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Double Parton Scattering Theory

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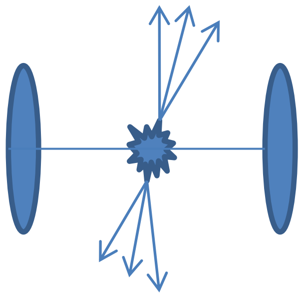
QCD@LHC 2011, St. Andrews, Scotland, 22nd August 2011

Based on work performed in collaboration with W. J. Stirling

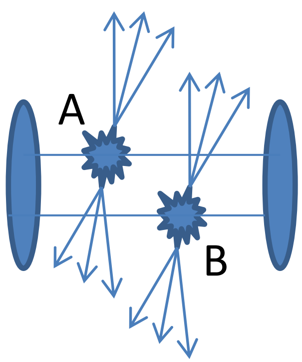
Outline

- Introduction to double parton scattering, and summary of the state of the subject prior to developments over the past year. 'dPDF framework' of Snigirev for describing DPS.
- Summary of our work showing that a compact analytic expression can be obtained for the 'DPS singular' part of a one-loop diagram. Use of this expression to show that dPDF framework appears to be treating the 'double parton splitting' or '1v1' part of the DPS cross section wrongly.
- What is wrong with the dPDF framework? Discussion of how the '1v1' piece should be treated correctly.
- How should the 'single parton splitting' or '2v1' piece be treated?
- Summary

Double Parton Scattering



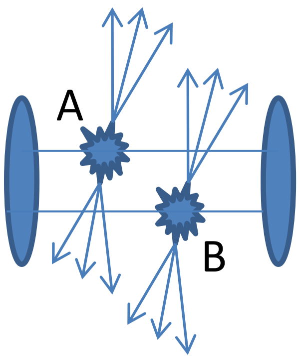
In the standard theoretical framework for p-p scattering, we assume that a given collection of hard outgoing particles can only have been produced from the collision of two partons, one from either proton. This is single parton scattering (SPS).



However, for certain final states, the possibility exists that the final state could have been produced as the result of two independent hard scatterings (**double parton scattering, or DPS**).

The total cross section for a given final state {A,B} to be produced via DPS is power suppressed with respect to the single scattering cross section. However, in the kinematic region where the transverse momenta of the particle subsets A and B is small, DPS gives a comparable contribution to SPS. Has been observed at the Tevatron [CDF Coll., Phys. Rev. D56 (1997) 3811, D0 Coll., Phys. Rev. D81 (2010) 052012].

Cross Section for DPS



Assuming only the factorisation of the hard processes A and B, the DPS cross section may be written as:

$$\sigma_D^{(A,B)} = \overset{\text{Symmetry factor}}{\frac{m}{2}} \sum_{i,j,k,l} \int \overbrace{\Gamma_h^{ik}\left(x_1, x_2, \mathbf{b}; Q_A, Q_B\right)\Gamma_h^{jl}\left(x_1', x_2', \mathbf{b}; Q_A, Q_B\right)}^{\text{Two-parton generalised PDF (2pGPD)}}$$
$$\times \underbrace{\hat{\sigma}_{ij}^{A}\left(x_1, x_1'\right)\hat{\sigma}_{kl}^{B}\left(x_2, x_2'\right)}_{\text{Parton level cross sections}} \mathrm{d}x_1\, \mathrm{d}x_1' \mathrm{d}x_2\, \mathrm{d}x_2' \mathrm{d}^2\mathbf{b}$$

The vector **b** in the 2pGPDs corresponds to the vector separation in transverse space between the two partons described by the 2pGPD.

DPS differs from SPS in that the cross section may not naturally be expressed in terms of PDFs depending only on *x* arguments. The 2pGPDs in the DPS cross section must share a common **b** in order that both pairs of partons can meet and interact – one cannot integrate independently over the transverse separation arguments of each PDF and obtain PDFs depending only on *x* arguments, as one can in the SPS case.

Assumptions made in Phenomenological Studies

1. Take Γ to be a product of longitudinal and transverse pieces .

$$\Gamma_h^{ik}(x_1, x_2, b; Q_A, Q_B) = D_h^{ik}(x_1, x_2; Q_A, Q_B)\, F_k^i(b)$$

Double parton distribution functions (dPDFs)

Parton pair density in transverse space (exponential or dipole form of radius R_p).

2. Assume that F does not depend on parton indices – i.e.

$$F_k^i(b) = F(b)$$

Then, if we define $\sigma_{eff} = \dfrac{1}{\int [F(b)]^2 d^2 b}$ we may write DPS cross section as:

$$\sigma_D^{(A,B)} = \frac{m}{2\sigma_{eff}} \sum_{i,j,k,l} \int D_h^{ik}(x_1, x_2; Q_A, Q_B) D_h^{jl}(x_1', x_2'; Q_A, Q_B)$$
$$\times \hat{\sigma}_{ij}^A(x_1, x_1') \hat{\sigma}_{kl}^B(x_2, x_2') \mathrm{d}x_1 \mathrm{d}x_1' \mathrm{d}x_2 \mathrm{d}x_2'$$

3. Neglect longitudinal correlations

$$D_h^{ij}(x_1, x_2; Q_A, Q_B) \approx D_h^i(x_1; Q_A) D_h^j(x_2; Q_B)$$
$$\Rightarrow \sigma_D^{(A,B)} \approx \frac{m}{2} \frac{\sigma_S^{(A)} \sigma_S^{(B)}}{\sigma_{eff}}$$

The ‘dPDF framework’

An alternative, supposedly more rigorous framework for calculating the p-p DPS cross section for the case $Q_A = Q_B = Q$ was proposed a number of years ago by Snigirev. He suggested that assumptions 1 and 2 on the previous slide approximately hold, such that the DPS cross section may be described in terms of dPDFs.

