B-physics from lattice QCD

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Introduction

- Brief review of heavy-light physics on the lattice
- Recent results for f_B, $|V_{cb}|$, $|V_{ub}|$ and $B-\overline{B}$ mixing

Heavy quarks on the lattice

The lattice cut-off is smaller than the heavy quark masses for realistic lattices. Some solutions: heavy quark effective theory(HQET), nonrelativistic QCD, step-scaling...

Fermilab Method:

Continuum QCD $\longrightarrow$ Lattice gauge theory
(using HQET)

nonrelativistic QCD method:

Continuum QCD $\longrightarrow$ Nonrelativistic QCD $\longrightarrow$ Lattice gauge theory

- Both methods require tuning parameters of the lattice action
- The currents and 4-quark operators must be matched as well. Typically this is done in lattice perturbation theory.

Other Approaches

The extrapolation method (Becirevic, et al, hep-lat/0002025; QCDSF, hep-lat/0701015):

In this case one simulates at masses around the charm quark and extrapolates to bottom with fit functions determined from HQET. Recently HPQCD has started using this type of approach with HISQ and ultrafine MILC lattices. Also, ETMC is using this type of approach, including data for a static b quark so that they are interpolating.

The step-scaling method (Guazzini, Sommer, and Tantalo, hep-lat/0609065):

One starts with a small volume where the b quark can be computed directly, where the finite size effects can be eliminated through step scaling functions which give the change of the observables when L is changed to $2L$.

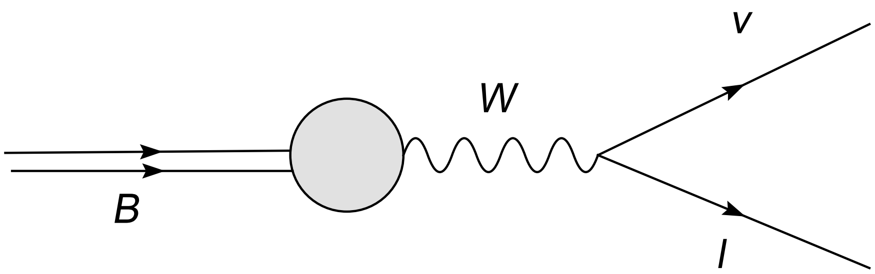
Matching Errors

One must estimate errors due to inexact matching of the lattice to the continuum.

In the Fermilab method, all errors associated with discretizing the action are combined. These errors are then estimated using knowledge of HQET power counting.

In the nonrelativistic QCD method, there are "relativistic errors" associated with using NRQCD [$O(\alpha_s \Lambda_{QCD}/m_Q)$, $O(\Lambda^2_{QCD}/m^2_Q)$], and "perturbation theory errors" associated with matching NRQCD to the lattice [$O(\alpha_s^2)$].

f_B



$$\mathcal{B}(B, \rightarrow \ell\overline{\nu}_\ell) = \frac{G_F^2 m_B m_\ell^2}{8\pi}\left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B \qquad (1)$$

