Kaon Physics and Lattice QCD

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RBC-UKQCD Collaboration

Lattice meets Phenomenology

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1. Introduction

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     - Details of RBC/UKQCD calculation left to Elaine Goode - Next Talk.
  5. The η-η' system.
  6. Conclusions
  7. Long-Distance Contributions in Kaon Physics.

- In lattice phenomenology we:
  1. Consider *What Next?* for "mature" quantities which are being calculated with "good" precision.
  2. Continue to extend the range of physical quantities which can be calculated.
  3. Need new ideas for some important phenomenological quantities which I don’t know how to start evaluating.
     - Non-leptonic B-decays.

- We understand how to calculate the spectrum, quark masses, and matrix elements of the form $\langle 0|O(0)|h\rangle$ and $\langle h_2|O(0)|h_1\rangle$. These continue to be calculated with ever improving precision.

Perturbation Theory

- The precision of lattice calculations is now reaching the point where we need better interactions with the N^nLO QCD perturbation theory community.
- The traditional way of dividing responsibilities is:

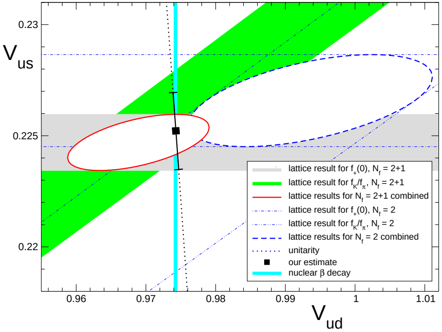
$$\text{Physics} = \underset{\substack{\uparrow \\ \text{Perturbative QCD}}}{C} \times \underset{\substack{\uparrow \\ \text{Lattice QCD}}}{\langle f\,|\,O\,|\,i\rangle}$$

- The two factors have to be calculated in the same scheme.
- Can we meet half way?

$$\begin{array}{c} \text{bare} \\ \text{lattice} \\ \text{operators} \end{array} \longrightarrow \ \textbf{?} \ \longleftarrow \begin{array}{c} \text{operators} \\ \text{renormalized} \\ \text{in } \overline{\text{MS}} \text{ scheme} \end{array}$$

- What is the best scheme for **?** (RI-SMOM, Schrödinger Functional, $\cdots$)?
- Recent examples of such collaborations following J.Gracey ...:
  - two-loop matching factor for m_q between the RI-SMOM schemes and $\overline{\text{MS}}$.
    M.Gorbahn and S.Jager, arXiv:1004:3997, L.Almeida and C.Sturm, arXiv:1004:4613
  - HPQCD + Karlsruhe Group in determination of quark masses.

2. V_{us} from Lattice Simulations



Flavianet Lattice Averaging Group

(preliminary)

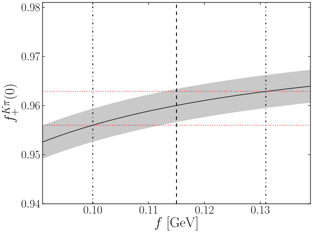
- Lattice calculations of f_K/f_π combined with the experimental widths $\Rightarrow V_{us}/V_{ud}$.
- Following the suggestion of Becirevic et al., precise lattice calculations of the $K_{\ell 3}$ form factor $f^+(0)$ are possible $\Rightarrow V_{us}$.

hep-ph/0403217

- Results are in remarkable agreement with SM.

- Currently the main uncertainty on $f^+(0)$ is due to the chiral extrapolation.

RBC-UKQCD, arXiv:1004:0886



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FLAG – Preliminary

- We have the two precise results:

$$\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.27599(59) \quad \text{and} \quad |V_{us} f_+(0)| = 0.21661(47)$$

Flavianet – arXiv:0801.1817

- We can view these as two equation for the four unknowns f_K/f_π, $f_+(0)$, V_{us} and V_{ud} .
- Within the Standard Model we also have the unitarity constraint:

$$|V_{ud}|^2 + |V_{us}|^2 \cancel{+ |V_{ub}|^2} = 1$$

- Thus we now have 3 equations for four unknowns.
- There has been considerable work recently in updating the determination of V_{ud} based on 20 different superallowed transitions. Hardy and Towner, arXiV:0812.1202

$$|V_{ud}| = 0.97425(22)\,.$$

- If we accept this value then we are able to determine the remaining 3 unknowns:

$$|V_{us}| = 0.22544(95), \quad f_+(0) = 0.9608(46), \quad \frac{f_K}{f_\pi} = 1.1927(59)\,.$$

FLAG - Preliminary

The results are remarkably consistent with the unitarity of the CKM Matrix

- Taking the experimental results for $K_{\ell 2}$ and $K_{\ell 3}$ decays and dividing by the $N_f = 2+1$ lattice values of f_K/f_π and $f^+(0)$ gives:

$$V_{ud}^2 + V_{us}^2 = 1.002(16)\,.$$

- If we combine the experimental results with the value of V_{ud} and the lattice values of $f^+(0)$ or f_K/f_π we find:

$$V_{ud}^2 + V_{us}^2 = 1.0000(7) \quad \text{or} \quad V_{ud}^2 + V_{us}^2 = 0.9999(7)\,.$$

SU(2) ChPT and $f_0(q^2_{\mathbf{max}})$; $q^2_{\mathbf{max}} = (m_K - m_\pi)^2$

m_π	$f_0(q^2_{\rm max})$
670 MeV	1.00029(6)
555 MeV	1.00192(34)
415 MeV	1.00887(89)
330 MeV	1.02143(132)

RBC-UKQCD ($N_f = 2+1$), arXiv:0710.5136

m_π	$f_0(q^2_{\rm max})$
575 MeV	1.00016(6)
470 MeV	1.00272(34)
435 MeV	1.00416(43)
375 MeV	1.00961(123)
300 MeV	1.01923(121)
260 MeV	1.03097(224)

ETM ($N_f = 2$), arXiv:0906.4728

- In the SU(2) chiral limit, $m_{ud} = 0$, we have the Callan-Treiman Relation

$$f_0(q^2_{\rm max}) = \frac{f_K}{f_\pi} \simeq 1.26.$$

- We have investigated whether the difference of the numbers in the table and 1.26 can be understood using SU(2) ChPT. J.Flynn & CTS, arXiv:0809.1229
  - The one-loop chiral logarithms have a large coefficient and are of the correct size to account for the difference. However they have the wrong sign!
  - There are linear and quadratic terms in m_π .
    They cannot be calculated in SU(2) ChPT, but estimating the LECs by converting results from SU(3) ChPT suggests that these terms have the correct sign and magnitude to account for the difference.
- The same features hold for $B \to \pi$ and $D \to \pi$ semileptonic decays.