The dPDFs in this framework evolve according to the ‘double DGLAP’ equation obtained by Shelest, Snigirev, and Zinovjev in 1982 [Phys. Lett. B 113:325]. At leading order, equation is:

$$\frac{dD_h^{j_1j_2}(x_1,x_2;t)}{dt} = \frac{\alpha_s(t)}{2\pi}\Bigg[\sum_{j_1'}\int_{x_1}^{1-x_2}\frac{dx_1'}{x_1'}D_h^{j_1'j_2}(x_1',x_2;t)P_{j_1'\to j_1}\left(\frac{x_1}{x_1'}\right)$$

$$+\sum_{j_2'}\int_{x_2}^{1-x_1}\frac{dx_2'}{x_2'}D_h^{j_1j_2'}(x_1,x_2';t)P_{j_2'\to j_2}\left(\frac{x_2}{x_2'}\right)$$

$$+\sum_{j'}D_h^{j'}(x_1+x_2;t)\frac{1}{x_1+x_2}P_{j'\to j_1j_2}\left(\frac{x_1}{x_1+x_2}\right)\Bigg]$$

$$\boxed{t = \ln\left(Q^2\right)}$$

Usual 1→1 splitting functions

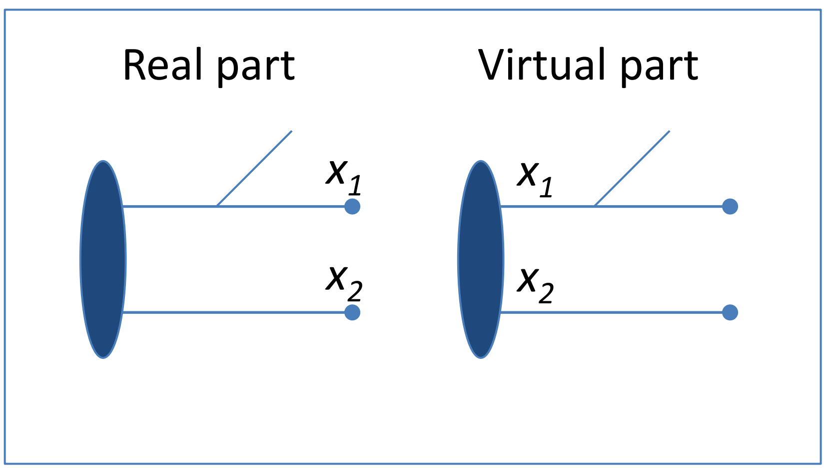
‘1→2’ splitting function

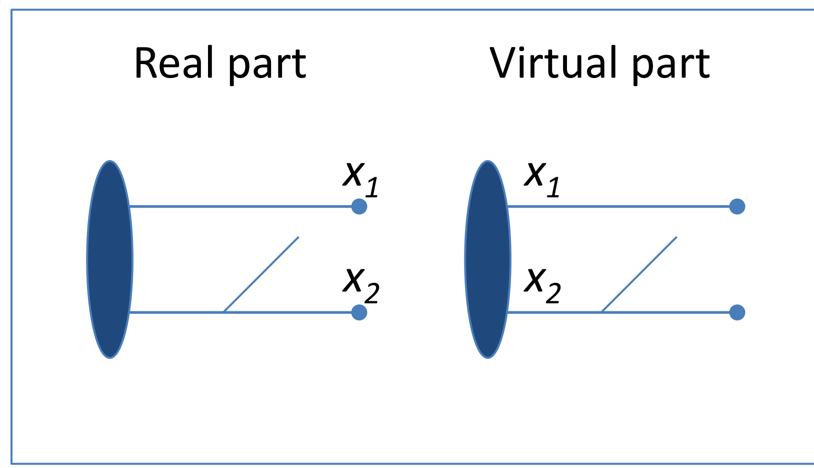
Single PDF

Pictorial Representation of the dDGLAP equation

$$\frac{dD_h^{j_1 j_2}(x_1,x_2;t)}{dt} = \frac{\alpha_s(t)}{2\pi}\Bigg[\boxed{\begin{aligned}&\sum_{j_1'}\int_{x_1}^{1-x_2}\frac{dx_1'}{x_1'}D_h^{j_1' j_2}(x_1',x_2;t)P_{j_1'\to j_1}\left(\frac{x_1}{x_1'}\right)\\ &+\sum_{j_2'}\int_{x_2}^{1-x_1}\frac{dx_2'}{x_2'}D_h^{j_1 j_2'}(x_1,x_2';t)P_{j_2'\to j_2}\left(\frac{x_2}{x_2'}\right)\end{aligned}}$$
$$+\sum_{j'}D_h^{j'}(x_1+x_2;t)\frac{1}{x_1+x_2}P_{j'\to j_1 j_2}\left(\frac{x_1}{x_1+x_2}\right)\Bigg]$$

'Independent branching' terms

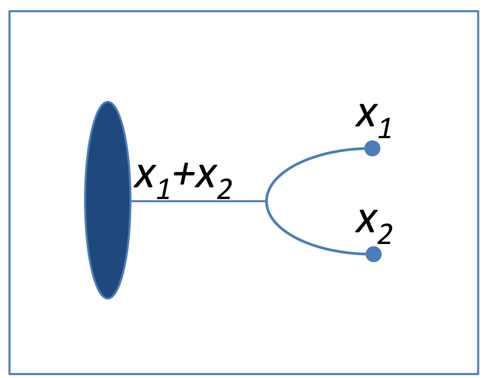




Pictorial Representation of the dDGLAP equation

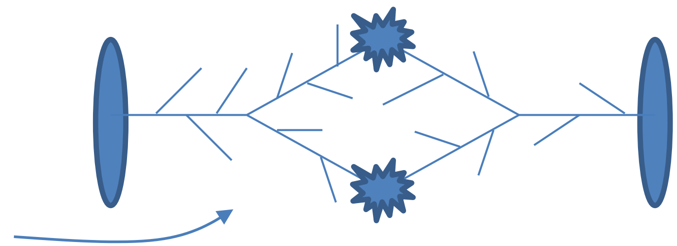
$$\begin{aligned}
\frac{dD_h^{j_1j_2}(x_1,x_2;t)}{dt} = \frac{\alpha_s(t)}{2\pi}\Bigg[&\sum_{j_1'}\int_{x_1}^{1-x_2}\frac{dx_1'}{x_1'}D_h^{j_1'j_2}(x_1',x_2;t)P_{j_1'\to j_1}\left(\frac{x_1}{x_1'}\right)\\
&+\sum_{j_2'}\int_{x_2}^{1-x_1}\frac{dx_2'}{x_2'}D_h^{j_1j_2'}(x_1,x_2';t)P_{j_2'\to j_2}\left(\frac{x_2}{x_2'}\right)\\
&\boxed{+\sum_{j'}D_h^{j'}(x_1+x_2;t)\frac{1}{x_1+x_2}P_{j'\to j_1j_2}\left(\frac{x_1}{x_1+x_2}\right)}\Bigg]
\end{aligned}$$

‘sPDF feed’ term



‘1v1’ contributions to DPS?