FNAL/MILC projects

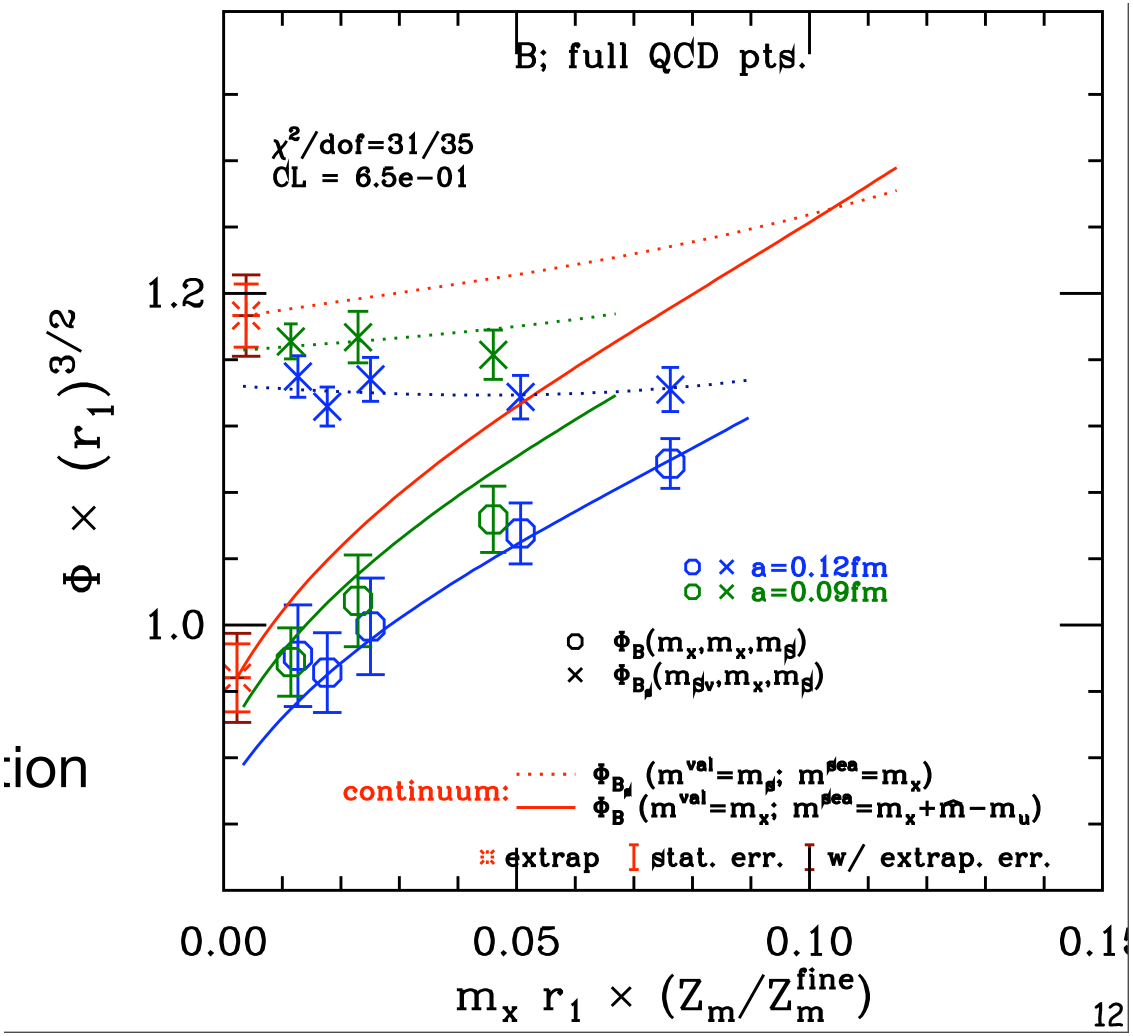
- Heavy-light spectrum calculations, Ludmilla Levkova, Steve Gottlieb, Carleton DeTar
- $B \to D^* \ell \nu$ form factor needed to obtain $|V_{cb}|$ from experiment, JL
- $B \to \pi \ell \nu$ form factor(s) needed to obtain $|V_{ub}|$ from experiment, Ruth Van de Water
- Heavy-light decay constants, Claude Bernard, Doug Toussaint, and Jim Simone
- Heavy-light bag parameters for $B - \overline{B}$ mixing, Todd Evans, Elvira Gamiz

Heavy-light simulation parameters

Table 1: Lattice parameters

a(fm)	$a\widehat{m}'/am'_s$	L(fm)	$m_\pi L$	$10/g^2$	Lat Dim	$\#$ Confs
≈ 0.15	$0.0194/0.0484$	2.4	5.5	6.586	$16^3 \times 48$	628
≈ 0.15	$0.0097/0.0484$	2.4	3.9	6.572	$16^3 \times 48$	628
≈ 0.12	$0.02/0.05$	2.4	6.2	6.79	$20^3 \times 64$	2000
≈ 0.12	$0.01/0.05$	2.4	4.5	6.76	$20^3 \times 64$	2000
≈ 0.12	$0.007/0.05$	2.4	3.8	6.76	$20^3 \times 64$	2000
≈ 0.12	$0.005/0.05$	2.9	3.8	6.76	$24^3 \times 64$	2000
≈ 0.09	$0.0124/0.031$	2.4	5.8	7.11	$28^3 \times 96$	2000
≈ 0.09	$0.0062/0.031$	2.4	4.1	7.09	$28^3 \times 96$	2000
≈ 0.09	$0.0031/0.031$	3.4	4.2	7.08	$40^3 \times 96$	1000
≈ 0.06	$0.0072/0.018$	3	6.33	7.480	$48^3 \times 144$	625
≈ 0.06	$0.0036/0.018$	3	4.49	7.470	$48^3 \times 144$	770
≈ 0.045	$0.0028/0.0140$	3	4.56	7.810	$64^3 \times 192$	860

