Global CKM fit(s) and lattice

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Introduction

End of the first B-factory era, Tevatron reign before LHC results

- Wealth of high-precision data on B-physics...
- ...entangling electroweak and strong physics
- Stringent test of SM, and in particular of Kobayashi-Maskawa mechanism for CP violation
- Opportunity to test (and constrain) extensions of SM through flavour (and especially B) physics data

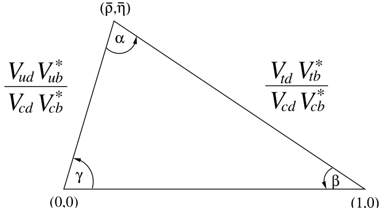
- (Very) good overall picture
- Interesting (but fluctuating !) hints of NP from B_s system

Global CKM fits relevant players in this game
both to check SM and provide directions for New Physics
provided hadronic physics well controlled

The big picture

In SM weak charged transitions mix quarks of different generations

Encoded in unitary CKM matrix $V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$



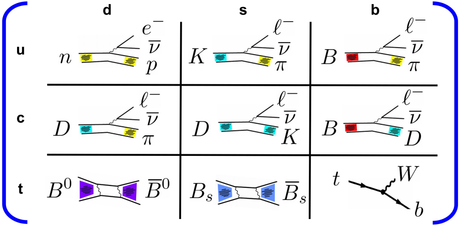
- 3 generations $\Longrightarrow$ 1 phase, only source of CP-violation in SM
- Wolfenstein parametrisation, defined to hold to all orders in λ and rephasing invariant

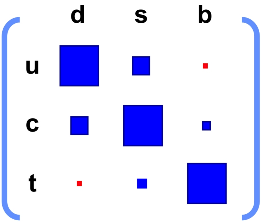
$$\lambda^2 = \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2} \qquad A^2\lambda^4 = \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2} \qquad \bar\rho + i\bar\eta = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

$\Longrightarrow$ 4 parameters describing the CKM matrix, to extract from data

A handle on these parameters

V =

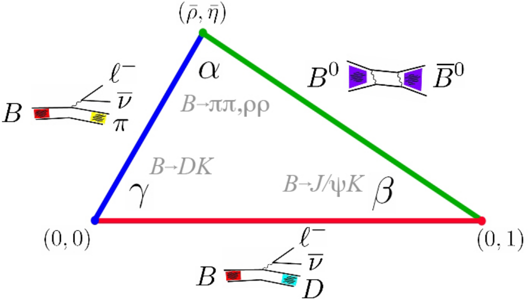




- $d \to u$: Nuclear physics (superallowed β decays)
- $s \to u$: Kaon physics (KLOE, KTeV, NA62)
- $c \to d, s$: Charm physics (CLEO-c, BESIII)
- $b \to u, c$ and $t \to d, s$: B physics (Babar, Belle, CDF/DØ, LHCb)
- $t \to b$: Top physics (CDF/DØ, ATLAS, CMS)

data = weak $\otimes$ QCD $\Longrightarrow$ Need for hadronic inputs (lattice) and deconvolution (statistics)

Some decays worth combining



Exp. uncertainties	
$B\to\pi\pi,\rho\rho$	α
$B\to DK$	γ
$B\to J/\Psi K_s$	β
$B\to J/\Psi\phi$	β_s
$K\to\pi\nu\bar\nu$	$(\bar\rho,\bar\eta)$

Th. uncertainties	
$B(b)\to D(c)\ell\nu$	$\lvert V_{cb}\rvert$ vs form factor (OPE)
$B(b)\to D(c)\ell\nu$	$\lvert V_{cb}\rvert$ vs form factor (OPE)
$M\to\ell\nu(\gamma)$	$\lvert V_{UD}\rvert$ vs f_M
ϵ_K	$(\bar\rho,\bar\eta)$ vs B_K
$\Delta M_d,\Delta M_s$	$\lvert V_{tb}V_{td,s}\rvert$ vs B_B
$B\to\ell^+\ell^-$	$\lvert V_{td,s}\rvert$ vs f_B

The inputs

CKMfitter CKM matrix within a frequentist framework ($\simeq \chi^2$ minimum)
+ specific scheme for systematic errors (Rfit)

data = weak $\otimes$ QCD $\Longrightarrow$ Need for hadronic inputs (often lattice)

$\|V_{ud}\|$	superallowed β decays	PRC79, 055502 (2009)
$\|V_{us}\|$	$K_{\ell 3}$ (Flavianet)	$f_+(0) = 0.963 \pm 0.003 \pm 0.005$
ϵ_K	PDG 08	$\hat{B}_K = 0.723 \pm 0.004 \pm 0.067$
$\|V_{ub}\|$	inclusive and exclusive	$\|V_{ub}\| \cdot 10^3 = 3.92 \pm 0.09 \pm 0.45$
$\|V_{cb}\|$	inclusive and exclusive	$\|V_{cb}\| \cdot 10^3 = 40.89 \pm 0.38 \pm 0.59$
Δm_d	last WA B_d-$\bar{B}_d$ mixing	$B_{B_s}/B_{B_d} = 1.05 \pm 0.01 \pm 0.03$
Δm_s	last WA B_s-$\bar{B}_s$ mixing	$B_{B_s} = 1.28 \pm 0.02 \pm 0.03$
β	last WA $J/\psi K^{(*)}$	
α	last WA $\pi\pi, \rho\pi, \rho\rho$	isospin
γ	last WA $B \to D^{(*)} K^{(*)}$	GLW/ADS/GGSZ
$B \to \tau\nu$	$(1.73 \pm 0.35) \cdot 10^{-4}$	$f_{B_s}/f_{B_d} = 1.199 \pm 0.008 \pm 0.023$
		$f_{B_s} = 228 \pm 3 \pm 17$ MeV

Statistical framework

$q = (A, \lambda, \bar{\rho}, \bar{\eta} \ldots)$ to be determined

- $\mathcal{O}_{\rm exp}$ experimental values of the observables
- $\mathcal{O}_{\rm th}(q)$ theoretical description in a model

In case of experimental (Gaussian) uncertainties, likelihoods and χ^2

$$\mathcal{L}(q) = \prod_{\mathcal{O}} \mathcal{L}_{\mathcal{O}}(q) \qquad \chi^2(q) = -2 \ln \mathcal{L}(q) = \sum_{\mathcal{O}} \left(\frac{\mathcal{O}_{\rm th}(q) - \mathcal{O}_{\rm exp}}{\sigma_{\mathcal{O}}} \right)^2$$

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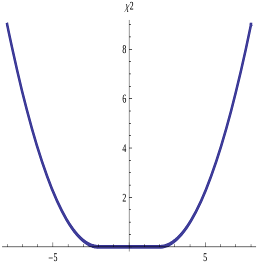
- Estimator $\hat{q}$ maximum likelood: $\chi^2(\hat{q}) = \min_q \chi^2(q)$
- Confidence level for a given q_0 (p-value for $q = q_0$) obtained from $\Delta\chi^2(q_0) = \chi^2(q_0) - \min_q \chi^2(q)$ (distributed like χ^2 law of *dim*(q))

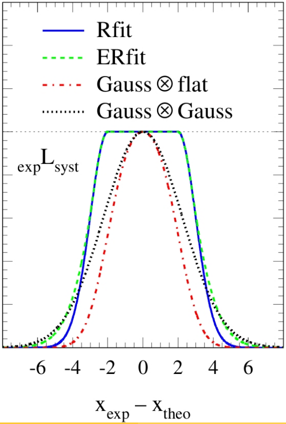
What to do in the case of systematic uncertainties,
which stand out of this picture ?

