

Near conformal gauge theories & W-TC



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September 2010, Durham

Overview

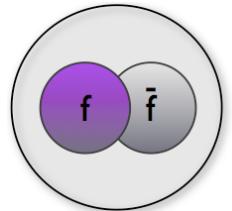
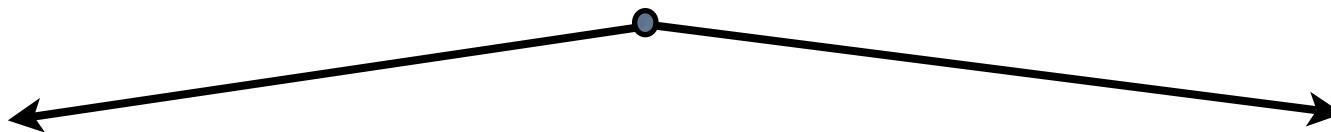
- ★ BSM, Technicolor, Walking-TC \Rightarrow study strongly coupled gauge theories (3)
- ★ General remarks gauge theories - conformal window SUSY & non-SUSY (4)
- ★ conformal gauge theories (CGT) -- observables? (1)

- ★ observables in mass-deformed CGT (6)
 - hyperscaling laws from RG
 - mass scaling from Feynmann-Hellmann thm
 - trajectory mass & decay constants
 - remarks on S-parameter
- Del Debbio & RZ
PRD'10 & arXiv:1009.2894
- ★ Lattice results (2 brief slides)
- ★ Epilogue

Beyond the SM

centered around the Higgs mechanism of SSB
⇒ W, Z masses; technical hierarchy problem?*

*Is the Higgs (object that unitarized $W_L W_L$ -scattering)
fundamental or composite?*

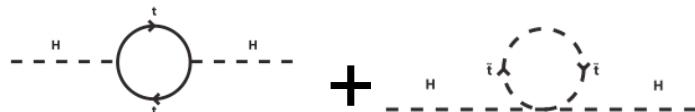


strong dynamics



Supersymmetry
opposite statistics partner

prototype



Technicolour
Higgs sector ⇒ Gauge theory

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes G_{TC}$$

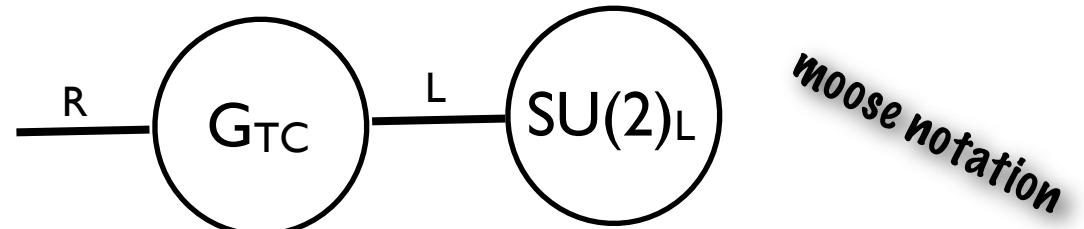
* Flavour sector, where real hierarchies are present, harder for model building

Technicolor

Susskind'79 Weinberg'79



- Higgs sector
→ **strongly** coupled gauge theory



moose notation



- χ -symmetry breaking:

$$\langle \bar{Q}_L Q_R \rangle \sim N_{TC} \Lambda_{TC}^3$$

$$\Lambda_{TC} \sim 4\pi F_T$$

$$F_T = v$$

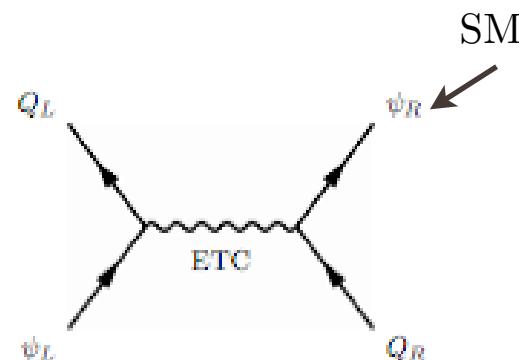
masses W,Z bosons through SSB (as in SM!)



