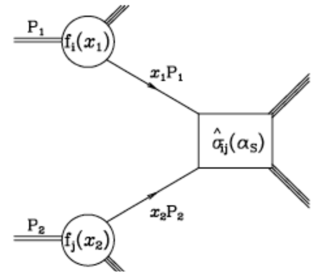
QCD for the LHC and the Tevatron

Keith Ellis
Fermilab

UK forum
September 2010

QCD improved parton model

Hard QCD cross section is represented as the convolution of a short distance cross-section and non-perturbative parton distribution functions. Physical cross section is formally independent of μ_F and μ_R



$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2\ f_i(x_1, \mu_F) f_j(x_2, \mu_F)\ \hat{\sigma}_{ij}(p_1, p_2, \alpha_S(\mu_R), Q^2, \mu_R, \mu_F).$$

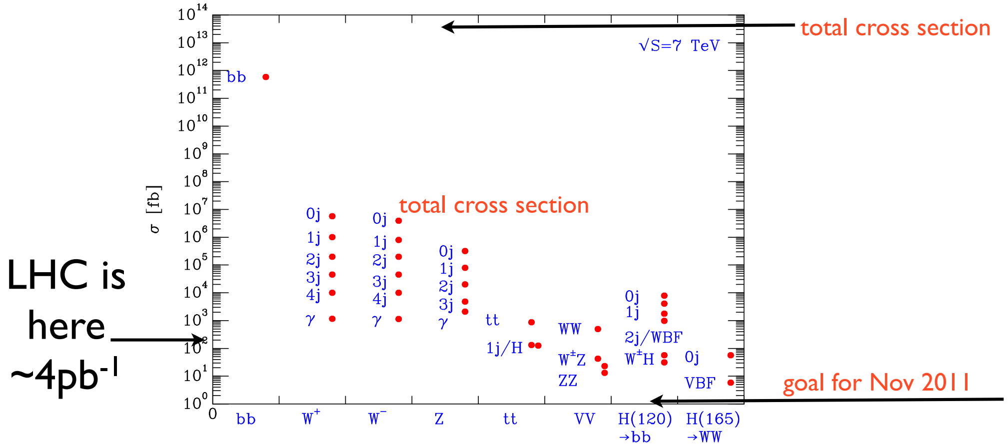
Physical cross section

Parton distributions

Strong Coupling

short-distance cross section σ in LO,NLO,...

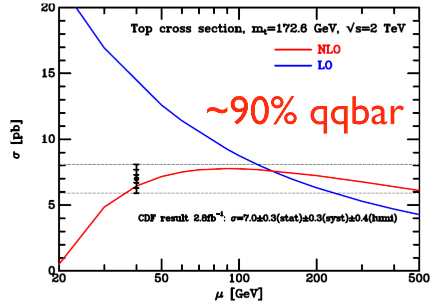
SM Ladder at 7 TeV

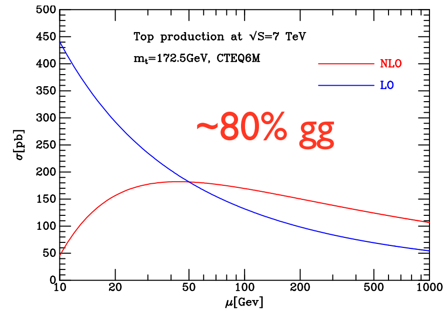


Includes decay of W/Z to one species of charged lepton and semi-leptonic decay of top (t → b l ν) (where applicable) and jets, Et> 25 GeV.

Why NLO?

- Less sensitivity to unphysical input scales, (eg. renormalization and factorization scales) at least formally.
- LO uncertainty becomes larger for multijet production, where Born approximation starts at high power of α_s^n
- NLO first approximation in QCD which gives an idea of suitable choice for μ.
- NLO has more physics, parton merging to give structure in jets, initial state radiation, more species of incoming partons enter at NLO.
- A necessary prerequisite for more sophisticated techniques which match NLO with parton showering.

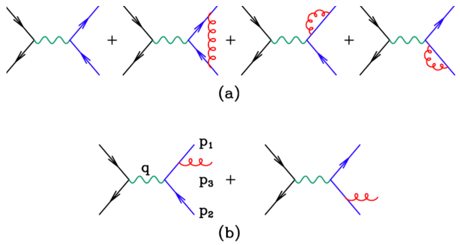




Ingredients of a NLO calculation

- Born process (LO).
- Interference of one-loop with LO
- Real radiation (also contributes to the two jet rate in the region of soft or collinear emission).
- Theoretical issues are efficient calculation of phase space and calculation of loop diagrams.

