Recent Progress in multi-leg calculations

Daniel Maître

CERN/IPPP

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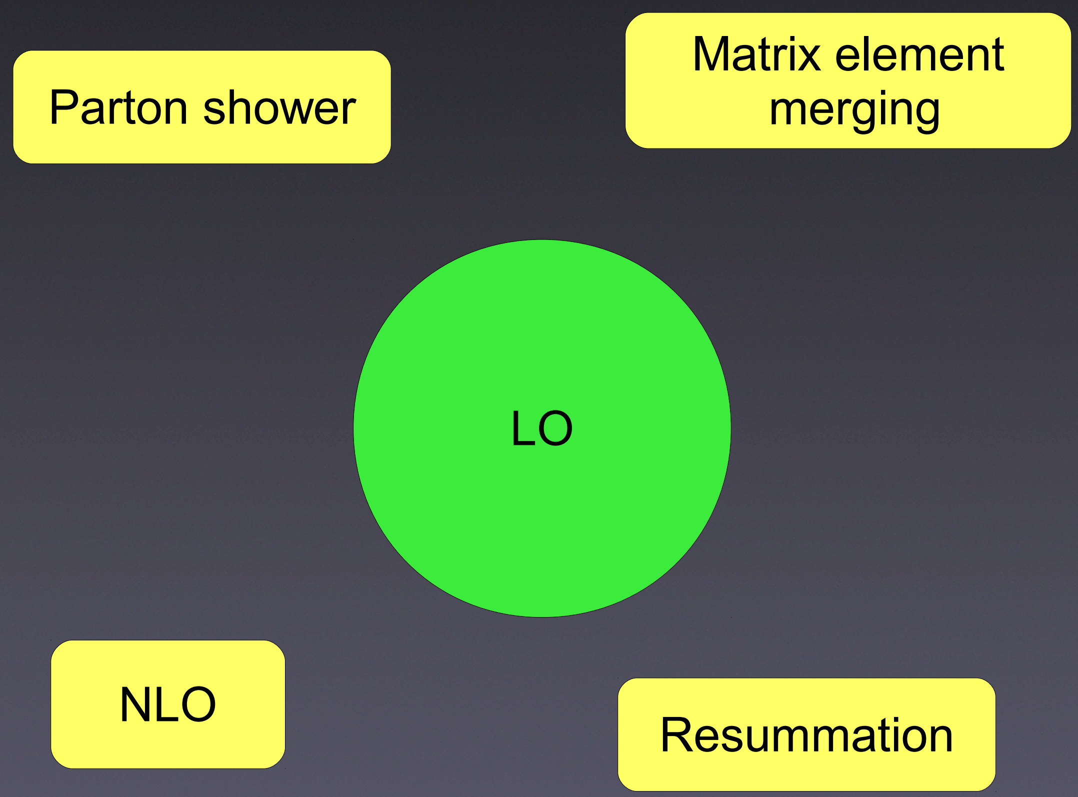
NLO Progress

- Concentrate on QCD
- Concentrate on fixed order
- Concentrate on (just one N)-LO

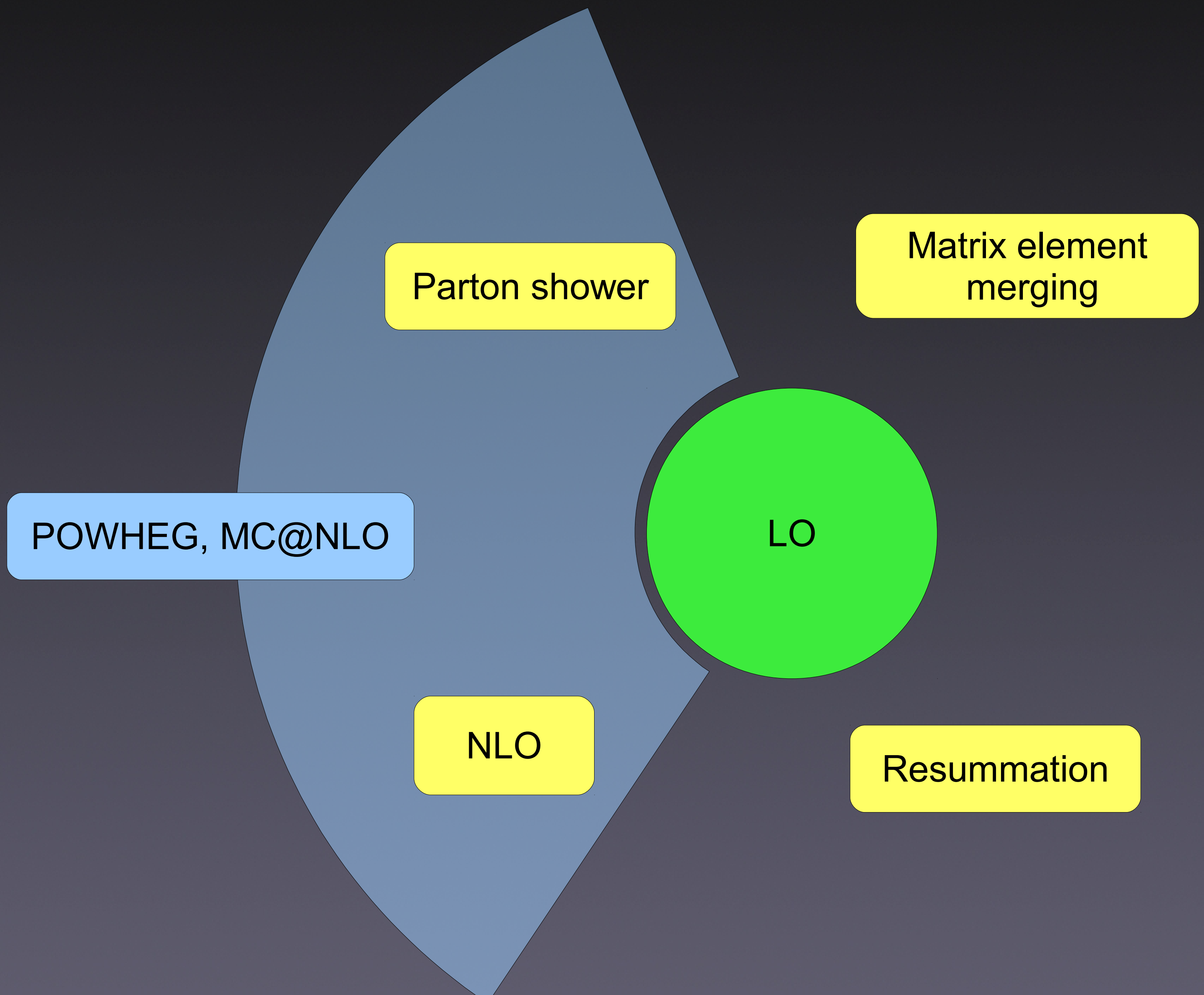
Outline

- Progress in high multiplicity NLO calculations
- Challenges
    - Conceptual
    - Practical
- Quick aside: W polarisation

