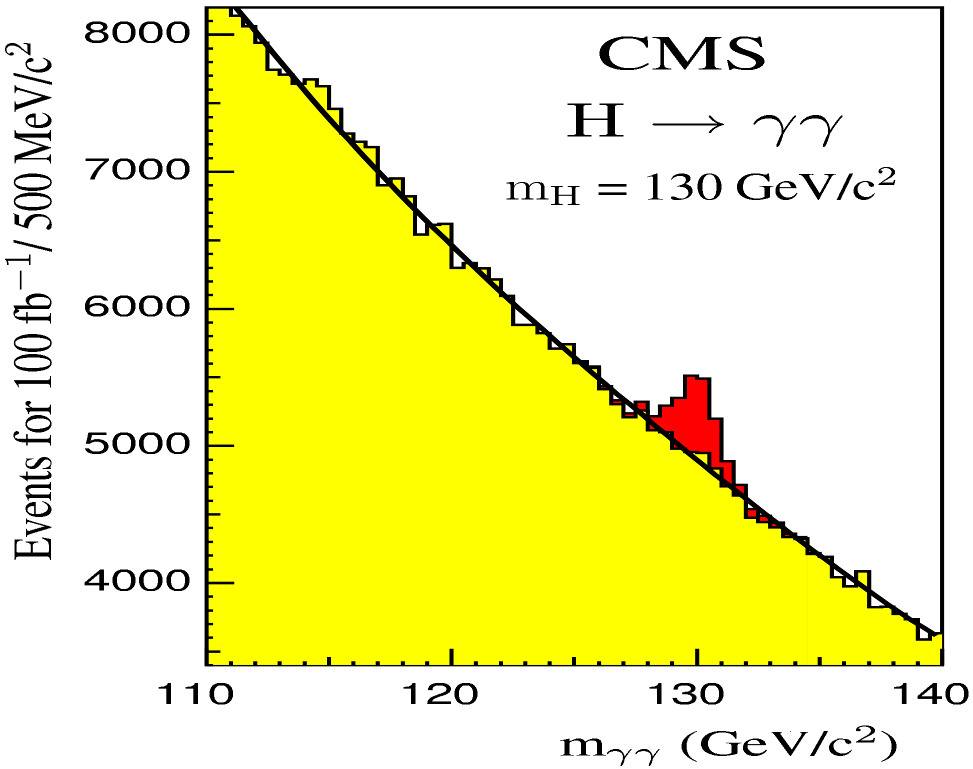
- ✗ $\mathrm{BR}(H \to \gamma\gamma) \approx 10^{-3}$
- ✗ large backgrounds from $q\bar{q} \to \gamma\gamma$ and $gg \to \gamma\gamma$
- ✓ but CMS and ATLAS will have excellent photon-energy resolution (order of 1%)

Look for two isolated photons.

$H \longrightarrow \gamma\gamma$

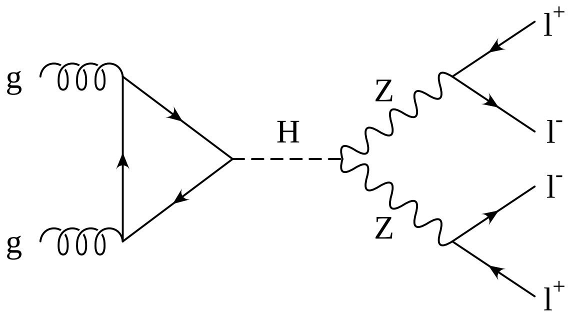
✓ Look for a narrow $\gamma\gamma$ invariant mass peak

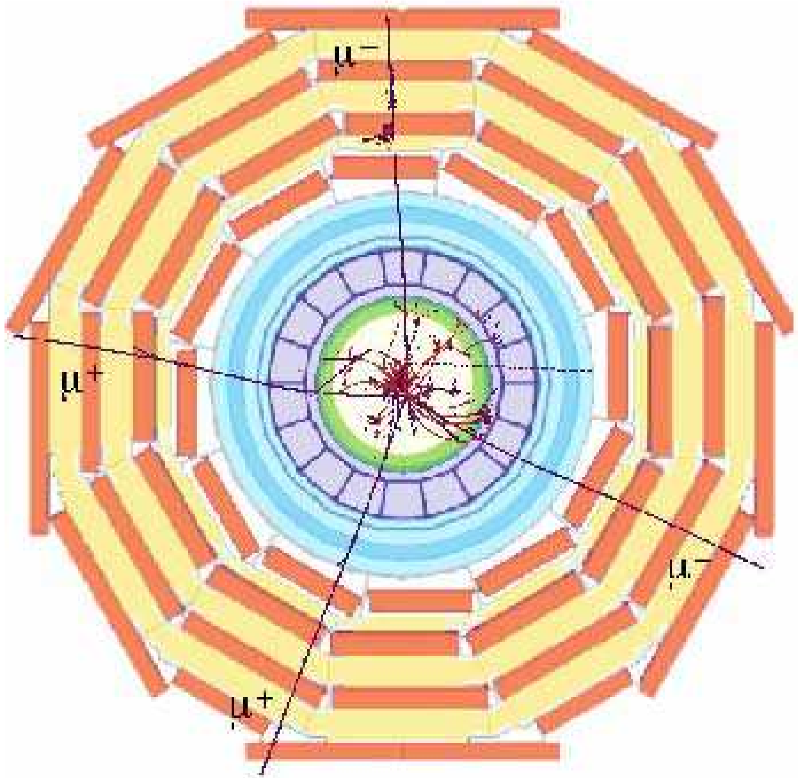
✓ extrapolate background into the signal region from sidebands.



$H \rightarrow ZZ \rightarrow \ell^+\ell^-\ell^+\ell^-$

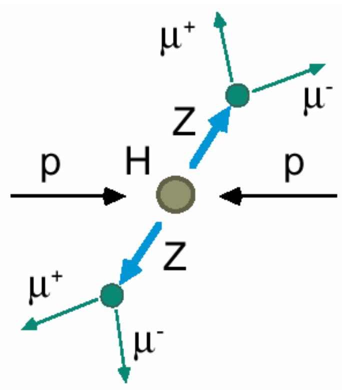
The gold-plated mode

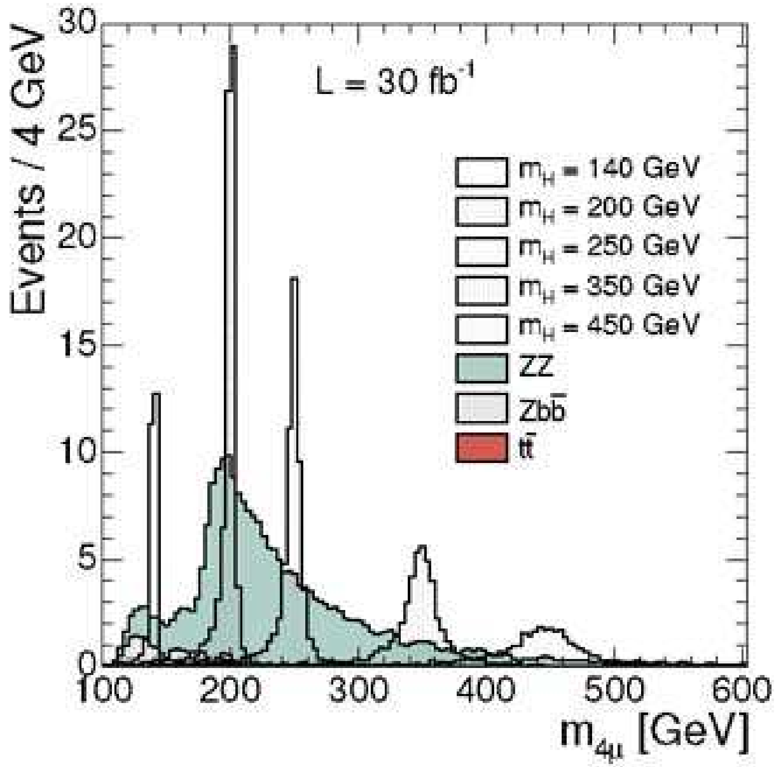




- ✓ This is the most important and clean search mode for $2m_Z < m_H < 600$ GeV.
- ✓ continuum, limited, irreducible background from $q\bar{q} \rightarrow ZZ$
- ✗ small BR$(H \rightarrow \ell^+\ell^-\ell^+\ell^-) \approx 0.15\%$ (even smaller when $m_H < 2m_Z$)

$H \to ZZ \to \ell^+\ell^-\ell^+\ell^-$





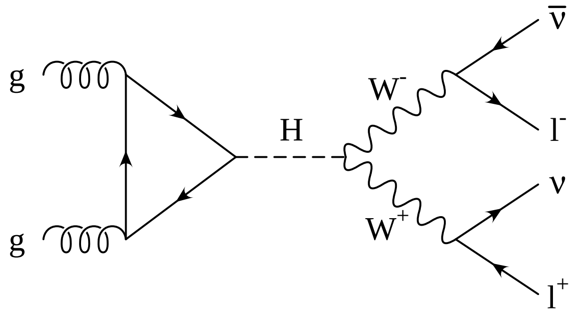
✓ invariant mass of the charged leptons fully reconstructed

For $m_H \approx 0.6$–1 TeV, use the "silver-plated" mode $H \to ZZ \to \nu\bar{\nu}\ell^+\ell^-$

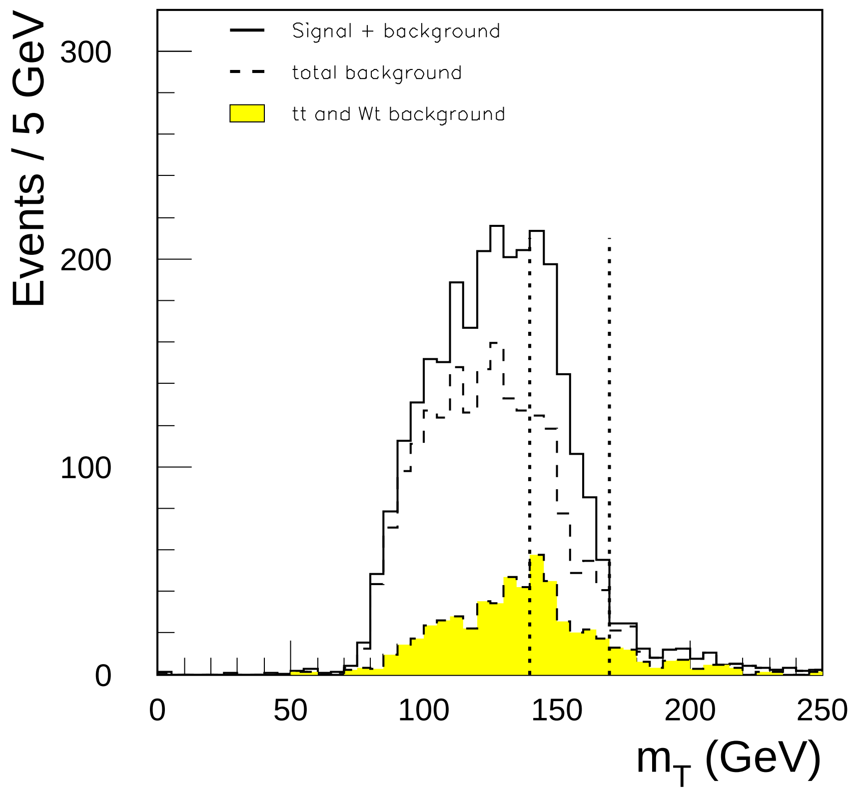
✓ $\mathrm{BR}(H \to \nu\bar{\nu}\ell^+\ell^-) = 6\ \mathrm{BR}(H \to \ell^+\ell^-\ell^+\ell^-)$