Rfit scheme

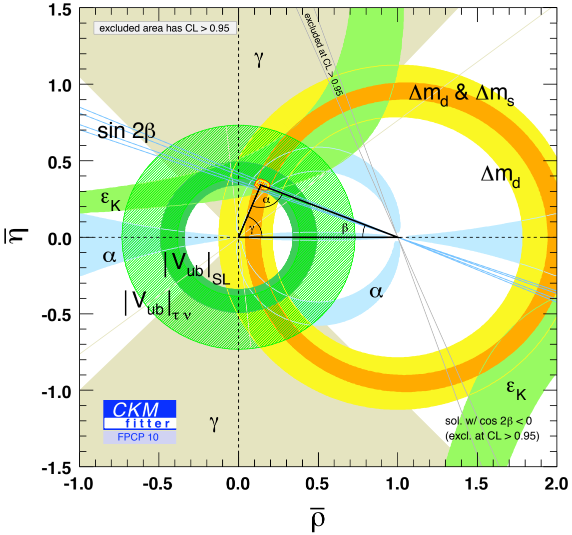
CKMfitter : Treatment of systematics within the Rfit scheme

- χ^2 with flat bottom (syst) and parabolic walls (stat)
- corresponding likelihood $\mathcal{L} = \exp(-\chi^2/2)$
- all values within range of syst treated on the same footing





The global fit



$|V_{ud}|, |V_{us}|$

$|V_{cb}|, |V_{ub}|$

$B \to \tau\nu$

$\Delta m_d, \Delta m_s$

ϵ_K

$\sin 2\beta$

α

γ

$$A = 0.8184^{+0.0094}_{-0.0311}$$
$$\lambda = 0.22512^{+0.00075}_{-0.00075}$$
$$\bar{\rho} = 0.139^{+0.027}_{-0.023}$$
$$\bar{\eta} = 0.342^{+0.016}_{-0.015}$$

(68% CL)

Hadronic inputs and lattice

Lattice averages

Consistent averages of lattice results for hadronic quantities needed
$\Longrightarrow$ we perform our own averages

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Consistent averages of lattice results for hadronic quantities needed
$\Longrightarrow$ we perform our own averages

- Collecting lattice results
  - only unquenched results with 2 or 2+1 dynamical fermions
  - papers and proceedings (but not preliminary results)
- Splitting error estimates into stat and syst
  - Stat : essentially related to size of gauge conf
  - Syst : fermion action, $a \to 0$, $L \to \infty$, mass extrapolations...
    added linearly when error budget available
- Potential problems
  - proceedings not always followed by peer-reviewed papers
  - some syst estimates controversial within lattice community (staggered action, extrapolations...)

An example: $\hat{B}_K$

Performing the above operations, we obtain the following table

Reference	N_f	Mean	Stat	Syst
JLQCD08	2	0.737	0.006	0.099
HPQCD/UKQCD06	2+1	0.830	0.025	0.245
RBC/UKQCD07	2+1	0.720	0.013	0.071
Laiho09	2+1	0.724	0.008	0.067
Our average		?	?	?

How to combine them ?

Averaging procedure

"Educated Rfit" used to combine the results, with different treament of statistical and systematic errors

- product of (Gaussian + Rfit) likelihoods for central value
- product of Gaussian (stat) likelihoods for stat uncertainty
- syst uncertainty of the combination
  = the one of the most precise method

Conservative, algorithmic procedure with internal logic for syst

- the present state of art cannot allow us to reach a better theoretical accuracy than the best of all estimates
  (combining 2 methods with similar syst does not reduce the intrinsic uncertainty encoded as a systematic)
- best estimate should not be penalized by less precise methods
  (opposed, e.g., to combined syst = dispersion of central values)

Our average for $\hat{B}_K$

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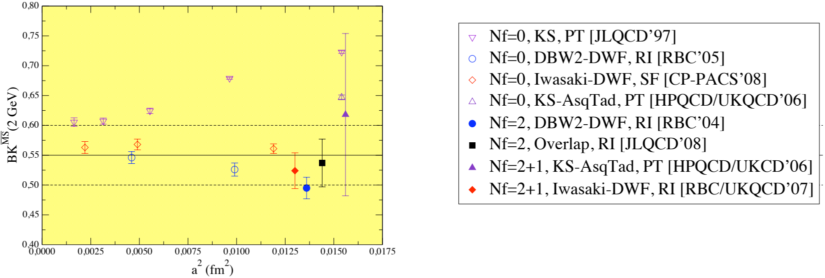
Our average		0.724	0.005	0.067

$\Longrightarrow$Used for B_d and B_s decay constants, bag parameters

Other lattice/flavour averages (1)

UTfit (different statistical approach)

- Quadratic combination of stat and syst, all treated Gaussian (range for syst earlier)
- Averages similar to Lattice conference plenary talks (speaker-dependent)

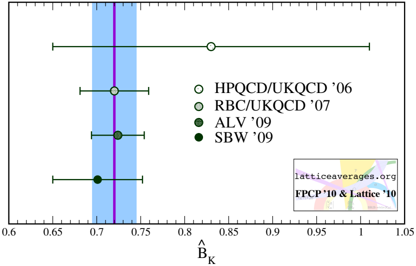


$$\hat{B}_K = 0.75 \pm 0.07$$ [Lubicz Tarantino 2008]

Other lattice/flavour averages (2)

www.latticeaverages.org [J. Laiho's talk]

- all errors treated as Gaussian
- takes into account correlations between different collaborations
- PDG-like prescription in case of incompatibilities



$$\hat{B}_K = 0.720 \pm 0.025$$

Comparing averaging methods

Modify our compilation to compare with `latticeaverages.org`

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Educated Rfit		0.724	0.005	0.067

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Modify our compilation to compare with `latticeaverages.org`

- Remove $N_f = 2$ results, add results preliminary/to be published

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BSW09	2+1	0.724	0.009	0.087
Educated Rfit		0.724	0.006	0.067

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BSW09 (prelim.)	2+1	0.701	0.019	0.088
Educated Rfit		0.723	0.007	0.067

Comparing averaging methods

Modify our compilation to compare with `latticeaverages.org`

- Remove $N_f = 2$ results, add results preliminary/to be published
- Add syst errors in quadrature

Reference	N_f	Mean	Stat	Syst
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RBC/UKQCD07	2+1	0.720	0.013	0.037
Laiho09	2+1	0.724	0.008	0.029
BSW09 (prelim.)	2+1	0.701	0.019	0.047
Educated Rfit		0.721	0.006	0.029
`latticeaverages.org`		0.720	0.025	

- CKM fitter current average: $\hat{B}_K = 0.724 \pm 0.005 \pm 0.067$
- Combining methods yield a syst smaller than syst of each ?