FNAL/MILC chiral fit



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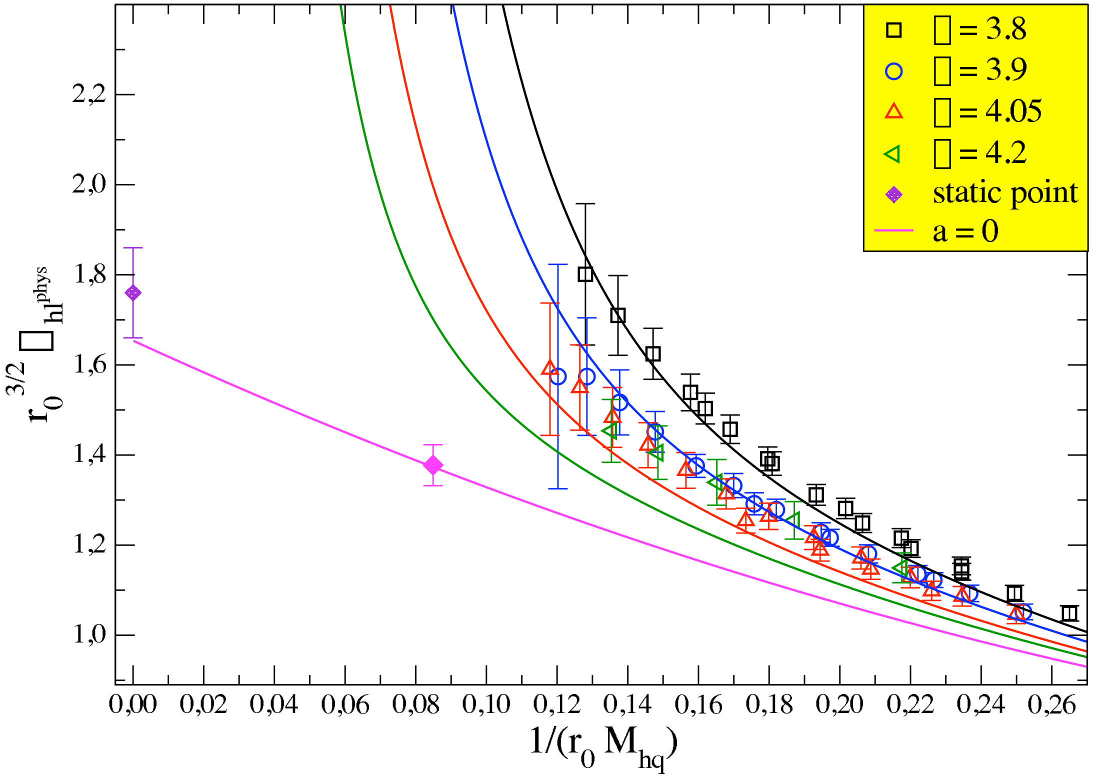
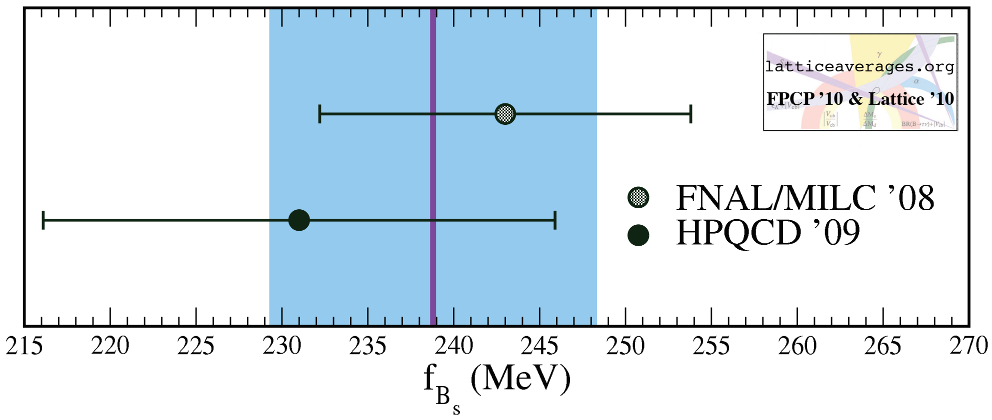
Results from ETMC