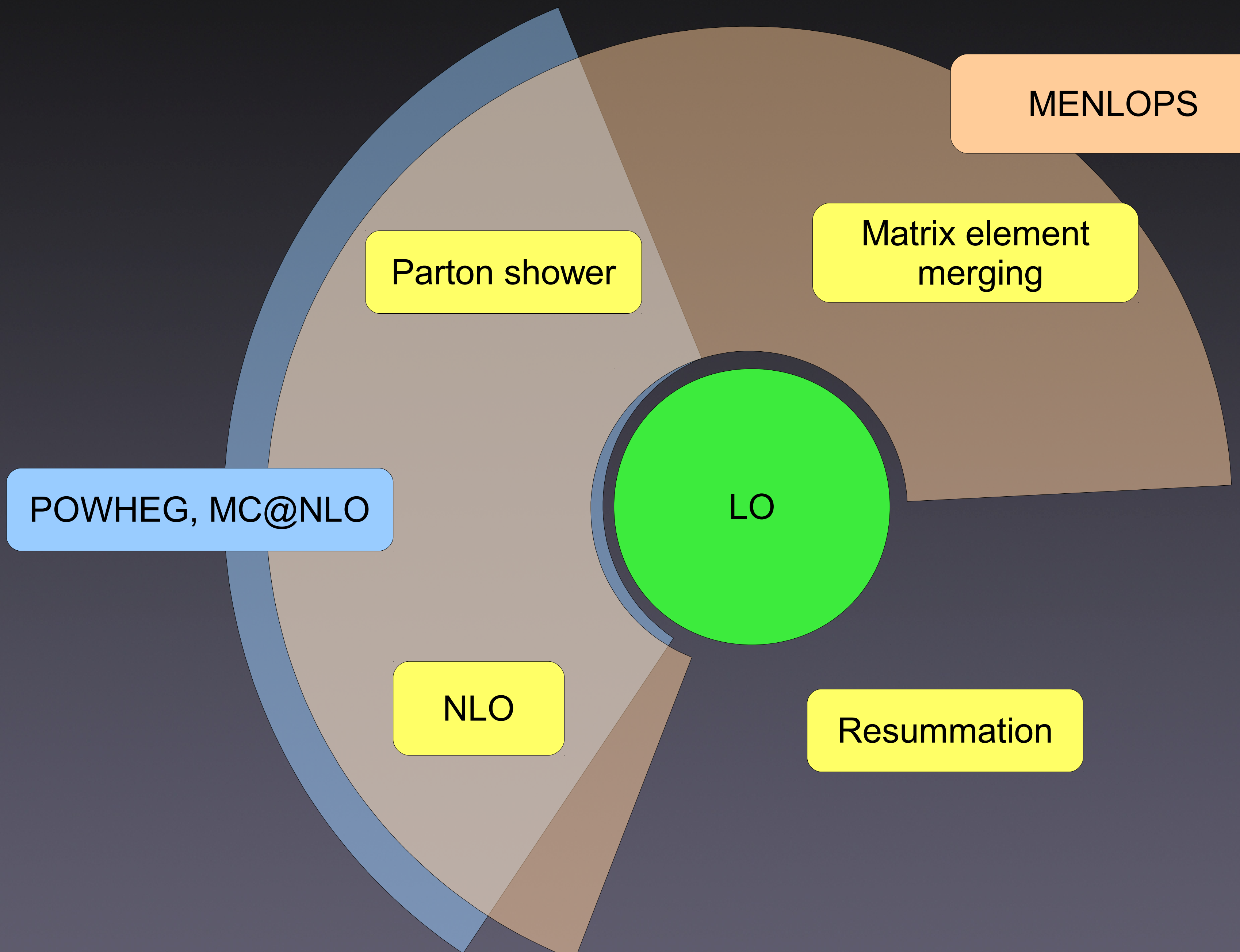
Beyond LO



Beyond LO


Parton shower
Matrix element merging
POWHEG, MC@NLO
LO
NLO
Resummation

Beyond LO


MENLOPS
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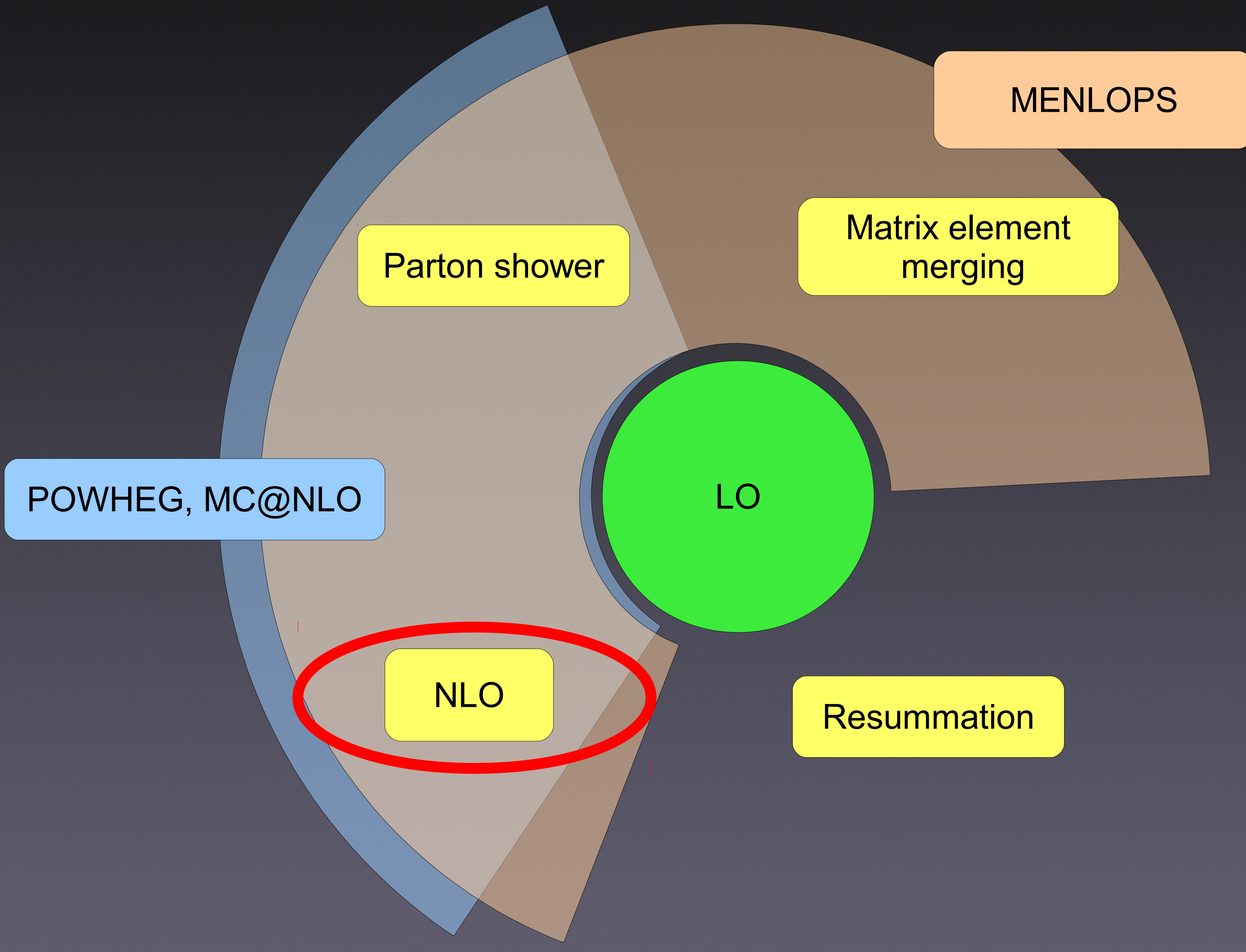
Beyond LO



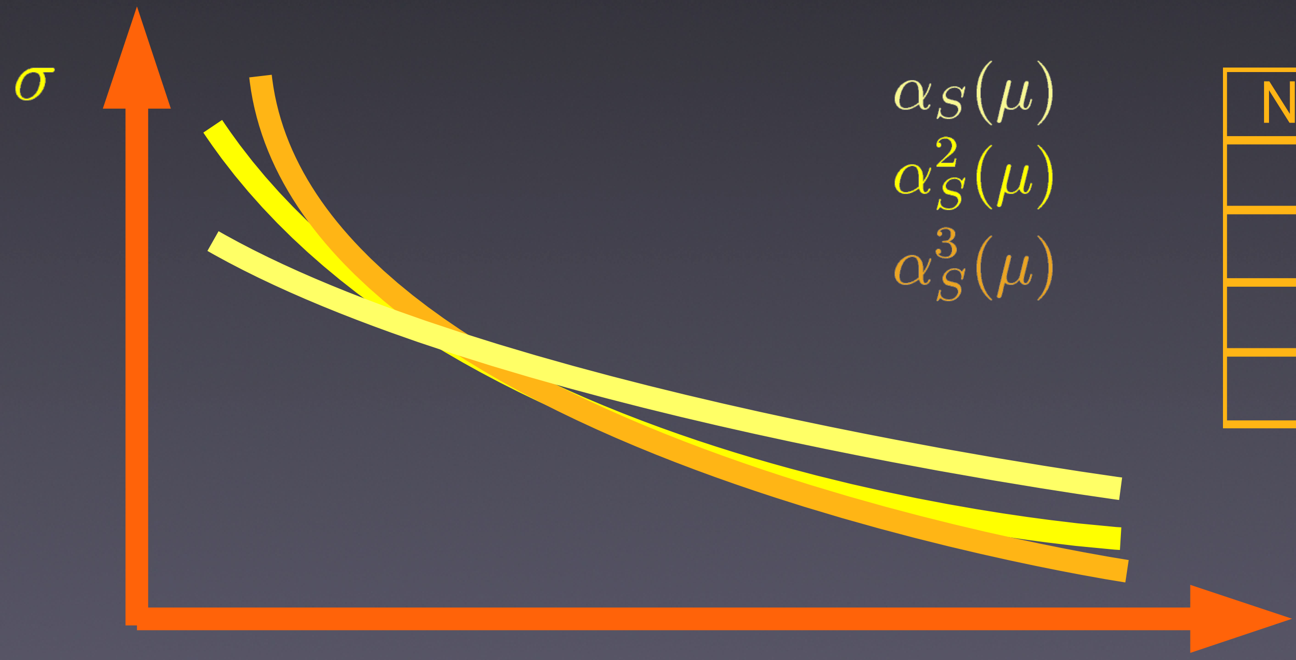
MENLOPS
Parton shower
Matrix element merging
POWHEG, MC@NLO
LO
NLO
Resummation

NLO Calculations

- Increased precision compared to LO
  - Absolute normalisation
  - Better shape of observables
    → more confidence in extrapolating background to signal region
  - Reduced factorisation and renormalisation scale dependence

Renormalization scale dependence

- Scale dependence increases with number of jets



Number of jets	LO
1	9%
2	28%
3	47%
4	64%

[from table I in arXiv:1009.2338]

Renormalization scale dependence

- Scale dependence increases with number of jets

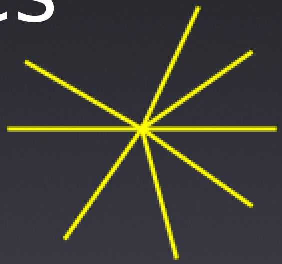
Number of jets	LO	NLO
1	9%	4.5%
2	28%	5.2%
3	47%	7.8%
4	64%	8.4%

[from table I in arXiv:1009.2338]

Theory prediction

- Generate a phase-space configuration with n final state particles

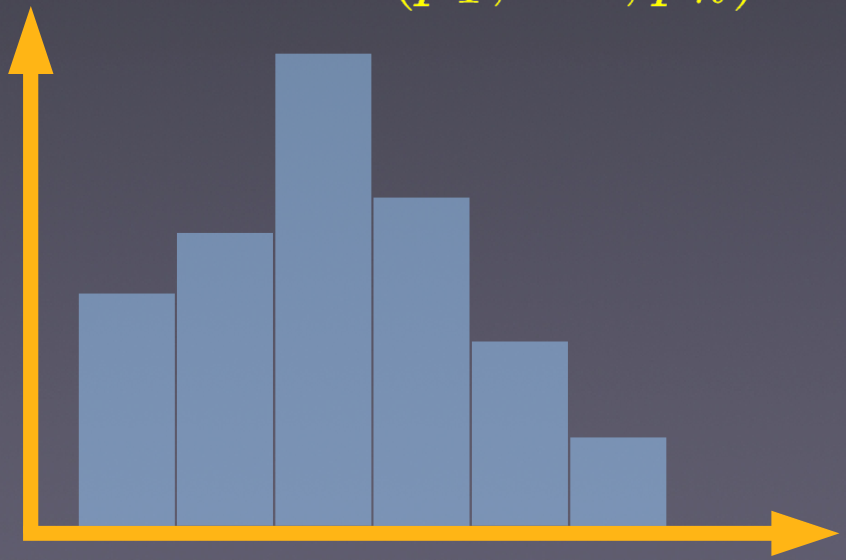
$$p_1, \ldots, p_n$$



- Compute value of the observable and weight

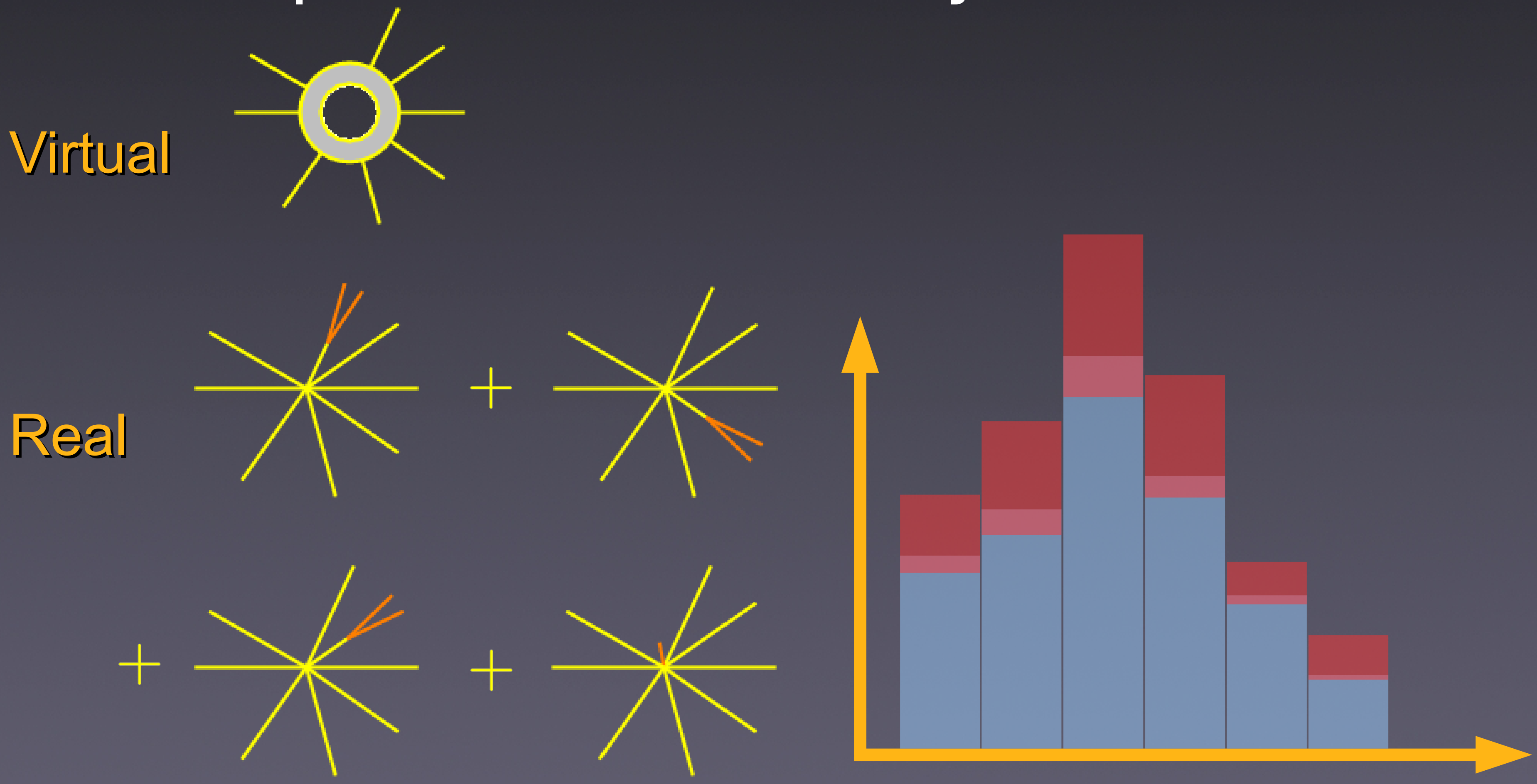
$$O(p_1, \ldots, p_n) \qquad W(p_1, \ldots, p_n)$$

