Progress in NNLO and multiloop calculations

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Outline

- Introduction
- NNLO calculations
- Multiloop calculations
- Summary and Outlook

Introduction

The dynamics of hard scattering processes is nowadays remarkably well described by perturbative QCD

Until few years ago the standard for QCD theoretical predictions was essentially limited to NLO (plus possibly the all-order resummation of some logarithmically enhanced terms)

LO predictions can give only the order of magnitude for cross sections and distributions:

- the scale of α_S is not defined
- jets ⇔ partons: jet structure starts to appear only beyond LO

NLO is thus the first order where reliable predictions can be obtained

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➡ **NNLO is thus the first order at which a reliable estimate of the error can be given**

Does it mean that NNLO calculations are essential for every process ?

Well, we can say that NNLO predictions are desirable at least in the following cases:

- For those processes whose NLO corrections are comparable to the LO contributions

  ➡ e.g. Higgs production at hadron colliders

- For those benchmark processes measured with high experimental accuracy

  - α_S measurements from e^+e^- event shape variables

  ➡ - W,Z hadroproduction

  - heavy quark hadroproduction

  ....

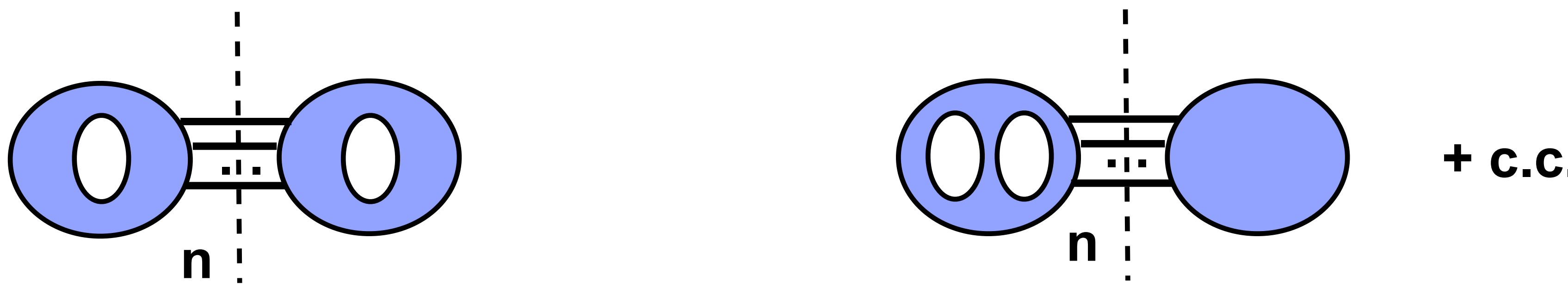
- For some important background processes

  ➡ e.g. vector boson pair production

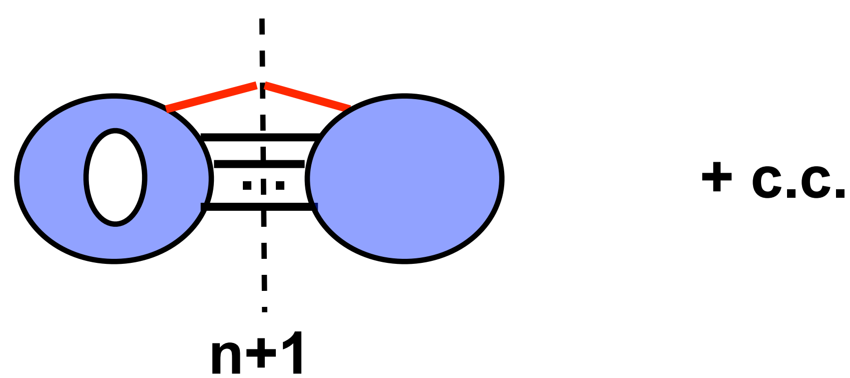
Ingredients of a NNLO calculation

Let us assume that the process involves n partons at LO ➡ we need:

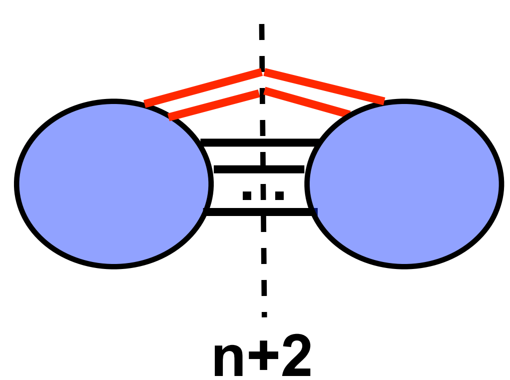
- Double virtual contribution with n resolved partons



- Real-virtual contribution with 1 unresolved parton



- Double-real contribution with 2 unresolved partons



Ingredients of a NNLO calculation

Difficulty: they are affected by different kinds of singularities

- UV sing. affect only virtual corrections ➡ removed by renormalization
- IR singularities present in all the three contributions

IR singularities cancel out in IR safe quantities

IR safe quantities are those that are independent of the presence of arbitrarily soft partons and independent on the individual momenta of a bunch of collinear partons

Unfortunately the pattern of the cancellation of IR singularities is much more involved than at NLO !

(Fully) inclusive processes

In the case of one-scale quantities double real, real virtual and double virtual contributions can be analytically computed and the singularities explicitly cancelled

- DIS structure functions

E. Zijlstra, W. Van Neerven (1992)

- Single hadron production

P.J.Rijken, W.L.Van Neerven (1997)
A.Mitov, S.Moch (2006)

- DY lepton pair production

R.Hamberg, W.Van Neerven, T.Matsuura (1991)

- Higgs boson production

R.Harlander, W.B. Kilgore (2002)
C. Anastasiou, K. Melnikov (2002)
V. Ravindran, J. Smith, W.L.Van Neerven (2003)

....

+

Vector boson rapidity distribution

➡ modelling the phase space constraint with an effective "propagator"

C.Anastasiou, K.Melnikov, L.Dixon,F.Petriello (2003)

But real experiments have finite acceptances !

What about more exclusive processes?

Many of the ingredients for NNLO corrections available since long time

Example: $e^+ e^- \to 3$ jets

- Tree amplitude for $e^+ e^- \to 5$ partons

K. Hagiwara, D. Zeppenfeld (1989)
F.A.Berends, W.Giele, H.Kuijf (1989)

- One-loop amplitude for $e^+ e^- \to 4$ partons

N. Glover, D. Miller (1996)
Z.Bern, L.Dixon, D.Kosower,S.Weinzierl (1996,1997)
J. Campbell, N. Glover, D. Miller (1997)

- Two-loop amplitude for $e^+ e^- \to 3$ partons

L.W. Garland et al. (2002)

Example: Drell-Yan

T.Matsuura, W.Van Neerven (1988)
R.Hamberg, W.Van Neerven, T.Matsuura (1991)

➡ Amplitudes known since more than 15 years !