- We also argue that information can be obtained at values of q^2 where the external pion is not soft, such as at the reference point $q^2 = 0$. J.Flynn, CTS, arXiV:0809.1229.

$$
\begin{aligned}
f^0(0) = f^+(0) &= F_+\left(1 - \frac{3}{4}\frac{m_\pi^2}{16\pi^2 f^2}\log\left(\frac{m_\pi^2}{\mu^2}\right) + c_+ m_\pi^2\right) \\
f^-(0) &= F_-\left(1 - \frac{3}{4}\frac{m_\pi^2}{16\pi^2 f^2}\log\left(\frac{m_\pi^2}{\mu^2}\right) + c_- m_\pi^2\right).
\end{aligned}
$$

- It is possible to calculate the chiral logarithm because this comes from a soft internal loop.
- The approach can be applied at other values of q^2.
- This idea has recently been extended to $K \to \pi\pi$ decays,

  J.Bijnens and A Celis, arXiV:0906.0302

  and to $B \to \pi$ and $D \to \pi$ semileptonic decays. J.Bijnens and I Jemos, arXiV:1006.1197
- Since the chiral extrapolation is a major source of systematic uncertainty for the lattice determination of V_{us} from $K_{\ell 3}$ decays, it is important to have all the possible theoretical information to guide us.

It would be useful to know the result at NNLO.

- It would be reassuring to confirm that it is possible to develop an effective theory in which hard and soft pions are separated.

Lunghi and Soni, arXiv:0803.4340 [hep-ph]

Buras and Guadagnoli, arXiv:0805.3887 [hep-ph], arXiv:0901.2056 [hep-ph]

Buras, Guadagnoli and Isisdori, arXiv:1002.3612 [hep-ph]

Within the standard model the indirect CP-Violation parameter

$$\varepsilon_K = \frac{2\eta_{+-}+\eta_{00}}{3}, \quad \eta_{ij} = \frac{\mathscr{A}(K_L \to \pi^i\pi^j)}{\mathscr{A}(K_S \to \pi^i\pi^j)},$$

can be written in the form

$$\varepsilon_K = \kappa_\varepsilon C_\varepsilon \hat{B}_K |V_{cb}|^2 |V_{us}|^2 \left(\frac{1}{2}|V_{cb}|^2 R_t^2 \sin 2\beta\, \eta_{tt} S_0(x_t) + R_t \sin\beta\,(\eta_{ct} S_0(x_c,x_t) - \eta_{cc}\, x_c)\right)$$

with

$$C_\varepsilon = \frac{G_F^2 f_K^2 M_{K^0} M_W^2}{6\sqrt{2}\pi^2\,(\Delta M_K)} \quad x_i = \bar{m}_i^2(\bar{m}_i), \quad R_t \simeq \frac{1}{\lambda}\frac{|V_{td}|}{|V_{ts}|}.$$

Two recent developments move the SM prediction downwards:

1. Precise lattice values of $\hat{B}_K$ are "low":

$$\hat{B}_K = 0.720(13)(37)$$ D.J.Antonio et al., RBC-UKQCD, hep-ph/0702042

$$\hat{B}_K = 0.724(8)(28)$$ C.Aubin, J.Laiho and R.Van de Water, arXiv:0905.3947

$$\varepsilon_K = \kappa_\varepsilon C_\varepsilon \hat{B}_K |V_{cb}|^2 |V_{us}|^2 \left(\frac{1}{2}|V_{cb}|^2 R_t^2 \sin 2\beta\, \eta_{tt} S_0(x_t) + R_t \sin\beta\, (\eta_{ct} S_0(x_c, x_t) - \eta_{cc}\, x_c)\right)$$

2 Write $\varepsilon_K = e^{i\phi_\varepsilon} \sin(\phi_\varepsilon)\left(\frac{\mathrm{Im}\, M_{12}^K}{\Delta M_K} + \xi\right)$, where

$$\xi = \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0} \quad \text{and} \quad \phi_\varepsilon = \arctan(2\Delta M_K/\Delta\Gamma) = (43.51 \pm 0.05)^\circ\,.$$

$\kappa_\varepsilon = 0.92 \pm 0.02$ is a correction factor taking into account the difference of ϕ_ε from 45° as well as the presence of the ξ term.

- Using the above values of $\hat{B}_K$ and κ_ε, Buras and Guadagnoli find: arXiv:0901.2056

$$|\varepsilon_K|^{\mathrm{SM}} = (1.78 \pm 0.25)\, 10^{-3} \quad \text{to be compared to} \quad |\varepsilon_K|^{\exp} = (2.229 \pm 0.012)\, 10^{-3}\,.$$

2σ "tension"

- The top-quark contribution to ε_K is the dominant one so that approximately:

$$|\varepsilon_K| \propto \kappa_\varepsilon\, f_K^2\, \hat{B}_K\, |V_{cb}|^4\, \xi_s^2\, \sin(2\beta)$$

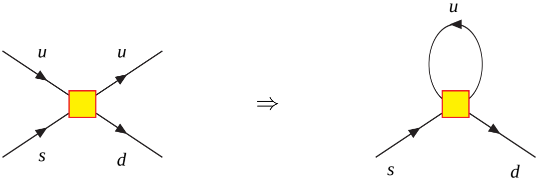
so that the prediction is very sensitive to $|V_{cb}| \stackrel{?}{=} (41.2 \pm 1.1)\, 10^{-3}$ and

$$\xi_s = \frac{f_{B_s}\sqrt{\hat{B}_s}}{f_{B_d}\sqrt{\hat{B}_d}} \stackrel{?}{=} 1.21 \pm 0.06$$

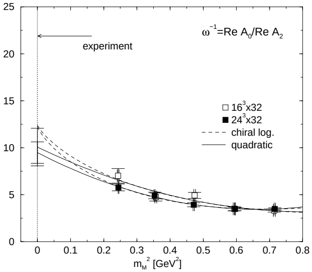
- At lowest order in the SU(3) chiral expansion one can obtain the $K \to \pi\pi$ decay amplitude by calculating $K \to \pi$ and $K \to$ vacuum matrix elements.
- In 2001, two collaborations published some very interesting (quenched) results on non-leptonic kaon decays in general and on the $\Delta I = 1/2$ rule and ε'/ε in particular:

Collaboration(s)	Re A_0/Re A_2	ε'/ε
RBC	25.3 ± 1.8	$-(4.0 \pm 2.3) \times 10^{-4}$
CP-PACS	9÷12	(-7 ÷ -2)$\times 10^{-4}$
Experiments	22.2	$(17.2 \pm 1.8) \times 10^{-4}$

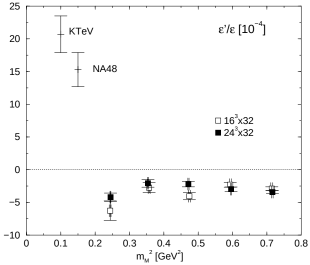
- This required the control of the *ultraviolet problem*, the subtraction of power divergences and renormalization of the operators – highly non-trivial.
  - Four-quark operators mix, for example, with two quark operators $\Rightarrow$ power divergences:



- Re A_0/Re A_2 as a function of the meson mass.



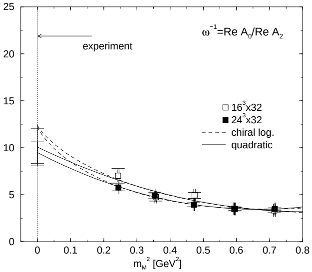
- ε'/ε as a function of the meson mass.