✓ the large E_T missing allows a measurement of the transverse mass

$H \longrightarrow WW \longrightarrow \ell^+ \bar{\nu} \ell^- \nu$



ATLAS TDR



$m_H = 170$ GeV

integrated luminosity = 20 fb^{-1}

- ✓ Exploit $\ell^+\ell^-$ angular correlations
- ✓ measure the transverse mass with a Jacobian peak at m_H

$$m_T = \sqrt{2\, p_T^{\ell\ell}\, \not{E}_T\, (1 - \cos(\Delta\Phi))}$$

- ✗ background and signal have similar shape $\Longrightarrow$ must know the background normalization precisely

Associated production search channels

- $pp \to t\bar{t}H \to t\bar{t}b\bar{b}$
  for $m_H < 120$–130 GeV

- $q\bar{q} \to WH\ ,\ ZH$ New and improved: Butterworth, Davison, Rubin, Salam, arXiv:0802.2470
  trigger on leptonic decay of W or Z, look for $H \to b\bar{b}$

New idea for WH and ZH associated production: concentrate on high $p_T(H) \gtrsim 200$ GeV:

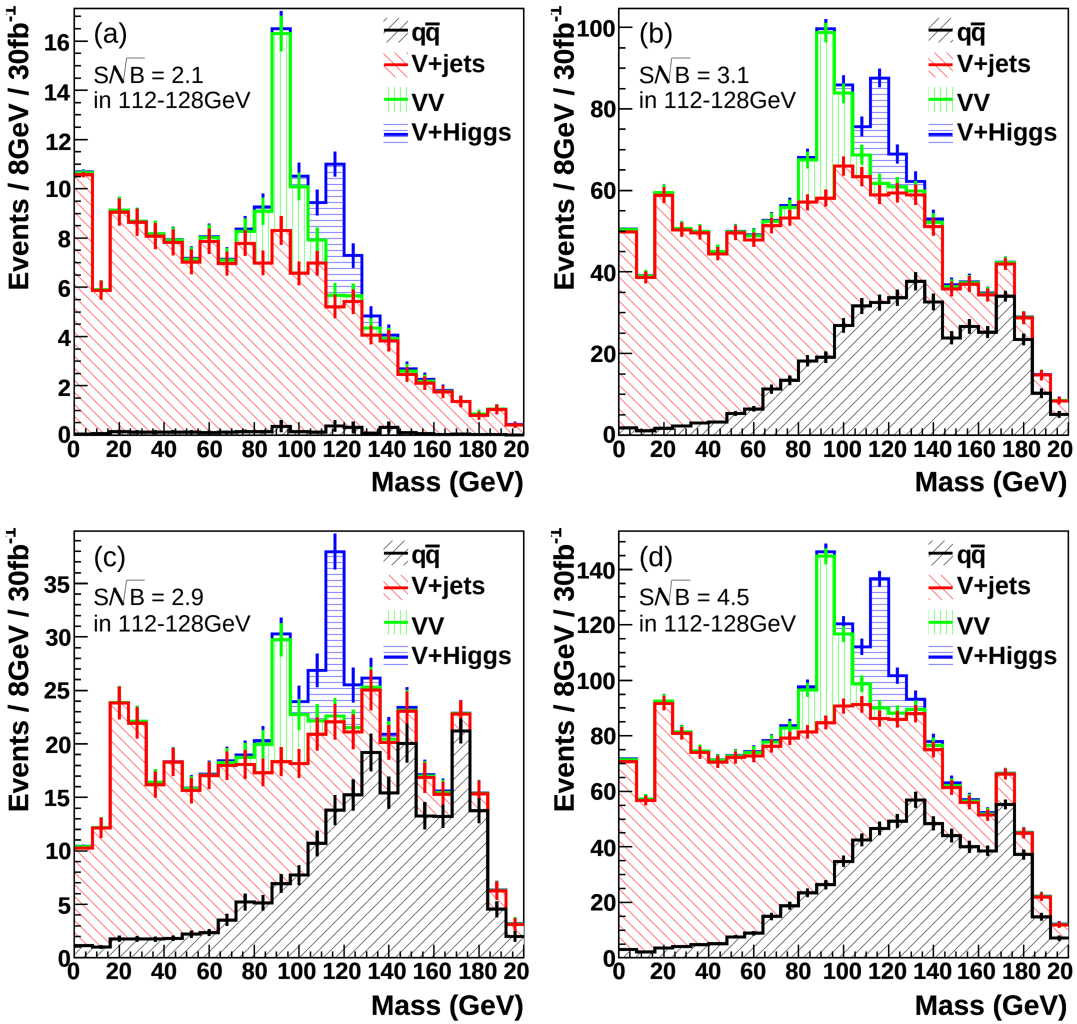
- $\Longrightarrow$ fat Higgs jet with $b\bar{b}(g)$ subjet structure
- small separation of b-quark jets from $H \to bb$ decay $\Longrightarrow$ better $b\bar{b}(g)$ invariant mass resolution
- lower background fraction than at low $p_T(H)$

Expected signal in HZ and HW at $p_T(H) > 200$ GeV

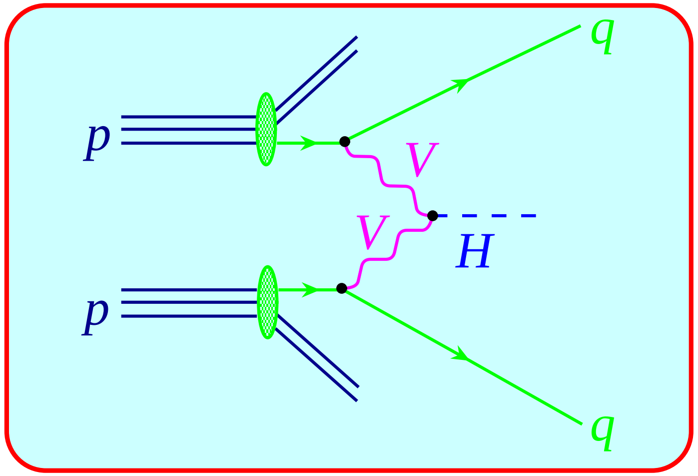
Example: $m_H = 120$ GeV, $\int Ldt = 30\ \text{fb}^{-1}$

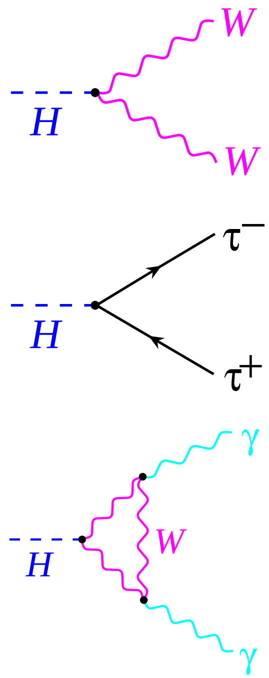
- Need excellent b tagging and non-b rejection efficiencies (assumed: 60% and 2% respectively)
- Search in
  (a) HZ with $Z\to ll$
  (b) HZ with $Z\to \nu\nu$ and
  (c) $WH\to l\nu b\bar{b}$ samples
- Promising signal with $30\ \text{fb}^{-1}$ when combining all 3 channels

Detailed studies with full detector simulation on the way



Vector Boson Fusion





$m_H > 120$ GeV

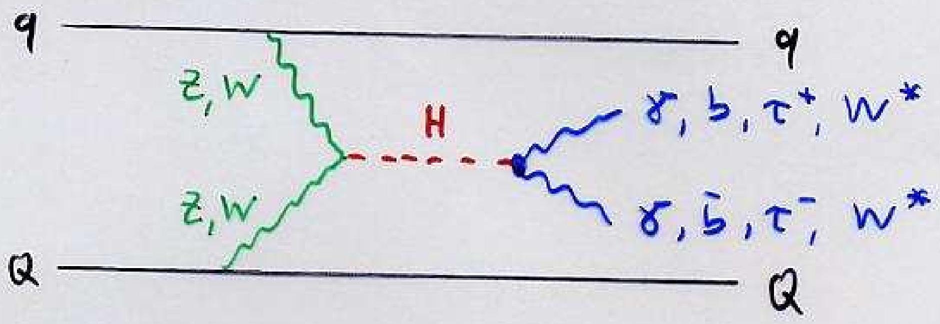
$m_H < 140$ GeV

$m_H < 150$ GeV

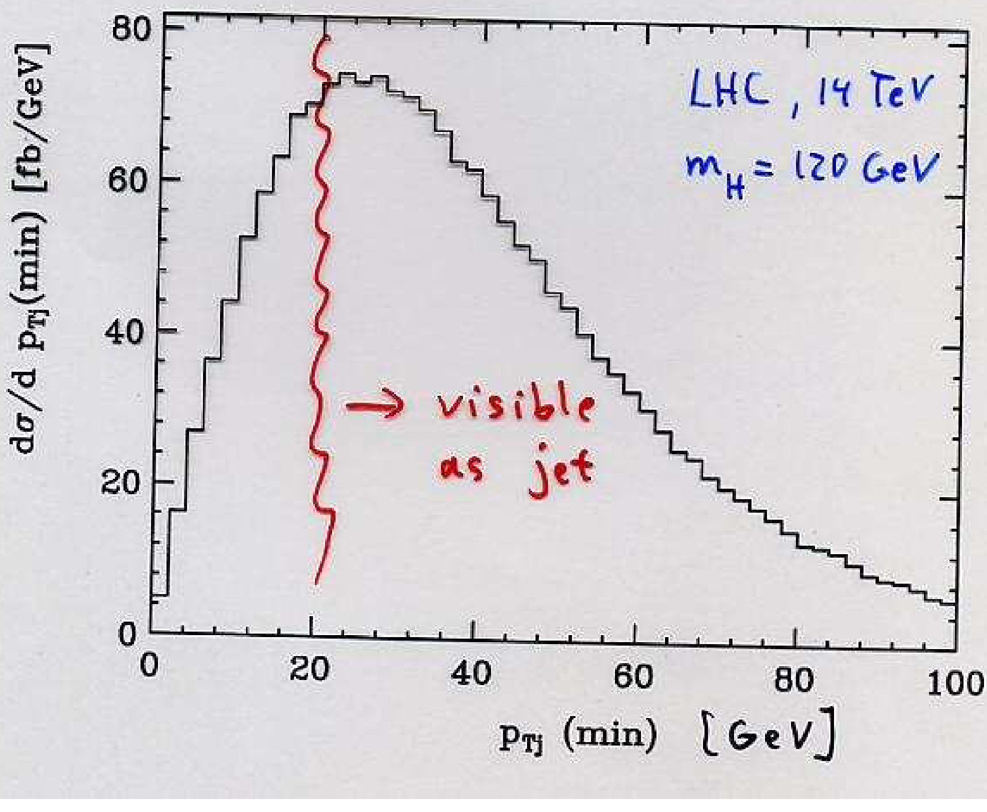
[Eboli, Hagiwara, Kauer, Plehn, Rainwater, D.Z. . . .]