- Fermion masses -> Extended TC

$$G_{SM} \times G_{TC} \subset G_{ETC}$$

Dimopoulos Susskind'79 Eichten Lane'80



breaking: $G_{ETC} \rightarrow G_{SM} \times G_{TC}$

$$\mathcal{L}^{\text{eff}} = \alpha_{ab} \frac{\bar{Q} T^a Q \bar{\psi} T^b \psi}{\Lambda_{ETC}^2} + \beta_{ab} \frac{\bar{Q} T^a Q \bar{Q} T^b Q}{\Lambda_{ETC}^2} + \gamma_{ab} \frac{\bar{\psi} T^a \psi \bar{\psi} T^b \psi}{\Lambda_{ETC}^2} + \dots$$

↑
SM fermion masses

↑
FCNC

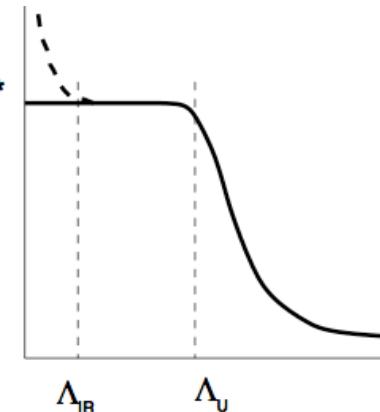
* QCD breaks $SU(2)_c$ spontaneously and gives mass to W boson (orders of mag. too small)

Issues -> Walking TC

- ★ Issues: 1. electroweak precision parameter $S \sim '95$ Lep
2. dynamical generation of fermion masses and FCNC Extended-TC

- ★ ‘Improvement’ Walking-TC: almost reaches IR fixed-point

1. $|S_{WTC}| \ll |S_{TC}|$ 2. $\gamma_{\text{mass}}^* \text{ large}$



- ★ Walking results enhancement of
No parametric definition (challenge)

$$\frac{\langle \bar{Q}Q \rangle_{\text{TC}}}{f_{\pi(\text{TC})}^3}$$

⇒ need to know more about strongly coupled near-conformal gauge theories ..

Gauge theories

- ★ what theorists can adjust:



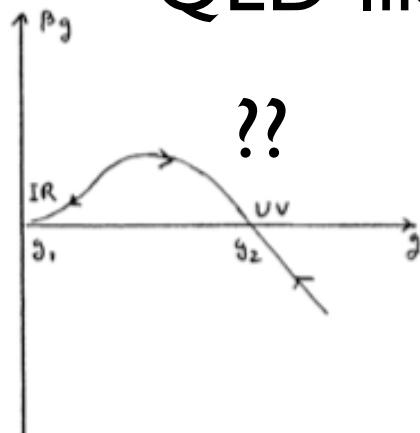
N_c N_F representation

- ★ one coupling theory:

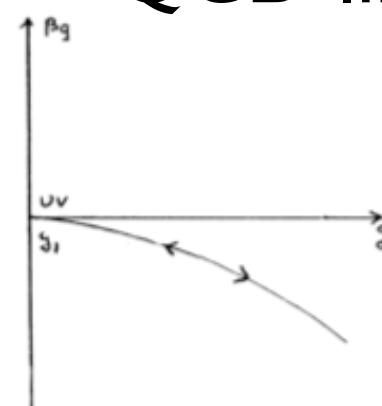
$g^* = 0$ either IR (QED-like) or UV (QCD-like asymptotic freedom) fixed-point

