Standard Model Candles, Theory Uncertainties and PDFs 2

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How straightforward is it in practice?

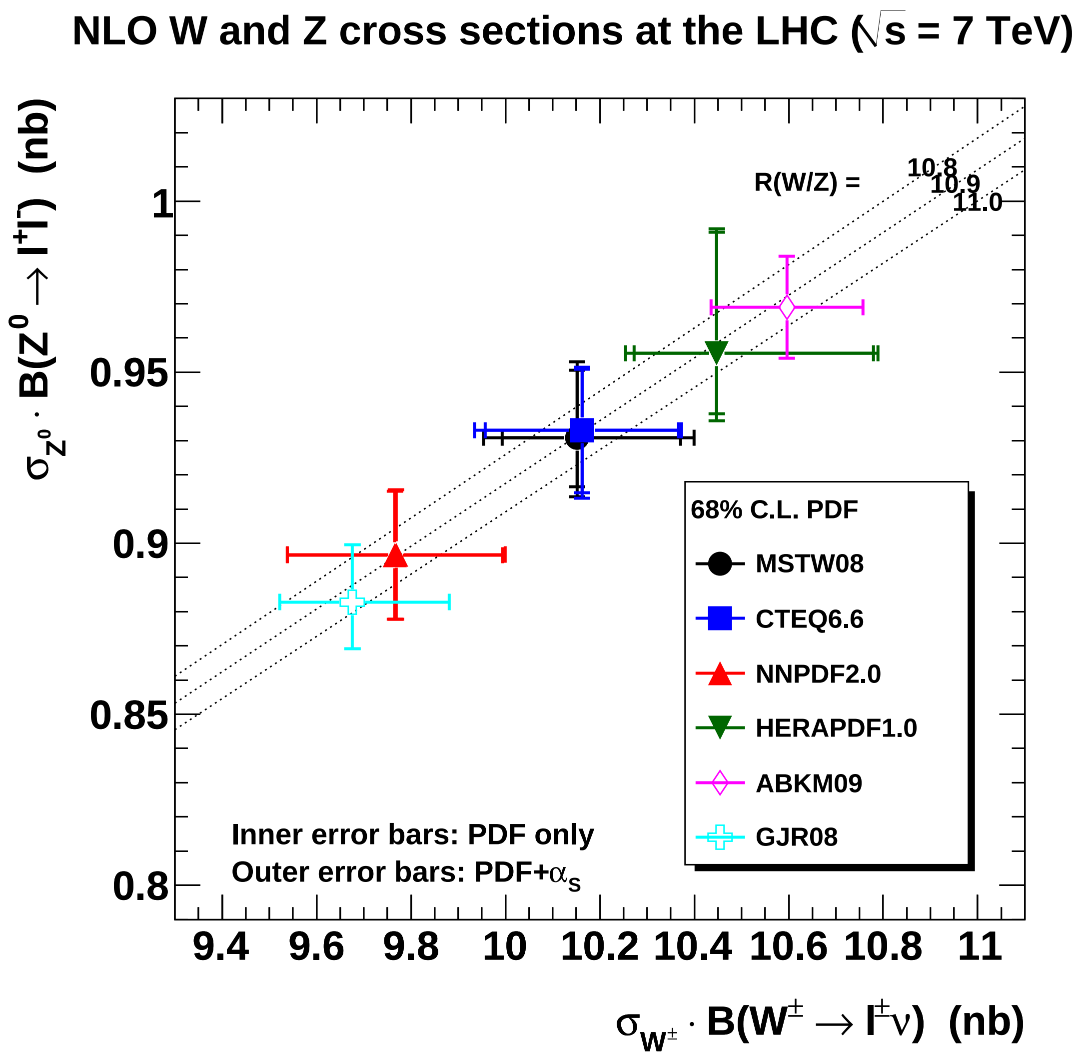
Predictions (Watt) for W and Z cross-sections for LHC with common NLO QCD and vector boson width effects, and common branching ratios, and at 7TeV.

Comparing all groups get significant discrepancies between them even for this benchmark process.

Can understand some of the systematic differences.

Some difference in W/Z ratio.

W, Z total cross-sections best-case scenario.



Sources of Variations/Uncertainty

It is vital to consider theoretical/assumption-dependent uncertainties:

- Methods of determining “best fit” and uncertainties.
- Underlying assumptions in procedure, e.g. parameterisations and data used.
- Treatment of heavy flavours.
- PDF and α_S correlations.

Responsible for differences between groups for extraction of fixed-order PDFs.

Different PDF sets

- MSTW08 – fit all previous types of data. Most up-to-date Tevatron jet data. Not most recent HERA combination of data. PDFs at LO, NLO and NNLO.

- CTEQ6.6 – very similar. Not quite as up-to-date on Tevatron data. PDFs at NLO. New – CT10 include HERA combination and more Tevatron data. Little changes.

- NNPDF2.0 – include all except HERA jet data (not strong constraint) and heavy flavour structure functions. Include HERA combined data. PDFs at NLO.

- HERAPDF2.0 – based entirely on HERA inclusive structure functions, neutral and charged current. Use combined data. PDFs at LO, NLO.

- ABKM09 – fit to DIS and fixed target Drell-Yan data. PDFs at NLO and NNLO. (Now prelim results using Tevatron jets).

- GJR08 – fit to DIS, fixed target Drell-Yan and Tevatron jet data. PDFs at NLO and NNLO.

Use of HERA combined data instead of original data slight increase in quarks at low x (depending on procedure).

Parton Fits and Uncertainties. Two main approaches.

Parton parameterization and Hessian (Error Matrix) approach first used by H1 and ZEUS, and extended by CTEQ. Now used in some form by all other than NNPDF.

$$\chi^2 - \chi^2_{min} \equiv \Delta\chi^2 = \sum_{i,j} H_{ij}(a_i - a_i^{(0)})(a_j - a_j^{(0)})$$

The Hessian matrix H is related to the covariance matrix of the parameters by

$$C_{ij}(a) = \Delta\chi^2 (H^{-1})_{ij}.$$

We can then use the standard formula for linear error propagation.

$$(\Delta F)^2 = \Delta\chi^2 \sum_{i,j} \frac{\partial F}{\partial a_i} (H)^{-1}_{ij} \frac{\partial F}{\partial a_j},$$

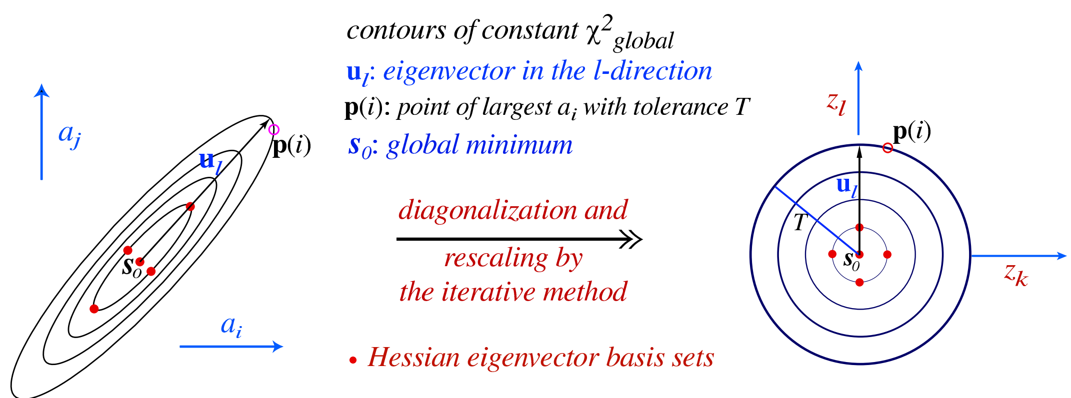
This is now the most common approach (sometimes *Offset method*).

Problematic due to extreme variations in $\Delta\chi^2$ in different directions in parameter space.

Solved by finding and rescaling eigenvectors of H leading to diagonal form

$$\Delta\chi^2 = \sum_i z_i^2$$

2-dim (i,j) rendition of d-dim (~20) PDF parameter space



(a) Original parameter basis

(b) Orthonormal eigenvector basis

Implemented by CTEQ, then others. Uncertainty on physical quantity then given by

$$(\Delta F)^2 = \sum_i \left(F(S_i^{(+)}) - F(S_i^{(-)})\right)^2,$$

where $S_i^{(+)}$ and $S_i^{(-)}$ are PDF sets displaced along eigenvector direction.

Question of choosing "correct" $\Delta\chi^2$ given complication of errors in full fit and sometimes conflicting data sets.

CTEQ use $\Delta\chi^2 \sim 40$ and MRST/MSTW use more complicated approach – results in $\Delta\chi^2 \sim 15$, for one σ. Other fits less global, keep to $\Delta\chi^2 = 1$.

- MSTW08 – 20 eigenvectors. Due to incompatibility of different sets and (perhaps to some extent) parameterisation inflexibility (little direct evidence for this) have inflated $\Delta\chi^2$ of $5-20$ for eigenvectors.

- CTEQ6.6 – 22 eigenvectors. Inflated $\Delta\chi^2$ of 40 for 1 sigma for eigenvectors (no normalization uncertainties in CTEQ6.6). CT10 have 26 eigenvectors, include normalization uncertainties and have a slightly modified tolerance criterion.

- HERAPDF2.0 – 9 eigenvectors. Use "$\Delta\chi^2 = 1$". Additional model and parameterisation uncertainties.

- ABKM09 – 21 parton parameters. Use $\Delta\chi^2 = 1$. Also α_S, m_c, m_b.

- GJR08 – 20 parton parameters and α_S. Use $\Delta\chi^2 \approx 20$. Impose strong theory constraint on input form of PDFs.

Perhaps surprisingly all get rather similar uncertainties for PDFs cross-sections.

Neural Network group (Ball *et al.*) limit parameterization dependence.

First part of approach, no longer perturb about best fit. Construct a set of Monte Carlo replicas $F_{i,p}^{art,k}$ of the original data set $F_{i,p}^{exp,(k)}$.

- REPLICAS FLUCTUATE ABOUT CENTRAL DATA:

$$F_{i,p}^{(art)(k)} = S_{p,N}^{(k)} F_{i,p}^{\text{exp}} \left(1 + r_p^{(k)} \sigma_p^{\text{stat}} + \sum_{j=1}^{N_{\text{sys}}} r_{p,j}^{(k)} \sigma_{p,j}^{\text{sys}}\right)$$

Where $r_p^{(k)}$ are random numbers following Gaussian distribution, and $S_{p,N}^{(k)}$ is the analogous normalization shift of the of the replica. Errors represented in variations of replicas. Fit to the data replicas obtaining PDF replicas $q_i^{(net)(k)}$ (follows Giele *et al.*)

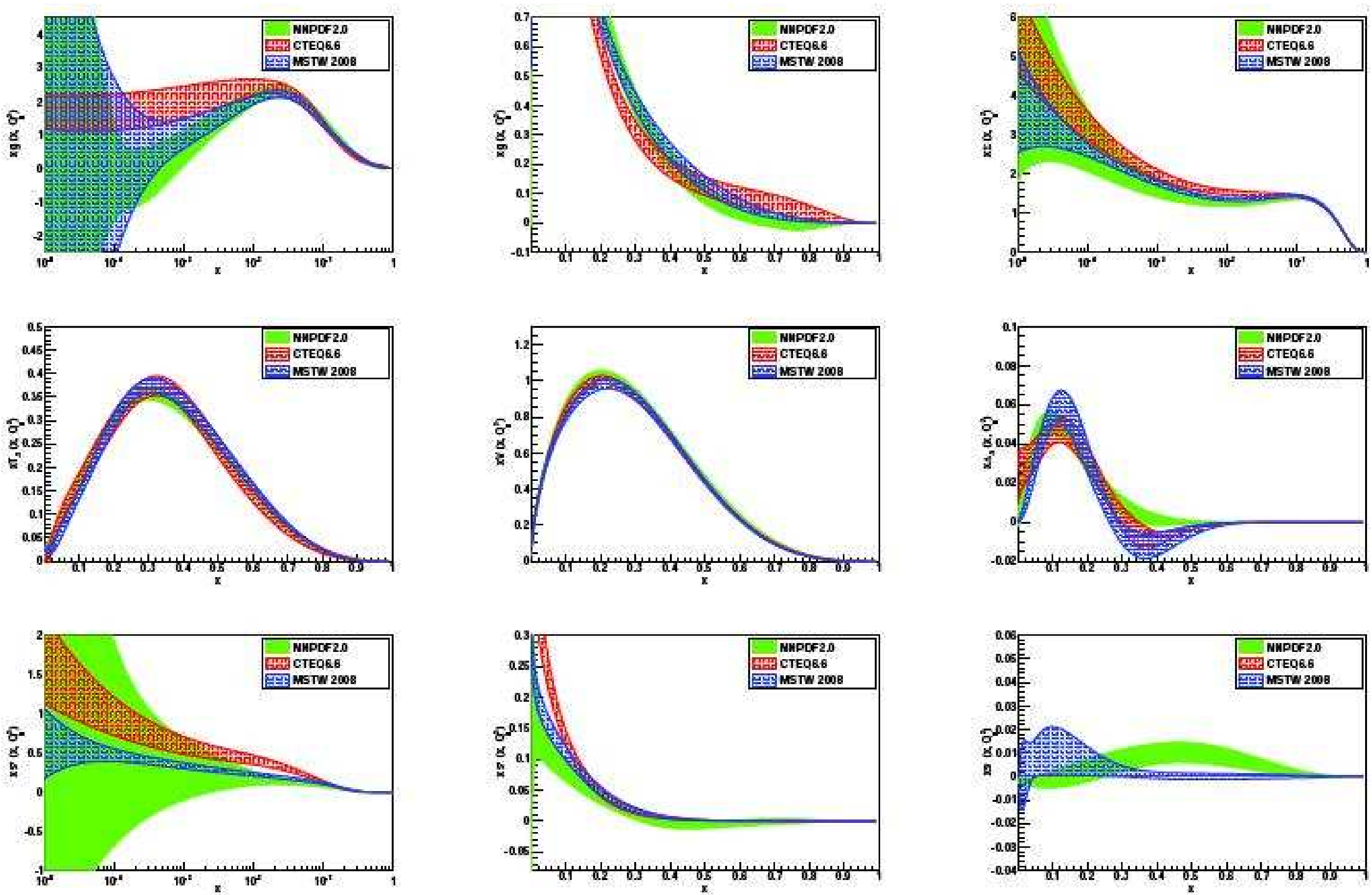
Mean μ_O and deviation σ_O of observable O then given by

$$\mu_O = \frac{1}{N_{rep}} \sum_1^{N_{rep}} O[q_i^{(net)(k)}], \quad \sigma_O^2 = \frac{1}{N_{rep}} \sum_1^{N_{rep}} (O[q_i^{(net)(k)}] - \mu_O)^2.$$

Eliminates parameterisation dependence by using a neural net which undergoes a series of (mutations via genetic algorithm) to find the best fit. In effect is a much larger sets of parameters – ~ 37 per distribution. Pre-processing exponents as $x \to 0$ and $x \to 1$used to aid convergence.

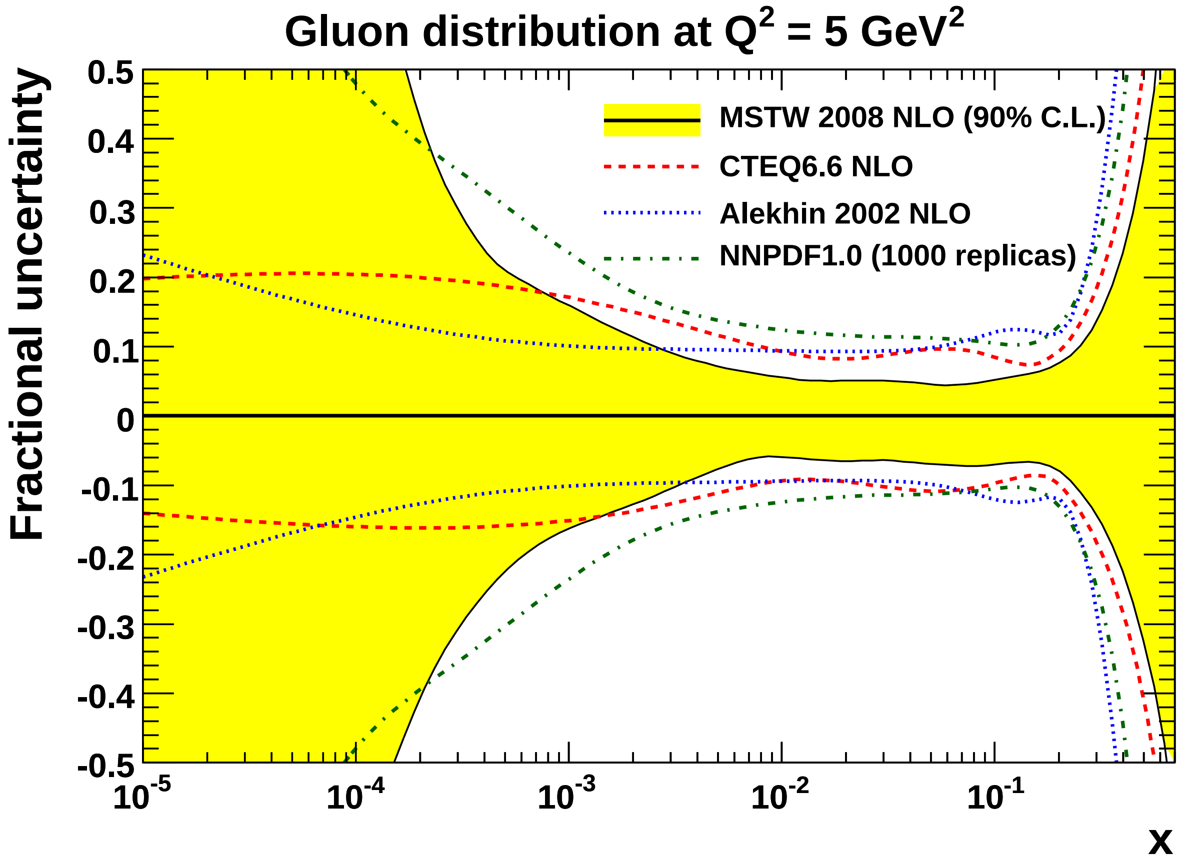
Each fit performed with half data fit (training) and half checked (validation) and stopped when comparison to validation set deteriorates.

Also reductions due to inclusion of new data.



NNPDF uncertainties pretty similar to other groups, with some particular exceptions.

Gluon Parameterisation - small x – different parameterisations lead to very different uncertainty for small x gluon.