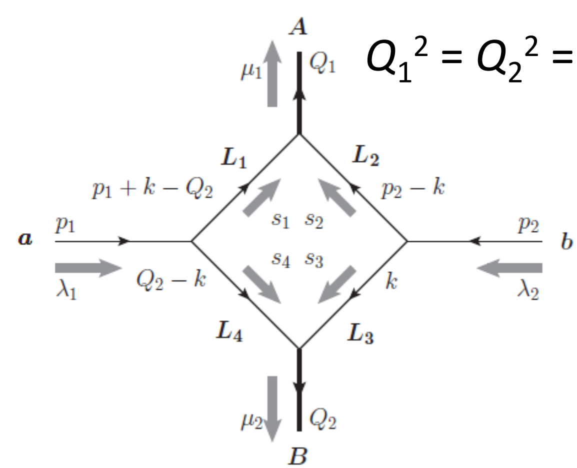
Under dPDF framework, there is a part of the DPS cross section that comes from multiplying two accumulated sPDF feed contributions together. Pictorially this piece looks like this:



According to the dPDF framework then, there is some part of this loop diagram which at the cross section level contains a DGLAP large logarithm $\alpha_s \log(Q^2/\Lambda^2)$ for **every** splitting (Λ being an IR cut-off). This same piece should contain all of the appropriate splitting functions convolved together.

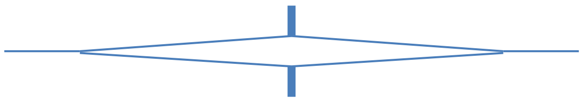
$Q_1^2 = Q_2^2 = Q^2 > 0$

Simplest example of a loop diagram with the given structure is the crossed box diagram on the right. Does the cross section expression for this contain a piece proportional to $(\alpha_s \log(Q^2/\Lambda^2))^2$?



Double Parton Scattering Singularity

We would expect this piece to come from the portion of the external and loop integrations in which the transverse momenta of the outgoing particles are small, and all internal loop particles are almost on shell and collinear.



This is the region around a specific pinch singularity (Landau singularity) in the crossed box integral known as the double parton scattering singularity.

DPS divergent part of a crossed box integral = expression for the part of the integral associated with the loop particles being almost on-shell and collinear, valid in the limit of small external transverse momenta. Expect this to diverge as external p_Ts $\rightarrow$ 0.

We have derived a simple analytical expression for the DPS divergent part of a crossed box diagram with arbitrary external and loop particles [JG and Stirling, JHEP 1106 048 (2011)]. Let us show how it is derived.

Derivation of DPS Divergence in Crossed Box

Numerator – depends on nature of particles in diagram

$$L = \int d^d k \frac{\mathcal{N}}{[k^2 + i\epsilon][(k - Q_2)^2 + i\epsilon][(p_1 + k - Q_2)^2 + i\epsilon][(p_2 - k)^2 + i\epsilon]}$$

Loop propagator denominators – universal to all crossed boxes

Decompose all vectors in terms of light-cone coordinates: $A = A^+ p + A^- n + \boldsymbol{A}$

$$p = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix} \propto p_1 \qquad n = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & -1 \end{pmatrix} \propto p_2$$

$$L = \int d^{d-2}\mathbf{k}\, dk^+ dk^- \frac{\mathcal{N}}{(2k^+k^- - \mathbf{k}^2 + i\epsilon)[2(k^+ - Q_2^+)(k^- - Q_2^-) - (\mathbf{k} - \mathbf{Q_2})^2 + i\epsilon]}$$
$$\times \frac{1}{[2(k^+ + Q_1^+)(k^- - Q_2^-) - (\mathbf{k} - \mathbf{Q_2})^2 + i\epsilon][2k^+(k^- - Q_1^- - Q_2^-) - \mathbf{k}^2 + i\epsilon]}$$

Derivation of DPS Divergence in Crossed Box

Perform k^- integration by contour method:

$$k_2^- = Q_2^- + \frac{(\boldsymbol{k}-\boldsymbol{Q})^2}{2(k^+ - Q_2^+)}$$

$$k_3^- = Q_2^- + \frac{(\boldsymbol{k}-\boldsymbol{Q})^2}{2(k^+ + Q_1^+)}$$

$$\begin{aligned}
L_{DPS} = &-2\pi i \int_{|\boldsymbol{k}|\ll Q_i^+,Q_i^-} d^{d-2}\boldsymbol{k} \int_{-Q_1^+}^{0} dk^+ \frac{\mathcal{N}\,|_{k^-=k_3^-}}{\left(2k^+Q_2^- + \frac{k^+(\boldsymbol{k}-\boldsymbol{Q_2})^2}{(k^++Q_1^+)} - \boldsymbol{k}^2 + i\epsilon\right)} \\
&\times \frac{1}{2(-Q_1^+ - Q_2^+)(\boldsymbol{k}-\boldsymbol{Q_2})^2\left(-2k^+Q_1^- + \frac{k^+(\boldsymbol{k}-\boldsymbol{Q_2})^2}{(k^++Q_1^+)} - \boldsymbol{k}^2 + i\epsilon\right)} \\
&+2\pi i \int_{|\boldsymbol{k}|\ll Q_i^+,Q_i^-} d^2\boldsymbol{k} \int_{0}^{Q_2^+} dk^+ \frac{\mathcal{N}\,|_{k^-=k_2^-}}{\left(2k^+Q_2^- + \frac{k^+(\boldsymbol{k}-\boldsymbol{Q_2})^2}{(k^+-Q_2^+)} - \boldsymbol{k}^2 + i\epsilon\right)} \\
&\times \frac{1}{2(Q_1^+ + Q_2^+)(\boldsymbol{k}-\boldsymbol{Q_2})^2\left(-2k^+Q_1^- + \frac{k^+(\boldsymbol{k}-\boldsymbol{Q_2})^2}{(k^+-Q_2^+)} - \boldsymbol{k}^2 + i\epsilon\right)}
\end{aligned}$$

Two terms corresponding to different k^+ ranges.