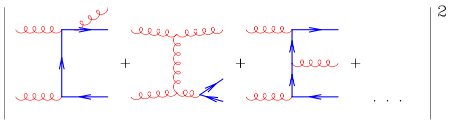
Example $e^+e^- \Rightarrow 2$ jets

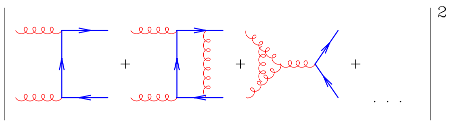


Heavy flavour production at NLO

At NLO $\sigma_H = O(\alpha_S^3)$

$$\frac{d}{d\ln\mu^2}\sigma_H = \alpha_s^4(\mu)$$


Real emission diagrams


Virtual emission diagrams

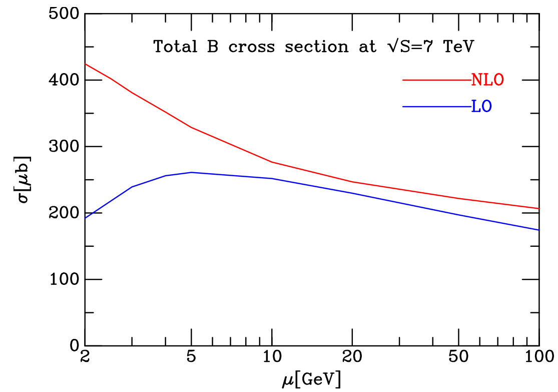
$$\hat{\sigma}_{ij}(s, m^2, \mu^2) = \frac{\alpha_S^2(\mu^2)}{m^2} f_{ij}\left(\rho, \frac{\mu^2}{m^2}\right)$$

$$\rho = \frac{4m^2}{s}, \quad \beta = \sqrt{1-\rho}$$

$$f_{ij}(\rho, \mu^2/m^2) = f_{ij}^{(0)}(\rho) + g^2(\mu^2)\left[f_{ij}^{(1)}(\rho) + \bar{f}_{ij}^{(1)}(\rho)\ln(\mu^2/m^2)\right]$$

$f_{ij}^{(1)}(\rho)$ now known analytically Czakon and Mitov, 0811.4119

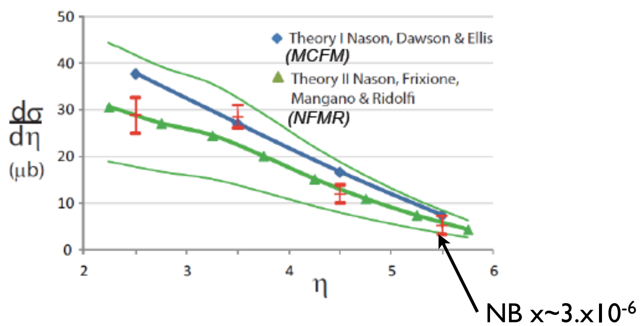
Scale dependence of B cross section

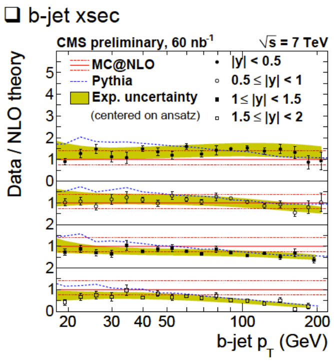


Perturbation series for b production at 7 TeV hardly much better behaved than at 1.96 TeV, small x regime.

B cross section

Sheldon Stone, LHCb ICHEP

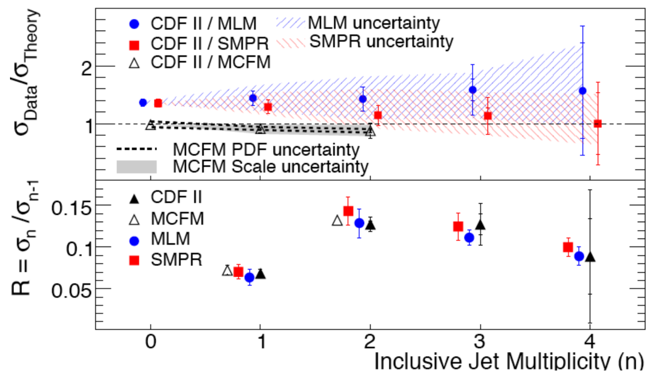




The theory of B production needs further work for this regime, but it is there is some measure of agreement