- The RBC and CP-PACS simulations were quenched, and relied on the validity of lowest order χPT in the region of approximately 400-800 MeV.
- Given the cancelations between different matrix elements (particularly O_6 and O_8) the negative value of ε'/ε is not such an embarrassment but

Must do better!

- Re A_0/Re A_2 as a function of the meson mass.



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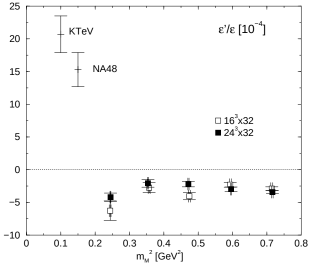

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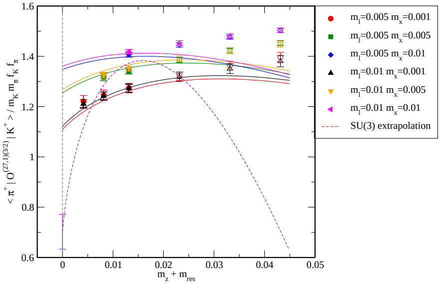
Unquenched Calculation

S.Li, Ph.D. thesis, RBC-UKQCD in preparation



$$O^{3/2}_{(27,1)} = (\bar{s}d)_L \left\{ (\bar{u}u)_L - (\bar{d}d)_L \right\} + (\bar{s}u)_L \, (\bar{u}d)_L$$

- RBC/(UKQCD) have repeated the calculation with the 24^3 DWF ensembles in the pion-mass range 240-415 MeV.
- For illustration consider the determination of α_{27}, the LO LEC for the (27,1) operator. Satisfactory fits were obtained, but again the corrections were found to be huge, casting serious doubt on the approach.
- Soft pion theorems are not sufficiently reliable $\Rightarrow$ need to compute $K \to \pi\pi$ matrix elements.

To arrive at this important conclusion required a truly major effort.

- J.Laiho et al. challenge this conclusion. Poster, Lattice conference

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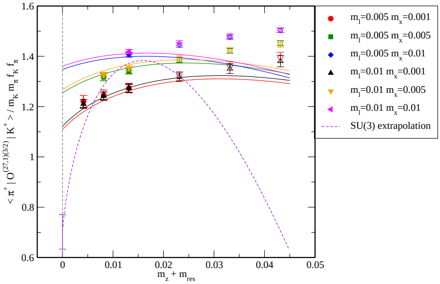

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$K \to (\pi\pi)_{I=2}$ decay amplitudes

- Preliminary results from the RBC/UKQCD study were presented by E.Goode and M.Lightman at
  - Lattice 2009: arXiv:0912.1667

    $\Delta I = 3/2,\ K \to \pi\pi$ Decays with Light, Non-Zero Momentum Pions
  - Lattice 2010:

    http://agenda.infn.it/contributionDisplay.py?contribId=11&sessionId=22&confId=2128

    $\Delta I = 3/2,\ K \to \pi\pi$ Matrix Elements with Nearly Physical Pion Masses
- Of course we would like to evaluate all the $K \to \pi\pi$ matrix elements in lattice simulations and reconstruct A_0 and A_2 and understand the $\Delta I = 1/2$ rule and the value of ε'/ε (see below).
- In the meantime however, we know Re A_0 and Re A_2 from experiment.
  I now attempt to demonstrate that we can also compute Re A_2.
- The experimental value of ε'/ε gives us one relation between Im A_0 and Im A_2, thus if we evaluate Im A_2 then within the standard model we know Im A_0 to some precision. Thanks to Andrzej Buras for stressing this to me.
  I also attempt to demonstrate that we can indeed compute Im A_2.
- I stress again that ultimately of course, we wish to do better than this.

See next section

All numerical results are preliminary.

- We need to be able to calculate $K \to \pi\pi$ matrix elements directly.
- The main theoretical ingredients of the *infrared* problem with two-pions in the s-wave are now understood.
- Two-pion quantization condition in a finite-volume

$$\delta(q^*) + \phi^P(q^*) = n\pi\,,$$

where $E^2 = 4(m_\pi^2 + q^{*\,2})$, δ is the s-wave $\pi\pi$ phase shift and ϕ^P is a kinematic function. M.Lüscher, 1986, 1991, $\cdots$.

- The relation between the physical $K \to \pi\pi$ amplitude A and the finite-volume matrix element M

$$|A|^2 = 8\pi V^2\,\frac{m_K E^2}{q^{*\,2}}\,\{\delta'(q^*) + \phi^{P\prime}(q^*)\}\,|M|^2\,,$$

where $\prime$ denotes differentiation w.r.t. q^* .

L.Lellouch and M.Lüscher, hep-lat/0003023; C.h.Kim, CTS and S.Sharpe, hep-lat/0507006; N.H.Christ, C.h.Kim and T.Yamazaki hep-lat/0507009

- Computation of $K \to (\pi\pi)_{I=2}$ matrix elements does not require the subtraction of power divergences or the evaluation of disconnected diagrams.
- In principle, we understand how to calculate the $\Delta I = 3/2$ $K \to \pi\pi$ matrix elements.
- Our aim is to calculate the matrix elements with as good a precision as we can.

- The operators whose matrix elements have to be calculated are:

$$\begin{aligned} O^{3/2}_{(27,1)} &= (\bar{s}^i d^i)_L \{(\bar{u}^j u^j)_L - (\bar{d}^j d^j)_L\} + (\bar{s}^i u^i)_L (\bar{u}^j d^j)_L \\ O^{3/2}_7 &= (\bar{s}^i d^i)_L \{(\bar{u}^j u^j)_R - (\bar{d}^j d^j)_R\} + (\bar{s}^i u^i)_L (\bar{u}^j d^j)_R \\ O^{3/2}_8 &= (\bar{s}^i d^j)_L \{(\bar{u}^j u^i)_R - (\bar{d}^j d^i)_R\} + (\bar{s}^i u^j)_L (\bar{u}^j d^i)_R \end{aligned}$$

- It is convenient to use the Wigner-Eckart Theorem: (Notation - $O^{\Delta I}_{\Delta I_z}$)

$$_{I=2}\langle \pi^+(p_1)\pi^0(p_2)\,|O^{3/2}_{1/2}|K^+\rangle = \frac{3}{2}\langle \pi^+(p_1)\pi^+(p_2)\,|O^{3/2}_{3/2}|K^+\rangle\,,$$

where
  - $O^{3/2}_{3/2}$ has the flavour structure $(\bar{s}d)\,(\bar{u}d)$.
  - $O^{3/2}_{1/2}$ has the flavour structure $(\bar{s}d)\,(\,(\bar{u}u) - (\bar{d}d)\,) + (\bar{s}u)\,(\bar{u}d)$.

- We can then use antiperiodic boundary conditions for the u-quark say, so that the $\pi\pi$ ground-state is $\langle \pi^+(\pi/L)\pi^+(-\pi/L)|$. C-h Kim, Ph.D. Thesis
  - Do not have to isolate an excited state.
  - Size (L) needed for physical $K \to \pi\pi$ decay halved (6 fm $\to$ 3 fm).