$|V_{ub}|$ inclusive and exclusive

Similar treatment to combine the two methods for $|V_{ub}|$:

- Incl. : $b \to u\ell\nu$ + Operator Product Expansion
- Excl. : $B \to \pi\ell\nu$ + Form factors (small q^2 LCSR, high q^2 lattice)

Inputs (first two from HFAG)

$$|V_{ub}|_{inc} = 4.32^{+0.21}_{-0.24} \pm 0.45$$

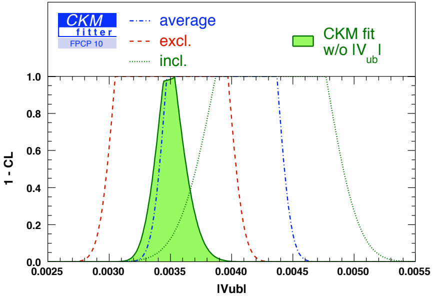
$$|V_{ub}|_{exc} = 3.51 \pm 0.10 \pm 0.46$$

$$|V_{ub}|_{ave} = 3.92 \pm 0.09 \pm 0.45$$

Output

$$|V_{ub}|_{global\ fit} = 3.53^{+0.15}_{-0.14}$$

$$|V_{ub}|_{indirect} = 3.53^{+0.15}_{-0.21}$$



with all values $\times 10^{-3}$

- Discrepancy incl./excl. depends on statistical treatment
- Improvement on the systematics for exclusive welcome

$|V_{cb}|$ inclusive and exclusive

Similar problems with $|V_{cb}|$ (individual values from HFAG)

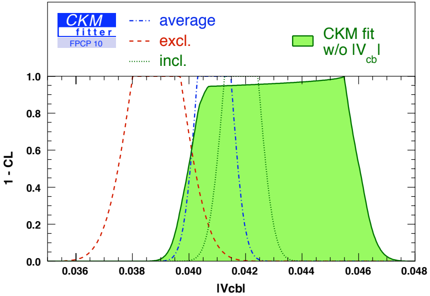
Inputs

$$|V_{cb}|_{inc} = 41.85 \pm 0.43 \pm 0.59$$
$$|V_{cb}|_{exc} = 38.85 \pm 0.77 \pm 0.84$$

$$|V_{cb}|_{ave} = 40.89 \pm 0.38 \pm 0.59$$

Output

$$|V_{cb}|_{global\ fit} = 41.48^{+0.38}_{-1.47}$$
$$|V_{cb}|_{indirect} = 45.48^{+0.78}_{-5.74}$$

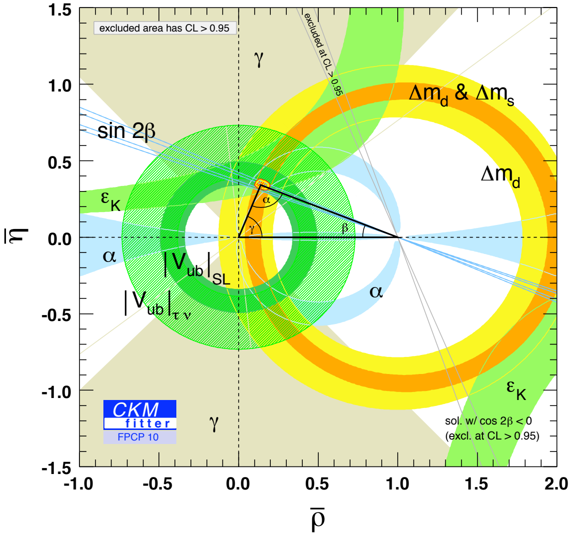


with all values $\times 10^{-3}$

- Ouput from the global fit, with/without $|V_{cb}|$ input (global/indirect)
- Improving systematics for exclusive would not help alone

Good or perfect agreement ?

The global fit



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$$\bar\eta = 0.342^{+0.016}_{-0.015}$$
(68% CL)

Relevant lattice quantities for the global fit

Will the global fit be impacted by an improved lattice input ?

- Remove input from the fit and predict its value (indirect)
- Check if uncertainty indirect vs input are comparable

Quantity	Indirect		Input	
f_{B_s}	$234.7^{+9.8}_{-8.3}$	(4%)	$228 \pm 3 \pm 17$	(9%)
f_{B_s}/f_{B_d}	$1.199^{+0.058}_{-0.044}$	(4%)	$1.199 \pm 0.008 \pm 0.023$	(3%)
$\hat{B}_{B_s}$	$1.136^{+0.103}_{-0.043}$	(7%)	$1.28 \pm 0.02 \pm 0.03$	(4%)
$\hat{B}_{B_s}/\hat{B}_{B_d}$	$1.156^{+0.075}_{-0.104}$	(8%)	$1.05 \pm 0.01 \pm 0.03$	(4%)
$\hat{B}_K$	$0.82^{+0.26}_{-0.14}$	(24%)	$0.724 \pm 0.004 \pm 0.067$	(10%)

- For decay constants, other constraints + global fit perform almost as well as input
- For bag parameters, improvement of lattice inputs crucial

Use the global fit to predict the values of decay csts using also

$$\mathcal{B}[M \to \ell\nu_\ell] = \frac{G_F^2 m_M m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_M^2}\right)^2 |V_{q_u q_d}|^2 f_M^2 \tau_M (1 + \delta_{em}^{M\ell 2})$$

Quantity	Process	Global fit	
f_K/f_π	$K_{\mu 2}/\pi_{\mu 2}$	$1.14^{+0.006}_{-0.008}$	(1%)
f_K	$K \to \ell\nu_\ell, \tau \to K\nu_\tau$	155^{+1}_{-2}	(1%)
f_D	$D \to \ell\nu_\ell$	206^{+11}_{-11}	(6%)
f_{D_s}	$D_s \to \ell\nu_\ell$	261^{+7}_{-7}	(3%)
f_B	$B \to \tau\nu$	197^{+10}_{-9}	(5%)

$\Longrightarrow$ To be compared with your favourite determination of decay cst

Semileptonic decays

Three moduli inputs of global fit from semileptonic decays

	Indirect		Input	
$\|V_{us}\|$	$0.2254^{+0.0010}_{-0.0010}$	(0.4%)	0.2246 ± 0.0012	(0.5%)
$\|V_{ub}\|$	$0.0035^{-0.0002}_{-0.0002}$	(4.2%)	$0.0039 \pm 0.0001 \pm 0.0005$	(14%)
$\|V_{cb}\|$	$0.0455^{+0.0008}_{-0.0055}$	(7.2%)	$0.0409 \pm 0.0004 \pm 0.0006$	(2.4%)

- $|V_{us}|$ and $|V_{cb}|$ input-driven
- $|V_{ub}|$ more constrained by the global fit than by its own input

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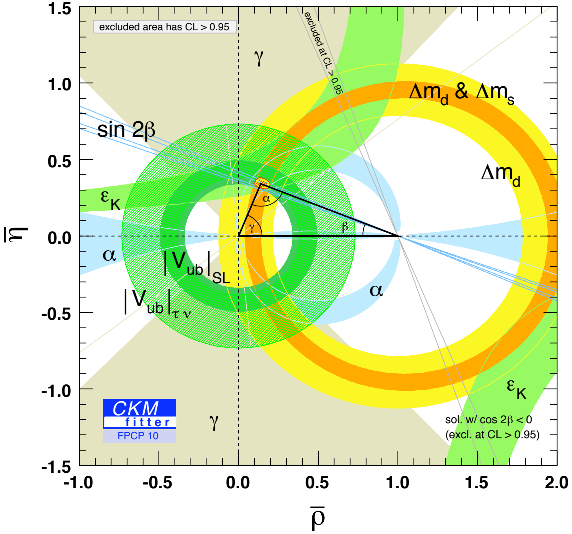
Two others can be predicted from global fit and experiment

