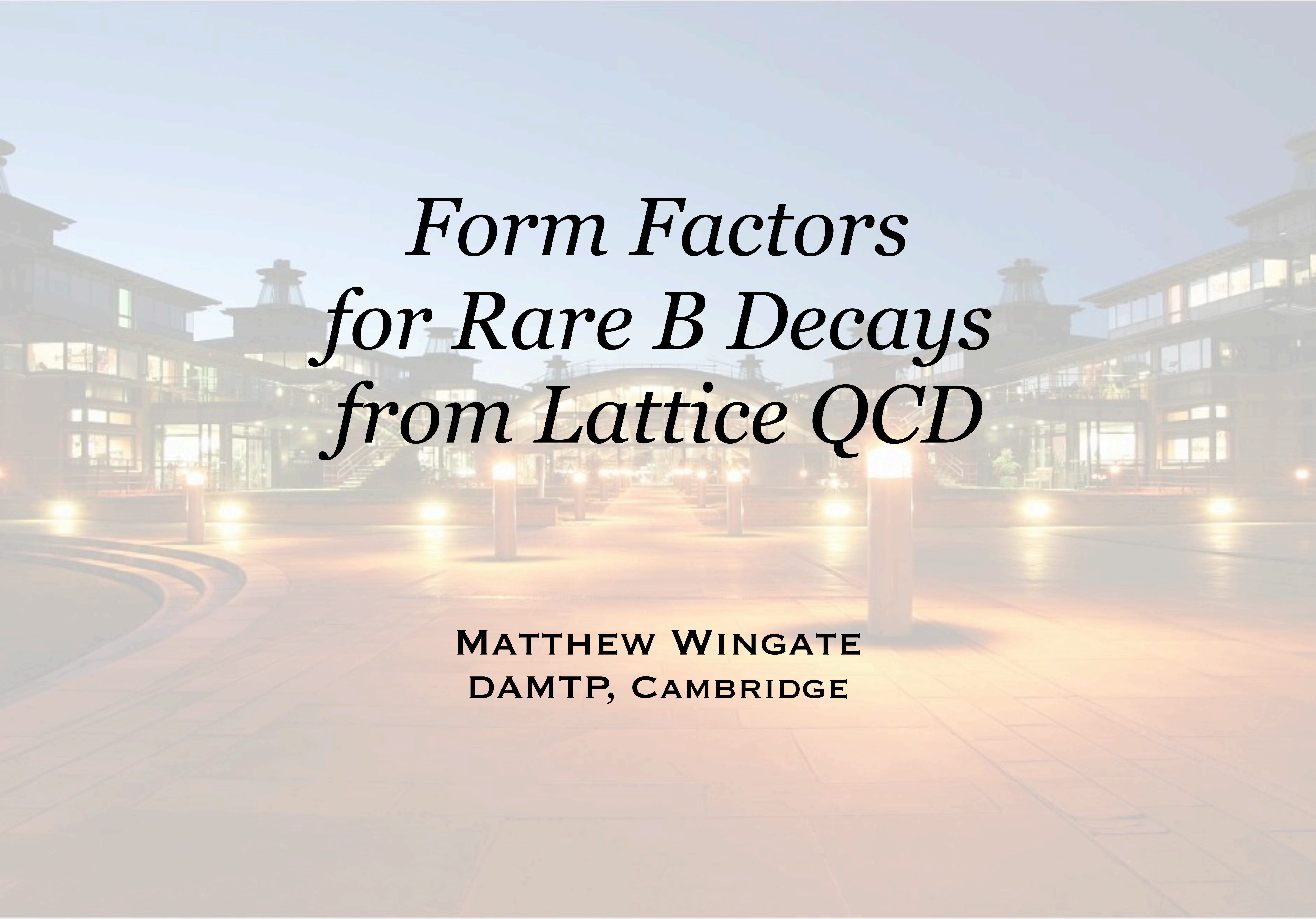

Form Factors for Rare B Decays from Lattice QCD

MATTHEW WINGATE
DAMTP, CAMBRIDGE

Outline

- Motivation
- Form factors
- Rare (FCNC) decays
- Preliminary results
- Summary

Two views of a weak decay

Phenomenologist:

Experimentalist:

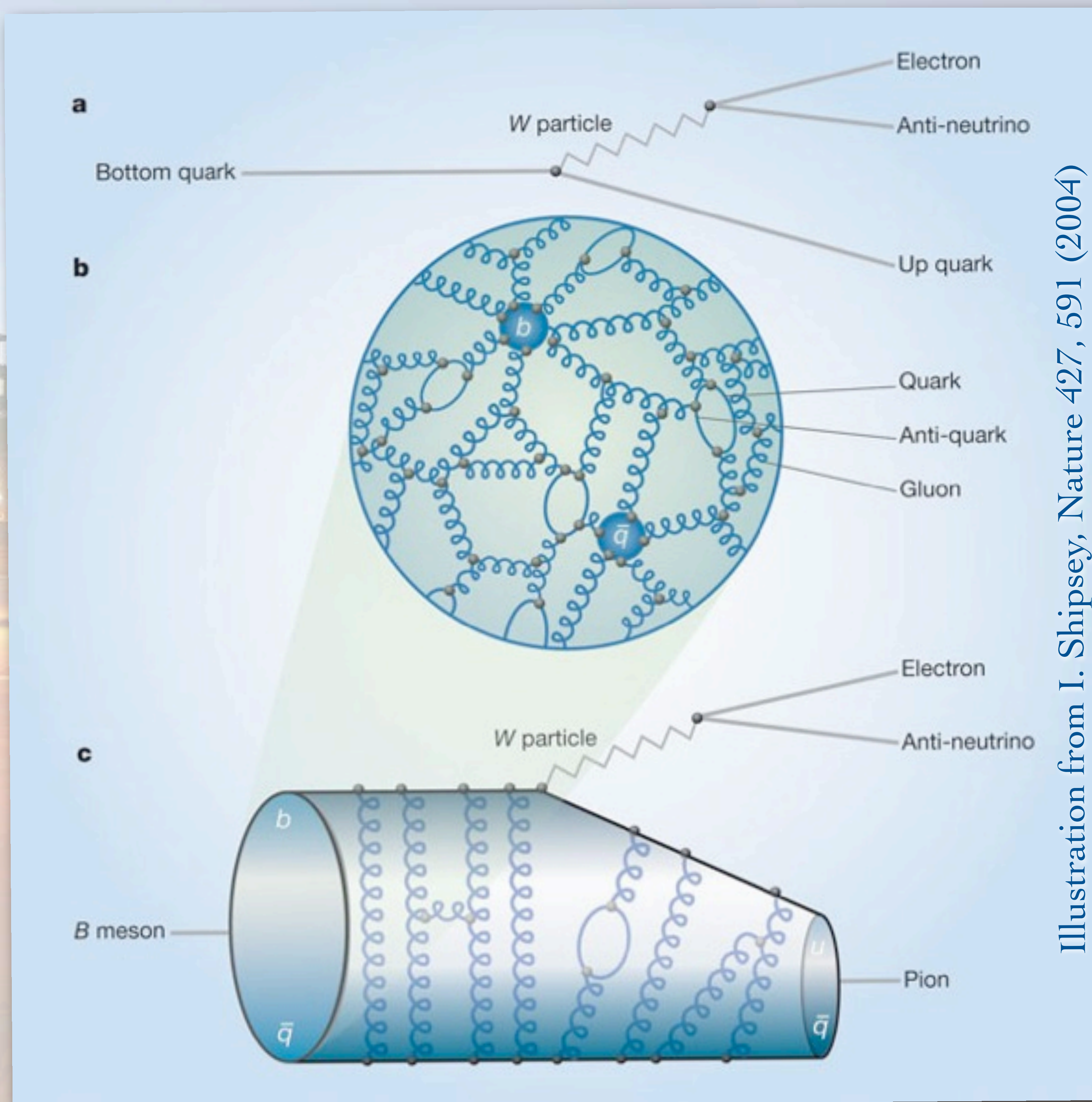

Illustration from I. Shipsey, Nature 427, 591 (2004)