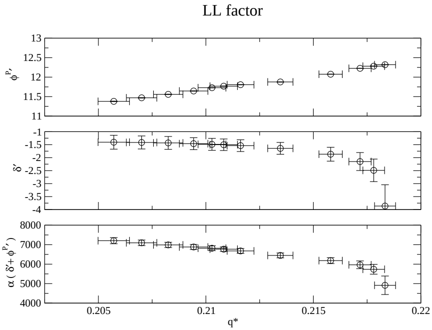
C.h. Kim and CTS, arXiv:1003.3191

- Use the Wigner-Eckart Theorem to relate the physical $K \to \pi^+\pi^0$ matrix element to that for $K \to \pi^+\pi^+$

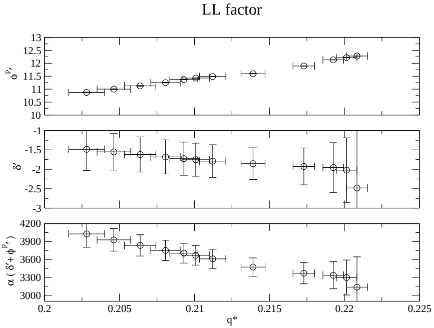
$$_{I=2}\langle \pi^+(p_1)\pi^0(p_2)\,|O^{3/2}_{1/2}|K^+\rangle = \frac{3}{2}\langle \pi^+(p_1)\pi^+(p_2)\,|O^{3/2}_{3/2}|K^+\rangle\,,$$

- Calculate the $K \to \pi^+\pi^+$ matrix element with the u-quark with twisted boundary conditions with twisting angle θ.
- Perform a Fourier transform of one of the pion interpolating operators with additional momentum $-2\pi/L$.
  The ground state now corresponds to one pion with momentum θ/L and the other with momentum $(\theta-2\pi)/L$.
- The corresponding $\pi\pi$ s-wave phase-shift can then be obtained by the Lüscher formula as a function of $\theta \Rightarrow$ this allows for the derivative of the phase-shift to be evaluated directly at the masses being simulated.
- We have carried this procedure out in an exploratory calculation. Fig
- Unfortunately this technique does not work for $K \to (\pi\pi)_{I=0}$ decays.

C.h. Kim and CTS, arXiv:1003.3191



$m_q = 0.004$



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- We have carried this procedure out in an exploratory calculation. Fig
- Unfortunately this technique does not work for $K \to (\pi\pi)_{I=0}$ decays.

- We argued that SU(2) ChPT can be used for $K_{\ell 3}$ form factors at $q^2=0$ (and other q^2 where the pion is not soft). J.Flynn, CTS, arXiV:0809.1229.

$$f^0(0)=f^+(0) = F_+\left(1-\frac{3}{4}\frac{m_\pi^2}{16\pi^2f^2}\log\left(\frac{m_\pi^2}{\mu^2}\right)+c_+m_\pi^2\right)$$
$$f^-(0) = F_-\left(1-\frac{3}{4}\frac{m_\pi^2}{16\pi^2f^2}\log\left(\frac{m_\pi^2}{\mu^2}\right)+c_-m_\pi^2\right).$$

- It is possible to calculate the chiral logarithm because this comes from a soft internal loop.
- This idea has been extended to $K\to\pi\pi$ decays, J.Bijnens and A Celis, arXiV:0906.0302

$$A_2=A_2^{\rm LO}\left(1-\frac{15}{4}\frac{m_\pi^2}{16\pi^2f^2}\log\left(\frac{m_\pi^2}{\mu^2}\right)\right)+\lambda_2\,m_\pi^2$$
$$A_0=A_0^{\rm LO}\left(1-\frac{3}{4}\frac{m_\pi^2}{16\pi^2f^2}\log\left(\frac{m_\pi^2}{\mu^2}\right)\right)+\lambda_0\,m_\pi^2\,,$$

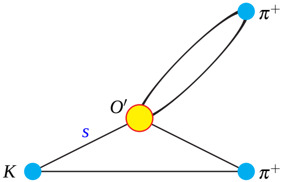
(and to $B\to\pi$ and $D\to\pi$ semileptonic decays. J.Bijnens and I Jemos, arXiV:1006.1197)

It would be useful to know the results at NNLO.

- It would be reassuring to confirm that it is possible to develop an effective theory in which hard and soft pions are separated.

Preliminary $\Delta I = 3/2$ Matrix Elements

RBC-UKQCD, M.Lightman and E.Goode, Lattice 2010



- The RBC/UKQCD strategy at this stage is to perform the simulations on a large lattice, $L \simeq 4.5$ fm, with light pions ($32^3 \times 64 \times 32$)

$$m_\pi \simeq 145\,\mathrm{MeV} \qquad \text{Unitary } m_\pi \simeq 180\,\mathrm{MeV}\,.$$

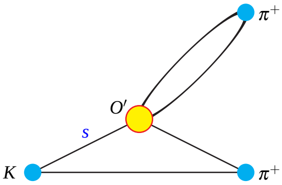
- The price is that the lattice is coarse, $a^{-1} \simeq 1.4$ GeV.
- With DWF, m_{res} increases as the coupling becomes stronger $\Rightarrow$ change the gauge action (from Iwasaki) by multiplying by the *Auxilliary Determinant* .

D.Renfrew, T.Blum, N.Christ, R.Mawhinney and P.Vranas, arXiv:0902.2587

R. Mawhinney, Lattice 2010

- This is tuned to suppress m_{res} but to maintain topology changing.

Preliminary $\Delta I = 3/2$ Matrix Elements – Cont.



- The masses and momenta are as follows:

Quantity	This Calculation	Physical
m_π	145.6(5) MeV	139.6 MeV
m_K	519(2) MeV	493.7 MeV
$E_{\pi\pi}(p_\pi \simeq 0)$	294(1) MeV	
$E_{\pi\pi}(p_\pi \simeq \sqrt{2}\pi/L)$	516(9) MeV	
$E_{\pi\pi}(p_\pi \simeq \sqrt{2}\pi/L) - m_K$	-2.7(8.3) MeV	

- The results presented here were obtained with 47 configurations (we are continuing to 100).

