Quark Masses from Lattice QCD

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HPQCD collaboration

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Thanks: The MILC collaboration for making their configurations publicly available.

Outline

- Motivation.
- Basics of Lattice QCD.
- Heavy quarks.
- Light quarks.
- QED effects.
- Conclusions and outlook.

Motivation

- Low-energy QCD is a strongly-coupled QFT. It also confines.
- We need non-perturbative tools to deal with it.
- Lattice QCD provides a non-perturbative definition of QCD. It also provides a first-principles quantitative calculational tool. And lately it is also becoming a precise tool.

Goals

- To make precise calculations in QCD.
- To determine the fundamental parameters of QCD: strong coupling constant and quark masses.

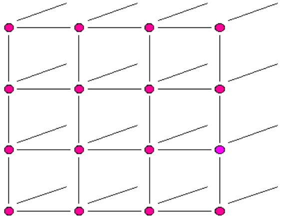
Basics of Lattice QCD

- We start with the (Euclidean) path integral:

$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu e^{-S_{QCD}[\psi,\bar{\psi},A]}$$

$$S_{QCD}[\psi,\bar{\psi},A] = S_G[A] + S_F[\psi,\bar{\psi},A] = S_G[A] + \bar{\psi}M[A]\psi$$

- We introduce a space-time lattice, with length L and lattice spacing a.



- We discretize the action: many possibilities.
- Now the path integral is finite-dimensional.
- High-dimensional integral, Euclidean space-time $\Rightarrow$ Montecarlo integration.

- We eliminate the lattice:
  - Take the infinite volume limit $L \to \infty$.
  - Take the continuum limit $a \to 0$.

Systematic Errors

- Finite volume: $m_\pi^{-1} \ll L$.
- Renormalization constants: The lattice is an ultraviolet regulator. In general, we need to calculate renormalization constants to relate quantities calculated in the lattice with quantities calculated in a different scheme.
- Chiral extrapolation: In practice, we are not able to simulate at physical values of the light quark masses $m_{u,d}$.
- Lattice spacing determination: Error in the determination of the lattice spacing in physical units.

Systematic Errors

Finite lattice spacing: we need simulations at different values of a, to extrapolate to the continuum limit $a \to 0$.

- To simulate at small values of a, while keeping the physical L constant is very expensive.
- Typically, error $\propto a, a^2$
- Improved actions (and operators) lead to smaller errors, making the extrapolation from a given set of lattice spacings more precise.
- ASQ(TAD) (S. Naik, the MILC collaboration, G.P. Lepage.)
  - Discretization errors $\approx \mathcal{O}(\alpha_s \mathrm{a}^2, \mathrm{a}^4)$.
- HISQ (E.F., Q. Mason, C. Davies, K. Hornbostel, G.P. Lepage, H. Trottier.)
  - Discretization errors $\approx \mathcal{O}(\alpha_s \mathrm{a}^2, \mathrm{a}^4)$.
  - Substantially reduced taste-changing.
  - Can be used to study heavy quarks.

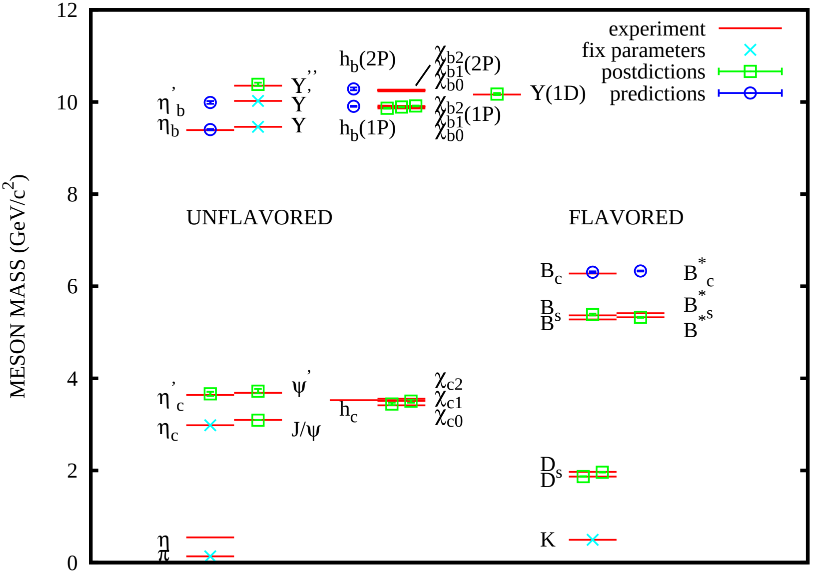
Fixing the parameters

The free parameters in the lattice formulation are fixed by setting a set of calculated quantities to their measured physical values.