Most assume single power x^{λ} at input $\rightarrow$ limited uncertainty. If input at low Q^2 λ positive and small-x input gluon *fine-tuned* to ~ 0. Artificially small uncertainty.

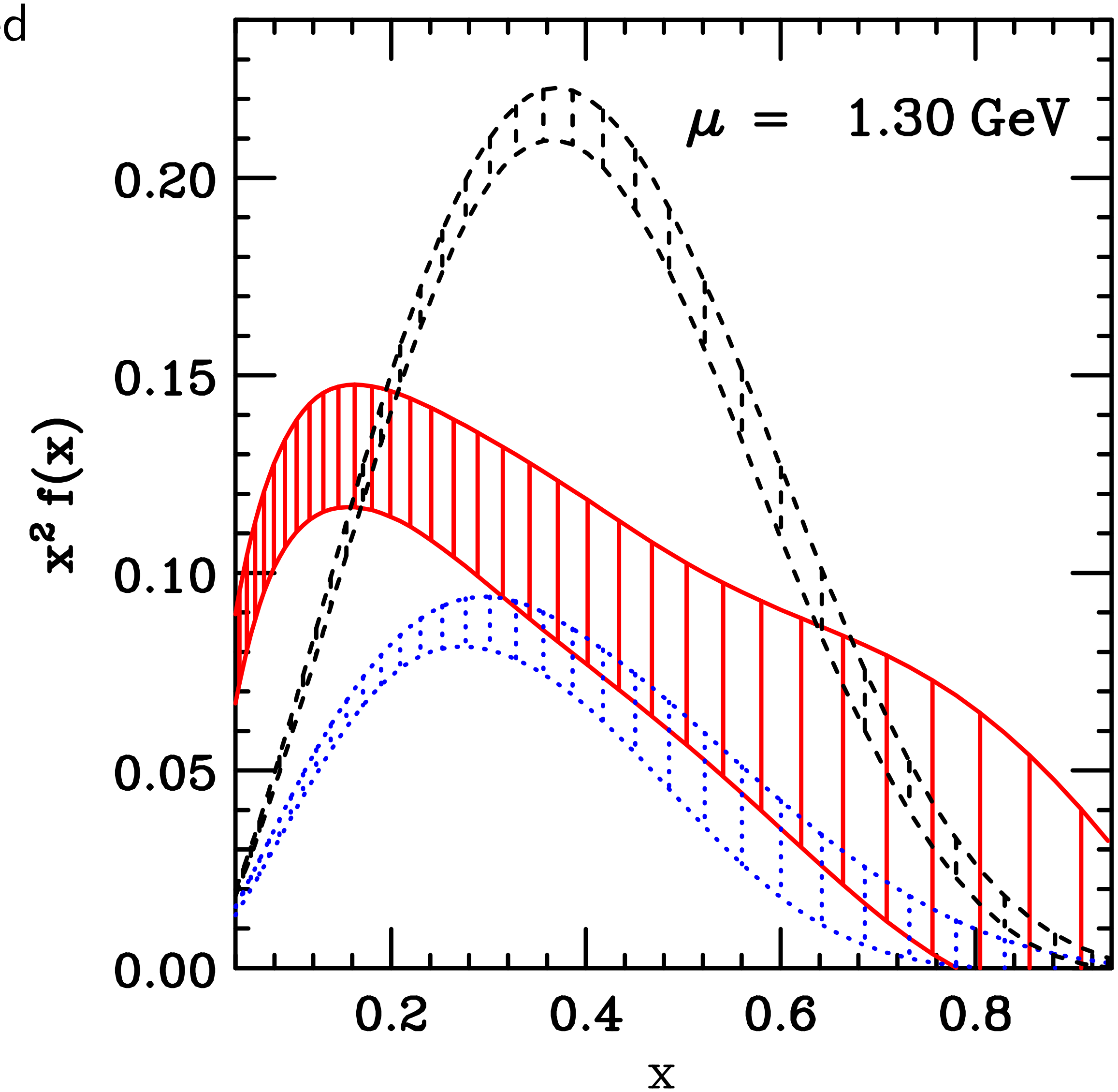
If $g(x) \propto x^{\lambda \pm \Delta\lambda}$ then $\Delta g(x) = \Delta\lambda \ln(1/x) * g(x)$.

MRST/MSTW and NNPDF more flexible (can be negative) $\rightarrow$ rapid expansion of uncertainty where data runs out.

Generally high-x PDFs parameterised so will behave like $(1-x)^\eta$ as $x \to 1$. More flexibility in CTEQ.

Very hard high-x gluon distribution (more-so even than NNPDF uncertainties).

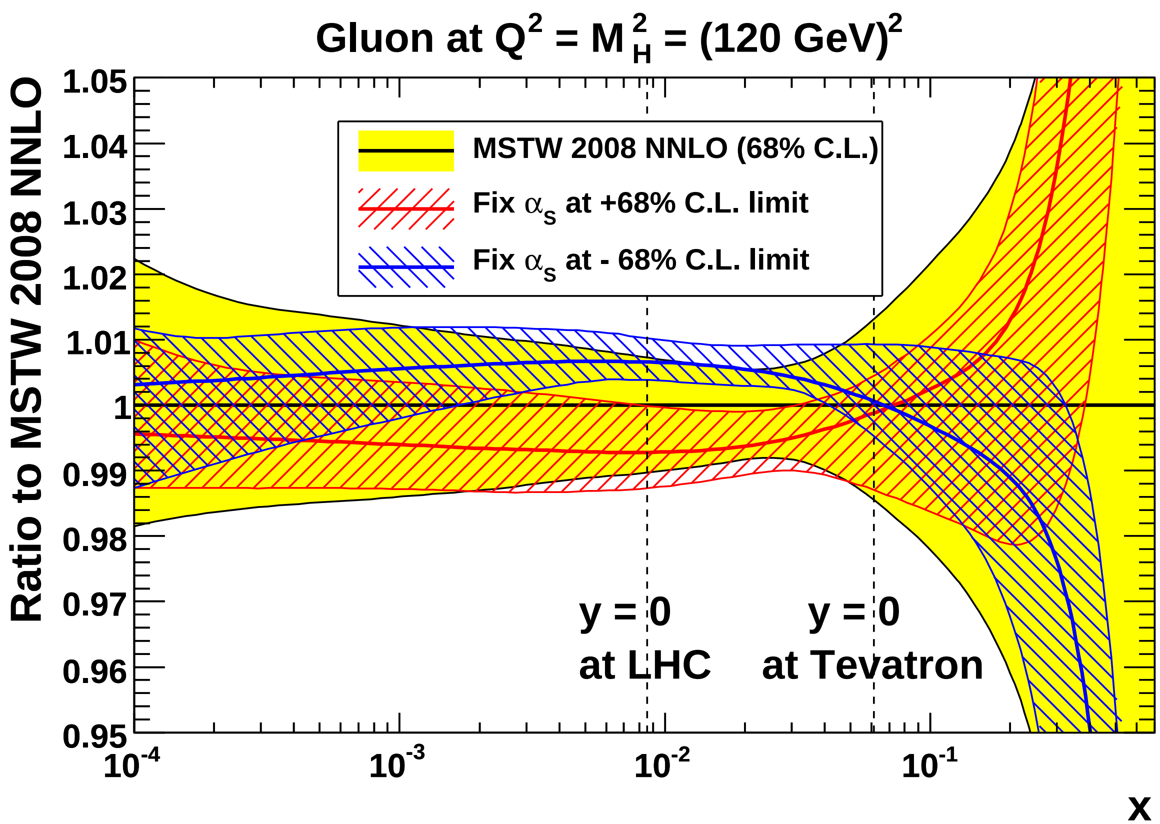
However, is gluon, which is radiated from quarks, harder than the up valence distribution for $x \to 1$?



PDF correlation with α_S.

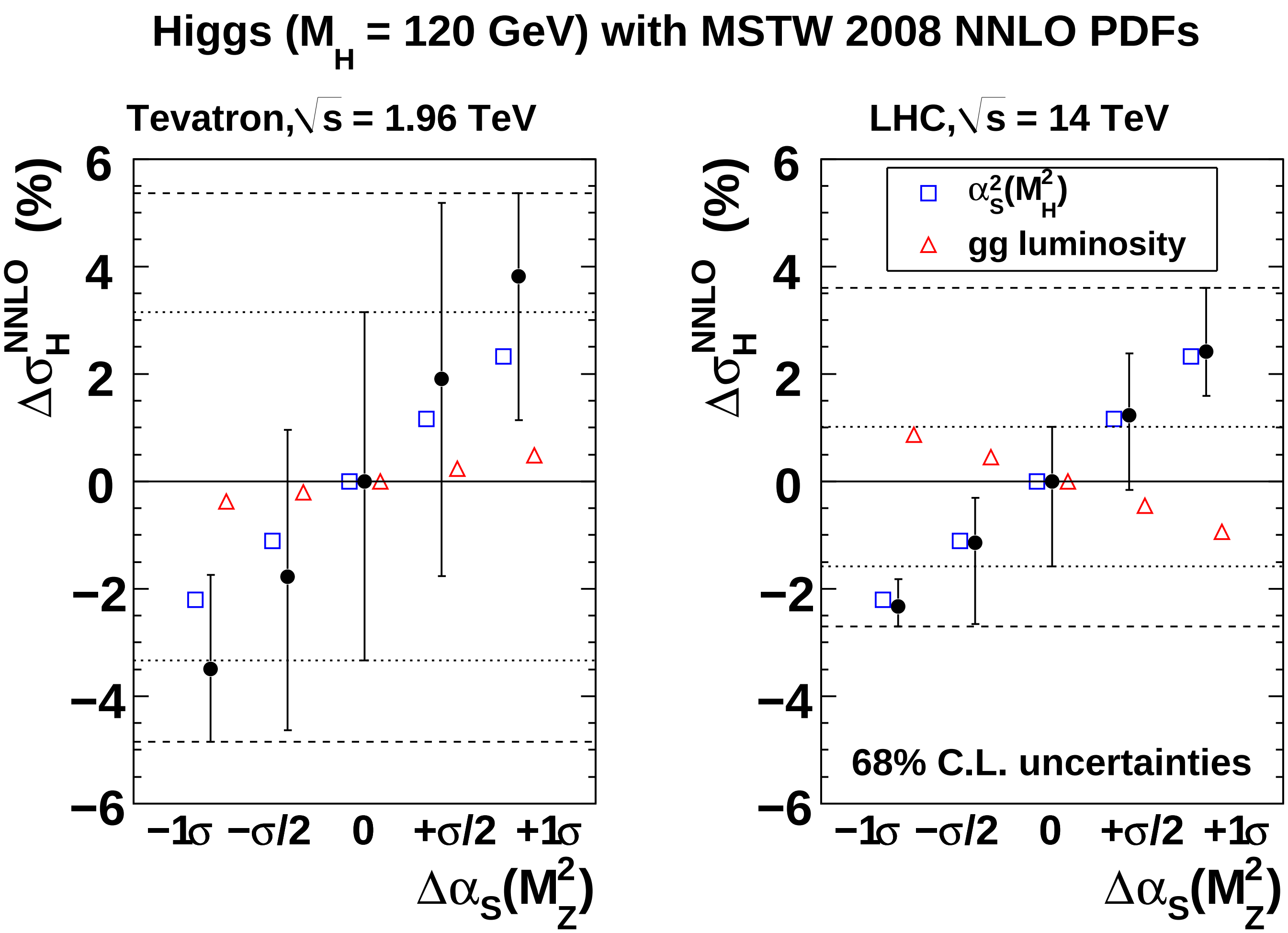
Can also look at PDF changes and uncertainties at different $\alpha_S(M_Z^2)$. Latter usually only for one fixed $\alpha_S(M_Z^2)$. Can be determined from fit, e.g. $\alpha_S(M_Z^2) = 0.1202^{+0.0012}_{-0.0015}$ at NLO and $\alpha_S(M_Z^2) = 0.1171^{+0.0014}_{-0.0014}$ at NNLO from MSTW.

PDF uncertainties reduced since quality of fit already worse than best fit.



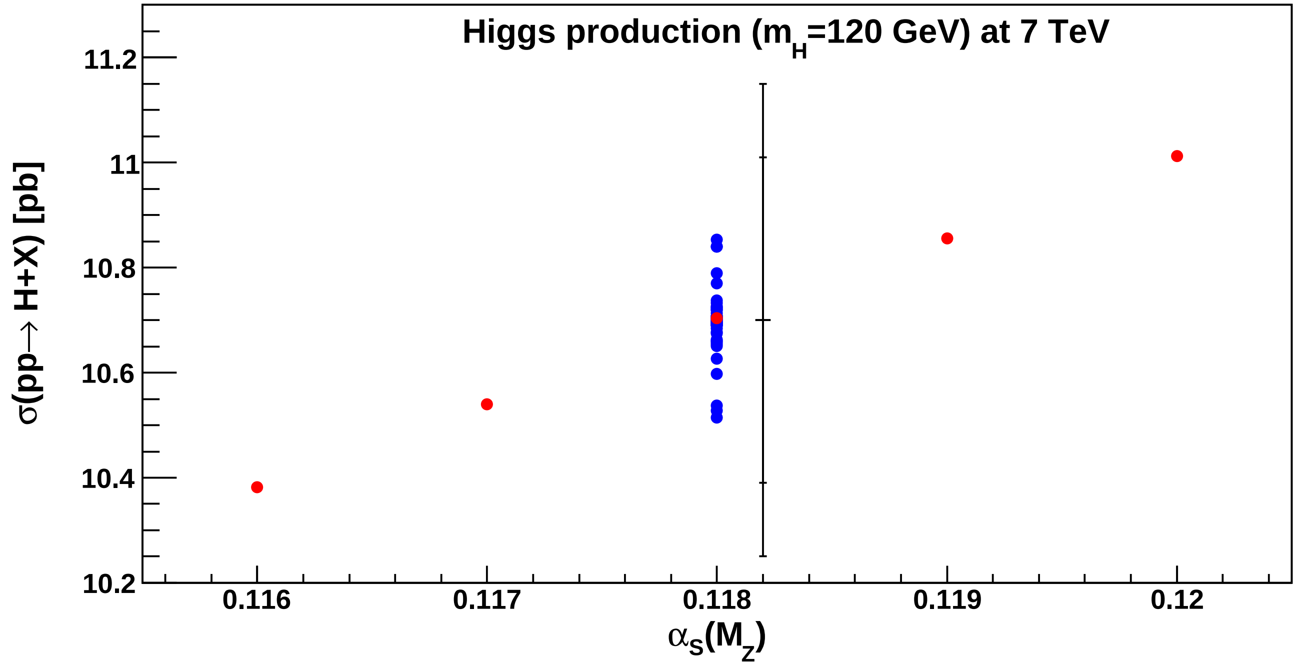
Expected gluon–$\alpha_S(M_Z^2)$ small–x anti-correlation $\rightarrow$ high-x correlation from sum rule.

NNLO predictions for Higgs $(120\mathrm{GeV})$ production for different allowed $\alpha_S(M_Z^2)$ values and their uncertainties.



Increases by a factor of $2-3$ (up more than down) at LHC. Direct $\alpha_S(M_Z^2)$ dependence mitigated somewhat by anti-correlated small-x gluon (asymmetry feature of *minor* problems in fit to HERA data). At Tevatron intrinsic gluon uncertainty dominates.

CTEQ have shown that up to Gaussian approx. for uncertainties (and some other caveats) α_S uncertainty accounted for by adding deviation from PDFs with upper and lower α_S limits (red) in quadrature with all other PDF eigenvectors (blue), seen below.



NNPDF advocate distributing PDF replicas according to probability of $\alpha_S(m_Z^2)$ taking that value based on some assumed central value and uncertainty, i.e.

$$N_{\rm rep}^{\alpha_S} \propto \exp\left(-\frac{(\alpha_S - \alpha_S^{(0)})^2}{2(\delta\alpha_S^{(68)})^2}\right),$$

All lead to roughly same results Vicini *et al.*

Heavy Quarks – Essential to treat these correctly. Two distinct regimes:

Near threshold $Q^2 \sim m_H^2$ massive quarks not partons. Created in final state. Described using **Fixed Flavour Number Scheme** (FFNS).

$$F(x, Q^2) = C_k^{FF}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2)$$

Does not sum $\ln^n(Q^2/m_H^2)$ terms, and not calculated for many processes beyond LO. Still occasionally used. Sometimes final state details in this scheme only.

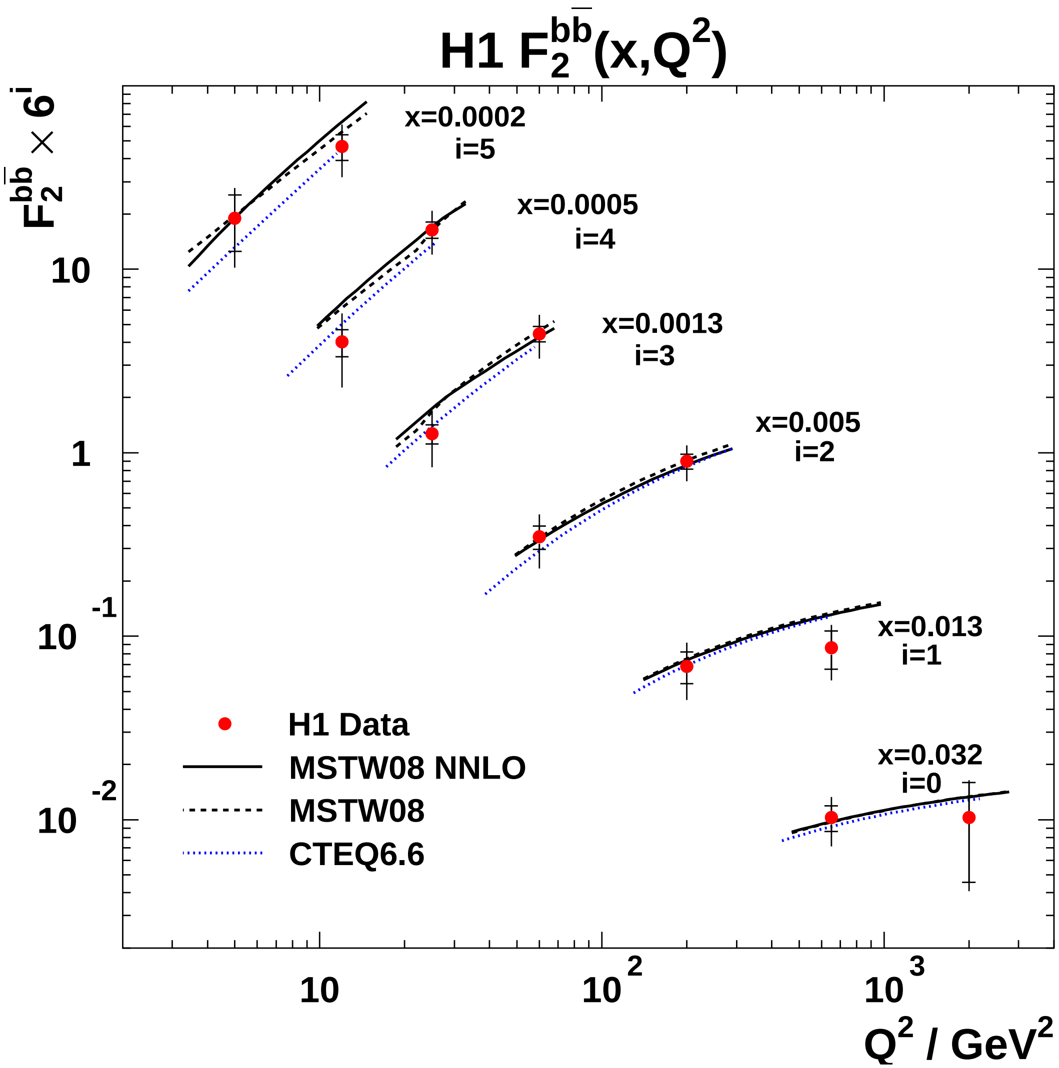
Alternative, at high scales $Q^2 \gg m_H^2$ heavy quarks like massless partons. Behave like up, down, strange. Sum $\ln(Q^2/m_H^2)$ terms via evolution. **Zero Mass Variable Flavour Number Scheme** (ZM-VFNS). Normal assumption in calculations. Ignores $\mathcal{O}(m_H^2/Q^2)$ corrections.

$$F(x, Q^2) = C_j^{ZMVF} \otimes f_j^{n_f+1}(Q^2).$$

Need a **General Mass Variable Flavour Number Scheme** (GM-VFNS) interpolating between the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$. Used by MRST/MSTW and more recently (as default) by CTEQ, and now also more regularly by H1,ZEUS.