- ★ focus AF-theories ($-\beta_0 < 0$) we know how to handle

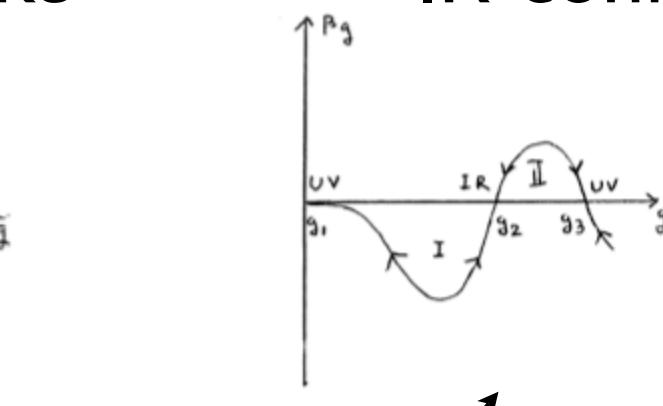
QED-like



QCD-like



IR-conformal

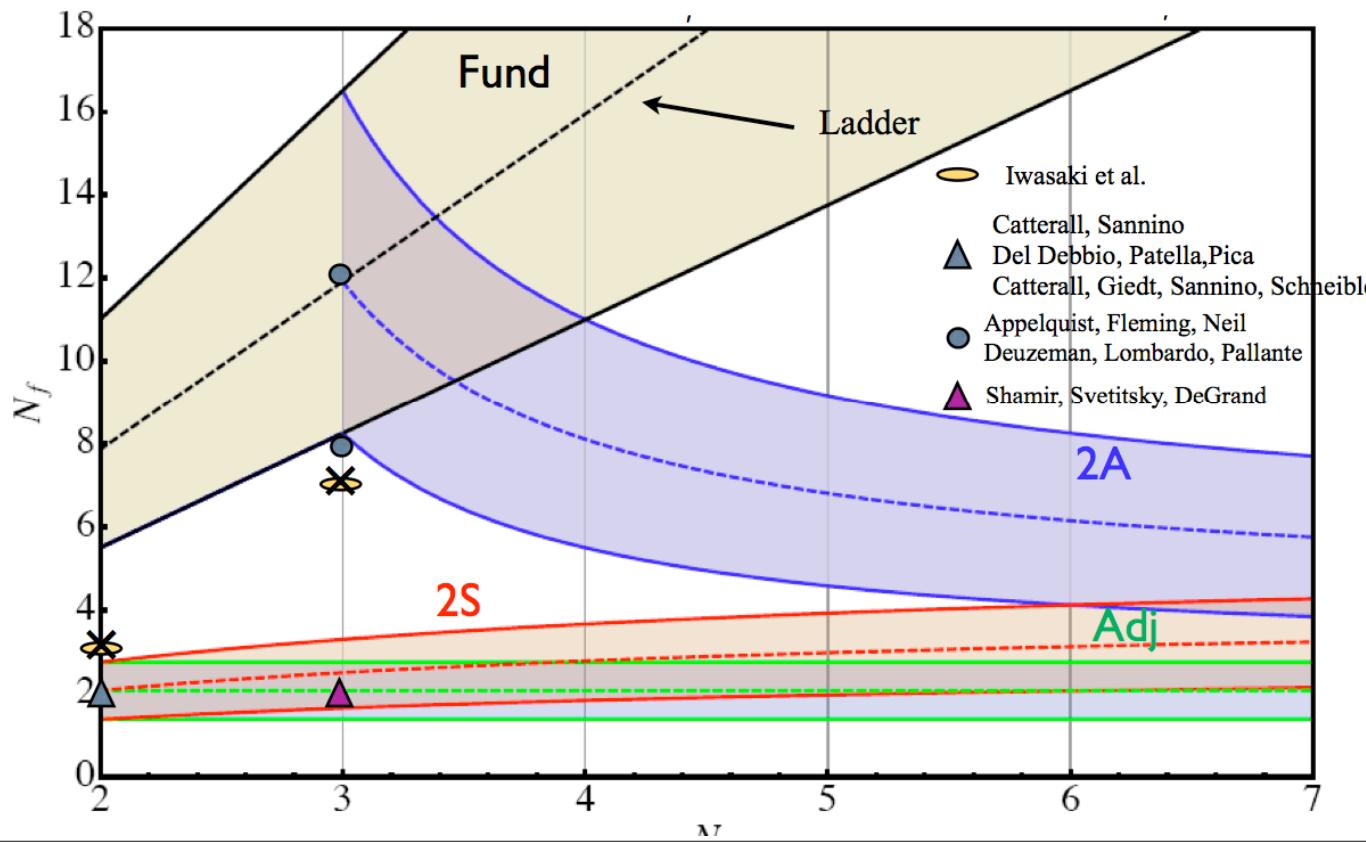


Facts non-SUSY Conformal Window

- ★ QCD chiral symmetry is broken (empirical) \Rightarrow not in CW!
- ★ Banks-Zaks'82 (Belavin-Migdal'76) perturbative IR fixed point (conformal)

