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New techniques and results in soft gluon physics

Work done with Gardi, Laenen, Magnea, Stavenga;
arXiv:0811.2067, arXiv:1008.0098, arXiv:1010.1860, arXiv:1102.0756

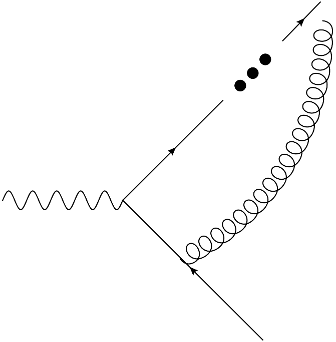
Higgs-Maxwell meeting, Edinburgh

Overview

- What are soft gluons?
- Why are they a problem?
- What is known already about them?
- New methods - path integrals, replica trick.
- Results and outlook.

Soft gluon physics

- In scattering amplitudes, get singularities due to gluon emission at large distances.



- Due to integrals over gluon positions:

$$\int d^n x$$

- Uncertainty principle $\Rightarrow$ equivalent to emission of zero energy gluons.

- Cancellation between real / virtual diagrams, but residual problems in cross-sections...

General structure of soft gluon emissions

- If ξ is the energy carried by soft gluons, typically get contributions:

$$\frac{d\sigma}{d\xi} = \sum_{n,m} \alpha^n \left[c^0_{nm} \frac{\log^m(\xi)}{\xi} + c^1_{nm} \log^m(\xi) + \ldots \right]$$

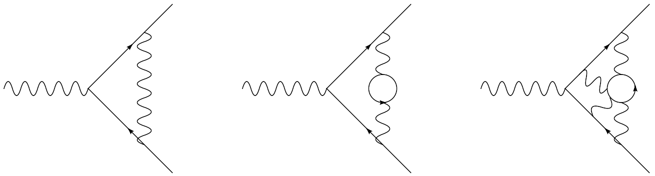
- First set of terms corresponds to *eikonal approximation*, in which momenta $k_i \rightarrow 0$ for all (soft) emissions. Well understood.
- Second set of terms is *next-to-eikonal* (NE) limit i.e. first order in k_i.
- Soft gauge bosons lead to a breakdown in perturbation theory!

Resummation of soft gluon logs

- In fact can sum up large logs to all orders.
- Many different methods exists (e.g. factorisation, SCET).
- I will follow the *web* approach (Gatheral, Frenkel, Taylor, Sterman).
- Tells us that soft photon amplitude has an exponential structure:

$$\mathcal{A} = \mathcal{A}_0 \exp\left[\sum G_c\right],$$

where $\mathcal{A}_0$ is the Born amplitude, and G_c are connected subdiagrams.

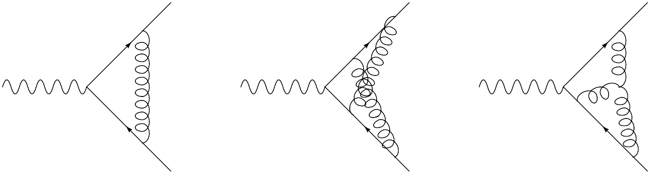


Soft resummation - nonabelian case

- Exponentiation generalisable to non-abelian theories, but structure is more complicated:

$$\mathcal{A} = \mathcal{A}_0 \exp\left[\sum \bar{C}_W W\right],$$

where W are *webs* (two-eikonal line irreducible subdiagrams).
- Webs have modified colour weights $\bar{C}_W$.



- More effort than abelian case, but still predicts eikonal logs to all orders.

Open problems

- Resummation of eikonal singularities has been very successful over many years.
- Would like more though. In particular:

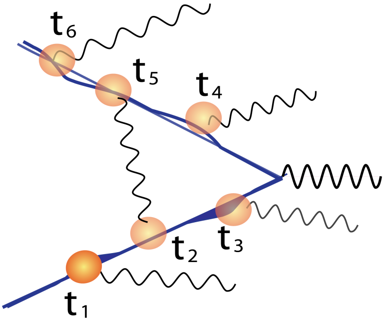
  What is the structure of webs for multiparton processes?

