QCD and jet physics

Lecture 1

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Introduction

- Defining jets
- Computing jet observables in and beyond pQCD
- Understanding jets
- Using jets

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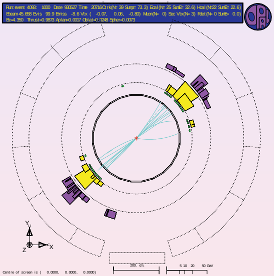
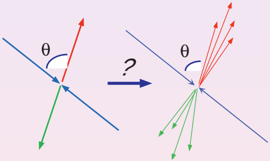
Why jets?

QCD is a weird theory ! Lagrangian involves partons which never make it to detectors.
Measured final state involves collimated sprays of energetic particles or jets!

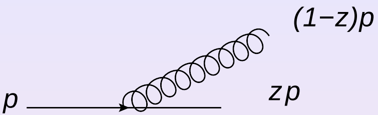




Looks like partons (like the YETI) leave some footprints.

Sterman TASI lectures

Theoretical issues



Need to calculate extra particle production. Probability for a parton to emit a soft and collinear gluon reads :

$$P = C_i \int \frac{\alpha_s((1-z)\theta)}{\pi} \frac{dz}{1-z} \frac{d\theta^2}{\theta^2}$$

This is singular so the answer diverges. A good sign ? If probability were just $\propto \alpha_s$ we would only see very few extra particles! But for calcs. need to introduce energy and

angular resolution.

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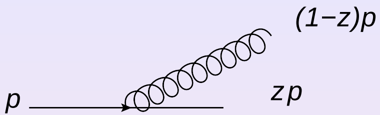

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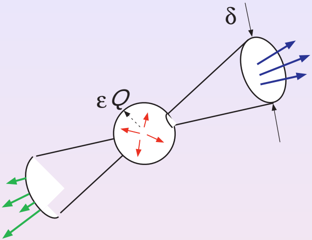
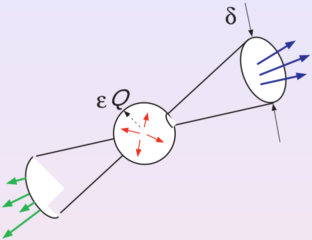
Early jet definitions



Define a dijet event by including anything below energy ϵ or within angle δ in dijet. Sterman and Weinberg 1978

Probability of particle production can be $\mathcal{O}(1)$. Probability of producing extra jet costs us α_s. Jet cross-sections computable in pQCD. But we need IRC safe jet definition at all orders.



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SW algorithm too basic. Where to place cones? What to do with overlapping cones? How to generalise to hadron collisions? More sophisticated cones were devised.

Snowmass accord developed laying out properties of an acceptable algorithm:

- Simple to implement in experimental analyses as well as theory calculations.
- Defined at any order in pQCD and yields finite results for rates at any order.
- Yields a cross-section relatively insensitive to hadronisation

ESW "More honoured in the breach than the observance!"

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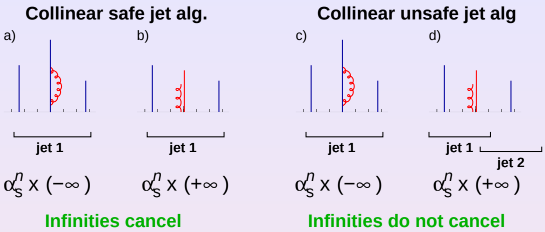
Long history of problems



Seeded cone - emission of a collinear parton changes the jet structure and leads to a divergence.

- Cone algorithms have been problematic including all the cones used at the Tevatron to date.
- There are also algorithms based on sequential recombination. These are IRC safe but have in the past not been commonly used at hadron colliders.
- Finally we have a set of algorithms of various kinds all of which satisfy Snowmass.