Various definitions possible. Versions used by MSTW (RT) and CTEQ (ACOT) have converged somewhat.

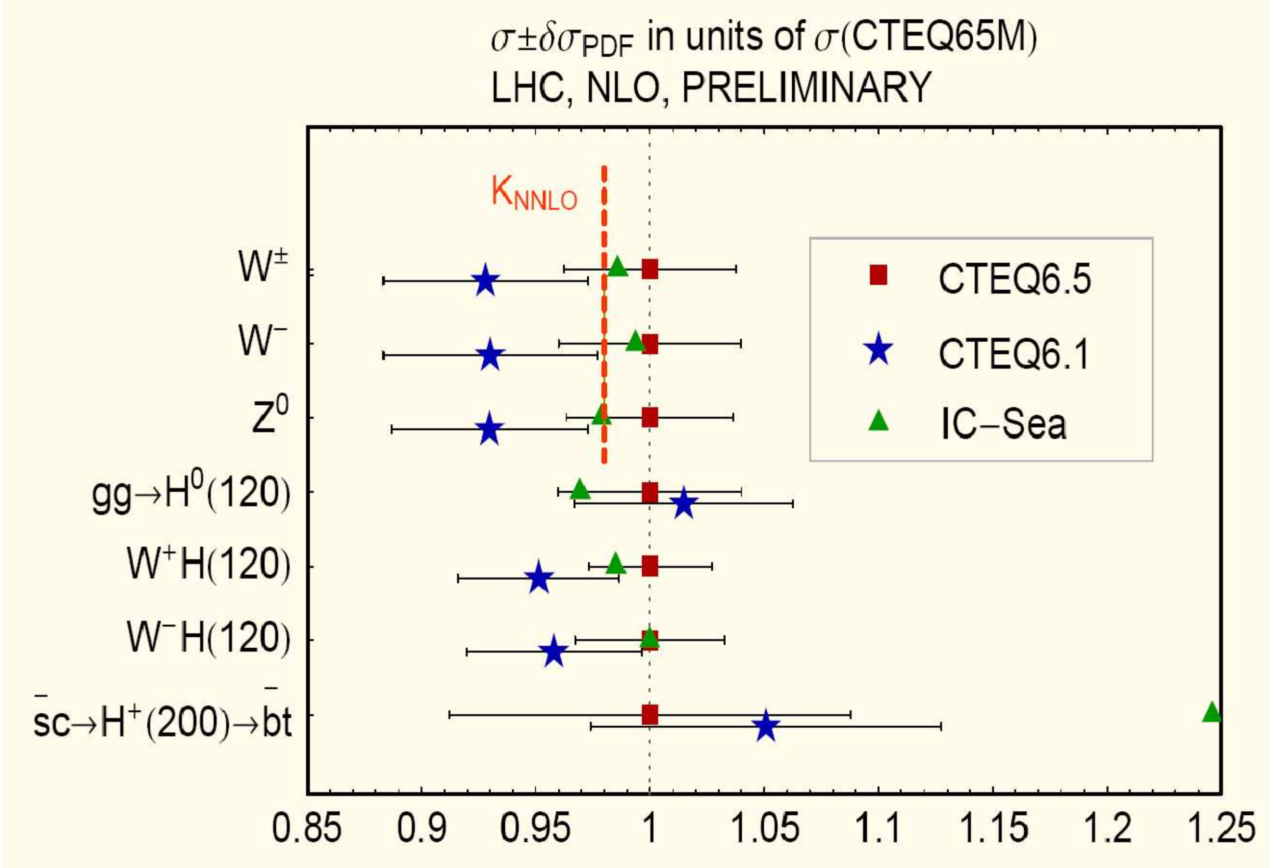
Various significant differences still exist as illustrated by comparison to most recent H1 data on bottom production.



Importance of using GM-VFNS instead of massless approach illustrated by CTEQ6.5.

Can be $> 8\%$ error in PDFs. Much more than scheme uncertainty.

Leads to large change in predictions using CTEQ partons at LHC of $5 - 10\%$.



The values of the predicted cross-sections at NLO for Z and a 120 GeV Higgs boson at the Tevatron and the LHC (latter for 14 TeV) as GM-VFNS altered.

PDF set	Tev		LHC	(14 TeV)
	σ_Z (nb)	σ_H(pb)	σ_Z (nb)	σ_H(pb)
MSTW08	7.207	0.7462	59.25	40.69
GMvar1	$+0.3\%$	-0.5%	$+1.1\%$	$+0.2\%$
GMvar2	$+0.7\%$	-1.1%	$+3.0\%$	$+1.5\%$
GMvar3	$+0.1\%$	-0.3%	$+1.1\%$	$+0.8\%$
GMvar4	$+0.0\%$	-0.1%	-0.4%	-0.2%
GMvar5	-0.1%	-0.1%	-0.5%	-0.3%
GMvar6	$+0.3\%$	-0.4%	$+1.6\%$	$+0.8\%$
GMvaropt	$+0.3\%$	-1.5%	$+2.0\%$	$+0.4\%$
ZM-VFNS	-0.7%	-1.2%	-3.0%	-3.1%
GMvarcc	$+0.0\%$	-0.1%	$+0.0\%$	-0.1%

Little more than 1% variation at Tevatron in σ_Z.

Up to $+3\%$ and -0.5% variation in σ_Z at the LHC. About half as much in σ_H due to higher average x sampled.

Most variation in ZM-VFNS.

The values of the predicted cross-sections at NNLO.

PDF set	Tev		LHC	(14 TeV)
	σ_Z (nb)	σ_H(pb)	σ_Z (nb)	σ_H(pb)
MSTW08	7.448	0.9550	60.93	50.51
GMvar1	$+0.1\%$	-0.5%	$+0.1\%$	-0.2%
GMvar2	$+0.3\%$	-0.8%	$+0.5\%$	$+0.1\%$
GMvar3	$+0.4\%$	-0.1%	$+0.5\%$	$+0.7\%$
GMvar4	$+0.0\%$	-0.2%	$+0.1\%$	-0.1%
GMvar5	$+0.1\%$	-0.3%	-0.2%	-0.2%
GMvar6	$+0.1\%$	-0.9%	$+0.3\%$	-0.2%
GMvaropt	$+0.4\%$	-0.2%	$+0.6\%$	$+0.8\%$
GMvarmod	-0.2%	-0.4%	-1.4%	-1.0%
GMvarmod′	$+0.0\%$	-0.7%	$+0.0\%$	$+0.1\%$

Maximum variations of order 1% at LHC. High-x gluon leads to 1% on σ_H at Tevatron.

Much improved stability compared to NLO.

Uncertainties due to m_c and m_b

Add uncertainties in quadrature with PDF parameter and α_S combined uncertainty.

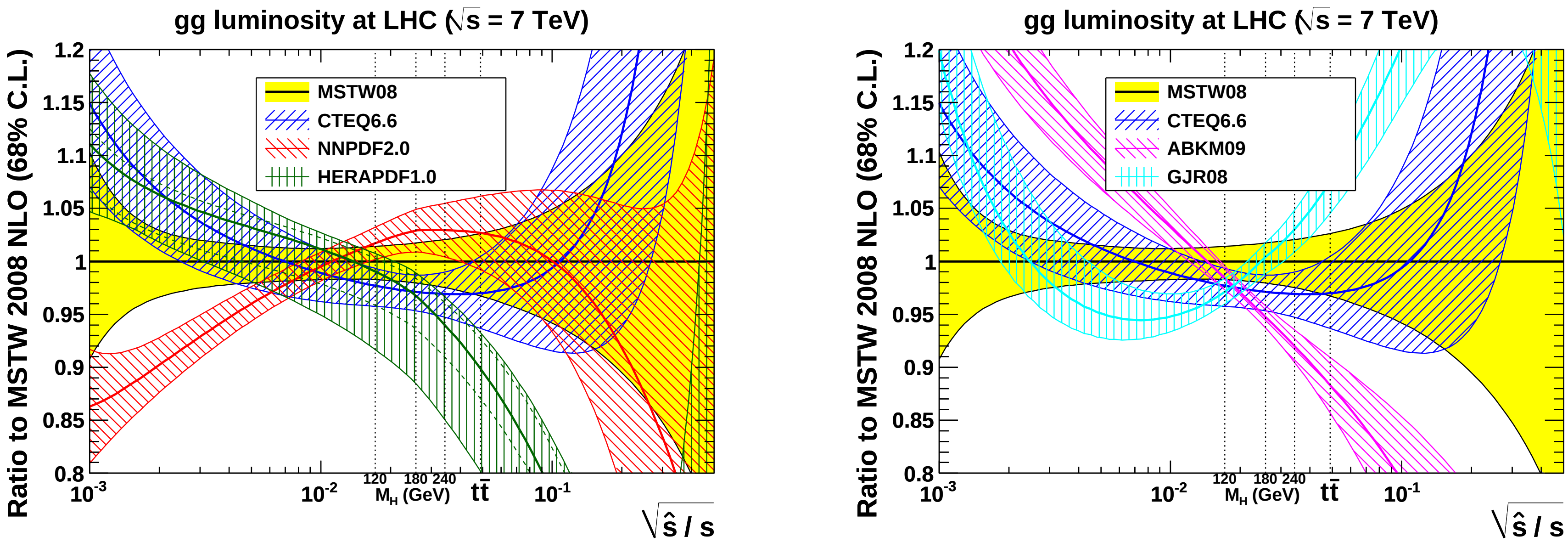
LHC, $\sqrt{s} = 7$ TeV	$B_{\ell\nu} \cdot \sigma^W$	$B_{\ell^+\ell^-} \cdot \sigma^Z$	σ^H
Central value	10.47 nb	0.958 nb	15.50 pb
PDF only uncertainty	$^{+1.7\%}_{-1.6\%}$	$^{+1.7\%}_{-1.5\%}$	$^{+1.1\%}_{-1.6\%}$
PDF+α_S uncertainty	$^{+2.5\%}_{-1.9\%}$	$^{+2.5\%}_{-1.9\%}$	$^{+3.7\%}_{-2.9\%}$
PDF+α_S+$m_{c,b}$ uncertainty	$^{+2.7\%}_{-2.2\%}$	$^{+2.9\%}_{-2.4\%}$	$^{+3.7\%}_{-2.9\%}$

LHC, $\sqrt{s} = 14$ TeV	$B_{\ell\nu} \cdot \sigma^W$	$B_{\ell^+\ell^-} \cdot \sigma^Z$	σ^H
Central value	21.72 nb	2.051 nb	50.51 pb
PDF only uncertainty	$^{+1.7\%}_{-1.7\%}$	$^{+1.7\%}_{-1.6\%}$	$^{+1.0\%}_{-1.6\%}$
PDF+α_S uncertainty	$^{+2.6\%}_{-2.2\%}$	$^{+2.6\%}_{-2.1\%}$	$^{+3.6\%}_{-2.7\%}$
PDF+α_S+$m_{c,b}$ uncertainty	$^{+3.0\%}_{-2.7\%}$	$^{+3.1\%}_{-2.8\%}$	$^{+3.7\%}_{-2.8\%}$

NNLO predictions for W, Z and Higgs ($M_H = 120$ GeV) total cross sections or 7 TeV LHC and 14 TeV LHC. Similar results in HERAPDF study Cooper-Sarkar.

α_S uncertainties more important, particularly for Higgs. Mass uncertainties significant, but least important of three effects, particularly for Higgs.

Predictions by various groups - parton luminosities – NLO. Plots by G. Watt.

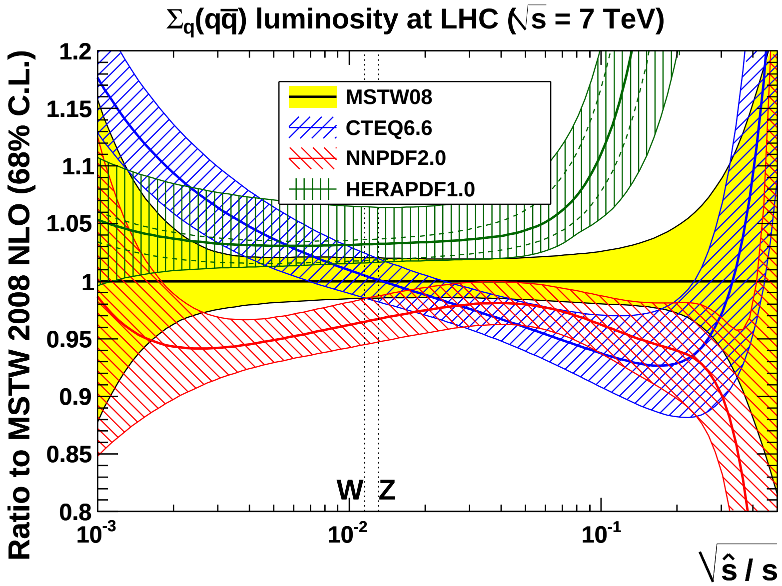


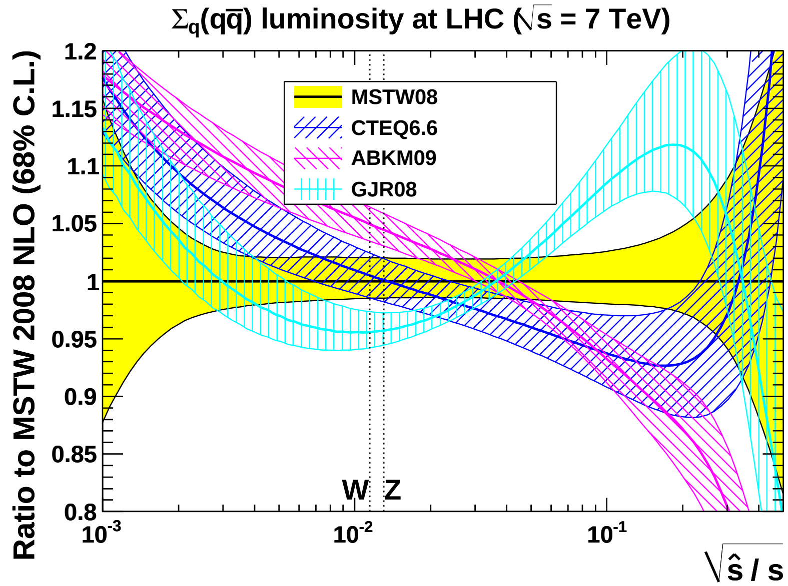
$$\frac{dL_{ij}}{d\hat{s}dy} = \frac{1}{s}\frac{1}{1+\delta_{ij}}[f_i(x_1)f_j(x_2) + f_i(x_1)f_j(x_2)] \quad \text{and integrate over } y$$

Cross-section for $t\bar{t}$ almost identical in PDF terms to 450GeV Higgs.

Also $H + t\bar{t}$ at $\sqrt{\hat{s}/s} \sim 0.1$.

Clearly some distinct variation between groups. Much can be understood in terms of previous differences in approaches.



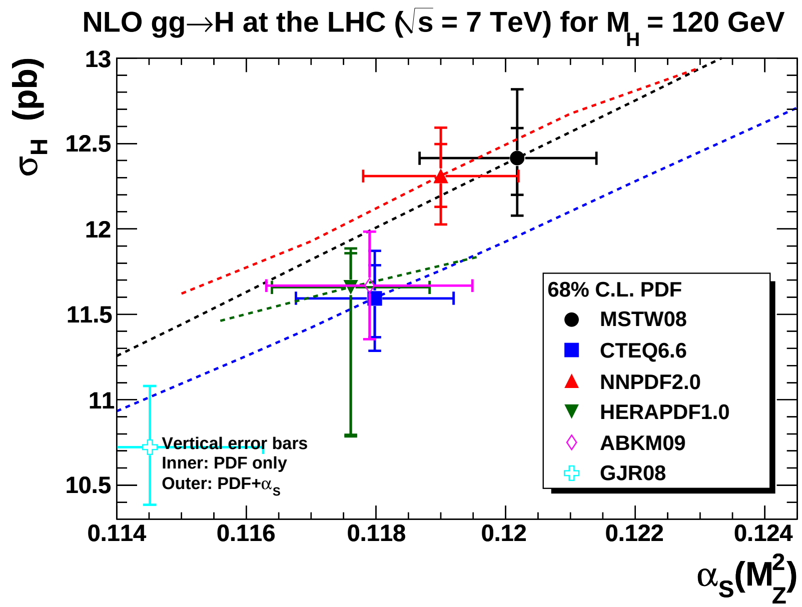


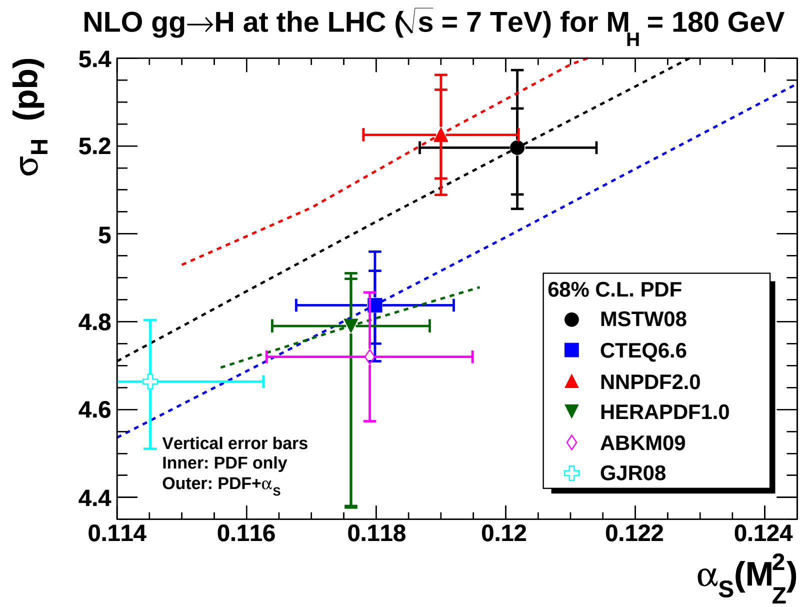
Many of the same general features for quark-antiquark luminosity. Some differences mainly at higher x.

