Matching Fixed Order and Parton Showers

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- Lecture 1: Matching Parton Showers to Next-to-Leading Order (NLOPS)
- Lecture 2: Merging Parton Showers with Multijet Matrix Elements (MEPS)

Outline I

- Parton Shower Monte Carlo (PSMC)
- Matching PSMC to Next-to-Leading Order (NLOPS)
  - Toy Model
  - MC@NLO
  - POWHEG
- Summary

Parton Shower Monte Carlo

Distribution of first (resolvable) emission:

LO (Born) — No (resolvable) emission — One emission

$$d\sigma_{\rm MC} = \underbrace{B\left(\Phi_B\right)\, d\Phi_B}_{\text{LO (Born)}} \left[\Delta_{\rm MC}\left(0\right) + \frac{R_{\rm MC}\left(\Phi_B,\Phi_R\right)}{B\left(\Phi_B\right)}\Delta_{\rm MC}\left(k_T\left(\Phi_B,\Phi_R\right)\right)\, d\Phi_R\right]$$

- MC Sudakov form factor:

$$\Delta_{\rm MC}\left(p_T\right) = \exp\left[-\int d\Phi_R \frac{R_{\rm MC}\left(\Phi_B,\Phi_R\right)}{B\left(\Phi_B\right)}\theta\left(k_T\left(\Phi_B,\Phi_R\right) - p_T\right)\right]$$

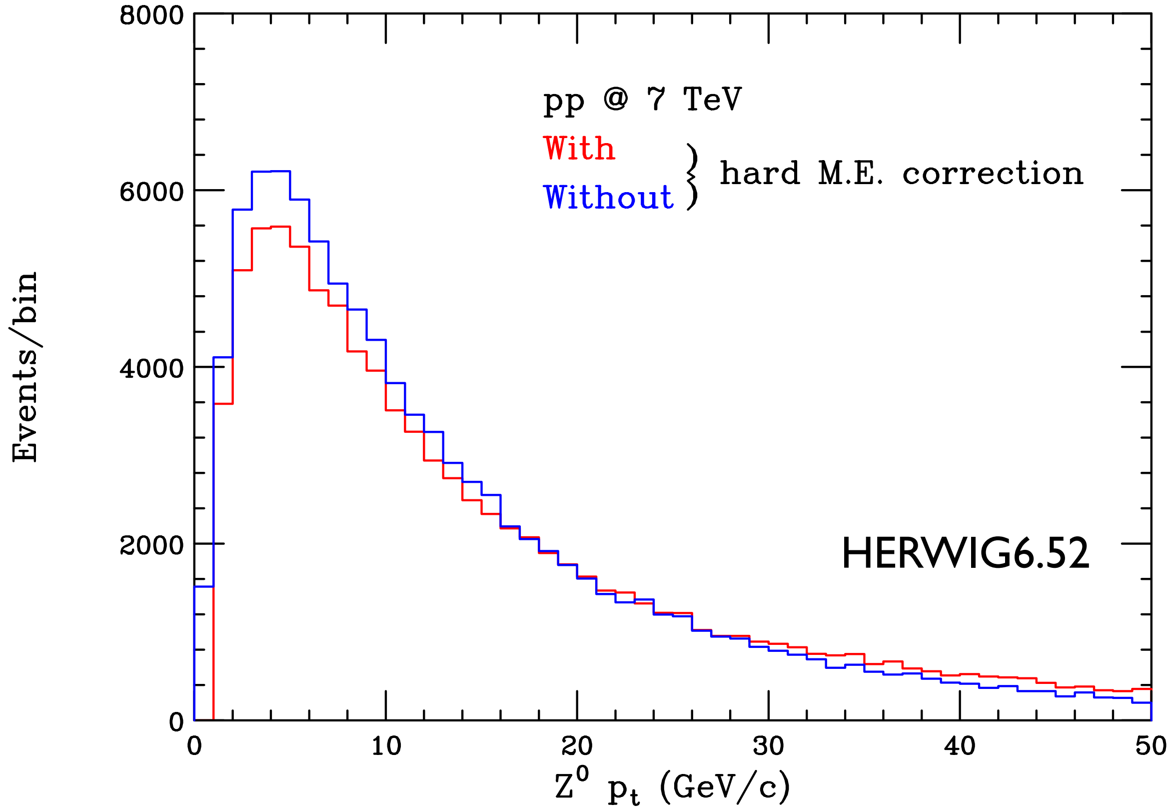
- Unitarity:

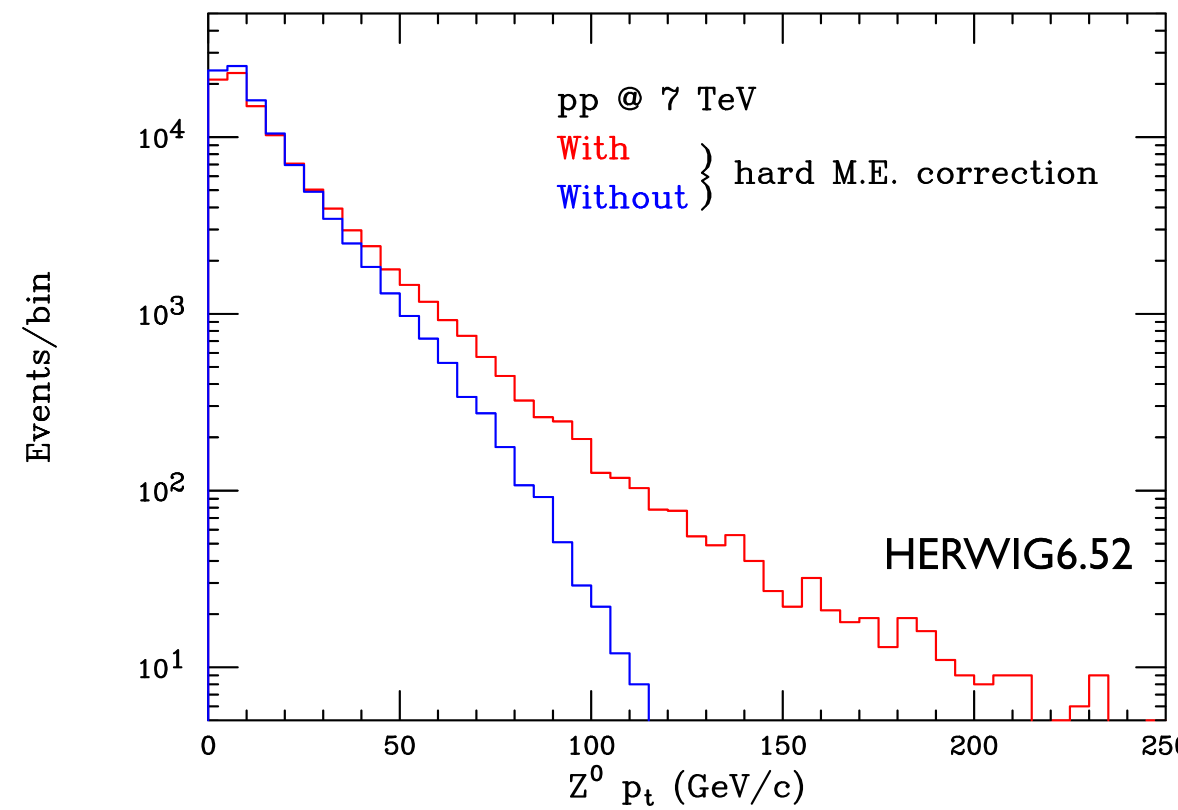
$$\int d\sigma_{\rm MC} = \int B\left(\Phi_B\right)\, d\Phi_B$$

- Expanded to NLO:

$$d\sigma_{\rm MC} = \left[B\left(\Phi_B\right) - \int R_{\rm MC}\left(\Phi_B,\Phi_R\right) d\Phi_R\right] d\Phi_B + R_{\rm MC}\left(\Phi_B,\Phi_R\right) d\Phi_B\, d\Phi_R$$

Parton Shower Monte Carlo





- Parton shower approximation
  - Bad for hard, wide-angle emission
- Hard matrix element correction: Z^0+parton
  - Not exact NLO

Toy Model

- Consider first a toy model that allows simple discussion of key features of NLO, of MC, and of matching between the two.
  - Assume a system can radiate massless "photons", energy x, with $0 \leq x \leq x_s \leq 1$, x_s being energy of system before radiation.
  - After radiation, energy of system is $x'_s = x_s - x$.
  - System can undergo further emissions, but photons themselves cannot radiate.

- Task of predicting an infrared-safe observable O to NLO amounts to computing the quantity

$$\langle O \rangle = \lim_{\epsilon \to 0} \int_0^1 dx\, x^{-2\epsilon} O(x) \left[\left(\frac{d\sigma}{dx}\right)_{\rm B} + \left(\frac{d\sigma}{dx}\right)_{\rm V} + \left(\frac{d\sigma}{dx}\right)_{\rm R} \right]$$

where Born, virtual and real contributions are respectively

$$\left(\frac{d\sigma}{dx}\right)_{\rm B,V,R} = B\delta(x)\,, \quad a\left(\frac{B}{2\epsilon} + V\right)\delta(x)\,, \quad a\frac{R(x)}{x}\,,$$

a is coupling constant, and $\lim_{x \to 0} R(x) = B$.