Despite this fact until recently the computation of the corresponding NNLO corrections could not be performed

The IR singularity structure of the three contributions has now been understood

S. Catani (1998); J.Campbell, N. Glover (1998)
S. Catani, MG (1999); Z.Bern, V. Del Duca, W. Kilgore, C. Schmidt (1999), D. Kosower, P. Uwer (1999), S. Catani, MG (2000)
G.Sterman, M. Tejeda-Yeomans (2002)

However the organization of the calculation into finite pieces that can be integrated numerically is still a formidable task

Two main strategies have been followed:

- **Sector decomposition**
- **Subtraction method**

Sector decomposition

K. Hepp (1966)
T. Binoth, G.Heinrich (2000,2004)
C.Anastasiou, K.Melnikov, F.Petriello (2004)

Sector decomposition as implemented by Anastasiou and collaborators works by dividing the integration region into sectors each containing a single singularity that can be made explicit by expansion into distributions

➡ This leads to a fully automated procedure by which the coefficients of the poles as well as finite terms can be computed numerically

The method has been successfully applied to a number of important fully exclusive NNLO computations

- Higgs and vector boson production in hadron collisions

C.Anastasiou, K.Melnikov, F.Petriello (2004)
K.Melnikov, F.Petriello (2004)

- NNLO QED computation of muon decay

C.Anastasiou, K.Melnikov, F.Petriello (2005)

- Semileptonic decay $b \longrightarrow c\, l\, \bar{\nu}_l$

K.Melnikov (2008)

Now able to compute distributions in a single run

F. Stoeckli (2009)

Example: FEWZ2.0

F.Petriello et al (2010)

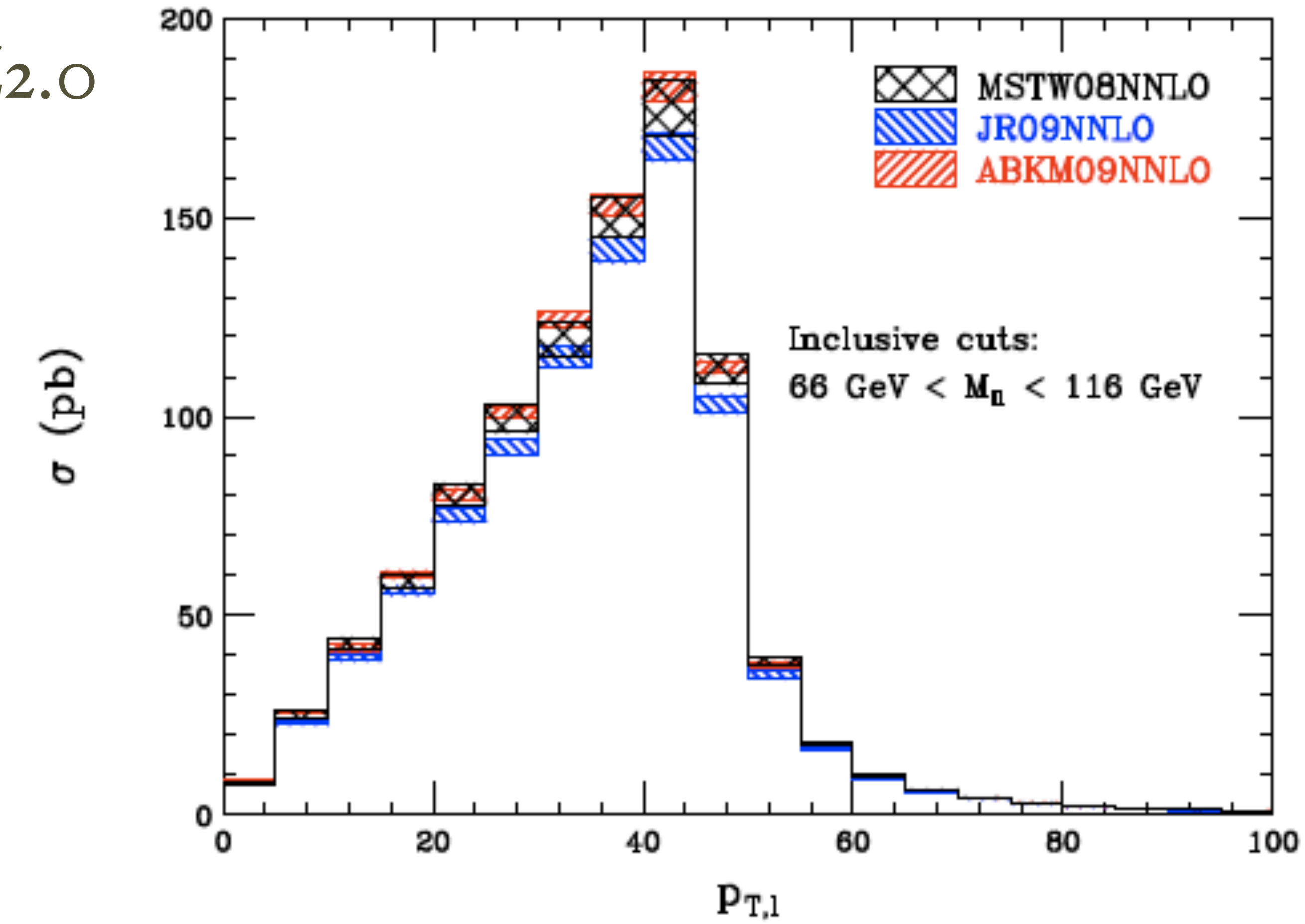


Figure 3: Bin-integrated cross sections for the lepton pseudorapidity (upper panel) and transverse momentum (lower panel) for all three NNLO PDF sets. Only a cut on the invariant mass 66 GeV $\leq M_{ll} \leq$ 116 GeV has been implemented. The bands indicate the PDF uncertainties for each set.