Canonical example W, Z production, but higher $\hat{s}/s$ relevant for WH or vector boson fusion.

All plots and more at http://projects.hepforge.org/mstwpdf/pdf4lhc

Variations in Cross-Section Predictions – NLO

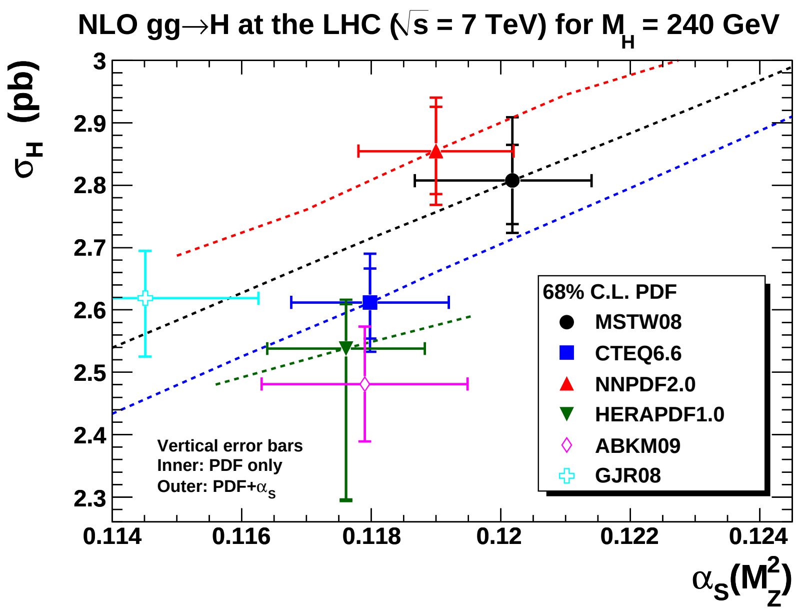


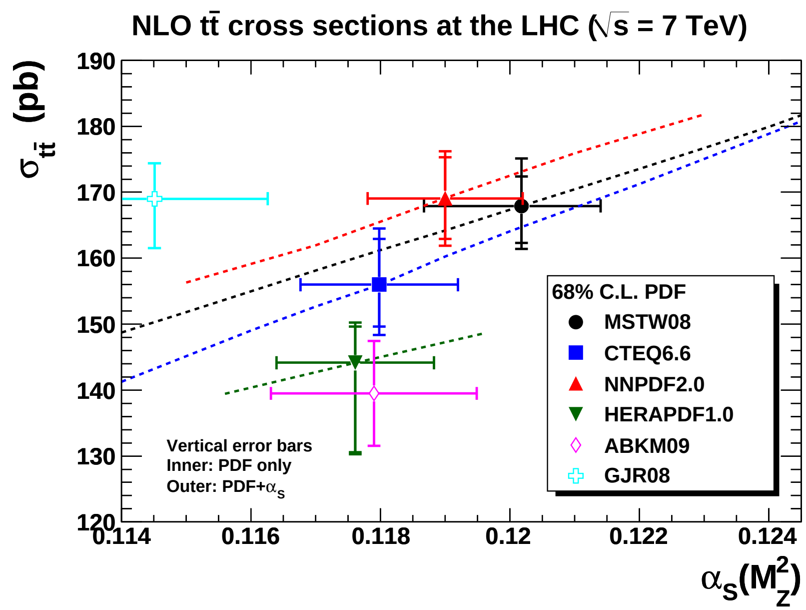


Dotted lines show how central PDF predictions vary with $\alpha_S(M_Z^2)$.

Plots based on PDF4LHC benchmark criteria, but from extensive independent study by G. Watt.

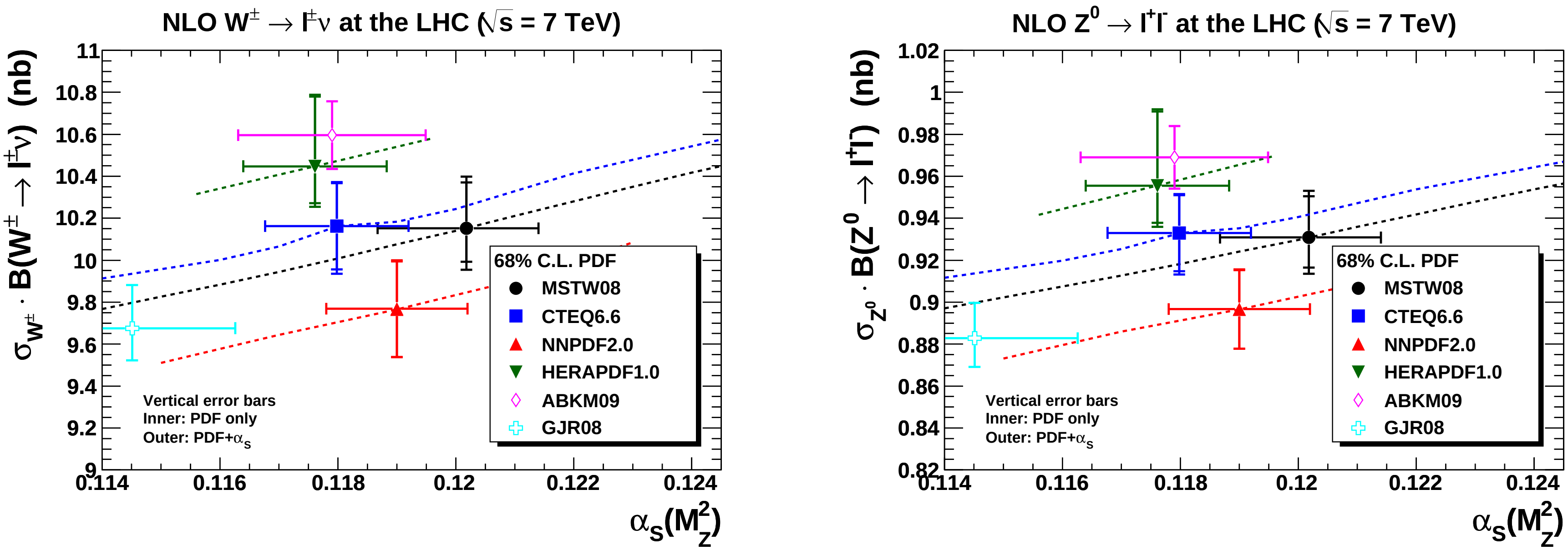
Clearly much more variation in predictions than uncertainties claimed by individual groups.





Excluding GJR08 amount of difference due to $\alpha_S(M_Z^2)$ variations $3-4\%$.

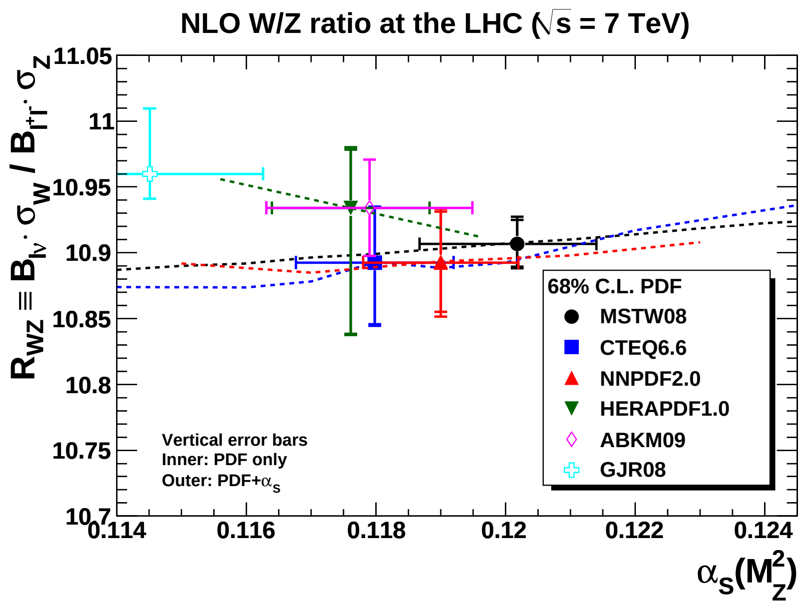
CTEQ6.6 now heading back towards MSTW08 and NNPDF2.0.

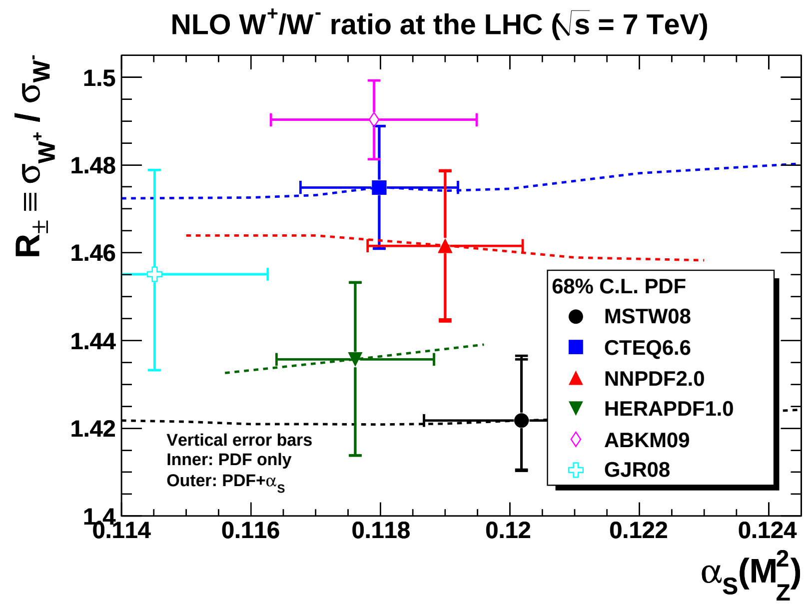


$W^+ + W^-$ cross-section. $\alpha_S(M_Z^2)$ dependence now more due to PDF variation with $\alpha_S(M_Z^2)$.

Again variations somewhat bigger than individual uncertainties.

Roughly similar variation for $\hat{s}$ up to a few times higher.





For W/Z values consistent but uncertainties vary. Largely due to strange uncertainty.

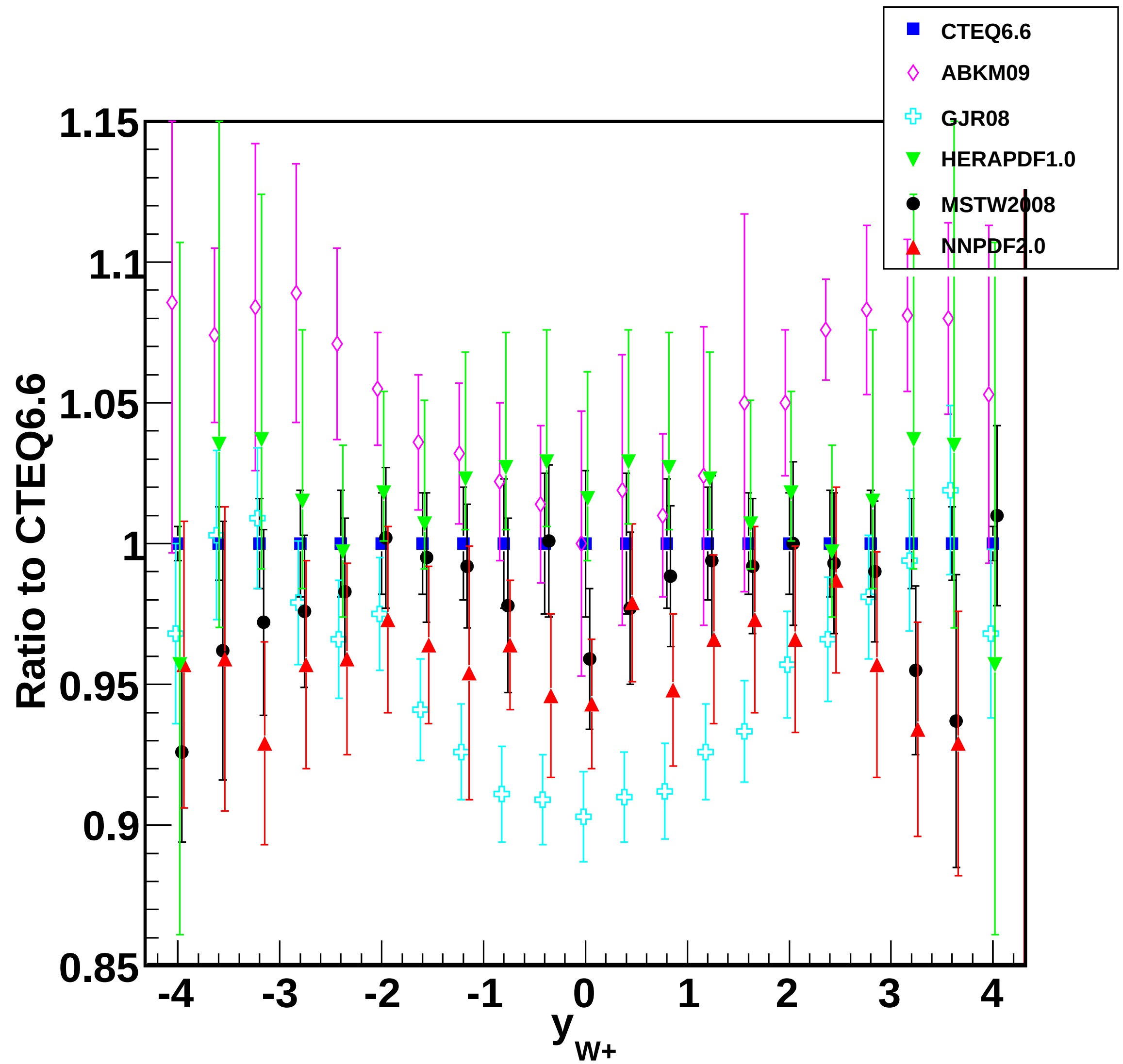
Quite a variation in ratio for W^+/W^-. Shows variations in flavour and quark-antiquark decompositions.

All plots and more at http://projects.hepforge.org/mstwpdf/pdf4lhc

Differences also clear in rapidity distributions.

Plot from PDF4LHC Interim Report.

Shape discriminating even if normalisation will be difficult for a while.

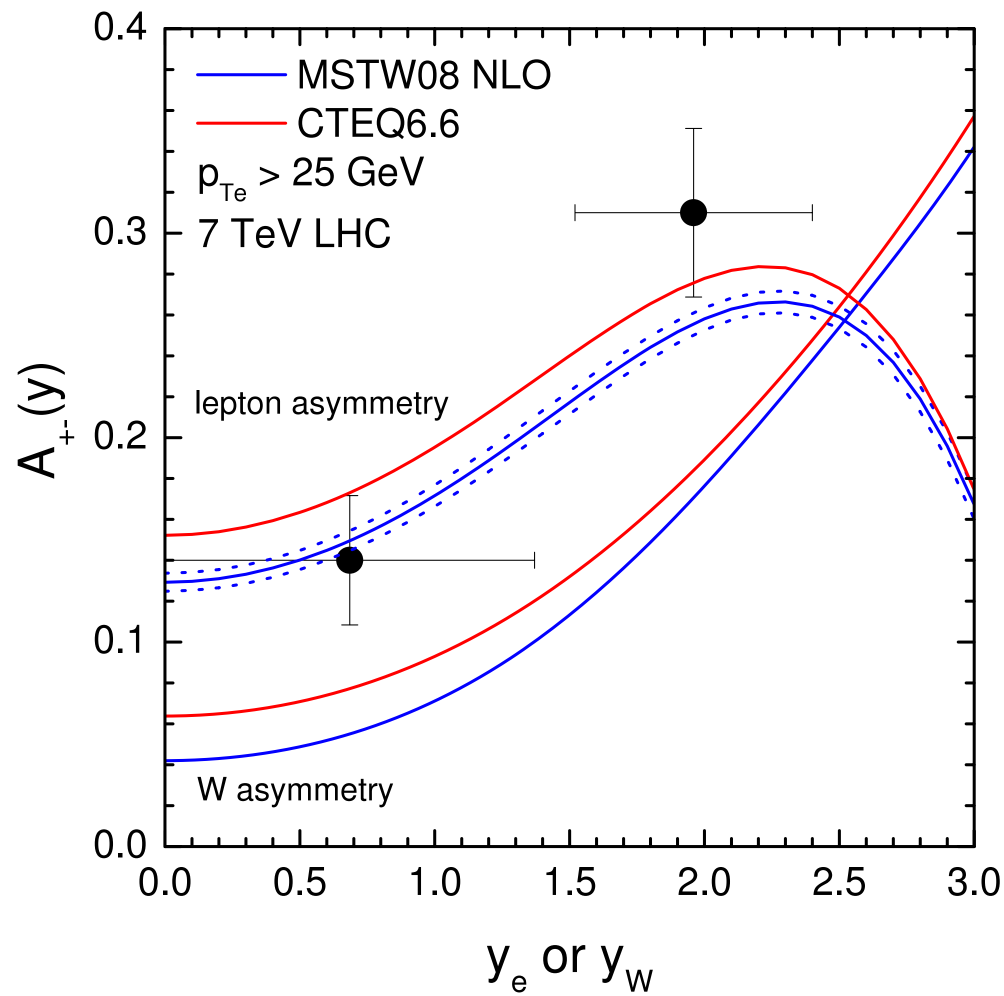


Translates into some significant differences in the more different W-asymmetry predictions.

