PHYSICS AT THE TERA SCALE

Helmholtz Alliance

Parton Shower and Multi Parton Interactions

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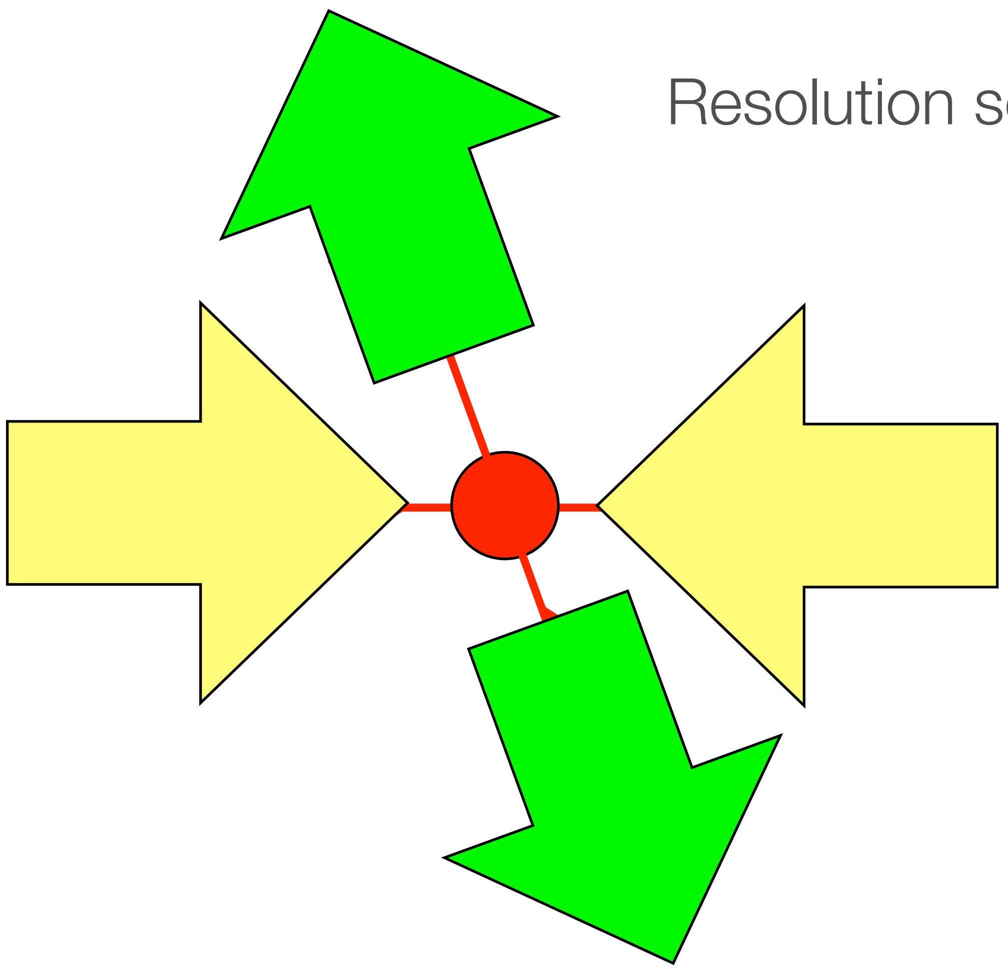
QCD@LHC

August 22, 2011, St Andrews, Scotland

Hadron-Hadron Collision

In hadron-hadron collision the picture is more complicated.



Resolution scale: 400 GeV

Decreasing the resolution scale more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

Important observation: The total cross section *is independent of* the resolution of the measurement (or detector).

We have to also consider the evolution of the final state jets.

Does perturbative QCD support this nice intuitive picture?

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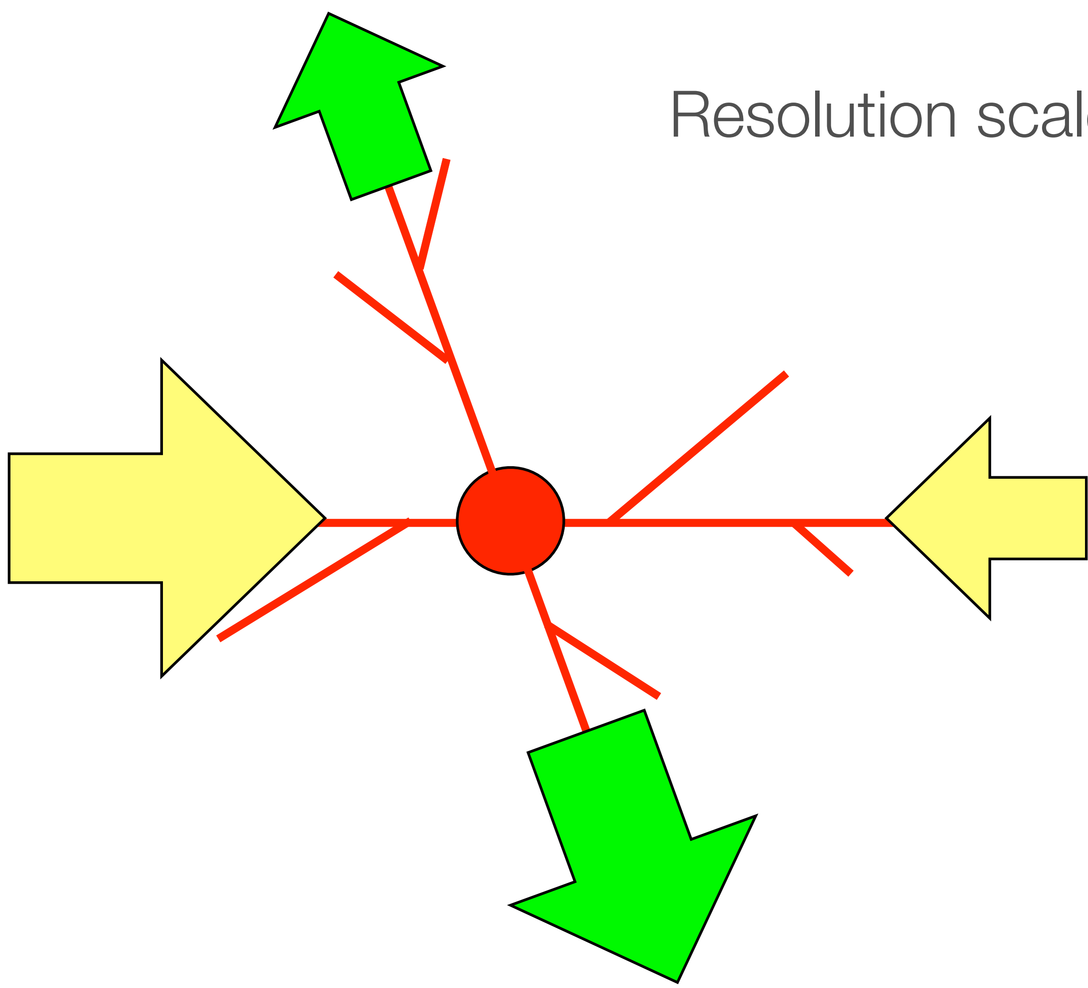

Resolution scale: 100 GeV

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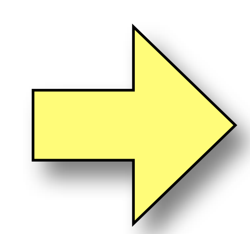

Cross section

The cross section is a phase space integral of all the possible matrix elements and the a convolution to the parton distribution functions.

$$\sigma[F] = \sum_m \int [d\{p,f\}_m] \overbrace{f_{a/A}(\eta_{\rm a},\mu_F^2)\, f_{b/B}(\eta_{\rm b},\mu_F^2)}^{\textit{parton distributions}} \frac{1}{2\eta_{\rm a}\eta_{\rm b} p_A\cdot p_B} \times \langle \mathcal{M}(\{p,f\}_m) | \underbrace{F(\{p,f\}_m)}_{\textit{observable}} \underbrace{|\mathcal{M}(\{p,f\}_m)\rangle}_{\textit{matrix element}}$$

- ✗ This is formally an all order expression and it is impossible to calculate out.
- ✗ We can do it at LO, NLO and in some cases NNLO level.
- ✗ Lots of complication with IR singularities.
- ✗ Lots of complication with spin and colors.
- ✓ The idea is to approximate the matrix elements using factorization properties of the QCD matrix element.

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$$\sigma[F] = \sum_m \int [d\{p,f\}_m] \, \mathrm{Tr}\{\underbrace{\rho(\{p,f\}_m)}_{\text{density operator in color} \otimes \text{spin space}} F(\{p,f\}_m)\}$$

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Approx. of the Density Operator

The $m+1$ parton physical state is represented by density operator in the quantum space and by the statistical state in the statistical space.

$$\rho\big(\{p,f\}_{m+1}\big) \Leftrightarrow \big|\rho(\{p,f\}_{m+1})\big)$$

This is based on the $m+1$ parton matrix elements. They are very complicated (especially the loop matrix elements). We try to approximate them by using their *soft collinear factorization properties*. For this we introduce operators in the statistical space:

Wide angle soft contributions

$$\big|\rho(\{\hat{p},\hat{f}\}_{m+1})\big) \approx \int_{t_m}^{\infty} dt\,\big[\underbrace{\mathcal{H}_C(t)} + \overbrace{\mathcal{H}_S(t)}\big]\big|\rho(\{p,f\}_m)\big)$$

Collinear and soft-collinear contribution

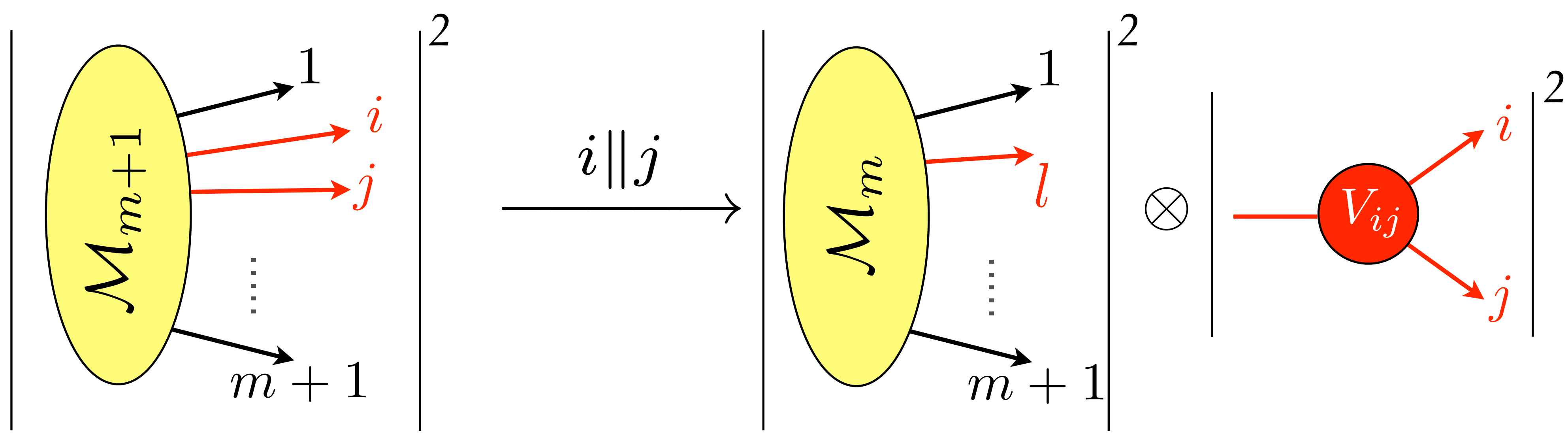
This parameter represents the hardness of the splitting. We will call it shower time.