- Bin



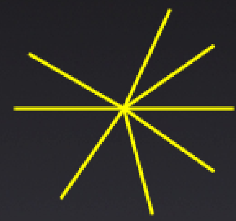
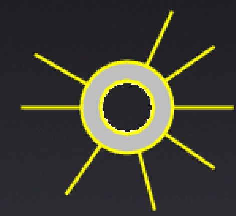
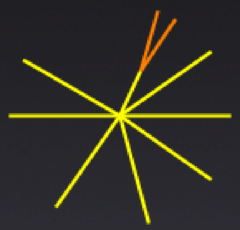
NLO Corrections

Consider (infrared safe) observable and add contributions that have an higher order in perturbation theory



NLO Corrections

NLO Cross section:

$\sigma_n^{NLO} = \int_n$  $+ \int_n$  $+ \int_{n+1}$ 

- Real & virtual corrections have infrared divergences
  - Virtual part has explicit divergences
  - Integral of the real part is divergent when particles become soft or collinear
- Combination is free of infrared divergences
- The cancellation is between objects living in two different phase spaces

NLO Subtraction

Introduce subtraction term σ_{n+1}^{sub}

- Same soft/collinear singularity structure as n+1 MEs

$$\int_{n+1} \left(\sigma_{n+1}^{real} - \sigma_{n+1}^{sub}\right)$$ is finite

- Easy enough to be integrated over the singular PS

$$\int_{n+1} \sigma_{n+1}^{sub} = \int_{n} \int_{1} \sigma_{n+1}^{sub} = \int_{n} \Sigma_{n}^{sub}$$

Numerical NLO Cross Section

Numerical NLO cross section

$$\sigma_n^{NLO} = \int_n \sigma_n^{tree} + \int_n \left(\sigma_n^{virt} + \Sigma_n^{sub}\right) + \int_{n+1} \left(\sigma_{n+1}^{real} - \sigma_{n+1}^{sub}\right)$$

Tree-like   Loop

Sometimes done by different programs
→ Binoth Les Houches Accord

Same ingredients enter the POWHEG formula

NLO wish list

Process	Completed
$pp \to VV + \text{jet}$	
$pp \to H + 2\text{jets}$	
$pp \to t\bar{t}b\bar{b}$	
$pp \to t\bar{t} + 2\text{ jets}$	
$pp \to VVb\bar{b}$	
$pp \to VV + 2\text{ jets}$	
$pp \to V + 3\text{ jets}$	
$pp \to VVV$	
$pp \to b\bar{b}b\bar{b}$	

NLO wish list (5 years ago)

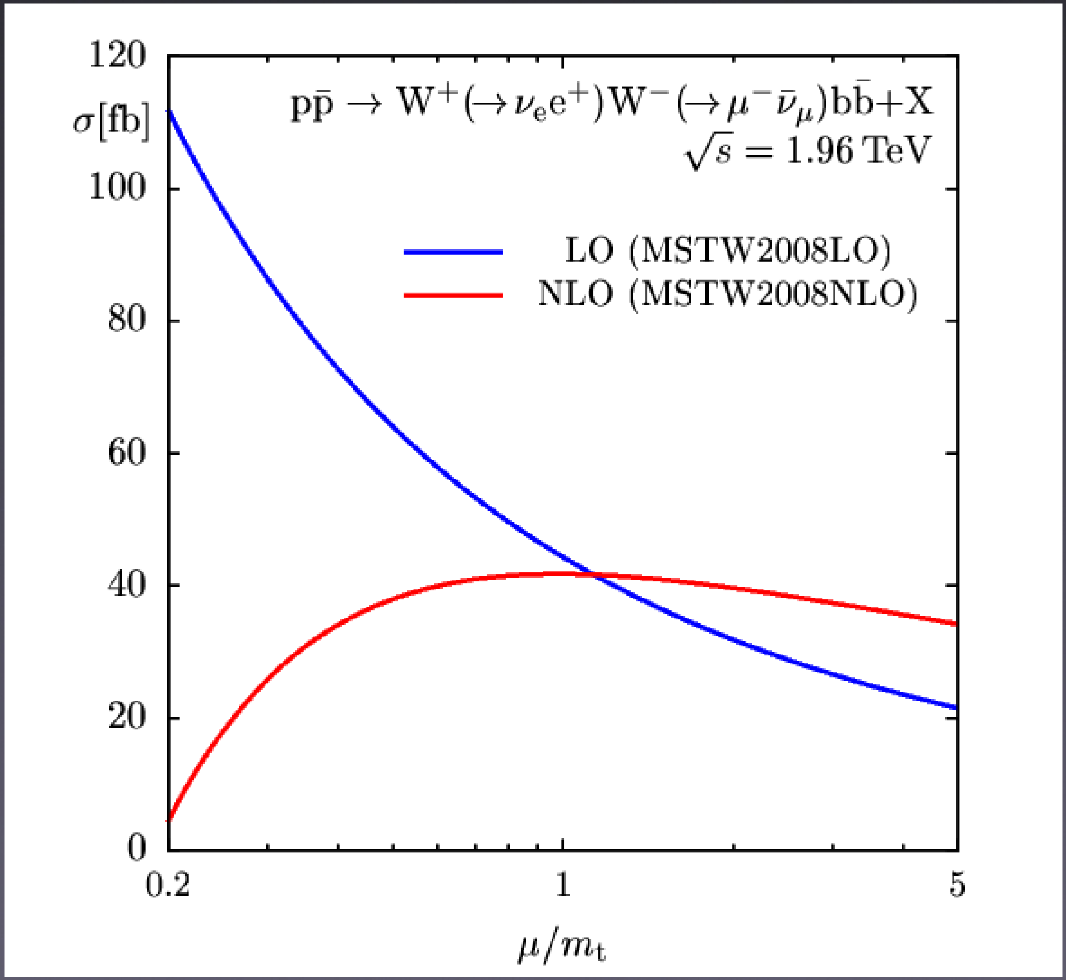
Process	Completed
$pp \to VV + \text{jet}$	
$pp \to H + 2\text{jets}$	Campbell, Ellis, Zanderighi (2006);
$pp \to t\bar{t}b\bar{b}$	
$pp \to t\bar{t} + 2 \text{ jets}$	
$pp \to VVb\bar{b}$	
$pp \to VV + 2 \text{ jets}$	
$pp \to V + 3 \text{ jets}$	
$pp \to VVV$	
$pp \to b\bar{b}b\bar{b}$	

NLO wish list now

Process	Completed
$pp \to VV + \text{jet}$	Dittmaier, Kallweit, Uwer(2008); Campbell, Ellis, Zanderighi (2007)
$pp \to H + 2\text{jets}$	Campbell, Ellis, Zanderighi (2006);Ciccolini, Denner, Dittmaier (2007)
$pp \to t\bar{t}b\bar{b}$	Bredenstein, Denner, Dittmaier, Pozzorini (2009) Bevilacqua, Czakon, Papadopoulos, Pittau, Worek (2009)
$pp \to t\bar{t} + 2\text{ jets}$	Bevilacqua, Czakon, Papadopoulos, Worek (2010)
$pp \to VVb\bar{b}$	$W^+W^-b\bar{b}$ Denner, Dittmaier, Kallweit, Pozzorini (2010)
$pp \to VV + 2\text{ jets}$	VBF: Bozzi, Jäger, Oleari, Zeppenfeld (2007) W^+W^+jj Melia, Melnikov, Röntsch, Zanderighi (2010)
$pp \to V + 3\text{ jets}$	Berger, Bern, Dixon, Febres Cordero, Gleisberg, Ita, Kosower, Maître (2009) Ellis, Melnikov, Zanderighi (2009)
$pp \to VVV$	Lazopoulos, Melnikov, Petriello (2007) Hankele, Zeppenfeld (2007) Binoth, Ossola, Papadopoulos, Pittau (2008)
$pp \to b\bar{b}b\bar{b}$	qq channel Binoth, Greiner, Guffanti, Reuter, Guillet, Reiter (2009)

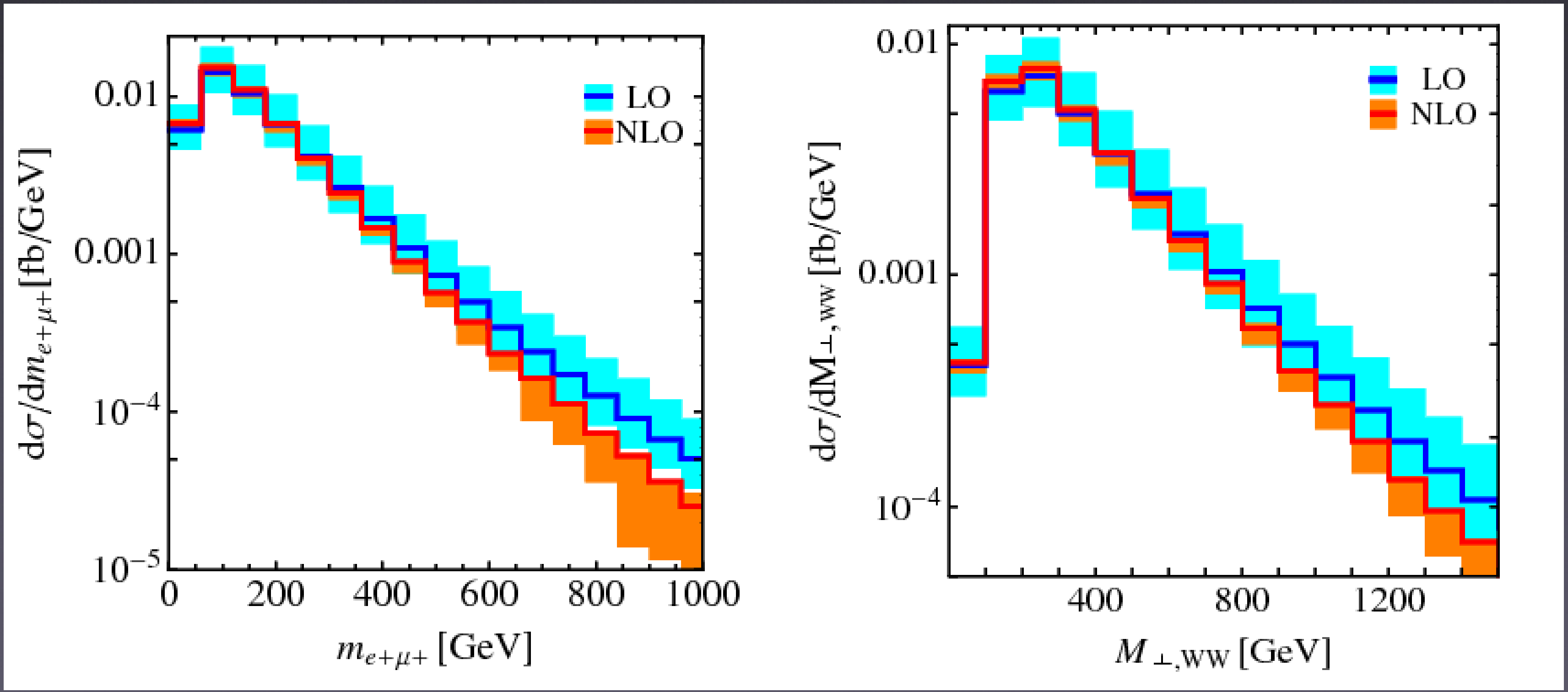
$W^+W^-b\bar{b}$

- Denner, Dittmaier, Kallweit, Pozzorini [ArXiv:1012.3975]