Derivation of DPS Divergence in Crossed Box

Perform k^- integration by contour method:

$$
\begin{aligned}
L_{DPS} = & -2\pi i \int_{|\boldsymbol{k}|\ll Q_i^+,Q_i^-} d^{d-2}\boldsymbol{k} \int_{-Q_1^+}^{0} dk^+ \frac{\mathcal{N}\,|_{k^-=k_3^-}}{\left(2k^+Q_2^- + \cancel{\frac{k^+(\boldsymbol{k}-\boldsymbol{Q_2})^2}{(k^++Q_1^+)}} - \boldsymbol{k}^2 + i\epsilon\right)} \\
& \times \frac{1}{2(-Q_1^+ - Q_2^+)(\boldsymbol{k}-\boldsymbol{Q_2})^2\left(-2k^+Q_1^- + \cancel{\frac{k^+(\boldsymbol{k}-\boldsymbol{Q_2})^2}{(k^++Q_1^+)}} - \boldsymbol{k}^2 + i\epsilon\right)} \\
& +2\pi i \int_{|\boldsymbol{k}|\ll Q_i^+,Q_i^-} d^{2}\boldsymbol{k} \int_{0}^{Q_2^+} dk^+ \frac{\mathcal{N}\,|_{k^-=k_2^-}}{\left(2k^+Q_2^- + \cancel{\frac{k^+(\boldsymbol{k}-\boldsymbol{Q_2})^2}{(k^+-Q_2^+)}} - \boldsymbol{k}^2 + i\epsilon\right)} \\
& \times \frac{1}{2(Q_1^+ + Q_2^+)(\boldsymbol{k}-\boldsymbol{Q_2})^2\left(-2k^+Q_1^- + \cancel{\frac{k^+(\boldsymbol{k}-\boldsymbol{Q_2})^2}{(k^+-Q_2^+)}} - \boldsymbol{k}^2 + i\epsilon\right)}
\end{aligned}
$$

$$k_2^- = Q_2^- + \cancel{\frac{(\boldsymbol{k}-\boldsymbol{Q})^2}{2(k^+-Q_2^+)}}$$

$$k_3^- = Q_2^- + \cancel{\frac{(\boldsymbol{k}-\boldsymbol{Q})^2}{2(k^++Q_1^+)}}$$

Crossed out terms are all negligible compared to accompanying terms in region around DPS singular point, where:

$$k^+, |\mathbf{k}-\mathbf{Q}_2|, |\mathbf{k}| << Q_i^+, Q_i^-$$

$$
\begin{aligned}
L_{DPS} \simeq & \frac{2\pi i}{2(Q_1^+ + Q_2^+)} \int_{|\boldsymbol{k}|\ll Q_i^+,Q_i^-} \frac{d^{d-2}\boldsymbol{k}}{(\boldsymbol{k}-\boldsymbol{Q_2})^2} \\
& \times \int_{-Q_1^+}^{Q_2^+} dk^+ \frac{\mathcal{N}\,|_{k^-=Q_2^-,k^+=0}}{\left(2k^+Q_2^- - \boldsymbol{k}^2 + i\epsilon\right)\left(-2k^+Q_1^- - \boldsymbol{k}^2 + i\epsilon\right)}
\end{aligned}
$$

Derivation of DPS Divergence in Crossed Box

$$L_{DPS} \simeq \frac{2\pi i}{2(Q_1^+ + Q_2^+)} \int_{|\boldsymbol{k}| \ll Q_i^+, Q_i^-} \frac{d^{d-2}\boldsymbol{k}}{(\boldsymbol{k} - \boldsymbol{Q_2})^2}$$
$$\times \int_{-Q_1^+}^{Q_2^+} dk^+ \frac{\mathcal{N}\,|_{k^- = Q_2^-, k^+ = 0}}{\left(2k^+ Q_2^- - \boldsymbol{k}^2 + i\epsilon\right)\left(-2k^+ Q_1^- - \boldsymbol{k}^2 + i\epsilon\right)}$$

Integral strongly peaked around $k^+ = 0$

$$L_{DPS} \simeq \frac{2\pi i}{2(Q_1^+ + Q_2^+)} \int_{|\boldsymbol{k}| \ll Q_i^+, Q_i^-} \frac{d^{d-2}\boldsymbol{k}}{(\boldsymbol{k} - \boldsymbol{Q_2})^2}$$
$$\times \int_{-\infty}^{\infty} dk^+ \frac{\mathcal{N}\,|_{k^- = Q_2^-, k^+ = 0}}{\left(2k^+ Q_2^- - \boldsymbol{k}^2 + i\epsilon\right)\left(-2k^+ Q_1^- - \boldsymbol{k}^2 + i\epsilon\right)}$$

Perform k^+ integration by contour method

$$L_{DPS} \simeq \frac{(2\pi i)^2}{2s} \int_{|\boldsymbol{k}| \ll Q_i^+, Q_i^-} \frac{d^{d-2}\boldsymbol{k}\,\mathcal{N}\,|_{k^- = Q_2^-, k^+ = 0}}{(\boldsymbol{k} - \boldsymbol{Q_2})^2 \boldsymbol{k}^2}$$

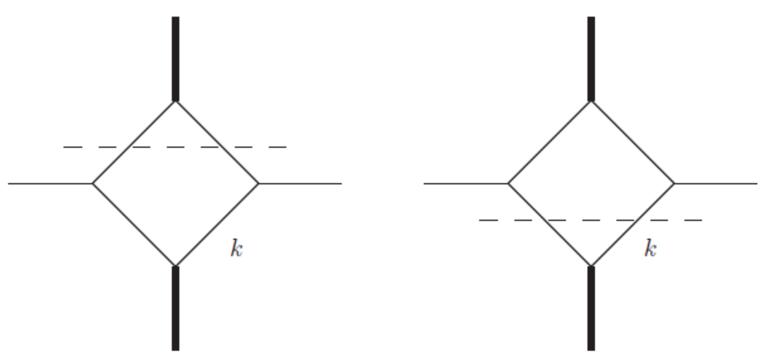
Compact expression!