Most measurements can be performed at the LHC with statistical accuracies on the measured cross sections times decay branching ratios, $\sigma \times$ BR, of order 10% (sometimes even better).

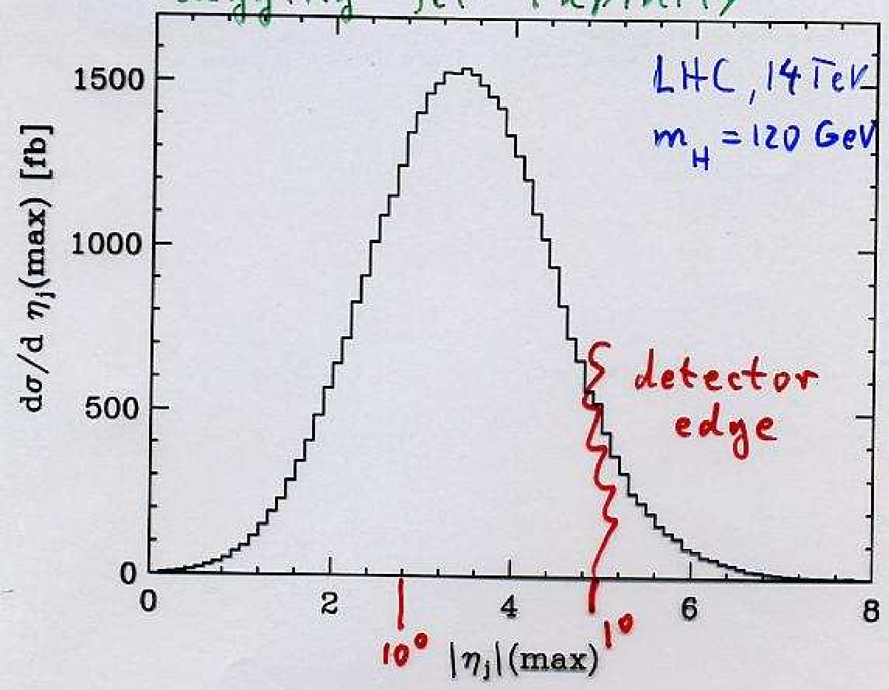
Characteristics of weak boson fusio



- scattered quarks lead to 2 forward tagging jets [Cahn, Kleiss, Stirlin



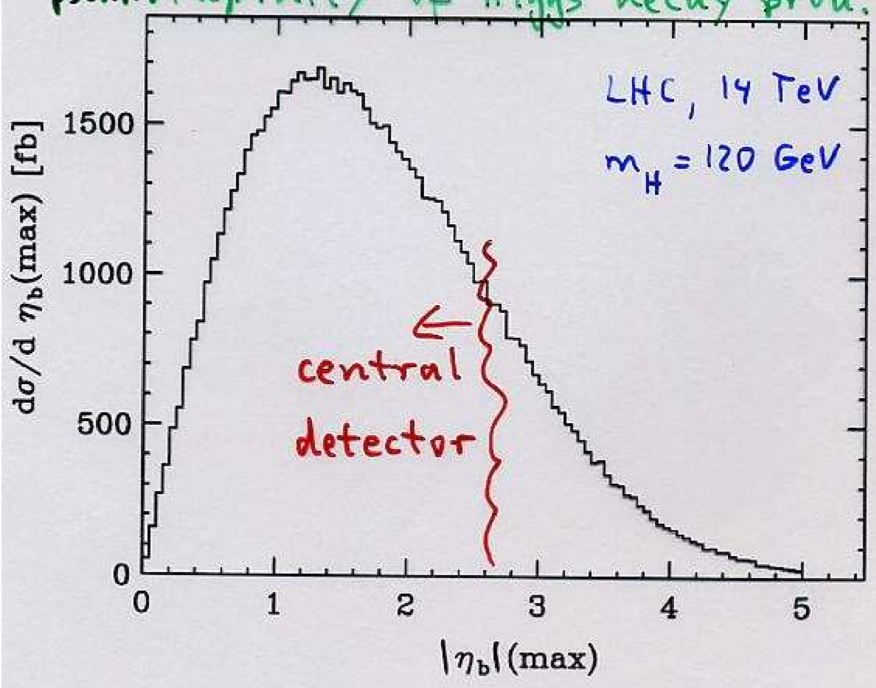
tagging jet rapidity



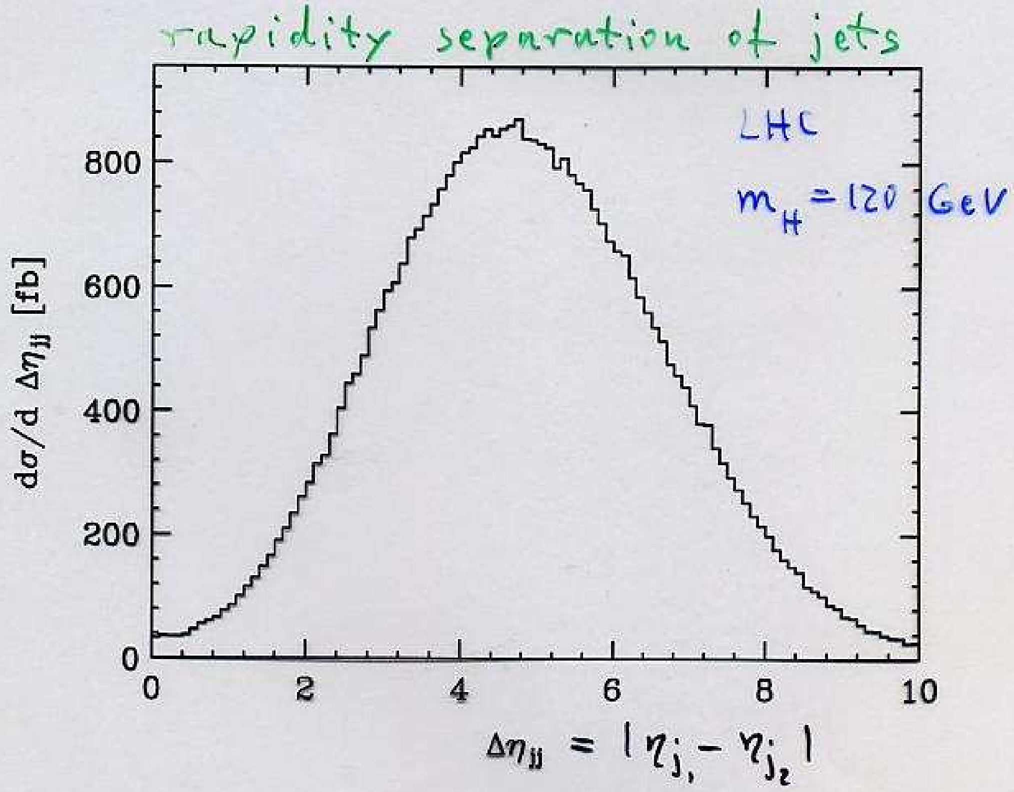
tagging je[illegible]
forward bu[illegible]
well inside
detector

$\eta = \frac{1}{2} \ln \frac{1+\cos}{1-\cos}$

pseudorapidity of Higgs decay prod.



Higgs deca[illegible]
products
are quite
central



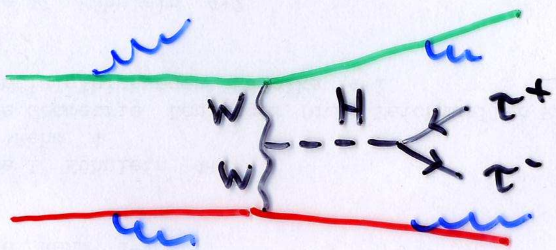
Tagging jets are typically far apart. Higgs decay products usually between 2 tagging jets

Central jet veto

- $t\bar{t}$ + jets background for $qq \to qqH$, $H \to W^+W^-$

  $\Rightarrow$ veto b-jets from $t \to bW$

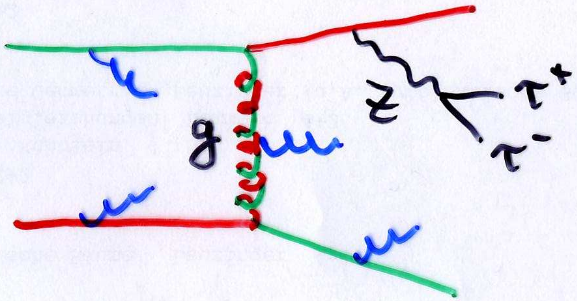
- t-channel color singlet exchange



"synchrotron" radiation between initial and final quark direction

$\Rightarrow$ central jets suppressed

- Major QCD backgrounds: t-channel color octet exch.