W + n-jet rates from CDF

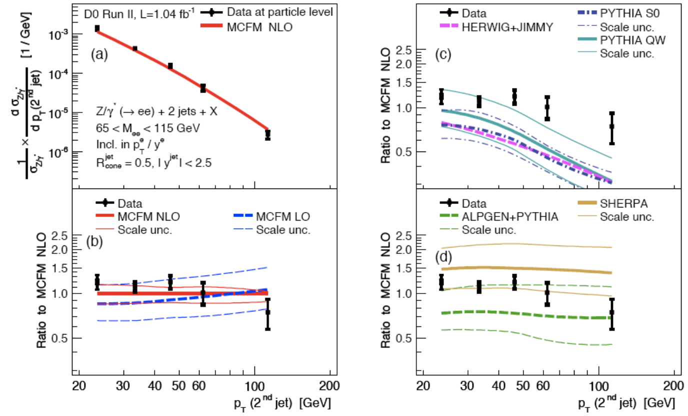
Aaltonen et al., arXiv: 0711.4044



Both uncertainty on rates and deviation of Data/Theory from 1 are smaller in MCFM (NLO) than in other calculations. The ratio R agrees well for all theory calculations, but only available from MCFM for n<3 in 2007.

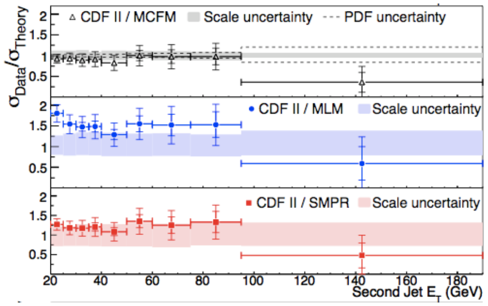
Z + jets results from D0

- MCFM, LO and NLO agrees with data.
- Shower-based generators show significant differences with data;
- matrix element + parton shower models agree in shape, but with larger normalization uncertainties.



W+n jet results at the Tevatron

W+0,1 and 2 jet rates from NLO (MCFM) have been compared successfully with data at the Tevatron.



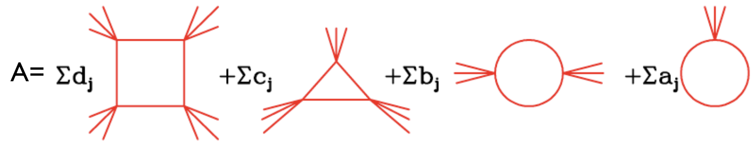
Success of W+1,2 jets predictions and the fact that LO uncertainties become larger as the number of jets increases -- strong motivation to calculate vector boson + 3,4.... jets

Industrial approach to NLO

- Preceding examples show us that NLO calculations can really improve the quality of the predictions.
- This will be important as we rediscover the SM at the LHC and also for the estimation of backgrounds to BSM physics.
- Backgrounds are best estimated from data, but in many circumstances it is helpful to have corroborating theoretical estimates.
- Hence an industrial style approach to NLO QCD is needed.

Ingredients in a one-loop calculation

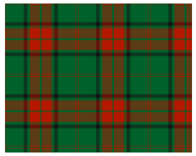
- For NLO calculations, any one-loop amplitude (no matter how many legs) can be written as a sum of sums of scalar boxes, triangles, bubbles and tadpoles



- The determination of the coefficients, d_j, c_j, b_j, a_j can be determined by semi-numerical methods, especially D-dimensional unitarity.

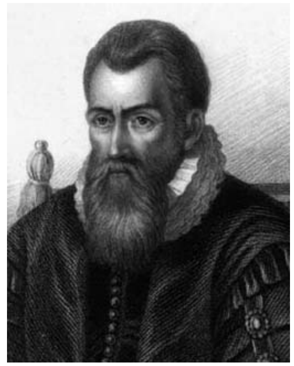
- The scalar integrals are all known analytically, see e.g. QCDLoop.fnal.gov, (Ellis,Zanderighi)

$$I_4^D(p_1^2,p_2^2,p_3^2,p_4^2;s_{12},s_{23};m_1^2,m_2^2,m_3^2,m_4^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma}$$
$$\times \int d^D l \, \frac{1}{(l^2-m_1^2+i\varepsilon)((l+q_1)^2-m_2^2+i\varepsilon)((l+q_2)^2-m_3^2+i\varepsilon)((l+q_3)^2-m_4^2+i\varepsilon)} ,$$



Scottish functions?

Logarithm



John Napier, 1550-1617

$$-\ln(1-x) \equiv \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \ldots$$

Dilogarithm

William Spence, 1777-1815

$$\mathrm{Li}_2(x) = -\int_0^x \frac{dz}{z} \ln(1-z) \equiv \frac{x}{1^2} + \frac{x^2}{2^2} + \frac{x^3}{3^2} + \ldots$$