	Global fit		Expt $\|V\| \times f(0)$		Pred $f_{D\to\pi/K}(0)$	
$\|V_{cd}\|$	$0.2250^{+0.0008}_{-0.0008}$	(0.3%)	0.150 ± 0.005	(3.4%)	0.666 ± 0.023	(3.3%)
$\|V_{cs}\|$	$0.9735^{+0.0002}_{-0.0002}$	(0.1%)	0.718 ± 0.008	(1.2%)	0.737 ± 0.009	(1.2%)

$\Longrightarrow$ Lattice close to that accuracy, will start being constraining

The global fit



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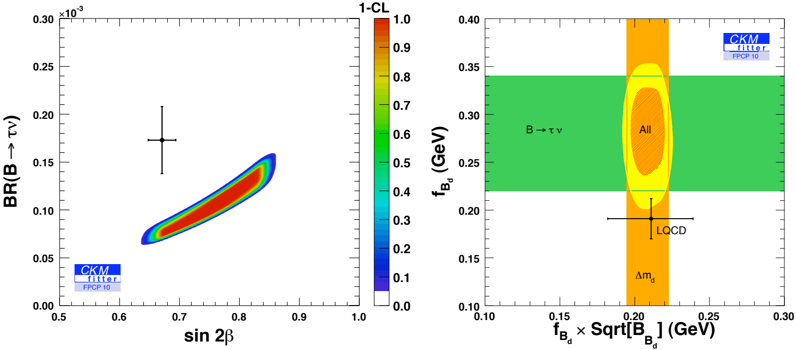
(68% CL)

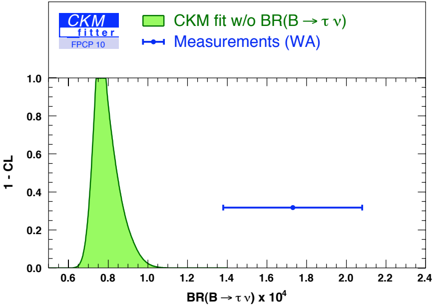
The global fit χ^2_{min} drops by $\sim 2.6\sigma$ $\sin 2\beta_{c\bar{c}}$ or $B \to \tau\nu$ removed



Issue not only the value of f_{B_d} since 2.6σ discrepancy from

$$\frac{B(B \to \tau\nu)}{\Delta m_d} = \frac{3\pi}{4} \frac{m_\tau^2 \tau_B}{m_W^2 \eta_B S[x_t]} \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \frac{\sin^2\beta}{\sin^2(\alpha+\beta)} \frac{1}{|V_{ud}|^2 B_{B_d}}$$

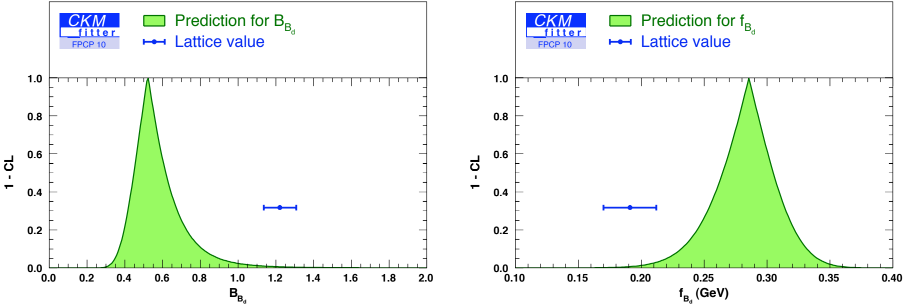
A few ways of escaping



- Change in measured $Br(B \to \tau\nu)$ (2.6 σ)

$$\begin{aligned} Br(B \to \tau\nu)_{w/o\ this\ input} &= (7.63^{+1.13}_{-0.61}) \cdot 10^{-3} \\ Br(B \to \tau\nu)_{meas} &= (17.3 \pm 3.5) \cdot 10^{-3} \end{aligned}$$

A few ways of escaping



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$$Br(B \to \tau\nu)_{meas} = (17.3 \pm 3.5) \cdot 10^{-3}$$

- Correlated change in lattice values for f_{B_d} (2.6 σ) and B_{B_d} (2.7 σ) [global fit yields "theory-free" predictions for both quantities]

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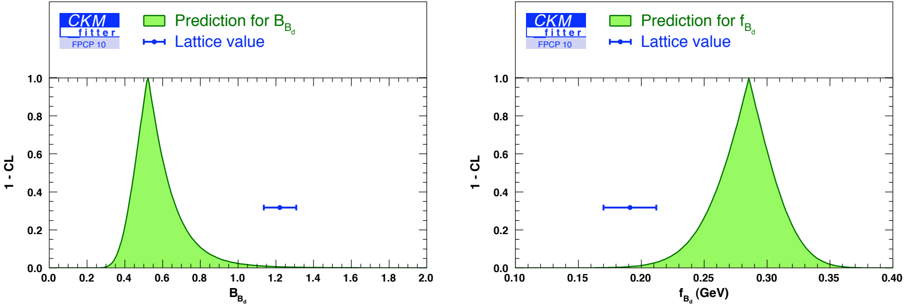

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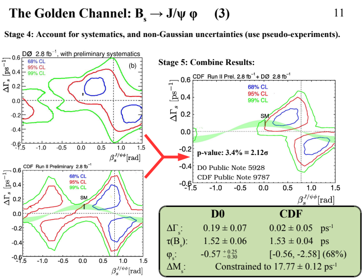
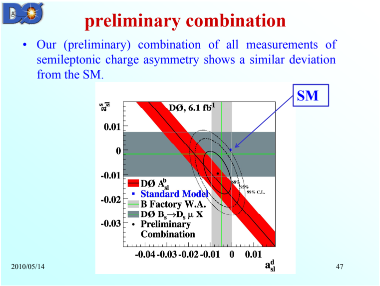
- Correlated change in lattice values for f_{B_d} (2.6 σ) and B_{B_d} (2.7 σ)
- New Physics in mixing ($\Delta F = 2$) or in tree ($\Delta F = 1$)

New Physics in meson mixing

Three discrepancies



	D0	CDF
$\Delta\Gamma_s$:	0.19 ± 0.07	0.02 ± 0.05 ps^{-1}
$\tau(B_s)$:	1.52 ± 0.06	1.53 ± 0.04 ps
φ_s:	$-0.57^{+0.25}_{-0.30}$	[-0.56, -2.58] (68%)
ΔM_s:	Constrained to 17.77 ± 0.12 ps^{-1}	



$(\Delta\Gamma_s, \phi_s)$ Linear comb of a^d_{SL} and a^s_{SL}

- $B \to \tau\nu$ vs $\sin 2\beta$ 1D constraint : 2.6 σ
- ϕ_s mixing phase from $B_s \to J/\Psi\phi$ (null test) 1D constraint : 2.1 σ
- Dimuon asym A_{SL} (null test) 1D constraint : 2.9 σ

$\Longrightarrow$Combination of χ^2 uncorrelated : 3.7 σ

[new CDF measurement of ϕ_s not included (yet)]

Neutral-B mixing

$$i\frac{d}{dt}\begin{pmatrix} |B_q(t)\rangle \\ |\bar{B}_q(t)\rangle \end{pmatrix} = \left(M^q - \frac{i}{2}\Gamma^q\right)\begin{pmatrix} |B_q(t)\rangle \\ |\bar{B}_q(t)\rangle \end{pmatrix}$$

M and Γ hermitian: mixing due to off-diagonal terms $M_{12}^q - i\Gamma_{12}^q/2$

$\Longrightarrow$Diagonalisation: physical $|B_{H,L}^q\rangle$ of masses $M_{H,L}^q$, widths $\Gamma_{H,L}^q$