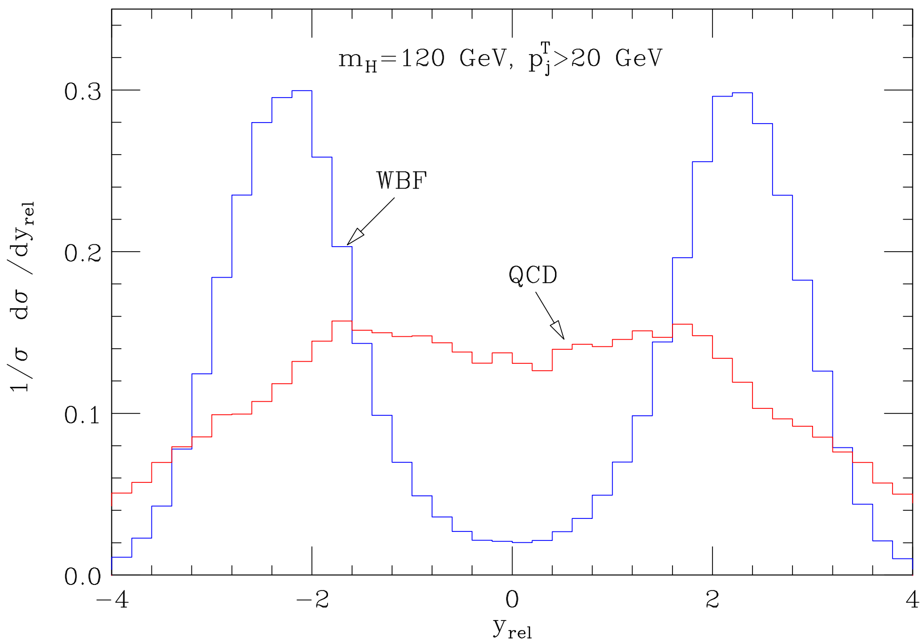


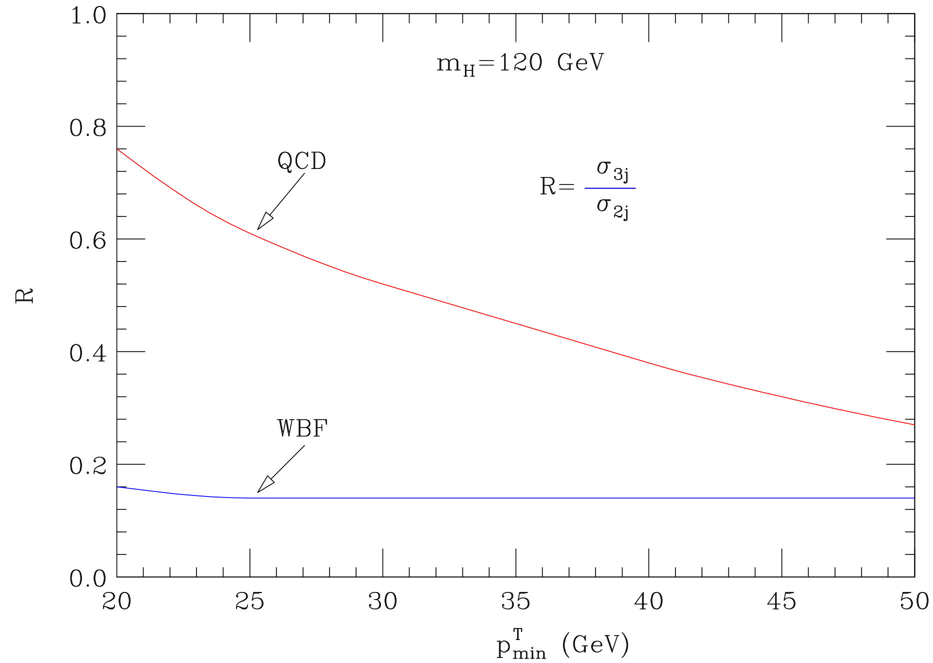
deflection of color charge by ~180° $\Rightarrow$ strong color acceleration

$\Rightarrow$ enhanced central gluon emis.

$\Rightarrow$ central jet veto suppresses QCD backgrounds to weak boson fusion

Central Jet Veto: $Hjjj$ from VBF vs. gluon fusion

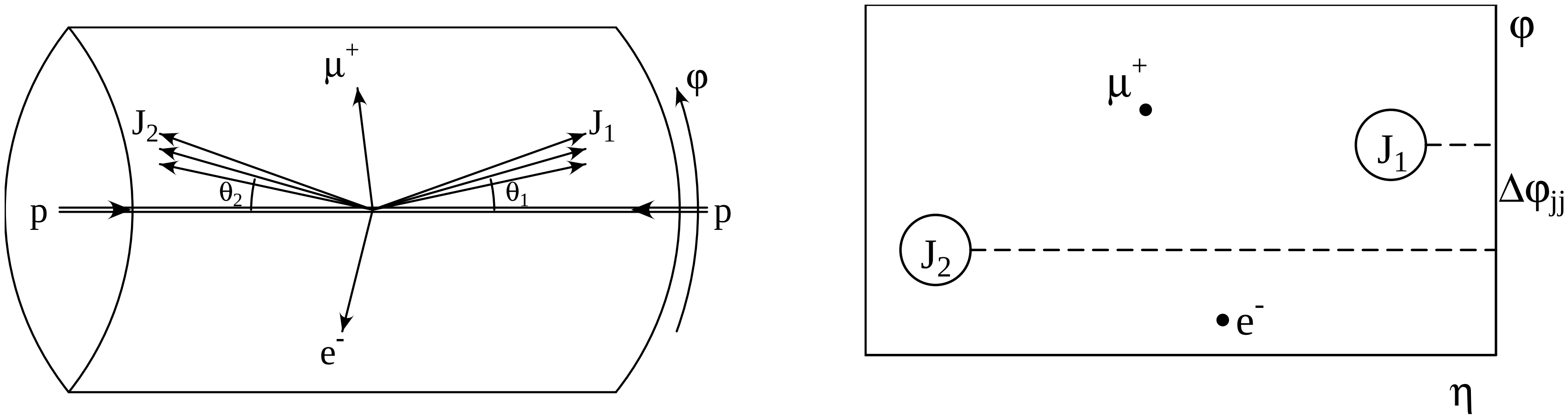




[Del Duca, Frizzo, Maltoni, JHEP 05 (2004) 064]

- Angular distribution of third (softest) jet follows classically expected radiation pattern
- QCD events have higher effective scale and thus produce harder radiation than VBF (larger three jet to two jet ratio for QCD events)
- Central jet veto can be used to distinguish Higgs production via GF from VBF

VBF signature



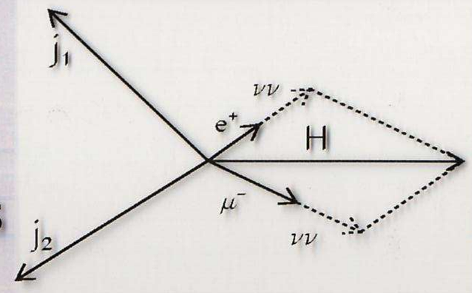
Characteristics:

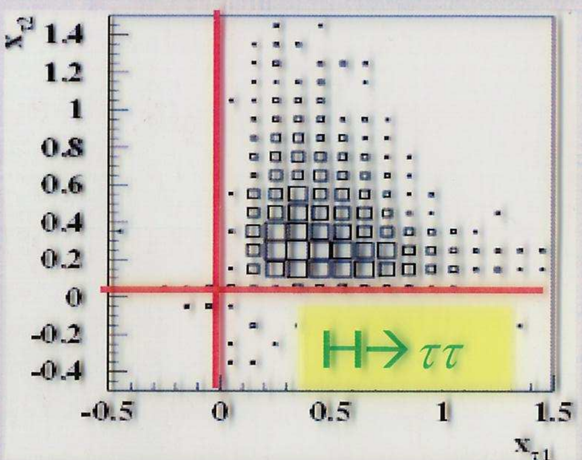
- energetic jets in the forward and backward directions ($p_T > 20$ GeV)
- large rapidity separation and large invariant mass of the two tagging jets
- Higgs decay products between tagging jets
- Little gluon radiation in the central-rapidity region, due to colorless W/Z exchange (central jet veto: no extra jets with $p_T > 20$ GeV and $|\eta| < 2.5$)

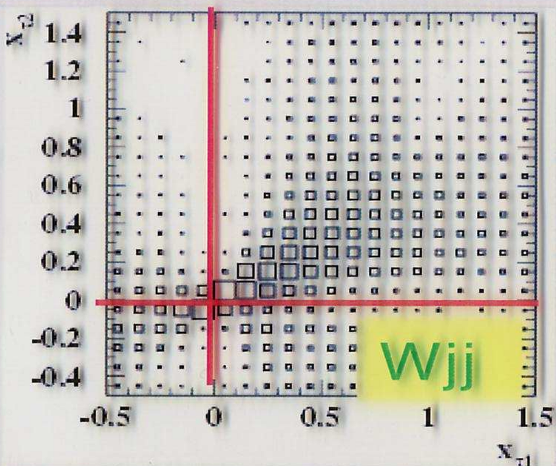
Weak Boson Fusion: $H \rightarrow \tau\tau$

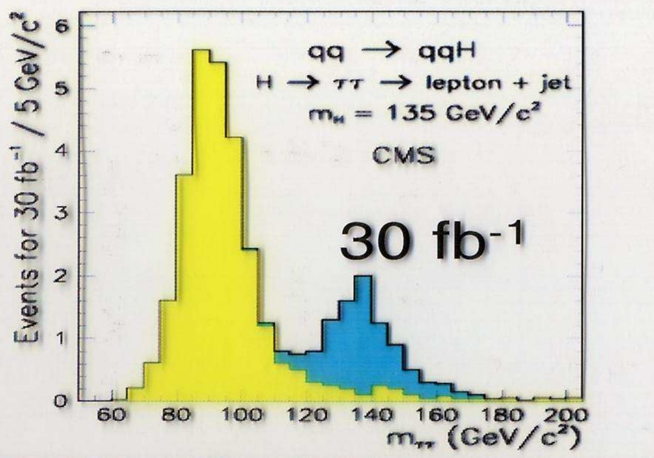
Mass can be reconstructed in collinear approximation