- In subtraction method, real contribution is written as:

$$\langle O \rangle_{\mathrm{R}} = aBO(0) \int_0^1 dx\, \frac{x^{-2\epsilon}}{x} + a \int_0^1 dx\, \frac{O(x)R(x) - BO(0)}{x^{1+2\epsilon}} \, .$$

Second integral is non-singular, so we can set $\epsilon = 0$:

$$\langle O \rangle_{\mathrm{R}} = -a\frac{B}{2\epsilon}O(0) + a \int_0^1 dx\, \frac{O(x)R(x) - BO(0)}{x}$$

- Therefore NLO prediction is:

$$\langle O \rangle_{\mathrm{sub}} = BO(0) + a \left[VO(0) + \int_0^1 dx\, \frac{O(x)R(x) - BO(0)}{x} \right]$$

- We rewrite this in a slightly different form:

$$\langle O \rangle_{\mathrm{sub}} = \int_0^1 dx \left[O(x)\frac{aR(x)}{x} + O(0)\left(B + aV - \frac{aB}{x} \right) \right]$$

Toy Monte Carlo

- In a treatment based on Monte Carlo methods, the system can undergo an arbitrary number of emissions (branchings), with probability controlled by the Sudakov form factor, defined for our toy model as follows:

$$\Delta(x_1, x_2) = \exp\left[-a \int_{x_1}^{x_2} dz \frac{Q(x)}{x}\right]$$

where $Q(x)$ is a monotonic function with the following properties:

$$0 \leq Q(x) \leq 1, \quad \lim_{x\to 0} Q(x) = 1, \quad \lim_{x\to 1} Q(x) = 0$$

$\Delta(x_1, x_2)$ is the probability that no photon be emitted with energy x such that $x_1 \leq x \leq x_2$.

Modified Subtraction

- We want to interface NLO to MC. Naive first try:

$$O(0) \quad \Rightarrow \quad \text{start MC with 0 real emissions: } \mathcal{F}^{(0)}_{\rm MC}$$

$$O(x) \quad \Rightarrow \quad \text{start MC with 1 emission at } x\text{: } \mathcal{F}^{(1)}_{\rm MC}(x)$$

so that overall generating functional is

$$\int_0^1 dx \left[\mathcal{F}^{(0)}_{\rm MC}\left(B + aV - \frac{aB}{x}\right) + \mathcal{F}^{(1)}_{\rm MC}(x)\frac{aR(x)}{x}\right]$$

- This is wrong: MC starting with no emissions will generate emission, with NLO distribution

$$\left(\frac{d\sigma}{dx}\right)_{\rm MC} = aB\frac{Q(x)}{x}$$

We must subtract this from second term, and add to first:

$$\begin{aligned}\mathcal{F}_{\rm MC@NLO} \quad = \quad & \int_0^1 dx \Bigg[\mathcal{F}^{(0)}_{\rm MC}\left(B + aV + \frac{aB[Q(x)-1]}{x}\right) \\ & + \mathcal{F}^{(1)}_{\rm MC}(x)\frac{a[R(x) - BQ(x)]}{x}\Bigg]\end{aligned}$$

$$\mathcal{F}_{\mathrm{MC@NLO}} = \int_0^1 dx \Big[\mathcal{F}_{\mathrm{MC}}^{(0)} \left(B + aV + \frac{aB[Q(x)-1]}{x} \right) + \mathcal{F}_{\mathrm{MC}}^{(1)}(x) \frac{a[R(x) - BQ(x)]}{x} \Big]$$

This prescription has several good features:

- $\mathcal{F}_{\mathrm{MC}}^{(0)} = \mathcal{F}_{\mathrm{MC}}^{(1)}$ to $\mathcal{O}(1)$, so added and subtracted terms are equal to $\mathcal{O}(a)$;
- Coefficients of $\mathcal{F}_{\mathrm{MC}}^{(0)}$ and $\mathcal{F}_{\mathrm{MC}}^{(1)}$ are now separately finite;
- Same resummation of large logs in $\mathcal{F}_{\mathrm{MC}}^{(0)}$ and $\mathcal{F}_{\mathrm{MC}}^{(1)} \Rightarrow \mathcal{F}_{\mathrm{MC@NLO}}$ gives same resummation as $\mathcal{F}_{\mathrm{MC}}^{(0)}$, renormalised to correct NLO cross section.

  Note, however, that some events may have negative weight.

MC@NLO

S Frixione & BW, JHEP 06(2002)029

finite virtual — divergent

$$d\sigma_{\rm NLO} = \left[B(\Phi_B) + V(\Phi_B) - \int \sum_i C_i(\Phi_B,\Phi_R)\, d\Phi_R\right] d\Phi_B + R(\Phi_B,\Phi_R)\, d\Phi_B\, d\Phi_R$$

$$\equiv \left[B + V - \int C\, d\Phi_R\right] d\Phi_B + R\, d\Phi_B\, d\Phi_R$$

$$d\sigma_{\rm MC} = B(\Phi_B)\, d\Phi_B \left[\Delta_{\rm MC}(0) + \frac{R_{\rm MC}(\Phi_B,\Phi_R)}{B(\Phi_B)} \Delta_{\rm MC}(k_T(\Phi_B,\Phi_R))\, d\Phi_R\right]$$

$$\equiv B\, d\Phi_B\, [\Delta_{\rm MC}(0) + (R_{\rm MC}/B)\, \Delta_{\rm MC}(k_T)\, d\Phi_R]$$

$$d\sigma_{\rm MC@NLO} = \left[B + V + \int (R_{\rm MC} - C)\, d\Phi_R\right] d\Phi_B\, [\Delta_{\rm MC}(0) + (R_{\rm MC}/B)\, \Delta_{\rm MC}(k_T)\, d\Phi_R]$$
$$+ (R - R_{\rm MC})\, \Delta_{\rm MC}(k_T)\, d\Phi_B\, d\Phi_R$$