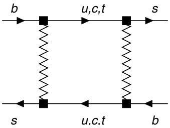
In terms of M_{12}^q, $|\Gamma_{12}^q|$ and $\phi_q = arg\left(-\frac{M_{12}^q}{\Gamma_{12}^q}\right)$ [small in SM]

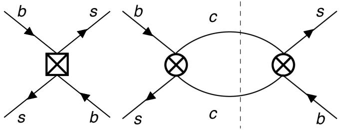
- Mass difference $\Delta M_q = M_H^q - M_L^q = 2|M_{12}^q|$
- Width difference $\Delta\Gamma_q = \Gamma_H^q - \Gamma_L^q = 2|\Gamma_{12}^q|\cos(\phi_q)$
- Asymmetry $a_{SL} = \frac{\Gamma(\bar{B}^q(t)\to\ell^+\nu X)-\Gamma(B^q(t)\to\ell^-\nu X)}{\Gamma(\bar{B}^q(t)\to\ell^+\nu X)+\Gamma(B^q(t)\to\ell^-\nu X)} = \frac{|\Gamma_{12}^q|}{|M_{12}^q|}\sin\phi_q$

p/q from mixing in time-dependent analysis of B decays

"phase" $\phi_{M_q} \simeq arg(M_{12}^{q*}) + O\left(\frac{|\Gamma_{12}^q|}{|M_{12}^q|}\right)$

Mixing observables in SM





[Beneke et al 96-98, Nierste and Lenz 06]

Effective Hamiltonian approach

- M_{12}^q dominated by dispersive part of top boxes
  - involve one operator at LO: $Q = \bar{q}\gamma_\mu(1-\gamma_5)b\bar{q}\gamma^\mu(1-\gamma_5)b$
- Γ_{12}^q dominated by absorptive part of charm boxes
  - non local contribution, expressed as expansion in $1/m_b$
  - involve two operators at LO: Q and $\tilde{Q}_S = \bar{q}_\alpha(1+\gamma_5)b_\beta\bar{q}_\beta(1+\gamma_5)b_\alpha$
  - right set for Γ_{12}, depending mainly on Q, taming $1/m_b$-corrections

$$\begin{aligned}
\Delta\Gamma_s &= f[f_{Bs}, B, \tilde{B}_S; \mu, m_b^{pow}, B_{1/m_b}\ldots] \\
\Delta\Gamma_s/\Delta M_s &= f[\tilde{B}_S/B; B_{1/m_b}, m_b^{pow}, \mu, \bar{m}_c\ldots] \\
a_{SL}^s &= f[\tilde{B}_S/B; |V_{ub}/V_{cb}|, \gamma, \mu, \bar{m}_c, B_{1/m_b}\ldots]
\end{aligned}$$

New Physics in $\Delta F = 2$

- $B \to \tau\nu$ vs $(\sin 2\beta, \Delta m_d)$
- ϕ_s from $B_s \to J/\Psi\phi$ and τ_{FS}
- A_{SL} from like-sign dimuon charge asymmetry

Why not New Physics only in $\Delta F = 2$ processes ?

- M_{12} dominated by short distances (top boxes) [affected by NP]
- Γ_{12} dominated by tree decays into charm [not affected by NP]
- Tree level (4 diff flavours) processes not affected by New Physics
- Model-independent parametrisation

$$\langle B_q|\mathcal{H}_{eff}^{SM+NP}|\bar{B}_q\rangle = \langle B_q|\mathcal{H}_{eff}^{SM}|\bar{B}_q\rangle \times [Re(\Delta_q) + i \cdot Im(\Delta_q)]$$

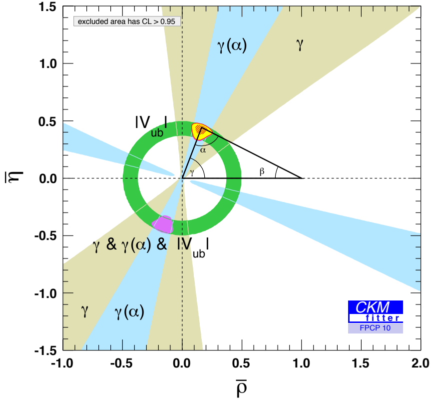
$\Longrightarrow$New physics only changing modulus and phase of M_{12}^d and M_{12}^s

[A. Lenz et al., 10]

Fixing the CKM part

Observables not affected by NP, used to fix CKM

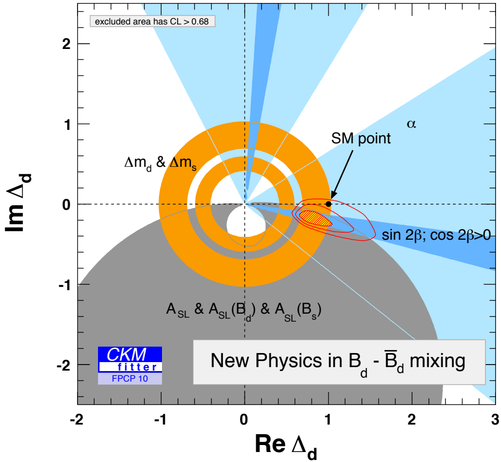
$|V_{ud}|, |V_{us}|, |V_{ub}|, |V_{cb}|, \gamma$ and $\gamma(\alpha) \equiv \pi - \alpha - \beta$ (ϕ_{B_d} cancels)



Observables affected by NP, used to determine Δ_d, Δ_s

- Neutral-meson oscillation $\Delta m_d, \Delta m_s$
- Lifetime difference $\Delta\Gamma_s$
- Time-dep asymmetries related to ϕ_{B_d}, ϕ_{B_s}
- Semileptonic asymmetries $a^d_{SL}, a^s_{SL}, A_{SL}$
- α (interference between decay and mixing)

B_d mixing

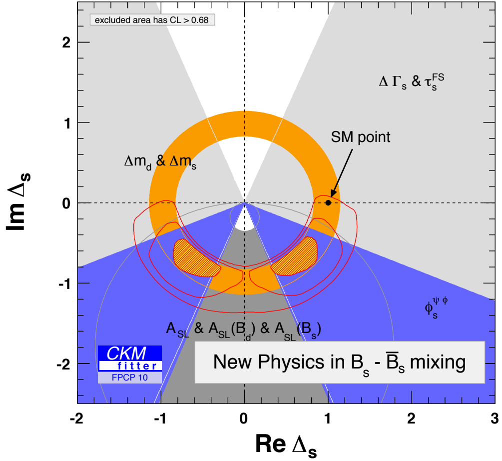


[Constraints 68% CL]

- Dominant const from β and Δm_d (2 rings from 2 sol for apex)
- $Br(B \to \tau\nu)$ shifts β constraint from real axis
- Disagreement with SM driven in same dir by $Br(B \to \tau\nu)$ and A_{SL}

2D SM hypothesis ($\Delta_d = 1 + i \cdot 0$): 2.5 σ

(w/o $B \to \tau\nu$: 1.1σ , w/o A_{SL}: 2.2 σ)



[Constraints 68% CL]