The total splitting operator is

$$\mathcal{H}_I(t) = \mathcal{H}_C(t) + \mathcal{H}_S(t)$$

Collinear Singularities

The QCD matrix elements have universal factorization property when two external partons become collinear

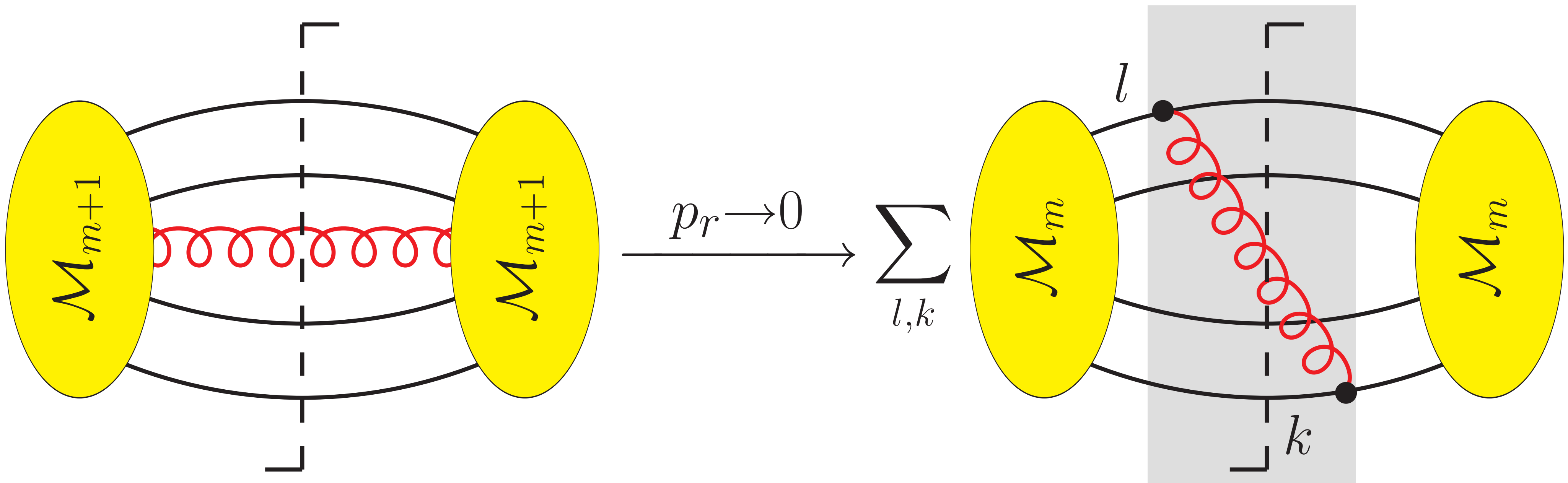


$$\mathcal{H}_C \sim \sum_l t_l \otimes t_l^\dagger \, V_{ij}(s_i, s_j) \otimes V_{ij}^\dagger(s_i', s_j') \Leftrightarrow \frac{\alpha_s}{2\pi} \sum_l \frac{1}{p_i \cdot p_j} P_{f_l, f_i}(z) + \dots$$

Altarelli-Parisi splitting kernels

Soft Singularities

The QCD matrix elements have universal factorization property when an external gluon becomes soft



$$\mathcal{H}_S \sim -\sum_{\substack{l,k \\ l \neq k}} \frac{\hat{p}_l \cdot \varepsilon(s)\, \hat{p}_k \cdot \varepsilon(s')}{\hat{p}_l \cdot \hat{p}_{m+1}\, \hat{p}_k \cdot \hat{p}_{m+1}}\, t_l \otimes t_k^\dagger$$

Soft gluon connects everywhere and the color structure is not diagonal; quantum interferences in the color space.

Resolvable Splittings

Let us consider a physical state at shower time t, $|\rho(t))$. This means every parton is resolvable at this time (this scale). Now, we apply the splitting operator:

$\mathcal{H}_I(t)$ operator changes
- the number of the partons, $m \to m+1$
- the color and spin structure
- flavors and momenta

$$|\rho^{\mathrm{R}}_{\infty}) = \int_t^{\infty} d\tau\, \mathcal{H}_I(\tau)\, |\rho(t))$$

This is good approximation if we allow only softer radiations than t, $\tau > t$

Now, let us consider a measurement with a resolution scale which correspond to shower time t'

$$|\rho^{\mathrm{R}}_{\infty}) \approx \underbrace{\int_t^{t'} d\tau\, \mathcal{H}_I(\tau)\, |\rho(t))}_{\text{Resolved radiations}} + \underbrace{\int_{t'}^{\infty} d\tau\, \mathcal{V}_I^{(\epsilon)}(\tau)\, |\rho(t))}_{\text{Unresolved radiations}}$$

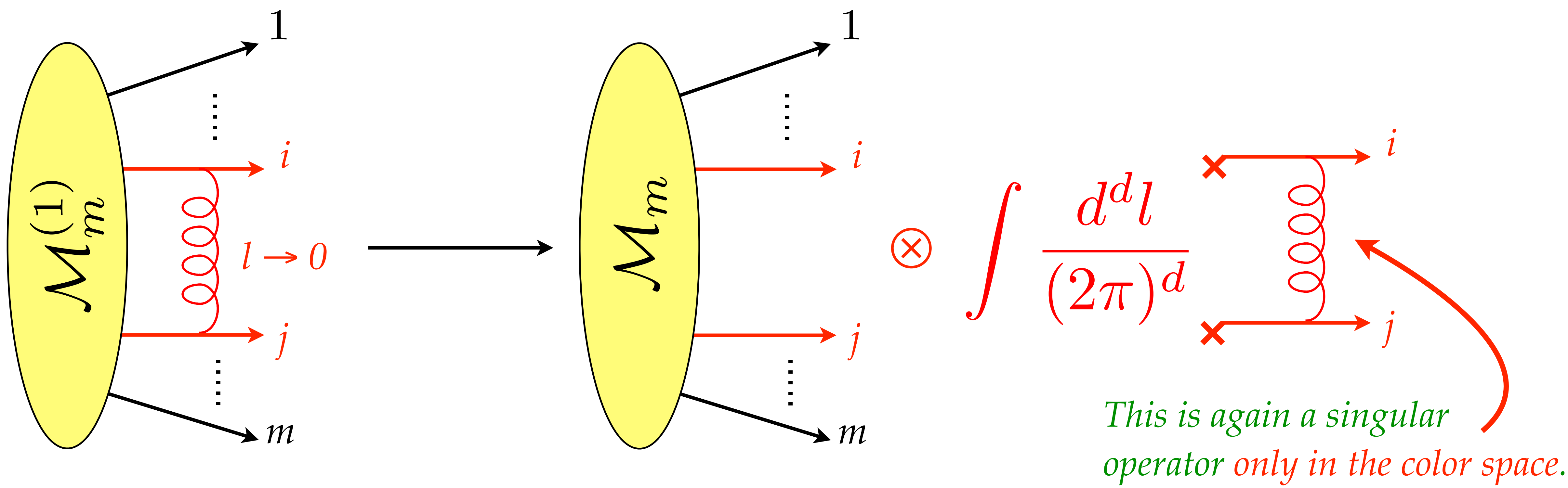
Unresolved radiations
This is a singular contribution

$\mathcal{V}_I(t)$ operator
- changes only the color structure
- $(1|\mathcal{V}_I(t) = (1|\mathcal{H}_I(t)$

What can we do about it?

Virtual Contributions

There is another type of the unresolvable radiation, *the virtual (loop graph) contributions.* We have *universal factorization properties* for the loop graphs. E.g.: in the soft limit, when the loop momenta become soft we have



We can use this factorization to *dress up* partonic states *with virtual radiation*. After careful analysis one can found that the virtual contribution can be approximated by

$$\big|\rho_\infty^{\mathrm{V}}\big) \approx - \int_t^\infty d\tau\, \mathcal{V}_I^{(\epsilon)}(\tau)\, \big|\rho(t)\big)$$

Same structure like in the real unresolved case but here with opposite sign.

Physical States

Combining the real and virtual contribution we have got

$$\big|\rho^{\rm R}_{\infty}\big) + \big|\rho^{\rm V}_{\infty}\big) = \int_t^{t'} d\tau\, \big[\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau)\big]\big|\rho(t)\big)$$

This operator dresses up the physical state with one real and virtual radiations that *is softer or more collinear than the hard state*. Thus the emissions are ordered. Now we can use this to build up physical states by considering all the possible way to go from t to t'.

$$\begin{aligned}\big|\rho(t')\big) = &\big|\rho(t)\big) \\ &+ \int_t^{t'} d\tau\, \big[\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau)\big]\big|\rho(t)\big) \\ &+ \int_t^{t'} d\tau_2\, \big[\mathcal{H}_I(\tau_2) - \mathcal{V}_I(\tau_2)\big] \int_t^{\tau_2} d\tau_1\, \big[\mathcal{H}_I(\tau_1) - \mathcal{V}_I(\tau_1)\big]\big|\rho(t)\big) \\ &+ \cdots \\ = &\underbrace{\mathbb{T}\exp\left\{\int_t^{t'} d\tau\, \big[\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau)\big]\right\}}_{\mathcal{U}(t',t)\ \textit{shower evolution operator}}\big|\rho(t)\big)\end{aligned}$$

$$\big|\rho(t')\big) = \mathcal{U}(t',t)\big|\rho(t)\big)$$

Full Splitting Operator

Very general splitting operator (*no spin correlation*) is

$$
\begin{aligned}
&\big(\{\hat p,\hat f,\hat c',\hat c\}_{m+1}\big|\mathcal{H}(t)\big|\{p,f,c',c\}_m\big) \\
&= \sum_{l=\mathrm{a},\mathrm{b},1,\dots,m} \delta\Big(t - T_l\big(\{\hat p,\hat f\}_{m+1}\big)\Big)\,\big(\{\hat p,\hat f\}_{m+1}\big|\mathcal{P}_l\big|\{p,f\}_m\big)\,\frac{m+1}{2} \\
&\quad\times \frac{n_\mathrm{c}(a)n_\mathrm{c}(b)\,\eta_\mathrm{a}\eta_\mathrm{b}}{n_\mathrm{c}(\hat a)n_\mathrm{c}(\hat b)\,\hat\eta_\mathrm{a}\hat\eta_\mathrm{b}}\,\frac{f_{\hat a/A}(\hat\eta_\mathrm{a},\mu_F^2)f_{\hat b/B}(\hat\eta_\mathrm{b},\mu_F^2)}{f_{a/A}(\eta_\mathrm{a},\mu_F^2)f_{b/B}(\eta_\mathrm{b},\mu_F^2)}\sum_k \Psi_{lk}(\{\hat f,\hat p\}_{m+1}) \\
&\quad\times \sum_{\beta=L,R}(-1)^{1+\delta_{lk}}\big(\{\hat c',\hat c\}_{m+1}\big|\mathcal{G}_\beta(l,k)\big|\{c',c\}_m\big)
\end{aligned}
$$

Splitting kernel is

$$
\Psi_{lk} = \frac{\alpha_\mathrm{s}}{2\pi}\,\frac{1}{\hat p_l\cdot\hat p_{m+1}}\left[A_{lk}\,\frac{2\hat p_l\cdot\hat p_k}{\hat p_k\cdot\hat p_{m+1}} + H_{ll}^{\mathrm{coll}}(\{\hat f,\hat p\}_{m+1})\right]
$$

Important:

$$
A_{lk} + A_{kl} = 1
$$

Color operator for gluon emission is

$$
\begin{aligned}
&\big(\{\hat c',\hat c\}_{m+1}\big|\mathcal{G}_R(l,k)\big|\{c',c\}_m\big) \\
&\qquad = {}_D\big\langle\{\hat c\}_{m+1}\big|t_l^\dagger\big|\{c\}_m\big\rangle\,\big\langle\{c'\}_m\big|t_k\big|\{\hat c'\}_{m+1}\big\rangle_D\ .
\end{aligned}
$$