Recently: improvement proposed to reduce the number of integrals based on non-linear transformations

C.Anastasiou, A. Lazopoulos, F. Herzog (2010)

Subtraction method

R.K. Ellis, D.A.Ross, A.E.Terrano (1981)
S.Frixione, Z.Kunszt, A. Signer (1995)
S.Catani, M. Seymour (1996)

$$d\sigma = \int_{n+1} r d\Phi_{n+1} + \int_n v d\Phi_n$$

$$d\sigma = \int_{n+1} \left(r d\Phi_{n+1} - \tilde{r} d\tilde{\Phi}_{n+1} \right) + \int_{n+1} \tilde{r} d\tilde{\Phi}_{n+1} + \int_n v d\Phi_n$$

Add and subtract a local counterterm with the same singularity structure of the real contribution that can be integrated analytically over the phase space of the unresolved parton

How to extend this procedure to NNLO ?

This absolutely non trivial issue has attracted quite an amount of work

Goal ➡ Formulate a general scheme that can be possibly applied to any process

D. Kosower (1998,2003,2005)
S. Weinzierl (2003)
S. Frixione, MG (2004)
A. & T. Gehrmann, N. Glover (2005)
G, Somogyi, Z. Trocsanyi, V. Del Duca (2005, 2007)

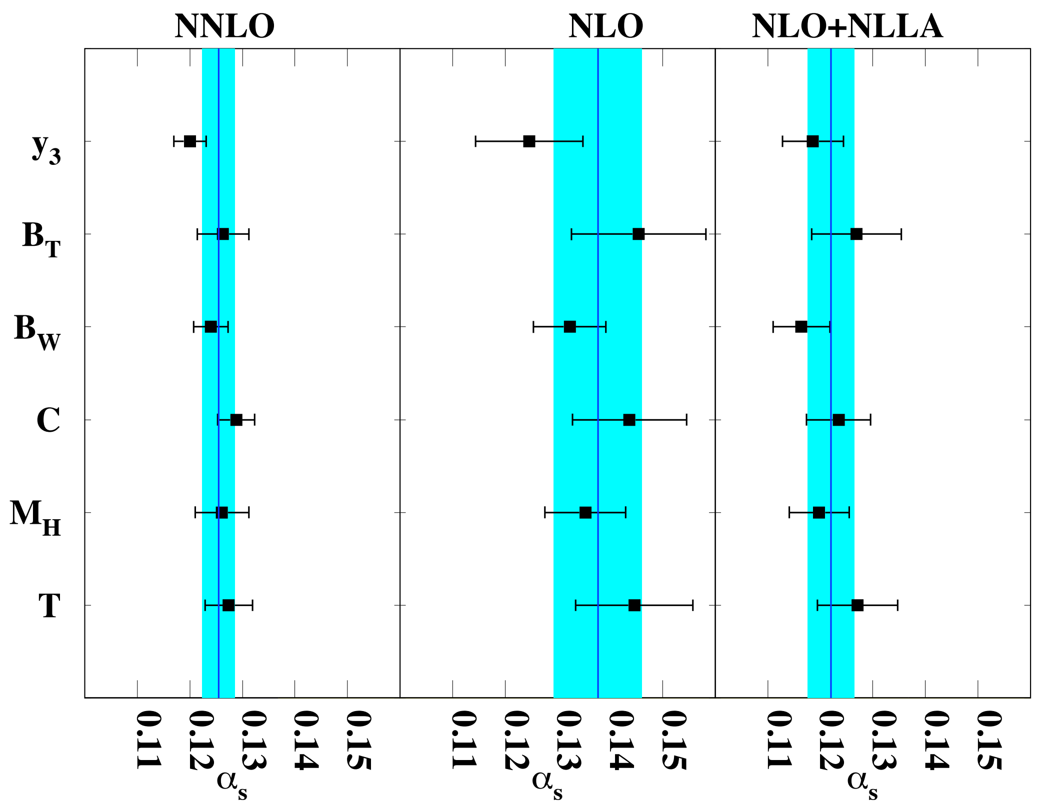
At present the only approach that has been proven to work is the antenna subtraction method by A. & T. Gehrmann and Glover

Counterterms constructed from antennae extracted from physical matrix elements

It led to the completion of the NNLO calculation of $e^+ e^- \to 3$ jets

Impressive achievement of a five years project !

A. & T. Gehrmann, N. Glover, G. Heinrich (2007)



Cross checked with independent implementation of the same method

S.Weinzierl (2008)

Important impact on α_S measurement

Now the method is being applied to hadron collisions

R. Boughezal, A.Gehrmann, M.Ritzmann (2010)
T.Gehrmann et al. (2010)
N.Glover, J.Pires (2010)
A.Gehrmann, G.Abelof (2011)

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A. & T. Gehrmann, N. Glover,
G. Heinrich (2007)

For such a tough calculation a fully independent check would be welcome

The approach by Trocsanyi et al. is based on the subtraction of counterterms constructed by the direct combination of the universal kernels controlling the soft and collinear singularities

- fully local counterterms → better numerical convergence
- analytic integration over unresolved partons much more difficult

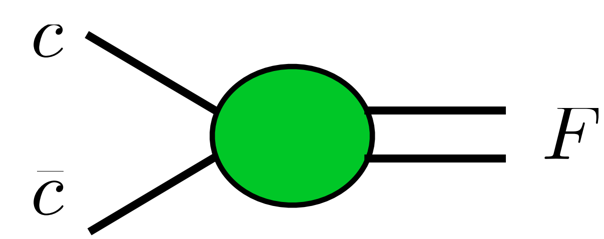
  → no complete result yet !

Hadron collisions: a shortcut

S. Catani, MG (2007)

Let us consider a specific, though important class of processes: the production of colourless high-mass systems F in hadron collisions (F may consist of lepton pairs, vector bosons, Higgs bosons......)

At LO it starts with $c\bar{c} \to F$



Strategy: start from NLO calculation of F+jet(s) and observe that as soon as the transverse momentum of the F $q_T \neq 0$ one can write:

$$d\sigma^F_{(N)NLO}|_{q_T \neq 0} = d\sigma^{F+\text{jets}}_{(N)LO}$$

Define a counterterm to deal with singular behaviour at $q_T \to 0$

But.....
the singular behaviour of $d\sigma^{F+\text{jets}}_{(N)LO}$ is well known from the resummation program of large logarithmic contributions at small transverse momenta

G. Parisi, R. Petronzio (1979)
J. Collins, D.E. Soper, G. Sterman (1985)
S. Catani, D. de Florian, MG (2000)

choose $d\sigma^{CT} \sim d\sigma^{(LO)} \otimes \Sigma^F(q_T/Q)$

where $\Sigma^F(q_T/Q) \sim \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \sum_{k=1}^{2n} \Sigma^{F(n;k)} \frac{Q^2}{q_T^2} \ln^{k-1} \frac{Q^2}{q_T^2}$