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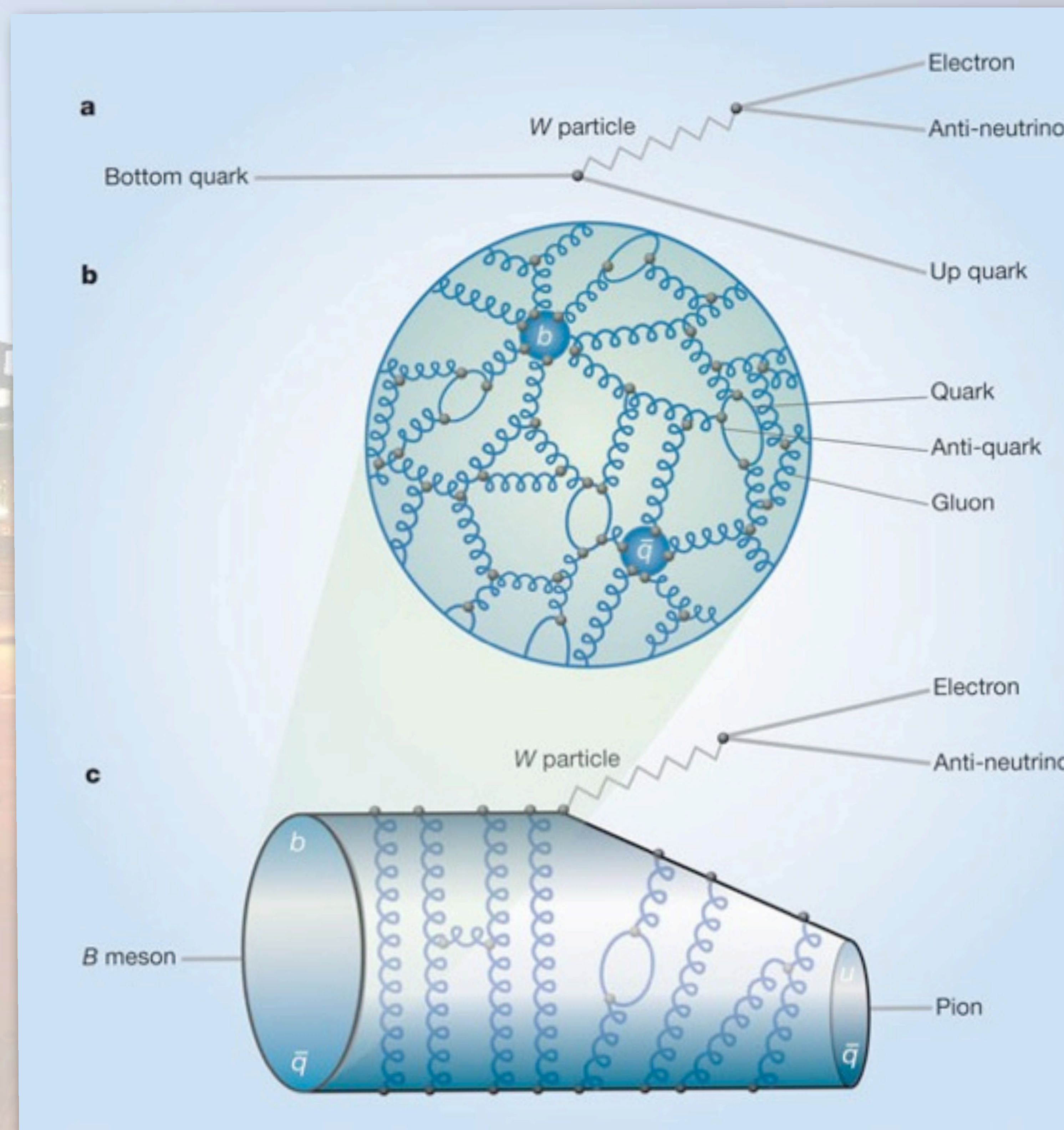

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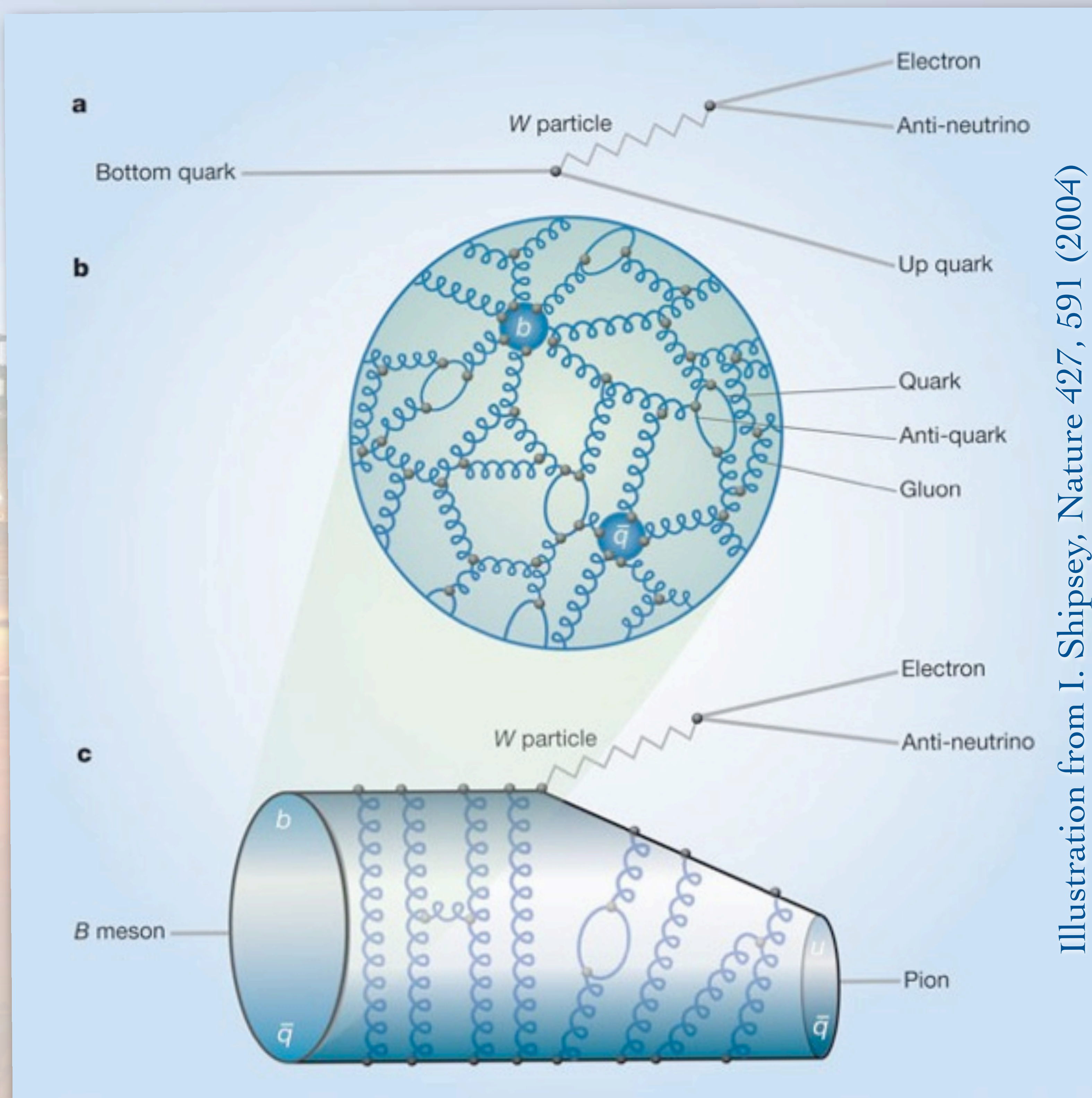

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Full set of form factors

Matrix element	Form factor	Relevant decay(s)
$\langle P \vert \bar{q}\gamma^\mu b \vert B\rangle$	f_+, f_0	$B \to \pi\ell\nu$ $B \to K\ell^+\ell^-$
$\langle P \vert \bar{q}\sigma^{\mu\nu}q_\nu b \vert B\rangle$	f_T	$B \to K\ell^+\ell^-$
$\langle V \vert \bar{q}\gamma^\mu b \vert B\rangle$	V	$\left\{ \begin{array}{l} B \to (\rho/\omega)\ell\nu \\ B \to K^*\ell^+\ell^- \end{array} \right.$
$\langle V \vert \bar{q}\gamma^\mu\gamma^5 b \vert B\rangle$	A_0, A_1, A_2	
$\langle V \vert \bar{q}\sigma^{\mu\nu}q_\nu b \vert B\rangle$	T_1	$\left\{ \begin{array}{l} B \to K^*\gamma \\ B \to K^*\ell^+\ell^- \end{array} \right.$
$\langle V \vert \bar{q}\sigma^{\mu\nu}\gamma^5 q_\nu b \vert B\rangle$	T_2, T_3	

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$\langle V \vert \bar{q}\sigma^{\mu\nu}\gamma^5 q_\nu b \vert B\rangle$	T_2, T_3	

... also make the spectator an s quark for B_s decays

$B \to \pi\, l\, \nu$, reviewed by J. Laiho (CKM2010)

BaBar result for $|V_{ub}|$ exclusive

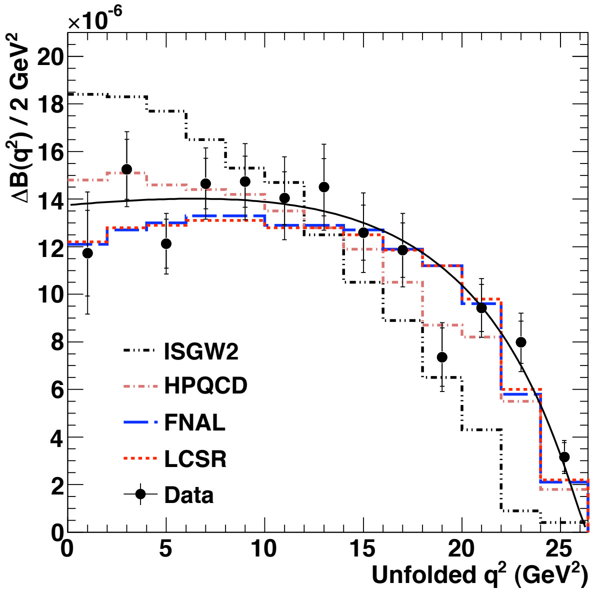
Talk by Martin Simard. Two different analyses, π-η analysis and π-ρ analysis.