- Scale: lattice spacing (equiv. coupling constant) a:
  - We use r_1, related to the heavy quark potential. The value of r_1/a is calculated with high precision on the lattice.
  - We use several quantities to then calculate r_1 in physical units: $m_{\Upsilon'} - m_{\Upsilon}$, $m_{D_s} - m_{\eta_c}/2$, f_{η_s}.

- Quark masses: $m_{u,d}$, m_s, m_c, m_b.
  Fixed by m_π, m_K, m_{η_c}, m_{η_b}.

Gold meson spectrum



Determination of quark masses

The lattice QCD parameters can be accurately tuned. The difficulty is converting to another renormalization scheme.

Direct method

- Conceptually straightforward: adjust the lattice parameters to obtain physical values for a set of quantities: $m_{q,latt}$.
- Now convert to your preferred scheme: $m_{q,\bar{MS}} = Z \;\; m_{q,latt}$
- Z can be calculated from lattice perturbation theory (or a non-perturbative matching.) Very difficult to reduce the errors. The best error with direct method in m_s is $\approx 5\%$ from α_s pert. theory. (Q. Mason et al, HPQCD, hep-lat/0511160.)

Fortunately we have some indirect methods: moments. And ratios of quark masses can be calculated much more accurately and are scheme independent up to lattice artifacts.

Heavy Quarks

- The discretization errors grow with the quark mass as powers of am.
- For a direct simulation, we need:

  $am_h \ll 1$ (heavy quarks)
  $La \gg m_\pi^{-1}$ (light quarks)

- Two scales. Difficult to do directly.
- Instead take advantage of the fact that m_h is large: $\Rightarrow$ effective field theory (NRQCD, HQET). Very successful for b physics.

Relativistic Heavy Quarks

A relativistic formulation has many advantages:

- The parameters of an effective theory have to be matched to QCD, which is both difficult and introduces another systematic error.
- If the action has enough symmetry, there are quantities which do not renormalize. For example, for staggered quarks, meson decay constants do not renormalize because of PCAC.
- Using the same formulation for the heavy and the light quarks is conceptually simpler, and allows, for example, to calculate accurate ratios of quark masses.
- Using the same formulation for the heavy and the light quarks also provides very stringent tests of the lattice methods, because there are very few free parameters: all the calculations should give the right answers once those are fixed.

Charm and bottom quark mass

Work in collaboration with:

K.G. Chetyrkin (Universität Karlsruhe)
J.H. Kühn (Universität Karlsruhe)
M. Steinhauser (Universität Karlsruhe)
C. Sturm (Brookhaven National Laboratory)

- Direct: bare lattice m_c (fixed through m_{η_c}) + lattice PT (2-loops). Very demanding. Combination of diagrammatic + high-β PT.
  Preliminary result: $m_c^{\overline{MS}}(3\text{GeV}) = 0.983(23)\text{GeV}$.

- Method of moments: lattice current-current correlators + high-order (α_s^3) cont. PT.

Method of moments

Compare derivatives of the vacuum polarization function (calculated in PT) with moments of of the experimental cross-section for heavy quark production in e^+e^- annihilation.

Can substitute experiments by lattice data: this allows the use of other currents beyond the vector current (for example, pseudoscalar current.)

$$
\begin{aligned}
G(t) &\equiv a^6 \sum_{\mathbf{x}} (am_{0h})^2 \langle 0 | j_5(\mathbf{x}, t) j_5(0,0) | 0 \rangle \\
j_5 &= \bar{\psi}_h \gamma_5 \psi_h
\end{aligned}
$$

Mass factors $\rightarrow$ independent of the cutoff in the continuum limit (PCAC):

$$
G_{lat}(t) \stackrel{a \to 0}{\to} G_{cont}(t)
$$

Method of moments

$$G_n = \sum_t (t/a)^n G(t)$$

Low n moments perturbative (m_h large).