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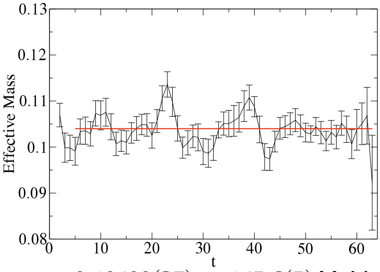
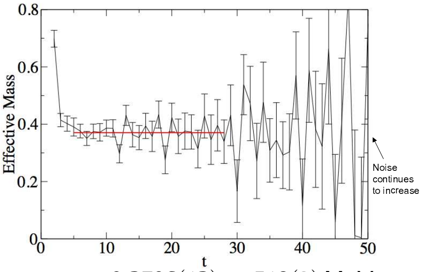
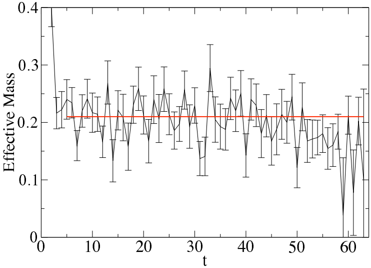


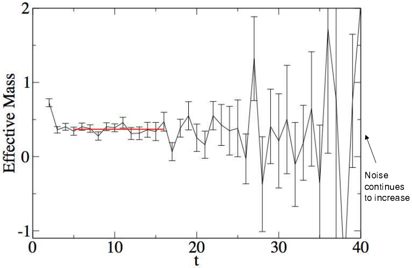
$m_\pi = 0.10400(37) \Rightarrow 145.6(5)$ MeV



$m_K = 0.3706(13) \Rightarrow 519(2)$ MeV



$E_{\pi\pi} = 0.2100(10) \Rightarrow 294(1)$ MeV



$E_{\pi\pi} = 0.3687(61) \Rightarrow 516(9)$ MeV

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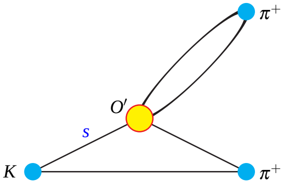
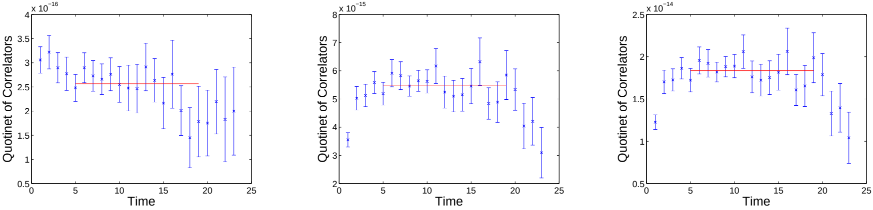


Source	Re(A_2) (10^{-8} GeV)
$t_K = 20$	1.52(12)
$t_K = 24$	1.52(10)
$t_K = 28$	1.71(13)
$t_K = 32$	1.35(22)
Weighted Average	1.56(7)
Experiment	1.5

Stat. error only



$O'^{3/2}_{(27,1)} = (\bar{s}d)_L\,(\bar{u}d)_L$ $\quad O'^{3/2}_{7} = (\bar{s}d)_L\,(\bar{u}d)_R$ $\quad O'^{3/2}_{8} = (\bar{s}^i d^j)_L\,(\bar{u}^j d^i)_R$

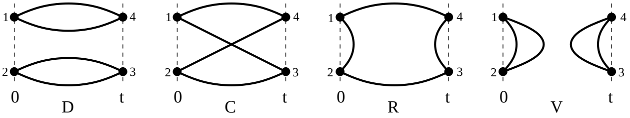
Sample plateaus for the matrix elements at matched kinematics ($p_\pi = \sqrt{2}p_{\rm min}$).

(Preliminary) Conclusions from $K \to (\pi\pi)_{I=2}$ study

- We have preliminary results for the $\Delta I = 3/2$ $K \to \pi\pi$ decay amplitude on 32^3 lattices with 2+1 flavours of DWF and the Iwasaki-DSDR gauge action.
- $m_\pi = 145.6(5)$ MeV, $m_K = 519(2)$ MeV, $E_{\pi\pi} = 516(9)$ MeV .
- Re $A_2 = 1.56(7)(25) \times 10^{-8}$ GeV .
  Error is dominated by lattice artefacts, a^{-3} on a coarse lattice.
- Im $A_2 = -9.6(4)(24) \times 10^{-13}$ GeV .
  In addition to lattice artefacts, we are in the process of performing the NPR for the EWP operators $O_{7,8}$ The result above is obtained by taking $Z_{ij} = 0.9(0.18)\delta ij$.
- Im A_2/Re $A_2 = -6.2(0.3)(1.3)\,10^{-5}$.
- We are confirming that these calculations are possible.
- Calculations of the $\Delta I = 1/2$ amplitudes are much more challenging - Next Section.

$K \to (\pi\pi)_{I=0}$ Decays

- The $I = 0$ final state has vacuum quantum numbers.
- Vacuum contribution must be subtracted; disconnected diagrams require statistical cancelations to obtain the $e^{-2m_\pi t}$ behaviour.
- Consider first the two-pion correlation functions, which are an important ingredient in the evaluation of $K \to \pi\pi$ amplitudes.



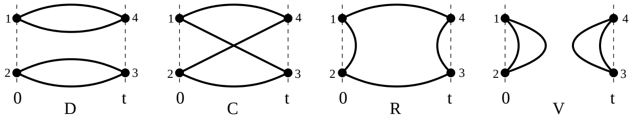
- For I=2 $\pi\pi$ states the correlation function is proportional to D-C.
- For I=0 $\pi\pi$ states the correlation function is proportional to 2D+C-6R+3V.

The major practical difficulty is to subtract the vacuum contribution with sufficient precision.

- We are performing high-statistics experiments on a $16^3 \times 32$ lattice, $a^{-1} = 1.73$ GeV, $m_\pi = 420$ MeV, propagators evaluated from each time-slice.

Qi Liu – Lattice 2010

Diagrams contributing to two-pion correlation functions



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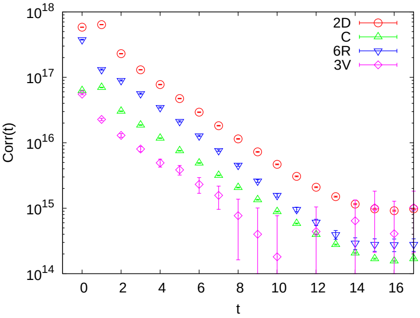
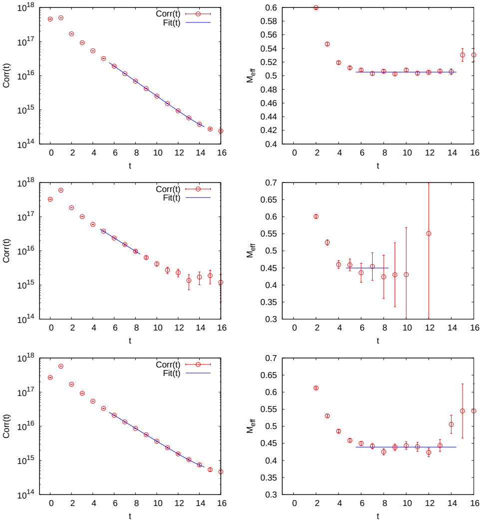