W+W- jj

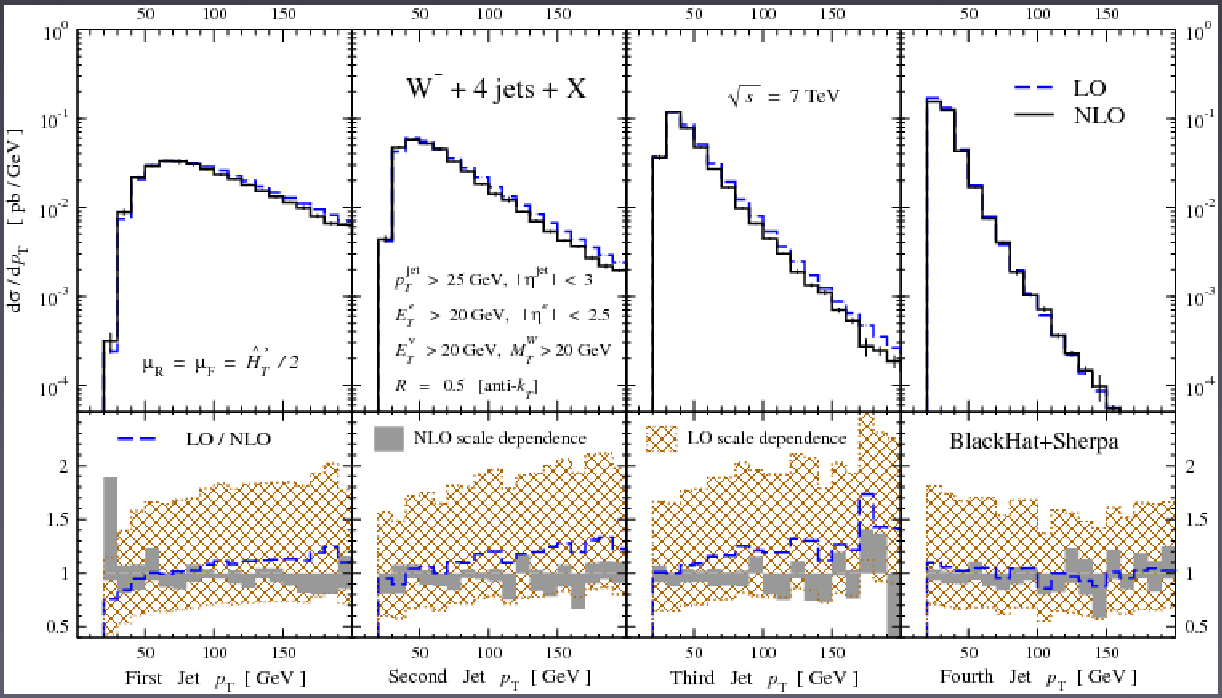
Melia, Melnikov, Röntsch, Zanderighi
[ArXiv:1007.5313]



$\mu = 150\,\mathrm{GeV}$ Scale variation $50\,\mathrm{GeV} < \mu < 400\,\mathrm{GeV}$

2 → 5 processes

- e+e → 5 jets
  Frederix, Frixione, Melnikov, Zanderighi
  [ArXiv:1008.5313]
- W+4 jets
  [Berger,Bern,Dixon,Febres Cordero,Forde,Gleisberg,Ita,Kosower,DM]
  [ArXiv:1009.2338]



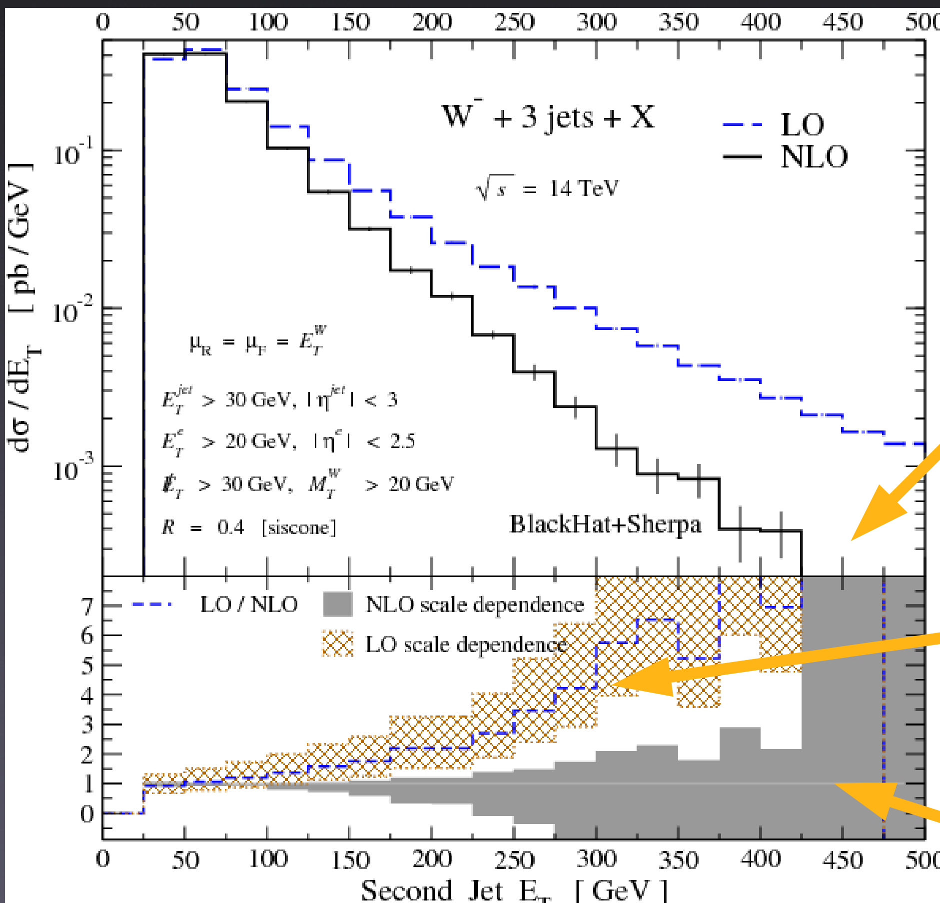
Challenges

- Conceptual challenges
  - Uncertainties estimation
    - Scale variation
    - PDF dependence
- Technical challenges
  - NLO computations are CPU expensive
- Technical and conceptual challenge
  - Merge NLO samples with different multiplicities

Scale choice

- Theory predictions depend on two unphysical scales
    - Renormalisation scale
    - Factorisation scale
- Due to the truncation of the perturbation series
- Want to choose a scale “typical” for the process
- Complicated processes have many scales

Scale choice



$$E_T^W = \sqrt{m_W^2 + p_T^2(W)}$$

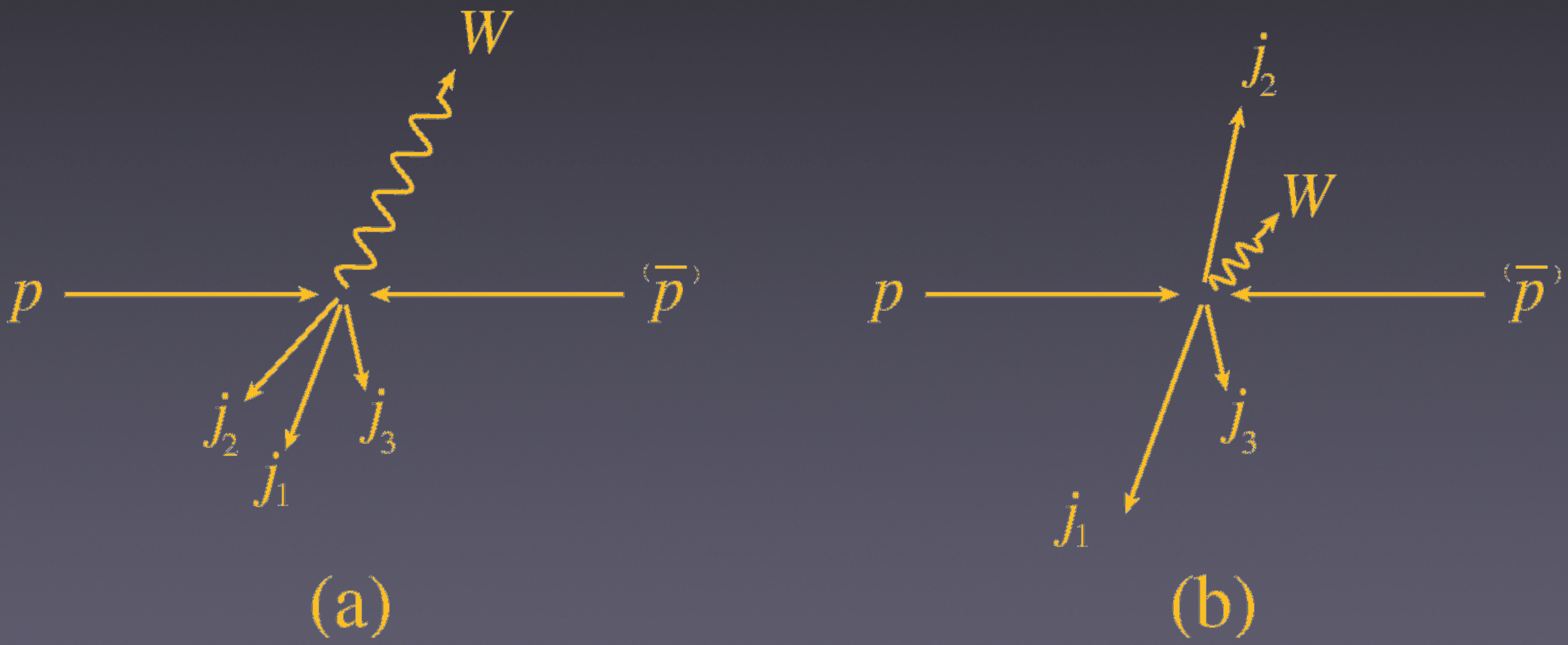
Differential cross section becomes negative

Large K factor and large dependence of the K factor

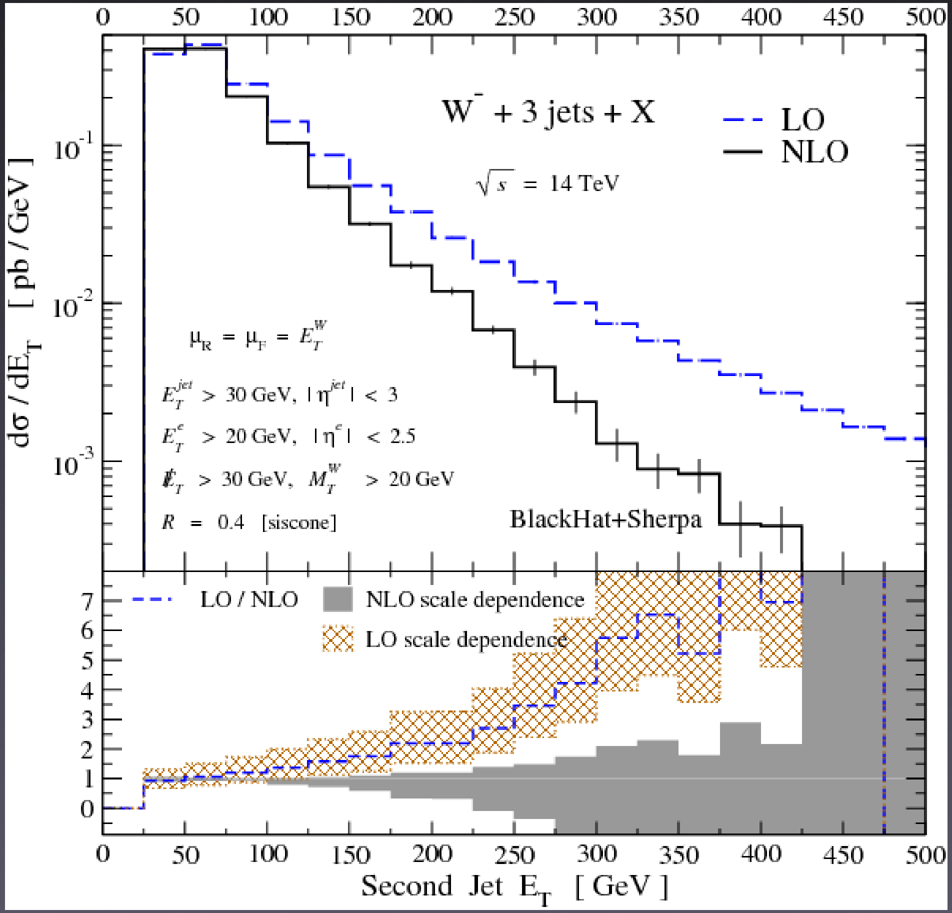
Large growth of the scale dependence of the NLO