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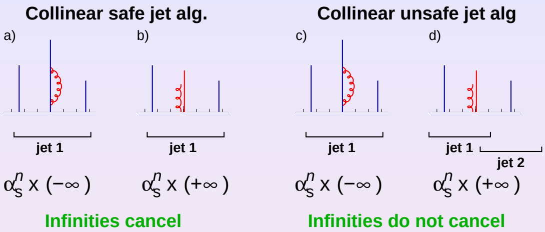

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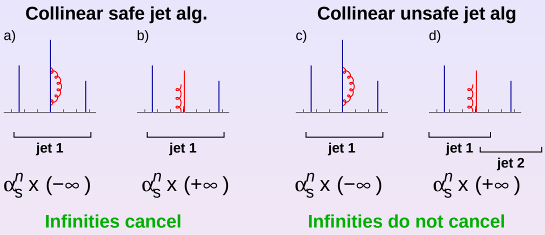

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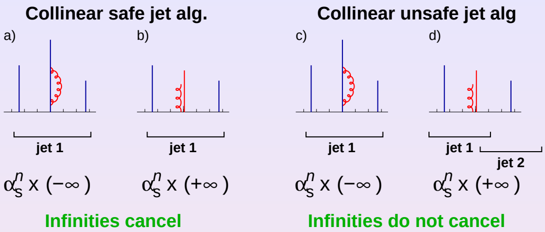

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Jet algorithms circa 2010

We shall only discuss IRC safe ones! Two main categories

1. Cone type : SISCONE (Seedless Infrared Safe Cone)
   Salam and Soyez 2007
2. Sequential Recombination based on a distance measure.
   - k_t or Durham algorithm
     Catani et. al 1993
   - Cambridge-Aachen
     Dokshitzer et. al 1997, Wobisch and Wengler 1998
   - Anti-k_t Cacciari, Salam, Soyez 2008.

Sequential Recombination

An example is the k_t algorithm for e^+e^-. Uses the distance measure

Definition

$$y_{ij} = 2\min\frac{(E_i^2, E_j^2)}{Q^2}\left(1 - \cos\theta_{ij}\right)$$

Reduces to relative k_t of softer particle if $\theta_{ij} \ll 1$. Related to QCD branching probability. Can be thought of as inverting a k_t ordered parton shower.

- Find the smallest y_{ij}. If this is smaller than $y_{\rm cut}$ merge i and j.
- Repeat until all objects are separated by more than $y_{\rm cut}$ and call the resulting entities jets.

For hadron colliders Incoming partons and divergences , no equivalent for Q, desire longitdinally invariant extensions.

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Hadron collider definitions

Most common is inclusive k_t algorithm with distance measures

Definition

$$d_{ij} = \min(p_{t,i}^2, p_{t,j}^2)\frac{\Delta_{ij}}{R^2}, \; \Delta_{ij} = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

$$d_{iB} = p_{t,i}^2$$

Ellis and Soper 1993 All quantites defined wrt beam. Note the introduction of a radius like parameter R.

- Find the smallest among d_{ij} and d_{iB}. If it is a d_{iB} call the object a jet and remove from list. If d_{ij} then merge i and j.
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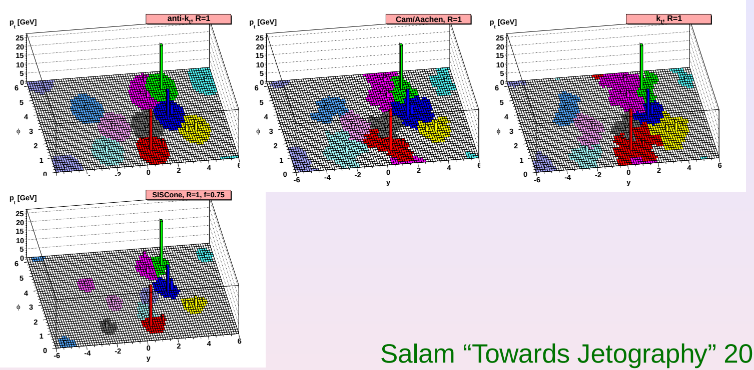
Other SR algorithms