Different fit and cut strategies. Fits to different numbers of modes, loose vs tight ν cut selection.
Results for $|V_{ub}|$ consistent within the different approaches. Results from π-η analysis:

Theory	q^2(GeV)^2	$\lvert V_{ub}\rvert(10^{-3})$
HPQCD	> 16	$3.24 \pm 0.13 \pm 0.16^{+0.57}_{-0.37}$
FNAL	> 16	$3.14 \pm 0.12 \pm 0.16^{+0.35}_{-0.29}$
LCSR	< 12	$3.70 \pm 0.07 \pm 0.09^{+0.54}_{-0.39}$

Exclusive $B \to \pi \ell \nu$ and $|V_{ub}|$

New result from Belle (talk by Kevin Varvell)



Warwick,

Results for $|V_{ub}|$

Using a BK parameterization for Belle experimental data $|V_{ub}|$ was extracted from the partial branching fraction for a number of different theories to give the normalization.

Simultaneous fit to lattice (Fermilab/MILC) and Belle q^2 dependence (using the z parameterization) leads to a model independent result of
$|V_{ub}| = (3.43 \pm 0.33) \times 10^{-3}$.

The same procedure with the latest BaBar data leads to
$|V_{ub}| = (3.14 \pm 0.07 \pm 0.09^{+0.35}_{-0.29}) \times 10^{-3}$.

$|V_{ub}|$ status update from LCSR

Talk by Patricia Ball.

LCSR yields value for $f_B f_+(q^2)$, and for consistency, calculation in progress to determine f_B from QCD sum rules. Test sensitivity to radiative corrections by doing calculation to order α_s^2.

Sum rule results for $|V_{ub}|$ have an $\sim 10\%$ irreducible theory uncertainty, but are still useful at the moment, given the current precision of the lattice and the tension with the inclusive determination.

Sum rule results (which use exclusive $B \to \pi \ell \nu$) tend to be less than 4 and in better agreement with lattice.

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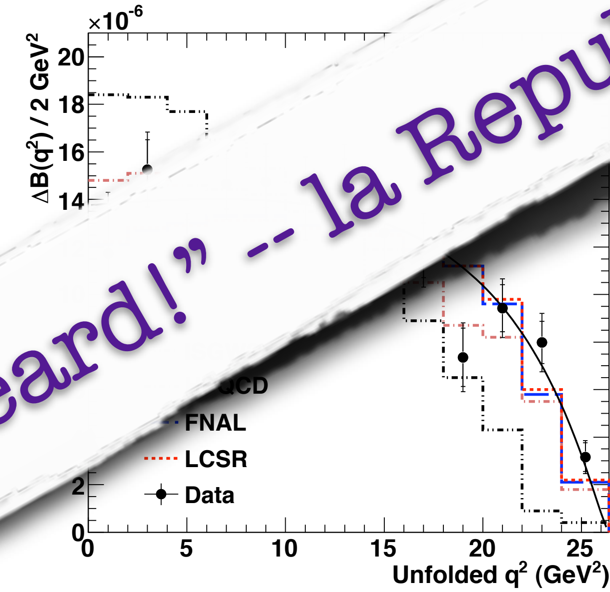

Warwick,

“ ... [best] review talk ... ever heard!” -- la Repubblica

Results for $|V_{ub}|$

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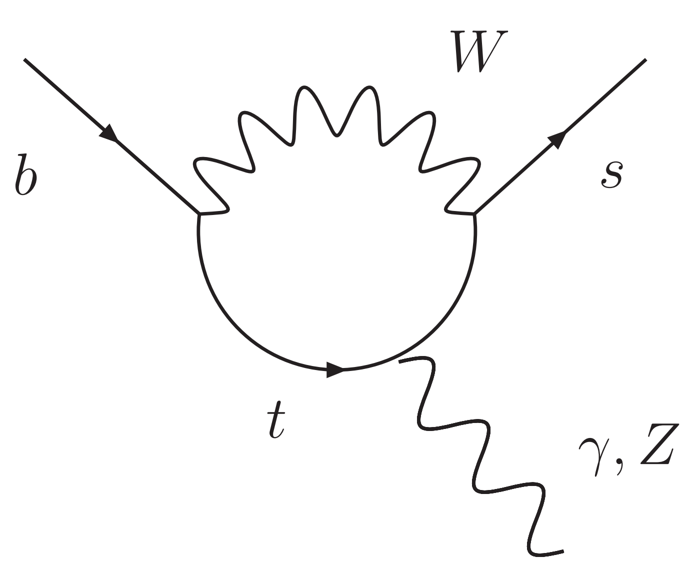
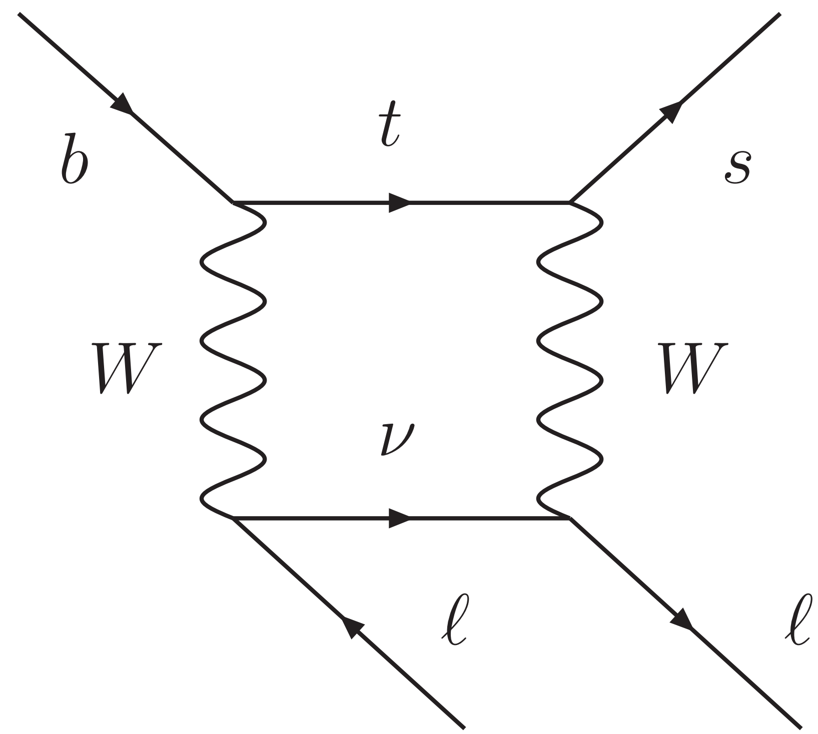
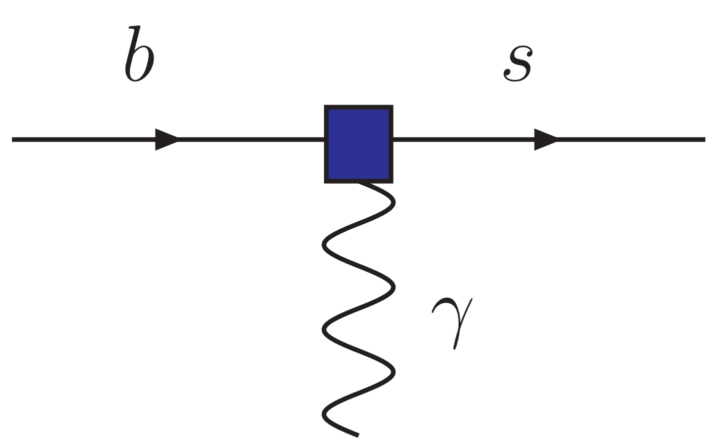
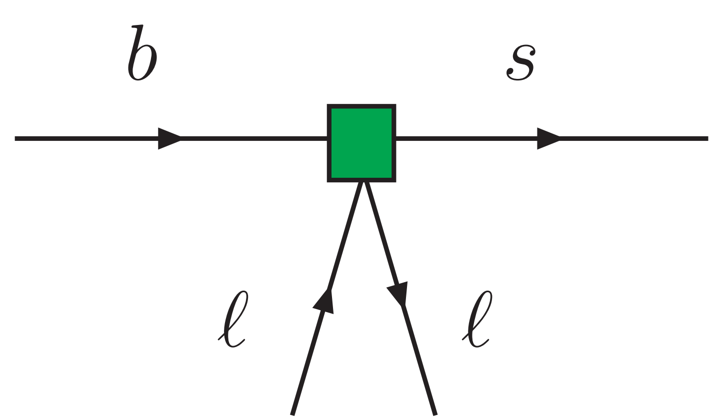
$b \rightarrow s$ is rare in the SM