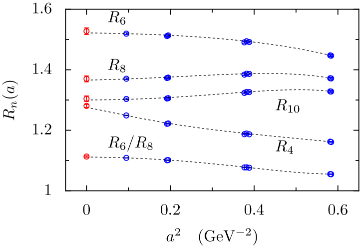
$$G_n = \frac{g_n(\alpha_{\bar{MS}(\mu)}, \mu/m_h)}{(am_h(\mu))^{n-4}}$$

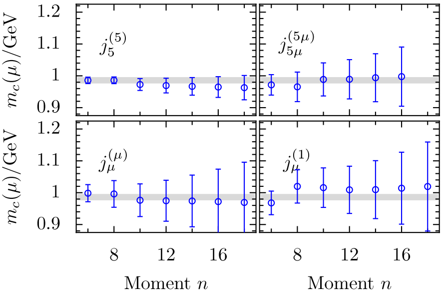
Better to use **reduced** moments:

$$R_n \equiv \begin{cases} G_4/G_4^{(0)} & n = 4 \\ \frac{am_{\eta_h}}{2am_{0h}} \left(G_n/G_n^{(0)}\right)^{1/(n-4)} & n \geq 6 \end{cases}$$

$$R_n = \begin{cases} r_4(\alpha_{\bar{MS}(\mu)}, \mu/m_h) & n = 4 \\ \frac{m_{\eta_h}}{2m_h(\mu)} r_n(\alpha_{\bar{MS}(\mu)}, \mu/m_h) & n \geq 6 \end{cases}$$

Method of moments



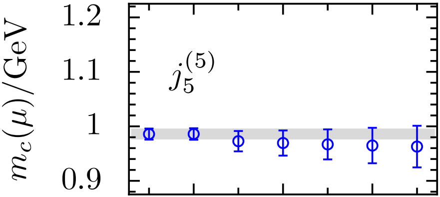


Different currents agree on m_c.

Pseudoscalar determination is the most precise.

Can also extract α_s.

Method of moments: m_c



$m_c^{n_f=4}(3GeV) = 0.986(6)GeV$

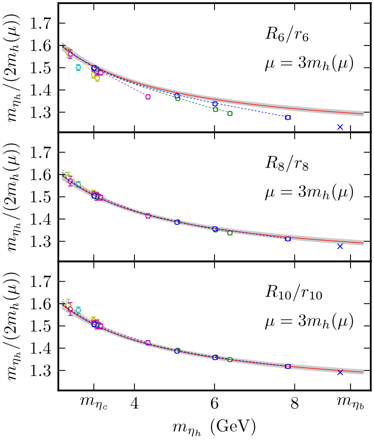
Agrees well with continuum determination from vector current and experimental $R(e^+e)$:

$mc^{n_f=4}(3GeV) = 0.986(13)GeV$

m_c error budget in %	
a^2 extrapolation	0.2
perturbation theory	0.4
$\alpha_{\bar{MS}}$ uncertainty	0.1
gluon condensate	0.2
statistical errors	0.2
errors in r_1/a	0.1
errors in r_1	0.1
$m_{u/d/s}$ extrapolation	0.2
Total	0.6

Method of moments: m_b

Calculate for $m_h > m_c$.



Slight extrapolation to m_b.

$m_b^{n_f=5}(10GeV) = 3.617(25)GeV$

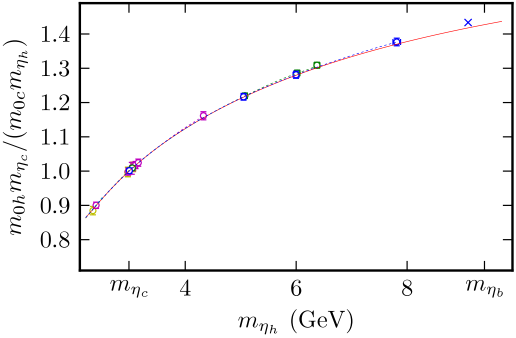
Agrees well with determination

from experimental $R(e^+e)$:

$m_b^{N_f=5}(10GeV) = 3.610(16)GeV$

Quark mass ratios

- Mass ratios can be calculated very accurately, in a purely non-perturbative way.
- We use the same action for all quarks $\Rightarrow$ the matching factors cancel up to lattice artifacts, which vanish when extrapolating



$$\frac{m_{q_1,latt}}{m_{q_2,latt}} \xrightarrow{a\to 0} \frac{m_{q_1,\overline{MS}}(\mu)}{m_{q_2,\overline{MS}}(\mu)}$$

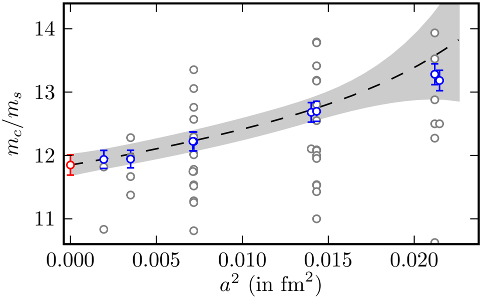
Quark mass ratios

- We can use this as a consistency check for our previous calculation:

$$\left(\frac{m_b}{m_c}\right)^{non-pert} = 4.49(4) \quad \left(\frac{m_b}{m_c}\right)^{pert} = 4.53(4)$$