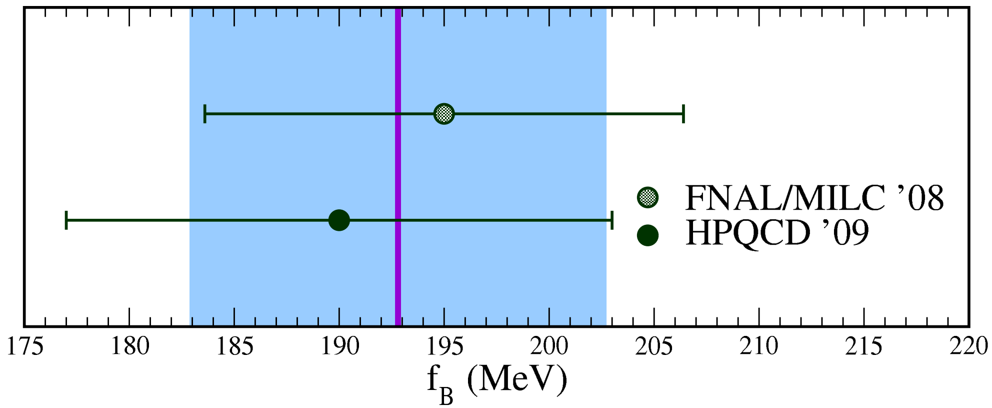


Update on B decay constants

quantity	FNAL/MILC(2010)	HPQCD(2009)	ETMC(2009)
f_{B_d} (MeV)	$211.8 \pm 6.3 \pm 5.5$	190 ± 13	191 ± 14
f_{B_s} (MeV)	$256.3 \pm 5.9 \pm 5.5$	231 ± 15	243 ± 14

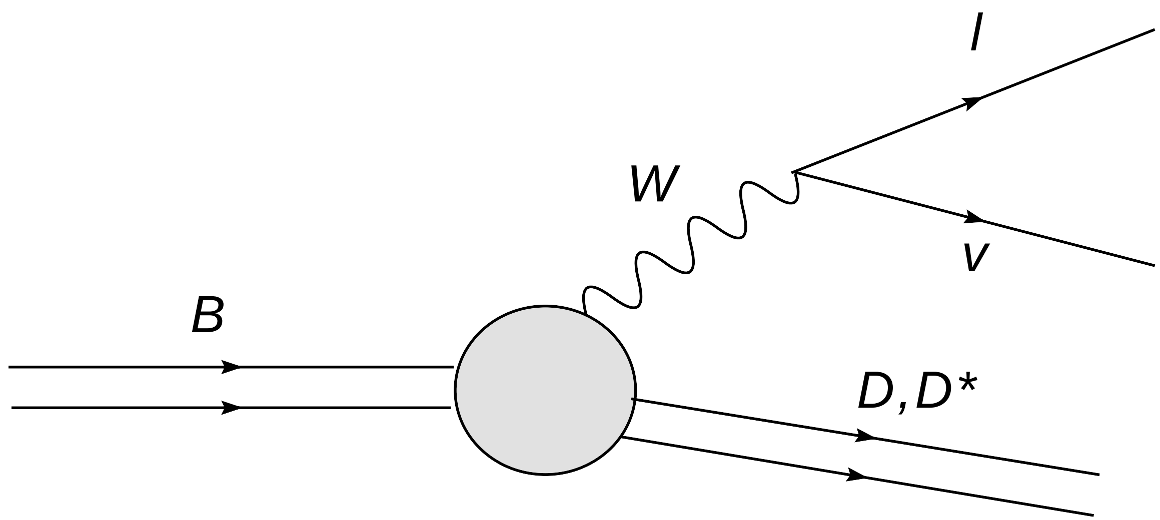
f_B, f_{B_s}





$f_{B_s} = 238.8 \pm 9.5, \quad f_B = 192.8 \pm 9.9$ MeV.

Charmed B semileptonic decays



Vertex proportional to $|V_{cb}|$. In order to extract it, nonperturbative input is needed.

Importance of $|V_{cb}|$

$|V_{cb}|$ is needed to constrain the apex of the unitarity triangle from kaon mixing. Given that

$$A = \frac{|V_{cb}|}{\lambda^2} \tag{2}$$

has $\approx 2\%$ error, we see that this contributes a 9% error to ϵ_K because it appears in the formula below to the fourth power.

$$|\epsilon_K| = C_\epsilon B_K A^2 \overline{\eta}\{-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\overline{\rho})\}$$

Given recent and expected progress in B_K, we must lower the errors on $|V_{cb}|$.

Methods for extracting $|V_{cb}|$

- Inclusive $b \to c\ell\nu$ can be calculated using the OPE and perturbation theory. Requires non-perturbative input from experiment: moments of inclusive form factor $\overline{B} \to X_c \ell \overline{\nu}_\ell$ as a function of minimum electron momentum. Theoretical uncertainties from truncating the OPE and PT, and also perhaps from duality violations.

- Exclusive $B \to D\ell\nu$ has an $\sim 8\%$ experimental error in the zero-recoil point. No problem in principle of going to small recoil on the lattice.

- Exclusive $B \to D^*\ell\nu$ is experimentally cleaner ($\sim 1.7\%$ experimental error at zero-recoil).