$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) Q_i(\mu)$$





Dominant operators

Decays

$$B \to K^* \gamma$$
$$B_s \to \phi \gamma$$
$$B \to (\rho/\omega)\gamma$$

$$B \to K^{(*)} \ell^+ \ell^-$$
$$B_s \to \phi \ell^+ \ell^-$$

$$\Lambda_b \to \Lambda \gamma$$
$$\Lambda_b \to \Lambda \ell^+ \ell^-$$

SM operators

$$Q_{7\gamma} = \frac{e}{8\pi^2} m_b \, \bar{s}_i \sigma^{\mu\nu} (1+\gamma_5) b_i F_{\mu\nu}$$

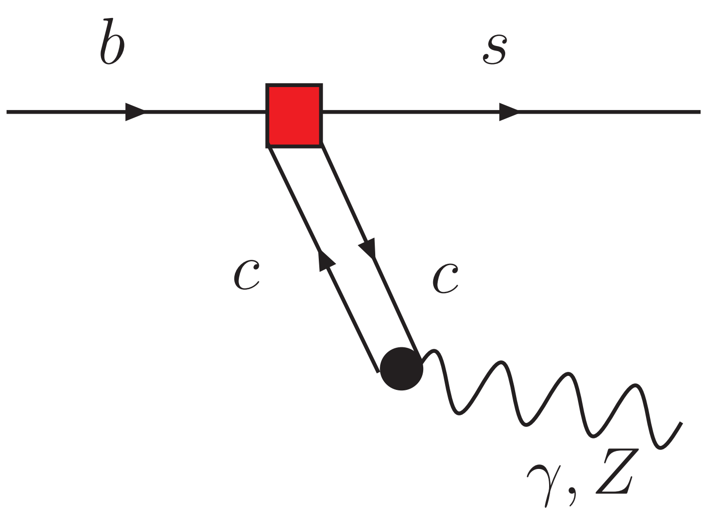
$$Q_{9V} = \frac{e}{8\pi^2} (\bar{s}\, b)_{V-A} (\bar{\ell}\, \ell)_V$$

$$Q_2 = (\bar{s}\, c)_{V-A} (\bar{c}\, b)_{V-A}$$

Long distance effects

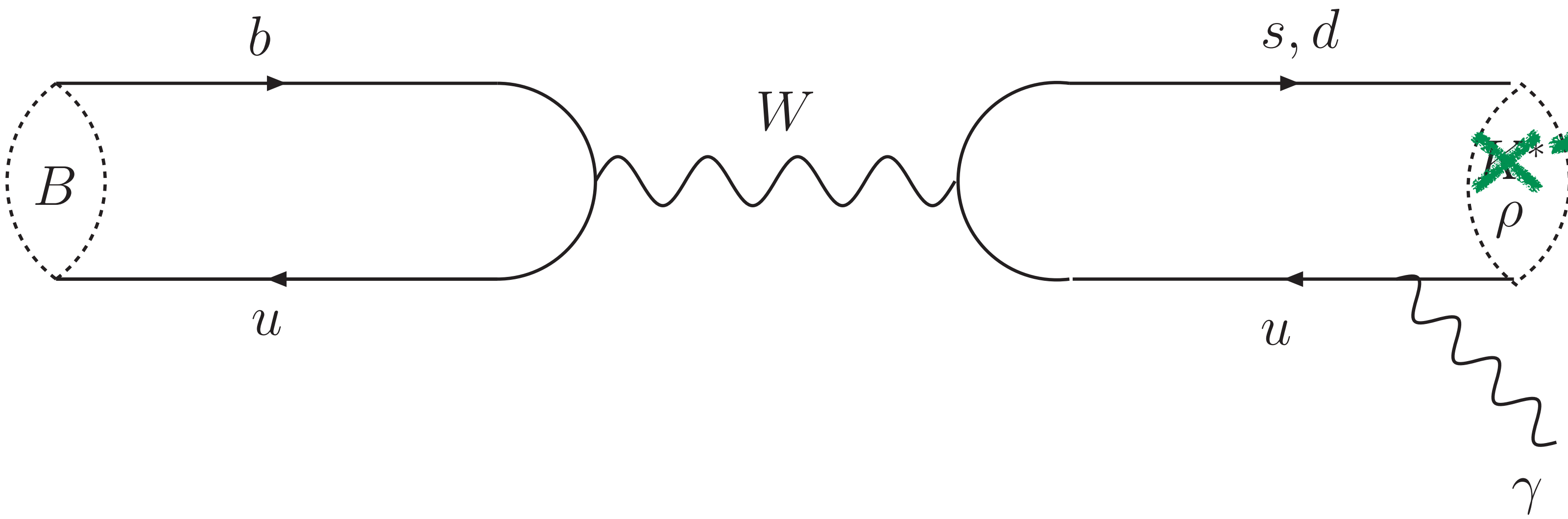
Phenomenological calculations necessary

Charmonium resonances



Khodjamirian, et al, PLB **402** (1997)
Grinstein & Pirjol, PRD **62** (2000), PRD **70** (2004)
Khodjamirian, et al, arXiv:1006.4945

Weak annihilation

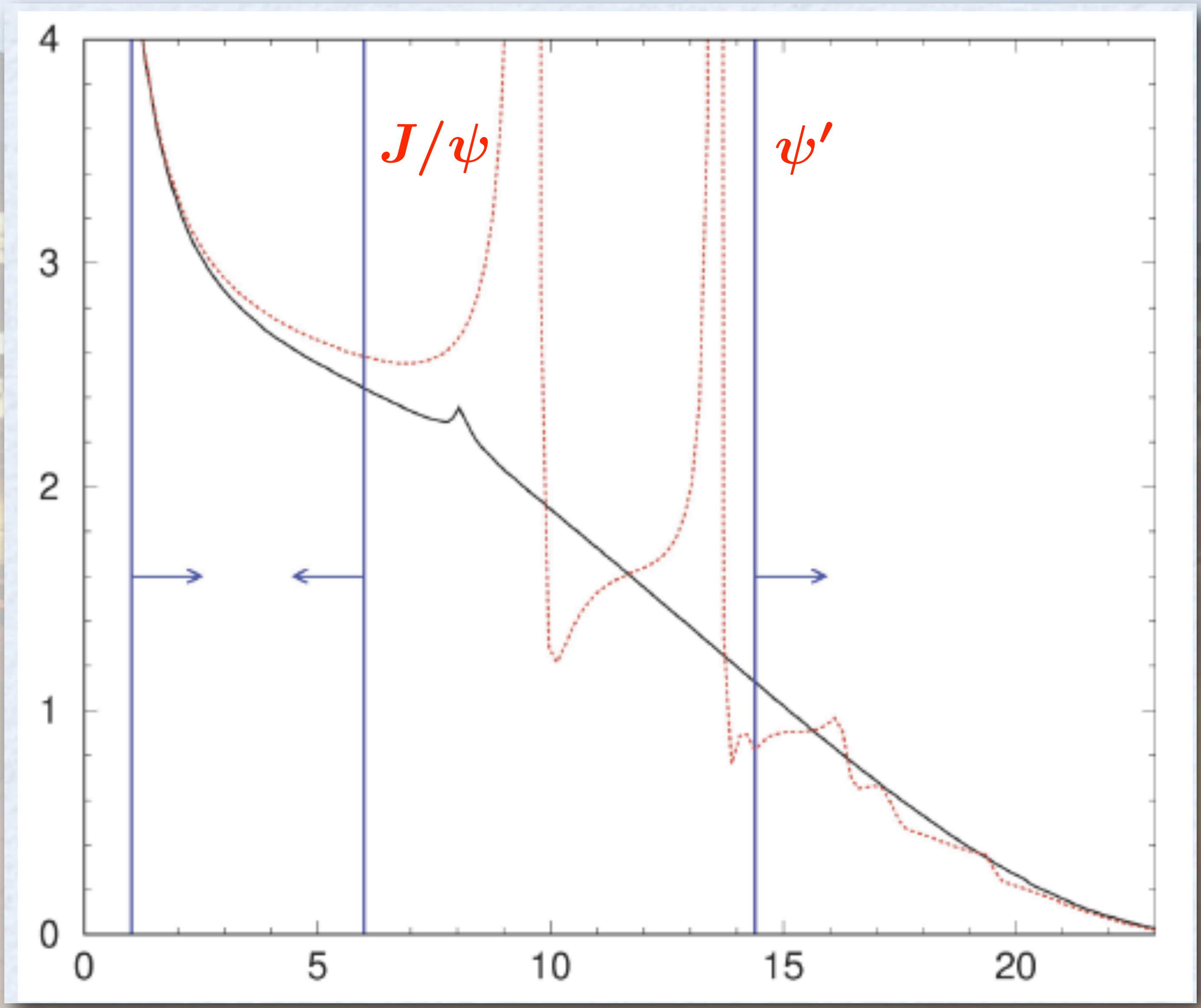


doubly Cabibbo-suppressed

Ball, Jones, Zwicky, PRD **75** (2007)