Cutkosky cuts of the box

$$L_{DPS} \simeq \frac{(2\pi i)^2}{2s} \int_{|\boldsymbol{k}| \ll Q_i^+, Q_i^-} \frac{d^{d-2}\boldsymbol{k}\, \mathcal{N}\,|_{k^-=Q_2^-, k^+=0}}{(\boldsymbol{k}-\boldsymbol{Q_2})^2 \boldsymbol{k}^2}$$

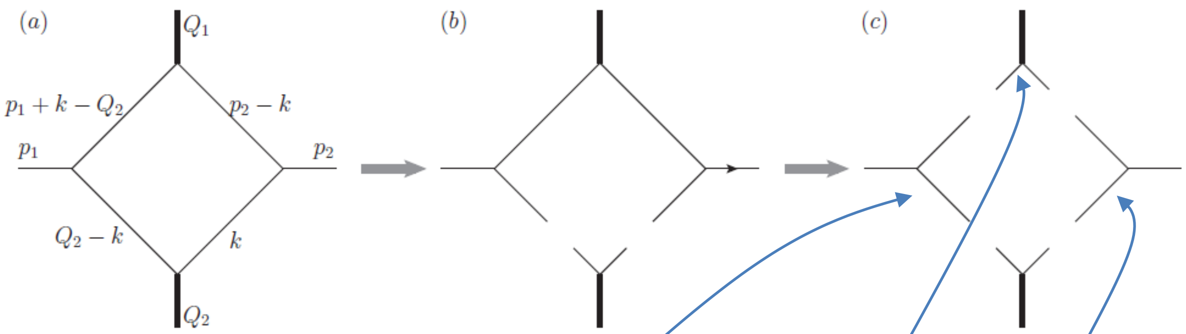
DPS divergence is in the real part of box integral – i.e. imaginary part of box amplitude since $i\mathcal{M} \propto L$

DPS divergent part of loop integral can also be found by taking sum of cuts in limit where external transverse momenta are small and internal particles are almost on shell.



Two cuts give the same contribution.

Decomposition of DPS divergent part of Crossed Box



$$L_{DPS}(\lambda_1\lambda_2\mu_1\mu_2) = \sum_{s_i,L_i}\int d^dk\,\delta(k^2)\delta((k-Q_2)^2)\Phi^{\lambda_2\to s_2s_3}_{b\to L_2L_3}(p_2;p_2-k,k)$$
$$\times\Phi^{\lambda_1\to s_1s_4}_{a\to L_1L_4}(p_1;p_1+k-Q_2,Q_2-k)\mathcal{M}^{s_3s_4\to\mu_2}_{L_3L_4\to B}(k,Q_2-k;Q_2)$$
$$\times\mathcal{M}^{s_1s_2\to\mu_1}_{L_1L_2\to A}(p_1+k-Q_2,p_2-k;Q_1)\left(\times\sqrt{Q_1^2/Q_2^2}\right)$$

Hard matrix elements – can evaluate with incoming off-shellness and transverse momentum = 0

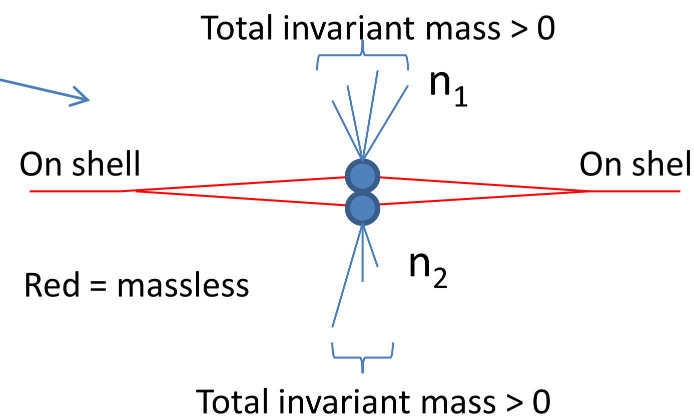
'Light-cone wavefunction to find L_2L_3 in b'

DPS divergent part of arbitrary one-loop integral

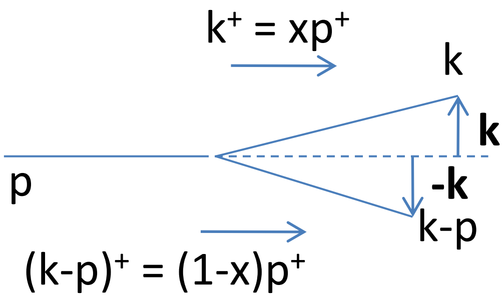
For crossed box: $$L_{DPS}(\lambda_1\lambda_2\mu_1\mu_2) = \sum_{s_i, L_i} \int d^d k \delta(k^2)\delta((k-Q_2)^2)\Phi^{\lambda_2 \to s_2 s_3}_{b \to L_2 L_3}(p_2; p_2 - k, k)$$
$$\times \Phi^{\lambda_1 \to s_1 s_4}_{a \to L_1 L_4}(p_1; p_1 + k - Q_2, Q_2 - k)\mathcal{M}^{s_3 s_4 \to \mu_2}_{L_3 L_4 \to B}(k, Q_2 - k; Q_2)$$
$$\times \mathcal{M}^{s_1 s_2 \to \mu_1}_{L_1 L_2 \to A}(p_1 + k - Q_2, p_2 - k; Q_1) \left(\times \sqrt{Q_1^2 / Q_2^2} \right)$$

Any one-loop diagram of this structure also has a DPS divergence.

To obtain DPS divergent part of an arbitrary one-loop diagram (of the appropriate character), replace 2→1 matrix elements by 2→n_1, 2→n_2 matrix elements above.