MSTW08 and CTEQ6.6 about the biggest discrepancy at low y.

HERAPDF diverges from others at highest y.

Possibly first real discriminating power from LHC measurements.



Deviations In predictions clearly much more than uncertainty claimed by each.

In some cases clear reason why central values differ, e.g. lack of some constraining data, though uncertainties then do not reflect true uncertainty.

Sometimes no good understanding, or due to difference in procedure which is simply a matter of disagreement, e.g. gluon parameterisation at small x affects predicted Higgs cross-section.

What is true uncertainty. Task asked of PDF4LHC group.

Interim recommendation take envelope of $global$ sets, MSTW, CTEQ NNPDF (check other sets) and take central point as uncertainty.

Not very satisfactory, but not clear what would be an improvement, especially as a general rule.

Usually not a big disagreement, and factor of about 2 expansion of MSTW uncertainty.

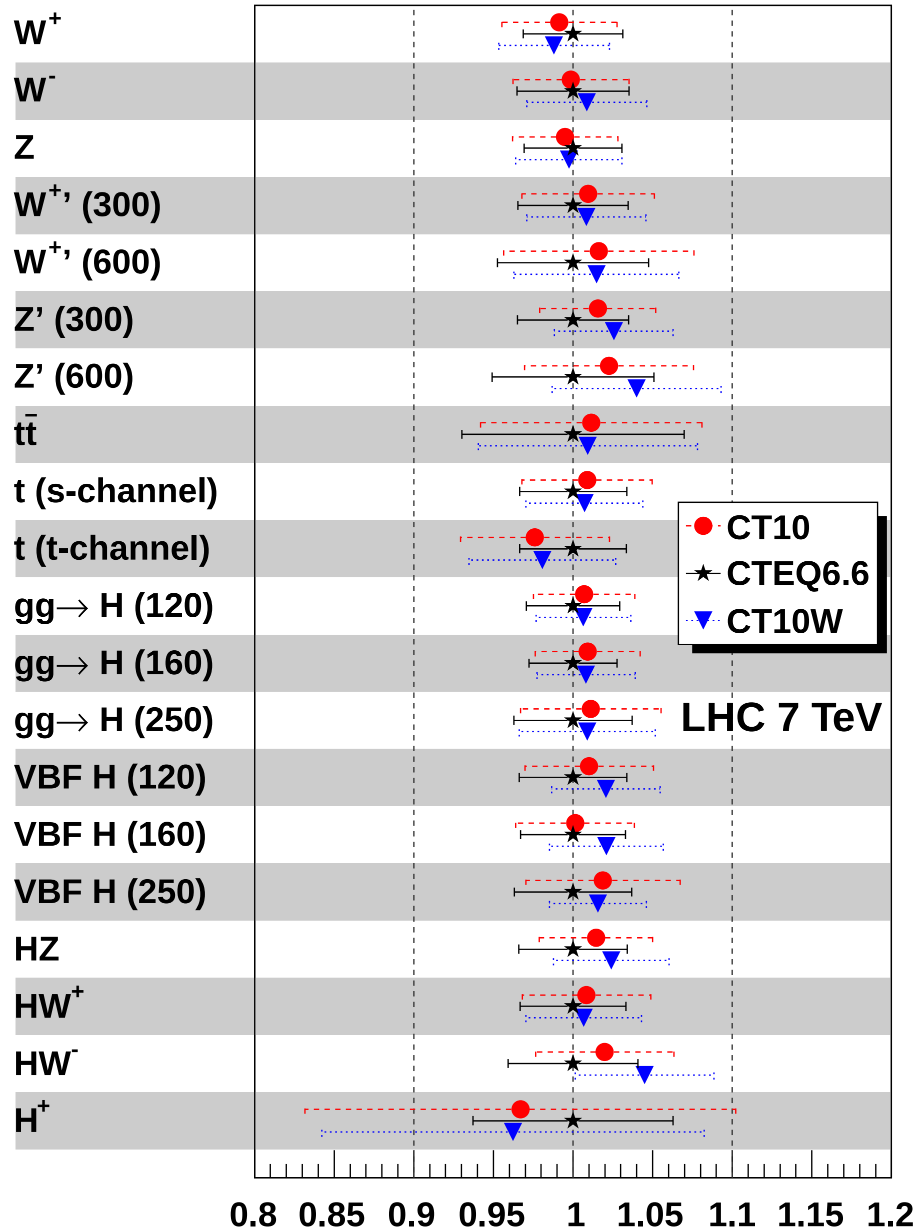
Very Recent Updates

MSTW find new combined HERA data lead to increase in W, Z by couple of $\%$. Less than 1% on Higgs (Tevatron and LHC).

CT10 (right) find change in W, Z very small (probably countered by gluon parameterisation change).

Slight increase in Higgs, $t\bar{t}$ (again probably gluon shape).

NNPDF find prelim GM-VFNS fits bring them closer to MSTW,CTEQ for W, Z.



Other sources of Uncertainty.

Also other sources which (mainly) lead to inaccuracies common to all fixed-order extractions.

- QED and Weak (comparable to NNLO ?) ($\alpha_s^3 \sim \alpha$). Sometime enhancements.
- Standard higher orders (NNLO – some sets available here.)
- Resummations, e.g. small x $(\alpha_s^n \ln^{n-1}(1/x))$, or large x $(\alpha_s^n \ln^{2n-1}(1-x))$ or equivalently summations in high-energy limit and threshold limit.
- low Q^2 (higher twist), saturation.

NNLO splitting functions now known. (Moch, Vermaseren and Vogt). Essentially full NNLO determination of partons now being performed (MSTW, ABKM,GJR,HERA), though heavy flavour not fully worked out in the fixed-flavour number scheme (FFNS) PDFs and jet cross-sections approximate. Improve consistency of fit very slightly, and reduces α_S.

Surely this is best, i.e. most accurate.

Yes, but ...... only know some hard cross-sections at NNLO.

Processes with two strongly interacting particles largely completed

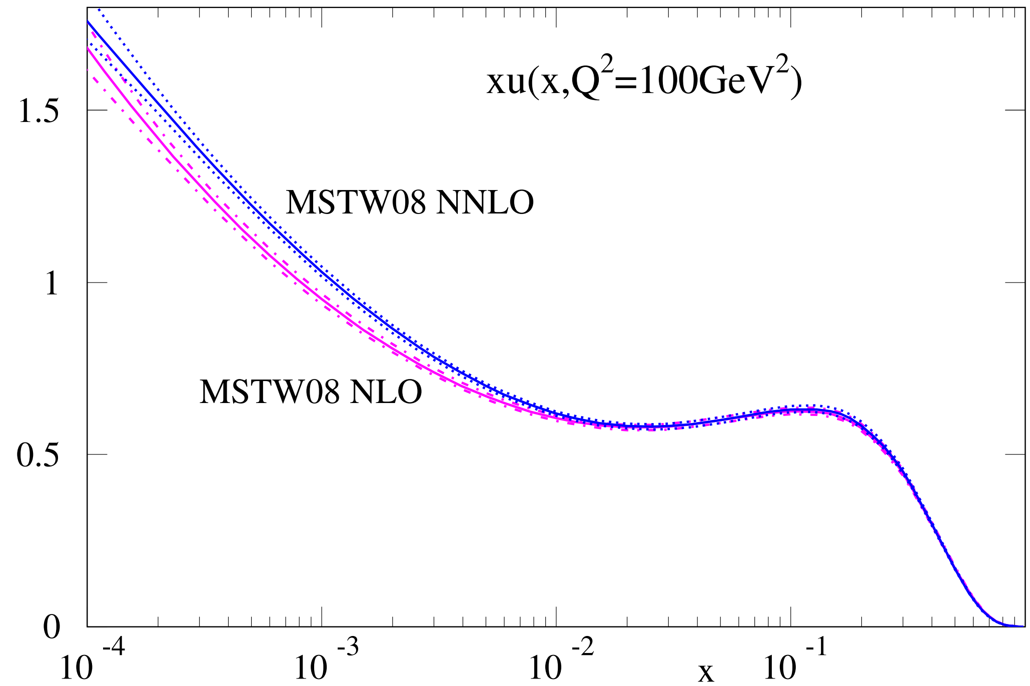
DIS coefficient functions and sum rules

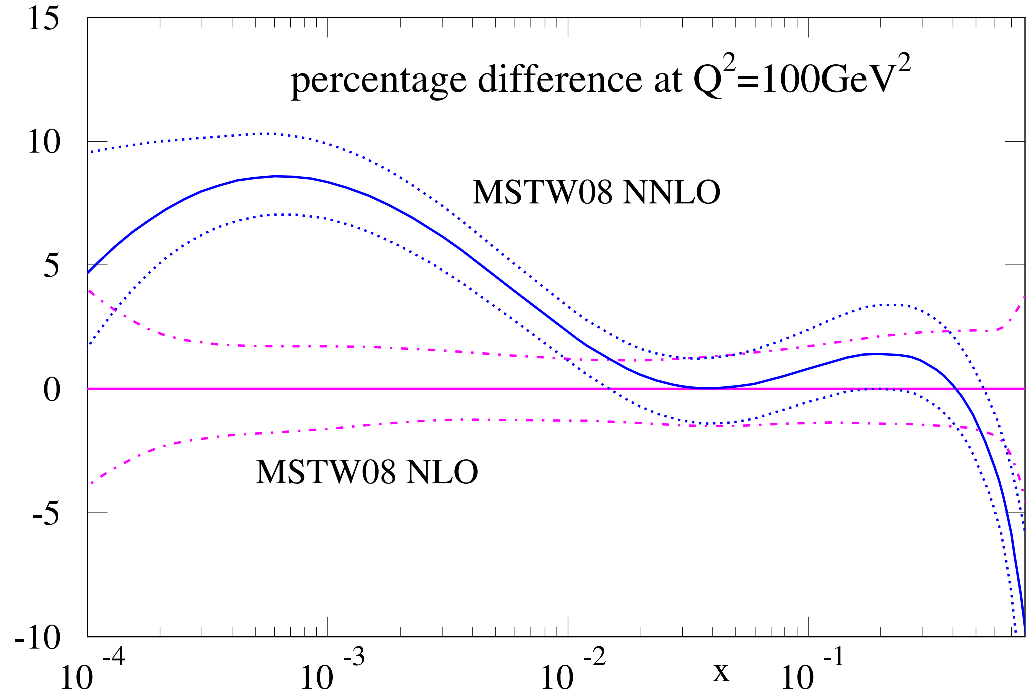
$pp(\bar{p}) \rightarrow \gamma^\star, W, Z$ (including rapidity dist.), H, A^0, WH, ZH.

But for many other final states NNLO not known. NLO still more appropriate.

Stability order-by-order.

Systematic difference between PDF defined at NLO and at NNLO.





Consideration of NNLO

Very good evidence that one should use NNLO if possible rather than NLO – many physical cross-sections, particularly $gg \to H$, not very convergent.

Fewer PDF sets available, can study differences between them better at NLO, but for central prediction need NNLO.

Related to issue of use and uncertainty of $\alpha_S(M_Z^2)$. Noted systematic change in value form fit as one goes from NLO to NNLO. Also highlighted in stability of predictions.

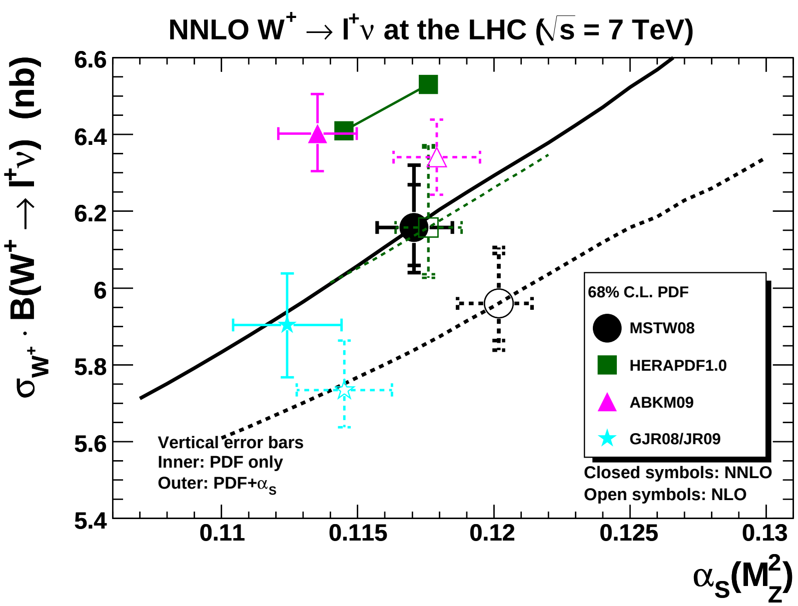
Consider percentage change from NLO to NNLO in MSTW08 predictions for best fit α_S compared to fixed $\alpha_S(M_Z^2) = 0.119$.

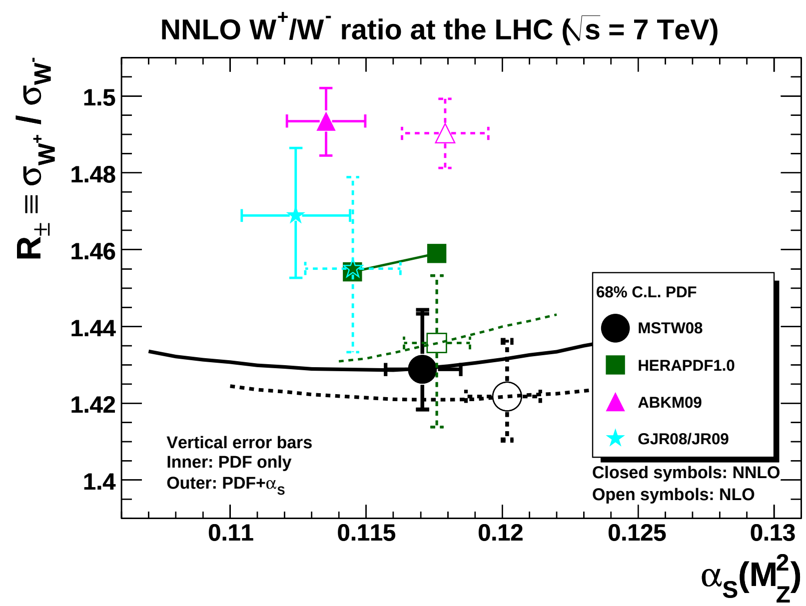
	$\sigma_{W(Z)}$ 7TeV	$\sigma_{W(Z)}$ 14TeV	σ_H 7TeV	σ_H 7TeV
MSTW08 best fit α_S	3.0	2.6	25	24
MSTW08 $\alpha_S = 0.119$	5.3	5.0	32	30

$\alpha_S(M_Z^2)$ is not a physical quantity. In (nearly) all PDF related quantities (and many others) shows tendency to decrease from order to order. Noticeable if one has fit at NNLO. Any settling on, or near common $\alpha_S(M_Z^2)$ **has** to take this into account.

Benchmark results at NNLO

Plots from not yet published results by G. Watt



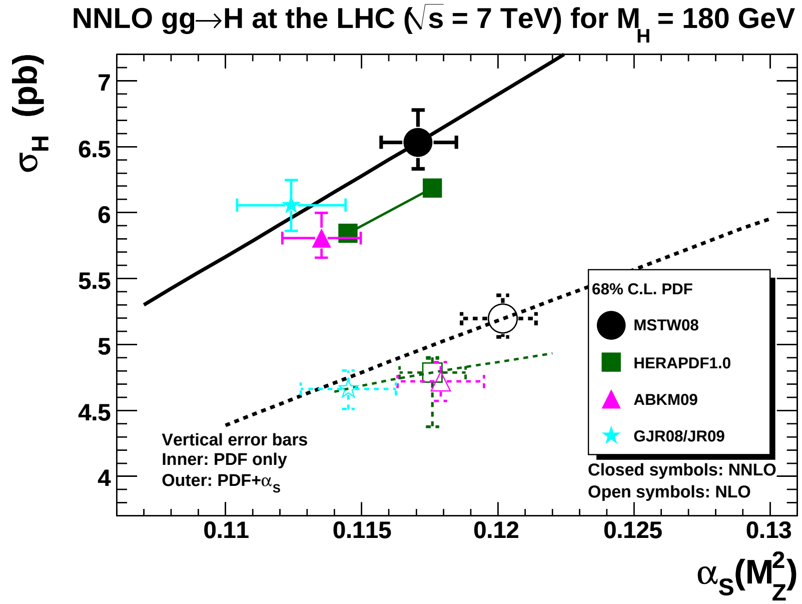


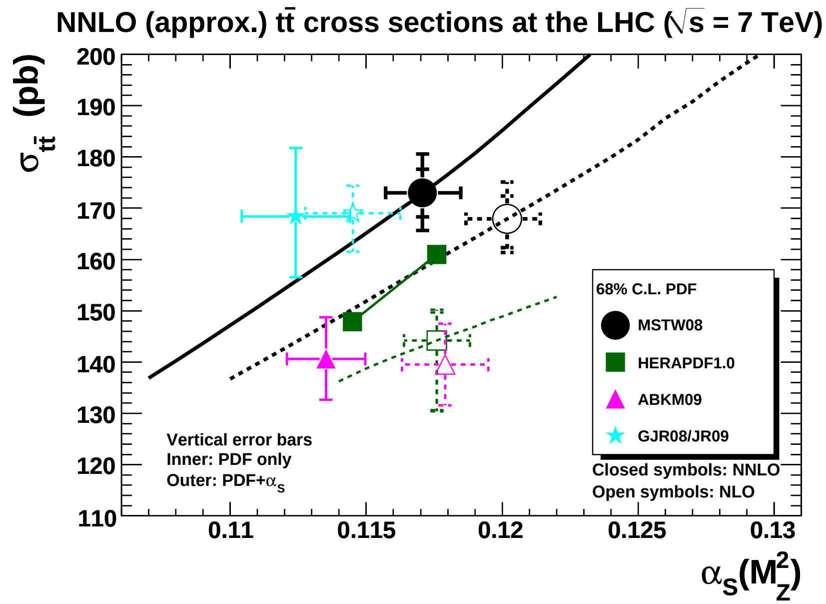
Differences between PDF sets very similar as at NLO, whereas differences from theoretical choices should diminish differences.

More stability if NNLO α_S lower than at NLO.

NNLO corrections have minor effect on asymmetries.

Plots from not yet published results by G. Watt





Differences between groups significant at NNLO, and similar to NNLO – parton luminosity compassion similar at NLO to NNLO.