  (Mitov, Sterman, Sung; Gardi, Laenen, Stavenga, White.)
- Another problem is what happens beyond the eikonal approximation (Grunberg et. al., Laenen et. al., Vogt et. al.).

  Can we systematically classify next-to-eikonal logarithms?
- Recently, we have developed a new path integral technique which can address both these questions.
- Complementary to other approaches.

Path Integral Method for Soft Gluon Resummation

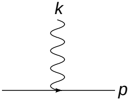
- Basic idea: rewrite a QCD scattering process in terms of *path integrals* over the emitting particle trajectories.



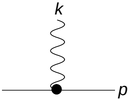
- Classical trajectories correspond to eikonal approximation (gluons have zero momentum).
- Expansion about classical trajectory gives NE corrections.

Path integral framework - results

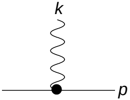
- Clear physical interpretation allows classification of NE effects.
- Find NE effective Feynman rules for use in resummation applications.



$$\frac{p^\mu}{p \cdot k},$$



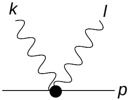
$$\frac{k^\mu}{2p \cdot k},$$



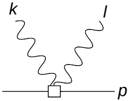
$$- k^2 \frac{p^\mu}{2(p \cdot k)^2}.$$

NE Feynman Rules

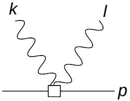
- Also get two gluon vertices:



$$+\frac{\eta^{\mu\nu}}{p\cdot(k+l)},$$



$$-\frac{l^{\mu}p^{\nu}p\cdot k+k^{\nu}p^{\mu}p\cdot l}{p\cdot(k+l)p\cdot kp\cdot l},$$



$$+\frac{p^{\mu}p^{\nu}k\cdot l}{p\cdot(k+l)p\cdot kp\cdot l}.$$

NE Feynman rules

- The Feynman rules are confirmed by an explicit diagrammatic proof (Laenen, Magnea, Stavenga, White).
- Have checked them by calculating NE effects in DY production, and confirming known results.
- Have proven that general structure of scattering amplitudes at NE order is:

$$\mathcal{M} = \mathcal{M}_0 \exp\left[\mathcal{M}^{E} + \mathcal{M}^{NE}\right] \times \left[1 + \mathcal{M}_{rem.}\right] + \mathcal{O}(NNE).$$

- Paves the way for full resummation of NE logarithms...

Multiparton webs

- Earlier we saw that two parton processes could be described in terms of *webs*.
- These were diagrams which entered the exponent of the soft gluon amplitude, thus generating large logs at all orders in α_S.
- Our new techniques allow us to extend this idea to multiparton processes.
- Derivation uses the *replica trick*, an elegant technique borrowed from statistical physics.
- Involves making N identical copies of the soft gluon sector in order to prove exponentiation!
- Here focus on results...

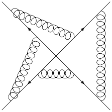
Multiparton webs - results

- We find that the exponent of the scattering amplitude contains terms of the form

$$\sum_{D,D'} \mathcal{F}_D R_{DD'} C_{D'},$$

  where the sum is over closed sets of diagrams related by gluon permutations, and $R_{DD'}$ is a *web-mixing matrix.*.
- The study of multiparton webs is thus equivalent to the study of these matrices.
- They encode all order colour and kinematic properties, entangled in a non-trivial way.

A four loop example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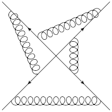
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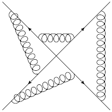
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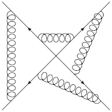
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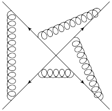
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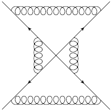
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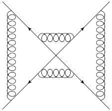
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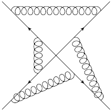
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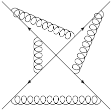
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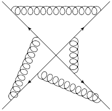
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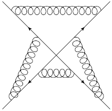
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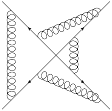
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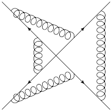
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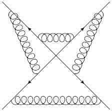
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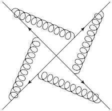
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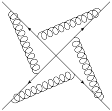
[[1,2],[2,3],[3,4],[4,1]]