Obtaining V_{cb} from $\overline{B} \to D^* l \overline{\nu}_l$

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 \sqrt{w^2 - 1} \times |V_{cb}|^2 \mathcal{G}(w) |\mathcal{F}_{B \to D^*}(w)|^2 \quad (3)$$

where $\mathcal{G}(w)|\mathcal{F}_{B\to D^*}|^2$ contains a combination of form-factors which must be computed non-perturbatively. $w = v' \cdot v$ is the velocity transfer from initial (v) to final state (v').

Calculating $B \to D^*$ form factor

$$\mathcal{F}_{B\to D^*}(1) = h_{A_1}(1), \tag{4}$$

$$\langle D^*(v)|\mathcal{A}^\mu|\overline{B}(v)\rangle = i\sqrt{2m_B 2m_{D^*}}\,\overline{\epsilon'}^\mu h_{A_1}(1). \tag{5}$$

$h_{A_1}(1)$ is constrained by heavy quark symmetry:

$$h_{A_1}(1) = \eta_A \left[1 - \frac{l_V}{(2m_c)^2} + \frac{2l_A}{2m_c 2m_b} - \frac{l_P}{(2m_b)^2}\right] \tag{6}$$

New calculation (Fermilab/MILC)

- We still use the Fermilab method to treat heavy quarks, as in the original quenched calculation of Hashimoto et al, hep-ph/0110253. Discretization effects are down by a power of $\alpha_s(\Lambda/m_Q)^2$.

- Double ratio is constructed which gives the answer more directly then original quenched calculation, allowing a cleaner determination and a huge savings in computing cost ($\sim$ factor of 10)

- The latest work uses four lattice spacings ($a \approx 0.15$ fm, $a \approx 0.12$ fm, $a \approx 0.09$ fm, $a \approx 0.06$ fm) and approximately four times the statistics.