x_τ = momentum fraction carried by tau decay products

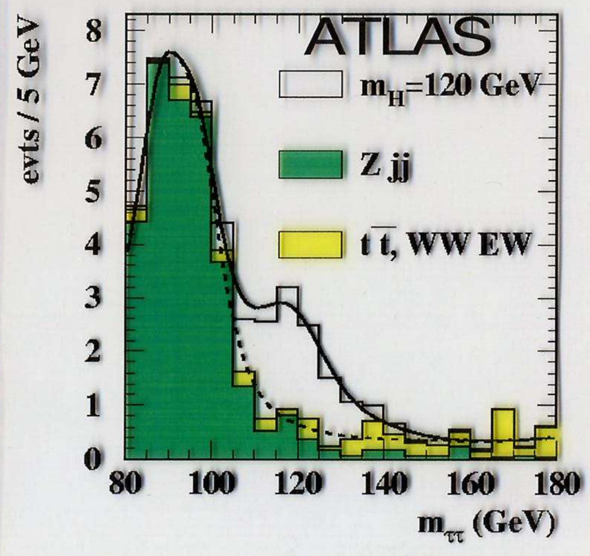








σ_M = 11 to 12 GeV



$H \rightarrow \tau\tau \rightarrow e\mu$ 30 fb^{-1}

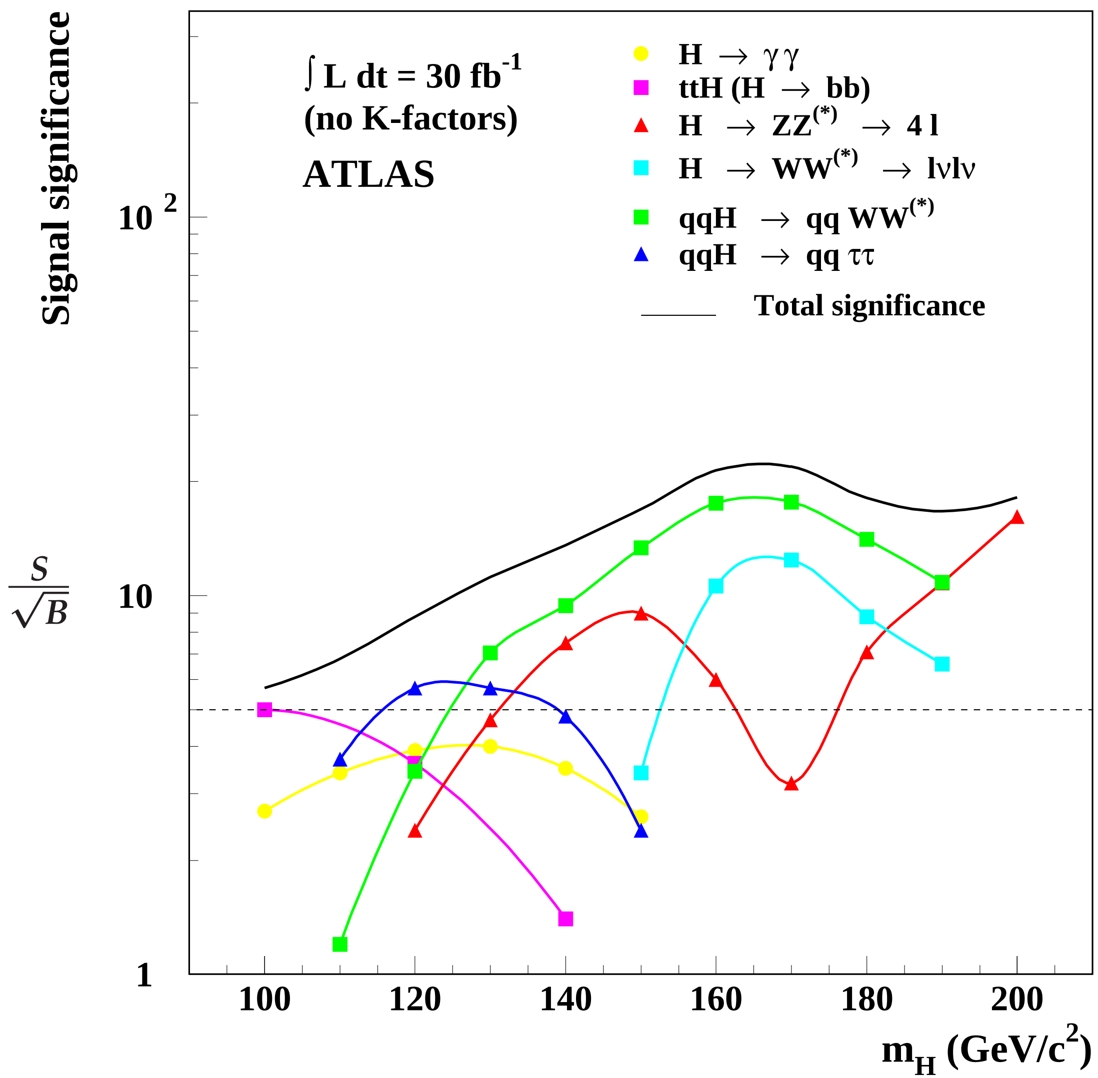
★ significance > 5 for 30 fb^{-1} and M_H=110 to 140 GeV ($\tau\tau \rightarrow e\mu$, $\tau\tau \rightarrow ll$, $\tau\tau \rightarrow l$ had)

★ background estimate: ~10%

for M_H>125 GeV from side bands

for M_H>125 GeV from normalisation of $Z \rightarrow \tau\tau$ peak

Higgs discovery potential



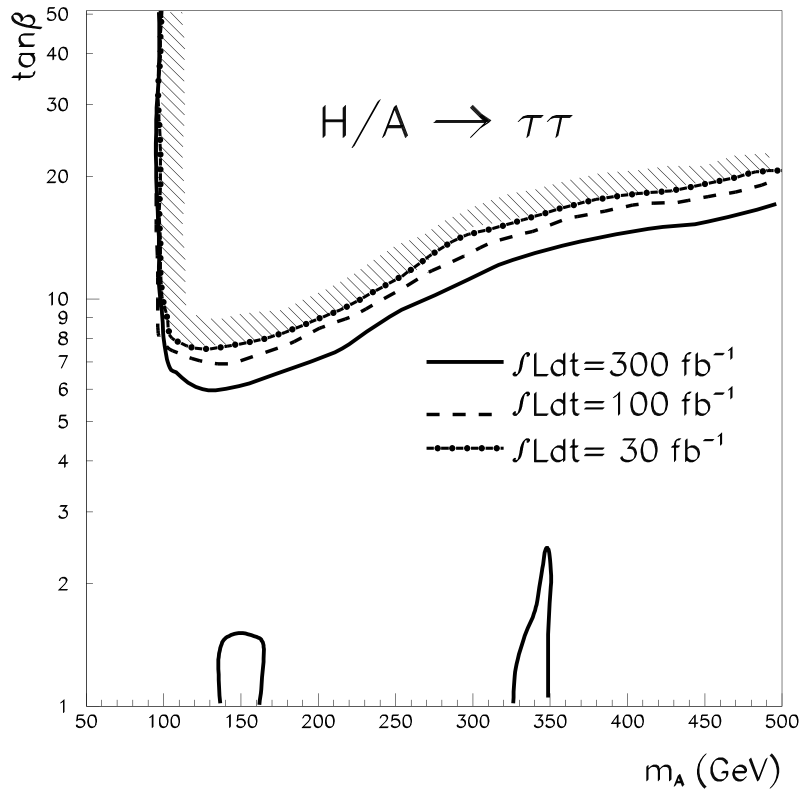
Reach for H/A discovery within MSSM

Enhancement of
Hbb and *Abb* coupling
by factor $\tan\beta$
compared to SM Higgs

$\Longrightarrow$ large production cross section for $pp \to \bar{b}bH/A$

$\Longrightarrow$ decay dominated by $H/A \to \bar{b}b,\ \tau^+\tau^-$

ATLAS TDR



5σ discovery contours

Reach for $H^{\pm}$ discovery within MSSM

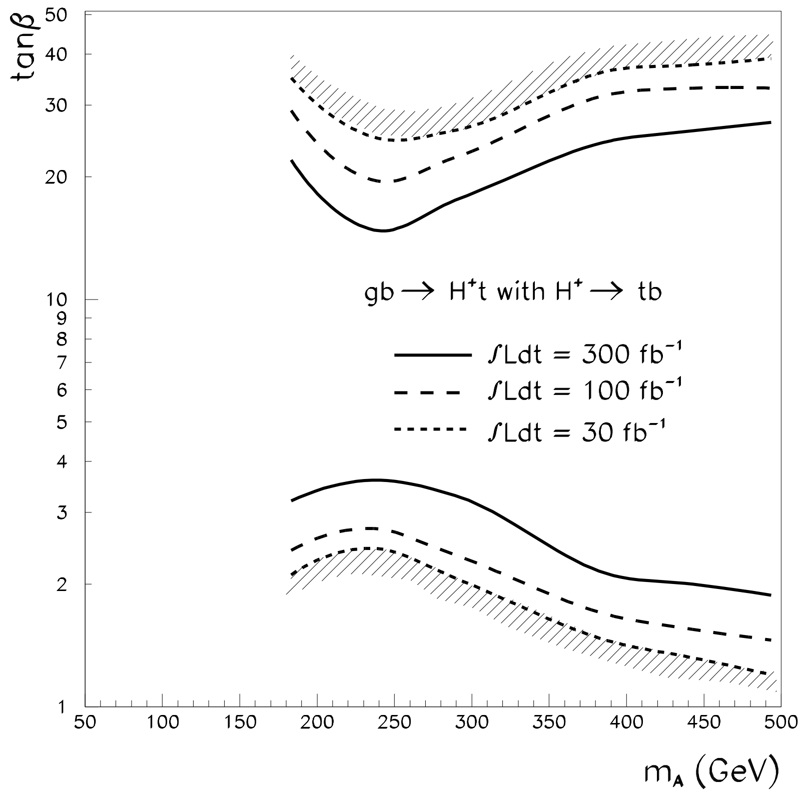
- For $m_{H^{\pm}} > m_t + m_b$ expect $H^{\pm} \to tb$ decay
- Dominant production process

$$gg \to H^{\pm} tb$$

b-quark has low p_T: $gb \to H^{\pm} t$ is dominant subprocess

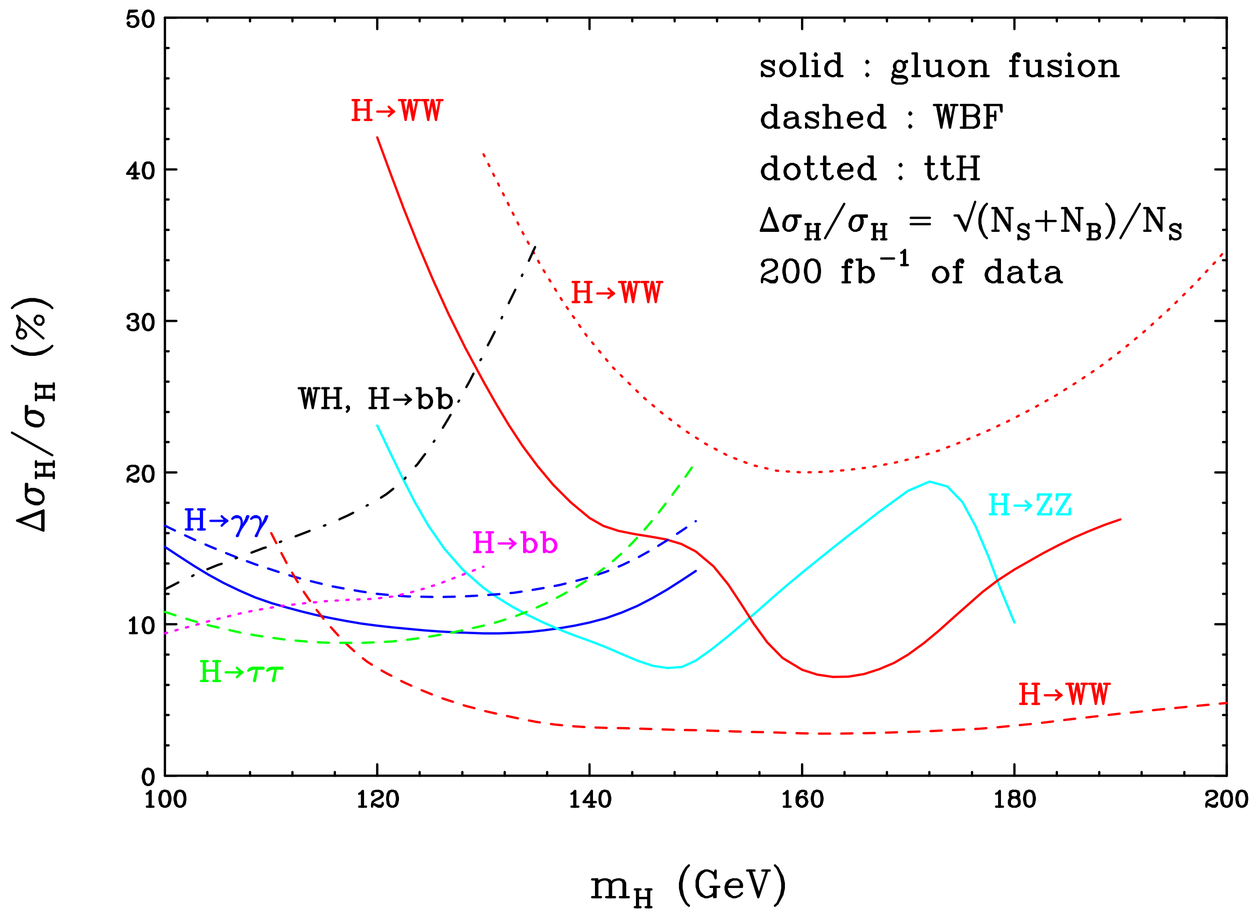
- Main background from $\bar{t}t(+\text{jets})$ production