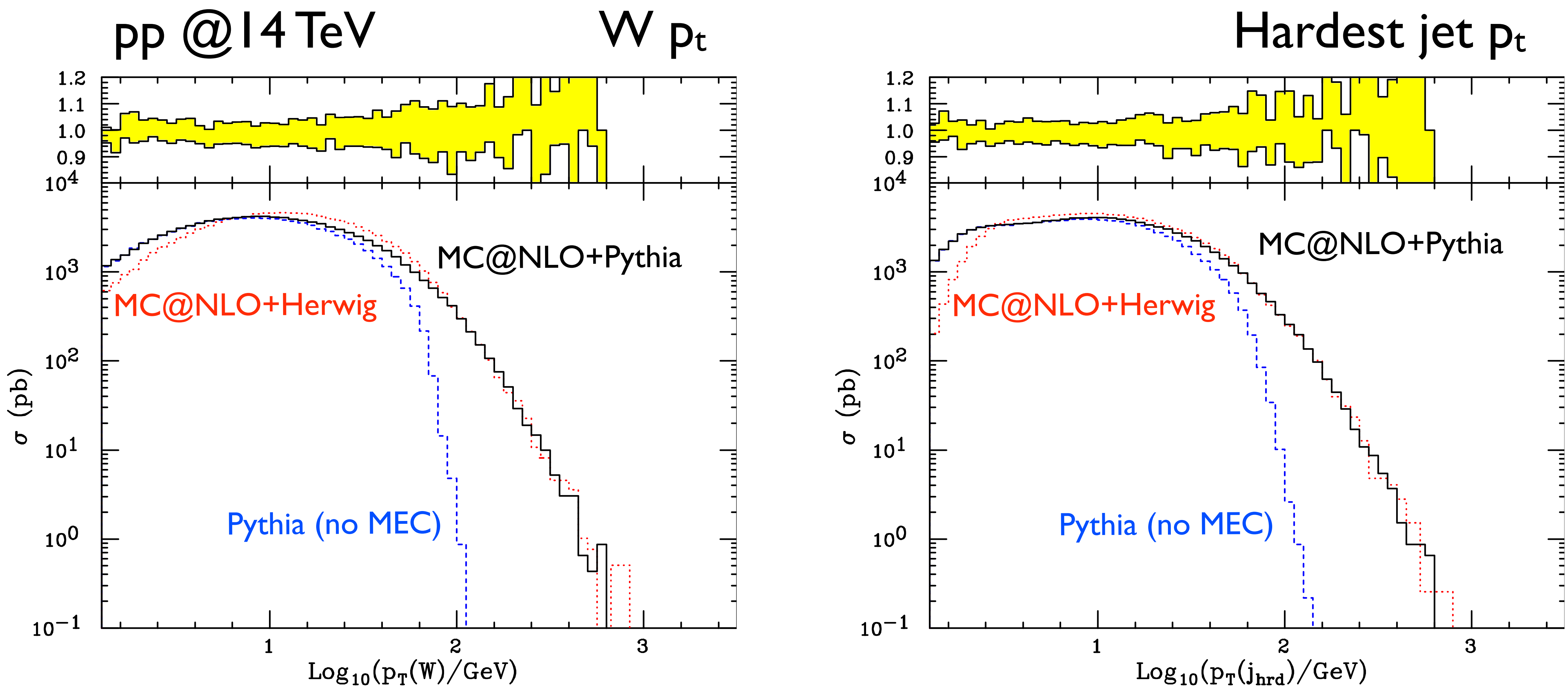
finite $\gtrless 0$

MC starting from no emission

MC starting from one emission

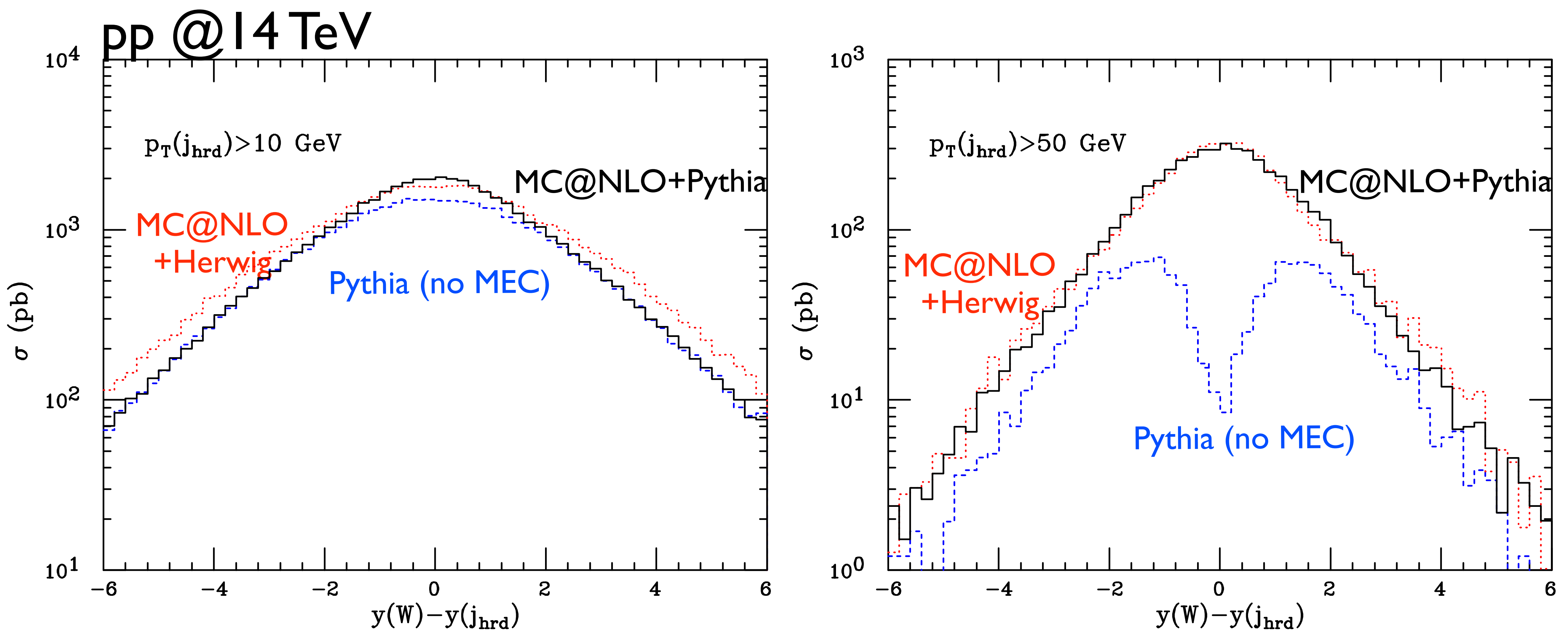
- Expanding gives NLO result

MC@NLO: W Production



- N.B. MC@NLO is MC-specific

MC@NLO: W Production



- N.B. NLO is only LO at high p_t

S Frixione & P Torrielli, JHEP 04(2010)110

POWHEG

P Nason, JHEP 11(2004)040

$$d\sigma_{\mathrm{PH}} = \overline{B}\left(\Phi_B\right) \mathrm{d}\Phi_B \left[\Delta_R\left(0\right) + \frac{R\left(\Phi_B, \Phi_R\right)}{B\left(\Phi_B\right)} \Delta_R\left(k_T\left(\Phi_B, \Phi_R\right)\right) \mathrm{d}\Phi_R\right]$$

$$\overline{B}\left(\Phi_B\right) = B\left(\Phi_B\right) + V\left(\Phi_B\right) + \int \left[R\left(\Phi_B, \Phi_R\right) - \sum_i C_i\left(\Phi_B, \Phi_R\right)\right] \mathrm{d}\Phi_R$$

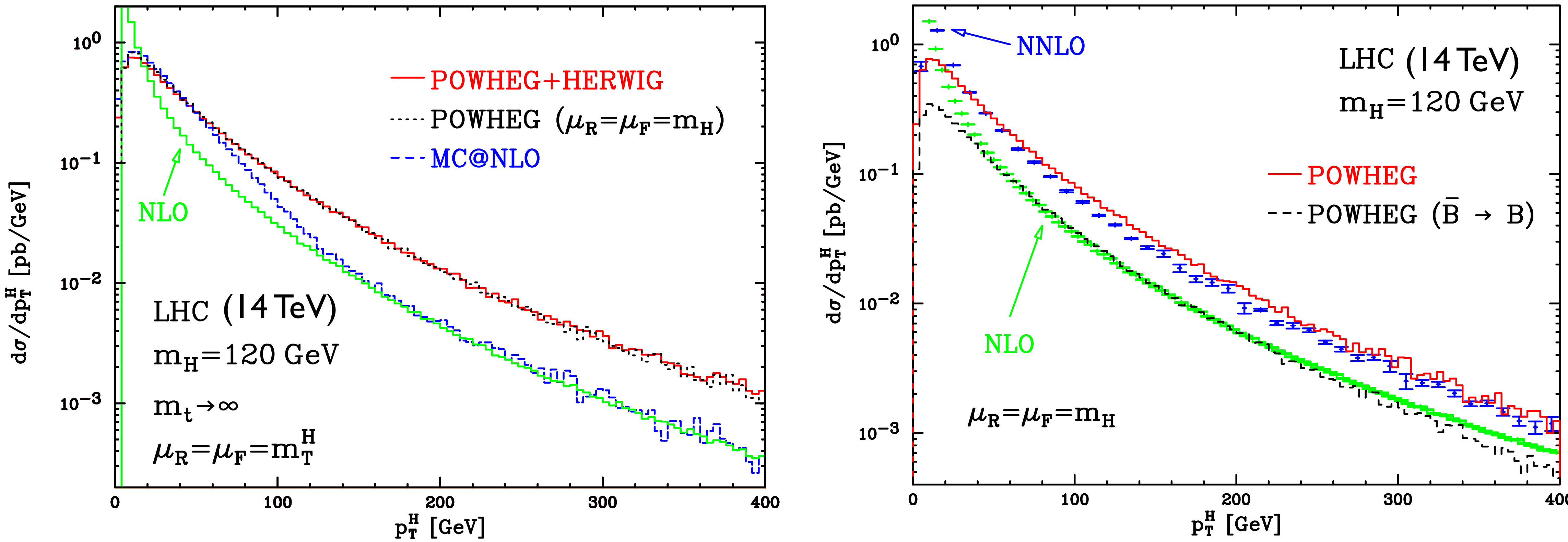
$$\Delta_R\left(p_T\right) = \exp\left[-\int \mathrm{d}\Phi_R \frac{R\left(\Phi_B, \Phi_R\right)}{B\left(\Phi_B\right)} \theta\left(k_T\left(\Phi_B, \Phi_R\right) - p_T\right)\right]$$

- NLO with (almost) no negative weights
- High p_t enhanced by $K = \overline{B}/B = 1 + \mathcal{O}(\alpha_S)$

arbitrary NNLO

POWHEG: Higgs Production

Alioli, Nason, Oleari, Re, JHEP 024(2009)002



- Large enhancement at high p_T
  - due to $\bar{B}/B$
- NNLO correction is indeed large (in this case ...)

POWHEG variation

$$\begin{aligned} \mathrm{d}\sigma'_{\mathrm{PH}} &= \overline{B}_S\left(\Phi_B\right) \mathrm{d}\Phi_B \left[\Delta_S\left(0\right) + \frac{R_S\left(\Phi_B,\Phi_R\right)}{B\left(\Phi_B\right)} \Delta_S\left(k_T\left(\Phi_B,\Phi_R\right)\right) \mathrm{d}\Phi_R\right] \\ &+ \left[R\left(\Phi_B,\Phi_R\right) - R_S\left(\Phi_B,\Phi_R\right)\right] \mathrm{d}\Phi_B \, \mathrm{d}\Phi_R \end{aligned}$$