Regions of applicability

$B \to X_s \ell^+ \ell^-$



$q^2(\mathrm{GeV}^2)$

Plot from E Lunghi's CKM2008 talk

- Short distance effects dominate at low q^2
- Short distance effects thought to dominate at large q^2

Form factors in $B \to K^{(*)} l^+ l^-$

- Differential branching fractions in q^2 regions
- Expt. in agreement with present SM estimates (Ali, Ball, Handoko, Hiller, PRD 61, 2000)
- Form factors needed as input for calculations of other observables: e.g. A_{FB} (when nonzero), F_L, $A_T^{(j)}$ (Bobeth, Hiller, van Dyk, arXiv:1006:5013)

Form factor definitions

$$q^\nu \langle K^*(p',\lambda)|\bar{s}\sigma_{\mu\nu} b|B(p)\rangle = 4\,T_1(q^2)\epsilon_{\mu\nu\rho\sigma} e_\lambda^{*\nu} p^\rho p'^\sigma ,$$

$$q^\nu \langle K^*(p',\lambda)|\bar{s}\sigma_{\mu\nu}\gamma_5 b|B(p)\rangle = 2i\,T_2(q^2)\left[e^*_{\lambda\mu}(M_B^2 - M_{K^*}^2) - \right.$$

$$\left.(e_\lambda^* \cdot q)(p+p')_\mu\right] + 2i\,T_3(q^2)(e_\lambda^* \cdot q)\left[q_\mu - \frac{q^2}{M_B^2 - M_{K^*}^2}(p+p')_\mu\right].$$

$$\langle K^*(p',\lambda)|\bar{s}\gamma^\mu b|B(p)\rangle = \frac{2i V(q^2)}{M_B + M_{K^*}}\epsilon^{\mu\nu\rho\sigma} e^*_{\lambda\nu} p'_\rho p_\sigma ,$$

$$\langle K^*(p',\lambda)|\bar{s}\gamma^\mu\gamma_5 b|B(p)\rangle = 2M_{K^*} A_0(q^2)\tfrac{e_\lambda^*\cdot q}{q^2} q^\mu$$

$$+(M_B + M_{K^*})A_1(q^2)\left[e_\lambda^{*\mu} - \tfrac{e_\lambda^*\cdot q}{q^2} q^\mu\right]$$

$$-A_2(q^2)\tfrac{e_\lambda^*\cdot q}{M_B + M_{K^*}}\left[p^\mu + p'^\mu - \tfrac{M_B^2 - M_{K^*}^2}{q^2} q^\mu\right] \qquad .$$

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$$\left.(e^*_\lambda \cdot q)(p+p')_\mu\right] + 2iT_3(q^2)(e^*_\lambda \cdot q)\left[q_\mu - \frac{q^2}{M \dots}\right.$$

$$\langle K^*(p',\lambda)|\bar{s}\gamma^\mu b|B(p)\rangle = \frac{2iV(q^2)}{M_B + M_{K^*}} \dots p_\rho p_\sigma,$$

Factor of 2 different from Becirevic, et al; Ball, et al.

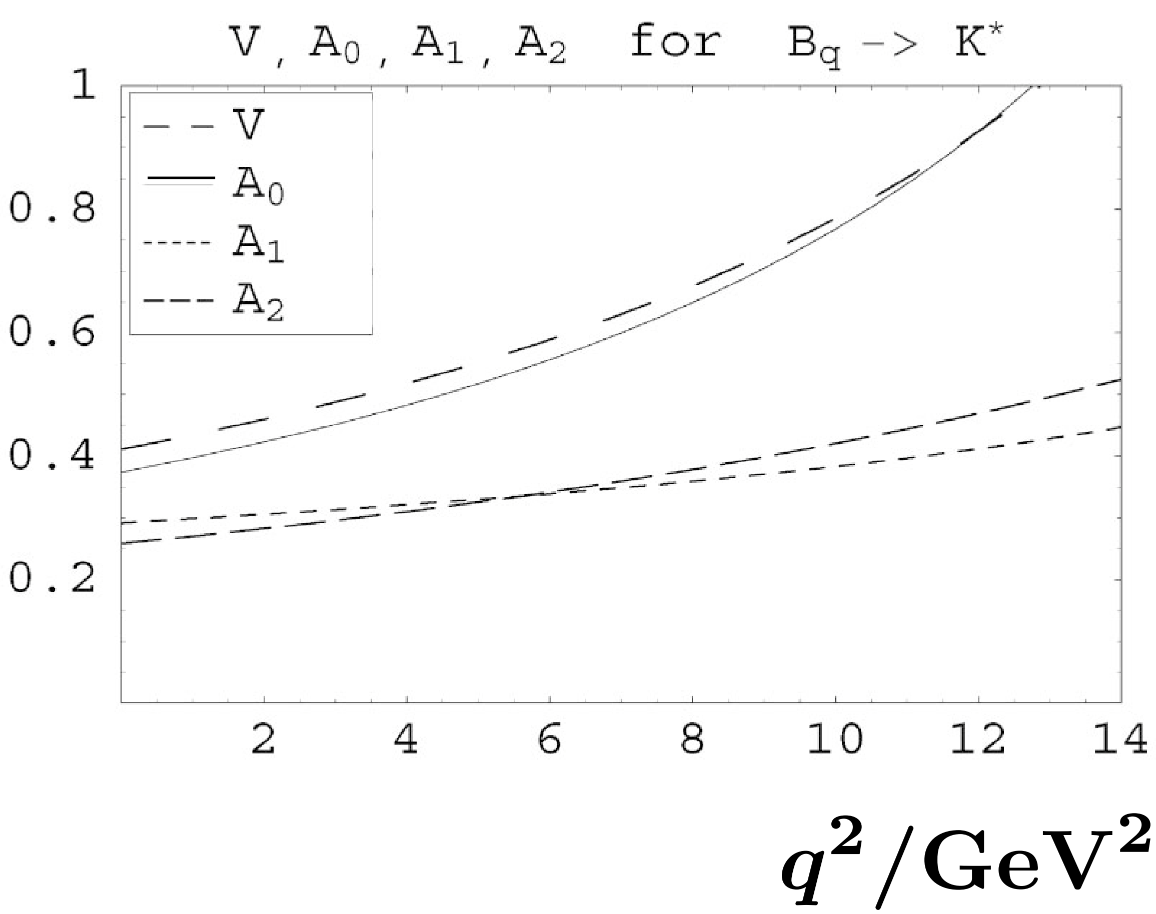
$$\langle K^*(p',\lambda)|\bar{s}\gamma^\mu\gamma_5 b|B(p)\rangle = 2M_{K^*}A_0(q^2)\frac{e^*_\lambda\cdot q}{q^2}q^\mu$$