ATLAS TDR



5σ discovery contours

Statistical and systematic errors at LHC for SM Higgs rate



Assumed errors in fits to couplings:

- QCD/PDF uncertainties
  - ±5% for VBF
  - ±20% for gluon fusion
- luminosity/acceptance uncertainties
  - ±5%

Measuring Higgs couplings at LHC

LHC rates for partonic process $pp \to H \to xx$ given by $\sigma(pp \to H) \cdot BR(H \to xx)$

$$\sigma(H) \times \mathrm{BR}(H \to xx) = \frac{\sigma(H)^{\mathrm{SM}}}{\Gamma_p^{\mathrm{SM}}} \cdot \frac{\Gamma_p \Gamma_x}{\Gamma} \,,$$

Measure products $\Gamma_p \Gamma_x / \Gamma$ for combination of processes ($\Gamma_p = \Gamma(H \to pp)$)

Problem: rescaling fit results by common factor f

$$\Gamma_i \to f \cdot \Gamma_i \,, \qquad \Gamma \to f^2 \Gamma = \sum_{obs} f \Gamma_i + \Gamma_{rest}$$

leaves observable rate invariant $\Longrightarrow$ no model independent results at LHC

Loose bounds on scaling factor:

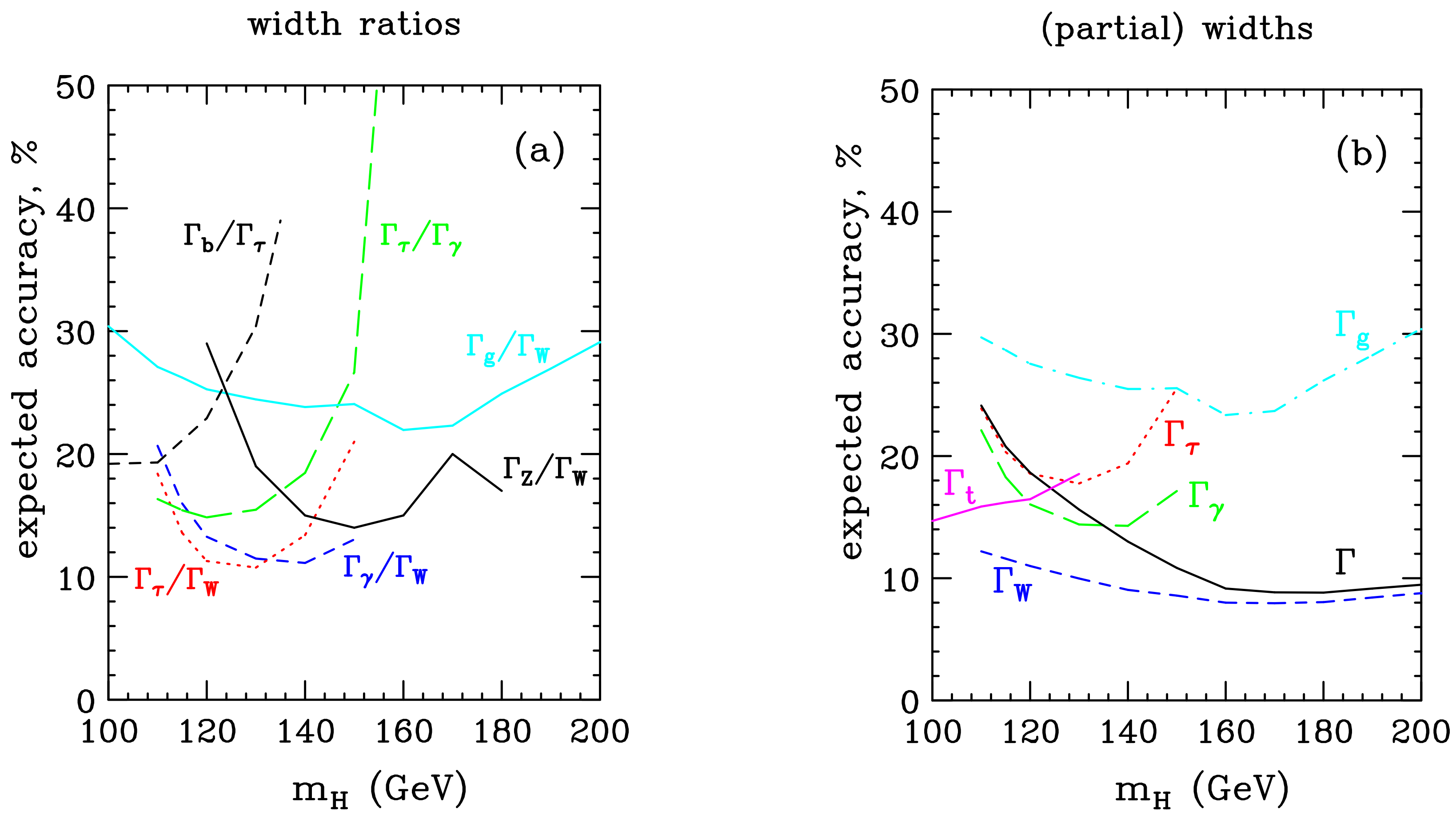
$$f^2 \Gamma > \sum_{obs.} f \Gamma_x \qquad \Longrightarrow \qquad f > \sum_{obs.} \frac{\Gamma_x}{\Gamma} = \sum_{obs.} BR(H \to xx) (= \mathcal{O}(1))$$

Total width below experimental resolution of Higgs mass peak ($\Delta m = 1 \ldots 20$ GeV)

$$f^2 \Gamma < \Delta m \qquad \Longrightarrow \qquad f < \sqrt{\frac{\Delta m}{\Gamma}} < \mathcal{O}(10 - 40)$$

Fit LHC data within constrained models

- $\frac{g_{H\tau\tau}}{g_{Hbb}} =$ SM value
- $\frac{g_{HWW}}{g_{HZZ}} =$ SM value
- no exotic channels



With 200 fb^{-1} measure partial width with 10–30% errors, couplings with 5–15% errors

Distinguishing the MSSM Higgs sector from the SM

Alternative: compare data to predictions of specific models

Example: m_H^{max} scenario of LEP analyses

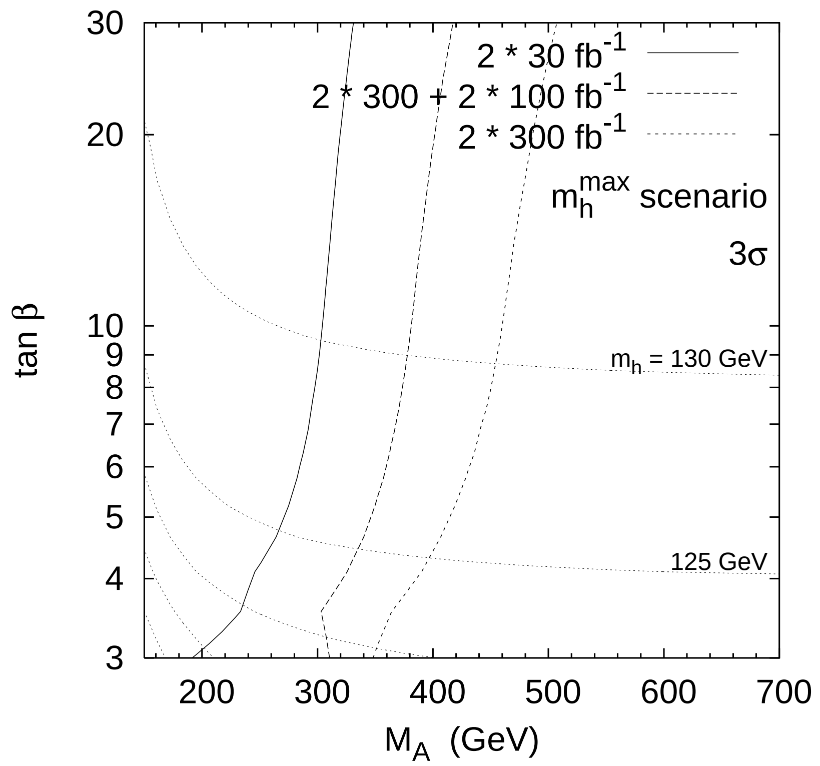
Consider modest m_A:

- decoupling almost complete for hWW and $h\gamma\gamma$ (effective) vertices
- enhanced hbb and $h\tau\tau$ couplings compared to SM increases total width of h

$\Longrightarrow$

- $\approx$ SM rates for $h \to \tau\tau$ in VBF
- suppressed $h \to \gamma\gamma$ and $h \to WW$ rates in VBF

3σ-effects or more at small m_A



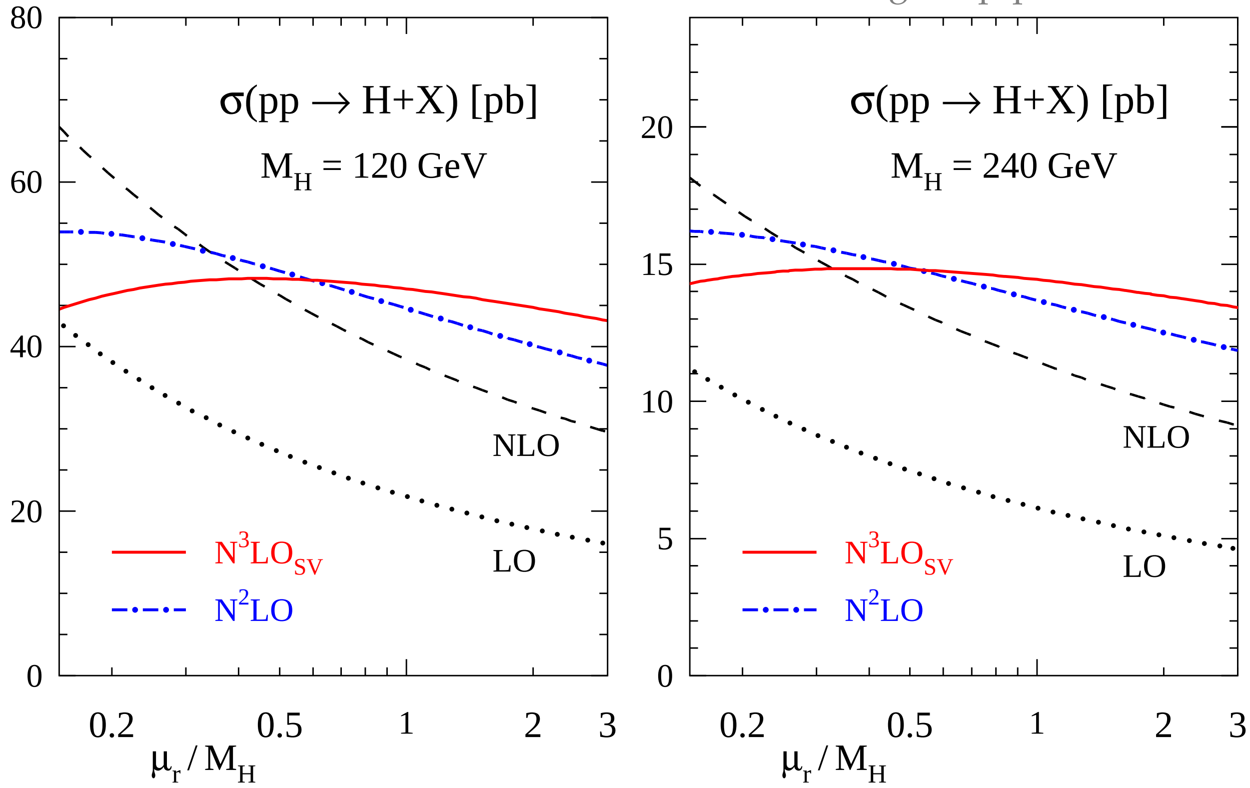
Corrections for Higgs production cross sections