Then the calculation can be extended to include the $q_T = 0$ contribution:

$$\boxed{d\sigma^F_{(N)NLO} = \mathcal{H}^F_{(N)NLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+\text{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO}\right]}$$

where I have subtracted the truncation of the counterterm at (N)LO and added a contribution at $q_T = 0$ to restore the correct normalization

The function $\mathcal{H}^F$ can be computed in QCD perturbation theory

$$\mathcal{H}^F = 1 + \left(\frac{\alpha_S}{\pi}\right)\mathcal{H}^{F(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{F(2)} + \ldots\ldots.$$

For a generic $pp \to F + X$ process:

- At NLO we need a LO calculation of $d\sigma^{F+\mathrm{jet(s)}}$ plus the knowledge of $d\sigma^{CT}_{LO}$ and $\mathcal{H}^{F(1)}$
  - the counterterm $d\sigma^{CT}_{LO}$ requires the resummation coefficients $A^{(1)}, B^{(1)}$ and the one loop anomalous dimensions
  - the general form of $\mathcal{H}^{F(1)}$ is known

D. de Florian, MG (2000)
G. Bozzi, S. Catani, D. de Florian, MG (2005)

- At NNLO we need a NLO calculation of $d\sigma^{F+\mathrm{jet(s)}}$ plus the knowledge of $d\sigma^{CT}_{NLO}$ and $\mathcal{H}^{F(2)}$
  - the counterterm $d\sigma^{CT}_{NLO}$ depends also on the resummation coefficients $A^{(2)}, B^{(2)}$ and on the two loop anomalous dimensions
  - we have computed $\mathcal{H}^{F(2)}$ for Higgs and vector boson production !

S. Catani, MG (2007)
S. Catani, L. Cieri, G.Ferrera, D. de Florian, MG (2009)

→ **this is enough to compute NNLO corrections for *any* process in this class provided F+jet is known up NLO and the two loop amplitude for $c\bar{c} \to F$ is known**

HNNLO

http://theory.fi.infn.it/grazzini/codes.html

HNNLO is a numerical program to compute Higgs boson production through gluon fusion in pp or $p\bar{p}$ collisions at LO, NLO, NNLO

- $H \to \gamma\gamma \quad (\texttt{higgsdec} = 1)$
- $H \to WW \to l\nu l\nu \quad (\texttt{higgsdec} = 2)$
- $H \to ZZ \to 4l$
  - $H \to e^+e^-\mu^+\mu^- \quad (\texttt{higgsdec} = 31)$
  - $H \to e^+e^-e^+e^- \quad (\texttt{higgsdec} = 32)$

→ includes appropriate interference contribution

The user can choose the cuts and plot the required distributions by modifying the `cuts.f` and `plotter.f` subroutines

Example: H→WW→lυlυ

MG (2008)

$p_T^{\min} > 25$ GeV $\quad m_{ll} < 35$ GeV $\quad \Delta\phi < 45^o$

35 GeV $< p_T^{\max} < 50$ GeV $\quad |y_l| < 2 \quad p_T^{\text{miss}} > 20$ GeV

Results for

$p_T^{\text{veto}} = 30$ GeV

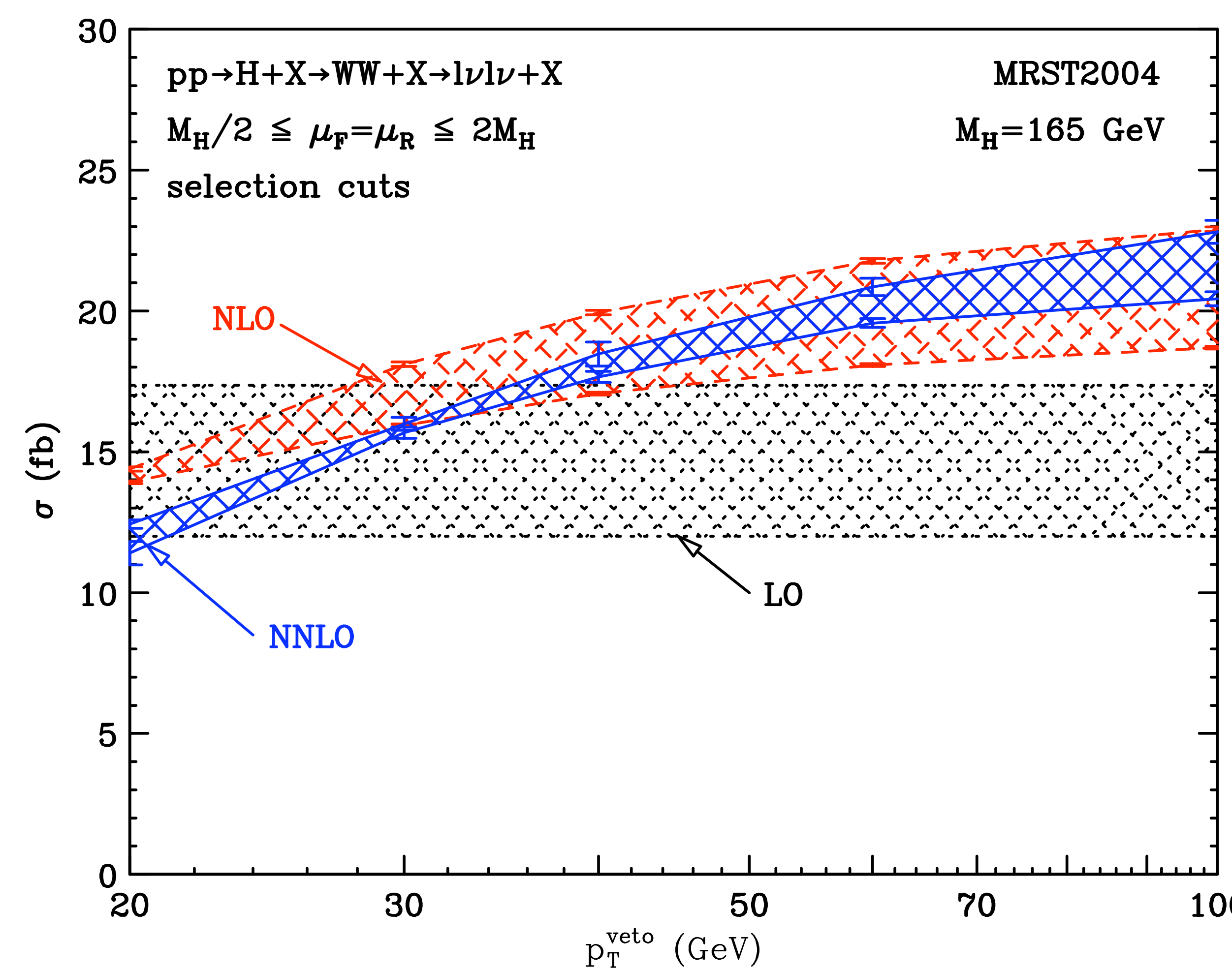
σ (fb)	LO	NLO	NNLO
$\mu_F = \mu_R = M_H/2$	17.36 ± 0.02	18.11 ± 0.08	15.70 ± 0.32
$\mu_F = \mu_R = M_H$	14.39 ± 0.02	17.07 ± 0.06	15.99 ± 0.23
$\mu_F = \mu_R = 2M_H$	12.00 ± 0.02	15.94 ± 0.05	15.68 ± 0.20