If β_0 tuned small $\frac{\alpha_s^*}{2\pi} = \frac{\beta_0}{-\beta_1} \ll 1$ $\beta(\alpha_s) = -\beta_0 \frac{\alpha_s^2}{2\pi} - \beta_1 \frac{\alpha_s^3}{(2\pi)^2} + \dots$

$\alpha_s^* \sim 0.02$



- upper line AF (ok)
- dashed Dyson-Schwinger
- lower unitary bound conjectured β -fct

*proof of
principle*

SUSY Conformal Window

★ Exact NSVZ'83 β -fct: from $\beta = 0$ get γ^*

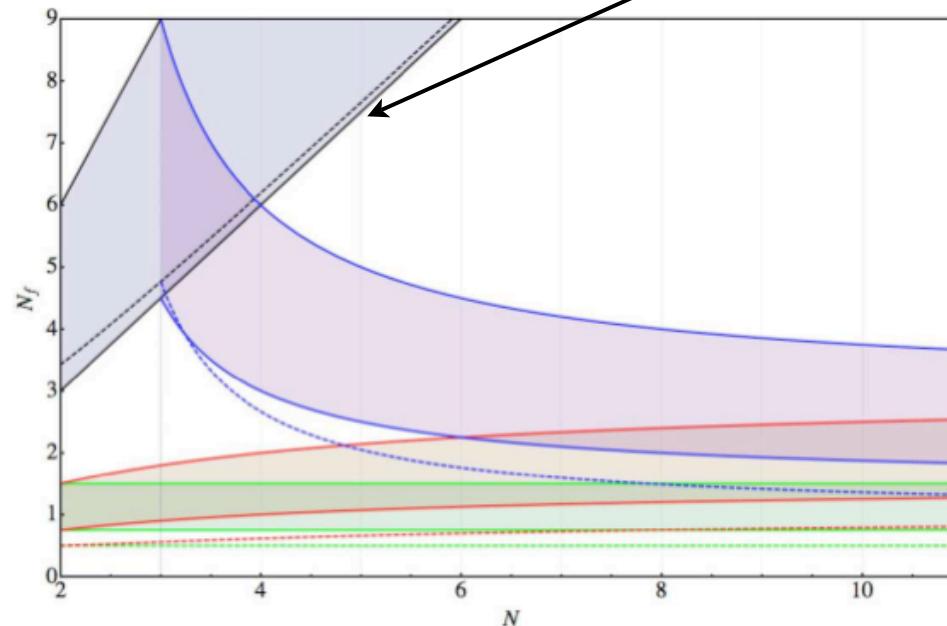
1. **Unitarity bound** on squark-bound state $\Delta_{QQ} = 2 - \gamma^* \geq 1 \Rightarrow \gamma^* \leq 1$

2. **Electric magnetic duality** $N_{\text{dual}} = N_F - N$

perturbative electric BZ-fixed point upper boundary (like QCD)

perturbative magnetic BZ-fixed point lower boundary !!