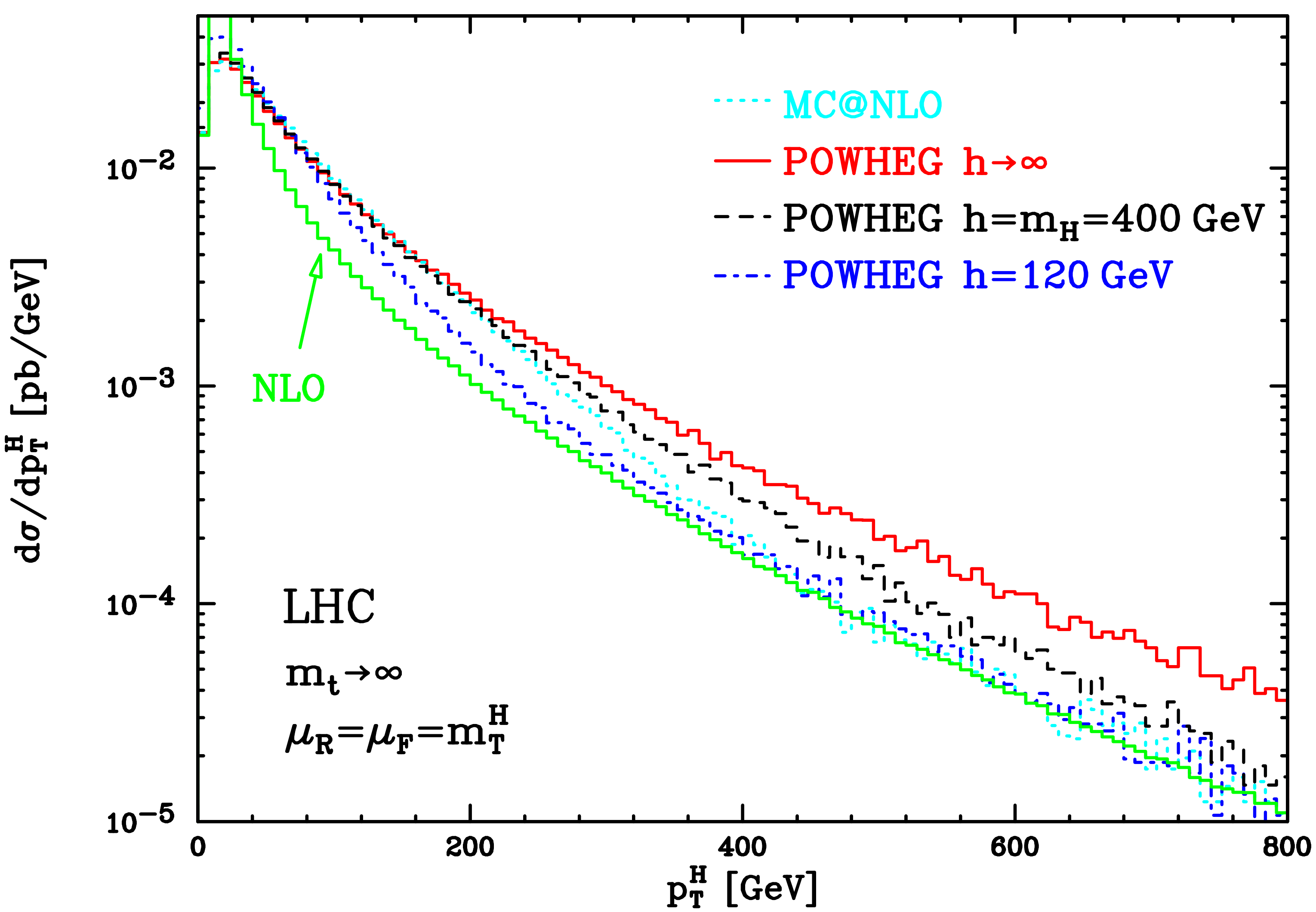
$$\overline{B}_S\left(\Phi_B\right) = B\left(\Phi_B\right) + V\left(\Phi_B\right) + \int \left[R_S\left(\Phi_B,\Phi_R\right) - \sum_i C_i\left(\Phi_B,\Phi_R\right)\right] \mathrm{d}\Phi_R$$

$$\Delta_S\left(p_T\right) = \exp\left[-\int \mathrm{d}\Phi_R \frac{R_S\left(\Phi_B,\Phi_R\right)}{B\left(\Phi_B\right)} \theta\left(k_T\left(\Phi_B,\Phi_R\right) - p_T\right)\right]$$

- R_S has soft & collinear singularities of R
- No change at NLO
- Close to MC@NLO when $R_S = R_{MC}$