Impact of higher order corrections strongly reduced by selection cuts

The NNLO band overlaps with the NLO one for $p_T^{\text{veto}} \gtrsim 30$ GeV

The bands do not overlap for $p_T^{\text{veto}} \lesssim 30$ GeV

NNLO efficiencies found in good agreement with MC@NLO



Anastasiou et al. (2008)

DYNNLO

http://theory.fi.infn.it/grazzini/dy.html

DYNNLO is a parton level MC program to compute vector boson production in pp or ppbar collisions up to NNLO in QCD perturbation theory

- $W^+ \to l^+\nu$ (nproc=1)
- $W^- \to l^-\nu$ (nproc=2)
- $Z \to l^+l^-$ (nproc=3)

The user can choose the cuts and plot the required distributions by modifying the cuts.f **and** plotter.f **subroutines**

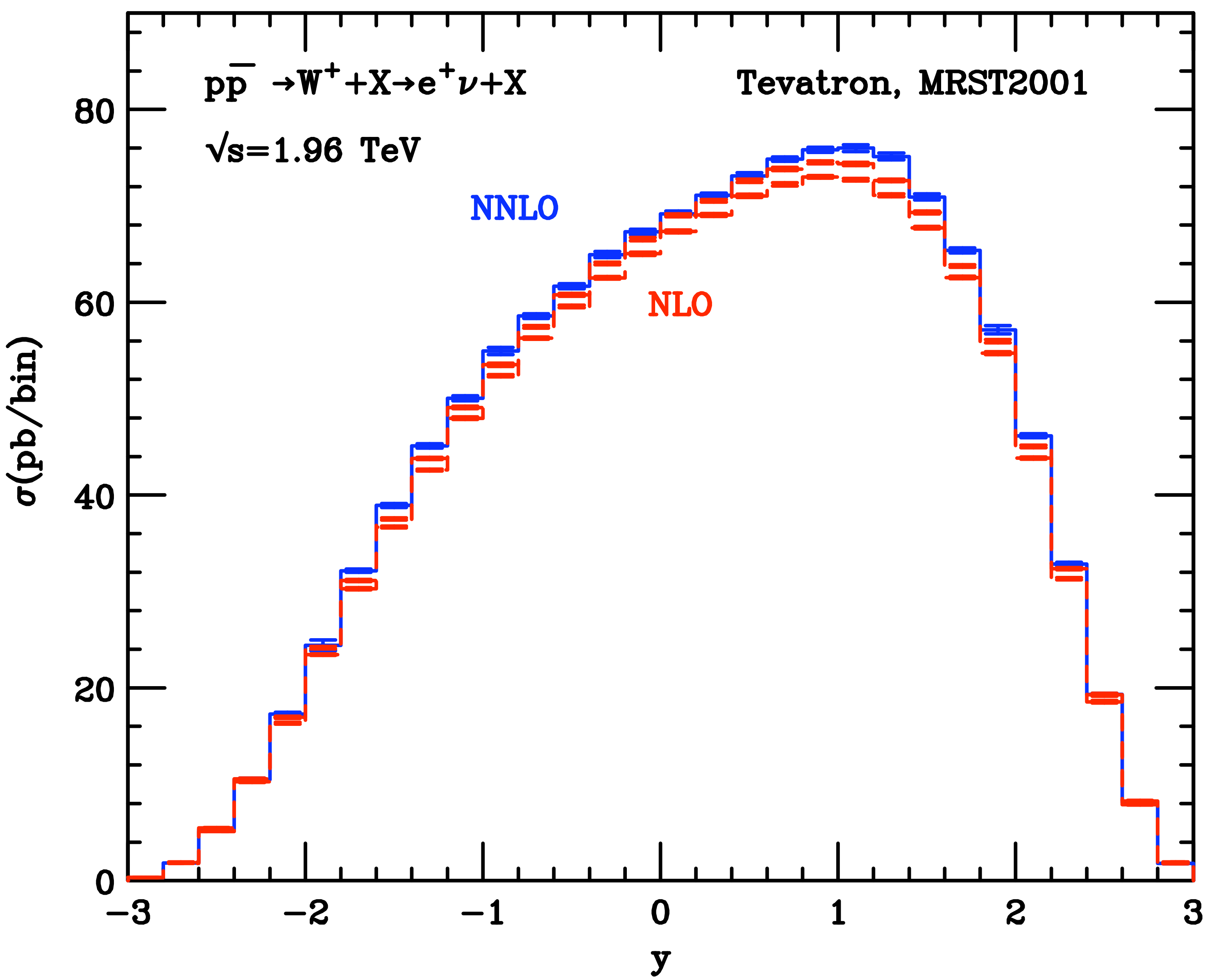
DYNNLO works exactly in the same way as HNNLO for Higgs production

Rapidity distribution of the vector boson

When no cuts are applied our numerical program provides the first independent check of the vector boson rapidity distribution up to NNLO

C.Anastasiou et al. (2003)

Tuned comparison for on shell W production at the Tevatron:



In this plot I compare the NNLO result with the NLO band (obtained by varying $\mu_F=\mu_R$ between 0.5 m_W and $2m_W$) and with the result by Anastasiou et al.