RBC/UKQCD, Qi Liu – Lattice 2010

Two-pion Correlation Functions

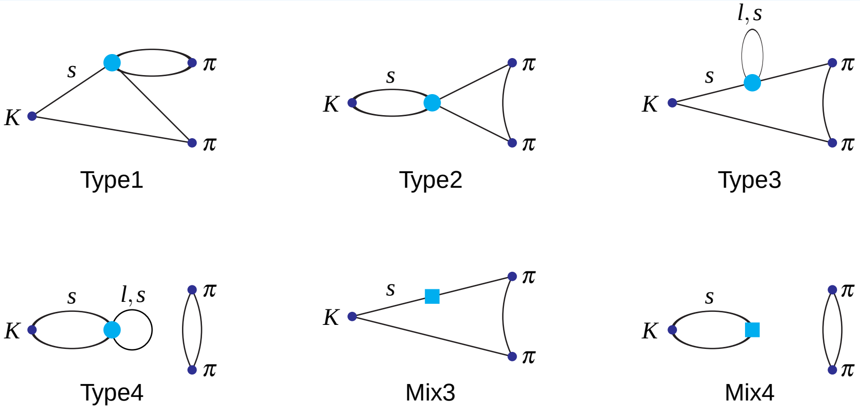


- $I = 2$ (Correlator and Effective Mass)
- $E_{\pi\pi} = 0.5054(15)$

- $I = 0$ (Correlator and Effective Mass)
- $E_{\pi\pi} = 0.450(17)$

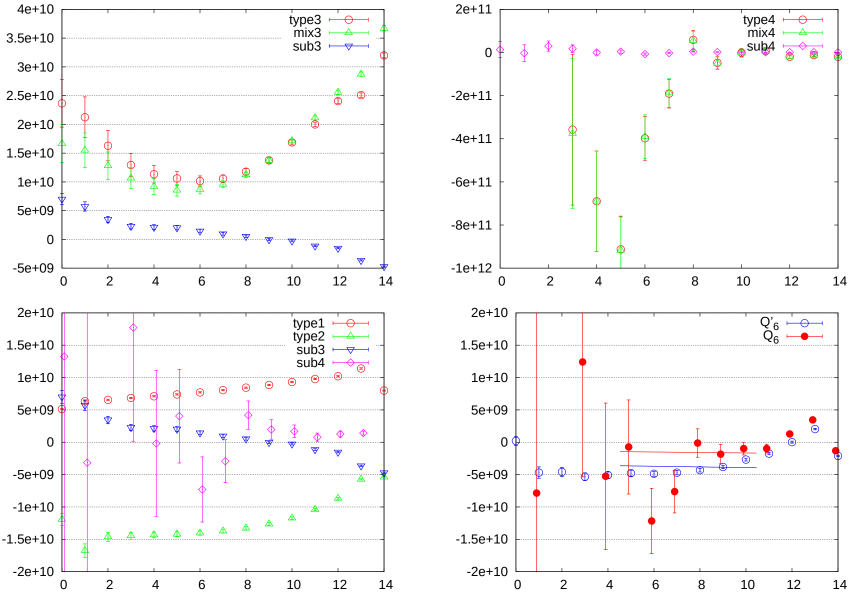
We are now doubling the statistics.

- $I = 0$ (Correlator - V and Effective Mass)
- $E_{\pi\pi} = 0.4392(59)$



- There are 48 different contractions and we classify the contributions into the 6 different types illustrated above.
- Mix3 and Mix4 are needed to subtract the power divergences which are proportional to matrix elements of $\bar{s}\gamma_5 d$.

Sample Results for $Q_6 = (\bar{s}^i d^j)_L \; \sum_q (\bar{q}^j q^i)_R$



Results from exploratory simulation at unphysical kinematics

RBC/UKQCD, Qi Liu – Lattice 2010

- These results are for the $K \to \pi\pi$ (almost) on-shell amplitudes with 420 MeV pions at rest:

Re A_0	$(3.0 \pm 0.8)\, 10^{-7}$ GeV
Im A_0	$-(2.9 \pm 2.2)\, 10^{-11}$ GeV
Re A_2	$(5.395 \pm 0.045)\, 10^{-8}$ GeV
Im A_2	$-(7.79 \pm 0.08)\, 10^{-13}$ GeV

- This is an exploratory exercise in which we are learning how to do the calculation.
- We are currently doubling the statistics to confirm our belief that a direct calculation appears to possible.
- The next stage is to proceed towards physical kinematics.

- From computations of $K \to \pi$ matrix elements in the pion mass-range 240-420 MeV using NLO ChPT, RBC/(UKQCD) conclude that they cannot determine the LO LEC for $K \to \pi\pi$ decays reliably
  $\Rightarrow$ need to calculate $K \to \pi\pi$ matrix elements directly.
- The computation of $K \to (\pi\pi)_{I=2}$ amplitudes is progressing well, with the preliminary result for Re A_2 is good agreement with the physical value.
  - Normalized Im A_2 available soon.
  - This will become a benchmark computation which will be improved in the coming years (finer lattices?).
- The exploratory results for $K \to (\pi\pi)_{I=0}$ decays encourage us to proceed to physical kinematics.

$\Rightarrow$ an understanding of the $\Delta I = 1/2$ rule and the value of ε'/ε.

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$K \to \pi\pi$ Decays - Conclusions

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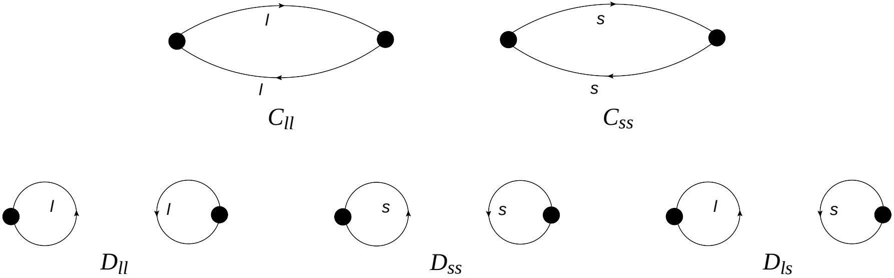
$\Rightarrow$ an understanding of the $\Delta I = 1/2$ rule and the value of ε'/ε.

RBC-UKQCD – arXiV:1002.2999

- A related topic is the study of the η-η' system.
- To study η and η' we also need to evaluate *disconnected* diagrams.



- Here l represents the u or d quark ($m_u = m_d$) and s the strange quark.

RBC-UKQCD – arXiV:1002.2999

- Let

$$O_l = \frac{\bar{u}\gamma_5 u + \bar{d}\gamma_5 d}{\sqrt{2}} \quad \text{and} \quad O_s = \bar{s}\gamma_5 s\,.$$

- We calculate the correlation functions

$$X_{\alpha\beta}(t) = \frac{1}{32}\sum_{t'=0}^{31} \langle\, O_\alpha(t+t')\,O_\beta(t')\,\rangle \quad \text{where} \quad \alpha,\beta = l,s\,.$$

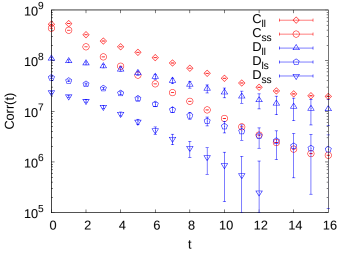
  - Sources are generated for each time slice (T=32).
  - $X_{ls} \neq 0$ because of the $D_{ls} = D_{sl}$ diagrams.