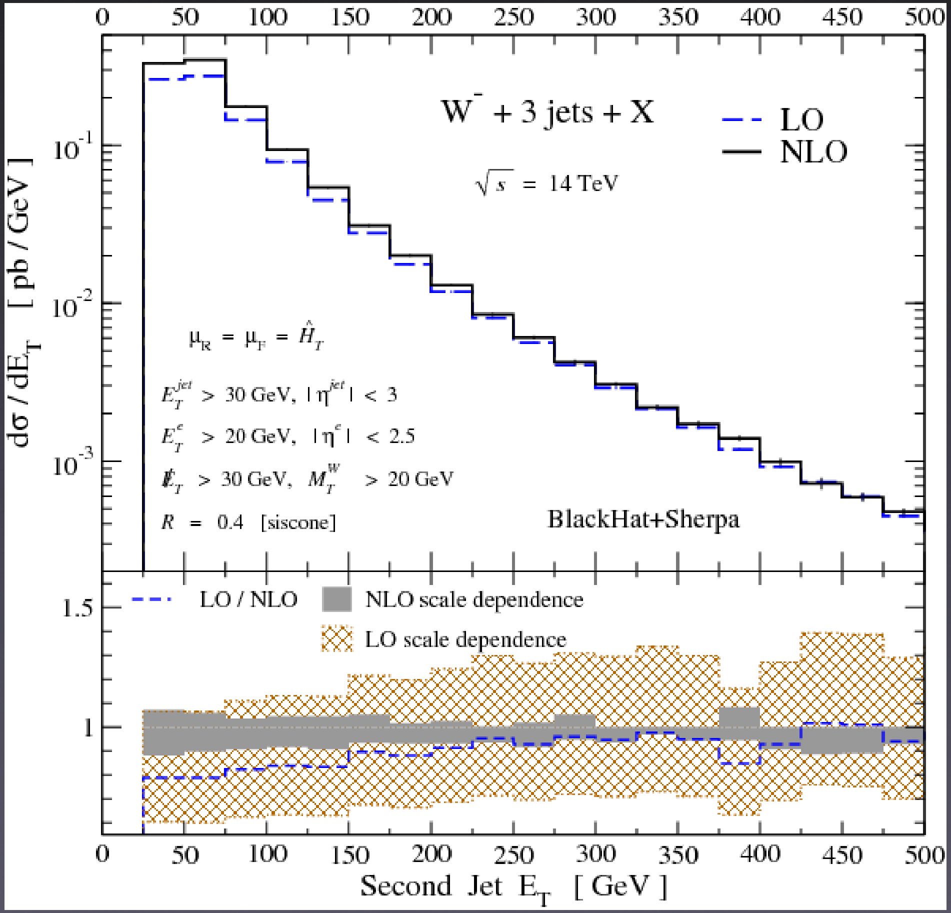
Scale choice

$$E_T^W = \sqrt{m_W^2 + p_T^2(W)} \qquad H_T = \sum_{j=1,2,3} E_{T,j}^{\text{jet}} + E_T^e + \not{E}_T$$



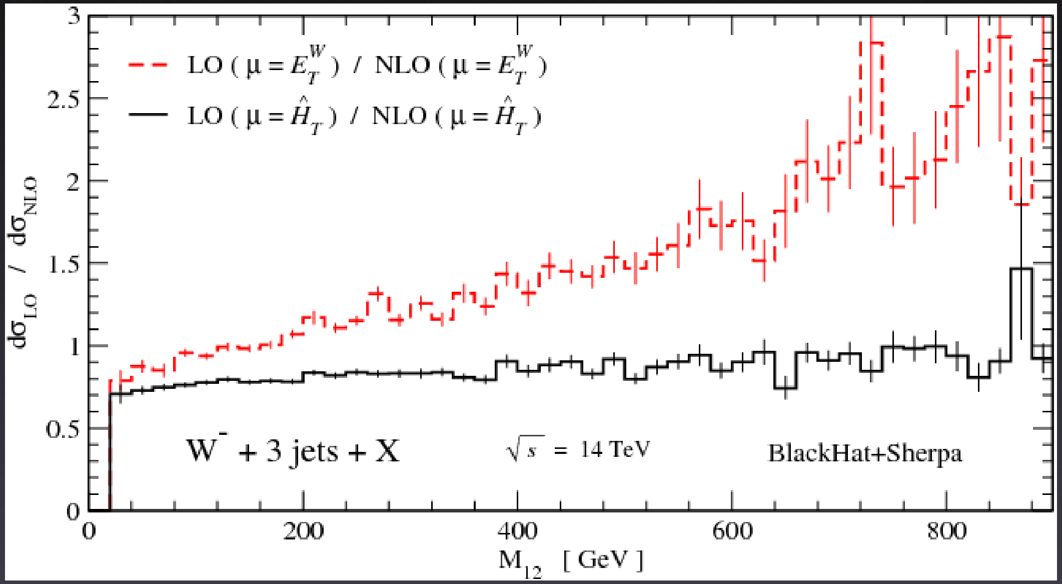
Scale choice



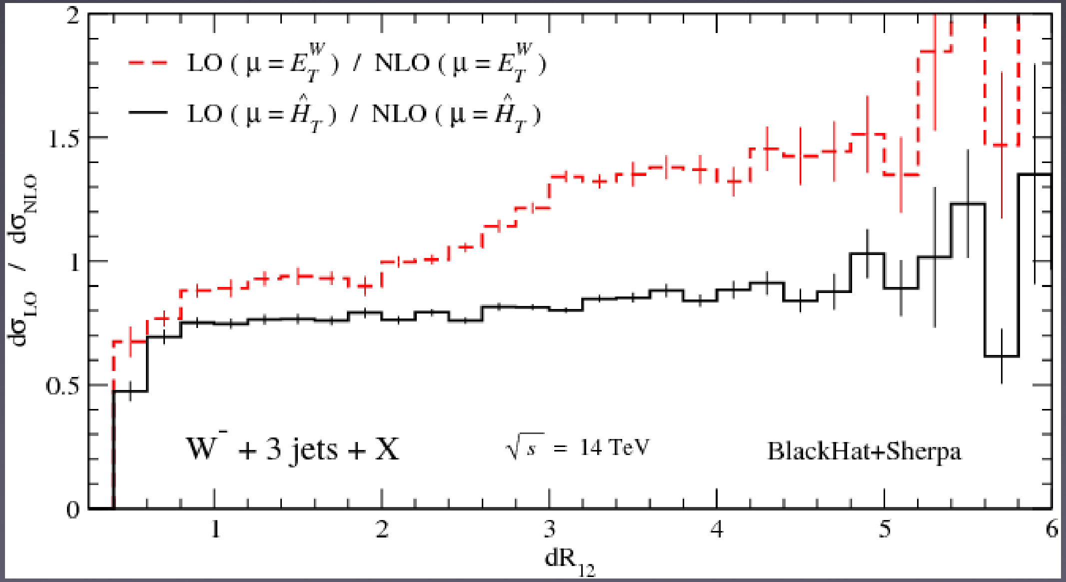


$$E_T^W = \sqrt{m_W^2 + p_T^2(W)} \qquad H_T = \sum_{j=1,2,3} E_{T,j}^{\text{jet}} + E_T^e + \not{E}_T$$

Scale choice



$$M_{ij}^2 = (p_i^{\rm jet} + p_j^{\rm jet})^2$$



- Does not work for all distributions!
- Distributions that are specifically sensitive to the W
- Choice of scale has more effect at LHC, but visible at Tevatron

$$\Delta R_{12} = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2}$$

Scale choice

- What dynamic scale shape?
  - CKKW, Average jet pt, H_T, ...
- Pitfalls at NLO
  - Scale choice should not introduce a bias for the observable considered
  - Scale choice should be “infrared safe” to insure cancellation of divergences in the real matrix elements
  - Different scale choice in subtraction and integrated subtraction might introduce finite terms

Technical challenges

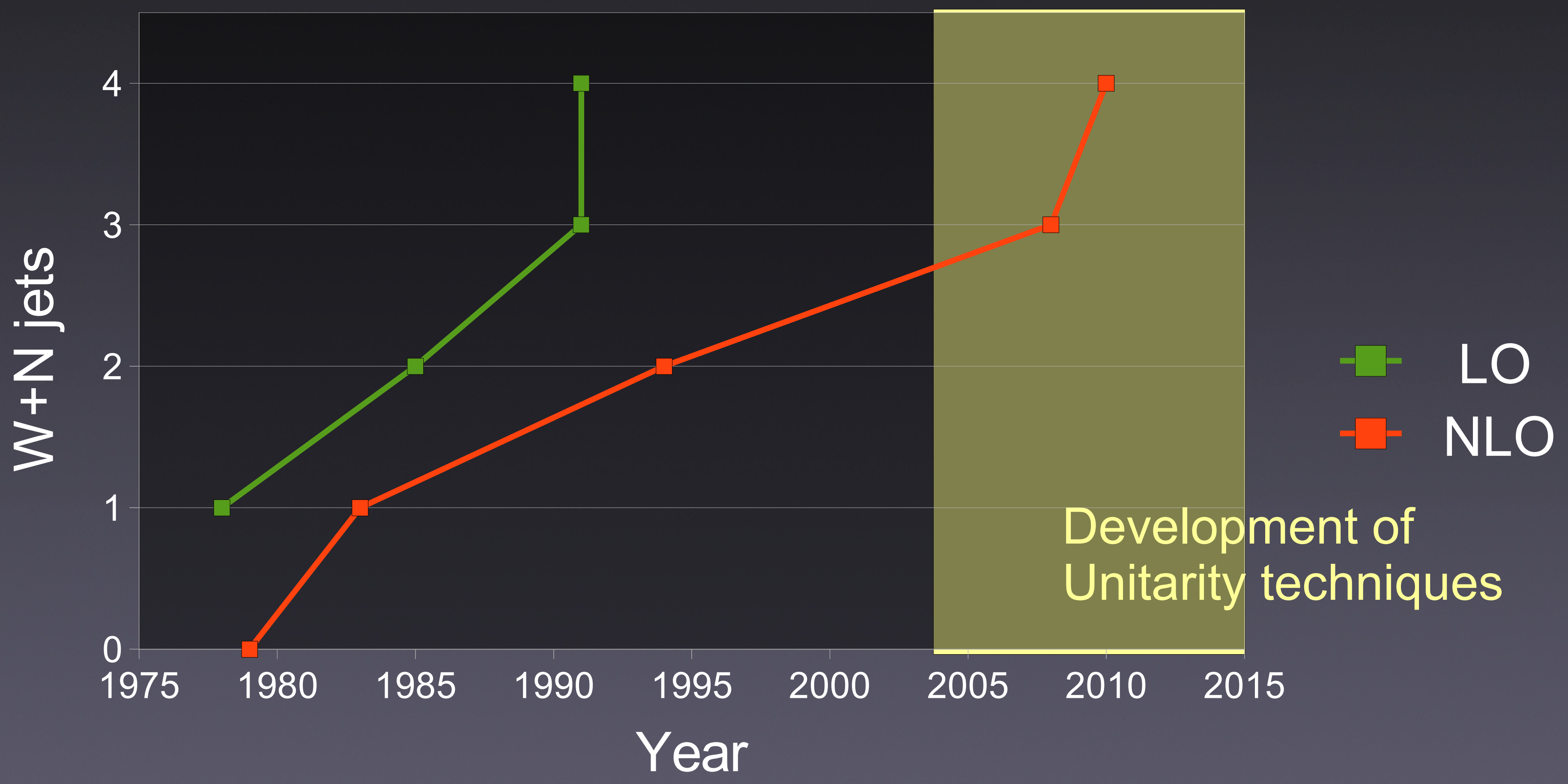
- Automation
- Numerical accuracy of the virtual ME
- NLO is CPU intensive
- Phase space integration (especially for the real part)
  - *n+1* phase space
  - Potentially many subtraction terms

Prospects

- Rapid progress for NLO computations
  - Automation of virtual matrix elements computation
  - Improvement of the efficiency of real part
  - Combination with Parton shower
    - Herwig++
    - MC@NLO
    - POWHEG Box
    - Sherpa
    - ...