Four loop mixing matrix ($\times 24$)

$$
\begin{pmatrix}
6 & -6 & 2 & 2 & -2 & 4 & -4 & 2 & -2 & -2 & -4 & 4 & -4 & 4 & 0 & 0 \\
-6 & 6 & -2 & -2 & 2 & -4 & 4 & -2 & 2 & 2 & 4 & -4 & 4 & -4 & 0 & 0 \\
2 & -2 & 6 & -2 & 2 & 4 & -4 & -2 & 2 & -6 & 4 & 4 & -4 & -4 & 0 & 0 \\
2 & -2 & -2 & 6 & 2 & 4 & -4 & -2 & -6 & 2 & -4 & -4 & 4 & 4 & 0 & 0 \\
-2 & 2 & 2 & 2 & 6 & 4 & -4 & -6 & -2 & -2 & 4 & -4 & 4 & -4 & 0 & 0 \\
2 & -2 & 2 & 2 & 2 & 4 & -4 & -2 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2 & 2 & -2 & -2 & -2 & -4 & 4 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & -2 & -2 & -2 & -6 & -4 & 4 & 6 & 2 & 2 & -4 & 4 & -4 & 4 & 0 & 0 \\
-2 & 2 & 2 & -6 & -2 & -4 & 4 & 2 & 6 & -2 & 4 & 4 & -4 & -4 & 0 & 0 \\
-2 & 2 & -6 & 2 & -2 & -4 & 4 & 2 & -2 & 6 & -4 & -4 & 4 & 4 & 0 & 0 \\
-2 & 2 & 2 & -2 & 2 & 0 & 0 & -2 & 2 & -2 & 4 & 0 & 0 & -4 & 0 & 0 \\
2 & -2 & 2 & -2 & -2 & 0 & 0 & 2 & 2 & -2 & 0 & 4 & -4 & 0 & 0 & 0 \\
-2 & 2 & -2 & 2 & 2 & 0 & 0 & -2 & -2 & 2 & 0 & -4 & 4 & 0 & 0 & 0 \\
2 & -2 & -2 & 2 & -2 & 0 & 0 & 2 & -2 & 2 & -4 & 0 & 0 & 4 & 0 & 0 \\
-18 & -6 & -6 & -6 & -18 & 12 & 12 & -6 & -18 & -18 & 12 & 12 & 12 & 12 & 24 & 0 \\
-6 & -18 & -18 & -18 & -6 & 12 & 12 & -18 & -6 & -6 & 12 & 12 & 12 & 12 & 0 & 24
\end{pmatrix}
$$

Web mixing matrices

- We can show the following interesting properties:
  1. Rows of web mixing matrices sum to zero i.e.

     $$\sum_{D'} R_{DD'} = 0.$$

     This is related to the fact that symmetric colour combinations do not exponentiate.
  2. The matrices are idempotent i.e. $R^2 = R$. This implies they have eigenvalues 0 and 1.
- These properties are intimately related to the cancellation of subdivergences in the exponent of the scattering amplitude (see also Sterman, Mitov, Sung).
- Proofs use the replica trick, and also results from enumerative combinatorics (Gardi, White).

Conclusion

- Path integral methods prove highly powerful in analysing soft gluons.
- Combined with the replica trick, a whole set of new results can be obtained.
- In particular:
  1. Classification of multiline webs (new mathematical structures!).
  2. Analysis of next-to-eikonal logarithms.
- Have carried out checks of the effective Feynman rules in Drell-Yan production.
- Paves the way for full resummation of higher-loop multiline and / or NE effects.
- Also more formal applications...

Further Applications

- There are various conjectures in QCD (and other field theories e.g. quantum gravity, $N = 4$ SYM) regarding exponents of scattering amplitudes.
- E.g. Sum over dipoles conjecture (Gardi, Magnea; Becher, Neubert), BDS conjecture (Bern, Dixon, Smirnov)...
- Also, soft gluons are related to Wilson lines, currently the subject of intense study (e.g. AdS/CFT dualities Alday, Maldacena).
- Our techniques could be used to efficiently calculate higher order contributions, and/or gain new insight beyond the planar limit.
- Also: factorization of soft graviton amplitudes (White, in prep.).
- Relations between enumerative combinatorics and QCD?