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Kaons and Fundamental Physics

- Determination of fundamental parameters
 - lepton universality
 - •CKM unitarity
 - mass determination
- Test suppression of top-dominated FCNCs
 - rare decays
 - •CP violation
- Interplay between SD & LD is essential

Leptonic and Semileptonic

Observables: $K(\pi) \rightarrow l \bar{\nu}_l \quad \& \quad K \rightarrow \pi \, l \bar{\nu}_l$

$$\frac{\Gamma(K_{\ell 2(\gamma)}^{\pm})}{\Gamma(\pi_{\ell 2(\gamma)}^{\pm})} = \underbrace{\left| \frac{V_{us}}{V_{ud}} \right|^2 \frac{f_K^2 m_K}{f_\pi^2 m_\pi} \left(\frac{1 - m_\ell^2 / m_K^2}{1 - m_\ell^2 / m_\pi^2} \right)^2 \times (1 + \delta_{em})$$
Lattice
$$\Gamma(K_{\ell 3(\gamma)}) = \frac{G_F^2 m_K^5}{192\pi^3} C_K S_{ew} |V_{us}|^2 f_+(0)^2 I_K^\ell (\lambda_{+,0}) \left(1 + \delta_{SU(2)}^K + \delta_{em}^{K\ell} \right)^2$$

CKM Unitarity



CKM Unitarity (Model Independent)

[Cirigliano et. al. `09]

$$\Lambda_{\rm NP} \gg M_{W}$$
 Neglect $\Im\left(\frac{M_{W}}{\Lambda_{\rm NP}}\right)$ corrections

Use SU(2) \otimes U(1) invariant operators [Buchmüller-Wyler `06] (plus U(3)⁵ flavour symmetry) $O_{lq}^{(3)} = (\bar{l}\gamma^{\mu}\sigma^{a}l)(\bar{q}\gamma_{\mu}\sigma^{a}q)$ $O_{ll}^{(3)} = \frac{1}{2}(\bar{l}\gamma^{\mu}\sigma^{a}l)(\bar{l}\gamma_{\mu}\sigma^{a}l)$

Constrained from EW precision data [Han, Skiba `05]

Redefine
$$\begin{array}{l} G_F(\mu \to e \, \nu \, \bar{\nu}) \to G_F(1 - 2 \bar{\alpha}_{ll}^{(3)}) \longrightarrow G_F^{\mu} \\ G_F(d \to u \, e \, \bar{\nu}) \to G_F(1 - 2 \bar{\alpha}_{lq}^{(3)}) \longrightarrow G_F^{SL} \end{array}$$

CKM Unitarity (Model Independent)

$$\mathbf{V}_{ud_{i}}^{PDG} = \frac{\mathbf{G}_{F}^{SL}}{\mathbf{G}_{F}^{\mu}} \mathbf{V}_{ud_{i}} \longrightarrow \Delta_{CKM} = 4\left(\overline{\alpha}_{ll}^{(3)} - \overline{\alpha}_{lq}^{(3)} + \dots\right)$$



Leptonic and Semileptonic

 $K(\pi) \to l \bar{\nu}_l \quad \& \quad K \to \pi \, l \bar{\nu}_l$

Observables

$$\begin{split} \mathsf{R}^{\mathsf{NA62}}_{\mathsf{K}} &= 2.500(16) \times 10^{-5} [\texttt{numbers}\\ \texttt{from KAON09]} \\ \mathsf{R}^{\mathsf{KLOE}}_{\mathsf{K}} &= 2.493(25)(19) \times 10^{-5} \end{split} \begin{array}{l} \text{New numbers}\\ \texttt{from NA62 in}\\ \texttt{agreement with SM} \end{split}$$

Test of lepton universality violation driven by experimental precision

Lepton Universality in the MSSM



But: finetuning of m_e necessary [Girrbach et. al. `09]

Model independent MLFV and GUT analysis [Isidori et. al. `09]

Lattice & Continuum: Masses

How do lattice results correspond to the continuum?