\Rightarrow weak-strong coupling duality \Rightarrow exist strongly coupled CGT (also from γ^*)



Two objectives

★ AF gauge theories $\approx 1/2$ non-CFT + $1/2$ CFT (SUSY)
(N.B. only known CFT in 4D are GT)

★ strong coupling -- value of γ^*

- SUSY $\mathcal{N} = 1$ tells $\gamma_* \leq 1$
- Dyson-Schwinger $\gamma_* \simeq 1$ then chiral symmetry breaks
- unitarity bound (Mack'77) $\gamma_* \leq 2$

Is the **unitarity bound** ever **reached?**

1. SUSY its because of the squark $\Delta_{QQ} = 2 - \gamma$
2. DS-eqs. truncation -- ladder approximation ... NJL

\Rightarrow we want answers \Rightarrow lattice simulations

size of
conformal window?
strongly coupled?
size γ_*

Obersvables of CFT?

Or how to identify a CFT

- I. Observables: vanishing β -function & $\langle O(x)O(0) \rangle \sim (x^2)^{-\Delta}$; $\Delta = d + \gamma_*$
2. Lattice computation finite m_{quark} (& Volume anyway)

⇒ look mass-deformed conformal gauge theories (mCGT)

$$\mathcal{L} = \mathcal{L}_{\text{CGT}} - m\bar{q}q$$

Obersvables in mCGT

- ★ Goal: analytic guidance for lattice (parametric laws)
- ★ If strongly coupled \Rightarrow hadronic spectrum \Rightarrow beloved hadronic observables

signature of such a theory: each hadronic observable

$$\mathcal{O} \sim m^{\eta_{\mathcal{O}}} , \quad \eta_{\mathcal{O}} > 0 , \eta = f(\gamma_*)$$

- ★ Let's settle some notation:

$$\Delta_{\mathcal{O}} = d_{\mathcal{O}} + \gamma_{\mathcal{O}} , \quad \text{scaling} = \text{physical} + \text{anomalous dimension}$$

$$\gamma_m = -\gamma_{\bar{q}q} , \quad \text{denoted by } \gamma_* \text{ at fixed-point}$$

$$\Rightarrow \Delta_{\bar{q}q} = 3 - \gamma_*$$

Hyperscaling laws

Consider matrix element: $\mathcal{O}_{12}(g, \hat{m}, \mu) \equiv \langle \varphi_2 | \mathcal{O} | \varphi_1 \rangle$

1. $\mathcal{O}_{12}(g, \hat{m}, \mu) = b^{-\gamma_{\mathcal{O}}} \mathcal{O}_{12}(g', \hat{m}', \mu') ,$

$$g' = b^{y_g} g \quad \hat{m}' = b^{y_m} \hat{m} , \quad y_m = 1 + \gamma_* , \quad y_g < 0 \text{ (irrelevant)}$$

2. $\mathcal{O}_{12}(\hat{m}', \mu') = b^{-(d_{\mathcal{O}} + d_{\varphi_1} + d_{\varphi_2})} \mathcal{O}_{12}(\hat{m}', \mu)$

RG-transformation*
 $\mu = b \mu'$
change physical units

3. Choose b s.t. $\hat{m}' = 1$

Hyperscaling relations

$$\Rightarrow \mathcal{O}_{12}(\hat{m}, \mu) \sim (\hat{m})^{(\Delta_{\mathcal{O}} + d_{\varphi_1} + d_{\varphi_2})/y_m}$$

* From Weinberg-like RNG eqs on correlation functions (widely used in critical phenomena)

physical states
no anomalous dim.

Applications:

$$\eta_{\mathcal{O}_{12}} = (\Delta_{\mathcal{O}} + d_{\varphi_1} + d_{\varphi_2})/y_m$$

★ vacuum condensates: $\langle \bar{q}q \rangle \sim m^{\frac{3-\gamma_*}{1+\gamma_*}}$, $\langle G^2 \rangle \sim m^{\frac{4}{1+\gamma_*}}$

*more later
on...*

★ decay constants: ($\Delta_H = d_H = -1$)