Unitarity for one-loop diagrams

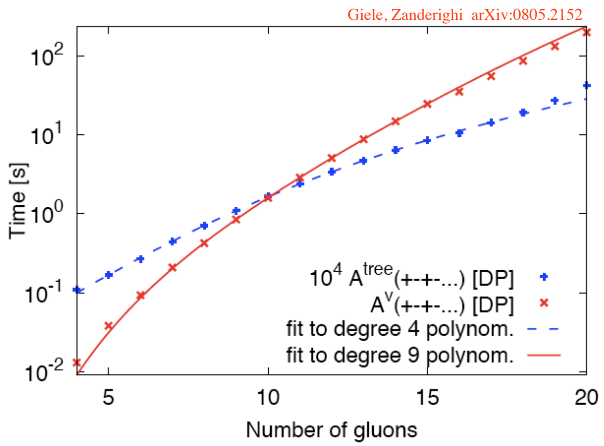
Important steps include:-

- First modern use of the idea Bern, Dixon,Kosower
- Cuts w.r.t. to loop momenta give (box) coefficients directly, complex momenta Cachazo, Britto, Feng
- OPP tensor reduction scheme, Ossola, Pittau, Papadopoulos
- Integrating the OPP procedure with unitarity Ellis, Giele, Kunszt
- D-dimensional unitarity Giele, Kunszt, Melnikov

One loop calculation of pure gluon amplitudes

Time to calculate one-loop amplitude scales as N^9 as expected. For small numbers of legs N=4,5,6 the times are of the order of 10's of milliseconds

4g:Ellis-Sexton(1985)
5g:Bern-Dixon-Kosower(1993)
6g:Ellis-Giele-Zanderighi(2006)

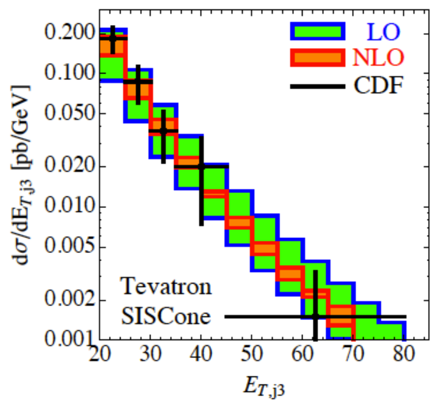


Overview: NLO results for W/Z+jets

Final state	Notes	Reference
W/Z	NLO only	MCFM
W/Z + 1 jet		MCFM
Wbb	massless b-quark	MCFM,hep-ph/9810489
Wbb	massive b-quark, no W decay	Febres et al, hep-ph/0606102, arXiv:0906.1923
Zbb	massless b-quark	MCFM,hep-ph/0006304
Zbb	massive b-quark, no Z decay	Febres et al, arXiv:0806.0808
W/Z + 2 jets	Virtual from BDK, hep-ph/9708239	MCFM,hep-ph/0202176, hep-ph/0308195
Wc	massive c-quark	MCFM,hep-ph/0506289
Zb	5-flavour scheme	MCFM,hep-ph/0312024
Zb + jet	5-flavour scheme	MCFM,hep-ph/0510362
W + 3 jets	adjusted leading colour(not yet in public code)	Ellis,Melnikov,Zanderighi, arXiv:0906.1445
W + 3 jets	full colour	Blackhat, arXiv:0907.1984
Z + 3 jets	full colour	Blackhat, arXiv:1004.1652
W^- + 4jets	leading colour (virtual)	Blackhat, arXiv:1009.2338

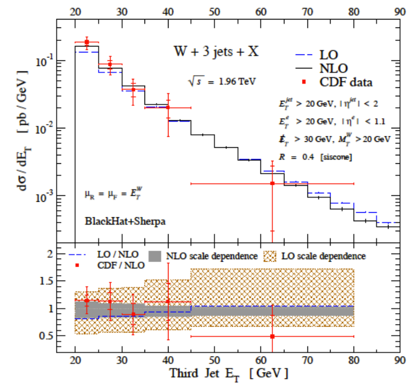
W+3jets at NLO

RKE, Melnikov, Zanderighi, 0906.1445



Adjusted leading colour approximation

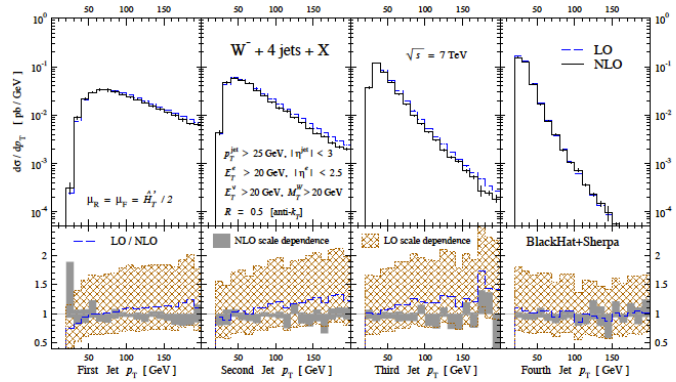
Berger et al, 0907.1984



Full treatment of colour

W^-+4jets at NLO at 7 TeV

Berger et al, 1009.2338

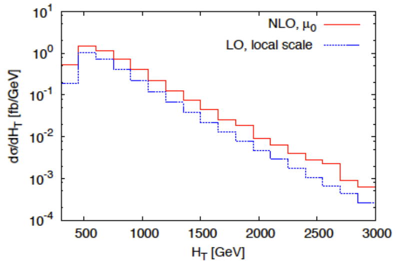


Currently only for W^- and in leading colour approximation for virtual amplitudes