Approx NNLO using HATHOR - (Aliev *et al*), includes scale-dependent parts and large threshold corrections at NNLO. Hence some theoretical uncertainty, but NNLO corrections not large at LHC.

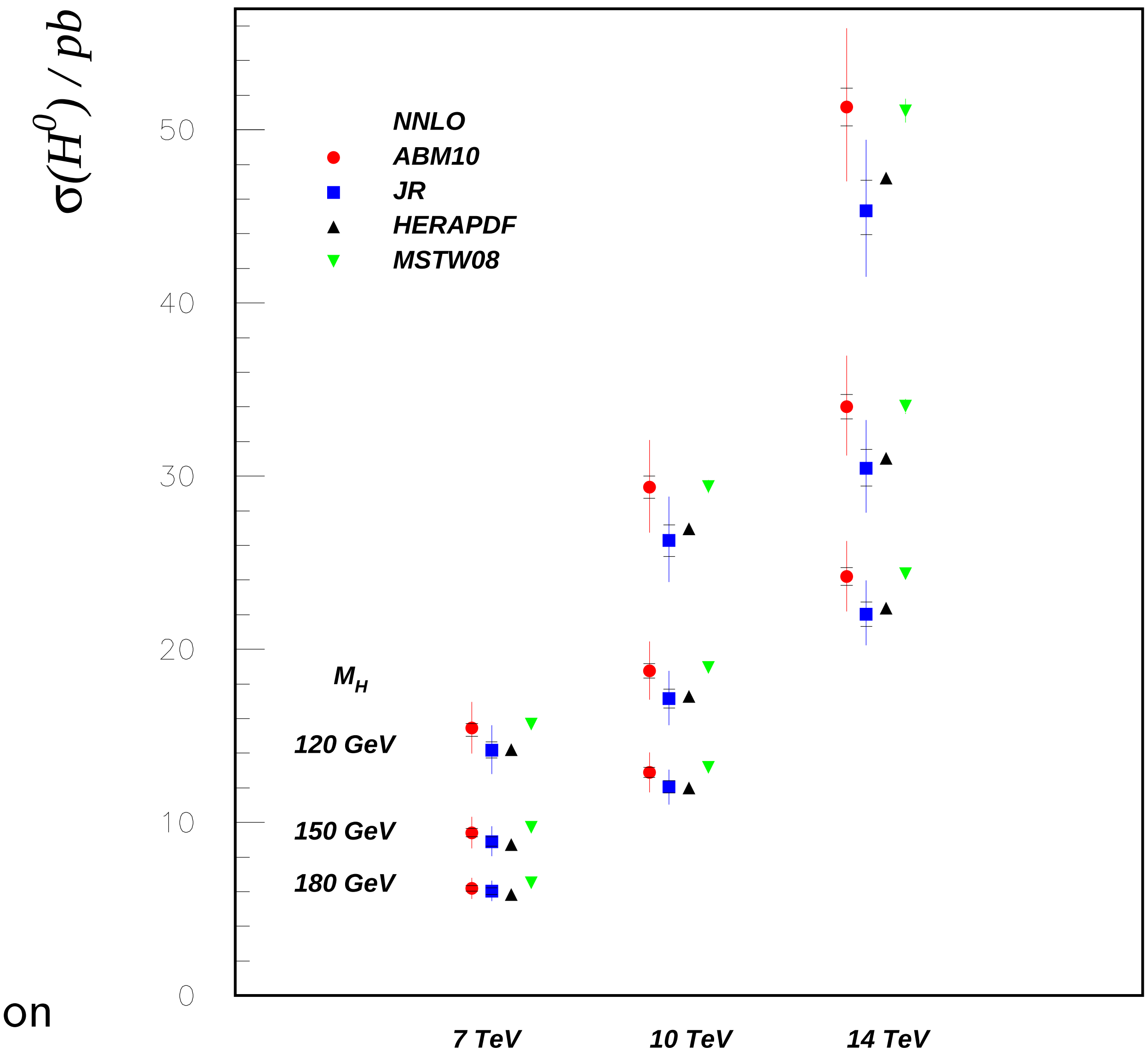
Top cross-section measurement potential discriminator of PDF sets, and correlated to Higgs predictions.

Similar study published by Alekhin *et al*.

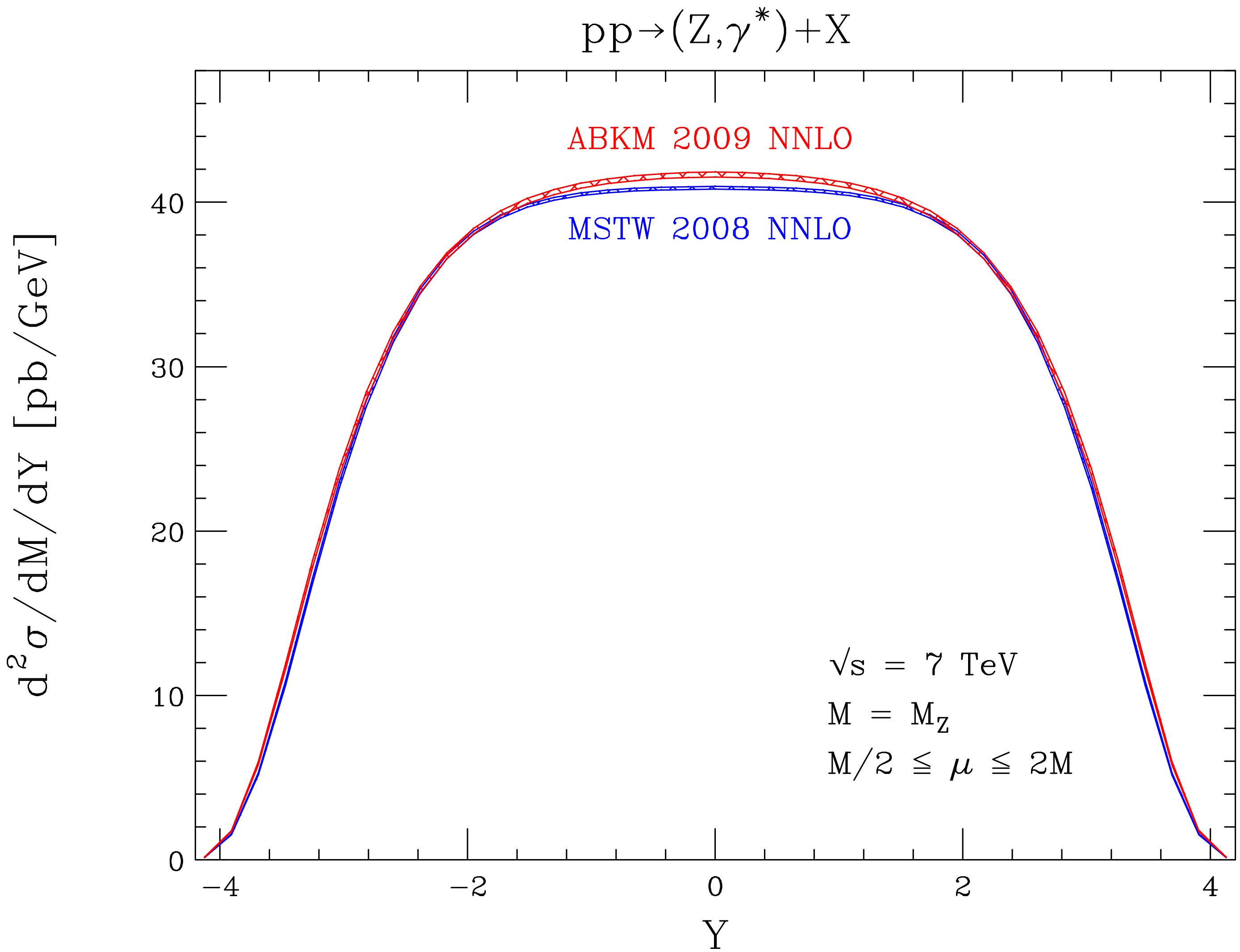
Difficult to compare PDF uncertainties meaningfully in this plot, but size of scale uncertainty illustrated for some sets.

Dominates Higgs cross-section, but very highly correlated between PDFs, i.e. overlap in uncertainties when included not strictly "agreement".

Consider full PDF uncertainty and then theory (scale) uncertainty separately. Correlation depends on process, but hopefully low.



Differences in rapidity distributions evident at NNLO.

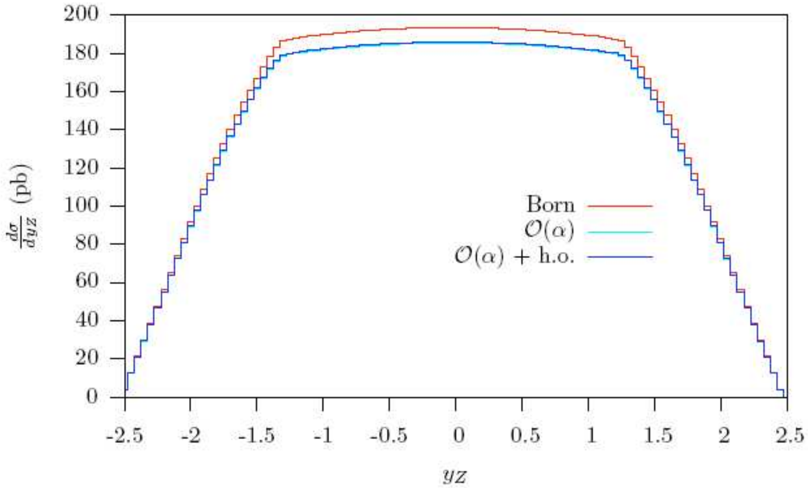


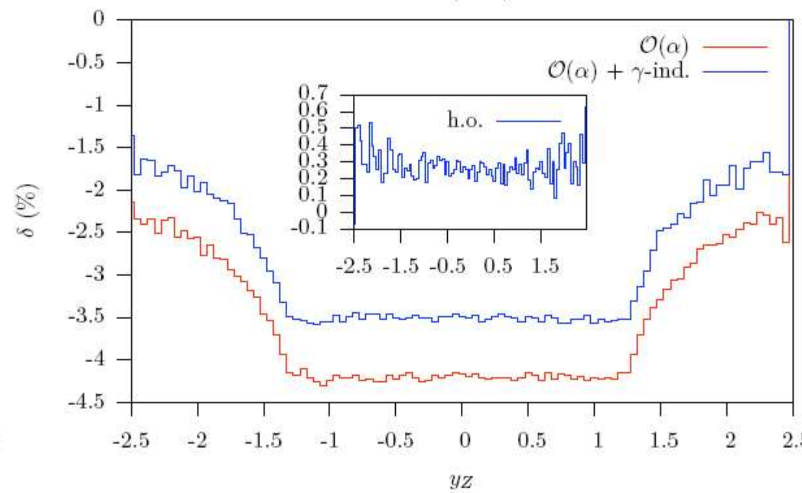
Based on (Anastasiou *et al.*), from Lance Dixon.

Differences bigger than uncertainties.

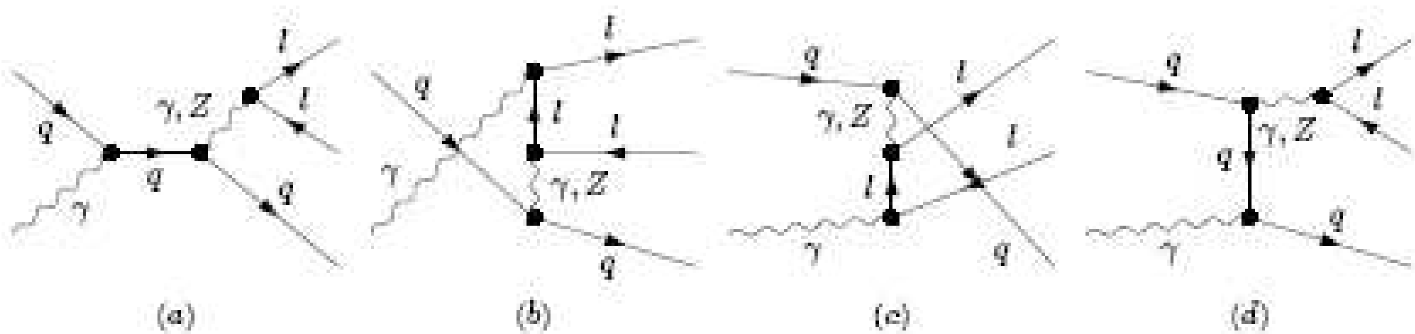
Electroweak corrections

Typically a few percent, e.g. Calone Calame *et al* who look at Drell-Yan processes.





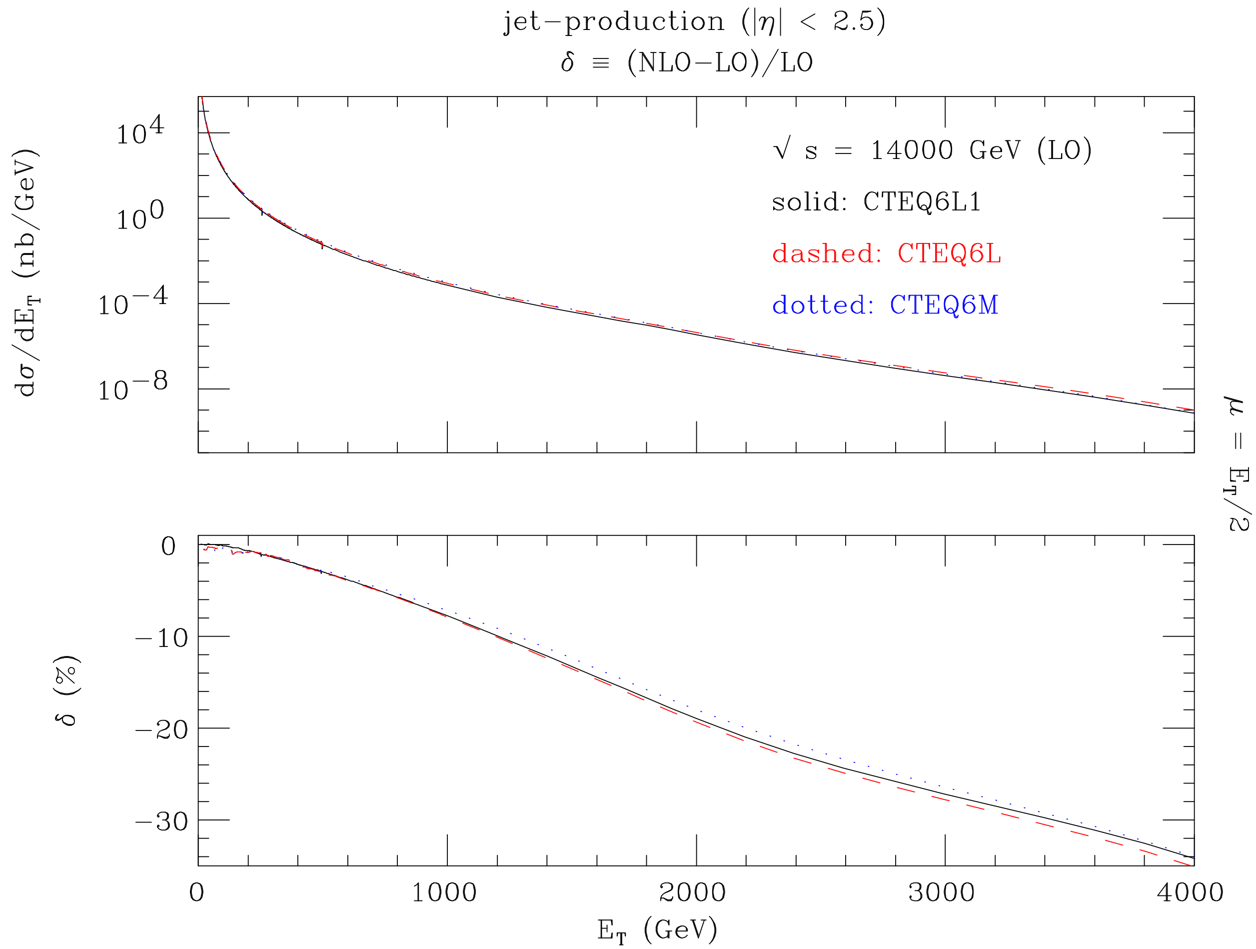
Also consider photon-induced processes. Requires the photon distribution of the proton. Currently only one QED-corrected pdf (MRST2004) set (leads to automatic isospin violation - reduces NuTeV anomaly).



Can also be a couple of percent (here in opposite direction).

Large Electroweak corrections

Jet cross-section a major example – calculation by Moretti, Nolten, Ross, goes like $(1 - \frac{1}{3} C_F \frac{\alpha_W}{\pi} \log^2(E_T^2/M_W^2))$.



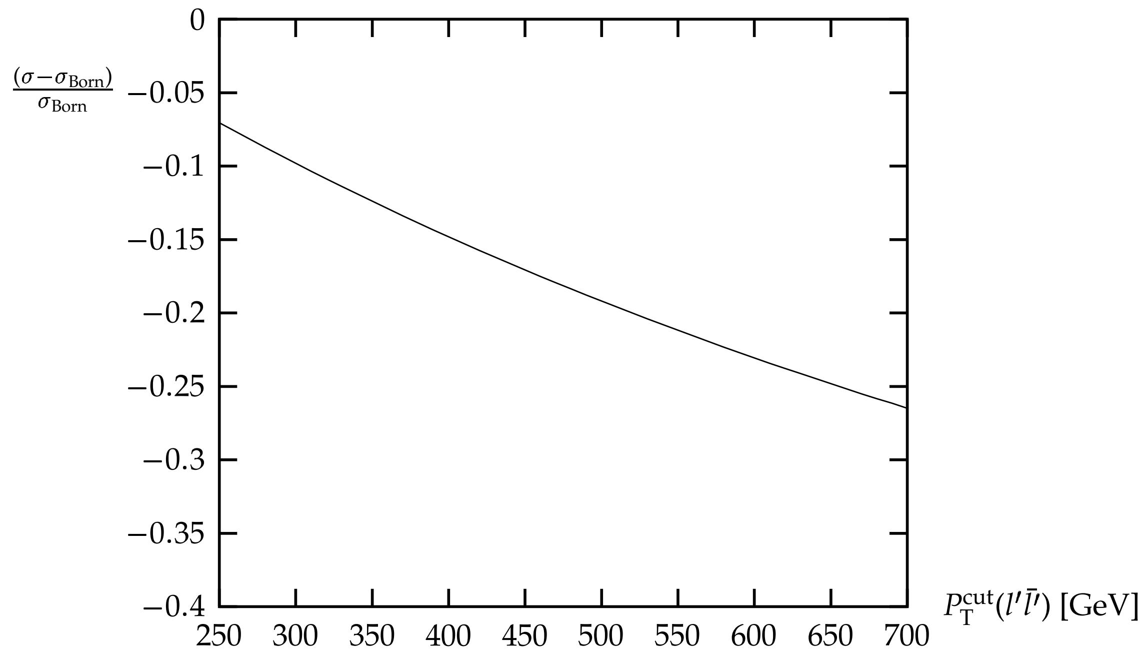
Big effect at LHC energies – $\log^2(E_T^2/M_W^2)$ a very large number. Up to 30%. Bigger than NLO QCD.