\mathcal{O}	def	$\langle 0 \mathcal{O} J^{P(C)}(p)\rangle$	$J^{P(C)}$	$\Delta_{\mathcal{O}}$	$\eta_{G[F]}$
S	$\bar{q}q$	G_S	0^{++}	$3 - \gamma_*$	$(2 - \gamma_*)/y_m$
S^a	$\bar{q}\lambda^a q$	G_{S^a}	0^+	$3 - \gamma_*$	$(2 - \gamma_*)/y_m$
P^a	$\bar{q}i\gamma_5 q$	G_{P^a}	0^-	$3 - \gamma_*$	$(2 - \gamma_*)/y_m$
V	$\bar{q}\gamma_\mu q$	$\epsilon_\mu(p)M_V F_V$	1^{--}	3	$1/y_m$
V^a	$\bar{q}\gamma_\mu \lambda^a q$	$\epsilon_\mu(p)M_V F_{V^a}$	1^-	3	$1/y_m$
A^a	$\bar{q}\gamma_\mu \gamma_5 \lambda^a q$	$\epsilon_\mu(p)M_A F_{A^a}$	1^+	3	$1/y_m$
		$ip_\mu F_{P^a}$	0^-	3	$1/y_m$

★ masses from Trace anomaly: $\theta_\alpha^\alpha|_{\text{on-shell}}^{q \neq 0} = \frac{1}{2}\beta G^2 + N_f m(1 + \gamma_m) \bar{q}q$
 $\beta = 0 \quad \& \quad \langle H(p)|H(k)\rangle = 2E_p \delta^{(3)}(p - k) \Rightarrow$

$$2M_h^2 = N_f(1 + \gamma_*)m\langle H|\bar{q}q|H\rangle$$

$$\sim m^{\frac{2}{(1+\gamma_*)}}$$

GMOR-like relation

■■■

- ★ Summarizing:
 - scaling laws for entire spectrum, decay constants & condensates
 - No goldstone boson due to χ -symmetry breaking

There is no chiral perturbation theory

- ★ Credits (presentation focused last paper):
 - lowest mass state Miransky '98
 - quark condensate (just stated) DeGrand'09
 - all lowest state results DelDebbio RZ'10 May (large time euclidian correlators)
 - all state results DelDebbio RZ'10 Sep

- ★ A point that can be clarified:
 - $M_H \sim m^{1/(1+\gamma^*)}$ looks a bit like heavy quark physics
 - The definite signature is $f_{P(B\text{-meson})} \sim m^{-1/2}$ whereas $f_{P(mCGT)} \sim m^{(2-\gamma^*)/(1+\gamma^*)}$

Mass scaling without RG

Del Debbio, RZ Sep'10

Hellmann-Feynman-Thm

$$\frac{\partial E_\lambda}{\partial \lambda} = \langle \psi(\lambda) | \frac{\partial \hat{H}(\lambda)}{\partial \lambda} | \psi(\lambda) \rangle$$

idea: $\frac{\partial \langle \psi(\lambda) | \psi(\lambda) \rangle}{\partial \lambda} = 0$

★ applied to our case:

$$m \frac{\partial M_h^2}{\partial m} = N_f m \langle H | \bar{q} q | H \rangle$$

★ combined with GMOR-like .. $m \frac{\partial M_H}{\partial m} = \frac{1}{1+\gamma_*} M_H$

$$M_H \sim m^{\frac{1}{1+\gamma_*}}$$

scaling law
without using RG!

Generalized Banks-Casher relation

★ Banks & Casher '80 a la Leutwyler & Smilga 92':

Green's function: $\langle q(x)\bar{q}(y) \rangle = \sum_n \frac{u_n(x)u_n^\dagger(y)}{m-i\lambda_n}$, where $D u_n = \lambda_n u_n$

$$\langle \bar{q}q \rangle_V = \frac{1}{V} \int dx \langle \bar{q}(x)q(x) \rangle = -\frac{2m}{V} \sum_{\lambda_n > 0} \frac{1}{m^2 + \lambda_n^2} \stackrel{V \rightarrow \infty}{=} -2m \int_0^\infty d\lambda \frac{\rho(\lambda)}{m^2 + \lambda^2}$$

★ UV-divergences later -- focus IR-physics

$$\langle \bar{q}q \rangle \stackrel{m \rightarrow 0}{\sim} m^{\eta_{\bar{q}q}} \Leftrightarrow \rho(\lambda) \stackrel{\lambda \rightarrow 0}{\sim} \lambda^{\eta_{\bar{q}q}}$$