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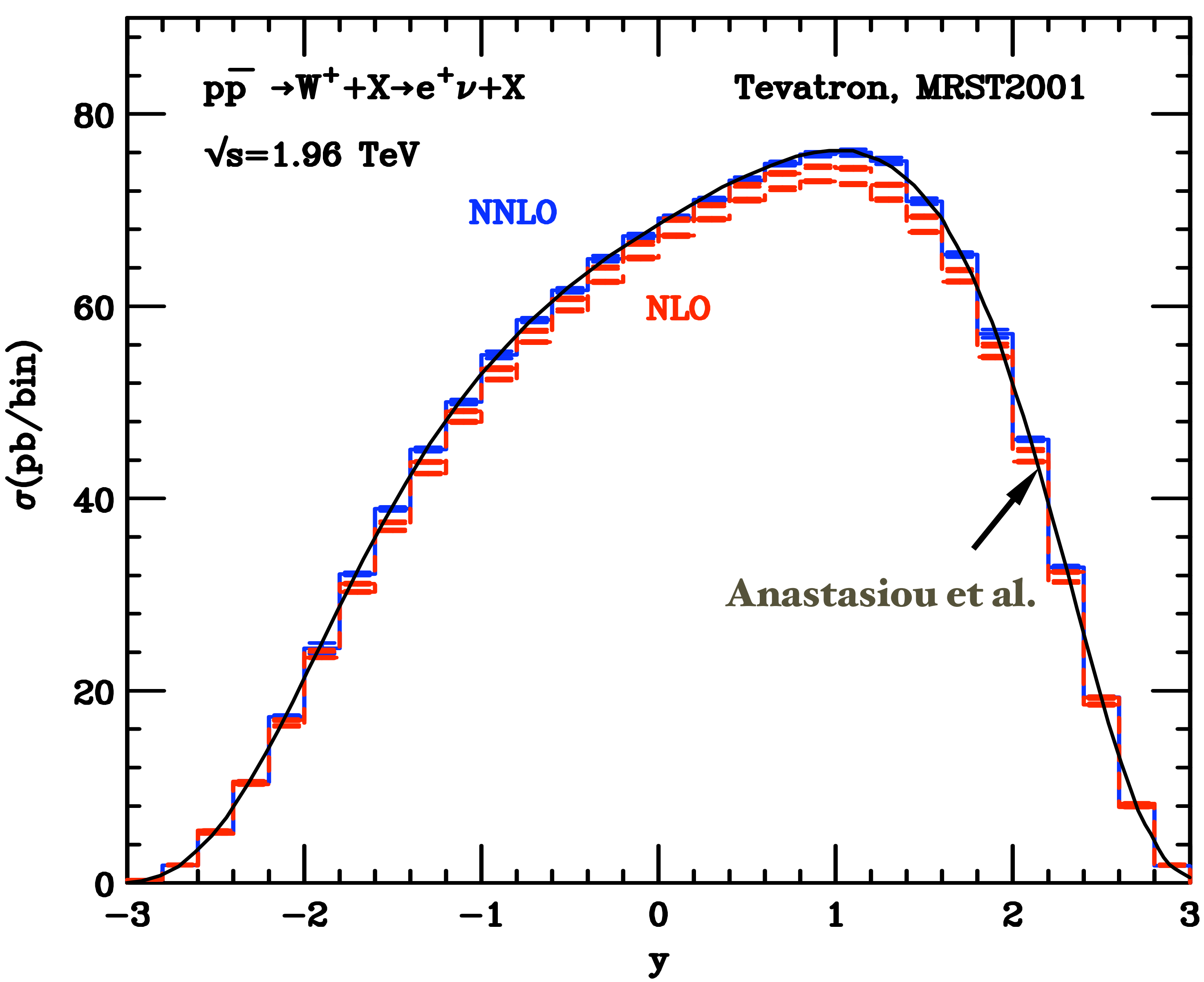

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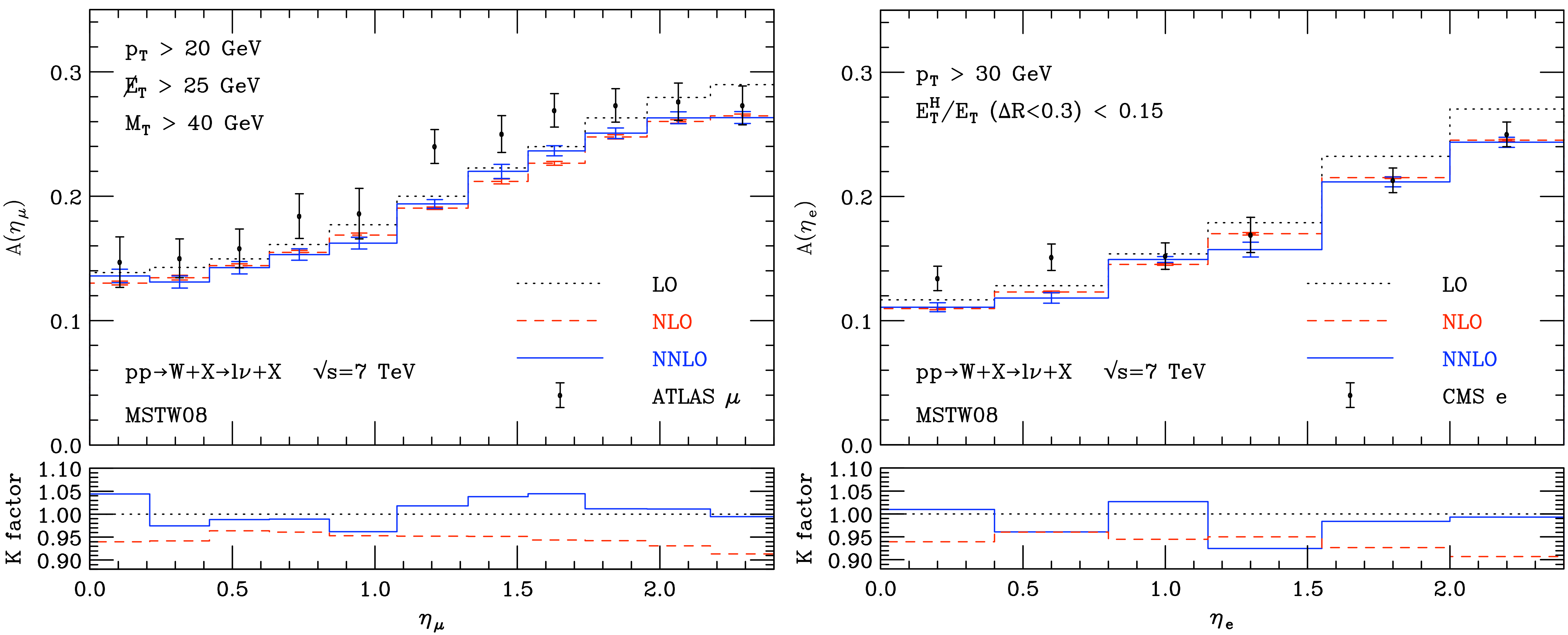
The agreement is good