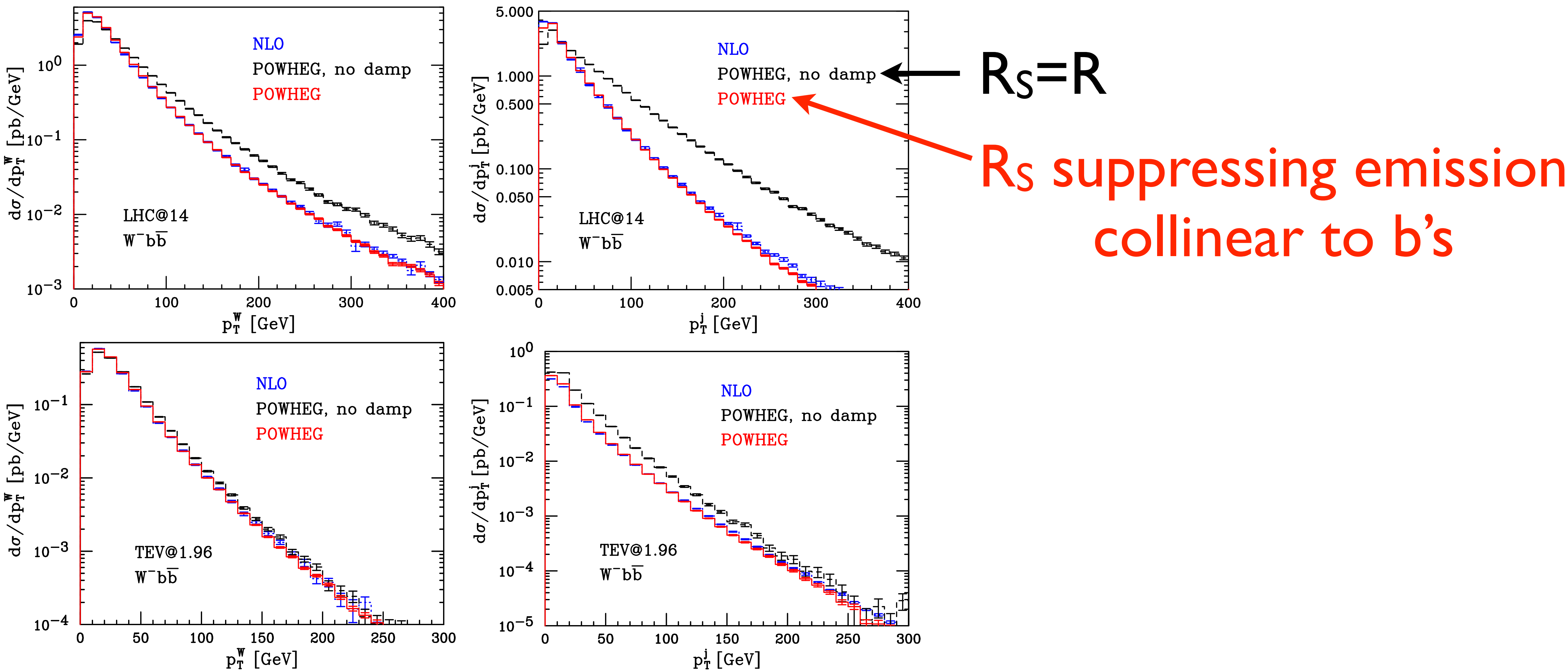
POWHEG variation

- Use $R_S = \dfrac{h^2}{p_T^2 + h^2} R$
- Varying h "tunes" NNLO



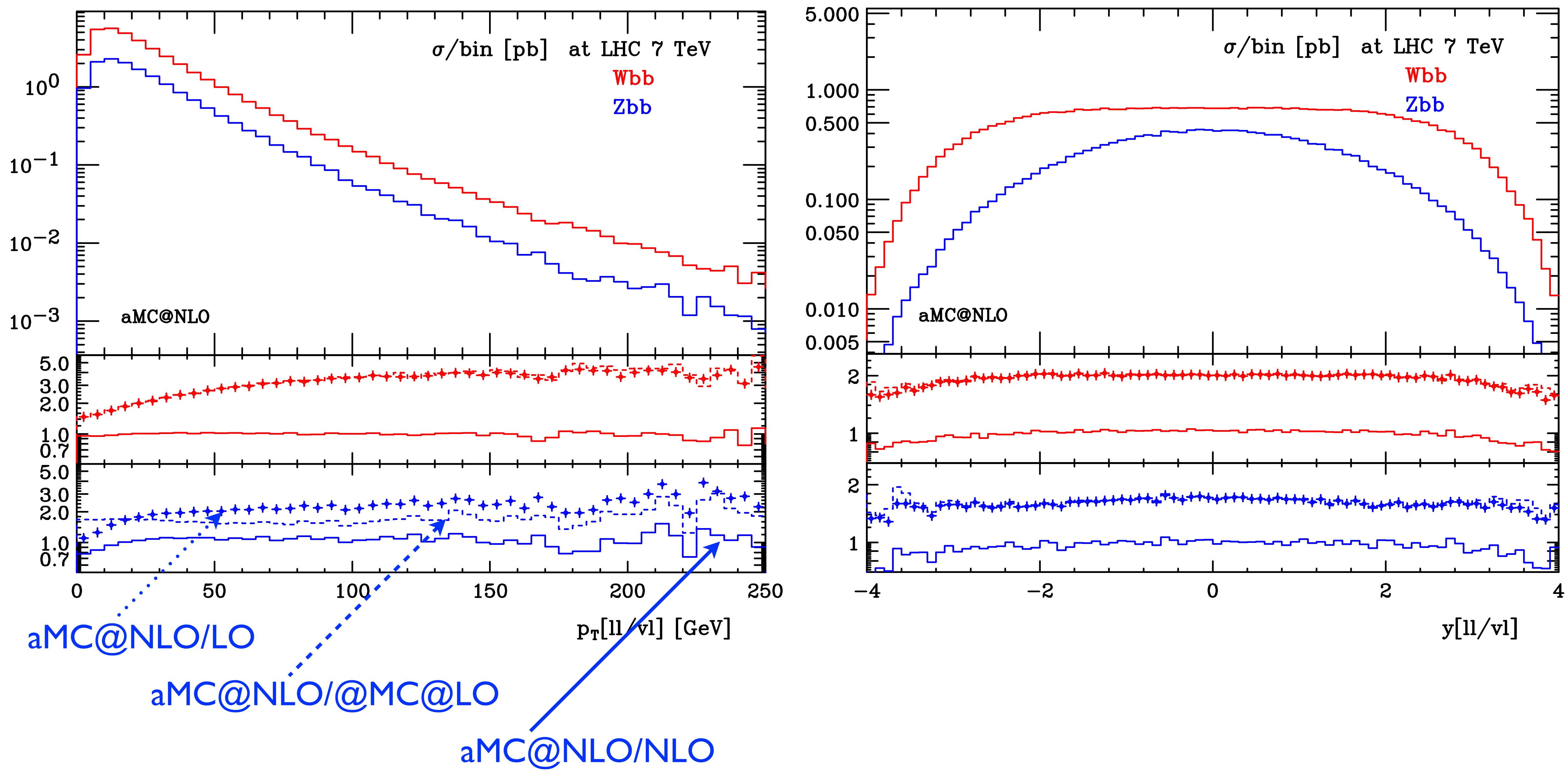
$Wb\bar{b}$ in POWHEG

Oleari & Reina, arXiv:1105.4488



- Damping gives results close to NLO at high p_T

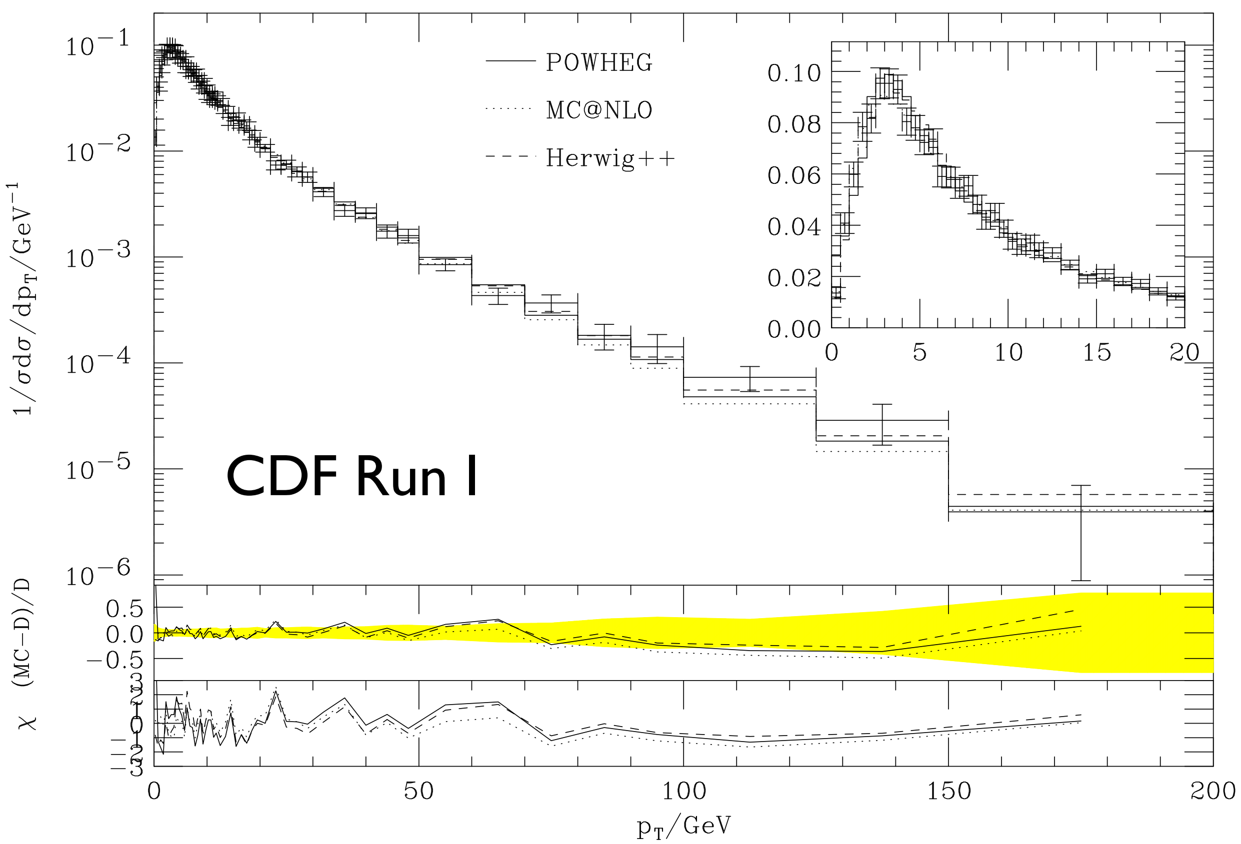
W$b\bar{b}$ in aMC@NLO