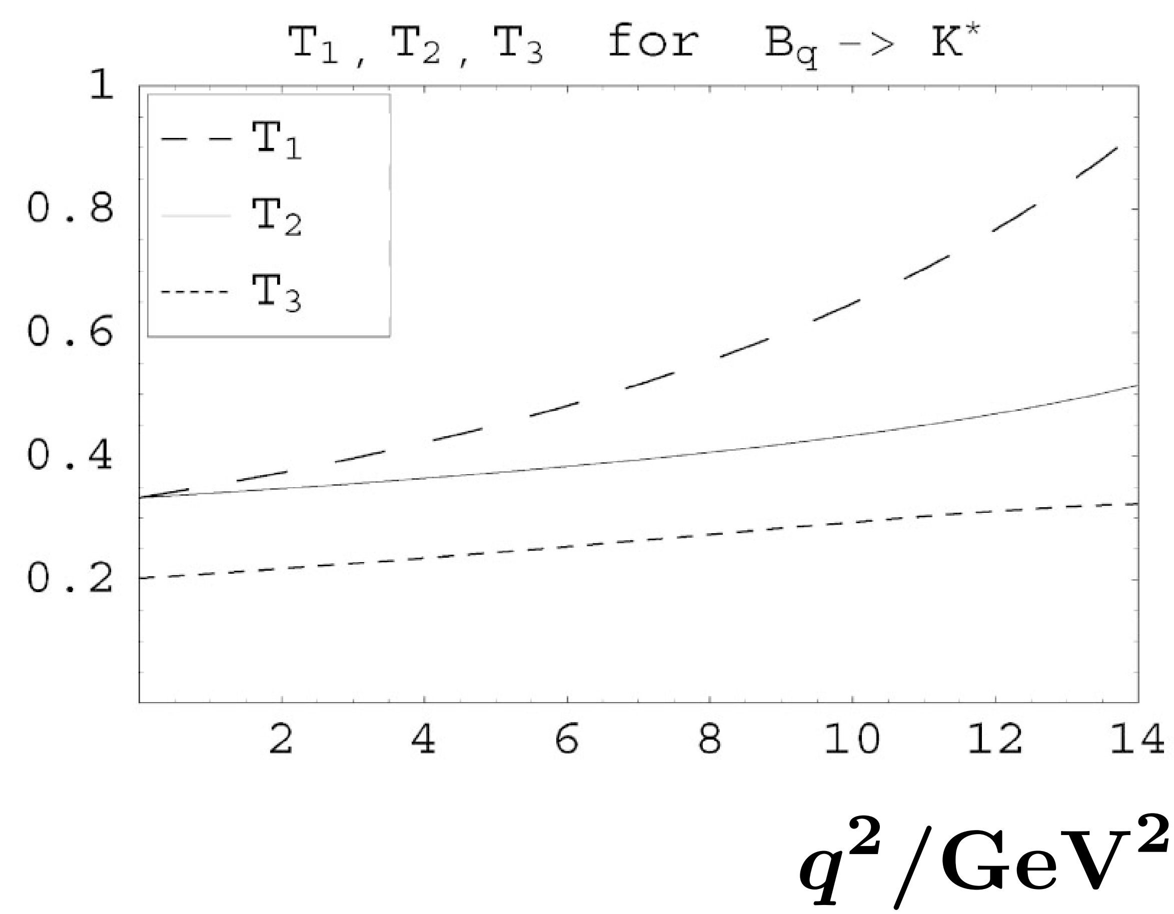
$$+(M_B+M_{K^*})A_1(q^2)\left[e_\lambda^{*\mu} - \frac{e^*_\lambda\cdot q}{q^2}q^\mu\right]$$

$$-A_2(q^2)\frac{e^*_\lambda\cdot q}{M_B+M_{K^*}}\left[p^\mu + p'^\mu - \frac{M_B^2-M_{K^*}^2}{q^2}q^\mu\right] \quad .$$

Sum rule determinations

Ball & Zwicky, Phys. Rev. D **71**, 014029 (2005)





ρ,ω final states in same paper; π, η, *K* final states in Ball & Zwicky, PRD **71**, 014015 (2005)

Early LQCD work on $B \to K^* \gamma$

- Bowler, *et al.* (UKQCD) (1994)
- Bernard, Hsieh, Soni (1994)
- Abada, *et al.* (APE) (1996)
- Bhattacharya and Gupta (1995)
- Del Debbio, *et al.* (UKQCD) (1998)

BLM quenched $B \to K^*$

Bećirević-Lubicz-Mescia, Nucl. Phys. B769, 31 (2007)

- Most recent study of form factor for $B \to K^* \gamma$
- Calculate with heavy quarks such that $m_H \approx m_D$
  - Allows calculation with $q^2 = 0$
  - Extrapolate using

$$T^{H\to V}(0) \times m_{H_s}^{3/2} = c_0 + c_1 m_{H_s}^{-1} + c_2 m_{H_s}^{-2}$$

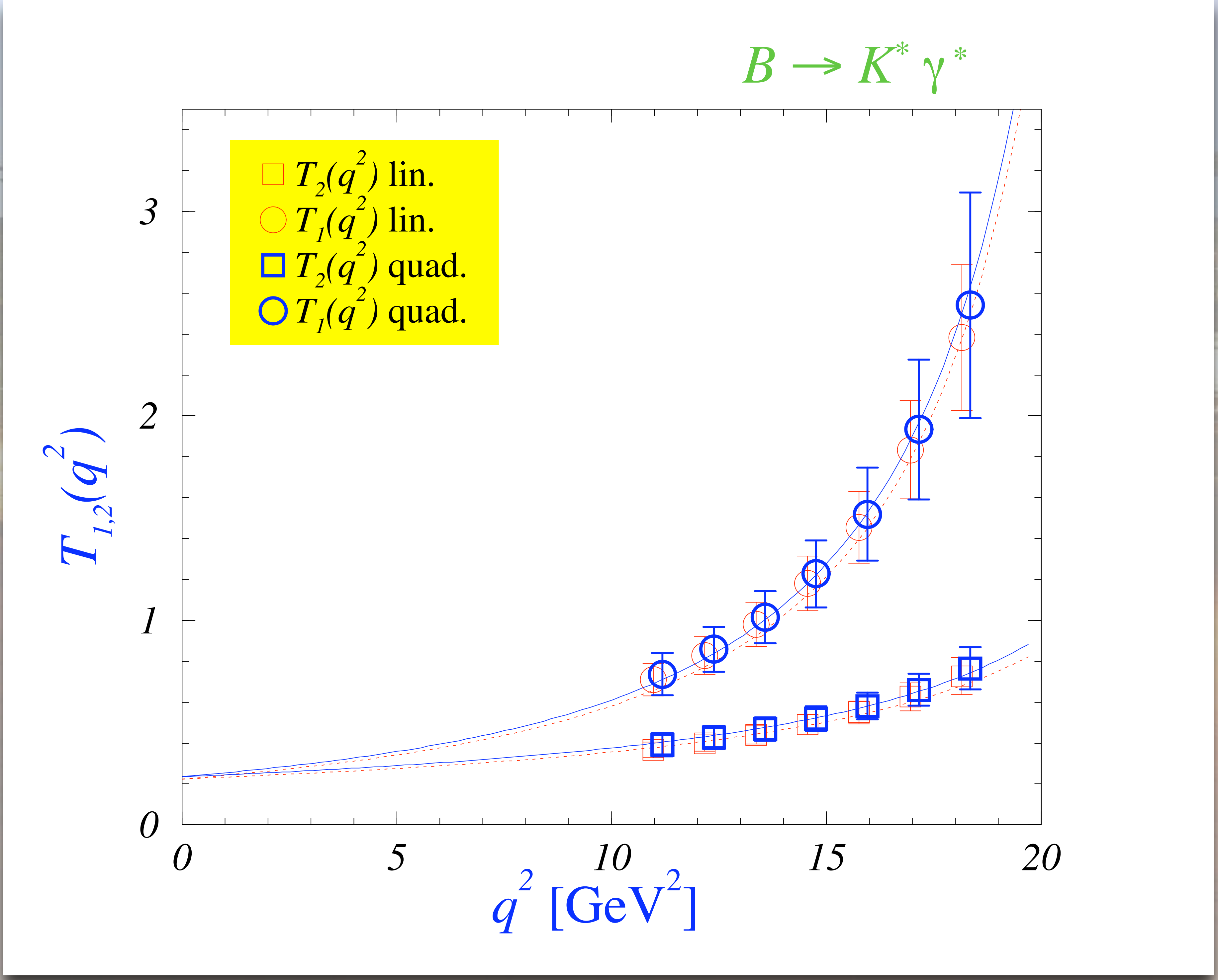
- Quenched result:

$$T^{B\to K^*}(q^2 = 0; \mu = m_b) = 0.24 \pm 0.03^{+0.04}_{-0.01}$$

$$T^{B\to K^*}(0)/T^{B\to \rho}(0) = 1.2 \pm 0.1$$

BLM quenched $B \to K^*$

Većirević-Lubicz-Mescia, Nucl. Phys. B769, 31 (2007)