Progress

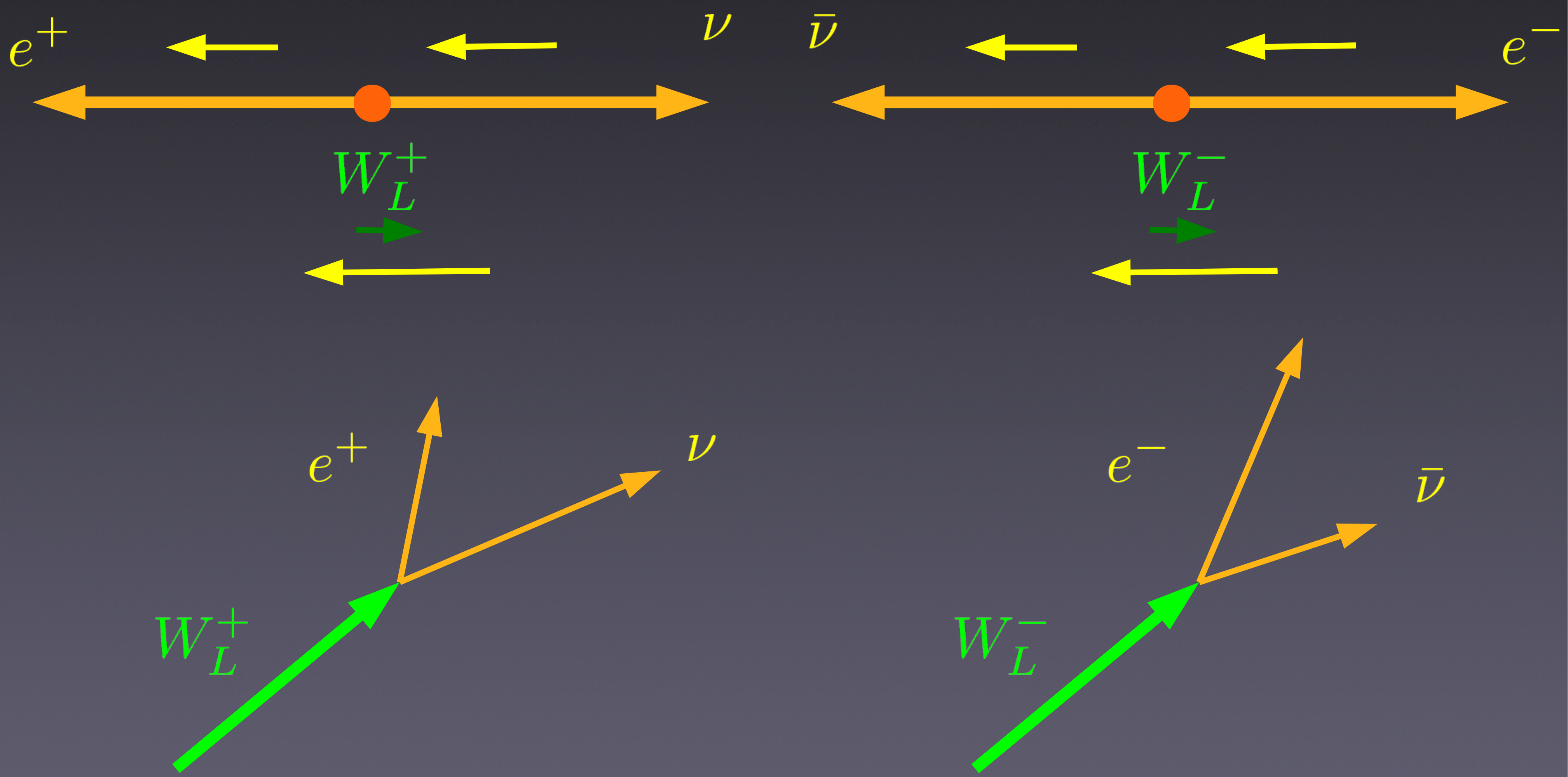
- W+jets NLO cross section calculation



W polarisation @ LHC

Left polarised *W*s

- Polarisation in the W flight direction

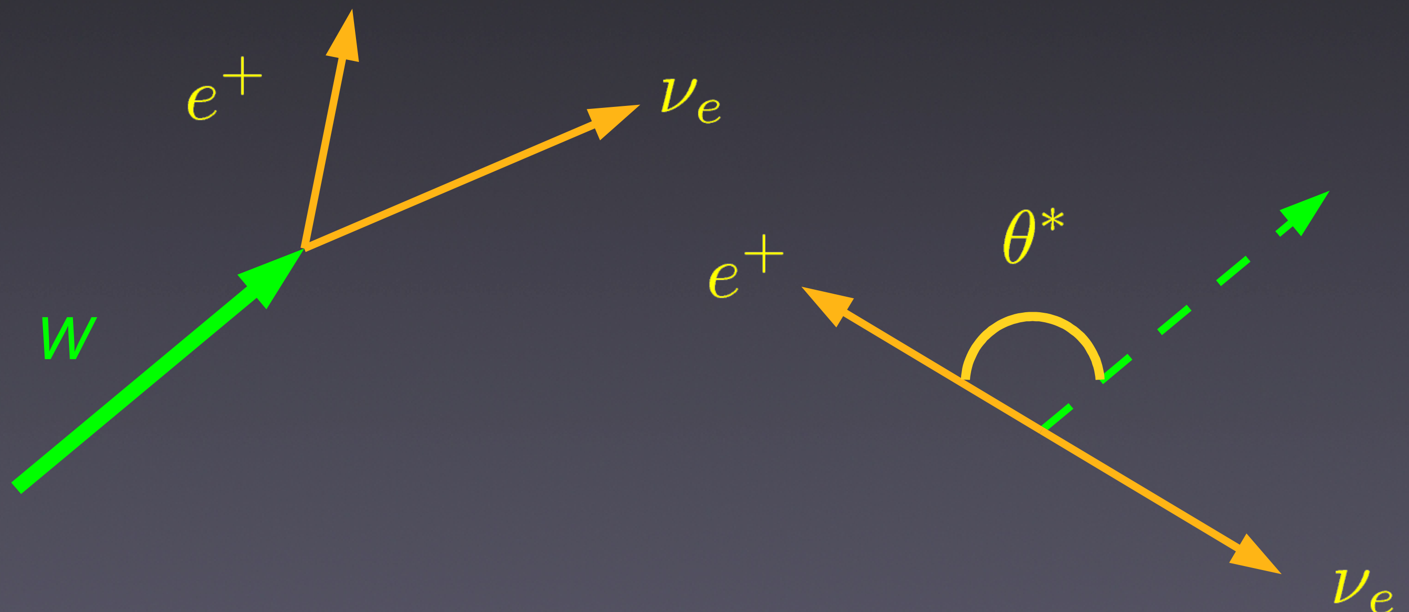


W polarisation @ LHC
[arXiv:1103.5445]

- Large transverse momentum W bosons the LHC are predominantly left-handed at the LHC (not the same as low *W* pT polarisation)
- Can be used to distinguish prompt W+jets from Ws produced in top pair decay, Higgs production or NP.
- Polarisation fractions are quite robust wit respect to radiative corrections.

W decay in W rest frame

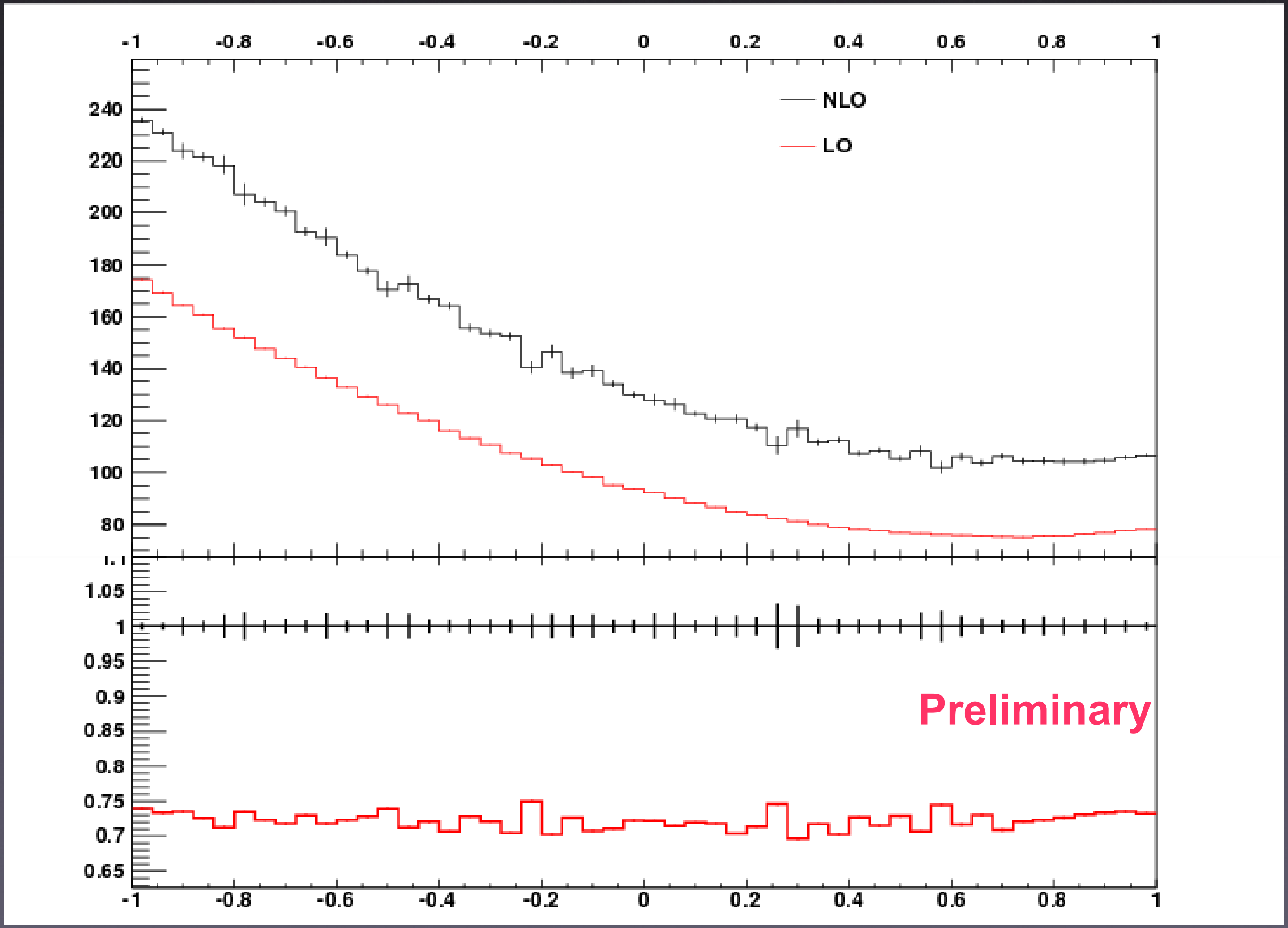
- Define θ^* as the angle of the charged lepton wrt the *W* flight direction in the *W* rest frame