Full Splitting Operator

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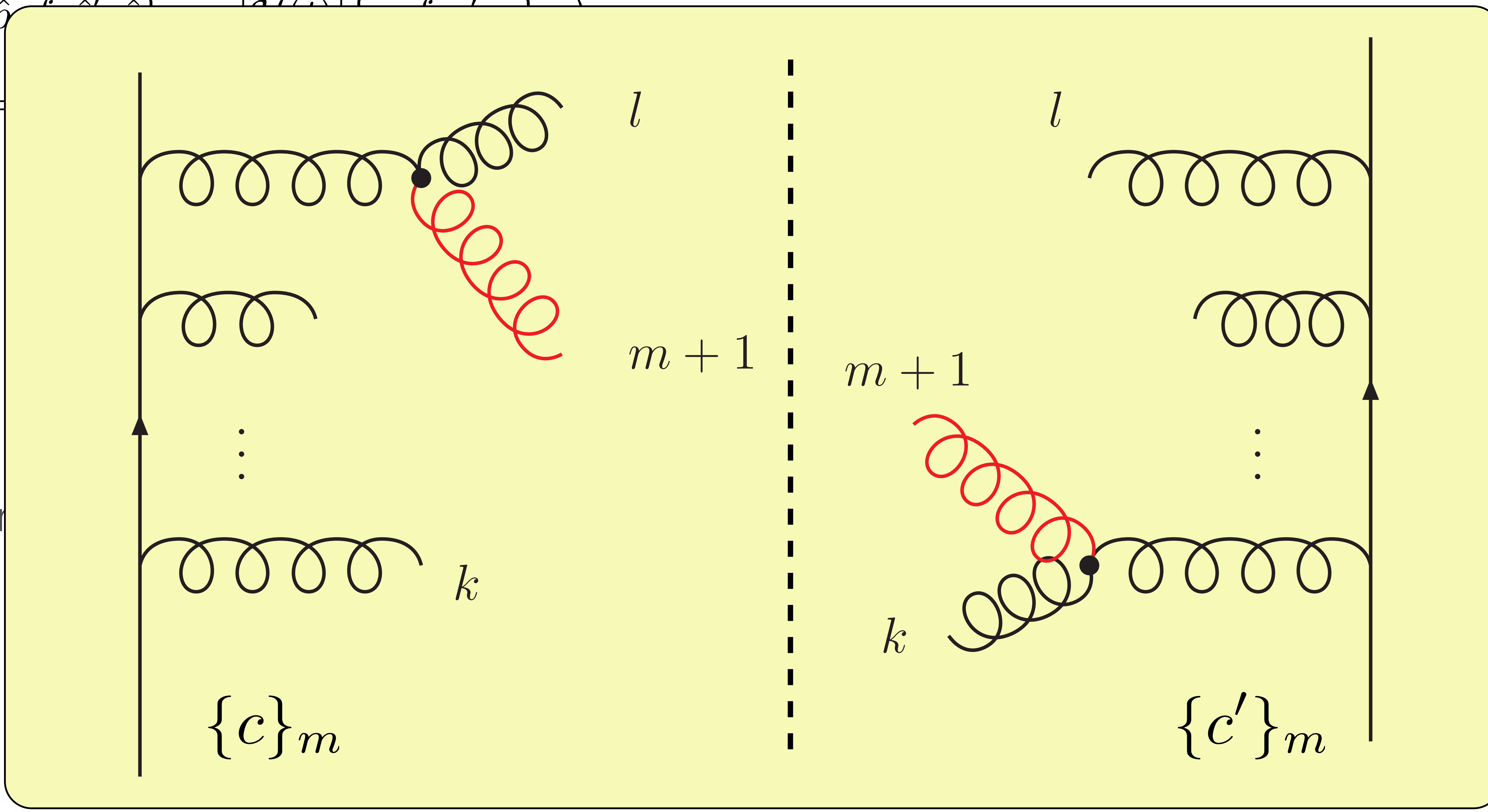
Splittir

Ψ_{lk}

1

Color

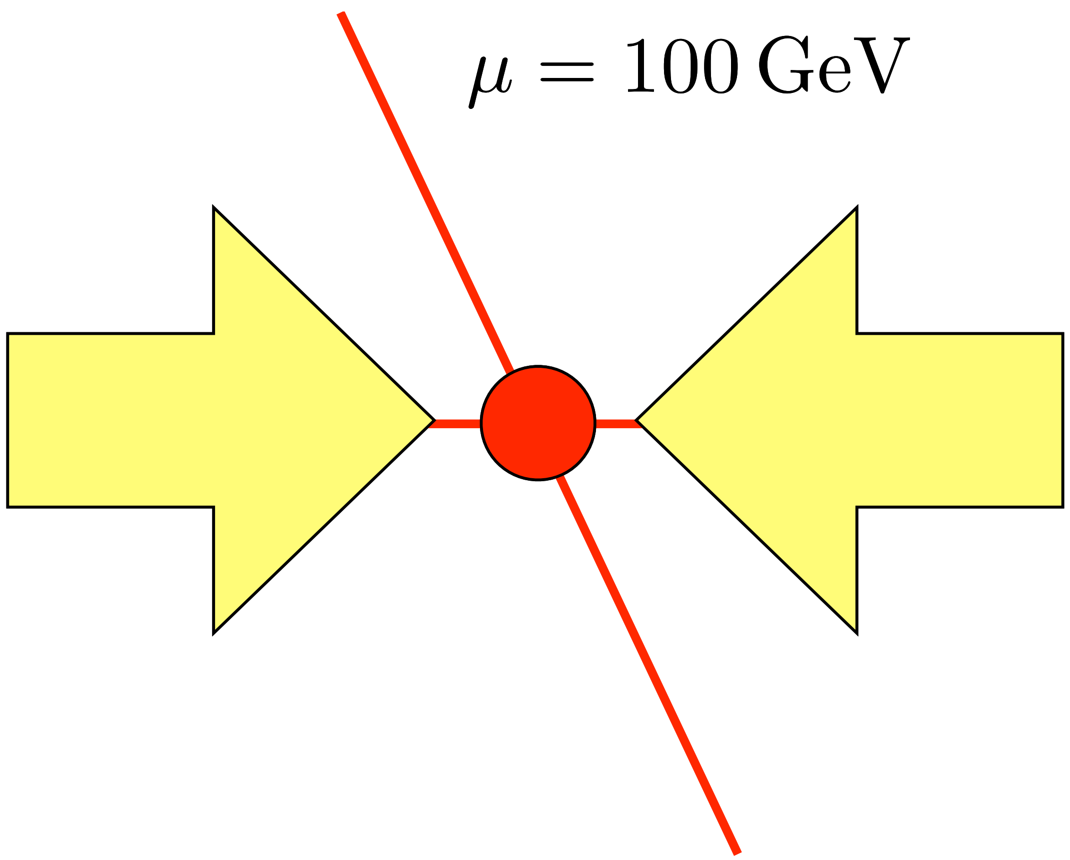


$$\left(\{\hat{c}',\hat{c}\}_{m+1}\middle|\mathcal{G}_R(l,k)\middle|\{c',c\}_m\right)$$

$$= {}_D\!\left\langle\{\hat{c}\}_{m+1}\middle|t_l^\dagger\middle|\{c\}_m\right\rangle\left\langle\{c'\}_m\middle|t_k\middle|\{\hat{c}'\}_{m+1}\right\rangle_D \; .$$

Multi Parton Interaction

Let us see how it looks at hadron collider



In hadron-hadron collision the parton distribution function also absorbs the contribution of the multiple interactions and joint interactions.

Our strategy:

- Identify factorazible singular contributions.
- Sum up the strongly ordered radiations.
- Minimize the number of the *ad-hoc* assumptions and tuning parameters.

Now, one can try to define the evolution operator in the following form

$$\mathcal{U}(t,t') = \mathbb{T}\exp\left\{\int_t^{t'} d\tau \left[\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau) + \sum_{\beta=\mathrm{MI,\,JI,}\ldots} \left\{\mathcal{H}_\beta(\tau) - \mathcal{V}_\beta(\tau)\right\}\right]\right\}$$

Single radiations
(IRS & FSR)

Everything else

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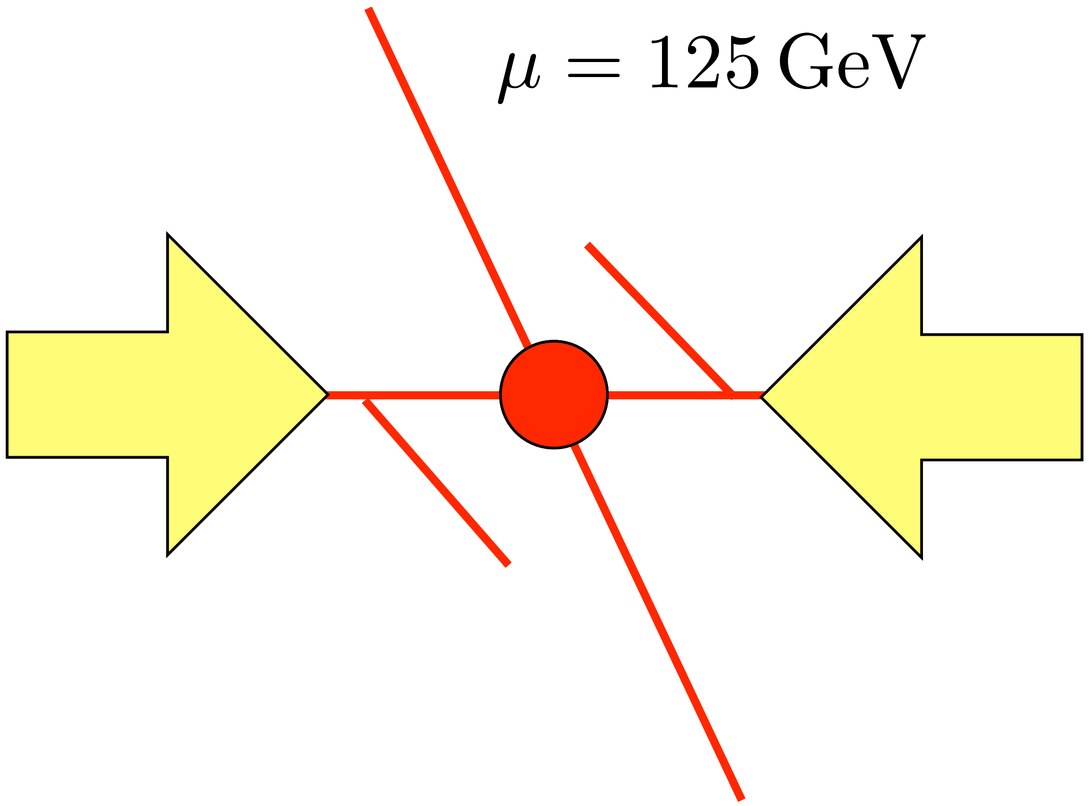