Similar to k_t but use different distance measure. SR algorithms can be defined together. Write distance measure as

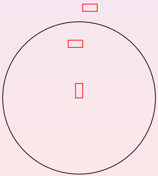
$$d_{ij} = \min(p_{t,i}^{2p}, p_{t,j}^{2p}) \frac{\Delta_{ij}}{R^2}$$

$p = 0$ is C/A algorithm while $p = -1$ is the anti-k_t algorithm. Note that C/A algorithm will invert an angular ordered shower while the anti-k_t is not straightforwardly related to branching dynamics.
A feature of anti-k_t is that soft reclusterings are not favoured over hard soft clusterings that happen earlier in the sequence.

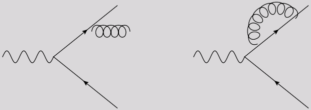
Appearance of hadron collider jets



Salam “Towards Jetography” 2009



Example



Dijet rate at leading order is σ_0. NLO involves vetoing real gluon emission above scale $Q^2 y_{\rm cut}$. Leading behaviour given by

$$\sigma_0 \, 2C_F \int \frac{\alpha_s}{\pi} \frac{d\omega}{\omega} \frac{d\theta^2}{\theta^2} \left[\Theta \left(Q^2 y_{\rm cut} - \omega^2 \theta^2 \right) - 1 \right]$$

.

Example

Thus one gets

$$-2C_F \int \frac{\alpha_s}{\pi} \frac{d\omega}{\omega} \frac{d\theta^2}{\theta^2} \Theta(\omega^2\theta^2 - Q^2 y_{\rm cut}) \sim -C_F \frac{\alpha_s}{2\pi} \ln^2 y_{\rm cut}$$

. Full NLO result reads

$$f_2 = 1 - C_F \frac{\alpha_s}{2\pi} \ln^2 y_{\rm cut} + C_1 \alpha_s + \alpha_s D(y_{\rm cut})$$

- If $y_{\rm cut} \ll 1$ then one needs to resum the logs.
- If $y_{\rm cut} \sim 1$ fixed order suffices.

If one has multiscale jet observable then may need to go beyond fixed order. Use resummation or parton shower+fixed order. Note at LHC possible huge hierarchy of scales $\sqrt{s}, p_T, m_t, M_H, \Lambda$ other than kinematical cuts such as veto scales!

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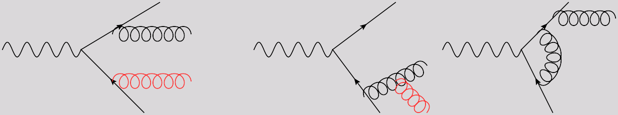
3 jet rate at NLO

Example

Diagrams involved both abelian and non-abelian

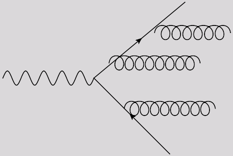


Leading double log result now reads

$$C_F \frac{\alpha_s}{2\pi} L^2 - \frac{\alpha_s^2}{\pi^2} L^4 \left(\frac{C_F^2}{4} + \frac{C_F C_A}{48} \right) + \cdots \; L = \ln \frac{1}{y_{\mathrm{cut}}}.$$

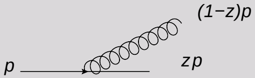
Again needs resummation at small y or parton shower.
Note p_T ordered parton shower gives wrong jet rates!
Webber 2010

Example



Jet energy or p_t used in kinematic reconstruction. Need to know how this relates to hard scale such as mass of heavy decaying particle or original parton. Study impact of PT radiation, ISR, UE and hadronisation.

Example



$$\delta p_t = (1-z)p_t - p_t = -zp_t, \; 1-z > z$$
$$\delta p_t = zp_t - p_t = -(1-z)p_t, \; z > 1-z$$

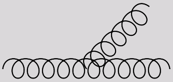
Note $\theta^2 > R^2$.