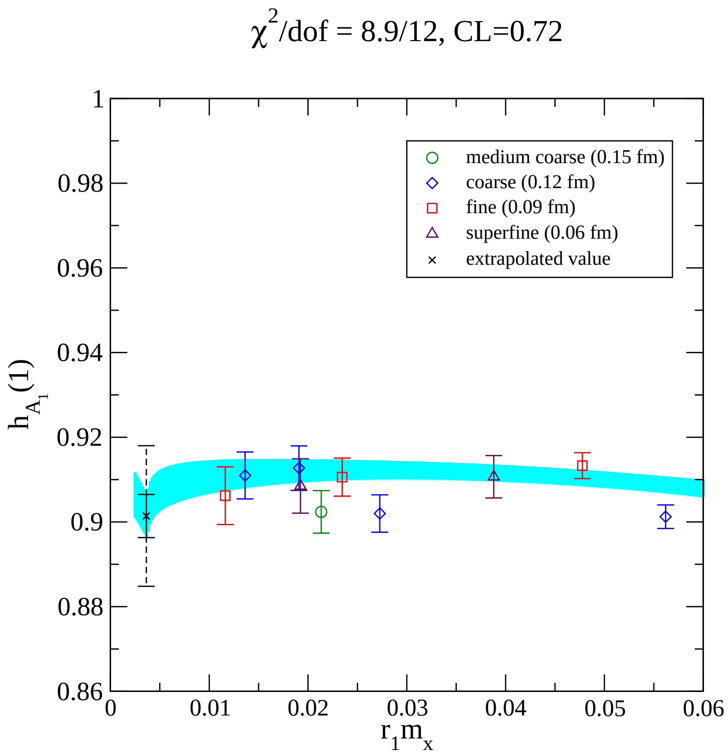
Staggered ChPT formula

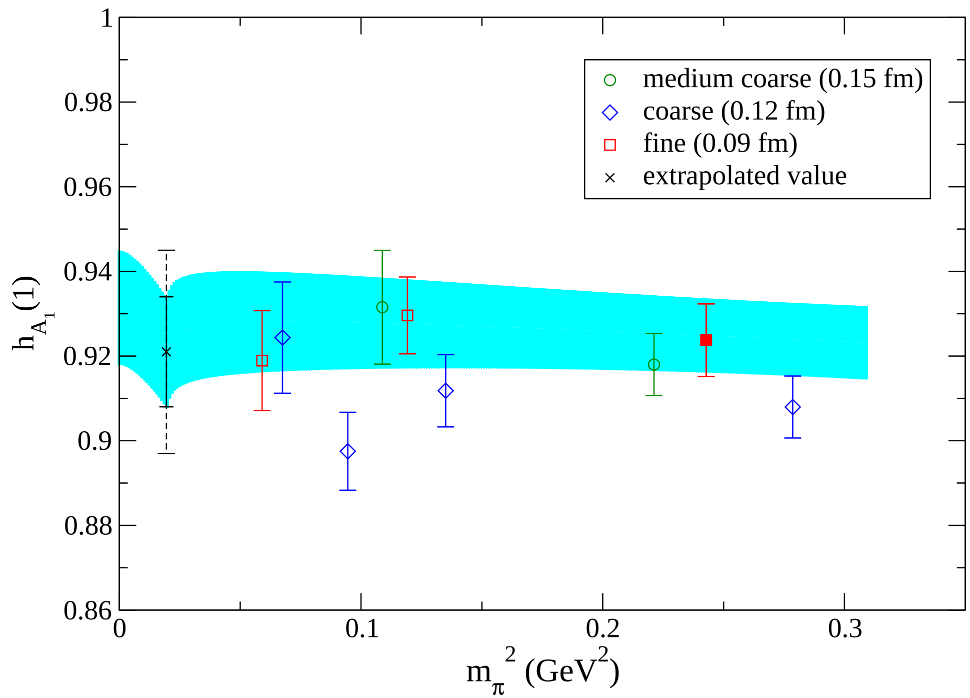
[from J.L. and Van de Water, PRD74 (2006) 034510]

$$
\begin{aligned}
h_{A_1}^{2+1}(1) \quad = \quad & 1 + X_A + \frac{g_\pi^2}{48\pi^2 f^2}\left[\frac{1}{16}\sum_B (2\overline{F}_{\pi_B} + \overline{F}_{K_B}) - \frac{1}{2}\overline{F}_{\pi_I} + \frac{1}{6}\overline{F}_{\eta_I}\right. \\
& + a^2\delta'_V \left(\frac{m_{S_V}^2 - m_{\pi_V}^2}{(m_{\eta_V}^2 - m_{\pi_V}^2)(m_{\pi_V}^2 - m_{\eta'_V}^2)}\overline{F}_{\pi_V} \right. \\
& + \frac{m_{\eta_V}^2 - m_{S_V}^2}{(m_{\eta_V}^2 - m_{\eta'_V}^2)(m_{\eta_V}^2 - m_{\pi_V}^2)}\overline{F}_{\eta_V} \\
& \left.\left. + \frac{m_{S_V}^2 - m_{\eta'_V}^2}{(m_{\eta_V}^2 - m_{\eta'_V}^2)(m_{\eta'_V}^2 - m_{\pi_V}^2)}\overline{F}_{\eta'_V} \right) + (V \to A) \right],
\end{aligned}
\tag{7}
$$

where a is the lattice spacing, δ'_V, g_π and X_A are constants, and $\overline{F}$ is a complicated function involving logs.

Chiral/Continuum Extrapolation





New (2010) vs Old (2008)

$|V_{cb}|$ exclusive

Updated result for $|V_{cb}|$ from $B \to D^* \ell \nu$ at CKM 2010.

$F(1) = 0.908(05)(9)(8)(9)(3)(3)$

Errors are statistics, $g_{DD^*\pi}$, χ extrapolation, HQ discretization, mass tunings, perturbative matching.

$|V_{cb}|F(1) \times 10^3 = 36.04 \pm 0.52$ from HFAG end of year 09 leads to

$|V_{cb}| = 39.7(7)(7) \times 10^{-3}$.

Errors are experiment and theory.

Compare with $41.85 \pm 0.73 \times 10^{-3}$ from inclusive. Discrepancy is smaller.