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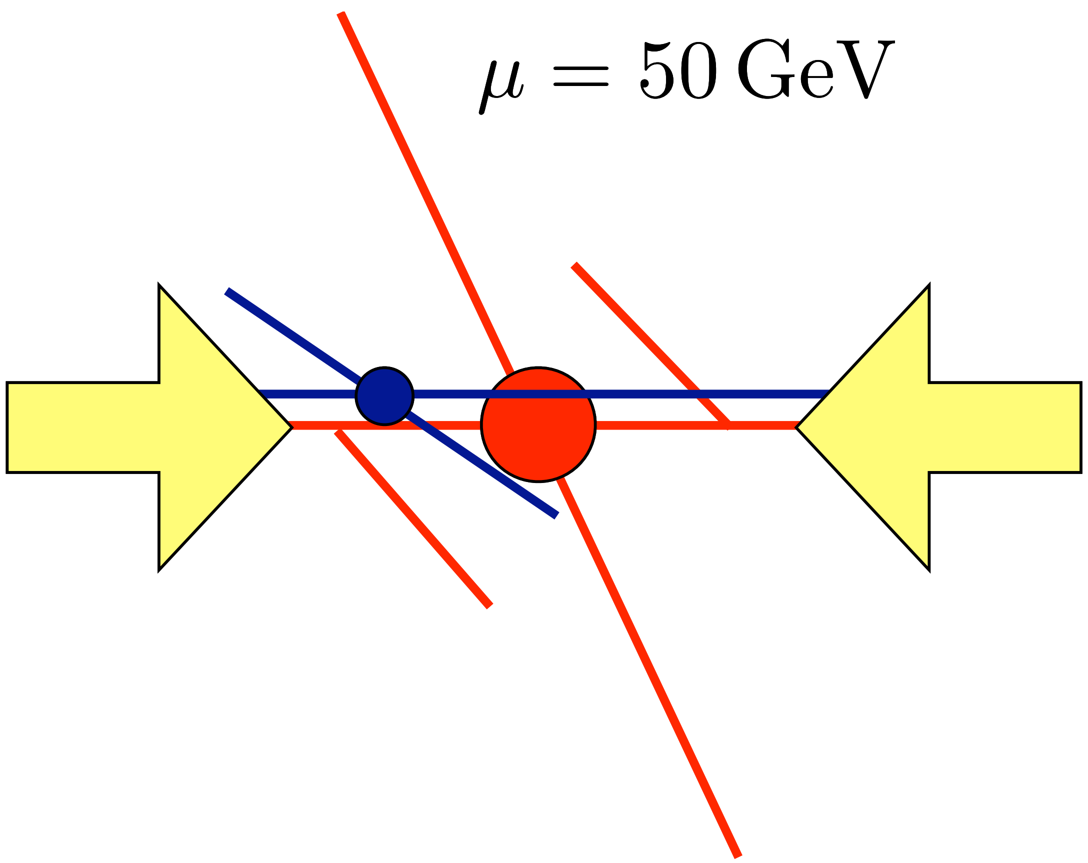

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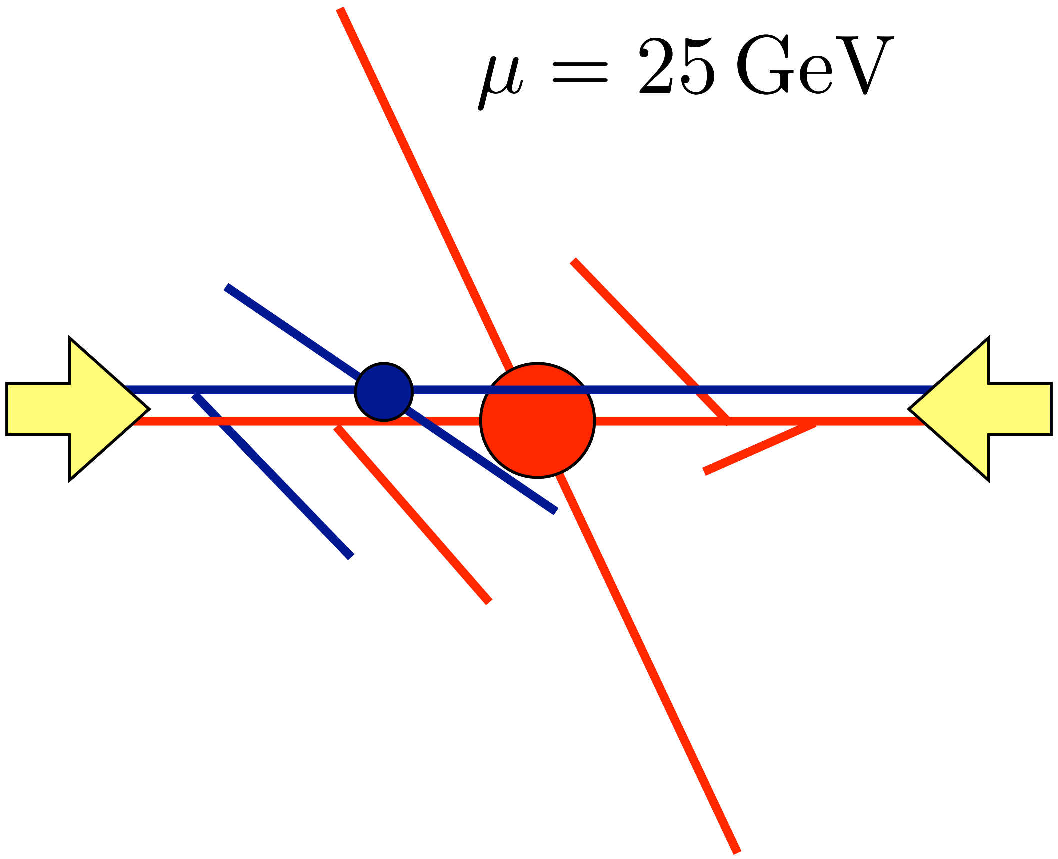

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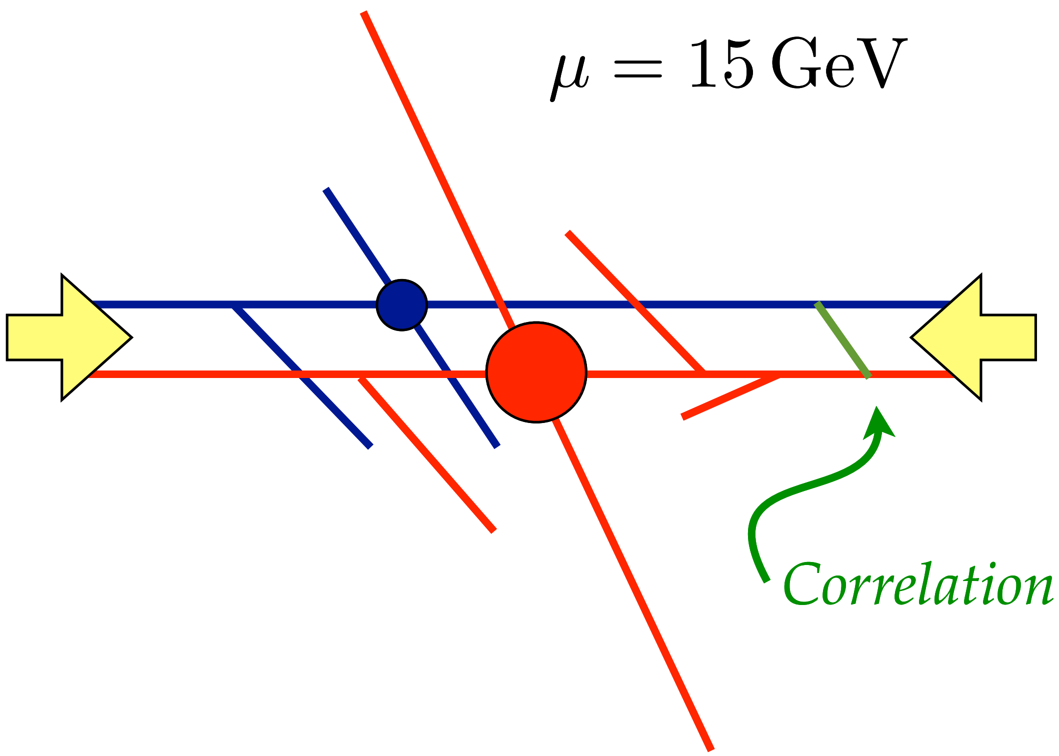

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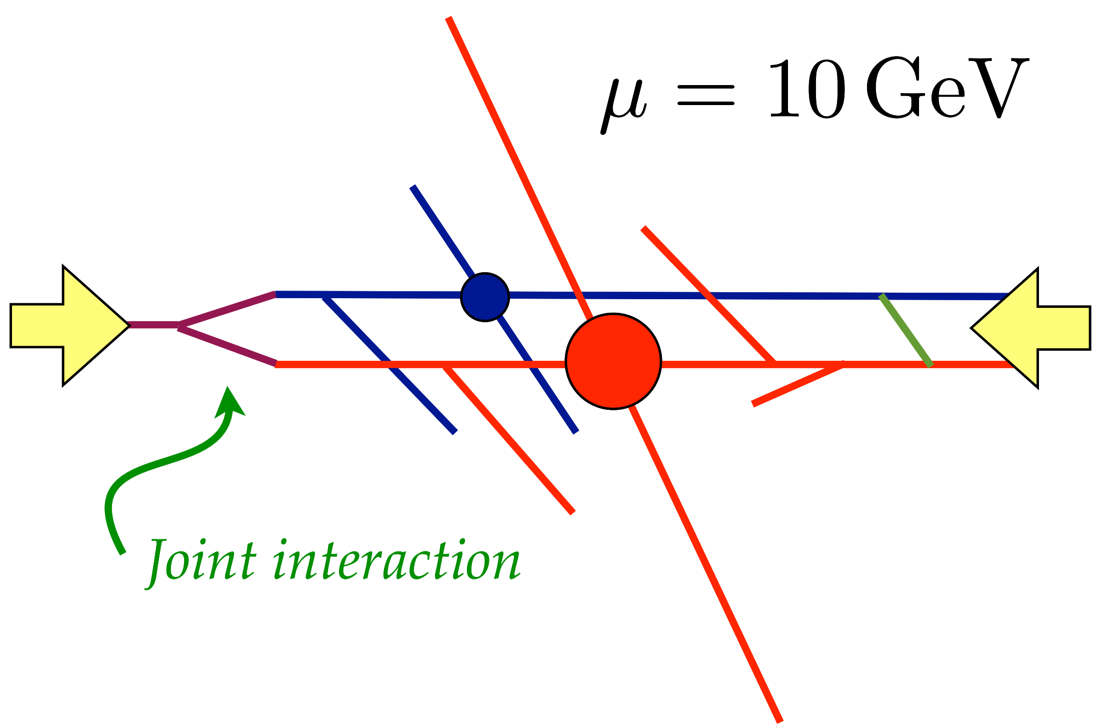

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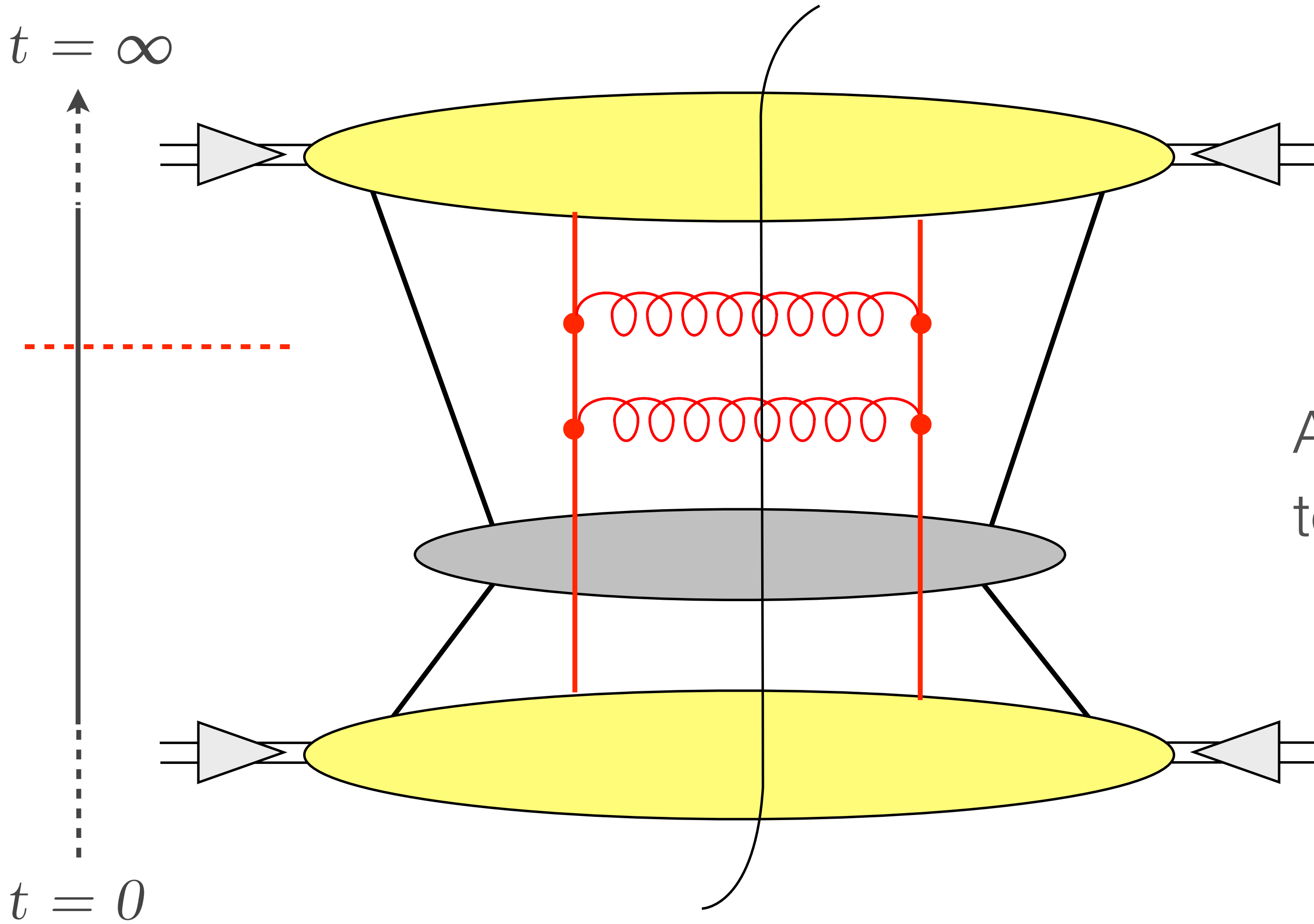


We need this at least at leading color level.