UKQCD quenched $B \to \rho l \nu$

Bowler, Gill, Maynard, Flynn, JHEP 05 (2004) 035

- Form factors A_0, A_1, A_2, V
- Extrapolate up in m_Q
- Partially integrated decay rate
- For $\mathbf{12.7\ GeV^2} < q^2 < \mathbf{18.2\ GeV^2}$

$$\Gamma_{PI}/|V_{ub}| \;=\; 4.9^{+12+\ 0}_{-10-14}\ \mathrm{ps}^{-1}$$

UKQCD quenched $B \to \rho l \nu$

Bowler, Gill, Maynard, Flynn, JHEP 05 (2004) 035

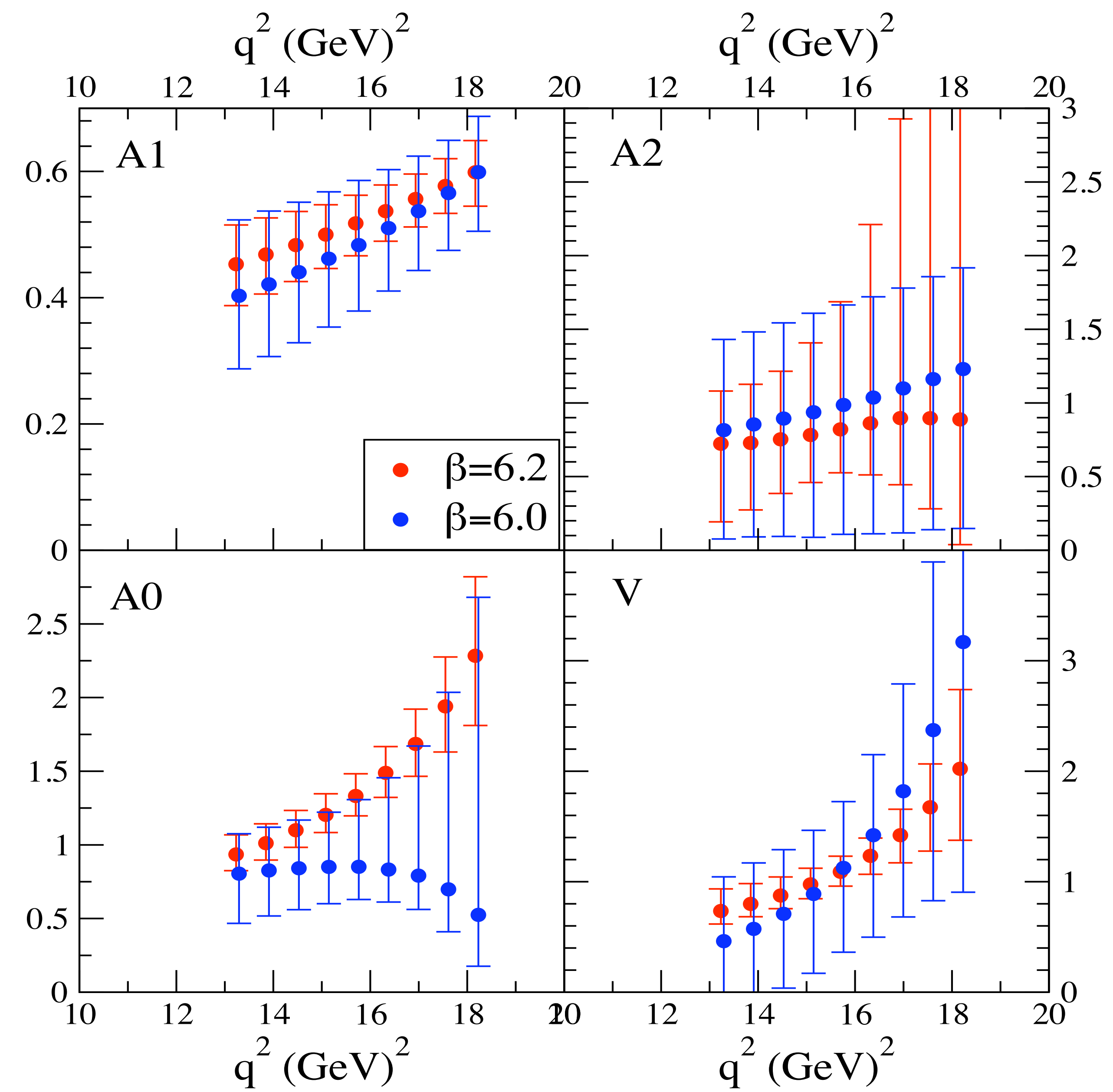


Figure 4: The form factors on both lattices. The vertical scale is different for each form factor.

Lattice NRQCD approach

with **Stefan Meinel, Zhaofeng Liu, Eike Müller,**
A. Hart, R. Horgan

- NRQCD formulation to calculate QCD dynamics of physically heavy *b* quark
- Matching to MSbar scheme in pert. th. (Hart, Horgan, Müller, in prep)
- Can work in lattice frame boosted relative to *B* (Horgan et al, PRD **80**, 2009)
- Stat. and EFT errors mandate working at low recoil
- $N_f = 2 + 1$ (MILC) configurations. No unquenched calculations of $B \to V$ form factors published yet.

Lattice data

Toward high statistics

MILC lattices (2+1 asqtad staggered)

	a(fm)	am_{sea}	Volume	$N_{conf} \times N_{src}$	am_{val}
coarse	$\sim$0.12	0.007/0.05	$20^3 \times 64$	2109×8	0.007/0.04
		0.02/0.05	$20^3 \times 64$	2052×8	0.02/0.04
fine	$\sim$0.09	0.0062/0.031	$28^3 \times 96$	1910×8	0.0062/0.031

Light meson momenta (units of $2\pi/L$)

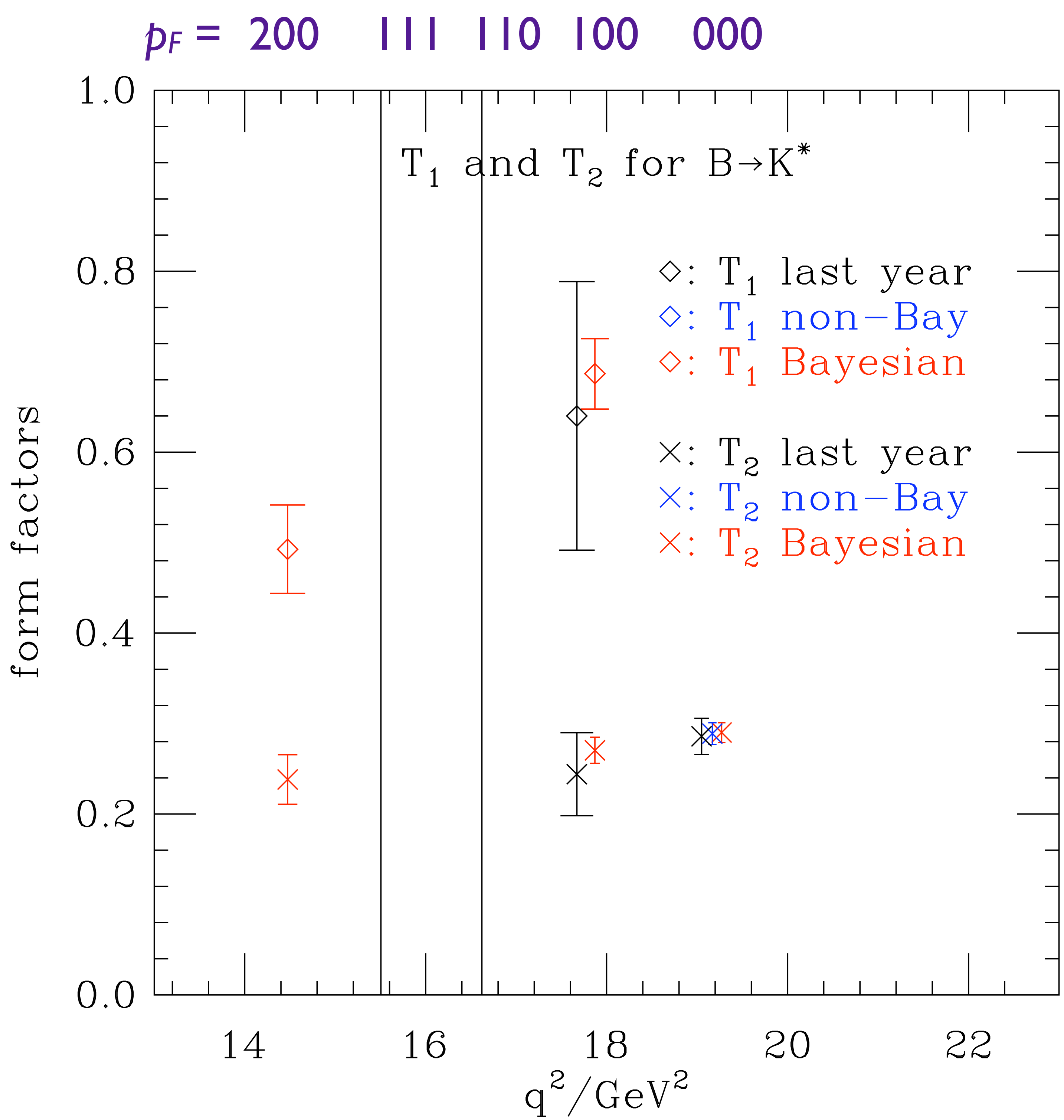
- $(p_x, p_y, p_z) = (0,0,0)$.
- $(\tilde{q}$,0,0), (0,$\tilde{q}$,0), (0,0,$\tilde{q}$), where $\tilde{q}$=1 or 2.
- (1,1,0), (1,-1,0), (1,0,1), (1,0,-1), (0,1,1), (0,1,-1).
- (1,1,1), (1,1,-1), (1,-1,1), (1,-1,-1).

Many Source/Sink separations (16 coarse, 22 fine)

So far, only v=0 NRQCD used (B at rest). Larger v (mNRQCD) next.