Light-cone wavefunctions



$$\Phi_{a\to bc}^{\lambda_1\to s_1 s_2}\left(p;k,k-p\right)=X_{a\to bc}^{\lambda_1\to s_1 s_2}\left(x\right)K_{a\to bc}^{\lambda_1\to s_1 s_2}\left(\mathbf{k}\right)$$

Square root of helicity dependent splitting function!

Transverse momentum dependent factor *K* contains a 1/$\mathbf{k}^2$ factor from propagator denominator, multiplied by a further factor coming from splitting matrix element.

Scalar ϕ^3 theory : splitting matrix element doesn't depend on **k.** For an arbitrary loop:

$$L_{DPS,\phi^3} \sim \int d^{d-2}\mathbf{k}\, K\left(\mathbf{k}-\mathbf{Q_2}\right)K\left(\mathbf{k}\right) \propto \int \frac{d^{d-2}\mathbf{k}}{\mathbf{k}^2\left(\mathbf{k}-\mathbf{Q_2}\right)^2} \propto \frac{1}{\mathbf{Q}_2^2} \quad \text{when d=4}$$

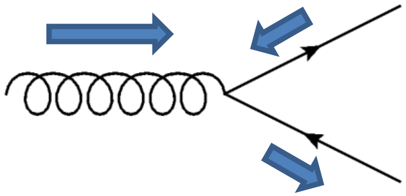
Power divergence at the DPS singular point which is unintegrable at the cross section level.

$$\int \frac{d^2\mathbf{Q}}{\mathbf{Q}^4} \to \infty$$

Light-cone wavefunctions

For any Standard Model massless particle splitting, matrix element is proportional to **k**.

Can show where this comes from for e.g. g $\longrightarrow$ q$\bar{q}$ graph:



Helicity conservation ➔ J_z of final state = 0 in collinear limit

J_z of initial state $=\pm 1$ ➔ splitting must be suppressed in collinear limit.

$$L_{DPS,SM} \sim \int d^{d-2}\mathbf{k}\, K(\mathbf{k}-\mathbf{Q}_2) K(\mathbf{k}) \sim \int_0^{Q^2} \frac{d^{d-2}\mathbf{k}\; \mathbf{k}\cdot(\mathbf{k}-\mathbf{Q}_2)}{\mathbf{k}^2(\mathbf{k}-\mathbf{Q}_2)^2} \sim \log\left(\frac{\mathbf{Q}_2^2}{Q^2}\right)$$

Strongly related to logarithmic scaling violations of parton distributions

Therefore the DPS divergence in any Standard Model loop diagram is always integrable!

$$\int_0^{Q^2} d^2\mathbf{Q}_2 \log^2\left(\frac{\mathbf{Q}_2^2}{Q^2}\right) = 2Q^2$$

No large logarithms dependent on the IR cut-off Λ! Seems to be inconsistent with prediction from dPDF framework.

What was wrong with the dPDF framework?

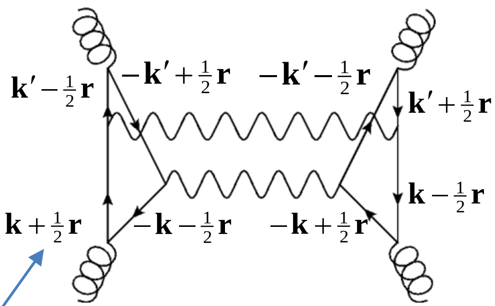
Contribution to pp ⟶ AB + X cross section coming from DPS divergent part of gg ⟶ AB crossed box:

$$\sigma_{pp\to gg\to AB+X,DPS}(s) = \int dX d\bar{X}\, f_g(X) f_g(\bar{X}) \hat{\sigma}_{gg\to AB,DPS}(\hat{s} = sX\bar{X})$$

$$\hat{\sigma}_{gg\to AB,DPS}(s) \propto \frac{1}{s}\int d^4q_1 d^4q_2 \delta(q_1^2 - M^2)\delta(q_2^2 - M^2)\delta^{(4)}(q_1 + q_2 - p_1 - p_2) \times |L_{DPS,gg\to AB}|^2$$

Insert expression for L_{DPS} and perform a number of changes of variable:

$$\sigma_{pp\to gg\to AB+X,DPS} \propto \int \prod_{i=1}^{2} dx_i d\bar{x}_i \hat{\sigma}_{q\bar{q}\to A}(\hat{s} = x_1\bar{x}_1 s)\hat{\sigma}_{q\bar{q}\to B}(\hat{s} = x_2\bar{x}_2 s) \times \int \frac{d^2\boldsymbol{r}}{(2\pi)^2}\Gamma_{q\bar{q}}|_{g\to q\bar{q}}(x_1, x_2, \boldsymbol{r})\Gamma_{q\bar{q}}|_{g\to q\bar{q}}(\bar{x}_1, \bar{x}_2, -\boldsymbol{r})$$



Have absorbed single PDFs and light-cone wavefunction parts of $|L|^2$ into these factors resembling 2pGPDs.

r = loop transverse momentum imbalance between amplitude and conjugate. This is the Fourier transform of the transverse separation between partons **b.**

What was wrong with the dPDF framework?

Write **r**-space 2pGPDs as Fourier transforms of **b**-space 2pGPDs, and then simplify expression:

$$\sigma_{pp\to gg\to AB+X,DPS} \propto \int \prod_{i=1}^{2} dx_i d\bar{x}_i \hat{\sigma}_{q\bar{q}\to A}(\hat{s} = x_1\bar{x}_1 s)\hat{\sigma}_{q\bar{q}\to B}(\hat{s} = x_2\bar{x}_2 s)$$
$$\times \int d^2\boldsymbol{b}\, \Gamma_{q\bar{q}}|_{g\to q\bar{q}}(x_1, x_2, \boldsymbol{b})\Gamma_{q\bar{q}}|_{g\to q\bar{q}}(\bar{x}_1, \bar{x}_2, \boldsymbol{b})$$

Looks exactly like the expression that we wrote down at the beginning of this talk! **However,** the ***b*** dependence of the Γ functions above is very different from an exponential or dipole form. In fact, for small ***b***:

$$\Gamma_{qq}\Big|_{g\to q\bar{q}}(x_1, x_2, \mathbf{b}) \sim 1/\mathbf{b}^2$$

Power law behaviour!