W charge asymmetry

Comparison of DYNNLO predictions to recent ATLAS and CMS data



G.Ferrera

NNLO corrections extremely small

NEW: $pp \to \gamma\gamma$ at NNLO

S. Catani, L. Cieri, D. de Florian,
G.Ferrera, MG (in progress)

Two loop amplitude available

C.Anastasiou, E.W.N.Glover,
M.E.Tejeda-Yeomans (2002)

$\gamma\gamma$ +jet at NLO available

Z.Nagy et al. (2003)

→ We can perform the NNLO calculation using hard coefficients obtained for Drell-Yan

Use Frixione isolation → no fragmentation contribution

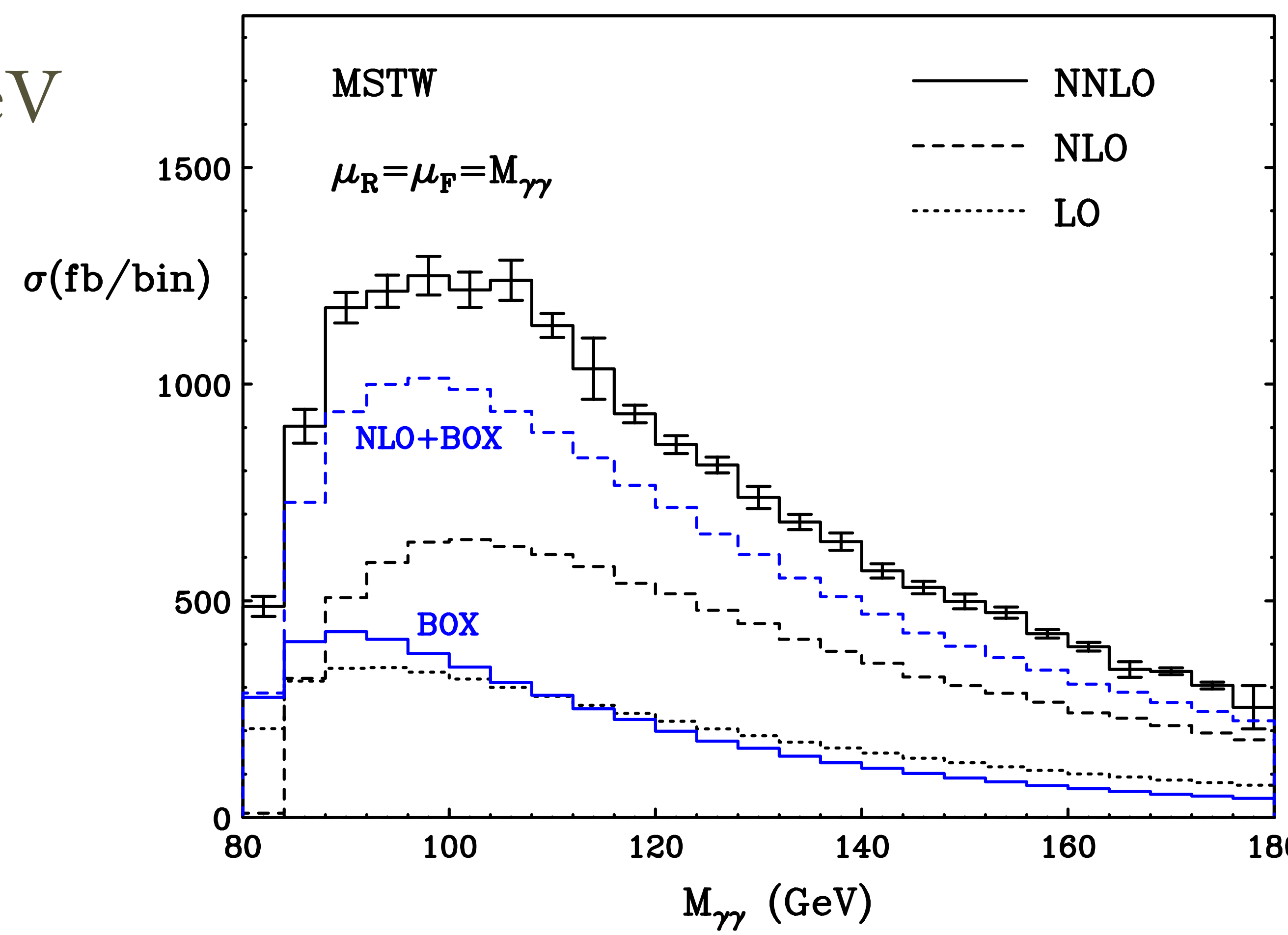
PRELIMINARY RESULTS LHC, $\sqrt{s}$=14 TeV

$p^{\gamma}_T \geq 40$ GeV $|\eta^{\gamma}| \leq 2.5$

60 GeV $\leq M_{\gamma\gamma} \leq$ 180 GeV

$$E_T^{had}(\delta) \leq \chi(\delta)$$

$$\chi(\delta) = \epsilon_\gamma E_T^\gamma \left(\frac{1-\cos(\delta)}{1-\cos(R_0)}\right)^n \qquad \begin{array}{l} n=1 \\ \epsilon_\gamma = 0.5 \\ R_0 = 0.4 \end{array}$$



Recent developments: ttbar

Why not to use subtraction and sector decompositon together ?

M.Czakon (2010,2011)

Consider the double real contribution to ttbar production and separate the singularities according to a FKS like approach

Make singular contributions explicit as usual

$$\int_0^1 \frac{d\lambda}{\lambda^{1-b\epsilon}} f(\lambda) \longrightarrow \int_0^1 d\lambda \left[\frac{f(0)}{b\epsilon} + \frac{f(\lambda)-f(0)}{\lambda^{1-b\epsilon}}\right]$$

Now use known universal structure of the singular behavior of the amplitude

S.Catani, MG (2000)

→ Double real contribution to NNLO ttbar total cross section obtained for the main partonic channels

Multiloop calculations

Multiloop results may be useful for:

- observables that are very precisely measured (e.g. sum rules, R...)
- understand infrared structure at higher orders and resummation at higher logarithmic accuracy