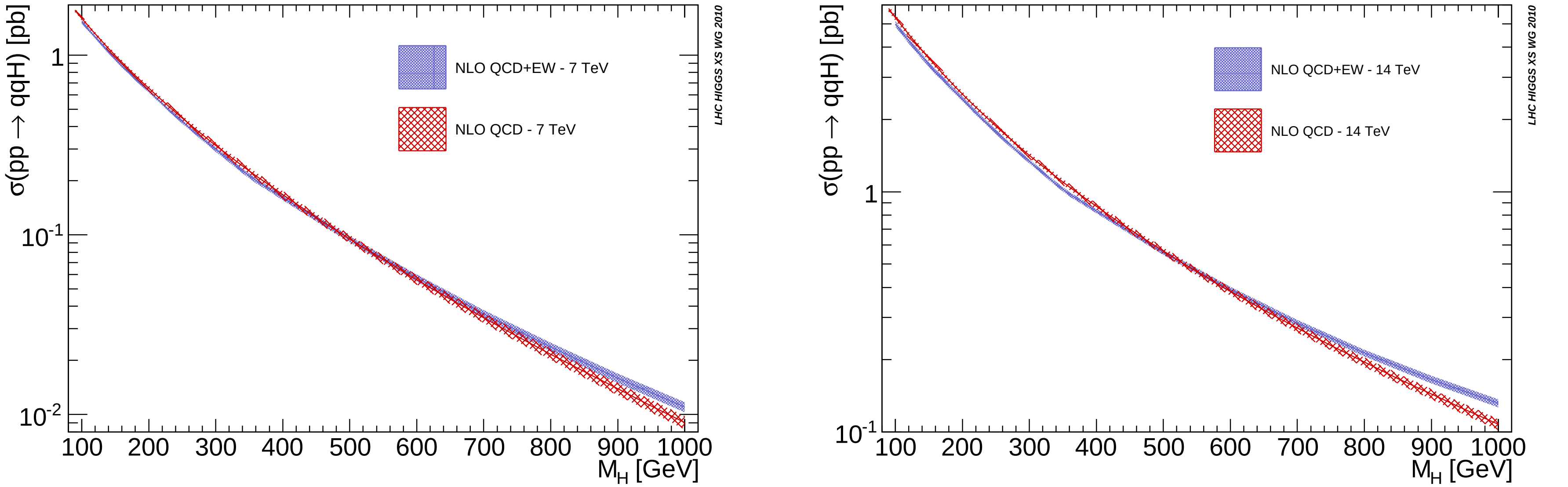
Similar results for corrections to other processes with a hard scale, e.g. Di-boson production (Accomando *et al*).

Plot shows fractional corrections as function of reconstructed Z transverse momentum in WZ production.

Same sort of corrections in large-p_T vector bosons in conjunction with jets (Kühn *et al*, Maina *et al*)...

$\ln(s/m_W^2)$ terms can also affect Γ_W extraction from the transverse mass distribution.





Effect of electroweak corrections to Higgs production from vector boson fusion.

Plots from Vector Boson Fusion section of Handbook of LHC Higgs Cross Sections.

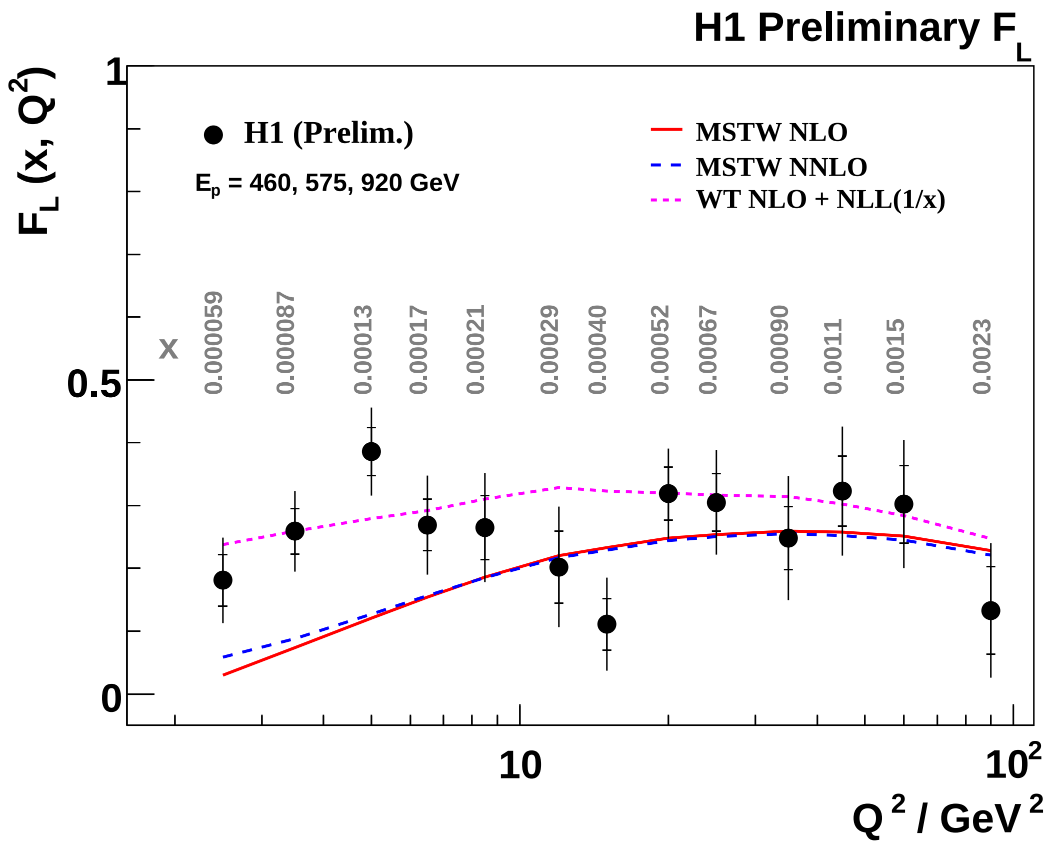
Small-x Theory

At each order in α_S each splitting function and coefficient function obtains an extra power of $\ln(1/x)$ (some accidental zeros in P_{gg}), i.e. $P_{ij}(x,\alpha_s(Q^2)), C_i^P(x,\alpha_s(Q^2)) \sim \alpha_s^m(Q^2)\ln^{m-1}(1/x)$.

Summed using BFKL equation (and a lot of work – Altarelli-Ball-Forte, Ciafaloni-Colferai-Salam-Stasto and White-RT)

Comparison to H1 prelim data on $F_L(x,Q^2)$ at low Q^2, only within White-RT approach, suggests resummations may be important.

Could possibly give a few percent effect on Higgs cross sections.

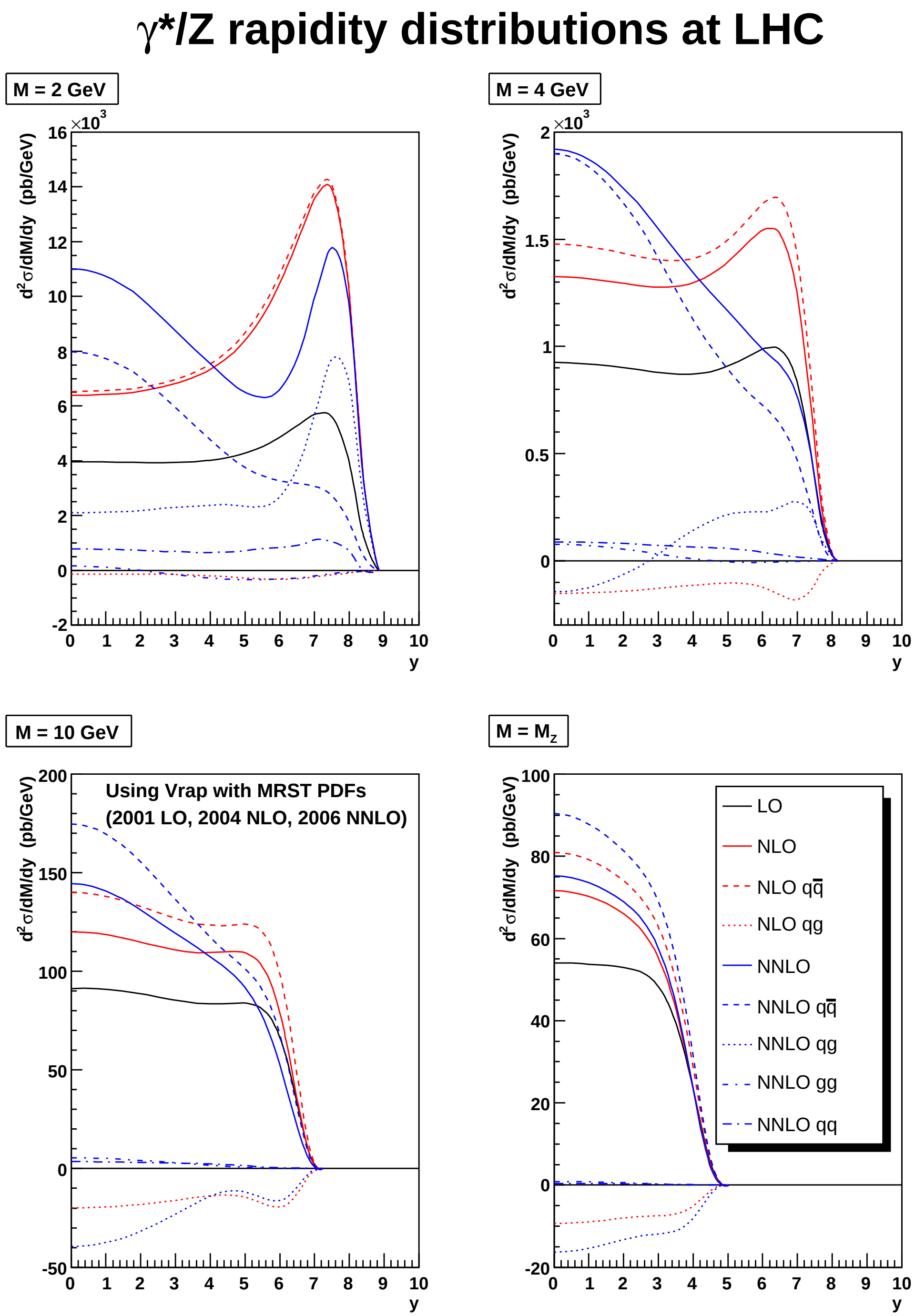


The region of large theory corrections is lower M^2 and high y.

However, this assumes perturbative prediction of Drell-Yan production is reliable.

As seen very large change in prediction from order to order, particularly for low M and high y.

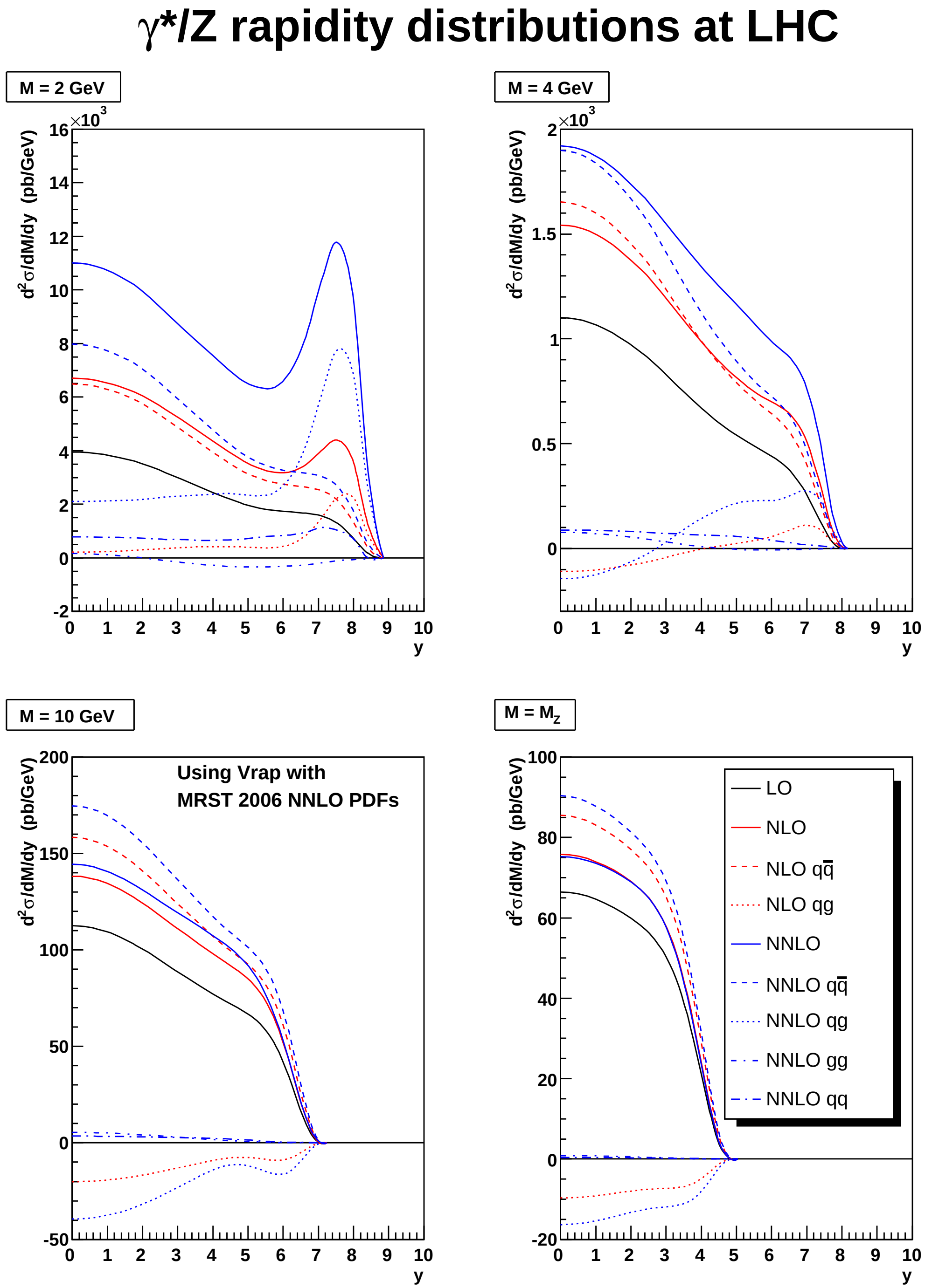
Problem with perturbative stability. Is this due to partons or cross-sections?



Keeping partons fixed while changing cross-sections (using MRST2006 NNLO partons) also shows part of instability due to partons. Unusual behaviour in very small x partons at NNLO. Due to similar high and low z terms in splitting functions.

Overall most obvious effect – large change in quark-gluon (and quark-quark) contributions at NNLO due to $1/z$ and $\ln(1-z)$ divergences in cross-sections appearing at this order.

Cross-section may be sensitive to resummations (high and low z) at lowest M and highest y. In region where measurements can be made?



Conclusions

We can calculate to NLO or NNLO in QCD and sometimes include electroweak corrections. Also need PDFs.

One can determine the parton distributions and predict cross-sections at the LHC, and the fit quality using NLO or NNLO QCD is fairly good.

Various ways of looking at *experimental* uncertainties on PDFs. Uncertainties $\sim 1-5\%$ for most LHC quantities. Major uncertainty in vector boson production. Ratios, e.g. W^+/W^- tight, and hopefully early constraint on partons.

Effects from input assumptions e.g. selection of data fitted, cuts and input parameterisation can shift central values of predictions significantly. Also affect size of uncertainties. Want balance between freedom and sensible constraints. Complete heavy flavour treatments essential in extraction and use of PDFs. α_S and PDFs heavily correlated.

Errors from higher orders estimated using scale variations, which should often be reasonable. Can dominate, e.g. Higgs. Resummation effects also potentially large. At LHC measurement at high rapidities, e.g. W, Z would be useful in testing understanding of QCD, and particularly quantities sensitive to low x at low scales, e.g. low mass Drell-Yan.

Comparison to Standard Model predictions at the LHC far from a straightforward procedure. Lots of theoretical issues to consider for real precision. Relatively few cases where Standard Model discrepancies will not require some significant input from QCD, PDF and electroweak physics to determine real significance.

However, does include pre-processing exponents as $x \to 1$ and $x \to 0$ to aid convergence of fit,

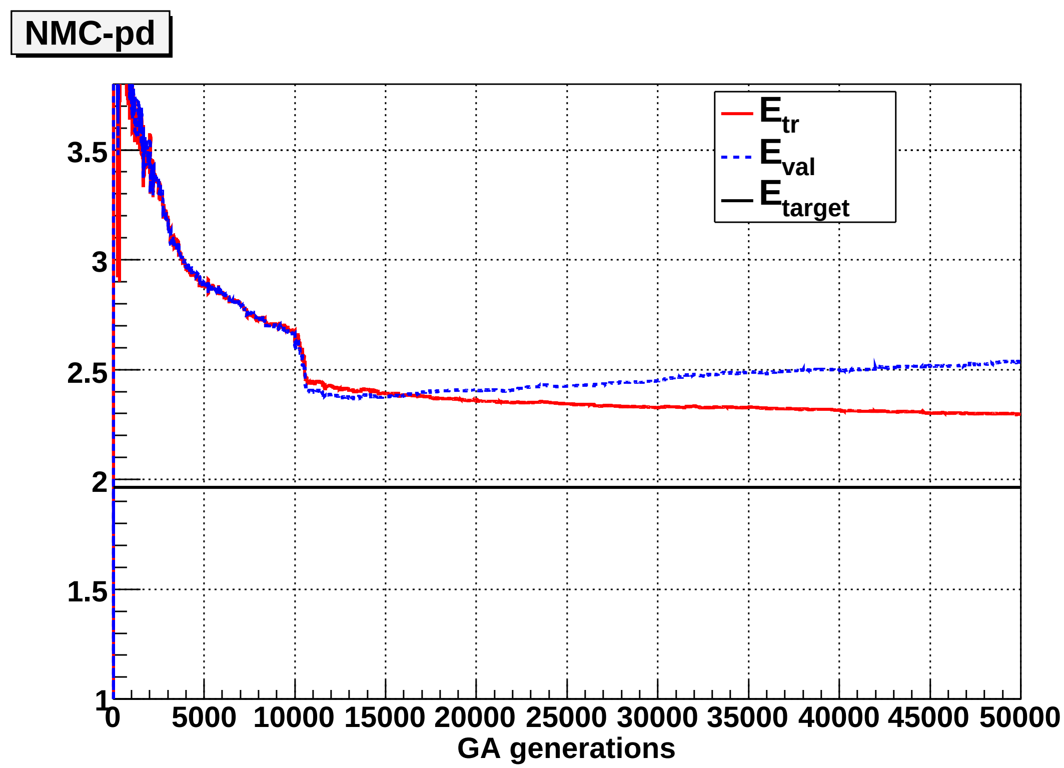
$$f(x, Q_0^2) = A(1-x)^m x^{-n} NN(x)$$

where n, m are in fairly narrow ranges, so overall behaviour guided at these extremes where data constraints vanish.