There is clearly a problem with assuming that the 2pGPDs factorise into a dPDF and a smooth function of size R_p. Need to take account of **b** (or **r**) dependence.

See also Diehl and Schafer [Phys. Lett. B 698 389 (2011)].

What ‘1v1’ contribution should one include in the DPS cross section?

Our proposal in JHEP 1106 048 was to remove the 1v1 contribution from DPS entirely. Also suggested by Blok, Dokshitzer, Frankfurt and Strikman (BDFS) [arXiv: 1106.5533].

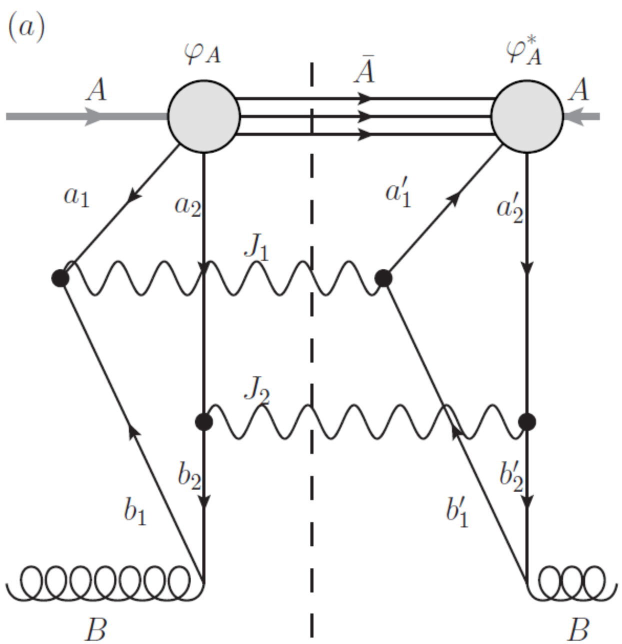
Our motivation for making this proposal was our finding that the 1v1 graphs do not contain large DGLAP logarithms at the cross section level → the main contribution to these graphs comes from the region in which transverse momenta (and virtualities) inside the loop are of order of the hard scale → these graphs should be regarded as pure SPS.

The motivation of the BDFS group was that the differential distribution for the 1v1 graphs to produce final states {A,B} is not peaked at small p_T of A and B, which is essentially the definition of what DPS is. Rather, it should be considered as a smooth background to DPS.

The two lines of reasoning are in fact very closely linked.

No double counting problem with SPS. Maybe the only sensible way of dealing with the 1v1 contribution?

What about the 2v1 contribution?



Take a similar approach as we did for the 1v1 graphs. Look at the simplest graph in which a single parton splits and then interacts with two 'independent' partons from a proton, and see if there is a large logarithm.

Need to use a wavefunction on the side with the two independent partons to represent the fact that the two partons are tied together in the same proton (see also BDFS [arXiv: 1106.5533]).

$$\sigma_{1v2}(s) = \sum_{s_i s_i' t_i t_i'} \int dx_1 dx_2 dy_1 dy_2 \hat{\sigma}_{\bar{q}q\to\gamma*}^{s_1,t_1;s_1',t_1';\mu_1}(\hat{s}=x_1y_1s)\hat{\sigma}_{q\bar{q}\to\gamma*}^{s_2,t_2;s_2',t_2';\mu_2}(\hat{s}=x_2y_2s)$$

$$\times \quad \Gamma_A^{s_1s_2,s_1's_2'}(x_1,x_2;\boldsymbol{b}=\boldsymbol{0}) \quad \Big[\underbrace{\frac{\alpha_s}{2\pi} P_{g\to q\bar{q}}^{\lambda\to t_2t_1,t_2't_1'}(y_2)\,\delta(1-y_1-y_2)}_{1 \longrightarrow 2 \text{ splitting function}} \int_{\Lambda^2}^{Q^2} \frac{d\boldsymbol{J_1}^2}{\boldsymbol{J_1}^2} \Big]$$

Required large logarithm

2pGPD of independent partons probed at **b** = **0**!

What about the 2v1 contribution?

As 2v1 part of DPS probes ***b*** distribution of independent partons differently from 2v2 part, σ_{eff} for 2v2 and 1v2 are different:

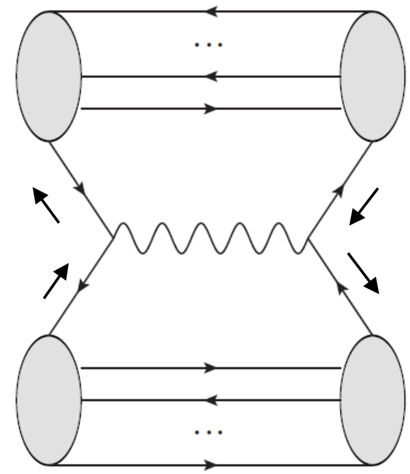
$$\frac{1}{\sigma_{eff,2v2}} \equiv \int d^2\boldsymbol{b}[F(\boldsymbol{b})]^2$$

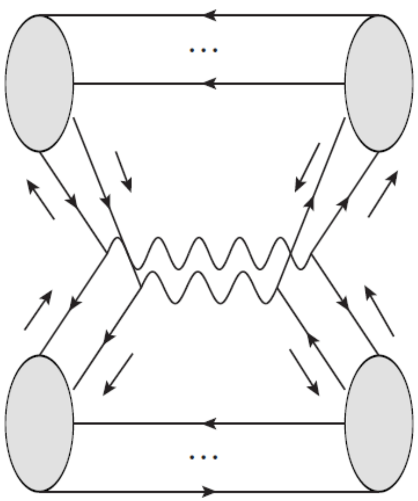
$$\frac{1}{\sigma_{eff,1v2}} \equiv \quad F(\boldsymbol{b}=\boldsymbol{0})$$

With a simple Gaussian form for F(**b**), each 2v1 part of the DPS cross section is enhanced by a factor of two due to the σ_{eff} factor.