Measurement of Higgs couplings from measured signal rates
$\Longrightarrow$ need QCD corrections to production cross sections. Much progress in recent years

- $gg \to H$ (all but NLO in $m_t \to \infty$ limit)
  - NLO for finite m_t: Graudenz, Spira, Zerwas (1993)
  - NNLO: Harlander, Kilgore (2001); Anastasiou, Melnikov (2002); Ravindran, Smith, van Neerven (2003)
  - N^3LO in soft approximation: Moch, Vogt (2005)
- Hjj by gluon fusion at NLO: Campbell, Ellis, Zanderighi (2006)
- Higgsstrahlung: implemented in MC@NLO Frixione, Webber
- weak boson fusion
  - distributions at NLO: Figy, Oleari, D.Z (2003); Campbell, Ellis, Berger (2004)
  - 1-loop EW corrections: Ciccolini, Denner, Dittmaier (2007)
  - approx. NLO QCD to $Hjjj$: Figy, Hankele, D.Z (2007)
- $\bar{t}tH$ associated production at NLO: Beenakker et al.; Dawson, Orr, Reina, Wackeroth (2002)
- $\bar{b}bH$ associated production at NLO: Dittmaier, Krämer, Spira; Dawson et al. (2003)

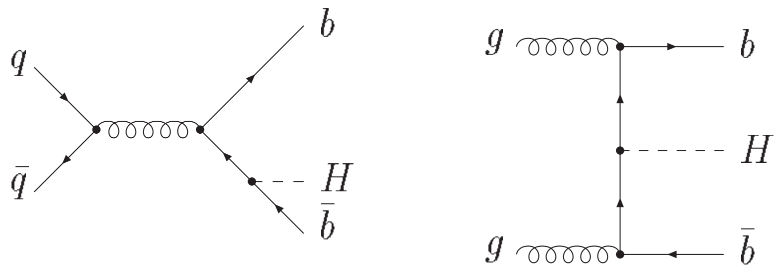
QCD corrections to $gg \to H$

Moch & Vogt, hep-ph/0508265

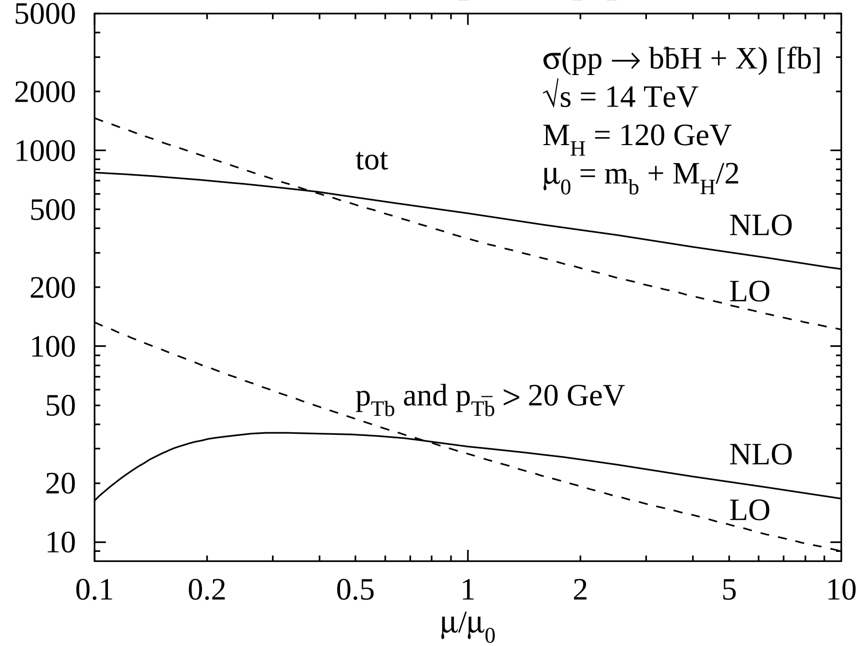


- ✓ Huge improvement in recent years
- ✓ Remaining scale uncertainty below 10%
- ✓ Uncertainty from gluon pdf $\approx 4 - 7\%$
- ✓ K-factor for cross section with cuts (central jet veto against $\bar{t}t$ background for $H \to WW$ search) has similar uncertainties [Anastasiou, Dissertori, Stöckli, Webber arXiv:0801.2682]

NLO QCD corrrections to $b\bar{b}H$ production



Dittmaier, Krämer, Spira hep-ph/0309204

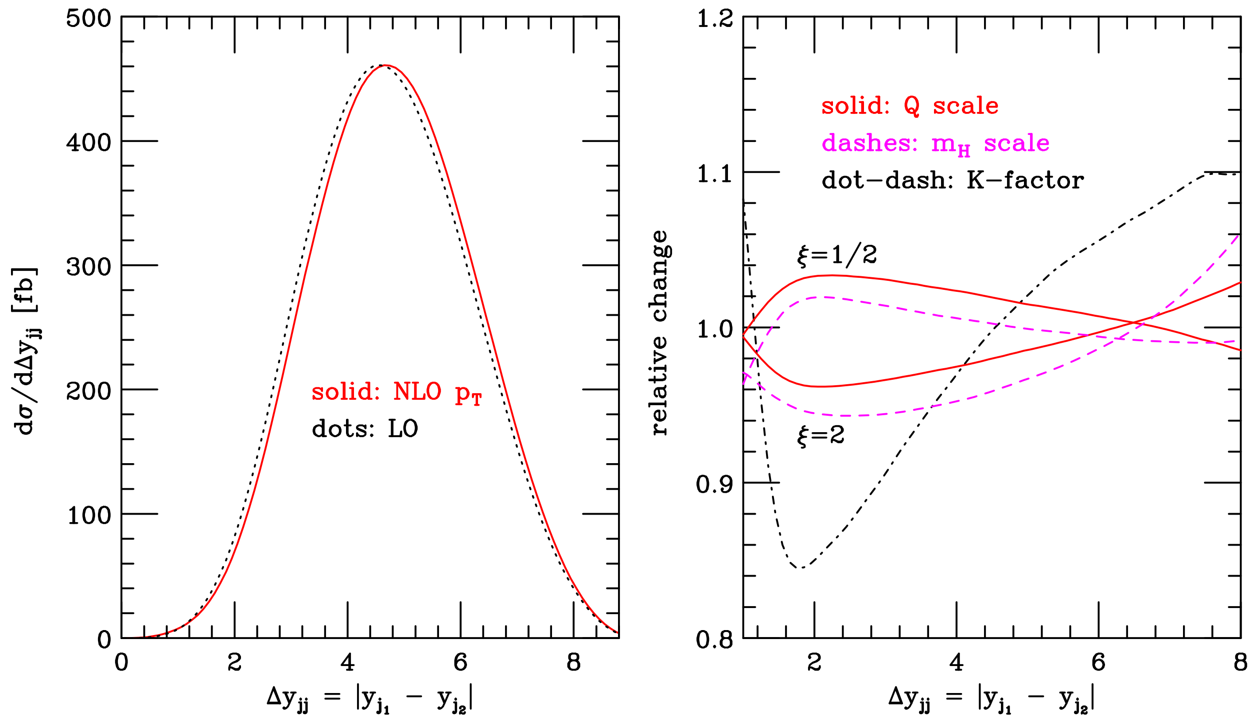


scale dependence of inclusive vs. double b-tagged cross section

- Discovery channel for H/A in the MSSM at sizeable tan β
- NLO corrections known for $\bar{b}bH$ final state
- b-quarks at low p_T: effective process is $\bar{b}b \to H$: cross section known at NNLO Harlander, Kilgore (2003)

NLO QCD corrections to VBF

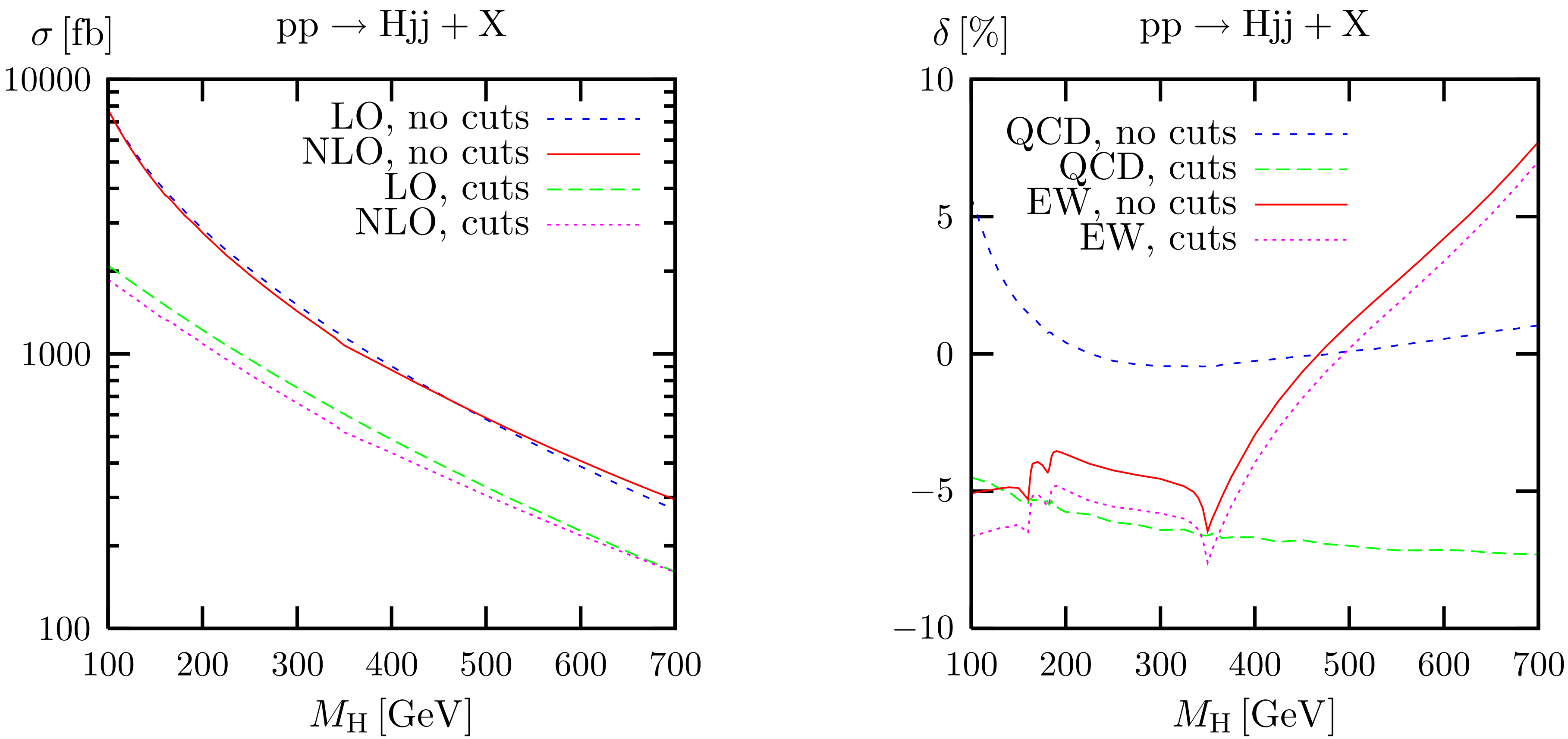
- ✓ Small QCD corrections of order 10%
- ✓ Tiny scale dependence of NLO result
  - $\pm 5\%$ for distributions
  - $< 2\%$ for σ_{total}
- ✓ K-factor is phase space dependent
- ✓ QCD corrections under excellent control
- ✗ Need electroweak corrections for 5% uncertainty