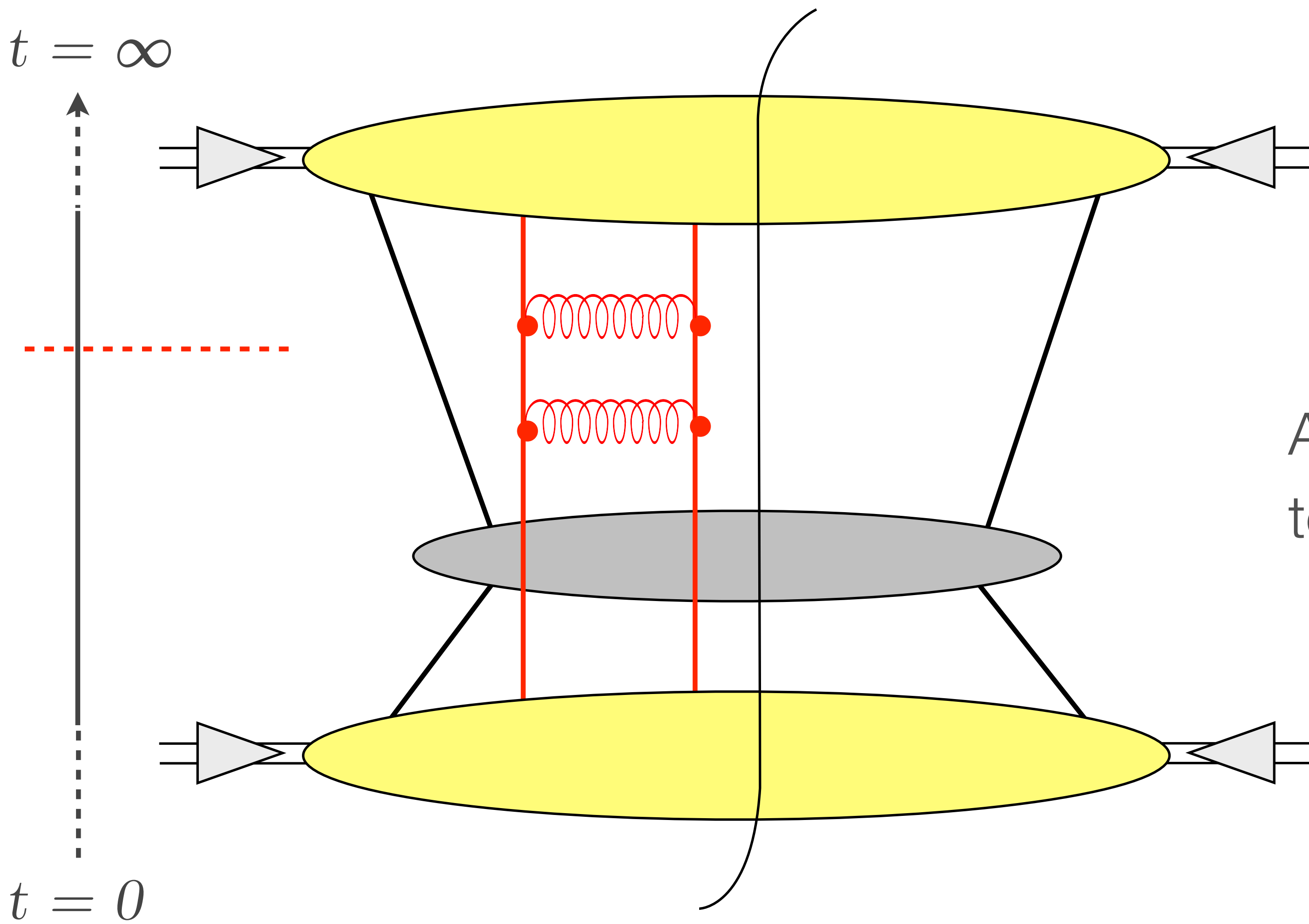
$$\mathcal{H}_{\rm MI}(t) = \left(\frac{\alpha_{\rm s}}{2\pi}\right)^2 \mathcal{O}(e^t)$$

Actually the real scaling is weaker due to the *power suppression*:

$$e^{t-t_0} \sim \frac{\Lambda^2_{\rm QCD}}{p^2_\perp}$$

- This is important in the very small pT regions and negligible in the large pT regions but it is hard to tell how import in the intermediate region. The cumulative effect could be sizable.
- Important to note that this is a kind of NLO contributions. Thus, compared to the standard shower this is also suppressed by an extra power of α_s.
- Requires multi parton PDF (mPDF).
- Implemented in Herwig & Pythia. (No “proper” mPDF implemented.)

Multi Parton Interaction



We need this at least at <u>leading color</u> level.

$$\mathcal{V}_{\rm MI}(t) = \left(\frac{\alpha_{\rm s}}{2\pi}\right)^2 \mathcal{O}(e^t)$$

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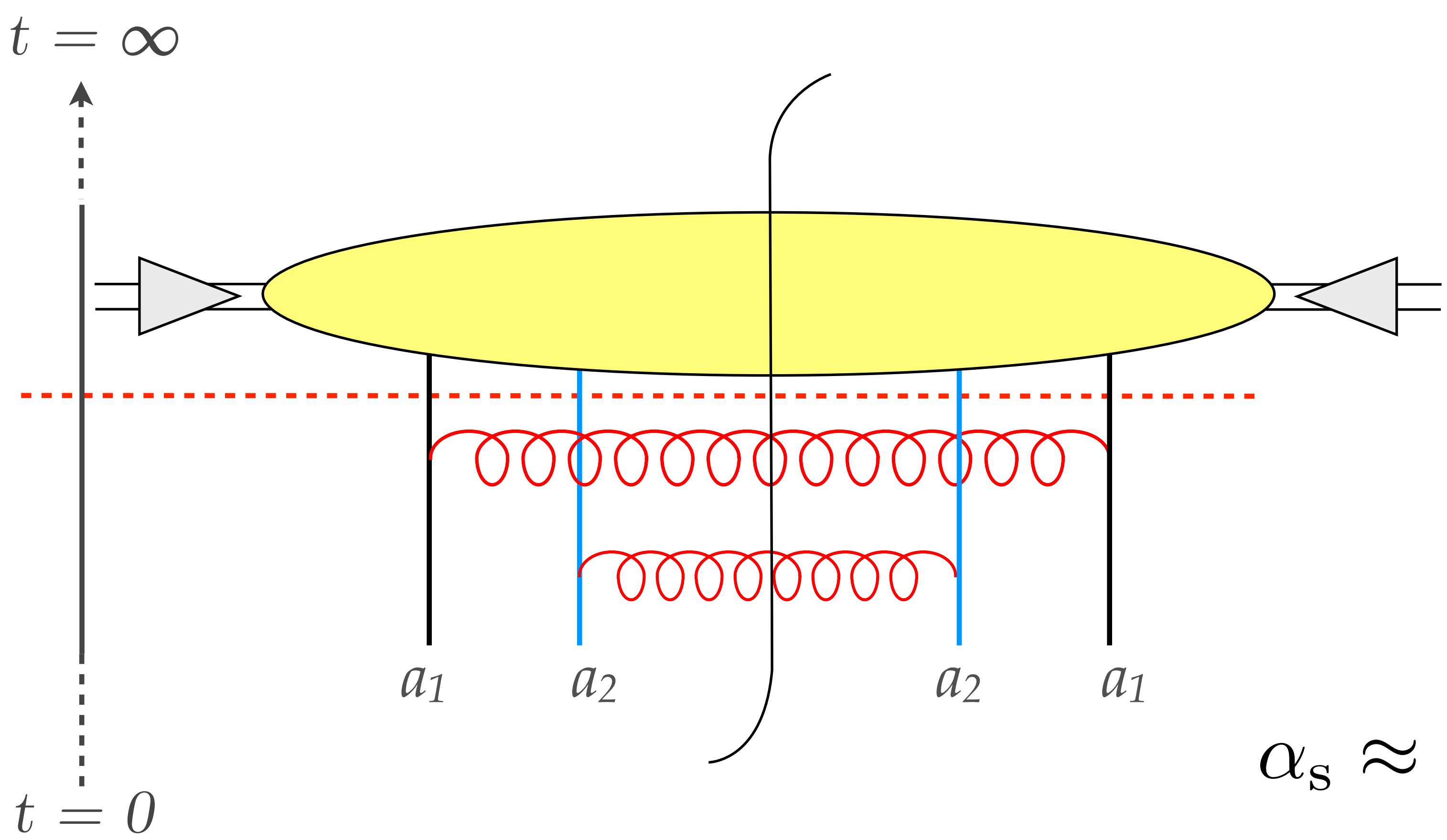
Standard IRS