Melnikov, Zanderighi 0910.3671

W+jets as a background

- Validate theoretical model with total cross section
- Extend theoretical model to New Physics kinematic region to estimate background.



$$\mu_0 = \sqrt{p_{T,W}^2 + m_W^2}$$

Danger! Explicit example of "ATLAS" style cuts shows ~100% NLO enhancements, whereas corrections to total cross section are quite moderate.

Also shows the importance of a choice of scale which reflects to kinematics of the event, because of enlarged phase space at LHC

cf Bauer and Lange 0905.4739

Kom, Stirling,arXiv:1004.3404

W^+/W^- Ratio

Define ratios without and with new physics contributions

$$R^{\pm}_{\rm SM}(n) = \frac{\sigma(W^+ + n \text{ jets})}{\sigma(W^- + n \text{ jets})}$$

$$R^{\pm}_{\rm exp.}(n) = \frac{\sigma(W^+ + n \text{ jets}) + \frac{1}{2}\sigma_{NP}}{\sigma(W^- + n \text{ jets}) + \frac{1}{2}\sigma_{NP}}$$

new physics contribution $$\frac{\sigma_{\rm NP}}{\sigma_{\rm SM}(W^+ + n \text{ jets}) + \sigma_{\rm SM}(W^- + n \text{ jets})} = \frac{2(R^{\pm}_{\rm SM} - R^{\pm}_{\rm exp.})}{(R^{\pm}_{\rm SM}+1)(R^{\pm}_{\rm exp.}-1)},$$

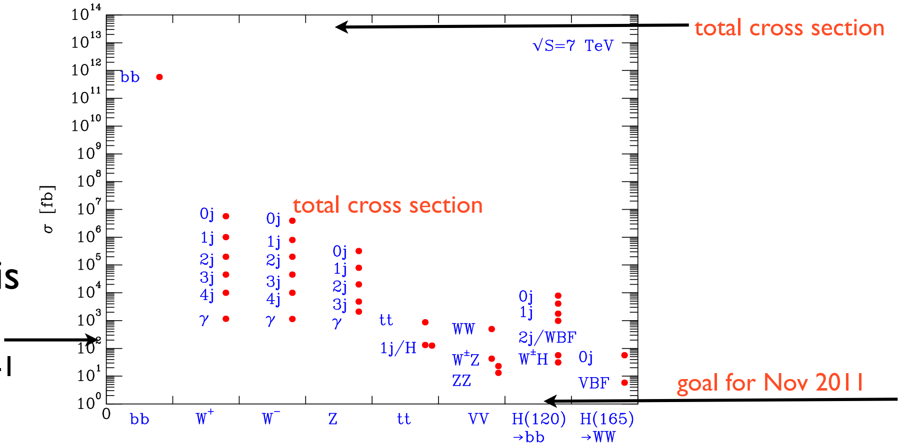
Berger et al, 1009.2338, $\sqrt{s}$=7TeV, anti-kt., R=0.5,pt_{jet}>25GeV, η_{jet}<3

# of jets	R_{SM} LO	R_{SM} NLO
0	1.656(0.001)	1.580(0.004)
1	1.507(0.002)	1.498(0.009)
2	1.596(0.003)	1.57(0.02)
3	1.694(0.005)	1.66(0.02)
4	1.817(0.001)	

(R_{SM} is rapidity dependent)

SM Ladder at 7 TeV

LHC is here ~4pb^{-1}

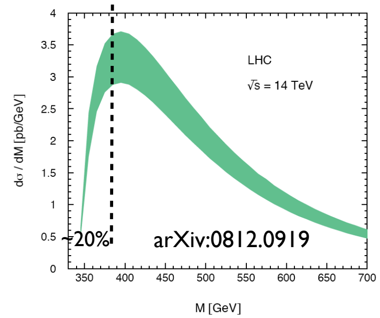


Includes decay of W/Z to one species of charged lepton and semi-leptonic decay of top (t → b l ν) (where applicable) and jets, Et> 25 GeV.