$m_H = 120$ GeV, typical VBF cuts

QCD + EW corrections to Hjj production

Cross sections without and with VBF cuts: $p_T(j) > 20$ GeV $\quad |y_{j_1} - y_{j_2}| > 4, \;\; y_{j_1} \cdot y_{j_2} < 0$

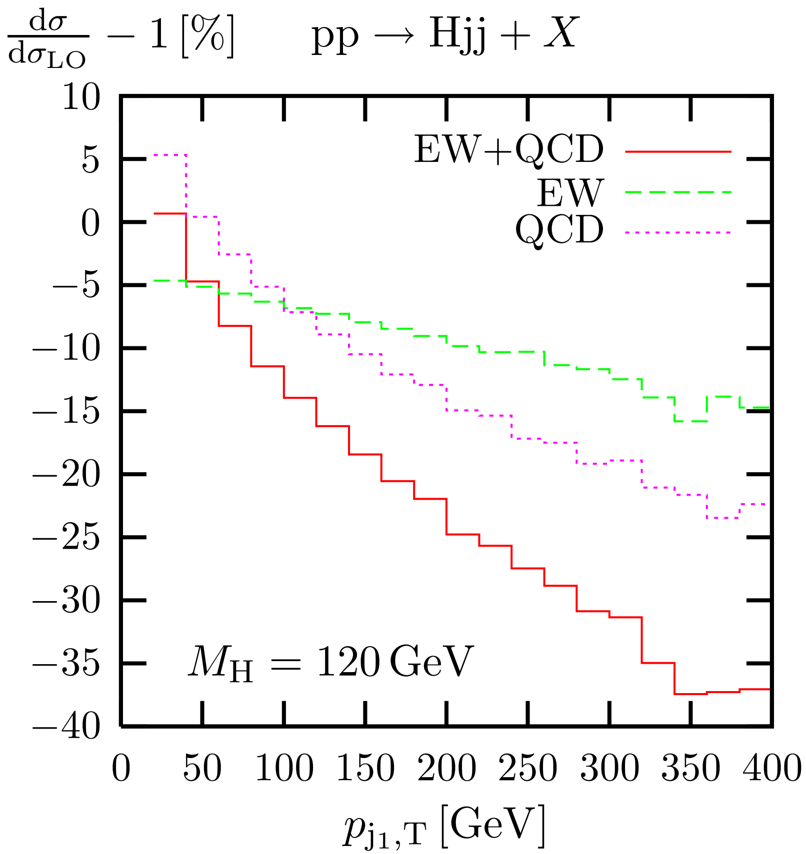


Ciccolini, Denner, Dittmaier, arXiv:0710.4749

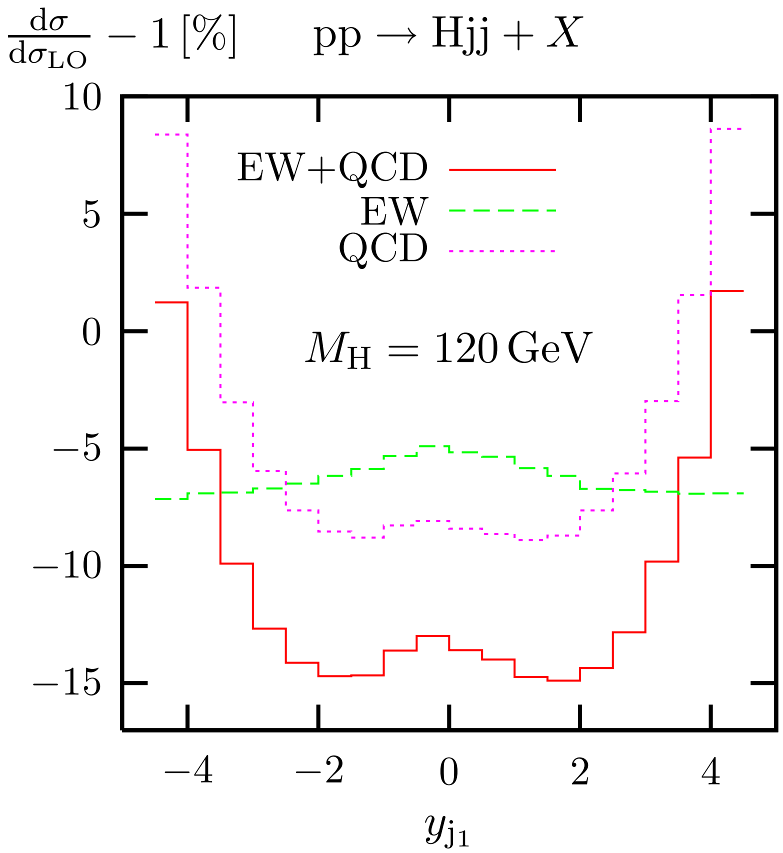
Relative size of 1-loop corrections

Consider distributions of hardest jet in the event:

p_T distribution



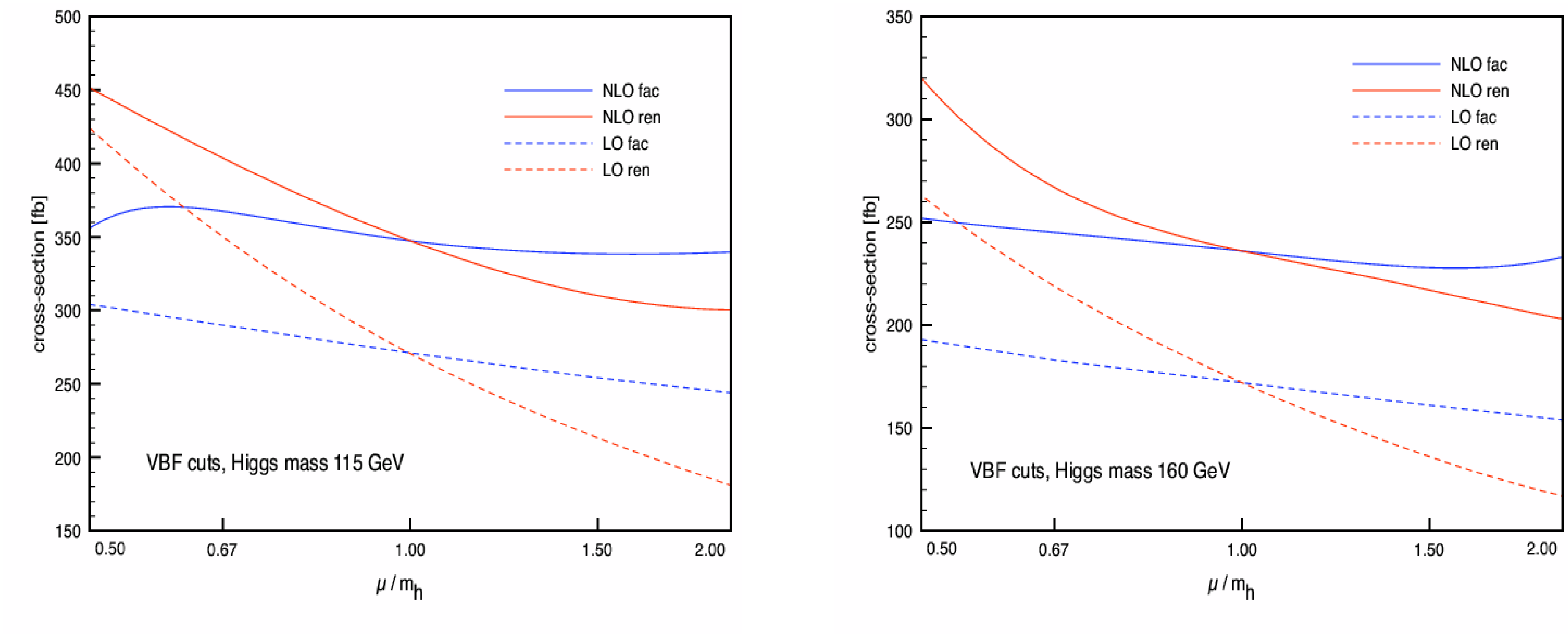
rapidity distribution



strong shape changes by QCD corrections, EW corrections affect mostly normalization

Hjj cross section for gluon fusion

Calculation of Hjj cross section at NLO in $m_t \to \infty$ limit by Campbell, Ellis, Zanderighi, hep-ph/0608194



- Modest increase of cross section at 1-loop: K-factor of order 1.2 - 1.4
- Reduced scale dependence at NLO: remaining scale uncertainty $\approx \pm 20\%$

Conclusions

- Spontaneous breaking of $SU(2) \times U(1)$ symmetry is largely untested experimentally $\Longrightarrow$ most important task for the LHC
- LHC will observe a SM-like Higgs boson in multiple channels, with $5 \ldots 20\%$ statistical errors
  $\Longrightarrow$ great source of information on Higgs couplings
- Absence of HVV and AVV couplings for the heavy H/A of the MSSM make their observation more challenging
  $\Longrightarrow$ Need large $\tan\beta$ rate enhancement for their discovery
- NLO QCD corrections and improved simulation tools are important for precise measurements with full LHC data.
- An exciting new era of particle physics is starting right now.