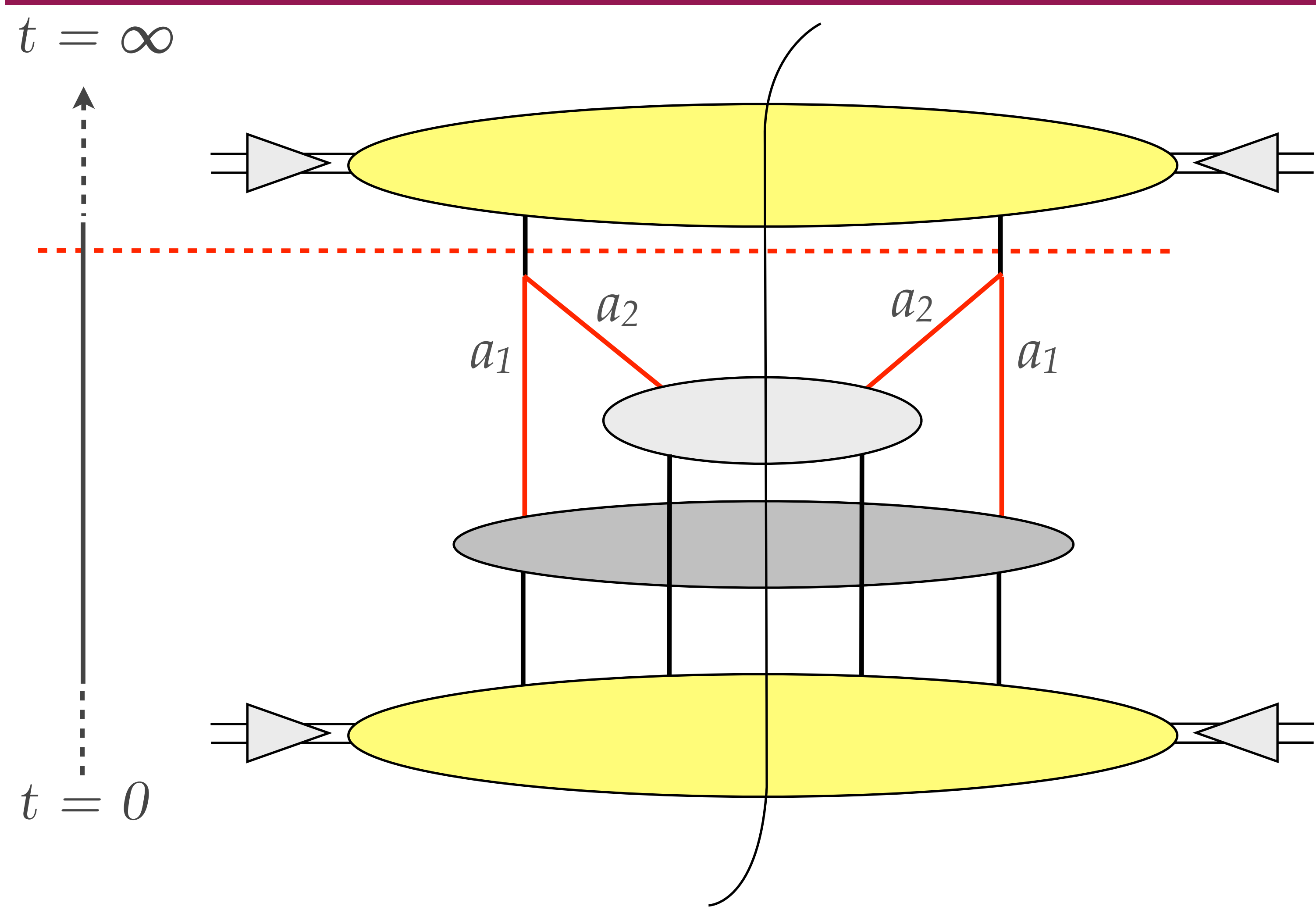


$$\mathcal{H}_{\mathrm{I}}(t) = \frac{\alpha_s}{2\pi}\mathcal{O}(t^2) + \frac{\alpha_s}{2\pi}\frac{1}{N_c^2}\mathcal{O}(t) + \left(\frac{\alpha_s}{2\pi}\right)^2\mathcal{O}(t^3)$$

$$\alpha_s \approx \frac{1}{N_c^2} \approx 0.1$$

- This is the standard shower evolution. Adds LL and NLL contributions. Not power suppressed.
- Since the MPI kernel is NLO contribution we should consider the standard shower at NLO level as well. (Just to be systematic.)
- If we consider NLO terms then we need subleading color contributions, too.
- Adds correction to the primary interaction as well as to the MPI contributions.
- It is implemented only at LO level in HERWIG & PYTHIA.

Joint Interaction



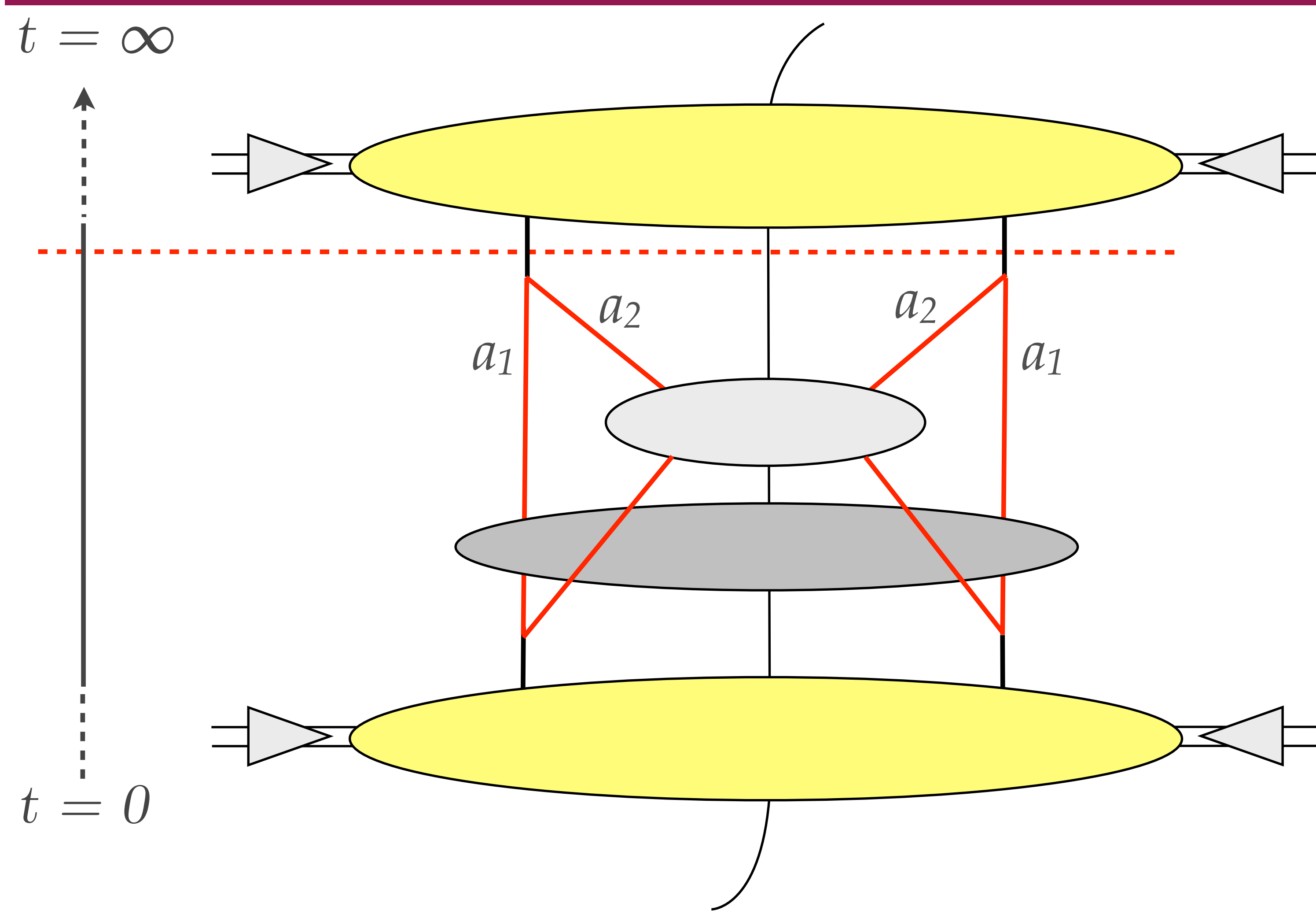
$$\mathcal{H}_{\rm JI}(t) = \frac{\alpha_s}{2\pi}\mathcal{O}(t) + \left(\frac{\alpha_s}{2\pi}\right)^2\mathcal{O}(t^2)$$

This is the most problematic contribution

$$\mathcal{V}_{\rm JI}(t) = 0$$

- This operator can be applied on states with at least two chains. (They are already power suppressed.)
- No corresponding factorizable virtual contribution. ➠ No associated Sudakov factor.
- There is some overlap with the NLO standard shower contribution.
- Some level it is implemented in PYTHIA.

Joint Interaction



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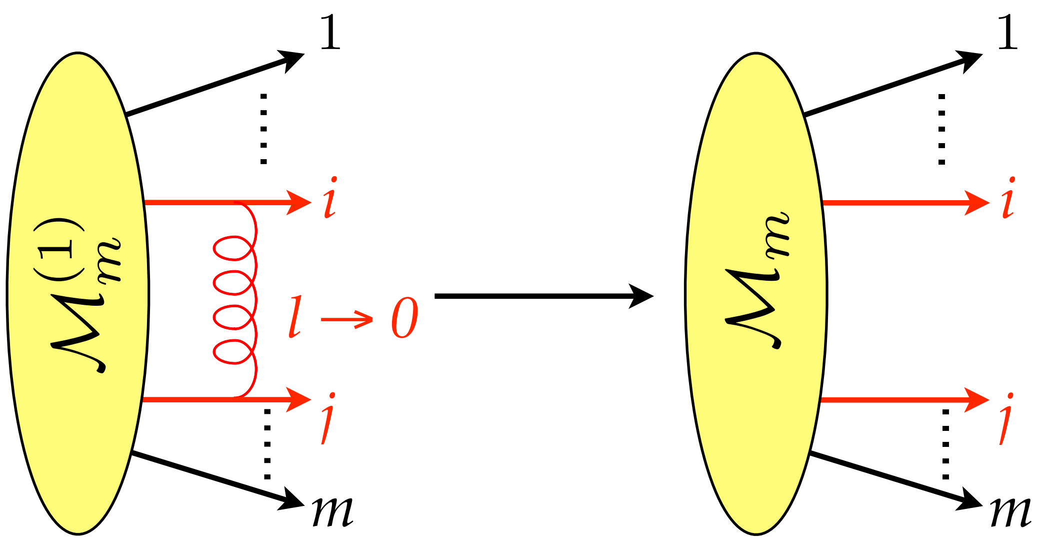
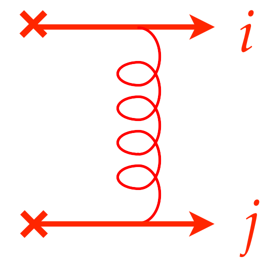
Virtual Contributions



This is a singular operator
in the color space.

$$\otimes \int \frac{d^d l}{(2\pi)^d}$$ 

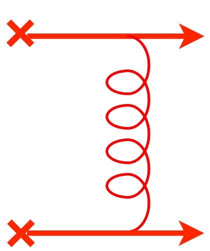
$$\Rightarrow \quad \mathcal{V}_I(t)$$

In standard parton shower this operator is obtained from the unitarity condition

$$\big(1\big|\mathcal{V}_I(t) = \big(1\big|\mathcal{H}_I(t) \quad \Rightarrow$$

Always real

But it turns out that we have imaginary contribution from the virtual graphs

$$\int \frac{d^d l}{(2\pi)^d}$$ 

$$\propto \mathcal{V}_I(t) + \underbrace{\mathrm{i}\pi\widetilde{\mathcal{V}}(t)}_{\text{Coulomb gluon}} \quad \text{and} \quad \big(1\big|\widetilde{\mathcal{V}}(t) = 0$$

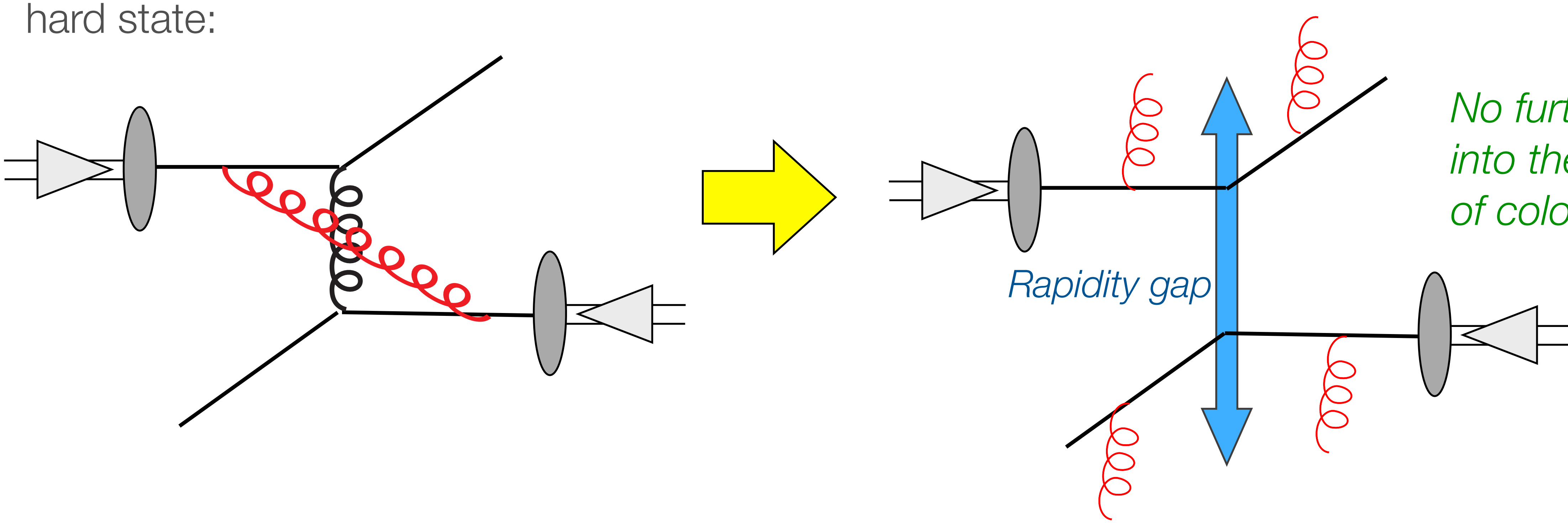
What can Coulomb gluon do?