Recent NLO results in top production

Final state	Notes	Reference
tt	top decay correlations	Melnikov et al, 0907.3090
tt	top decay correlations	Campbell et al, MCFM
tt+1jet	top decay correlations	Dittmaier et al, arXiv:0807.1223 , 0810.0452, 0905.2299
tt+1jet	top decay correlations	Melnikov et al, 1004.3284
ttγ		Duan Peng-Fei et al, 0907.1324
ttH		Beenakker et al, hep-ph/0107081
ttH		Reina et al, hep-ph/0109066,hep-ph/0305087
ttZ		Lazopoulos et al. , arXiv:0804.2220
tt+2jets		Bevilacqua et al,1002.4009

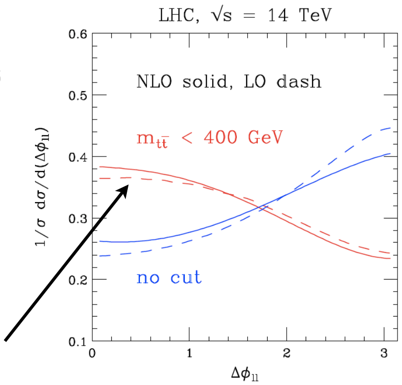
Investigation of effect of NLO on spin correlations



By imposing a cut on M<400GeV we can reveal the spin correlations in top production

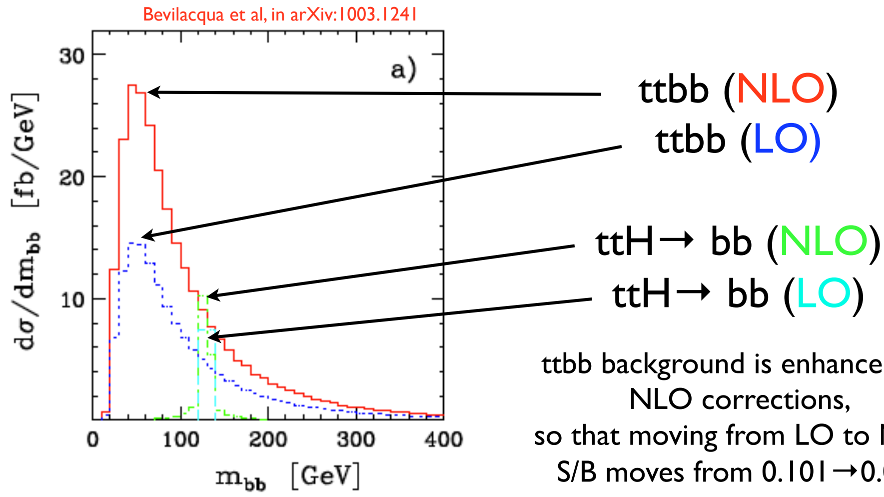
c.f. Mahlon and Parke arXiv:1001.3422

- At 14 TeV, 20% of cross section passes this cut (ie 2×10^5fb)
- Events produced per fb^{-1}: 2×10^5fb
- dilepton (e, μ) branching fractions $4/81$
- Efficiency 10%
- Overall 1000 $(e\mu)$ events per fb^{-1}



- NLO effects have modest effect on lepton-lepton angular correlation

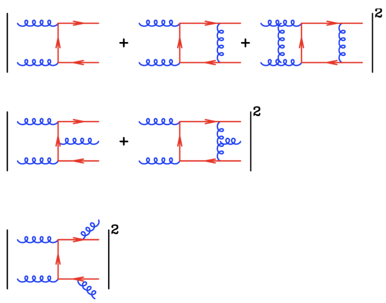
NLO estimates of S/B in ttH production



ttbb background is enhanced by NLO corrections, so that moving from LO to NLO S/B moves from 0.101→0.079

Progress on the NNLO Top quark cross section

* Motivation: Scale dependence is dominant error at LHC.
* Standard candle for gg flux.



Loop-by-loop, Anastasiou, 0809.1355
Korner et al, arXiv:, 0802.0106, 0809.3980
2-loop amplitudes, qqbar
Czakon, arXiv:0803.1400

tt+jet, Dittmaier arXiv:0810.0452

Analytic results at two loop for the gluon gluon channel not yet known, but structure of the IR poles are known, Ferroglia. arXiv:0908.3676, Becher&Neubert, Mitov, Ferroglia, Gardi & Magnea

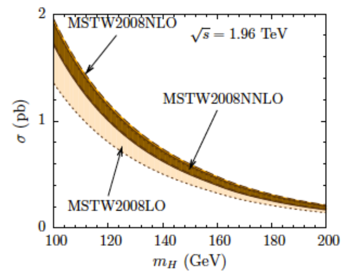
Higgs boson at 1.96TeV

Two contrasting views on the uncertainty
on the gluon-gluon fusion Higgs cross section