BaBar result for $|V_{ub}|$ exclusive

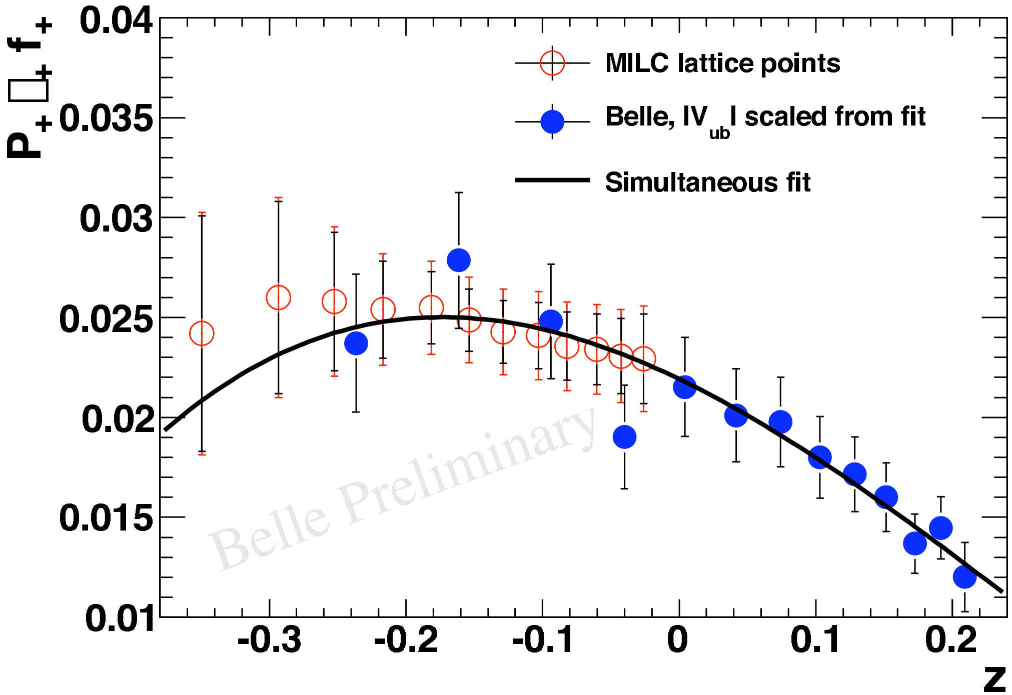
Talk by Martin Simard. Two different analyses, π-η analysis and π-ρ analysis.

Different fit and cut strategies. Fits to different numbers of modes, loose vs tight ν cut selection.
Results for $|V_{ub}|$ consistent within the different approaches. Results from π-η analysis:

Theory	q^2(GeV)^2	$\lvert V_{ub}\rvert(10^{-3})$
HPQCD	> 16	$3.24 \pm 0.13 \pm 0.16^{+0.57}_{-0.37}$
FNAL	> 16	$3.14 \pm 0.12 \pm 0.16^{+0.35}_{-0.29}$
LCSR	< 12	$3.70 \pm 0.07 \pm 0.09^{+0.54}_{-0.39}$

Exclusive $B \to \pi \ell \nu$ and $|V_{ub}|$

New result from Belle (talk by Kevin Varvell at CKM 2010)