Monday, August 22, 2011

Coulomb Gluon

1. Coulomb gluon changes the color configuration and the color flow. It is pure virtual contribution, thus it is unresolvable. *It does the same thing what color reconnection does.*

2. It always make color correlation between the two incoming partons. Let's consider a color octet hard state:



No further radiation into the gap because of color coherence.

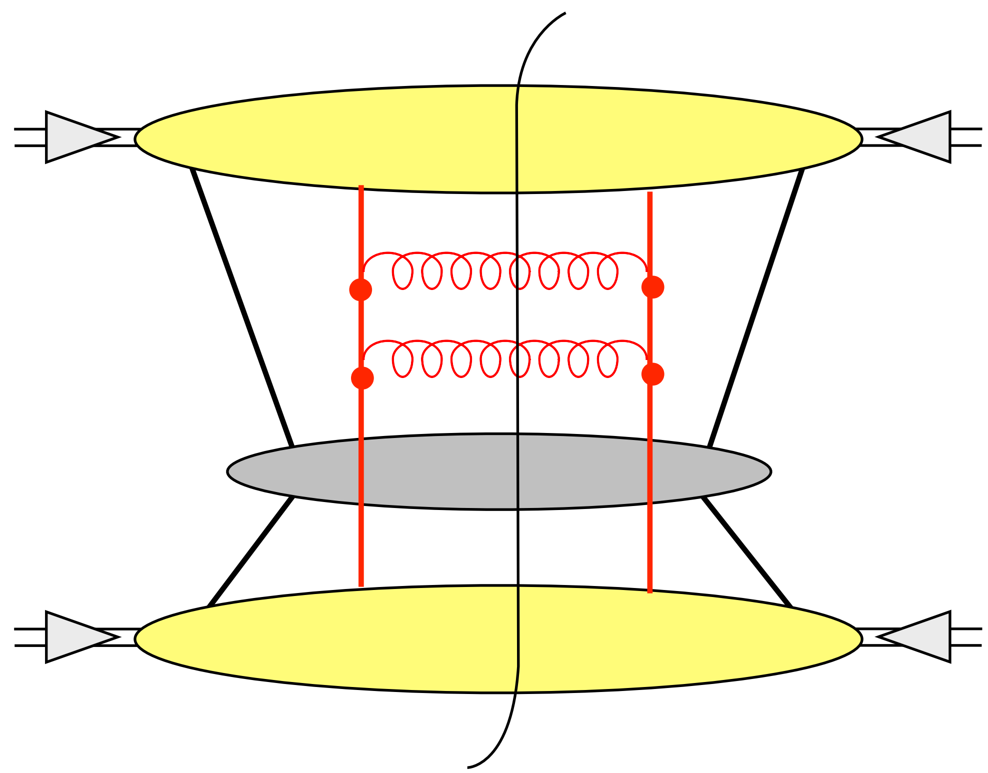
This is a contribution to the diffractive events.

3. Leads to *"Super Leading Logs"* in the case of some non-global observables.

Do we have Coulomb like contribution in the MPI virtual graphs?

MPI: Coulomb Gluon

In the MPI part the “resolvable” radiation comes from extra 2 →2 scattering. This is very singular in the low pT region. This singularity must be cancelled by the corresponding virtual graps.

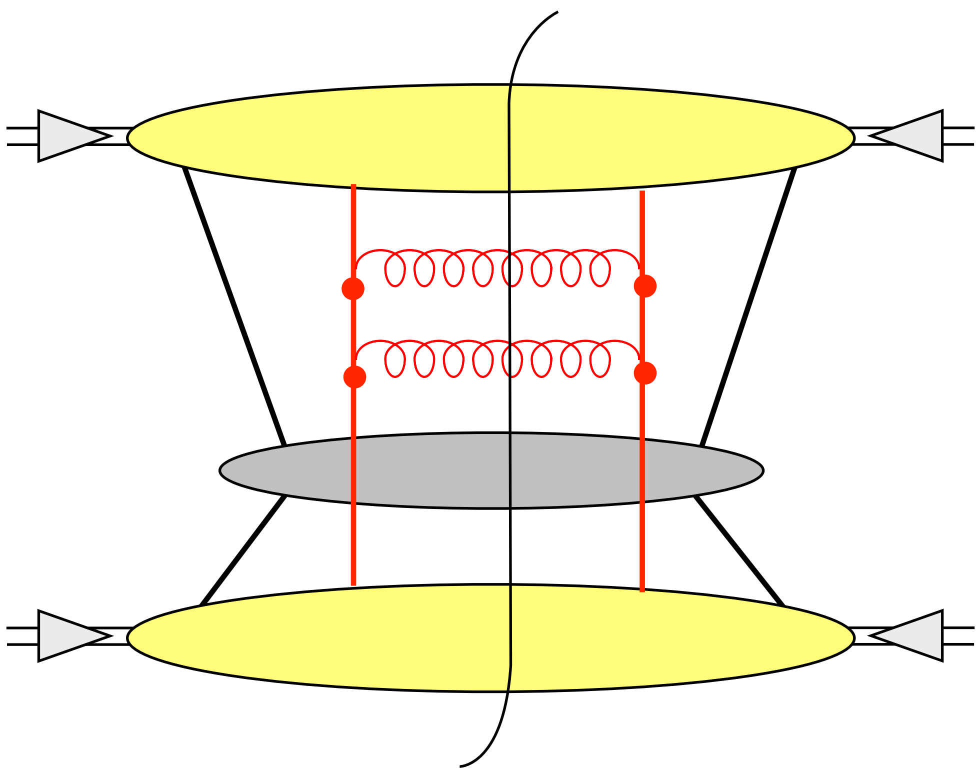


Real 2 →2 scattering adds two extra jets

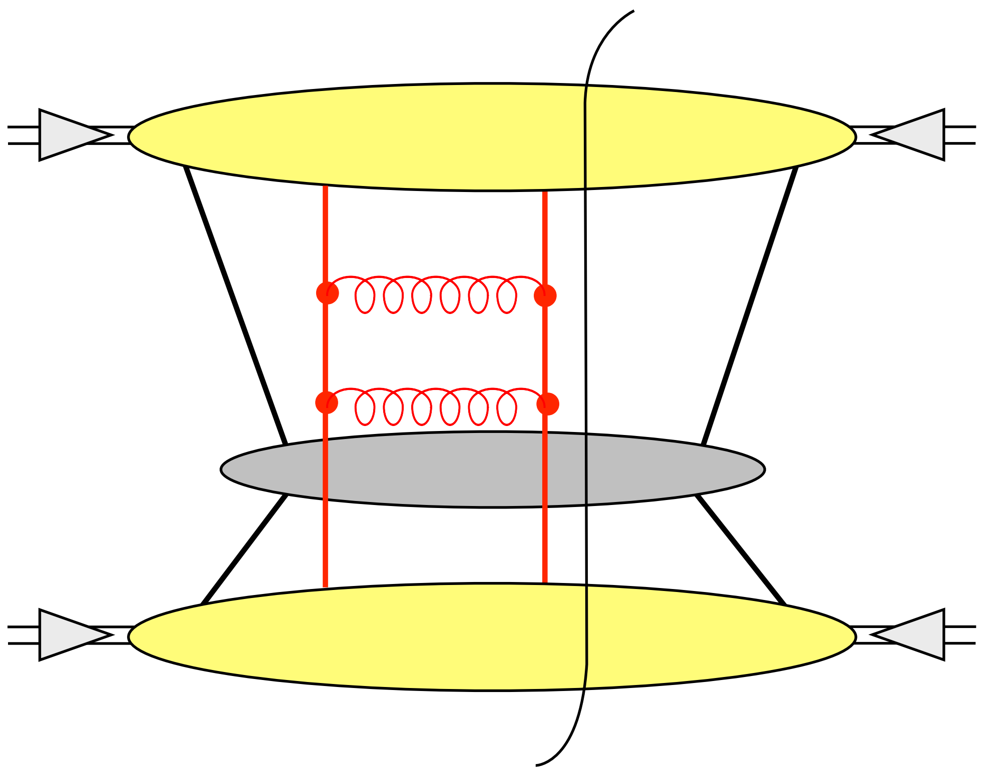
Pythia and Herwig put this graph into a simple probabilistic framework and exponentiate the extra 2→2 process.

MPI: Coulomb Gluon

In the MPI part the "resolvable" radiation comes from extra 2 ➝2 scattering. This is very singular in the low pT region. This singularity must be cancelled by the corresponding virtual graps.



Real 2 ➝2 scattering adds two extra jets

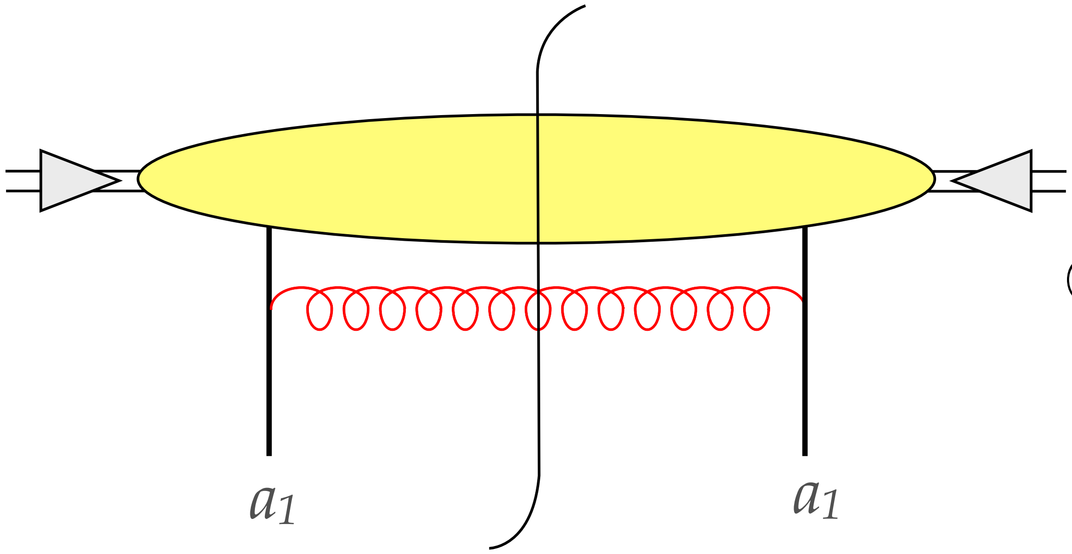


Corresponding virtual graph.
This is a forward elastic scattering contribution.
It can produce Coulomb gluon term ➠ *Color reconnection effect*

Pythia and Herwig put this graph into a simple probabilistic framework and exponentiate the extra 2➝2 process.