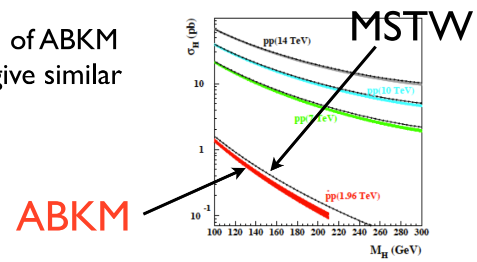
Ahrens et al, (ABNY) 0808.3008.0809.4283,1008.3162 $\sigma_{\mathrm{ABNY}}(M_H = 165) = 385^{+6}_{-2}\,{}^{+30}_{-32}$ fb

Baglio and Djouadi, (BD)1003.4266,1009.1363 $\sigma_{\mathrm{BD}}(M_H = 165) = 377^{+154}_{-135}$ fb

Source of uncertainty	ABNY	BD
Scale variation	3%(N³LL)	+15%/-20%(NNLO)
PDF	5-10%	25% (including α_s)
α_s	6% (not strong correlation with PDF)	strongly correlated (included with PDF)

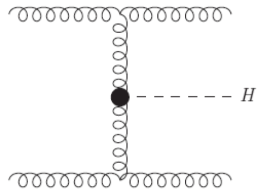


Major source of discrepancy is inclusion of ABKM parton distribution, MSTW and CTEQ give similar results.

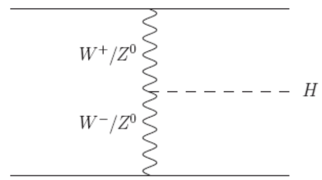


Long(?) term solution : find the Higgs!

Higgs production with 2 jets



Parton-parton fusion



Vector boson fusion

NLO corrections to parton-parton fusion, H+2 jets calculated in the $m_t \rightarrow \infty$ approximation were presented in arXiv:0608194v2 (Campbell, Ellis, Zanderighi).

$A)\quad 0 \rightarrow H q \bar{q} q' \bar{q}' g\ ,$

$B)\quad 0 \rightarrow H q \bar{q} g g g\ ,$

$C)\quad 0 \rightarrow H g g g g g\ .$

$$\mathcal{L}_{\text{eff}} = \frac{1}{4} A (1 + \Delta) H G^a_{\mu\nu} G^{a\,\mu\nu}\ ,$$

Higgs + 2 jets

- arXiv:0608194v2 was based on a semi-numerical method of calculation of virtual corrections. Code was never released.
- now updated in arXiv:1001.4495 (Campbell,Ellis,Williams), to use compact, analytic expressions for virtual amplitudes Badger,Berger,Campbell, Del Duca, Dixon,Glover,Ellis,Mastrolia,Risager,Sofianatos,Williams,Zanderighi
- Much faster code, obtainable in MCFMv5.7 or greater,
- ~5ms per virtual point, (2.66GHz iMac, gfortran, no opt.)
- Fast enough to include Higgs decays, such as $H \to WW^* \to ll\nu\nu$.

Higgs + 2 jet phenomenological impact

* Higgs + 2 jets important at LHC as "background" to VBF.
* However also important at the Tevatron to calculate the signal rate for m_H for various jet bins

ADGSW arXiv:0905.3529	LO	NLO	NNLO
Higgs+0 jet	✔	✔	✔
Higgs+1 jet	✔	✔	
Higgs+2 jets	✔		

CEW arXiv:1001.4495	LO	NLO	NNLO
Higgs+0 jet	✔	✔	
Higgs+1 jet	✔	✔	
Higgs+2 jets	✔	✔	

m_H [GeV]	160
Γ_H [GeV]	0.0826
σ_{LO} [fb]	$0.345^{+92\%}_{-44\%}$
σ_{NLO} [fb]	$0.476^{+35\%}_{-31\%}$
Finite m_t correction, R	1.113 ± 0.003

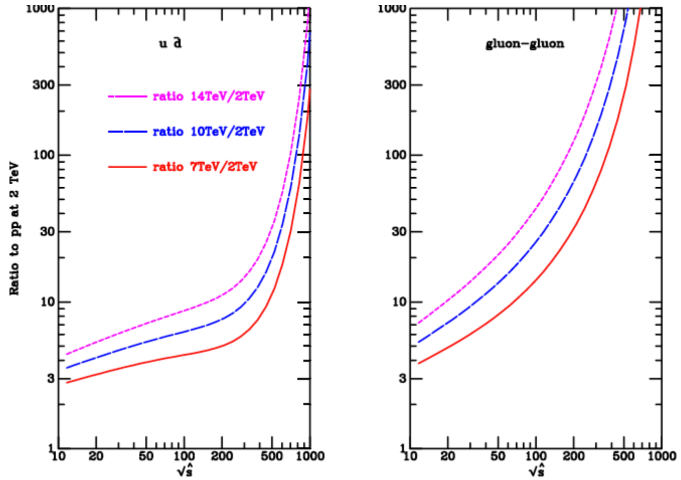
Calculate the total uncertainty by multiplying the Higgs+n jet fraction by the uncertainty in that jet bin.