Lattice perturbation theory

• Non-perturbative renormalisation Transform to \overline{MS} scheme using perturbation theory



momentum configuration plus 2 point function: fix mass and field renormalisation

Conversion RI/SMOM→MS



off shell: Λ^2/q^2 suppression and good convergence



Rare Kaon Decays

FCNCs which are dominated by top-quark loops:

 $\begin{array}{ll} b \rightarrow s: & b \rightarrow d: & s \rightarrow d: \\ |V_{tb}^* V_{ts}| \propto \lambda^2 & |V_{tb}^* V_{td}| \propto \lambda^3 & |V_{ts}^* V_{td}| \propto \lambda^5 \end{array}$

CKM suppression: enhanced sensitivity to NP

how can we suppress the light quark contribution?

Quadratic GIM:
$$\lambda \frac{m_c^2}{M_W^2}$$

CP violation: $Im(V_{cs}^*V_{cd})$

GIMnastics

Quadratic GIM suppresses light quark contribution





No couplings to γ s: $\mathbf{K} \to \pi \nu \bar{\nu}$



• Dominant Operator: $Q_{\nu} = (\bar{s}_L \gamma_{\mu} d_L)(\bar{\nu}_L \gamma^{\mu} \nu_L)$



$K_L \rightarrow \pi^0 \bar{\nu} \nu$: Effective Hamiltonian



CP violating

[Bord, Gorbahn, Stamou `10]

Only top quark contributes: $H_{eff} = \frac{4G_F}{\sqrt{2}} \frac{\alpha V_{ts}^* V_{td}}{2\pi \sin^2 \Theta_W} X(x_t) Q_v$

Use isospin symmetry and normalise to: $K^+ \rightarrow \pi^0 e^+ \nu$

$$\mathfrak{Br}(\mathsf{K}_{\mathsf{L}} \to \pi^{0} \bar{\nu} \nu) = \kappa_{\mathsf{L}} \left(\frac{\mathsf{Im}(\mathsf{V}_{\mathsf{ts}}^* \mathsf{V}_{\mathsf{td}})}{\lambda^5} \mathbf{X}(\mathsf{x}_{\mathsf{t}}) \right)^2$$

$K_L \rightarrow \pi^0 \nu \bar{\nu}$: Theoretical Status



Experiment: $< 6.7 \times 10^{-8}$ [E391a '08] => K0T0

$$\begin{split} \mathsf{K}^{+} &\to \pi^{+} \, \bar{\nu} \, \nu \, \text{and} \, \mathsf{K}_{L} \to \pi^{0} \, \bar{\nu} \, \nu \\ & \text{Different from } \mathsf{K}_{L} \to \pi^{0} \, \bar{\nu} \, \nu \\ \bullet \, \mathsf{CP \ conserving: \ Top \& \ charm \ contribute} \\ & \hspace{1cm} \mathcal{B}r \left(\mathsf{K}^{+} \to \pi^{+} \nu \bar{\nu}(\gamma)\right) = \mathsf{\kappa}_{+}(1 + \Delta_{EM}) \\ & \hspace{1cm} \times \left| \frac{\mathsf{V}_{ts}^{*} \mathsf{V}_{td} \mathsf{X}_{t}(\mathfrak{m}_{t}^{2}) + \lambda^{4} \mathrm{Re} \mathsf{V}_{cs}^{*} \mathsf{V}_{cd} \left(\frac{\mathsf{P}_{c}(\mathfrak{m}_{c}^{2}) + \delta \mathsf{P}_{c,u}}{\lambda^{5}} \right) \right|^{2} \\ & \hspace{1cm} \left| \frac{\mathfrak{m}_{c}^{2}}{\mathfrak{M}_{W}^{2}} \right|^{2} \text{ suppression lifted by } \log(\frac{\mathfrak{m}_{c}}{\mathfrak{M}_{W}}) \frac{1}{\lambda^{4}} \end{split}$$

Like in
$$K_L \rightarrow \pi^0 \, \bar{\nu} \, \nu$$

 \mathcal{M}^2_W

- Only Q_{ν} : Quadratic GIM & Isospin symmetry
- Top quark contribution like in $\, {\ensuremath{\mathsf{K}}}_L \to \pi^0 \, \bar{\nu} \, \nu$