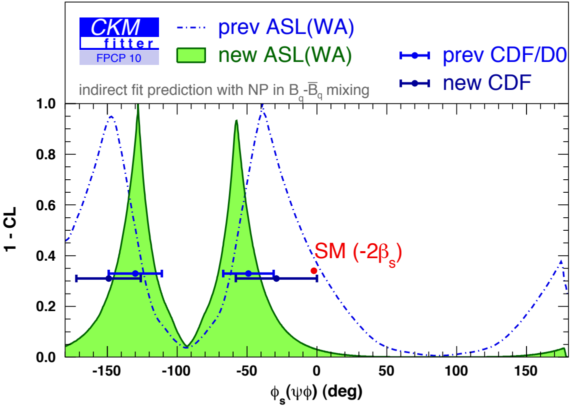
[New CDF meas. of ϕ_s not included (yet)]

- Dominant constraints from Δm_s, ϕ_s and A_{SL}
- Disagreement with SM driven in same dir by ϕ_s and A_{SL}

2D SM hypothesis ($\Delta_s = 1 + i \cdot 0$): 2.7σ

(w/o $B \to \tau\nu$: 2.7 σ, w/o A_{SL}: 1.9 σ, w/o ϕ_s: 1.7 σ)

Output of the fit: ϕ_s and SM hypothesis



- p-value for $\Delta_d = \Delta_s = 1$ 4D hypothesis is 3.4 σ
- . . . before the new measurement of CDF on ϕ_s (will decrease)
- other scenarios for NP in $\Delta F = 2$, linking B_d, B_s and K

Theoretical inputs where lattice matters

$$\begin{aligned}
\Delta\Gamma_s &= f[f_{Bs}, B, \tilde{B}_S; \mu, m_b^{pow}, B_{1/m_b} \ldots] \\
\Delta\Gamma_s/\Delta M_s &= f[\tilde{B}_S/B; B_{1/m_b}, m_b^{pow}, \mu, \bar{m}_c \ldots] \\
a_{SL}^s &= f[\tilde{B}_S/B; |V_{ub}/V_{cb}|, \gamma, \mu, \bar{m}_c, B_{1/m_b} \ldots]
\end{aligned}$$

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$$\Delta\Gamma_s/\Delta M_s = f[\tilde{B}_S/B; B_{1/m_b}, m_b^{pow}, \mu, \bar{m}_c\ldots]$$
$$a_{SL}^s = f[\tilde{B}_S/B; |V_{ub}/V_{cb}|, \gamma, \mu, \bar{m}_c, B_{1/m_b}\ldots]$$

- B_s decay constant for $\Delta\Gamma_s$, bag parameter for ΔM_s
- Bag parameters for scalar operators from quenched lattice estimate [Becirevic et al., 02]

  $$\tilde{B}'^s_S(m_b)/\tilde{B}'^d_S(m_b) = 1.00 \pm 0.03 \qquad \tilde{B}'^s_S(m_b) = 1.40 \pm 0.13$$

  $\Longrightarrow$ Lattice on $\langle B_s|\bar{q}_\alpha(1+\gamma_5)b_\beta\bar{q}_\beta(1+\gamma_5)b_\alpha|\bar{B}_s\rangle$? [HPQCD 06 ?]
- $1/m_b$ suppressed operators: bag parameters (vacuum insertion approximation) and power correction scale

  $$B_{Ri}(m_b) = 1.0 \pm 0.5 \qquad m_b^{\rm pow} = 4.70 \pm 0.10$$

  $\Longrightarrow$ Lattice on $\langle B_s|\bar{q}_\alpha\vec{D}_\rho\gamma^\mu(1-\gamma_5)D^\rho b_\alpha\bar{q}_\beta\gamma_\mu(1-\gamma_5)b_\beta|\bar{B}_s\rangle/m_b^2$?

Conclusions

Conclusions

SM Global fit

- (almost) all constraints in very good agreement
- issue of hadronic quantities: how to average ?
- improvement on systematics for bag parameters, $|V_{us}|$, $|V_{ub}|$?
- central value of B_{B_d}, exclusive $|V_{ub}|$, $|V_{cb}|$?
- a role for $D \to K/\pi\ell\nu$ in global fits ?

NP in $\Delta B = 2$ affecting M_{12}^q only

- several discrepancies from SM ($B \to \tau\nu$ vs $\sin(2\beta)$, ϕ_s, A_{SL}
- $\Delta_d = \Delta_s = 1$ 4D SM hypothesis at 3.4 σ
- ... before the new measurement of CDF on ϕ_s
- leading and $1/m_b$-suppressed $\Delta B = 2$ operators ?

Illustration of the potentialities of flavour physics
for New Physics searches
if we have a good control of hadronic inputs

More

CKM_FPCP10.html 5/21/10 9:55 PM

CKMfitter global fit results as of FPCP 10:

- Wolfenstein parameters
- UT angles and sides
- UT_s angle and apex
- CKM elements
- Theory parameters
- Rare branching fractions (B→lν, B→ll)

Wolfenstein parameters and Jarlskog invariant:

Observable	Central ± 1 σ	± 2 σ	± 3 σ
A	0.8184 [+0.0094 -0.0311]	0.818 [+0.019 -0.040]	0.818 [+0.028 -0.048]
λ	0.22512 [+0.00075 -0.00075]	0.2251 [+0.0015 -0.0015]	0.2251 [+0.0022 -0.0022]
ρbar	0.139 [+0.027 -0.023]	0.139 [+0.051 -0.033]	0.139 [+0.063 -0.042]
ηbar	0.342 [+0.016 -0.015]	0.342 [+0.031 -0.027]	0.342 [+0.046 -0.037]
J [10^{-5}]	2.98 [+0.15 -0.19]	2.98 [+0.30 -0.23]	2.98 [+0.45 -0.28]

UT angles and sides:

Observable	Central ± 1 σ	± 2 σ	± 3 σ
sin 2α	-0.02 [+0.12 -0.14]	-0.02 [+0.18 -0.27]	-0.02 [+0.24 -0.34]
sin 2α (meas. not in the fit)	-0.254 [+0.328 -0.050]	-0.254 [+0.419 -0.097]	-0.25 [+0.48 -0.14]
sin 2β	0.686 [+0.023 -0.019]	0.686 [+0.046 -0.033]	0.686 [+0.069 -0.046]
sin 2β (meas. not in the fit)	0.831 [+0.013 -0.037]	0.831 [+0.026 -0.110]	0.831 [+0.038 -0.177]
|sin (2β+γ)|	0.933 [+0.025 -0.027]	0.933 [+0.042 -0.045]	0.933 [+0.051 -0.063]
α [deg]	90.5 [+4.1 -3.6]	90.5 [+7.8 -5.2]	90.5 [+9.9 -6.8]
α [deg] (meas. not in the fit)	97.4 [+1.5 -9.5]	97.4 [+2.9 -12.1]	97.4 [+4.3 -13.9]
α [deg] (dir. meas.)	89.0 [+4.4 -4.2]	89.0 [+9.1 -8.3] ∥ 178.3 [+1.7 -5.6] ∥ 0 [+5 -0]	89 [+21 -13] ∥ 178.3 [+1.7 -13.8] ∥ 0 [+12 -0]
β [deg]	21.65 [+0.92 -0.75]	21.7 [+1.9 -1.3]	21.7 [+2.8 -1.8]
β [deg] (meas. not in the fit)	28.11 [+0.69 -1.84]	28.1 [+1.4 -5.0]	28.1 [+2.1 -7.7]
β [deg] (dir. meas.)	21.15 [+0.90 -0.88]	21.1 [+1.8 -1.7]	21.1 [+2.8 -2.6]

file:///Users/descotes/Documents/Talks/2010/fpcp10/plots/Num/CKM_FPCP10.html Page 1 of 4

More plots and results available on http://ckmfitter.in2p3.fr

J. Charles, Theory
O. Deschamps, LHCb
SDG, Theory
R. Itoh, Belle
A. Jantsch, ATLAS
H. Lacker, ATLAS
A. Menzel, ATLAS
S. Monteil, LHCb
V. Niess, LHCb
J. Ocariz, BaBar
S. T'Jampens, LHCb
V. Tisserand, BaBar/LHCb
K. Trabelsi, Belle

Back-up

Statistical method

Observables depend on q (relevant param) and μ (nuisance param)

- Minimise $\chi^2(q,\mu)$ w.r.t μ and subtract the absolute minimum to get the test statistics $t = (q)$ [likelihood ratio]
- Assume that the true value of μ is μ_t and compute PDF$[t|q,\mu_t]$
- Compute CL$_{\mu_t}(q)$ through Toy Monte Carlo

$$CL_{\mu_t}(q) = \int_0^{t(q;\mathrm{data})} dt\ \mathrm{PDF}[t|q,\mu_t]$$

How to treat nuisance parameters ?