Interference contributions to proton-proton DPS

In proton-proton SPS, only one parton leaves each proton, interacts, and then returns
→ interacting parton must return with the same quantum numbers as it left with such that it can recombine with spectators to form original proton
→ No interference contribution to proton-proton SPS





In proton-proton DPS, fact that interacting partons must recombine with spectators to form original proton only imposes conditions on 'sum' of quantum numbers of active partons
→ Possibility of interference diagrams in which flavour, spin or colour are swapped between active partons, provided that a swap in the opposite direction occurs for the other proton.

Polarised PDF contributions to proton-proton DPS

In proton-proton DPS, there exists the possibility of having contributions to the cross section associated with polarized 2pGPDs, *even when the colliding protons are unpolarized*!

Reason for this: there may be correlations in helicity *between* the two active partons!

e.g. $$\Delta q_1 \Delta q_2 = \underbrace{q_1\!\uparrow q_2\!\uparrow + q_1\!\downarrow q_2\!\downarrow}_{\text{Same spin}} - \underbrace{q_1\!\uparrow q_2\!\downarrow - q_1\!\downarrow q_2\!\uparrow}_{\text{Opposing spin}}$$

If probability to find two quarks with same spin differs from probability to find two quarks with opposing spins, $\Delta q_1 \Delta q_2 \neq 0$

Similarly – contributions associated with colour correlations between partons.

Issues of interference & spin/colour correlations raised by Diehl and Schafer [Phys. Lett. B 698 389 (2011)].

Outstanding questions and further work

- We have made a distinction between ‘independent’ parton pairs and those arising from a perturbative splitting. In this picture we need to choose some initial scale Q_0 at which the proton only contains independent pairs, and choose 2pGPDs dictating the distributions of these pairs at this scale. What scale should we choose for Q_0 (presumably something rather close to Λ_{QCD}), and what 2pGPDs should we use at this scale?

- How large are the interference and polarized contributions? A numerical study would be useful, but constructing input forms for interference/polarized 2pGPDs is difficult. One cannot construct approximate input forms for these based on products of single PDFs/GPDs as one does for the unpolarised distributions.

Summary

- We have derived a compact analytical expression for the DPS divergence in an arbitrary one-loop diagram. Using this expression we have shown that the DPS divergent part of a one-loop diagram does not behave as is anticipated by the dPDF framework of Snigirev.
- The majority of the contribution to a 1v1 loop graph comes from the region in which the particles inside the loop have virtualities and transverse momenta of order of the hard scale. Seems to suggest that we should consider these graphs as pure SPS.
- Calculation of a simple 1v2 graph indicates that 1v2 diagrams should be included as part of DPS, but with a different σ_{eff}. Simple model calculations indicate $1/\sigma_{eff}$ for 1v2 graphs is about twice as big as that for 2v2 graphs.
- There are interference and polarised contributions to DPS, even in unpolarised pp scattering.

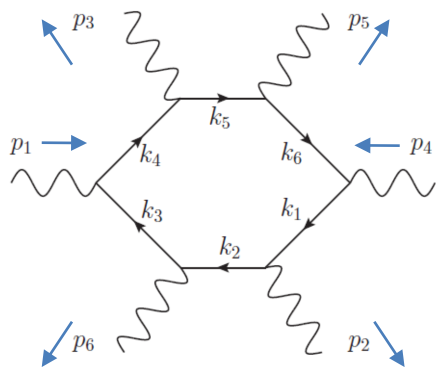
Backup Slides

DPS divergences in Six Photon Amplitude

Our analytical expression for the DPS singularity of a one-loop diagram can be used to **explain** interesting behaviour of amplitudes around DPS singular points that has been observed using 'traditional' NLO multileg integration techniques.

e.g. Six photon amplitude

Take all helicities as incoming, label helicity amplitude as $\lambda_1\lambda_2\ \lambda_3\lambda_4\ \lambda_5\lambda_6$



This is just one diagram contributing to the amplitude, which has a DPS singularity at $\mathbf{P}_\Sigma = \mathbf{p}_3 + \mathbf{p}_5 = 0$

NLO multileg community observed several interesting things about the six photon helicity amplitudes:

1. No helicity amplitude diverges at $\mathbf{P}_\Sigma = 0$ as $1/\mathbf{P}_\Sigma^2$, as was expected by some authors.
2. The NMHV $- - -$ +++ amplitude is finite at $\mathbf{P}_\Sigma = 0$.
3. The MHV $-$ ++ $-$ ++ amplitude is also finite at $\mathbf{P}_\Sigma = 0$.

Bernicot, arXiv:0804.1315,
Bern et. al., arXiv:0803.0494

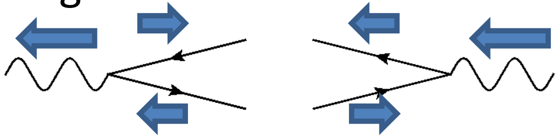
DPS divergences in Six Photon Amplitude

1. No helicity amplitude diverges at $\mathbf{P}_\Sigma = 0$ as $1/\mathbf{P}_\Sigma{}^2$, as was expected by some authors.

Already explained. Associated with angular momentum nonconservation at both $\gamma \longrightarrow q\bar{q}$ vertices in collinear limit.

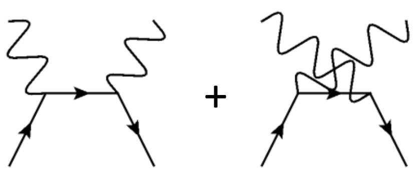
2. The NMHV $- - -\,+++$ amplitude is finite at $\mathbf{P}_\Sigma = 0$.

Overall J_z nonconservation between $\gamma\gamma$ initial state and $q\bar{q}q\bar{q}$ intermediate state in collinear limit weakens DPS divergence such that it is finite.



3. The MHV $-\,++\,-\,++$ amplitude is perfectly finite at $\mathbf{P}_\Sigma = 0$.

There are four graphs giving a DPS divergence at the point $\mathbf{P}_\Sigma = 0$. The matrix elements to be used in the calculation of the DPS divergent parts of the **sum** of these graphs are the sum of the following two graphs:



= full matrix element for $q\bar{q} \longrightarrow \gamma\gamma$. For MHV amplitude studied, photons have same helicity in both matrix elements, and go to zero by MHV rules for QED.