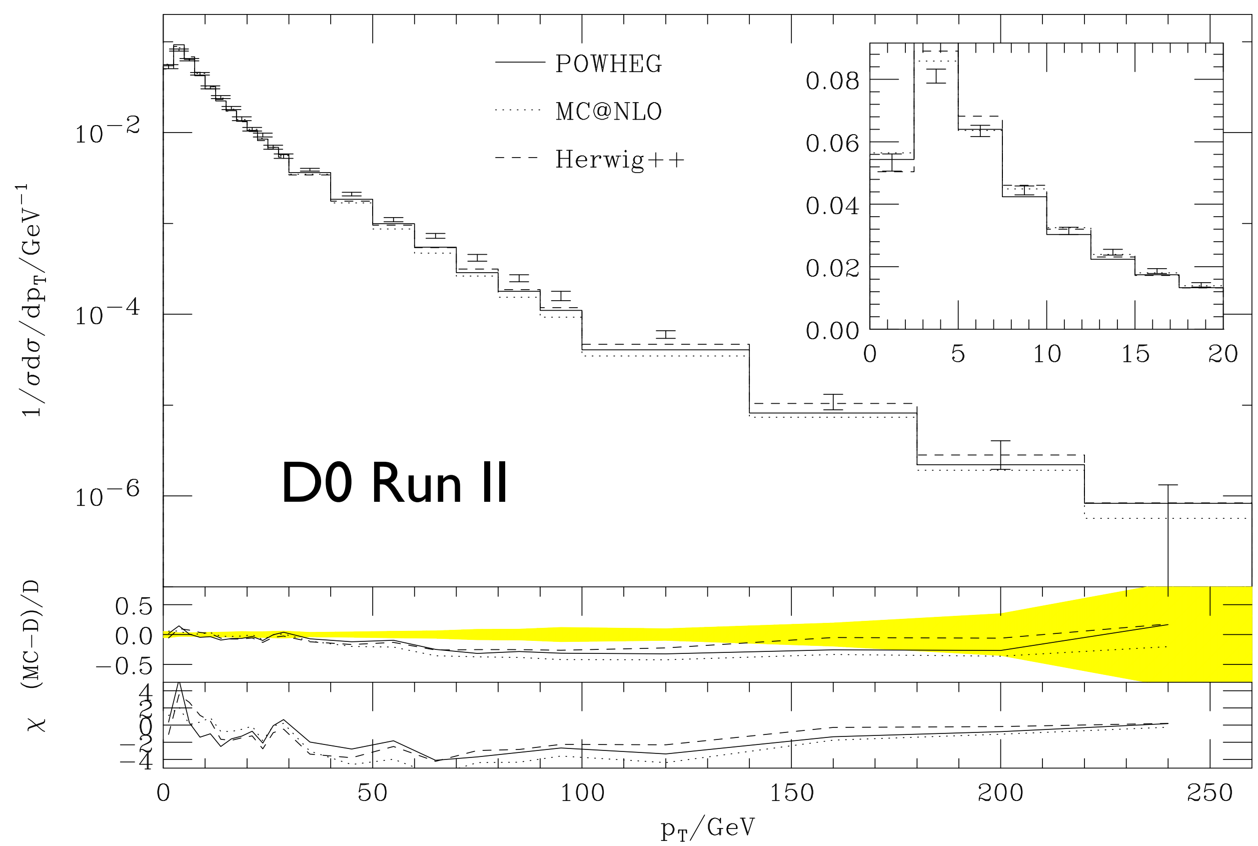


- aMC@NLO uses MADLOOP+MADFKS for NLO
- Always close to NLO at high p_T

Frederix et al., arXiv:1106.6019

Z^0 p_t at Tevatron

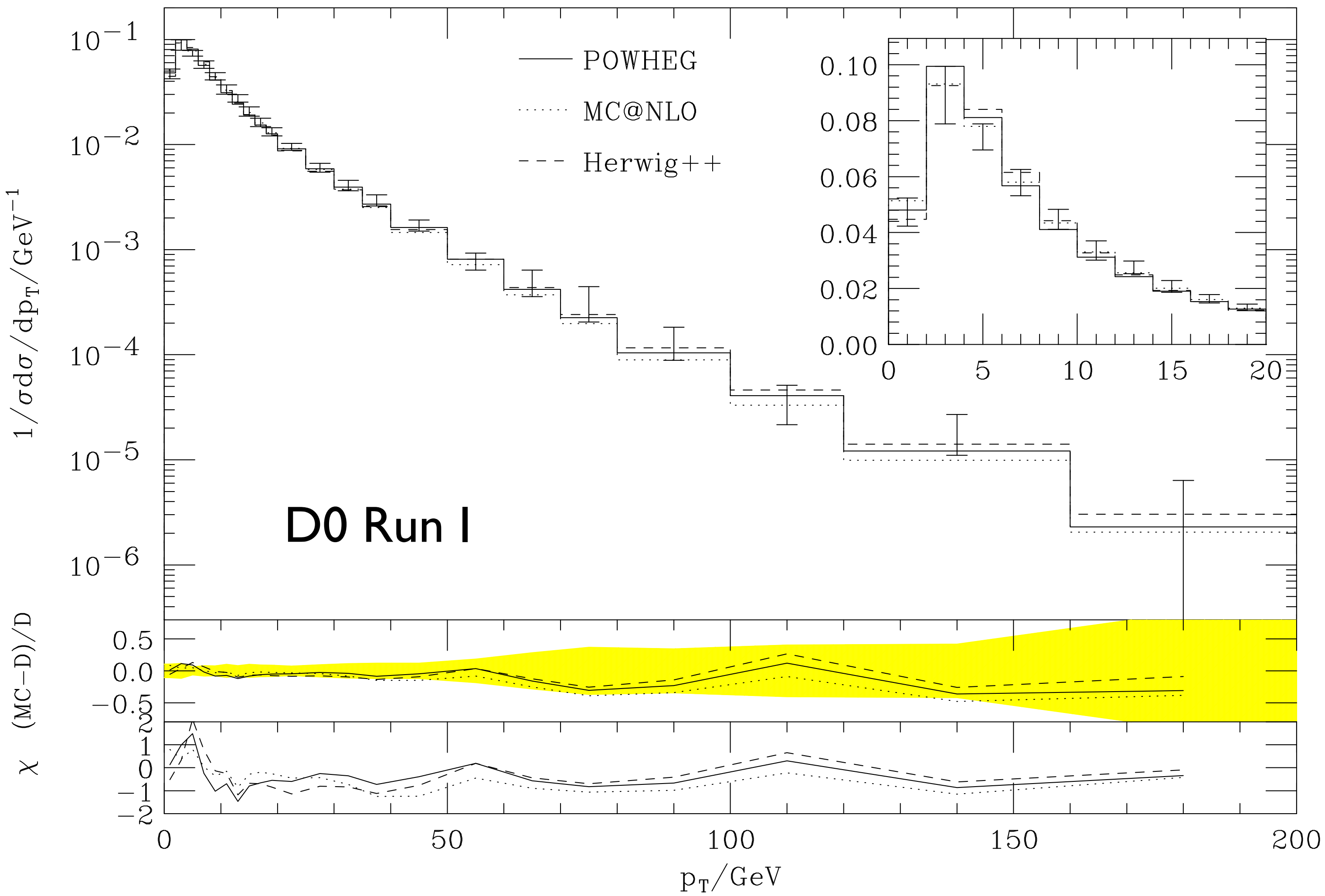




- NLO is only LO at high p_t

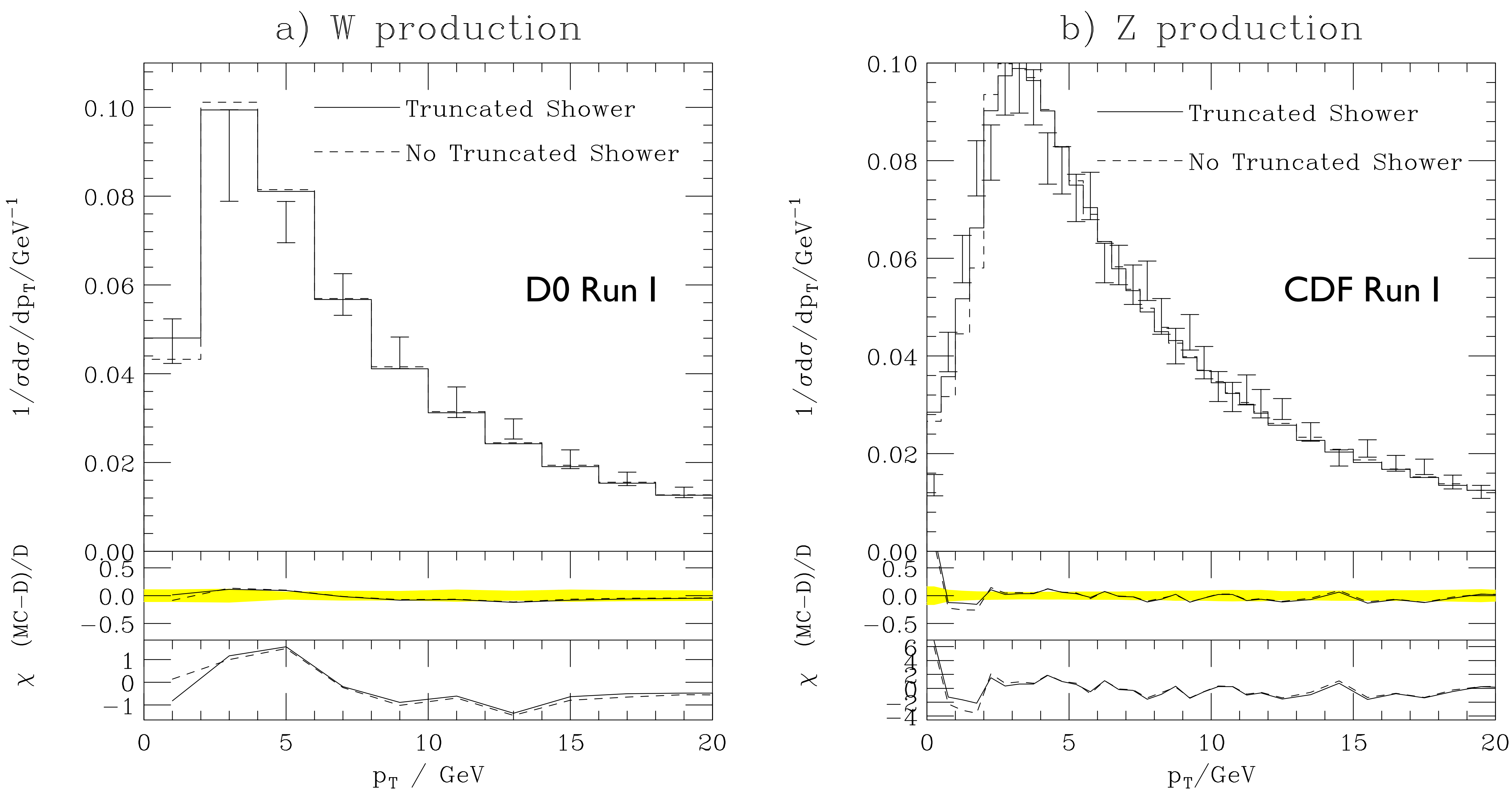
Hamilton, Richardson, Tully JHEP10(2008)015