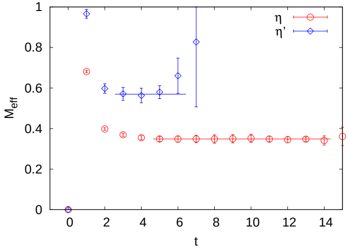
- The four correlation functions correspond to the diagrams as follows:

$$\begin{pmatrix} X_{ll} & X_{ls} \\ X_{sl} & X_{ss} \end{pmatrix} = \begin{pmatrix} C_{ll} - 2D_{ll} & -\sqrt{2}D_{ls} \\ -\sqrt{2}D_{sl} & C_{ss} - D_{ss} \end{pmatrix}.$$

- The usual expectation that disconnected diagrams and the resulting mixing are small does not apply here.

RBC-UKQCD – arXiV:1002.2999





- We diagonalize $X(t)$ at each t:

$$X(t)=A^{\mathrm{T}}\begin{pmatrix} e^{-m_\eta t} & 0 \\ 0 & e^{-m_{\eta'} t}\end{pmatrix}A\,,\quad \text{where}\quad A=\begin{pmatrix}\langle\eta\,|\,O_l\,|0\rangle & \langle\eta\,|\,O_s\,|0\rangle \\ \langle\eta'\,|\,O_l\,|0\rangle & \langle\eta'\,|\,O_s\,|0\rangle\end{pmatrix}$$

- To be more precise we diagonalize $X(t_0)^{-1}X(t)$. Lüscher and Wolff (1990)

- In the standard phenomenological treatment of $\eta - \eta'$ mixing

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |8\rangle_{\rm sym} \\ |1\rangle_{\rm sym} \end{pmatrix}$$

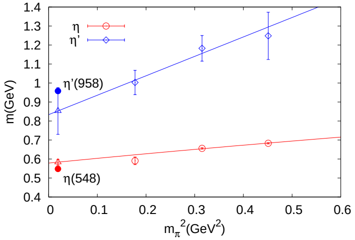
- In the O_8 and O_1 basis

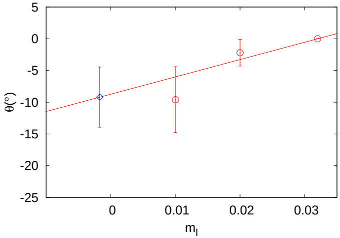
$$A = \begin{pmatrix} \sqrt{Z_8}\cos\theta & -\sqrt{Z_1}\sin\theta \\ \sqrt{Z_8}\sin\theta & \sqrt{Z_1}\cos\theta \end{pmatrix} \quad \text{where} \quad {}_{\rm sym}\langle a|O_b|0\rangle = \sqrt{Z_a}\delta_{ab}\,.$$

- If this model is correct then the columns of A are orthogonal. We find for the dot product - $-0.009(49)$ for $m_l = 0.01$ and $0.008(24)$ for $m_l = 0.02$.
- The mixing angle can be determined from

$$\frac{A_{\eta 1}A_{\eta' 8}}{A_{\eta 8}A_{\eta' 1}} = -\tan^2\theta\,.$$

RBC-UKQCD – arXiV:1002.2999





- We find $m_\eta = 583(15)$ MeV and $m_{\eta'} = 853(123)$ MeV and $\theta = -9.2(4.7)^\circ$. (Statistical errors only.)
- To our accuracy, our calculation demonstrates that QCD can explain the relatively large mass of the ninth pseudoscalar meson and its small mixing with the SU(3) octet state.
- There is plenty more to do!

6. Conclusions

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- At this workshop we have seen lattice contributions to much beautiful phenomenology, both in improved precision and in the extension of computations beyond the standard quantities.

  Recent years: Quenched $\Rightarrow \gtrsim 500\,$MeV pions $\Rightarrow$ "Almost physical pions"

- This improvement has to be continued vigorously if precision flavour physics is to play a complementary role to large $p_\perp$ discovery experiments at the LHC in unraveling the next level of fundamental physics
- We do not know how to compute some important phenomenological quantities.
- At the 1989 Lattice Conference in Capri, Ken Wilson made the seemingly pessimistic prediction that it will take about 30 years to have precision Lattice QCD.

  We have 9 years left, but are well on our way now.

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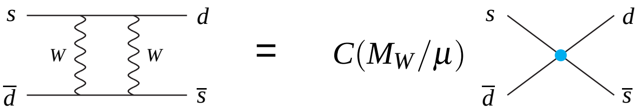
We have 9 years left, but are well on our way now.

- We are used to calculating the short distance contributions to physical processes.

  For example in neutral-kaon mixing:



- In many cases the short-distance contribution is the dominant term, but long-distance contributions are not always negligible:
  - If GIM suppression is logarithmic.
  - CKM enhancement (even if GIM suppression is power like).
- As lattice computations of the short-distance contributions become more precise we should try to learn how to compute these long-distance contributions effectively. Early thoughts in this direction include:
  - Rare Kaon Decays. G.Isidori, G.Martinelli, P.Turchetti, hep-lat/0506026
  - Neutral Kaon Mixing. N.Christ, Lattice 2010

Isidori, Martinelli, Turchetti, hep-lat/0506026

- $K \to \pi\ell^+\ell^-$ Decays. The main non-perturbative correlators for these decays are:

G.Ecker, A.Pich, E.de Rafael, (1987);

G. D'Ambrosio, G.Ecker, G.Isidori, J.Portolés, hep-ph/9808289

$$-i\int d^4x\, e^{-iq\cdot x}\,\langle \pi^j(p)\,|T\left\{J^\mu_{\rm em}(x)\,[Q^u_i(0)-Q^c_i(0)]\right\}\,|K^j(k)\rangle\,,$$

where $q=k-p$ is the momentum transfer and Q_i (i=1,2) are four quark operators.

- $K^+\to\pi^+\nu\bar\nu$ Decays. Suppression of long-distance effects is partially compensated by a large CKM coefficient and the dominant T-product is:

G.Buchalla, A.Buras, hep-ph/9308272, hep-ph/9901288;

A.Falk, A.Lewandowski, A.Petrov, hep-ph/0012099

$$-i\int d^4x\, e^{-iq\cdot x}\,\langle \pi^+(p)\,|T\left\{J^\mu_Z(x)\,[Q^u_i(0)-Q^c_i(0)]\right\}\,|K^+(k)\rangle\,.$$

- Without the presence of the Q_i the calculation is just the by-now standard one of $K\to\pi$ form-factors.
- With q^2 below any physical threshold, IMT avoid considering the corresponding Minkowski->Euclidean issues.

Isidori, Martinelli, Turchetti, hep-lat/0506026

- The generic calculation is of the correlation functions

$$-i\int d^4x\, e^{-iq\cdot x}\,\langle 0\,|\,\phi_\pi(t_\pi,\vec{p})J^\mu_X(x)\,[Q^u_i(0)-Q^c_i(0)]\,\phi^\dagger_K(t_K,\vec{k})\,|\,0\,\rangle\,,$$

  with $t_\pi > 0$ and $t_K < 0$.
- The main issue discussed in IMT is that of renormalization and the subtraction of power divergences.
  - Mixing of operators Q_i with lower dimensional operators. $\surd$
  - Contact terms between the Q_i and the interpolating operators - spectral analysis needed. $\surd$
  - Contact terms between the Q_i and currents depend on the currents.