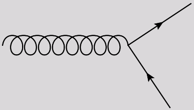
$$\langle \delta p_t \rangle_q = -\frac{C_F \alpha_s}{2\pi} p_t \int_{R^2}^{1} \frac{d\theta^2}{\theta^2} \frac{1+z^2}{1-z} \min\left[(1-z), z\right]$$

This gives

$$\langle \delta p_t \rangle_q = -C_F \frac{\alpha_s}{\pi} p_t \ln \frac{1}{R} \left(2 \ln 2 - \frac{3}{8}\right)$$

Example





For a gluon jet one replaces

$$C_F \frac{1+z^2}{1-z} \to C_A \left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z) \right) + T_R n_f \left(z^2 + (1-z)^2 \right)$$

to obtain

$$\langle \delta p_t \rangle_g = -\frac{\alpha_s}{\pi} p_t \ln \frac{1}{R} \left[C_A \left(2 \ln 2 - \frac{43}{96} \right) + T_R n_f \frac{7}{48} \right]$$

δp_t for jets

Our calculations imply

$$\frac{\langle \delta p_t \rangle_q}{p_t} = -0.43\alpha_s \ln \frac{1}{R}$$
$$\frac{\langle \delta p_t \rangle_g}{p_t} = -1.02\alpha_s \ln \frac{1}{R}$$

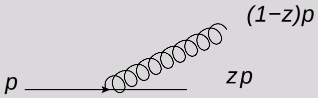
For $R = 0.4$ quark jet will have 5 percent less and gluon jet 11 percent less p_t than parent parton.

- Above results are subject to significant finite R and higher order changes.
- SISCONE has different recombination. Draw cone centred on $p_1 + p_2$ and require one parton to fall outside it. Gives similar result with $R_{kt} \sim 1.3 R_{\mathrm{SIS}}$

Jet Masses

Of interest e.g for boosted heavy particle searches.

Example



Average jet mass for quark and gluon jets.

Invariant mass $M_j^2 = (p+k)^2 = 2p.k$ is just $z(1-z)p_t^2\theta^2$ which gives

$$C_F \frac{\alpha_s}{2\pi} \int \frac{1+z^2}{1-z} \frac{d\theta^2}{\theta^2} z(1-z)p_t^2\theta^2$$

Here we need $\theta^2 < R^2$.

Example

Gives

$$\langle M_j^2 \rangle_q = \frac{3}{8} C_F \frac{\alpha_s}{\pi} R^2 P_t^2$$

.

For gluons coefficient is $\frac{7}{20} C_A + \frac{1}{20} n_f T_R$. Scale of jet is RP_t.

Rule of thumb $\sqrt{\langle M_j^2 \rangle} \approx 0.2 RP_t$ S.D.Ellis et.al 2008

Jet Mass Distribution

Example

Compute LO jet mass distribution as follows

$$\frac{1}{\sigma}\frac{d\sigma}{dM^2} = C_F \frac{\alpha_s}{2\pi}\int \frac{1+z^2}{1-z}\frac{d\theta^2}{\theta^2}\delta(p_t^2 z(1-z)\theta^2 - M^2)$$

which gives

$$\frac{1}{M^2}\frac{C_F\alpha_s}{\pi}\ln\left(\frac{R^2P_t^2}{M^2}\right)$$

.

For $RP_t \gg M$ resummation may be needed. More complex than at LEP but recently done in a small R approx.

Banfi, MD, Khelifa-Kerfa, Marzani

Conclusions

- Learnt about the need for jets, IRC safety in jet definitions.
- Discussed modern day (2010) jet definitions for e^+e^- and hadron collisions.
- Carried out calculations of jet rates
- Developed simple estimates for impact of radiation on jet p_t and invariant masses.

In the next lecture we shall bravely move on to address NP effects (using PT tools !). Then discuss how to put all this to use!