$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta^*} = \frac{3}{8}(1-\cos\theta^*)^2 f_L + \frac{3}{8}(1+\cos\theta^*)^2 f_R + \frac{3}{4}\sin^2\theta^* f_0$$

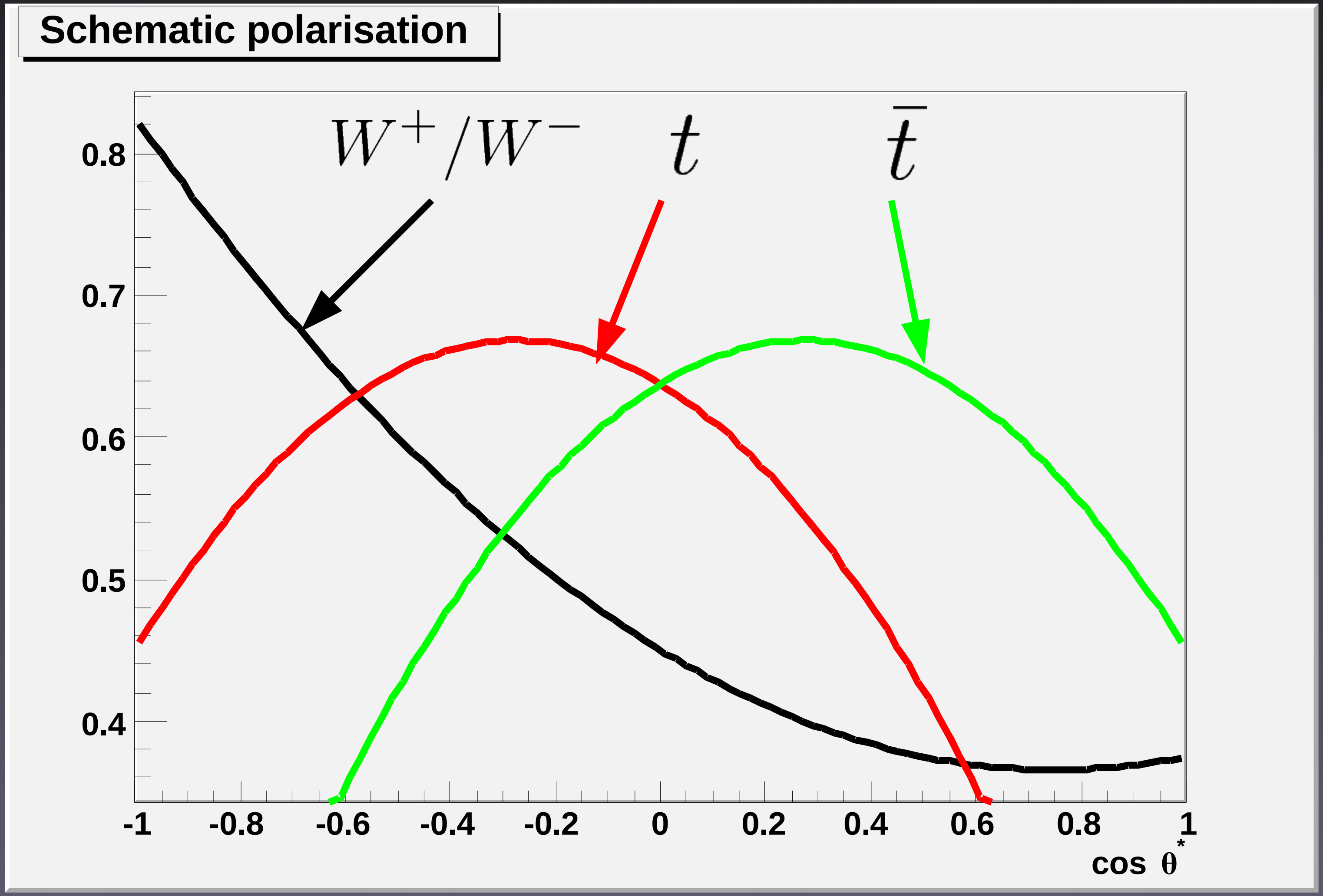
Cos Theta* distribution

- Left polarisation clearly visible



W+1 jet, MSTW08, pT *W > 50 GeV*

W polarisation



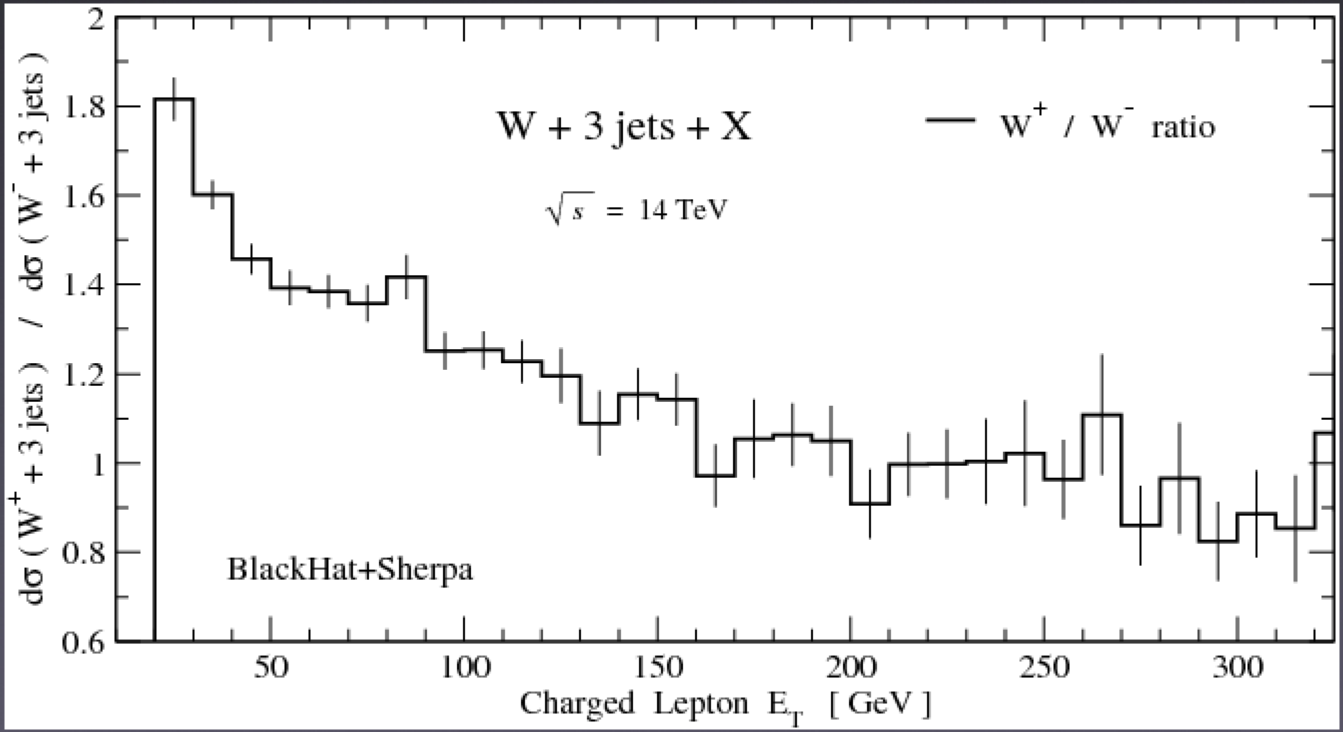
Conclusions

- NLO predictions are becoming available for many high multiplicity processes
- New frontier $2 \rightarrow 5$
- There are still effects to be discovered

Backup

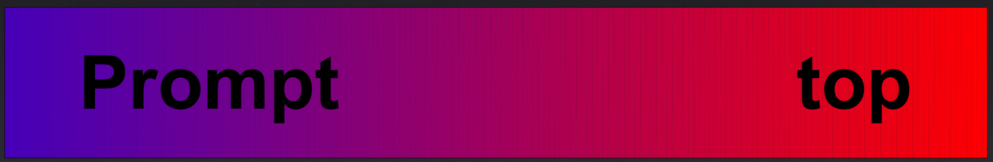
“Discovery”

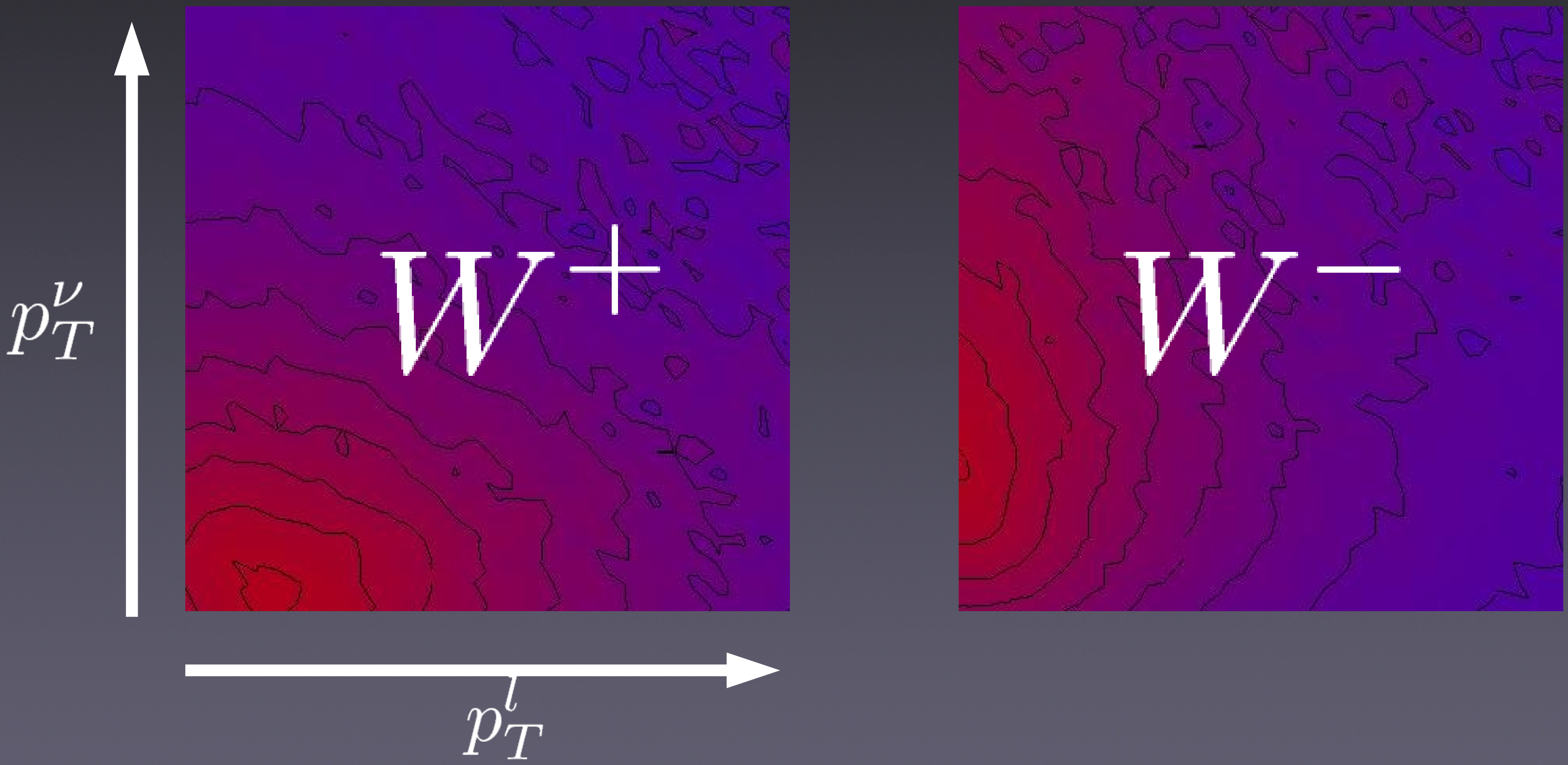
- Ratio of the charged lepton Et distributions e^+/e^-