Leading order (HQET) current presently used.
$1/m_b$ current matrix elements computed, matching calc. in progress

Preliminary results



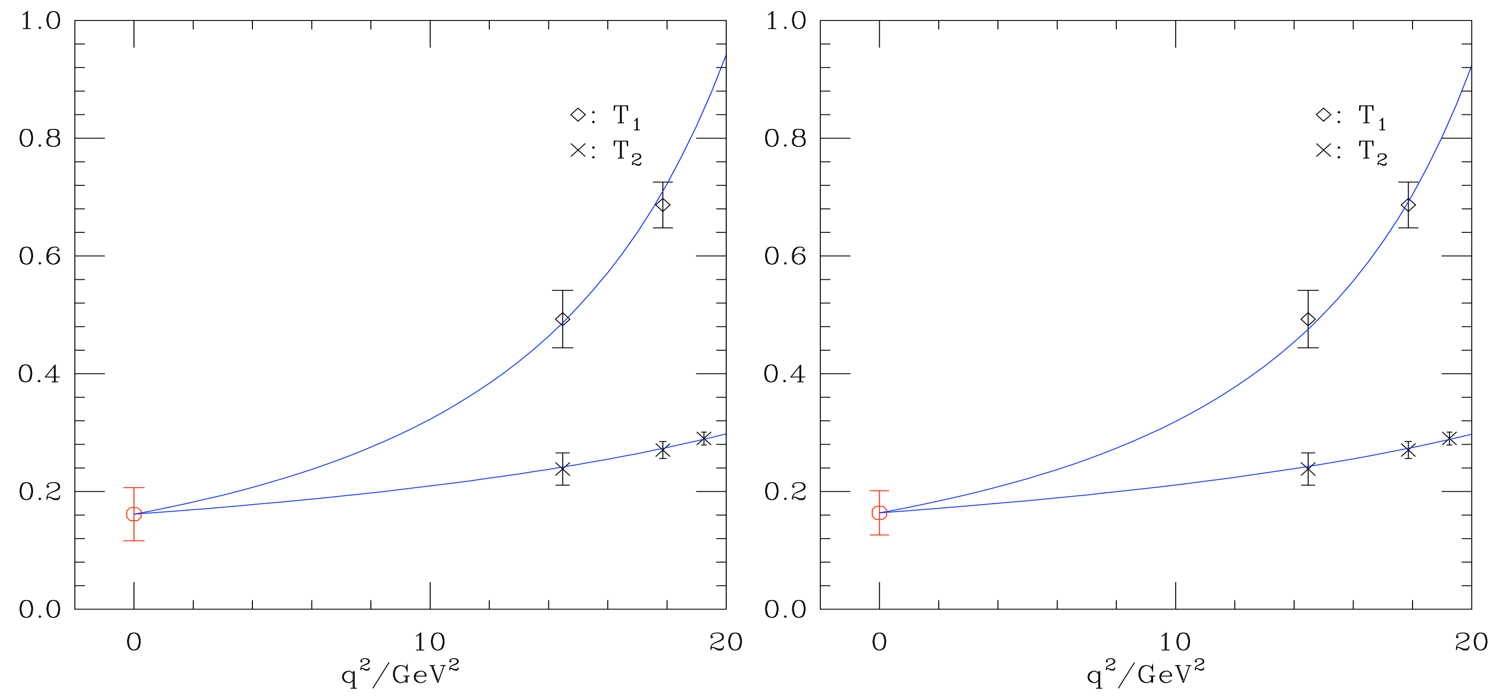
Z. Liu, CKM2010

Preliminary results

Extrapolation of T_1 and T_2 to $q^2 = 0$

Pole dominance [Becirevic & Kaidalov (2000), Ball & Zwicky (2005), Becirevic et al. (2007)]

$$T_1(q^2) = \frac{T(0)}{(1-\tilde{q}^2)(1-\alpha\tilde{q}^2)}, \quad T_2(q^2) = \frac{T(0)}{1-\tilde{q}^2/\beta}, \quad \tilde{q}^2 = q^2/M^2_{B_s^*}.$$



$T(0) = 0.161(45)$ if $M_{B_s^*}$ is a free parameter (left graph).
$T(0) = 0.164(38)$ if $M_{B_s^*} = 5.4158$ GeV is fixed from PDG2010.

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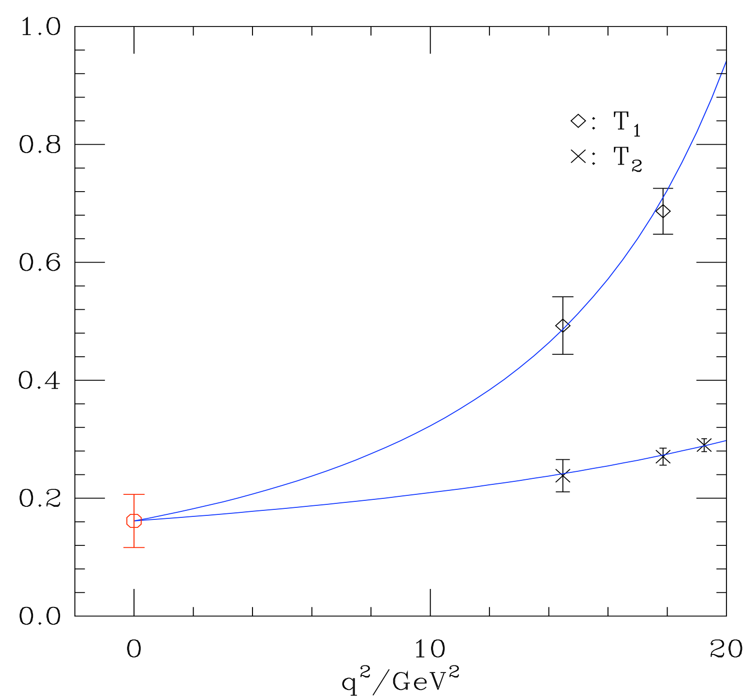
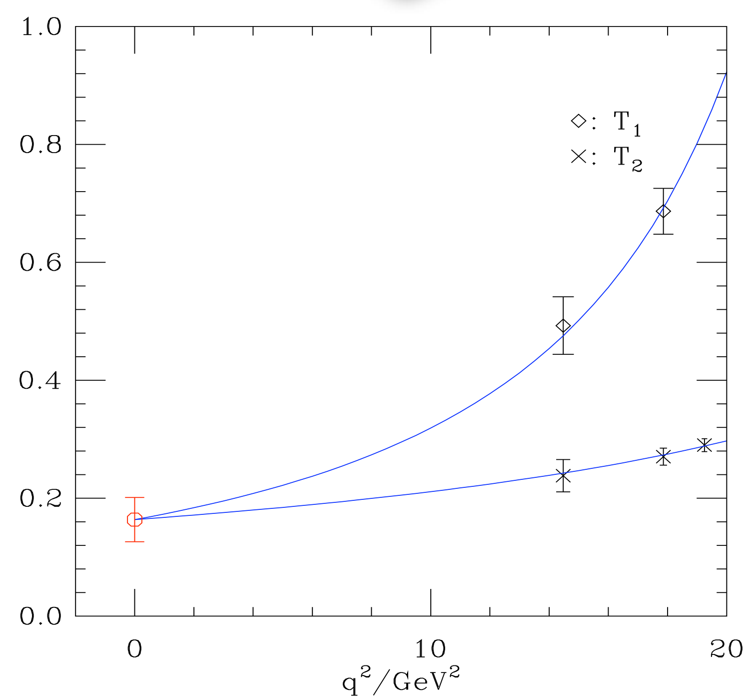
Multiply by 2 to compare Becirevic, et al; Ball, et al.





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Z. Liu, CKM2010

Form factor shapes

- With limited data now, try Becirevic-Kaidalov, Ball-Zwicky formulae
- Much work done on (z) series expansion recently

Boyd, Grinstein, Lebed, PRL (1995); Boyd, Savage, PRD (1997); Caprini, Lellouch, Neubert; Arneson et al, PRL (2005); Bharucha, Feldmann, Wick, arXiv:1004.3249

- HPQCD $D \to K$ paper: generalized expansion to simultaneously fit q^2, m_q, & a dependences

Na et al, arXiv:1008.4562

Light quark mass extrapolation

- In present LQCD calculations, masses and kinematics are such that ρ and K^* are stable
- Could there be large threshold effects in matrix elements?
- Simpler to study ρ and K^* decay constants
- Internal consistency check: $|V_{ub}|$ from $B \to \pi l \nu$ and $B \to \rho l \nu$. Agreement would suggest extrapolations trustworthy within errors.

Summary

- Calculations in high gear now
- Lattice complications
  - Worse statistics than pseudoscalar final states
  - Light quark mass extrapolations through thresholds
- Phenomenology essential
  - Estimate effects of long distance effects
  - Check of sum rule results, esp. at large q^2
- Being further from perfection is no excuse for giving up!