W p_t at Tevatron



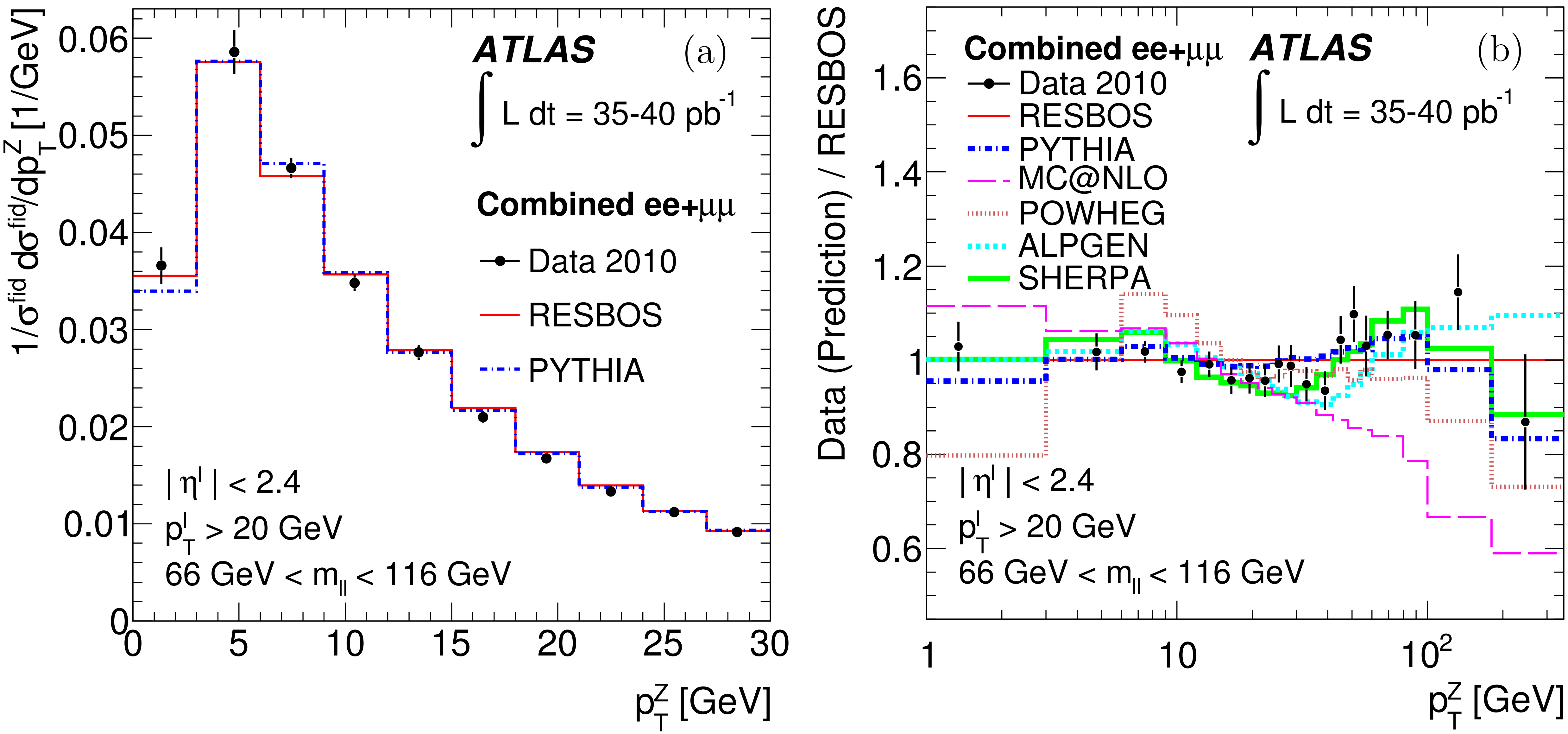
- All agree (tuned) at Tevatron

Truncated shower



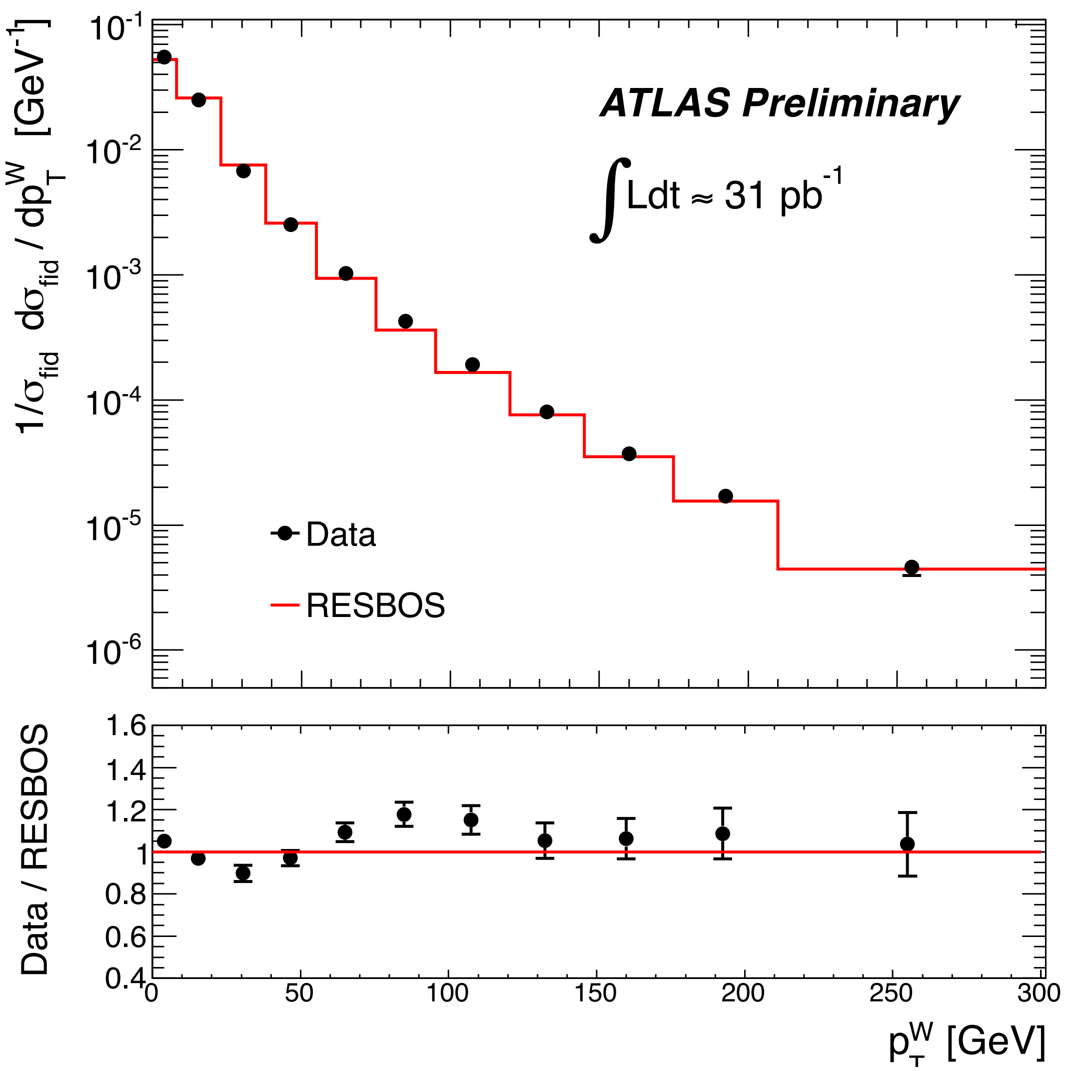
- POWHEG highest p_t emission not always first
  - must add 'truncated' shower at wider angles

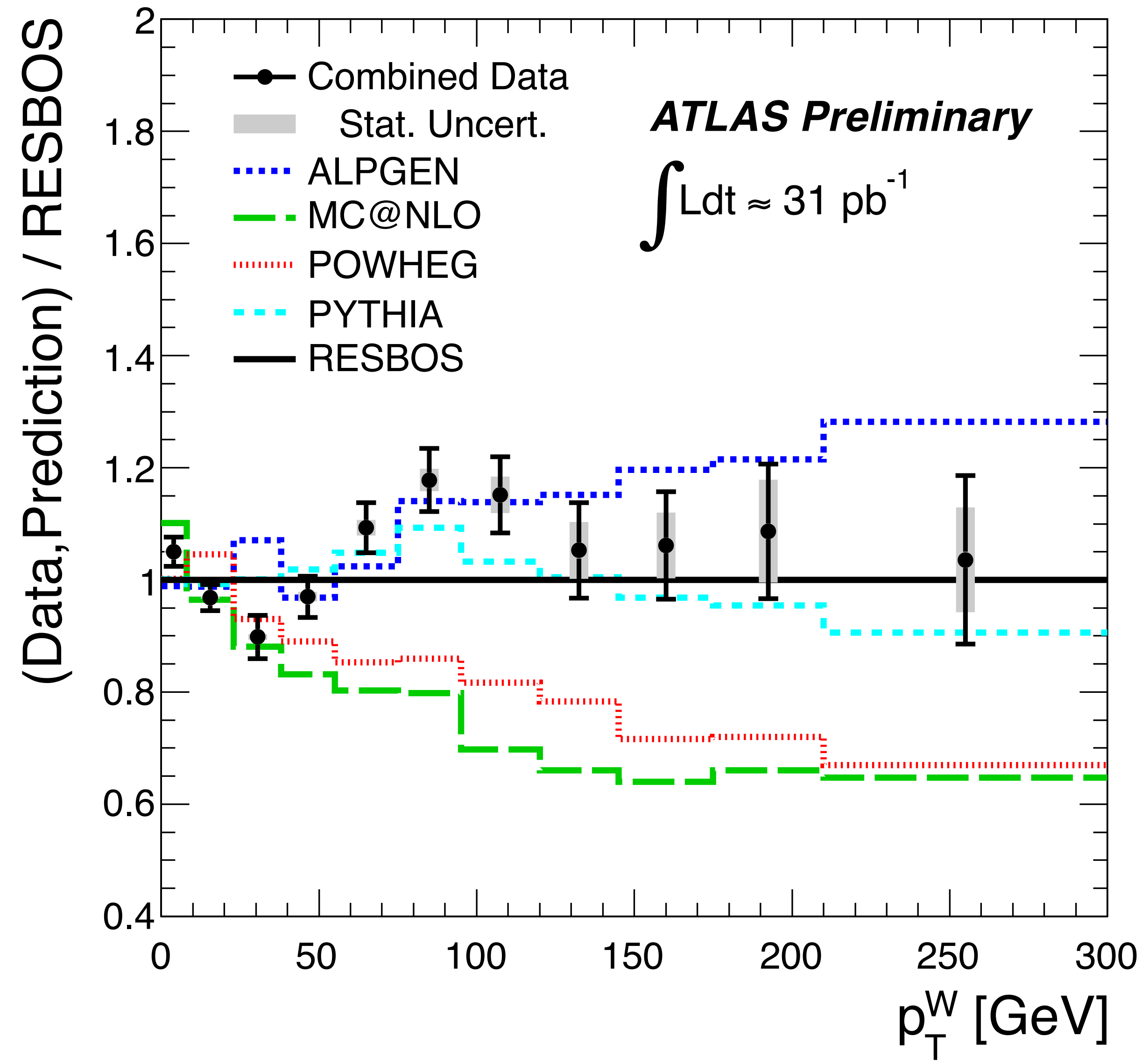
Z^0 p_t at LHC (7 TeV)



- RESBOS = p_T resummation (not EG)
- NLOPS gives deficit at high p_T ?