★ QCD : $\eta_{\bar{q}q} = 0 \Rightarrow \rho(0) = -\pi \langle \bar{q}q \rangle$

Banks, Casher'80

mCGT: another way to measure anomalous dimension

DeGrand'09
DelDebbio RZ'10 May

Mass & decay constant trajectory

★ At large- N_c neglect width $\rightarrow g_{H_n} \equiv \langle 0 | \mathcal{O} | H_n \rangle$ (decay constant)

$$\Delta(q^2) \sim \int_x e^{ixq} \langle 0 | \mathcal{O}(x) \mathcal{O}(0) | 0 \rangle = \sum_n \frac{|g_{H_n}|^2}{q^2 + M_{H_n}^2}$$

★ In limit $m \rightarrow 0$

$$\Delta(q^2) = \int_0^\infty \frac{ds s^{1-\gamma_*}}{q^2 + s} + \text{s.t.} \propto (q^2)^{1-\gamma_*}$$

★ Solution are given by:

$$M_{H_n}^2 \sim \alpha_n m^{\frac{2}{1+\gamma_*}}, \quad g_{H_n}^2 \sim \alpha'_n (\alpha_n)^{1-\gamma_*} m^{\frac{2(2-\gamma_*)}{1+\gamma_*}}$$

where α_n arbitrary function (corresponds freedom change of variables in \int)

★ QCD expect $\alpha_n \sim n$ (linear radial Regge-trajectory) (few more words)

★ For those who know: resembles deconstruction Stephanov'07
difference physical interpretation of spacing due to scaling spectrum

remarks S-parameter

Analytical guidance S-parameter: $S = 4\pi\Pi_{V-A}(0) - \text{pion pole}$

$$(q^\mu q^\nu - q^2 g^{\mu\nu}) \delta_{ab} \Pi_{V-A}(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T(V_a^\mu(x) V_b^\nu(0) - (V \leftrightarrow A)) | 0 \rangle$$

$$\Pi_{V-A}(q^2) \simeq \frac{f_V^2}{m_V^2 - q^2} - \frac{f_A^2}{m_A^2 - q^2} - \frac{f_P^2}{m_P^2 - q^2} + \dots$$

(conspiracy) module cancellations improve on ...