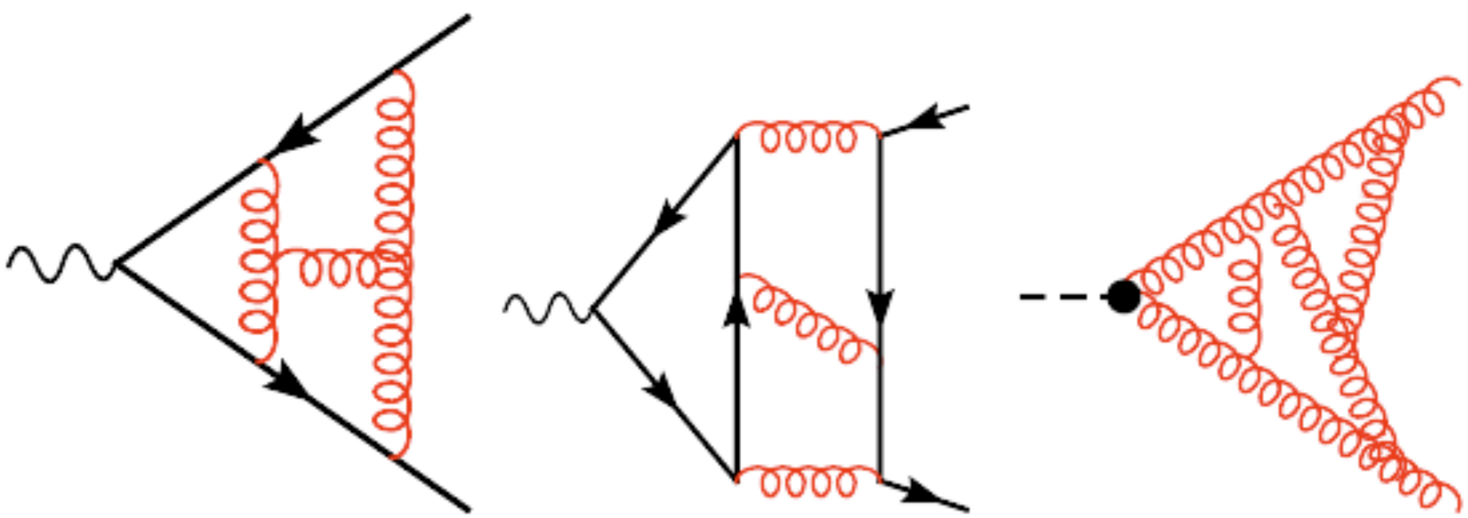
Current frontier:

- One-loop: $2 \to n$ ➡ talk by Maitre
- Two-loop: $2 \to 2$, limit being the different scales in loop integral
- Three-loop: $1 \to 2$, limit being integrals
- Four-loop: $1 \to 1$, limit being integrals

Multiloop calculations: recent results

Three-loop form factors computed both for quark and gluon

P. Baikov, K. Chetyrkin, A.V. Smirnov, V.A. Smirnov, M. Steinhauser (2009)
E.W.N. Glover, T. Huber, N. Ikizlerli, C. Studerus, T.Gehrmann (2010)



Relevant for DIS, Drell-Yan and Higgs production at hadron colliders at N^3LO

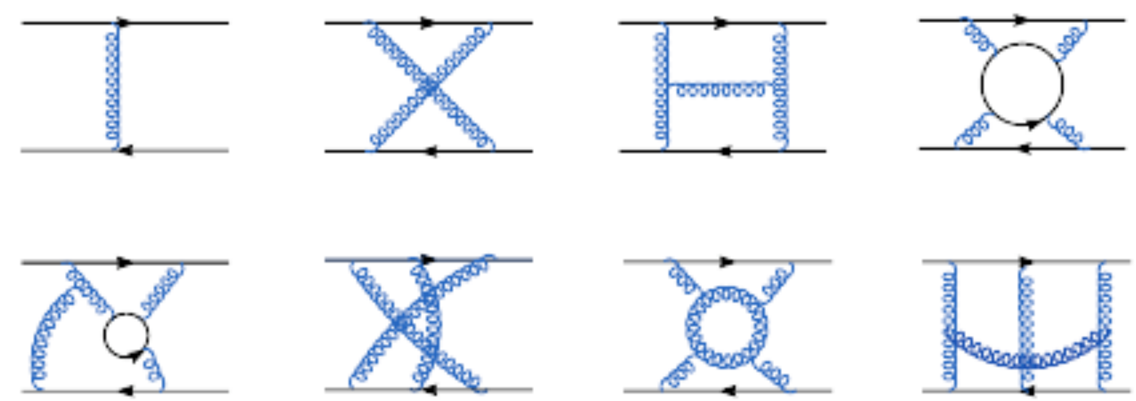
Can be used to extract higher order resummation coefficients

Three-loop static quark potential

M.Steinhauser, A.Smirnov, V.Smirnov (2010)

Potential between two heavy quarks
important quantity in QCD

- It enters top-quark production near threshold
- b and c bound states



Four-loop two point functions:

Recent results:

- Hadronic Z and τ decays to $O(\alpha_S{}^4)$
- polarized Bjorken sum rule

P. Baikov, K.Chetyrkin, J.Kuhn (2010)

$$
\begin{aligned}
C^{Bjp} &= 1-a_s+(-4.583+0.3333\,n_f)\,a_s^2 \\
&+ a_s^3\,(-41.44+7.607\,n_f-0.1775\,n_f^2)\,a_s^3 \\
&+ (-479.4+123.4\,n_f-7.697\,n_f^2+0.1037\,n_f^3)\;a_s^4
\end{aligned}
$$

Summary & Outlook

- Fully exclusive NNLO calculations are important in many cases

  - they provide a precise estimate of higher order corrections when cuts are applied

  - the corresponding acceptances can be compared with those obtained with standard MC event generators

- After some years of work the first fully exclusive NNLO computations have appeared, most notably

  - Higgs and vector boson production in hadron collisions

  - $e^+ e^- \to 3$ jets

- A powerful method, based on sector decomposition, complements the more traditional approach of the subtraction method

- What are the next NNLO calculations that could be performed ?
  - Vector boson pair production

    → important background for Higgs boson searches

    - $\gamma\gamma$ — D. de Florian et al. (2011)
    - WW
      - WW+jet at NLO done — J. Campbell et al. (2007), S.Dittmaier et al. (2008)
      - two loop correction known for $m^2_W << s,t,u$ — M. Chachamis et al. (2007)
  - Heavy quark production
    - ttbar+jet at NLO done — S. Dittmaier et al. (2007)
    - one loop squared known — M.Rogal et al. (2008), C.Anastasiou, Aybat (2008)
    - two loop amplitude in progress:
      - qqbar amplitude known numerically and analytically for fermionic and planar contributions — M. Czakon (2008)
      - gg amplitude known analytically for leading color contribution — R. Bonciani et al. (2010)
  - Jets in hadron collisions ?

- Multiloop calculations:

  - Three-loop quark and gluon form factors

  - Three-loop static heavy quark potential

  - Four loop hadronic Z and τ decays

  - Four loop correction to the Bjorken sum rule