- Leverage the precision on m_c to get to m_s and m_l.

$$\frac{m_c}{m_s} = 11.85(16) \Rightarrow m_s(2GeV) = 92.4(1.5)Mev$$



Light quarks

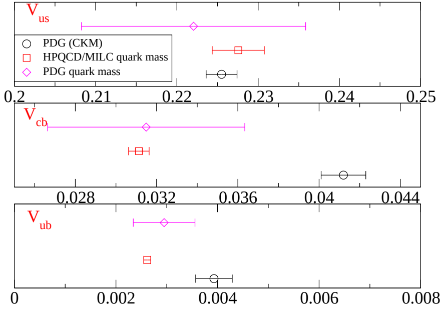
Using MILC's values for the ratios of the strange to the light quark masses, $\frac{m_s}{m_l} = 27.2(3)$, and of u and d quark masses, $\frac{m_u}{m_d} = 0.42(4)$ we obtain:

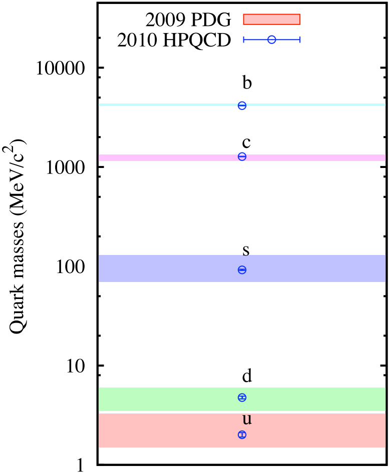
$$\bar{m}_l{}^{(3)}(2Gev) = 3.40(7)Mev$$
$$\bar{m}_u^{(3)}(2Gev) = 2.01(14)Mev$$
$$\bar{m}_d^{(3)}(2Gev) = 4.79(16)Mev$$

Quark masses: summary

Using accurate heavy quark masses and accurate ratios we can leverage to a 1.5% m_s accuracy.

Accurate ratios rule out some quark mass matrix models based on textures.





HPQCD arXiv:1004.4285.

C. McNeile arXiv:1004.4985.

QED effects

In the light quark sector we have strong em. and isospin-violating contributions. They are usually estimated phenomenologically. Simulations with quenched QED are starting to get a handle on these effects from the lattice (Blum et al, 1006:1311; also simulations by MILC.)
Violations of Dashen's theorem are usually parameterized by

$$\left(M^2_{K^\pm} - M^2_{K^0}\right)_{em} = (1 + \Delta_E)\left(M^2_{\pi^\pm} - M^2_{\pi^0}\right)_{em}$$

Blum et al find a value for Δ_E roughly of order $\mathcal{O}(1)$, in line with phenomenological estimates.
They find

$$\begin{aligned}
\left(M^2_{\pi^\pm} - M^2_{\pi^0}\right)^{LO+NLO}_{em} &= 4.50(23)Mev \;\; \text{exp. } 4.5936(5)Mev \\
\left(M^2_{K^\pm} - M^2_{K^0}\right)^{LO+NLO}_{em} &= 1.87(10)Mev \;\; \text{exp. } -3.937(28)Mev \\
\left(M^2_{K^\pm} - M^2_{K^0}\right)_{m_u - m_d} &= -5.840(96)Mev
\end{aligned}$$

Conclusions

- The use of a highly improved quark action and fine enough lattices provides a very good way of doing precision calculations in heavy-heavy and heavy-light systems. In particular it can be used to obtain the most accurate determination of the m_c mass to date.
- Using the same, relativistic action for heavy and light quarks allows us to leverage the precision results for the heavy quark masses to obtain precision results for the light quark masses.
- It provides a stringent test of the lattice methods, because by fixing a small number of parameters in the lattice formulation we should reproduce a large number of experimental measurements.

Outlook

- With 0.03 fm ensembles we should be able to simulate at the b mass without extrapolation. This is a promising way of achieving precision b physics calculations from the lattice.
- Precise results using different fermion discretizations are arriving, and should give us increased confidence that we have the systematic errors under control.
- Simulations which include em. effects (at least in the quenched approximation) are starting to appear. They should provide a handle on em and isospin-breaking effects from first principles.