- plug-in: identify μ_t assume μ_t value of μ minimising $\chi^2(q,\mu)$ on real data
- supremum minimise w.r.t μ_t to get CL(q) - quite general, but too conservative (minimum sometimes for unphysical μ) and computing demanding

ϵ_K defined in $K \to \pi\pi$ [dominated by $I = 0$ from $\Delta I = 1/2$ rule]

$$\eta_{ab} = \frac{A(K_L \to \pi^a \pi^b)}{A(K_S \to \pi^a \pi^b)} \qquad \epsilon_K = \frac{\eta_{00} + 2\eta_{+-}}{3} \qquad \epsilon'_K = \frac{-\eta_{00} + \eta_{+-}}{3}$$

related to $K\bar{K}$ mixing: $|K_{S(L)}\rangle = [(1+\bar\epsilon)|K^0\rangle \mp (1-\bar\epsilon)|\bar{K}^0\rangle]/\sqrt{1+\bar\epsilon^2}$

$$\epsilon_K = \bar\epsilon + \xi = \sin\phi_\epsilon e^{i\phi_\epsilon}\left[\frac{\mathrm{Im} M_{12}}{\Delta M} + \xi\right] \qquad \xi = \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0}$$

with strong-phase separation $A(K_0 \to (\pi\pi)_I) = A_I e^{i\delta_I^{strong}}$

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with strong-phase separation $A(K_0 \to (\pi\pi)_I) = A_I e^{i\delta_I^{strong}}$

- $\epsilon_K, \Delta M_K$ from experiment, $\phi_\epsilon = \arctan(-2\Delta M/\Delta\Gamma) \simeq \pi/4$
- $\mathrm{Im} M_{12}$ from effective Hamiltonian
  - keep only lowest-dimension contribution $\mathrm{Im} M_{12}^{(6)}$
  - top/charm boxes $\otimes$ $\hat{B}_K \propto \langle \bar{K}^0|(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}|K^0\rangle$

$$\epsilon_K \simeq C_\epsilon \hat{B}_K \lambda^2 \bar{\eta}^2 |V_{cb}|^2 [|V_{cb}|^2 (1-\bar{\rho})\eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_t, x_c) - \eta_{cc} S_0(x_c)]$$

Inami-Lim fncs $S_0(x_q = m_q^2/m_W^2)$, C_ϵ normalisation

ϵ_K and κ_ϵ

$$\epsilon_K = \sin\phi_\epsilon e^{i\phi_\epsilon}\left[\frac{\mathrm{Im}M_{12}}{\Delta M} + \xi\right] = \kappa_\epsilon \frac{e^{i\phi_\epsilon}}{\sqrt{2}}\frac{\mathrm{Im}M_{12}^{(6)}}{\Delta M_K} \text{ with } \kappa_\epsilon \neq 1$$

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- $\phi_\epsilon = 43.5 \pm 0.7^\circ \neq 45^\circ$

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- $\phi_\epsilon = 43.5 \pm 0.7^\circ \neq 45^\circ$
- $\xi = \mathrm{Im} A_0 / \mathrm{Re} A_0$ [Buras Guadagnoli 2008,2009]
  $\mathrm{Re} A_0$ from expt, but $\mathrm{Im} A_0$ ($I = 0$ QCD penguins, hard to compute)
  - directly from lattice ? [Ch. Sachrajda's talk]
  - linked through ϵ'/ϵ to $\mathrm{Im} A_2$ ($I = 2$ ew penguins, "easier" to bound)

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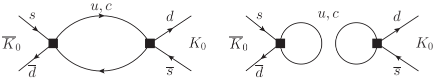
- Higher-dim corr to $\mathrm{Im}M_{12}$ [Buras Guadagnoli Isidori 2010]



$$M_{12} = \langle K_0|H_{\text{eff}}^{|\Delta S|=2}|\bar{K}_0\rangle - \langle K_0|\int d^4x \mathrm{Re}[iT[H_{\text{eff}}^{|\Delta S|=1}(x)H_{\text{eff}}^{|\Delta S|=1}(0)]]|\bar{K}_0\rangle$$

- Leading from $d = 6$ operator $(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$ in $\Delta S = 2$
- $d = 8$ corrections from two insertions of $\Delta S = 1$, estimated in χPT

ϵ_K and κ_ϵ

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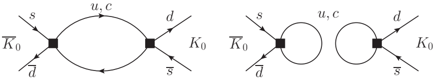

$$M_{12} = \langle K_0|H_{\text{eff}}^{|\Delta S|=2}|\bar{K}_0\rangle - \langle K_0|\int d^4x \mathrm{Re}[iT[H_{\text{eff}}^{|\Delta S|=1}(x)H_{\text{eff}}^{|\Delta S|=1}(0)]]|\bar{K}_0\rangle$$

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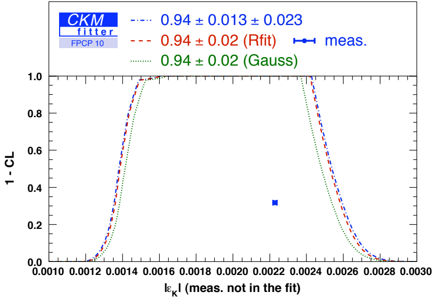
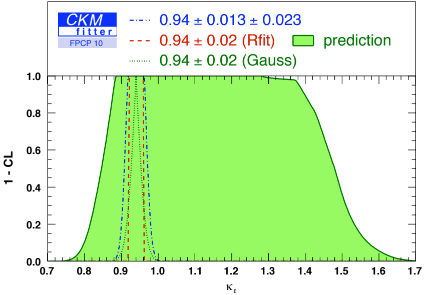
Separate stat/syst: $\kappa_\epsilon = 0.940 \pm 0.013 \pm 0.023$ (BGI: 0.94 ± 0.2)

$|\epsilon_K|$ in global fit (1)

- Assume something on κ_ϵ, compute ϵ_K and compare with expt
- Use measurement of ϵ_K and the global fit to study κ_ϵ





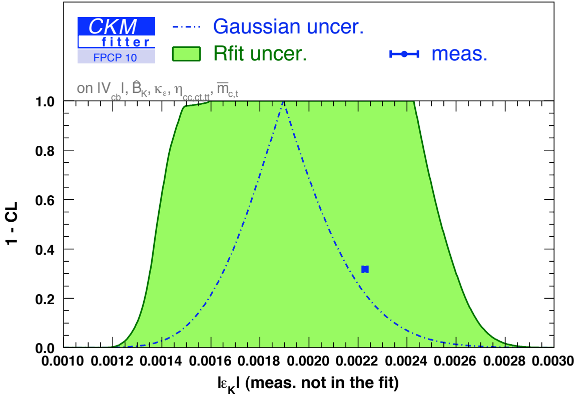
$\Longrightarrow$ The correction κ_ϵ does not spoil the quality of the global fit