    For $K\to\pi\ell^+\ell^-$ decays, gauge invariance $\Rightarrow$ no power divergences. GIM mechanism not necessary

    For $K^+\to\pi^+\nu\bar{\nu}$ decays, GIM used to cancel power divergences and the linear divergence is absent in regularizations which preserve chiral symmetry .
- General arguments checked by one-loop perturbative calculations.
- IMT believe that their results open a new field of interesting physical applications to the lattice community.

N.Christ - Lattice 2010

Start with the correlation function:

$$C(t_3,t_b,t_a,t_0) = \frac{1}{2}\sum_{t_1,t_2=t_a}^{t_b} \langle 0\,|\,\phi_K(t_3)\,T\{H(t_2)H(t_1)\}\,\phi_K(t_0)\,|\,0\rangle$$

where

$$H(t) = \sum_{\vec{x}} \mathscr{H}^{\Delta S=1}(t,\vec{x})\,,$$

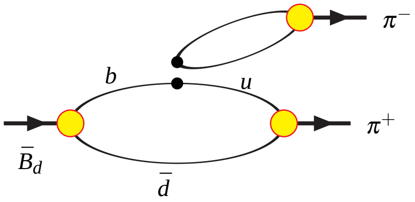
and $t_3 \gg t_b > t_a \gg t_0$.

- By calculating C for sufficiently large $t_b - t_a$ we obtain ΔM:

$$C(t_3,t_b,t_a,t_0) \simeq -Z_K^2\,\Delta M\,(t_b-t_a)\,e^{-m_K(t_3-t_0)}\,.$$

- Subtraction of short-distance contribution.
- Finite-volume corrections included à la Lellouch-Luscher.
- “With sufficient computing power a calculation of $m_{K_S} - m_{K_L}$ is possible".

- A huge amount of information has been obtained about decay rates and CP-asymmetries for $B \to M_1 M_2$ decays (over 100 channels).
- With just a few exceptions (e.g. CP-asymmetry in $B \to J/\Psi K_s$) our ability to deduce fundamental information about CKM matrix elements is limited by our inability to quantify the non-perturbative strong interaction effects.
- Most approaches were based on **Naive Factorization:**

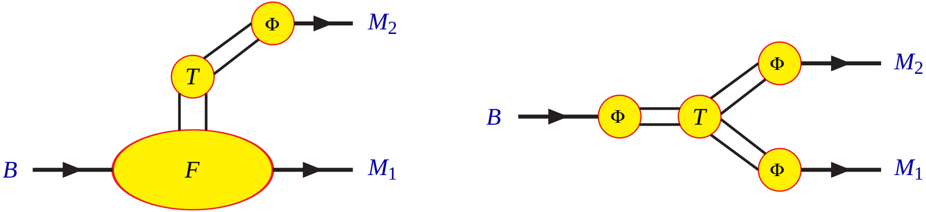


$$\langle \pi^+ \pi^- \,|\, (\bar{u}b)_{V-A}\, (\bar{d}u)_{V-A} \,|\, \bar{B}_d \rangle \to \langle \pi^- \,|\, (\bar{d}u)_{V-A} \,|\, 0 \rangle\, \langle \pi^+ \,|\, (\bar{u}b)_{V-A} \,|\, \bar{B}_d \rangle$$

- $\langle \pi^- \,|\, (\bar{d}u)_{V-A} \,|\, 0 \rangle$ is known (f_π).
- $\langle \pi^+ \,|\, (\bar{u}b)_{V-A} \,|\, \bar{B}_d \rangle$ is known in principle ($F_0^{B\to\pi}(m_\pi^2)$).
- No rescattering in the final state. No strong phase-shifts.
- μ dependence does not match on the two sides.

- In 1999 we realized that in the limit $m_b \to \infty$, the long distance effects *factorise* into simpler universal quantities:

M.Beneke, G.Buchalla, M.Neubert, CTS (BBNS)



$$\langle M_1, M_2 | O_i | B \rangle = \sum_j F_j^{B\to M_1}(m_2^2) \int_0^1 du\, T_{ij}^I(u) \Phi_{M_2}(u) + \quad (M_1 \leftrightarrow M_2)$$

$$+ \int_0^1 d\xi\, du\, dv\, T_i^{II}(\xi, u, v)\, \Phi_B(\xi)\, \Phi_{M_1}(v)\, \Phi_{M_2}(u)$$

Implications of Factorization

- The significance and usefulness of the factorization formula stems from the fact that the non-perturbative quantities which appear on the RHS are much simpler than the original matrix elements which appear on the LHS.
  They either reflect universal properties of a single meson state (the light-cone distribution amplitudes) or refer to a $B \to$ meson transition matrix element of a local current (form-factor).
- Conventional (naive) factorization is recovered as a rigorous prediction in the infinite quark-mass limit (i.e. neglecting $O(\alpha_s)$ and $O(\Lambda_{\text{QCD}}/m_b)$ corrections).
- Perturbative corrections to naive factorization can be computed systematically. The results are, in general, non-universal (i.e. process dependent).
- All strong interaction phases are generated perturbatively in the heavy quark limit.
- The factorization formulae are valid up to $O(\Lambda_{\text{QCD}}/m_b)$ corrections.
- Many observables of interest for CP-violation become accessible. The precision of the calculations is limited by our knowledge of the wave-functions and of the power corrections.
- For a comprehensive study of 96 PP and PV decay modes see

Beneke and Neubert, hep-ph/0308039.

- The main limitation of the factorization framework is due to the fact that m_b is not so large, so that CKM and chiral enhancements to non-factorizable $O(\Lambda_{\rm QCD}/m_b)$ terms are important.
- At present we do not know how to begin computing $B \to M_1 M_2$ matrix elements!
  - Many intermediate states contribute.
- What can lattice simulations contribute to the factorization formula:
  - Parton distribution amplitudes of light mesons (at least the low moments) $\surd$.
  - $B \to M$ form-factors $\surd$.
  - Parton distribution amplitudes of B-meson X.
- I now briefly explain why we have not been able to compute ϕ^B or its moments.

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$$\phi^B_{\alpha\beta}(\tilde{k}_+) = \int dz_-\, e^{i\tilde{k}_+ z_-} \langle 0 | \bar{u}_\beta(z)[z,0] b_\alpha(0) | B\rangle \Big|_{z^+, z_\perp = 0}$$

- ϕ^B is convoluted with the perturbative hard-scattering amplitude T_i^{II} $\Rightarrow$ we need

$$\frac{\sqrt{2}}{\lambda_b} = \int_0^\infty \frac{d\tilde{k}_+}{\tilde{k}_+}\, \phi^B_+(\tilde{k}_+)\,.$$

  (In higher orders of perturbation theory factors containing $\log(\tilde{k}_+)$ appear.)
- At large $\tilde{k}_+$, $\phi^B(\tilde{k}_+) \sim 1/\tilde{k}_+$, but the convolution is finite.
- Positive moments of $\phi^B(\tilde{k}_+)$, which can be written in terms of local operators, diverge as powers of $1/a \Rightarrow$ need a technique to subtract these divergences with sufficient precision.
- We need new theoretical ideas for the lattice to contribute to $B \to M_1 M_2$ decays.