Prompt *W*s vs *W*s from top decay

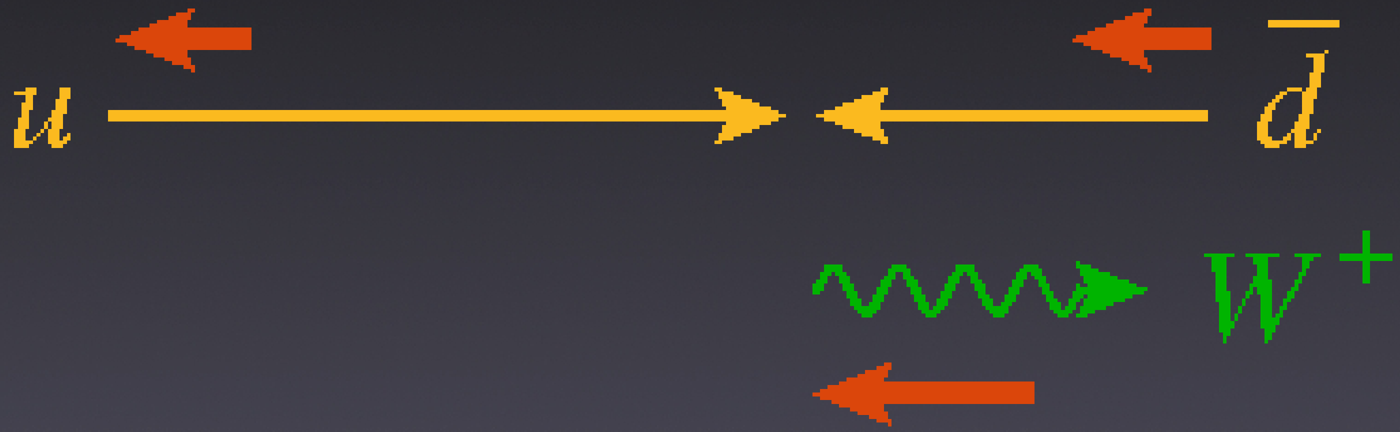
$$\frac{\frac{d^2\sigma}{dp_T^\nu p_T^l}(t\bar{t} \to W^\pm + 3\text{jets})}{\frac{d^2\sigma}{dp_T^\nu p_T^l}(W^\pm + 3\text{jets})}$$





Small pT polarisation

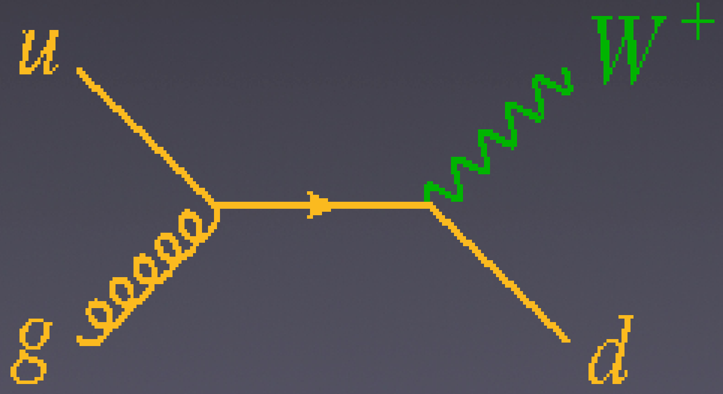
- At lowest order largest contribution



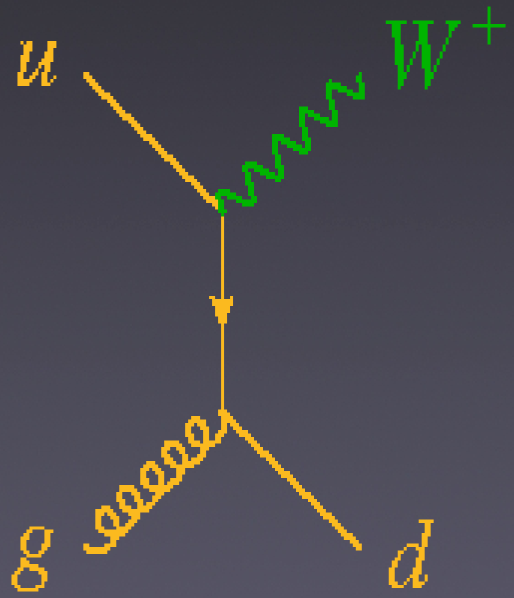
W mostly in the direction of the *u* quark because $u(x) > \bar{d}(x)$ → W left-polarised

Large pT Ws

- Three production processes at lowest order $ug \to W^+ d \ > \ u\bar{d} \to W^+ g \ > \ \bar{d}g \to W^+ \bar{u}$
- Consider only diminant $ug \to W^+ d$
- Two channels



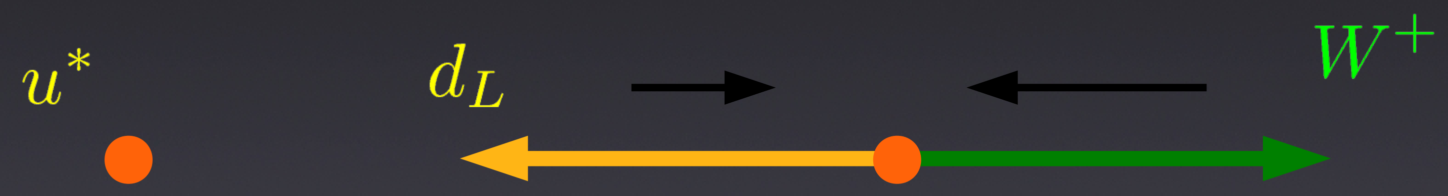
s-channel



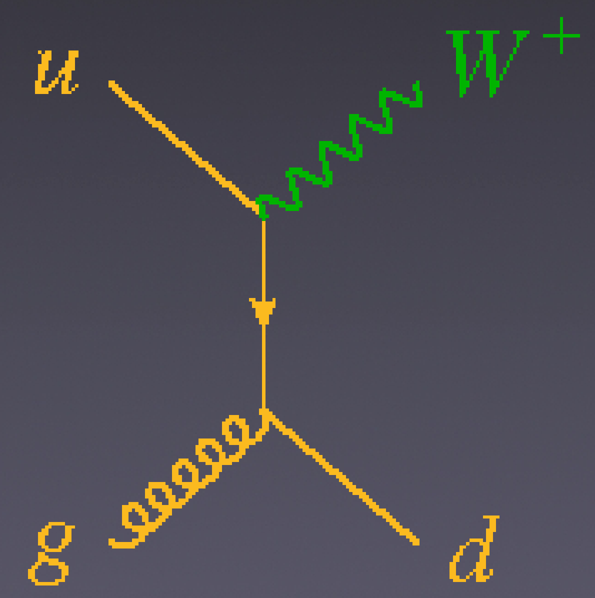
t-channel

Large Wpt

- *s*-channel yields 100% left polarized (in partonic CMS)



- *t*-channel

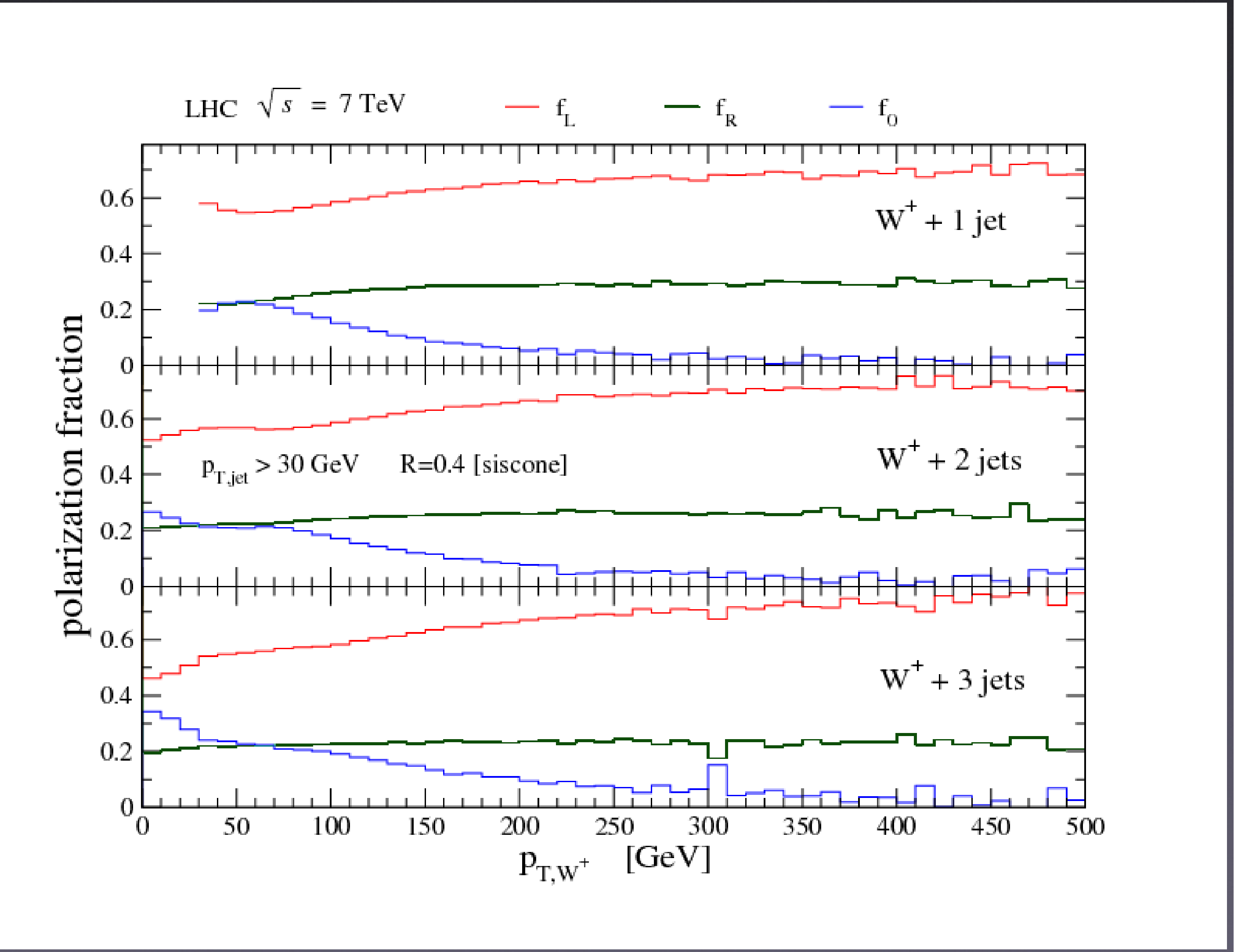


  - *u* is left-handed
  - If *g* left-handed
    t-channel can be 0
    → left polarisation
  - If *g* right-handed mixed
    polarisations but ME
    smaller (1/4) at large *W* pT

Polarisation fractions

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta^*} = \frac{3}{8}(1-\cos\theta^*)^2 f_L + \frac{3}{8}(1+\cos\theta^*)^2 f_R + \frac{3}{4}\sin^2\theta^* f_0$$

- Polarisation is not much affected by the number of jets and increases as the W pT increases



Automated implementations

- Different automated implementations
  - TevJet [Seymour,Tevlin]
  - Sherpa [Gleisberg,Krauss]
  - MadDipole [Frederix,Gehrmann,Greiner]
  - AutoDipole [Hasegawa,Moch,Uwer]
  - Dipoles [Czakon,Papadopoulos,Worek]
  - MadFKS [Frederix,Frixione,Maltoni,Stelzer]
  - POWEG BOX [Alioli,Oleari,Nason,Re]
  - ...

Real Corrections

- Different techniques
  - Catani-Seymour
  - Frixione-Kunszt-Signer
  - Phase-space slicing
  - Antenna subtraction
  - ...