0-jet % 1-jet % 2-jet %

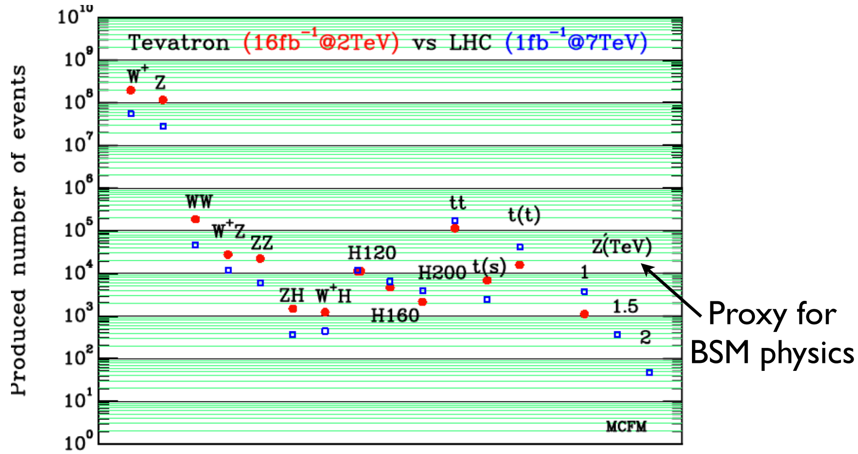
$$\frac{\Delta N_{\text{signal}}(\text{scale})}{N_{\text{signal}}} = 60\% \cdot \left({}^{+5\%}_{-9\%}\right) + 29\% \cdot \left({}^{+24\%}_{-23\%}\right) + 11\% \cdot \left({}^{+35\%}_{-31\%}\right) = \left({}^{+13.8\%}_{-15.5\%}\right)$$

(Corresponding uncertainty using LO Higgs + 2 jet is +20%,-17%).

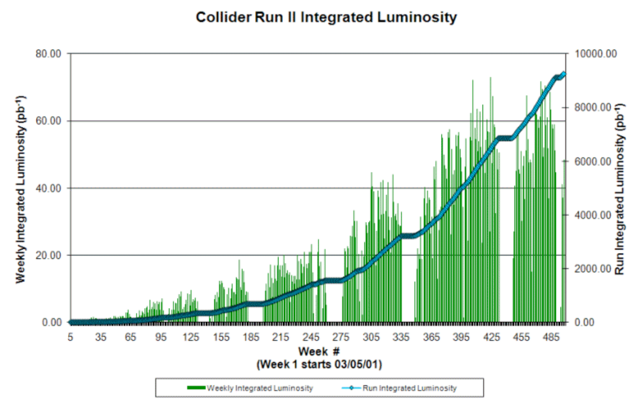
Parton luminosities
Tevatron vs LHC



16 fb^{-1}@1.96TeV vs 1 fb^{-1}@7TeV



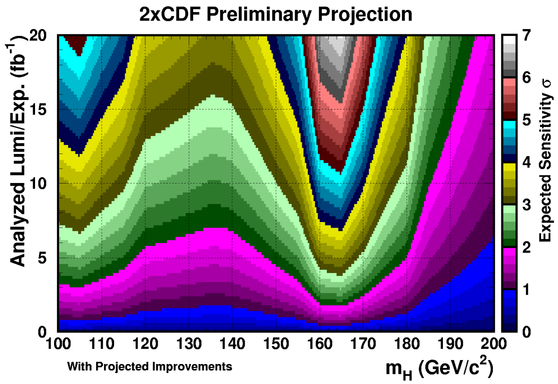
Tevatron Luminosity



Average weekly luminosity exceeds $50pb^{-1}$!

Data taking efficiency CDF (~85%) and D0 (~92%)

Higgs search projections at the Tevatron



Konigsberg, La Thuile

If the Tevatron were run for three more years it could accumulate 16fb^{-1} and provide 3σ evidence for the low mass Higgs boson

This would complement the information available from the LHC

Conclusions

- Several new QCD calculations performed this year give needed information for LHC operating at $\sqrt{s}$=7 TeV and accumulating 1fb^{-1}
- Operations at higher luminosity/energy will require NLO calculations with higher numbers of external legs. New semi-numerical techniques make these calculations possible.
- For qqbar initiated physics at a mass scale below 200 GeV, the Tevatron with 10fb^{-1} is superior to the LHC at $\sqrt{s}$=7 TeV with 1fb^{-1}.
- It would imprudent to shut the Tevatron before this reach has clearly been surpassed.