$$\Pi_{V-A}^{\text{W-TC}}(0) \sim O(m^{-1})$$

$$\Pi_{V-A}^{\text{mCGT}}(0) \sim O(m^0)$$

$$\Pi_{V-A}^{\text{mCGT}}(q^2) \sim \frac{m^{2/y_m}}{q^2}$$

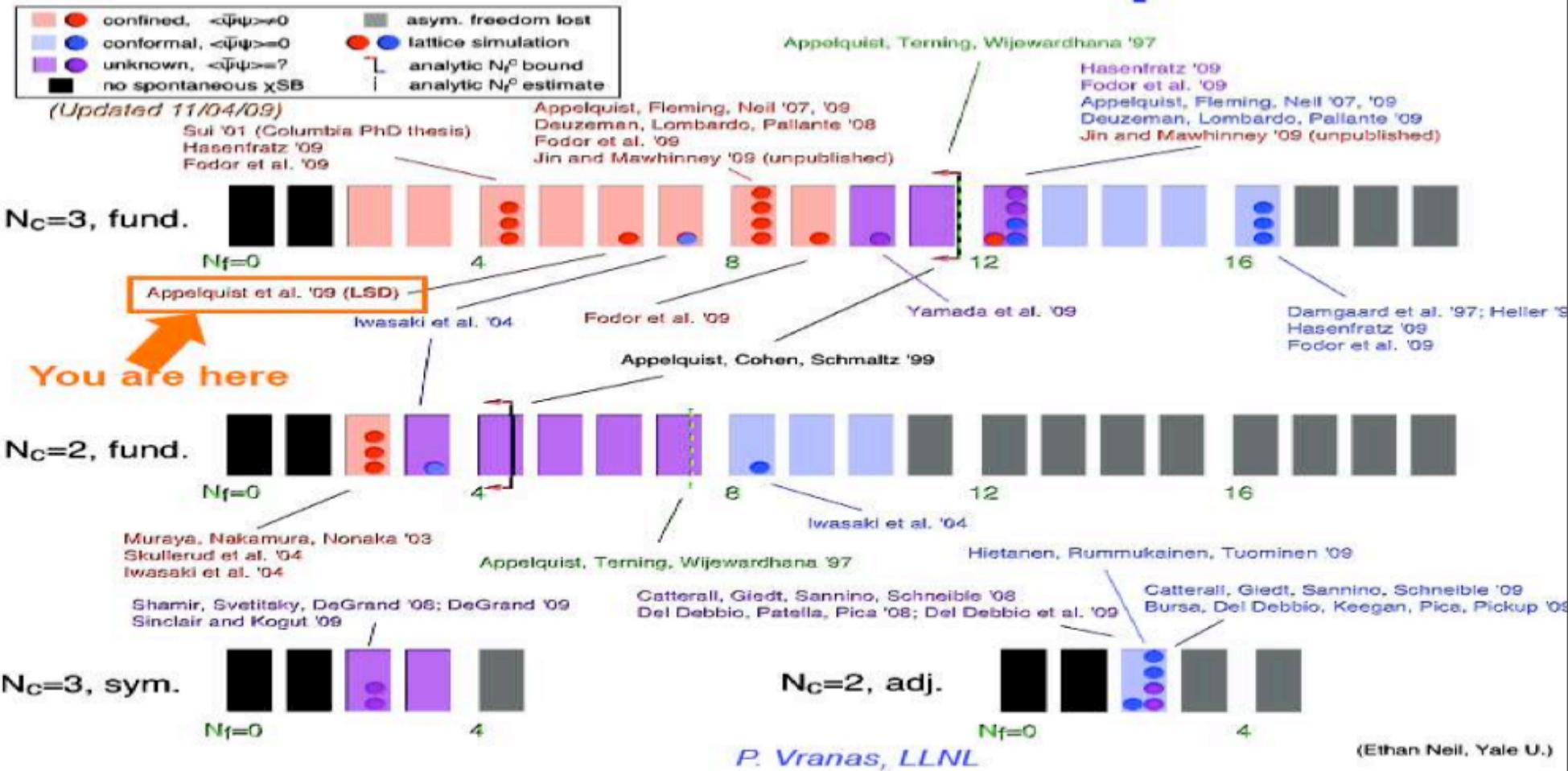
for $-q^2 \gg (\Lambda_U)^2$
Sannino'10 free theory

⇒ lattice determination coming soon (already some market)

Lattice results (due apologies for brevity)

Not Quite the

Current landscape



..... a few points without plots:(

- ★ Ca 7(2) groups UK Swansea/Edbgh), Finnland, Holland, Lin & Onugi
USA (LSD,deGRand, Knuti,Fodor, Caterall & Sannino)
- ★ Measure β -fct (stepsize scaling)
problem: $m \neq 0$ so not fixed-pt β -fct not physical
measuring zero (cancellations)
- ★ measure enhancement $\langle QQ \rangle / f_\pi^3$ (LSD) parametric control?
- ★ Longterm mass/decay constant scaling parametric control should work
- ★ See reasonable results scaling in $0^+, 1^-$ more noisy 0^{++} (as usual)
- ★ typically $\gamma^* \sim 0.4(3?)$ not too large (upper bound difficult)
- ★ so-called MinimalWTC looks conformal \Rightarrow conformal-TC model building

Epilogue

★ Issues not covered:

1. mCGT philosophy has not entered model building: Conformal Technicolor
2. formation of quark condensate & interpretation of IR & UV cut-off

★ Model-building: size of conformal window -- how large anomalous dimension

★ More theoretical questions: trajectories etc

★ Maybe by understanding mCGT better we learn sthg about QCD

Thanks for your attention!

Backup slides ...