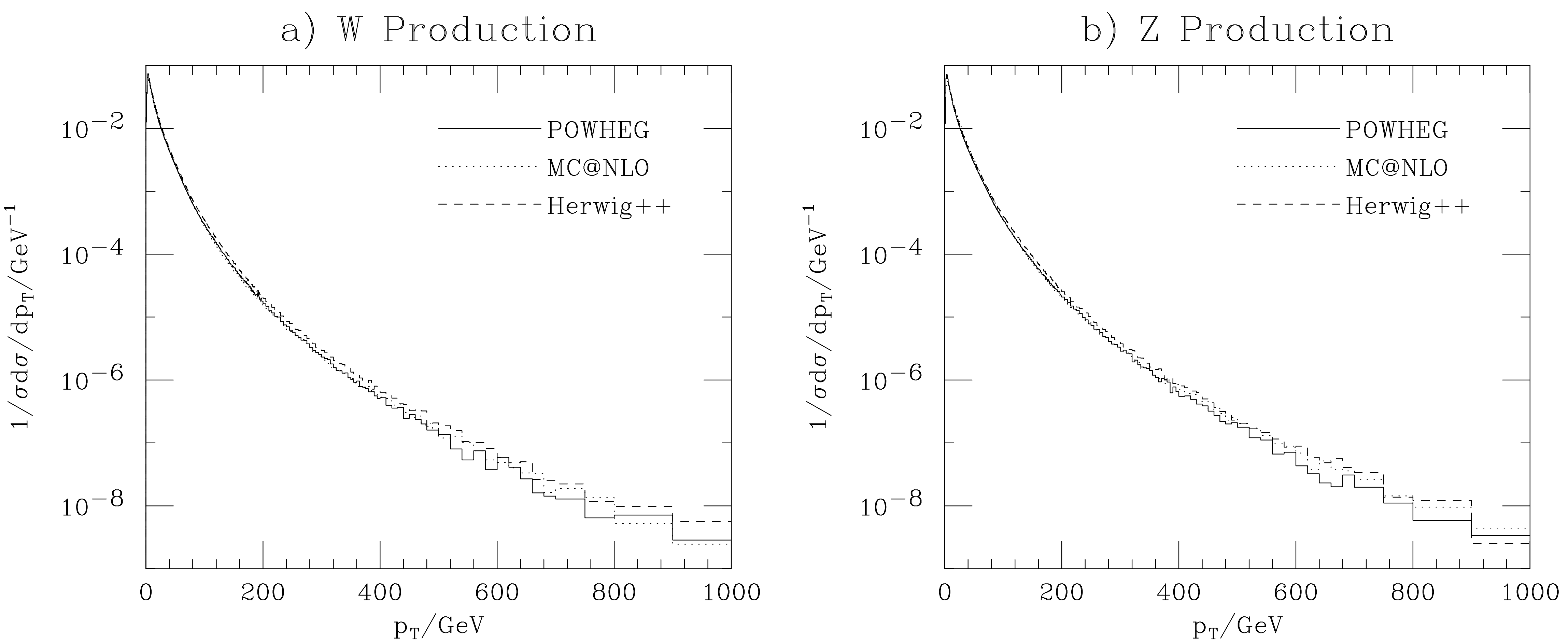
W p_t at LHC (7 TeV)





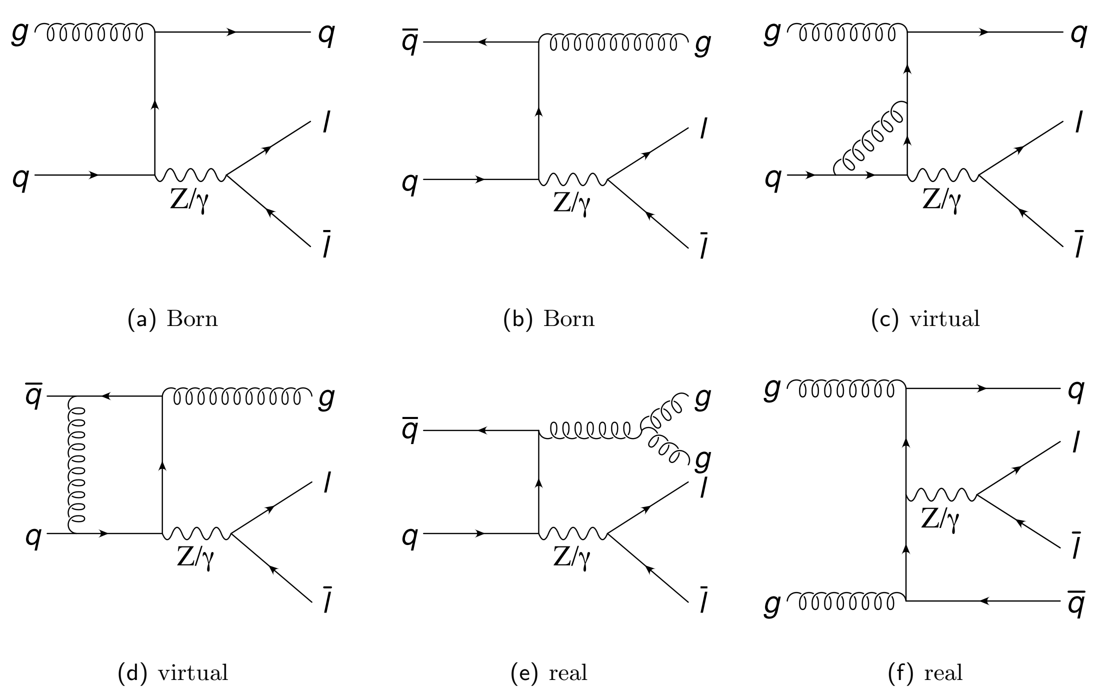
- Again, NLOPS deficit at high p_T ?

W & Z^0 p_t at LHC (14 TeV)

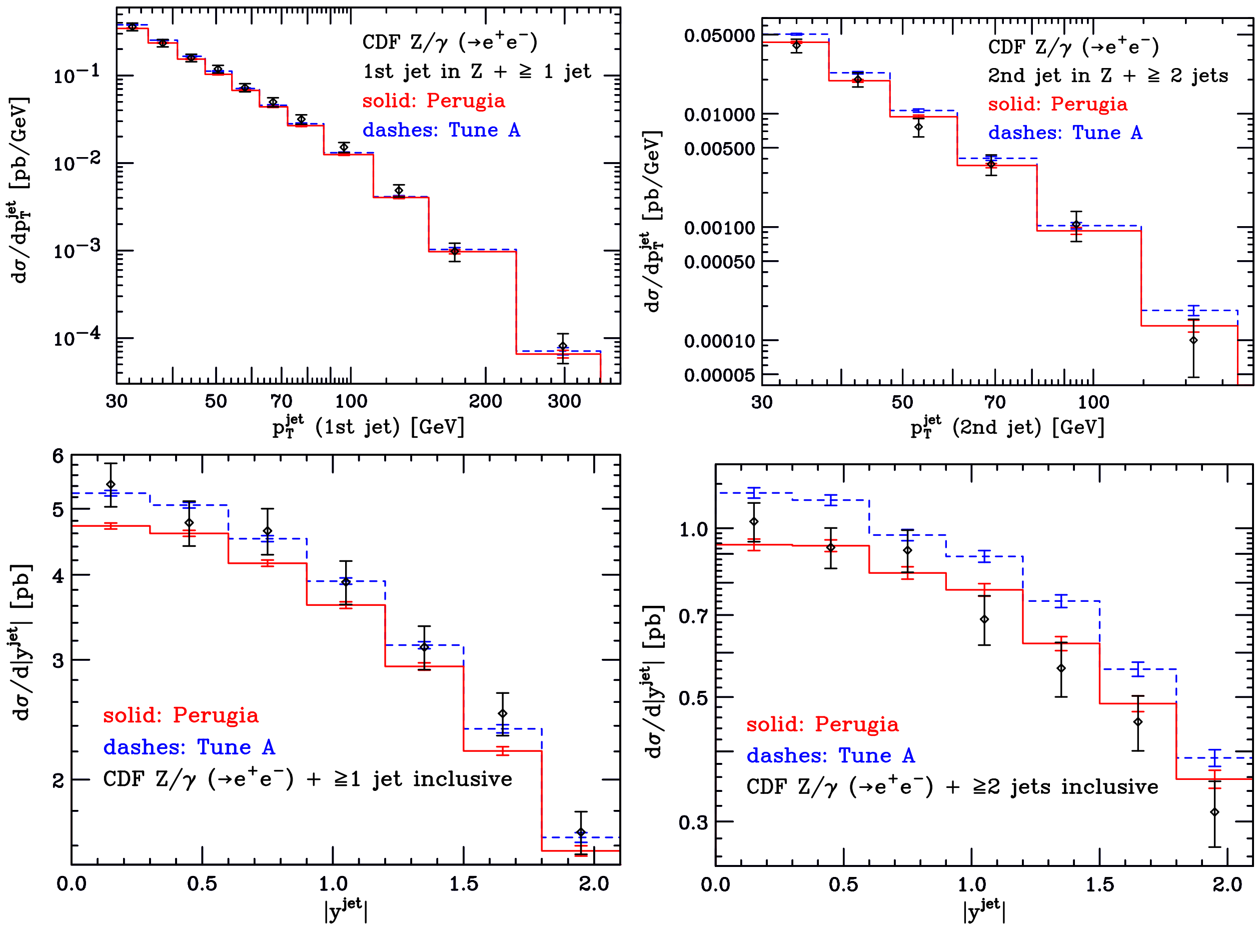


- MC@NLO & POWHEG still in fair agreement at 14 TeV

Z^0+1 jet POWHEG



Sample graphs



- Cut now needed on 'underlying Born' p_t of Z^0
- Good agreement with CDF (not so good with D0)
- First jet is now NLO, second is LO (times $\bar{B}/B$ ...)

Alioli, Nason, Oleari, Re, 1009.5594

Summary of Lecture 1

- Matching PSMC & NLO means NLO must be modified to avoid double counting
- MC@NLO: modify subtraction method
  - Some negative weights (“counter-events”)
  - Close to NLO at high p_T
- POWHEG: generate hardest emission at NLO
  - (Almost) no negative weights
  - May enhance high p_T
- Both describe Tevatron data well
  - Some high p_T deficits at LHC?