$K^+ \rightarrow \pi^+ \bar{\nu} \nu$ charm contribution



$$\bar{s}_i \gamma^{\mu} (1 - \gamma_5) q_j \ \bar{q}_j \gamma_{\mu} (1 - \gamma_5) d_i$$

$$\bar{s}_i \gamma^{\mu} (1 - \gamma_5) q_i \ \bar{q}_j \gamma_{\mu} (1 - \gamma_5) d_j$$

$$\bar{c}\gamma^{\mu}\gamma_5 c \ \bar{\nu}\gamma^{\mu}(1-\gamma_5)\nu$$

$$\bar{s}\gamma^{\mu}(1-\gamma_5)q \ \bar{\nu}_l\gamma_{\mu}(1-\gamma_5)l$$
$$\bar{l}\gamma^{\mu}(1-\gamma_5)\nu_l \ \bar{q}\gamma_{\mu}(1-\gamma_5)d$$

• Resum $\log \frac{m_c}{M_W}$ in P_c

P_c at NNLO: ±2.5% (theory) [Buras, Gorbahn, Haisch, Nierste ´06]

NLO EW [Brod, Gorbahn`08]

Long Distance Contribution



No GIM below the charm quark mass scale q^2/m_c^2 higher dimensional operators UV scale dependent

One loop CHPT calculation approximately cancels this scale dependence [Isidori, Mescia, Smith `05]

Also: box-type diagrams considered (from two semileptonic operator insertions) cancelation is more complicated

 $\delta P_{c,u} = 0.04 \pm 0.02$ [Isidori, Mescia, Smith `05]

One Current & One Operator



$K^+ \rightarrow \pi^+ \bar{\nu} \nu$ Long distance

- Matrix element extracted from K₁₃ decays [Mescia, Smith '07]
- $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is $K^+ \rightarrow \pi^+ \nu \bar{\nu} (\gamma)$ QED radiative corrections included:

 $\Delta_{\text{EM}}(\text{E}_{\gamma} < 20 \text{MeV}) = -0.003$ • Uncertainty in $\kappa_{+}(1 - \Delta_{\text{EM}})$ reduced by $\frac{1}{7}$

- Below charm scale: Dimension 8 operators [Falk et. al. '01]
- Together with light quarks: $\delta P_{c,u} = 0.04 \pm 0.02$ [Isidori, Mescia, Smith '05]

$K^+ \rightarrow \pi^+ \bar{\nu} \nu$ Error Budget

Theory error budget $\mathcal{B}_{K^+} = 0.822(69)(29) \times 10^{-10}$



Experiment [E787, E949 '08]

 $\mathfrak{Br}_{\mathsf{K}^+} = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$

Talk by Goadzovski on NA62 => 10%

$K_L \rightarrow \pi^0 l^+ l^-$: Three Contributions







$K_L \rightarrow \pi^0 l^+ l^-$: Three Contributions



$K_L \rightarrow \pi^0 l^+ l^-$: Improvements

- Measure both $\mathfrak{Br}_{e^+e^-}$ and $\mathfrak{Br}_{\mu^+\mu^-}$: [Mescia et. al. '06] Disentangle short distance contribution (y_{7V} , y_{7A})
- Dominant theory error in a_s : Forward backward asymmetry. [Mescia, Smith, Trine '06] Better measurement of $K_S \rightarrow \pi^0 l^+ l^-$. [Smith '07]



$$\begin{array}{ll} \mbox{[KTEV '04]} & \mbox{[KTEV '00]} \\ Br_{e^+e^-} & Br_{\mu^+\mu^-} \\ < 28 \times 10^{-11} & < 38 \times 10^{-11} \end{array}$$

One Current & One Operator



What about the two photon contribution?

€_K :Indirect CPViolation



- In almost all old analysis: $\phi_{\varepsilon} = 45^{\circ}$ and $\xi = 0$
- In reality: $\xi \neq 0$ $\phi_e \neq 45^{\circ}$ [Andrivash et. al.'04]

$$|\epsilon_{\rm K}^{\rm SM}| = |\epsilon_{\rm K}|(\phi_{\rm c} = 45^{\circ}, \xi = 0)$$

+ similar contribution as $\delta P_{c,u}$ in ε_K

 $\kappa_{\epsilon} = 0.94 \pm 0.02$ [Buras, Guadagnoli, Isidori `10]

Calculation of $M_{12}^{K} = \langle K^{0} | \mathcal{H}_{eff}^{\Delta S=2} | \bar{K}^{0} \rangle$