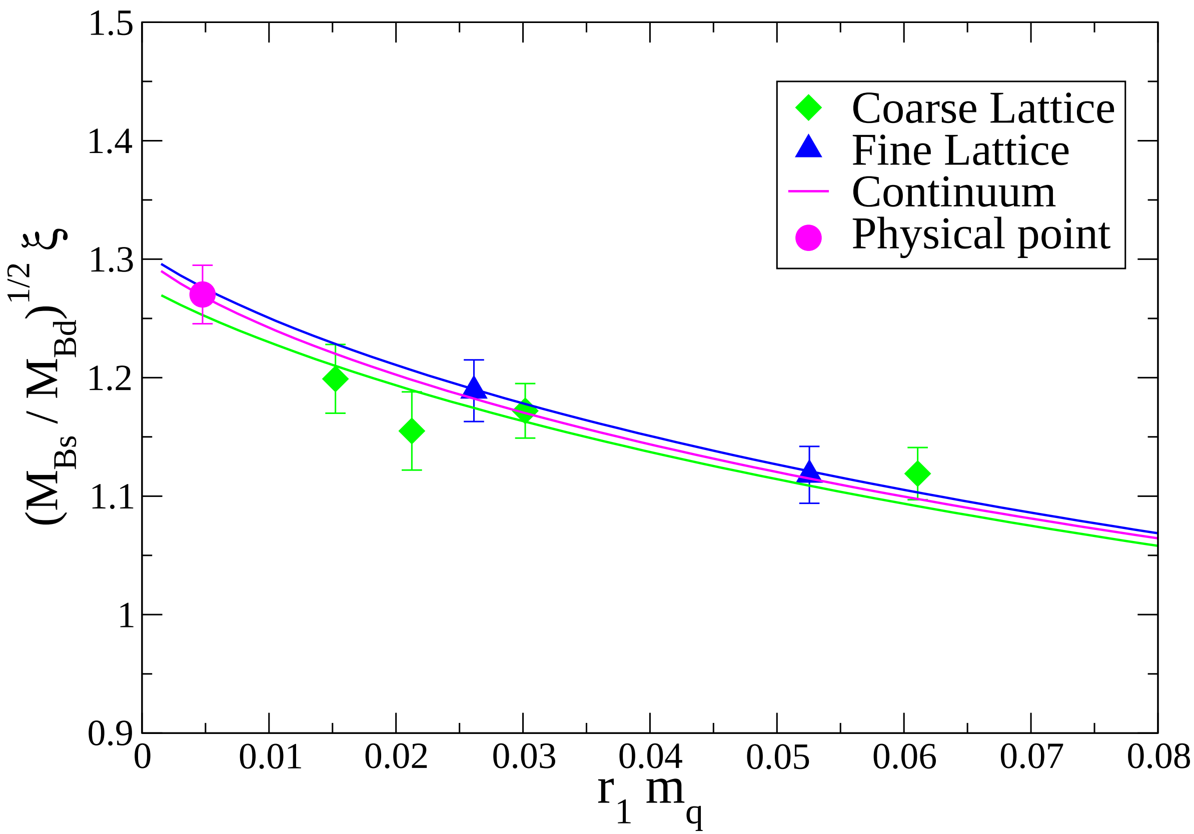
Results for $|V_{ub}|$

Using a BK parameterization for Belle experimental data $|V_{ub}|$ was extracted from the partial branching fraction for a number of different theories to give the normalization.

Simultaneous fit to lattice (Fermilab/MILC) and Belle q^2 dependence (using the z parameterization) leads to a model independent result of $|V_{ub}| = (3.43 \pm 0.33) \times 10^{-3}$.

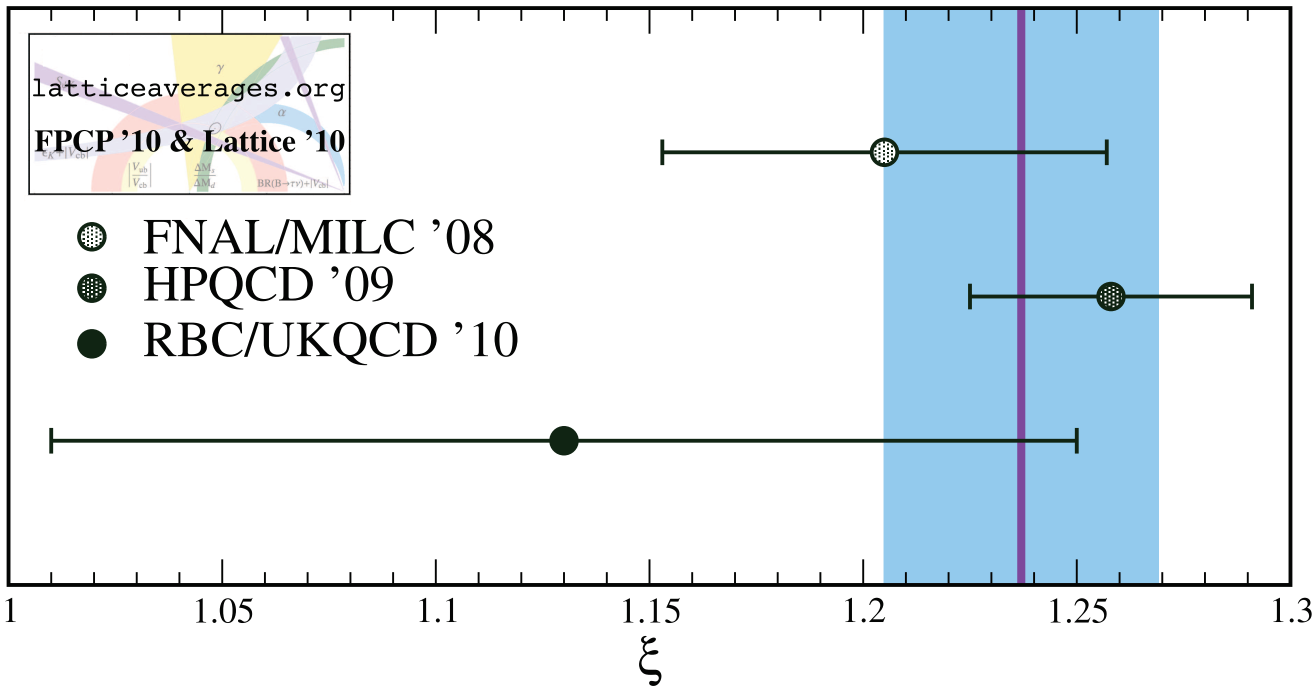
The same procedure with the latest BaBar data leads to $|V_{ub}| = (3.14 \pm 0.07 \pm 0.09^{+0.35}_{-0.29}) \times 10^{-3}$.

$B - \overline{B}$ mixing



HPQCD result (arXiv:0902.1815).

ξ



$\xi = 1.237 \pm 0.032.$

Conclusion

The lattice can provide useful constraints on CKM physics and searches for new physics. Few percent level accuracy is needed, and it is beginning to be achieved in the simplest weak-matrix elements.

It will be essential to have many results from different groups using different methods!