- $\kappa_\epsilon = 0.94 \pm 0.02$ (Gaussian)
- $\kappa_\epsilon = 0.94 \pm 0.02$ (Rfit)
- $\kappa_\epsilon = 0.940 \pm 0.013 \pm 0.023$

$|\epsilon_K|$ in global fit (2)

Impact of the statistical treatment of theoretical inputs on ϵ_K

κ_ϵ and $|V_{cb}|, \hat{B}_K, \eta_{ct,cc,tt}, \bar{m}_{c,t}$



- Gaussian error : 1.3 σ discrepancy
- Rfit error : no discrepancy

ϵ_K and κ_ϵ (1)

$$\epsilon_K = \sin\phi_\epsilon e^{i\phi_\epsilon}\left[\frac{\mathrm{Im}M_{12}}{\Delta M} + \xi\right] = \kappa_\epsilon \frac{e^{i\phi_\epsilon}}{\sqrt{2}}\frac{\mathrm{Im}M_{12}^{(6)}}{\Delta M_K} \text{ with } \kappa_\epsilon \neq 1$$

1) $\phi_\epsilon = 43.5 \pm 0.7^\circ \neq 45^\circ$ $\qquad (\kappa_\epsilon)_\phi = 0.97 \pm 0.01$

2) $\xi = \mathrm{Im}A_0/\mathrm{Re}A_0$ $\qquad$ [Buras Guadagnoli 2008,2009]

- $\mathrm{Re}A_I$ from experiment (no theory for $\Delta I = 1/2$ rule)
- $\mathrm{Im}A_I$ from $\mathcal{H}_{\mathrm{eff}}^{\Delta S=1}$: $\mathrm{Im}A_I \propto \sum y_i \langle(\pi\pi)_I|Q_i|K\rangle$
  with y_i Wilson coefficients and Q_i $\Delta S = 1$ operators
- $\mathrm{Im}A_0$ (ξ) dominated by QCD penguin $\langle Q_6\rangle_0$ (not known)
- $\mathrm{Im}A_2$ dominated by electroweak penguin $\langle Q_8\rangle_2$ (lattice, sum rules)
- ϵ'/ϵ involves both $\mathrm{Im}A_0$ and $\mathrm{Im}A_2$

$$\langle Q_6\rangle_0 = f[(\epsilon'/\epsilon)_{\mathrm{exp}}, \langle Q_8\rangle_2] \qquad \xi = g[\langle Q_6\rangle_0] = g'[(\epsilon'/\epsilon)_{\mathrm{exp}}, \langle Q_8\rangle_2]$$

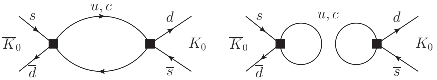
$$(\kappa_\epsilon)_{\phi+\xi} = 0.918 \pm 0.013|_{\epsilon'/\epsilon,\phi_\epsilon} \pm 0.007|_{Q_6,Q_8}$$

κ_ϵ (2)

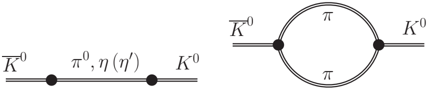
3) Higher-dim corr to $\mathrm{Im}M_{12}$ [Buras Guadagnoli Isidori 2010]

$$M_{12} = \langle K_0 | H_{\mathrm{eff}}^{|\Delta S|=2} | \bar{K}_0 \rangle - \langle K_0 | \int d^4x \mathrm{Re}[iT[H_{\mathrm{eff}}^{|\Delta S|=1}(x) H_{\mathrm{eff}}^{|\Delta S|=1}(0)]] | \bar{K}_0 \rangle$$

- Leading from $d = 6$ operator $(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$ in $H_{\mathrm{eff}}^{|\Delta S|=2}$
- $d = 8$ corr from $H_{\mathrm{eff}}^{|\Delta S|=2}$ m_K^2/m_c^2-suppressed wrt $d = 6$ charm [2%]
- $d = 8$ corrections from two insertions of $H_{\mathrm{eff}}^{|\Delta S|=1}$



computed using χPT for
$\Delta S = 1$ transitions:
$\xi \to \rho\xi$ with $\rho = 0.6 \pm 0.3$



$(\kappa_\epsilon)_{\mathrm{tot}} = 0.940 \pm 0.013 \pm 0.023$

slightly more conservative than BGI (0.94 ± 0.2)

ξ correction to κ_ϵ (1)

[Buras 1998]

Computing $\mathrm{Im}A_0$ and $\mathrm{Im}A_2$ from $\Delta S = 1$ Hamiltonian provides:

$$\frac{\epsilon'}{\epsilon} = \mathrm{Im}\lambda_t[P^{(1/2)} - P^{(3/2)}]$$

$P^{(1/2)} = \frac{G_F \omega}{2|\epsilon|\mathrm{Re}A_0} \sum y_i \langle Q_i \rangle_{I=0}(1 - \Omega_{\eta+\eta'})$ $\qquad P^{(3/2)} = \frac{G_F}{2|\epsilon|\mathrm{Re}A_0} \sum y_i \langle Q_i \rangle_{I=2}$
where y_i Wilson coefficients, isospin breaking part $\Omega_{\eta+\eta'} = 0.25 \pm 0.05$

$\epsilon'/\epsilon = N_0 + N_1 \cdot R_6 + N_2 \cdot R_8$ [Buras and Jamin, 2003]

- N_0, N_1, N_2 numbers coming from λ_t, experimental $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$ and assumptions on bag parameters
- R_6 and R_8 proportional to bag parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$

$$R_6 = R_6[(\epsilon'/\epsilon)_{\mathrm{exp}}, R_8 = 1.0 \pm 0.2]$$

ξ correction to κ_ϵ (2)

[Buras and Guadagnoli 2009]

Ratio of contribution of ImA_2 over that of ImA_0 in ϵ'/ϵ

$$\Omega_1 = \frac{N_2 \cdot R_8}{|N_0 + N_1 \cdot R_6|} \qquad \Omega_2 = \frac{N_0 + N_2 \cdot R_8}{|N_1 \cdot R_6|}$$

(first estimate preferred because cst term N_0 dominated by $I = 0$)

Ω to disentangle ImA_2 and ImA_0 in ϵ'/ϵ, then compute $\xi = \mathrm{Im}A_0/\mathrm{Re}A_0$

R_8	ϵ'/ϵ	R_6	Ω_1	Ω_2	$(\kappa_{\epsilon,1})_\phi$	$(\kappa_{\epsilon,2})_\phi$
1	$1.65 \cdot 10^{-3}$	1.28	0.32	0.38	0.921	0.915
1	$1.39 \cdot 10^{-3}$	1.15	0.36	0.43	0.926	0.921
1	$1.91 \cdot 10^{-3}$	1.40	0.29	0.35	0.915	0.910
1.2	$1.65 \cdot 10^{-3}$	1.35	0.37	0.42	0.917	0.912
0.8	$1.65 \cdot 10^{-3}$	1.20	0.28	0.35	0.924	0.919

$$(\kappa_\epsilon)_{\phi+\xi} = 0.918 \pm 0.013 \pm 0.007$$