Box diagram with internal u,c,t



$$\lambda_i \lambda_j A(x_i, x_j)$$

$$\lambda_{i} = V_{is}^{*} V_{id}$$

plus GIM:



Gives three different contributions for

$$\begin{split} \mathcal{M}_{12}^{\mathsf{K}} &= \langle \mathsf{K}^{0} | \mathcal{H}_{eff}^{\Delta S=2} | \bar{\mathsf{K}}^{0} \rangle \\ & \uparrow \\ & \mathsf{Caveat: first only SD} \end{split}$$

$$\begin{split} \mathcal{H} &\propto \left[\lambda_t^2 \eta_t S(x_t) \right] \quad \mbox{top} \\ + 2\lambda_c \lambda_t \eta_{ct} S(x_c, x_t) \mbox{charm top} \\ + \lambda_c^2 \eta_c S(x_c) \right] b(\mu) \tilde{Q} \quad \mbox{charm} \end{split}$$

 $\tilde{Q} = (\bar{s}_L \gamma_\mu d_L) (\bar{s}_L \gamma^\mu d_L)$

Calculation of $\mathcal{M}_{12}^{\mathsf{K}} = \langle \mathsf{K}^0 | \mathcal{H}_{eff}^{\Delta S=2}$

d





charm $(\log x_c)^0$ hard GIM $(\alpha_s \log x_c)^n$ $\alpha_{s}(\alpha_{s}\log x_{c})^{n}$

-12%

17.7%

η_{ct} : Charm Top at LO





- The Leading Order result $(\alpha_s \log x_c)^n \log x_c$ starts with a $\log x_c$
- Tree level matching
- One-loop Renormalistion
 Group Equation

$$\begin{split} \mathfrak{m}_{c}^{2}\lambda_{c}(\lambda_{c}-\lambda_{u})\,\log\frac{\mathfrak{m}_{c}}{M_{W}}\\ \to \mathfrak{m}_{c}^{2}\lambda_{c}\lambda_{t}\tilde{Q}\log x_{c} \end{split}$$

η_{ct} : Charm Top beyond LO













- One-loop matching at μ_t
- One-loop matching at μ_c
- Two-loop RG running
- Plus d=6 operators NLO [Herrlich, Nierste]
- NNLO: RGE and matching for d=6 operators RGE: [MG, Haisch `04], Matching: [Bobeth, et. al. `00]
- O(10000) diagrams were calculated [Brod, Gorbahn `10]

η_{ct} at NNLO



Long Distance Contribution

 $\epsilon_{\rm K}$ LD of the matrix element is known precisely

$$\int d^{4}x \langle K^{0} | H^{|\Delta S|=1}(x) H^{|\Delta S|=1}(0) | \bar{K}^{0} \rangle$$

$$\int d^{4}x \langle K^{0} | H^{|\Delta S|=1}(x) H^{|\Delta S|=1}(0) | \bar{K}^{0} \rangle$$

$$\int dispersive part$$

$$\epsilon_{K} = e^{i\phi_{\epsilon}} \sin \phi_{\epsilon} \left(\frac{\operatorname{Im}(M_{12}^{K})}{\Delta M_{K}} + \xi \right)$$

$$estimated form \epsilon'$$

estimated in CHPI

no higher dimensional operators and scale cancellation

put everything in: $\kappa_{e} = 0.94 \pm 0.02$ [Buras, Isidori, Guadgnoli `10]

|EK| and Error Budget



 $|V_{cb}| = 406(13) \times 10^{-4}$

Experiment [PDG `10]: $|\epsilon_K| \stackrel{\text{exp.}}{=} 2.229(12) \times 10^{-3}$

New Physics & New Operators

So far we mostly discussed the SM background

New operators are generated by your favourite NP model

Talks by Petrov and Jäger

4 more operators



Donnerstag, 16. September 2010

S



 $K \to \pi \nu \, \bar{\nu}$: very clean and sensitive to short distances Lattice could clarify the long distance contribution to K⁺

the same for \mathcal{E}_{K} (thanks to the improvement by Lattice) Matrix elements for NP?

Closer contact of the perturbative and lattice community could be beneficial (quark masses)

Many more exciting things in kaon physics: ϵ' , unitarity, lepton univ.