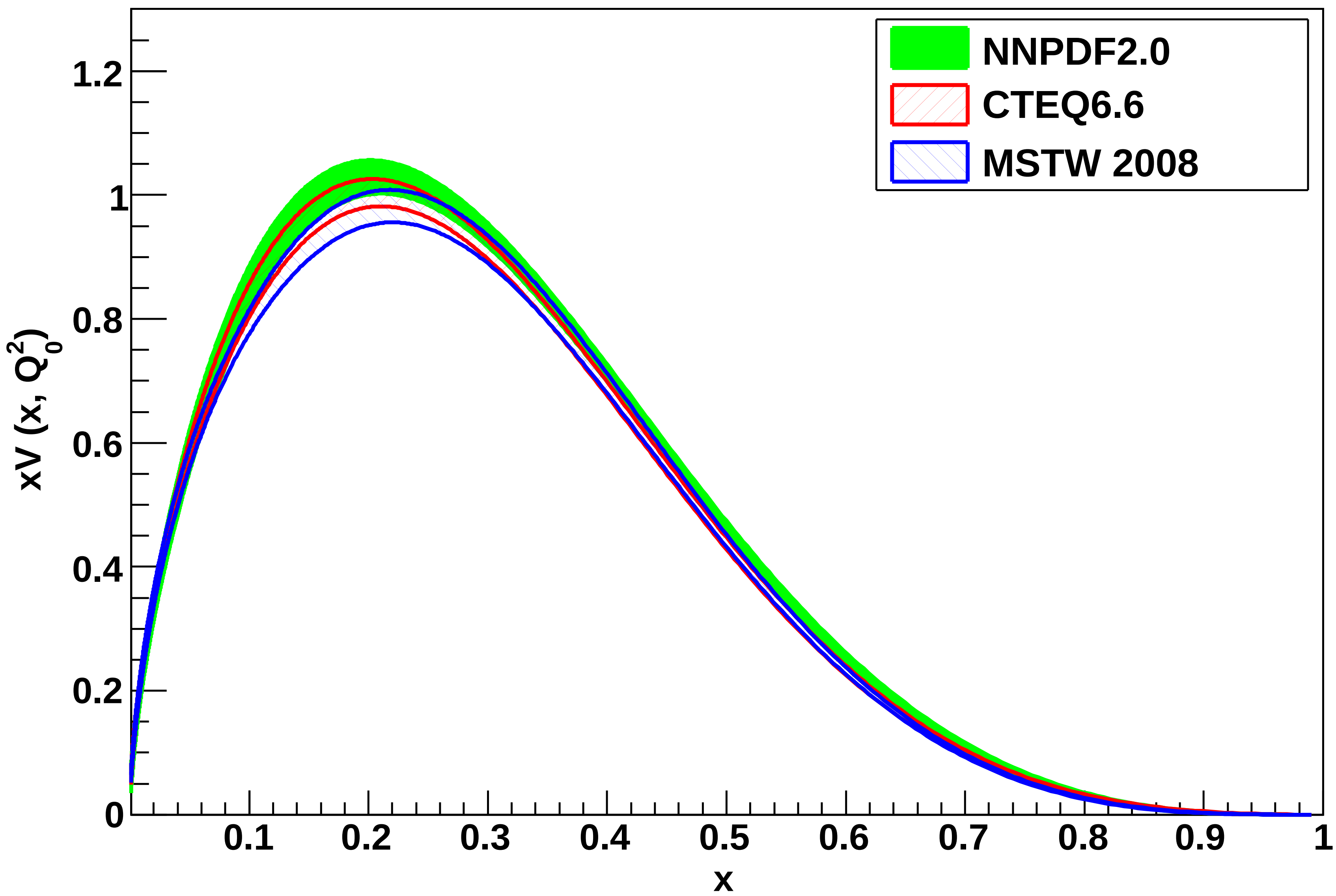
Split data sets randomly into equal size *training* and *validation* sets.

Fit until quality of fit to validation set starts to go up, even though training set still (hopefully slowly) improving.

Criterion for stopping the fit depends on different data sets.



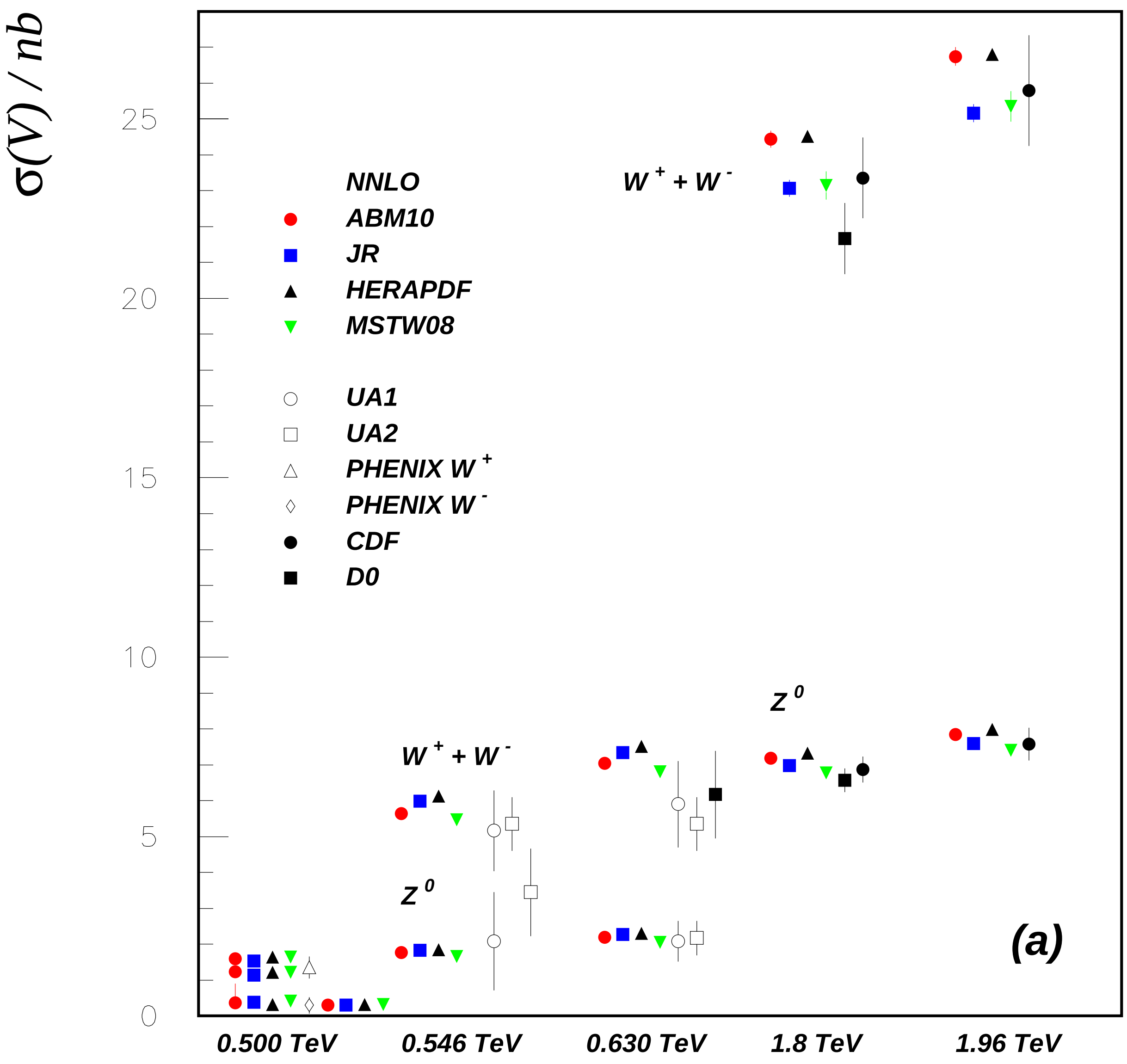
Uncertainty has depended on stopping criteria.



Uncertainties on, e.g. valence quarks not notably different to other groups at all.

Study published by Alekhin *et al*.

Note consistent normalisation difference between CDF and D0.



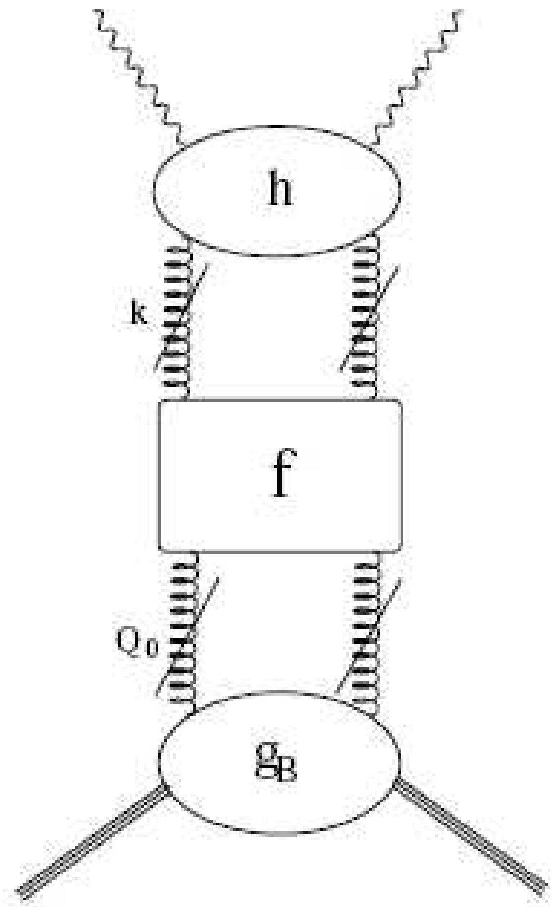
Small-x Theory

Reason for this instability – at each order in α_S each splitting function and coefficient function obtains an extra power of $\ln(1/x)$ (some accidental zeros in P_{gg}), i.e. $P_{ij}(x,\alpha_s(Q^2)), C_i^P(x,\alpha_s(Q^2)) \sim \alpha_s^m(Q^2)\ln^{m-1}(1/x)$.

BFKL equation for high-energy limit

$$f(k^2,x) = f_I(Q_0^2)+\int_x^1 \frac{dx'}{x'}\bar{\alpha}_S\int_0^\infty \frac{dq^2}{q^2}K(q^2,k^2)f(q^2,x),$$

where $f(k^2,x)$ is the unintegrated gluon distribution $g(x,Q^2) = \int_0^{Q^2}(dk^2/k^2)f(x,k^2)$, and $K(q^2,k^2)$ is a calculated kernel known to NLO.



Physical structure functions obtained from

$$\sigma(Q^2,x) = \int(dk^2/k^2)\,h(k^2/Q^2)f(k^2,x)$$

where $h(k^2/Q^2)$ is a calculable impact factor.

The global fits usually assume that this is unimportant in practice, and proceed regardless.

Fits work well at small x, but could improve.

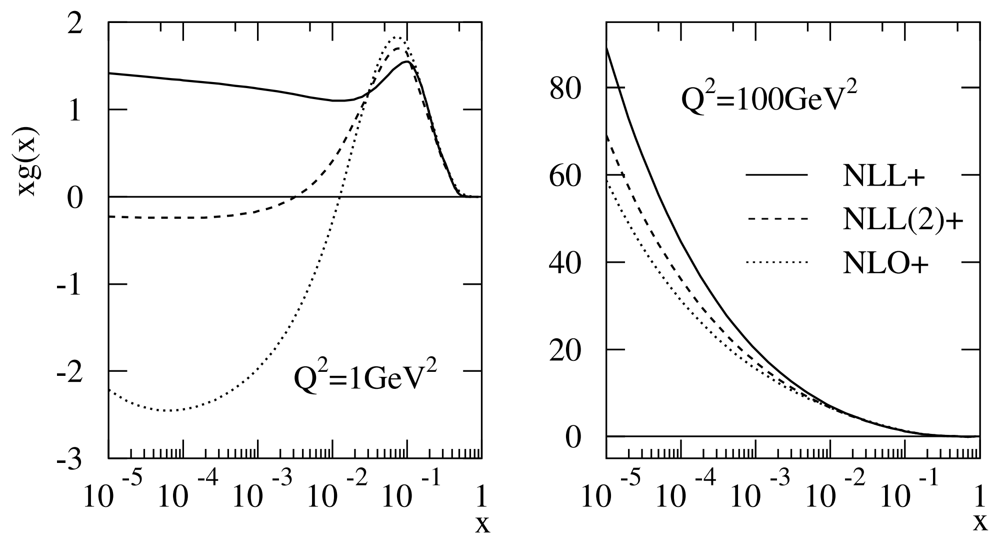
Good recent progress in incorporating $\ln(1/x)$ resummation Altarelli-Ball-Forte, Ciafaloni-Colferai-Salam-Stasto and White-RT.

Include running coupling effects and variety (depending on group) of other corrections

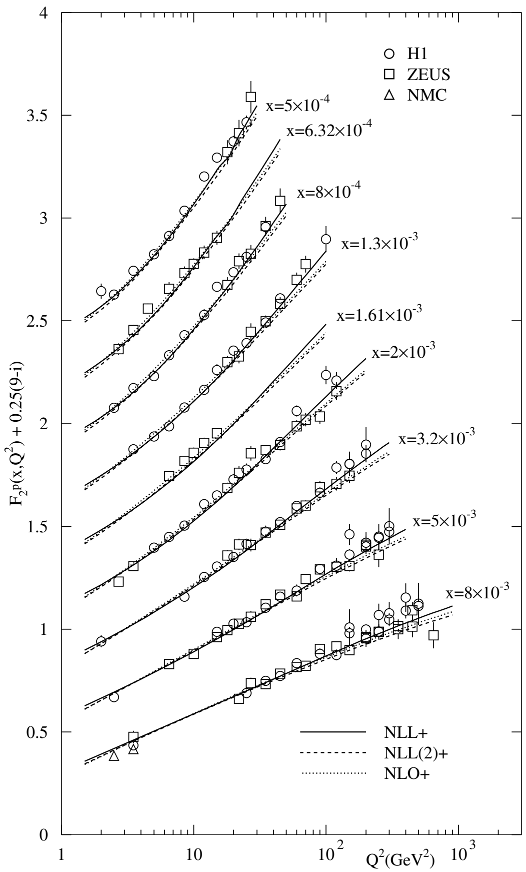
By 2008 very similar results coming from the competing procedures, despite some differences in technique.

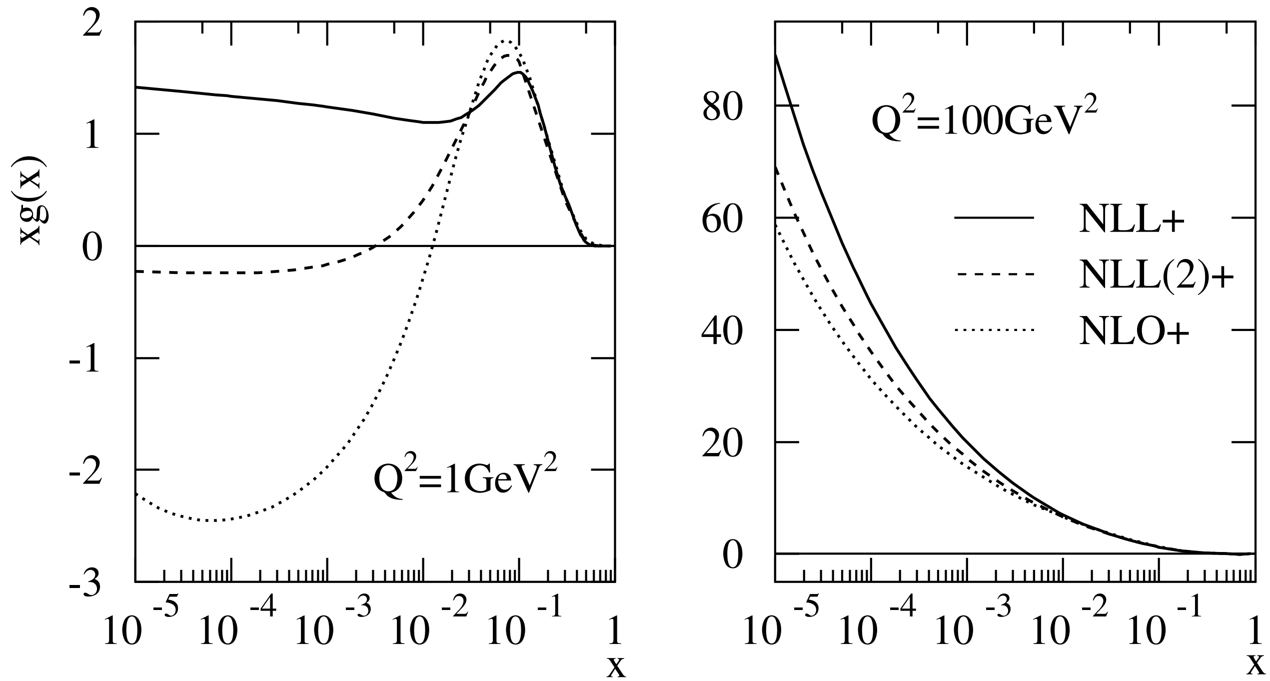
Full set of coefficient functions still to come in some cases, but splitting functions comparable.

Note, in all cases NLO corrections lead to dip in functions below fixed order values until slower growth (running coupling effect) at very small x.



A fit to data with NLO plus NLO resummation, with heavy quarks included (White,RT) performed.





→ moderate improvement in fit to HERA data within global fit, and change in extracted gluon (more like quarks at low Q^2).

Together with indications from Drell Yan resummation calculations (Marzani, Ball) few percent effect quite possible.