Single Parton PDF

The PDF has a operator product definition



$$\propto 2p^+ \int \frac{dz^-}{2\pi} e^{\mathrm{i}xz^-p^+} \langle p|\bar{q}(0)q(z)|p\rangle$$

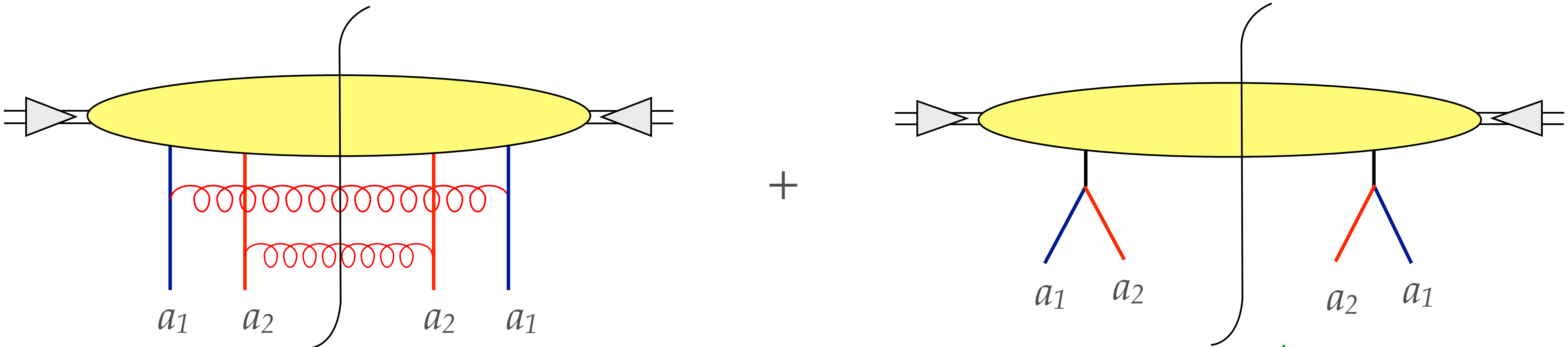
This expression is UV divergent and needed to be renormalized.

$$\mu \frac{d}{d\mu} f_{a/H}(x,\mu) = \sum_b [P_{a,b} \otimes f_{b/H}](x,\mu)$$

The UV singularity in the PDF corresponds to the IR singularity in hard part of the cross section. Everything is consistent.

How does it work in the mPDF case?

Multi Parton PDF



transverse separation

$$\propto 2p^+ \int \frac{dz_2^-}{2\pi}\, e^{ix_2 z_2^- p^+} \int \frac{dz_1^-}{2\pi}\, e^{ix_1 z_1^- p^+} \langle p|\bar{q}(-\tfrac{1}{2}z_2)\Gamma_2 q(\tfrac{1}{2}z_2)\bar{q}(y-\tfrac{1}{2}z_1)\Gamma_1 q(y+\tfrac{1}{2}z_1)|p\rangle$$

This operator is also UV divergent and needed to be renormalized. RGE provides the generalized DGLAP equation.

For $y \neq 0$ we have a homogeneous DGLAP equation, there is no contribution from 2→4 transitions

$$\frac{d}{dt}F(x_i, y) = P \otimes_{x_1} F + P \otimes_{x_2} F$$

For $\int dy\, F(x,y)$ we have contribution from 2→4 transitions

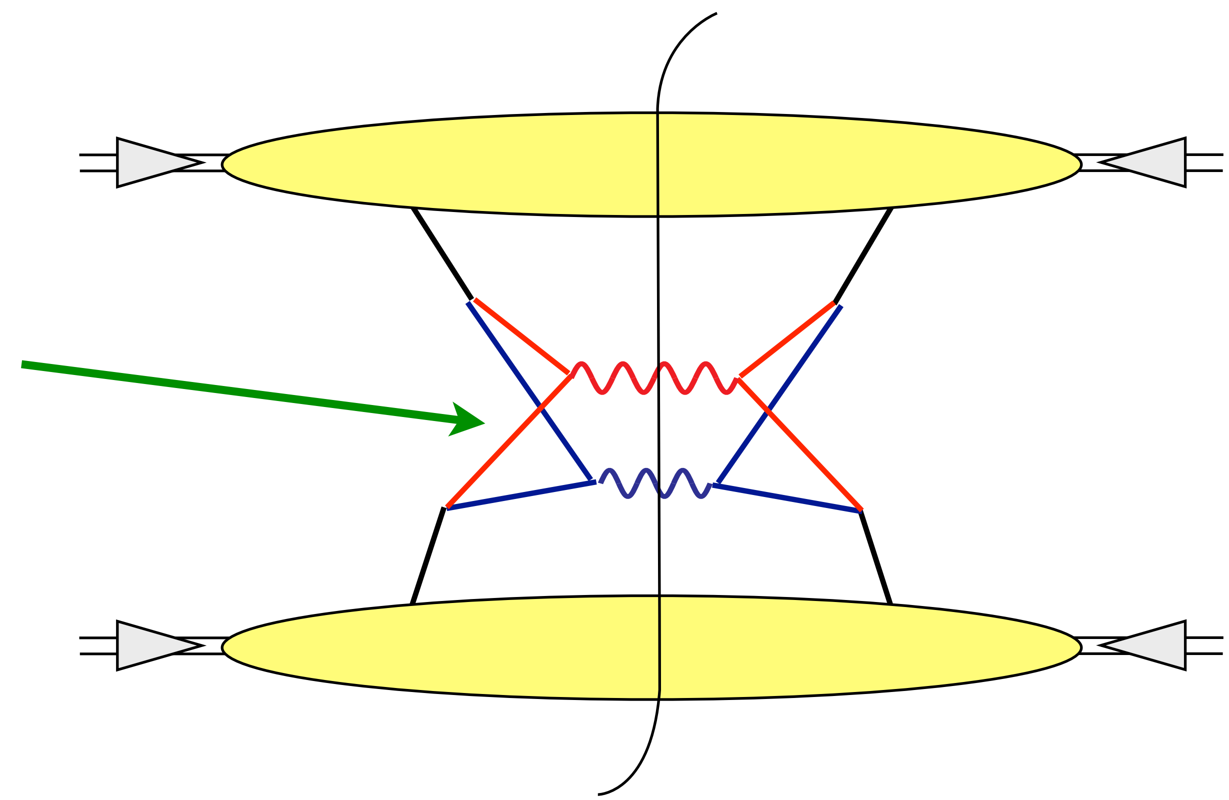
Marcus Diehl

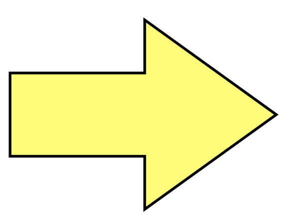
Multi Parton PDF

Let us study the 2→4 transitions in the hard matrix elements. In this example we have double Z boson production

There is a 1-loop graph in this process. This loop integral is perfectly finite, there is *NO IR singularities*.

This configuration would be a NLO effect in the standard shower.

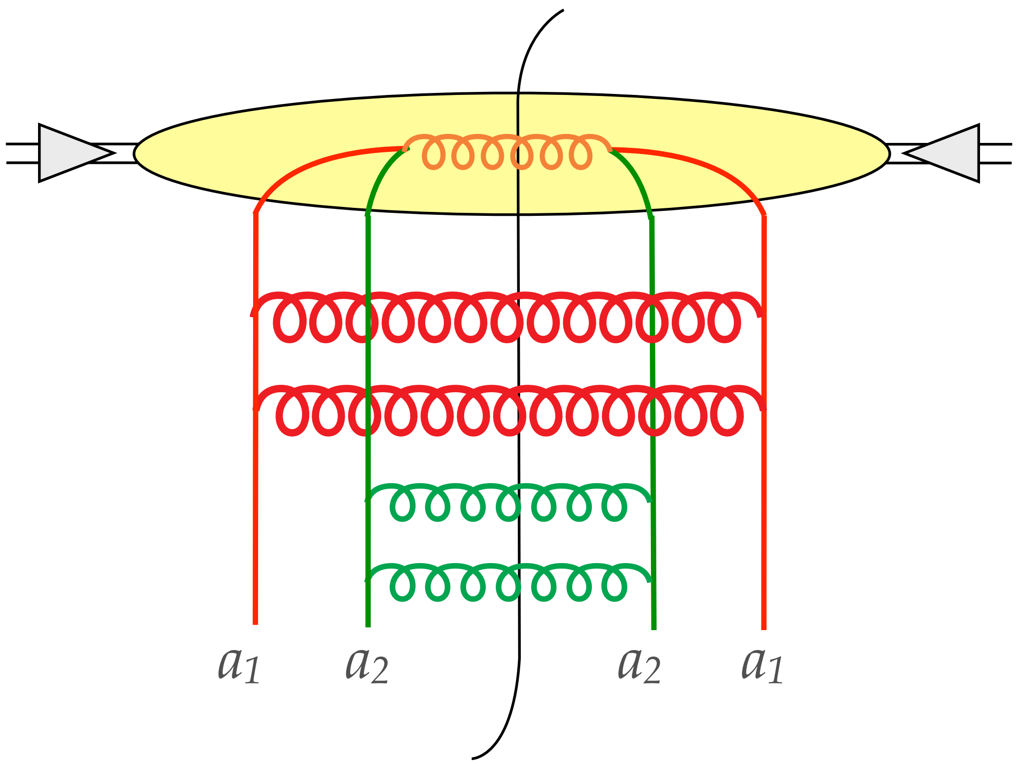




The MPI and the NLO shower has some overlap region. One should arrange the calculation in such a way to avoid double counting...

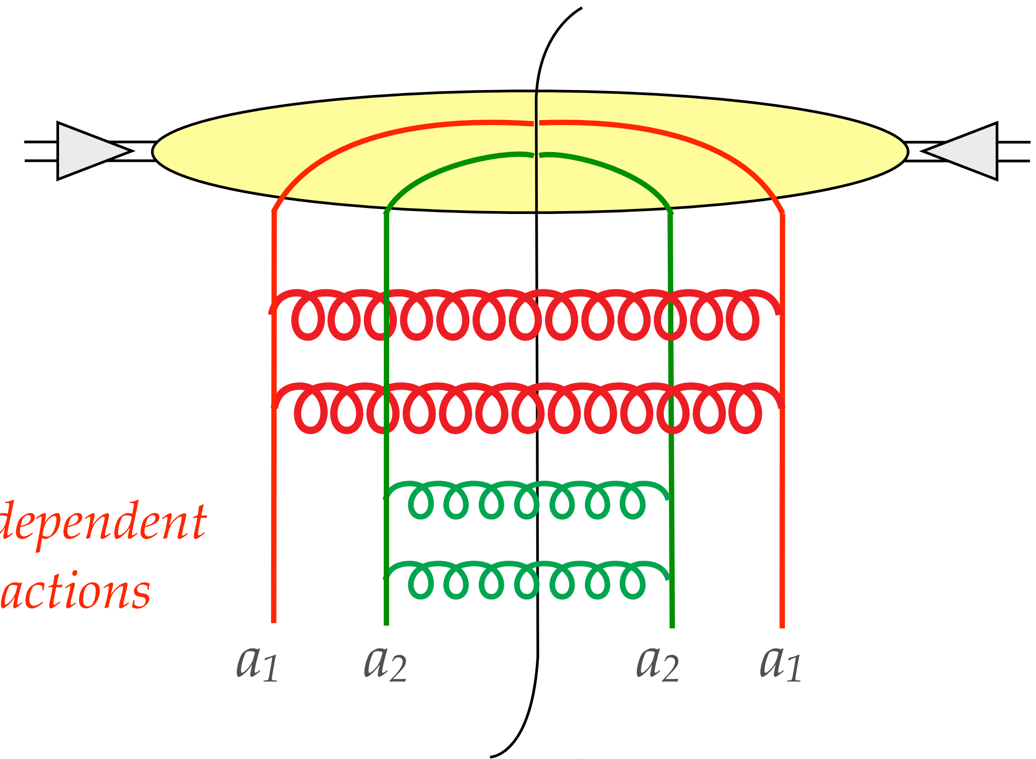
Color of mPDF

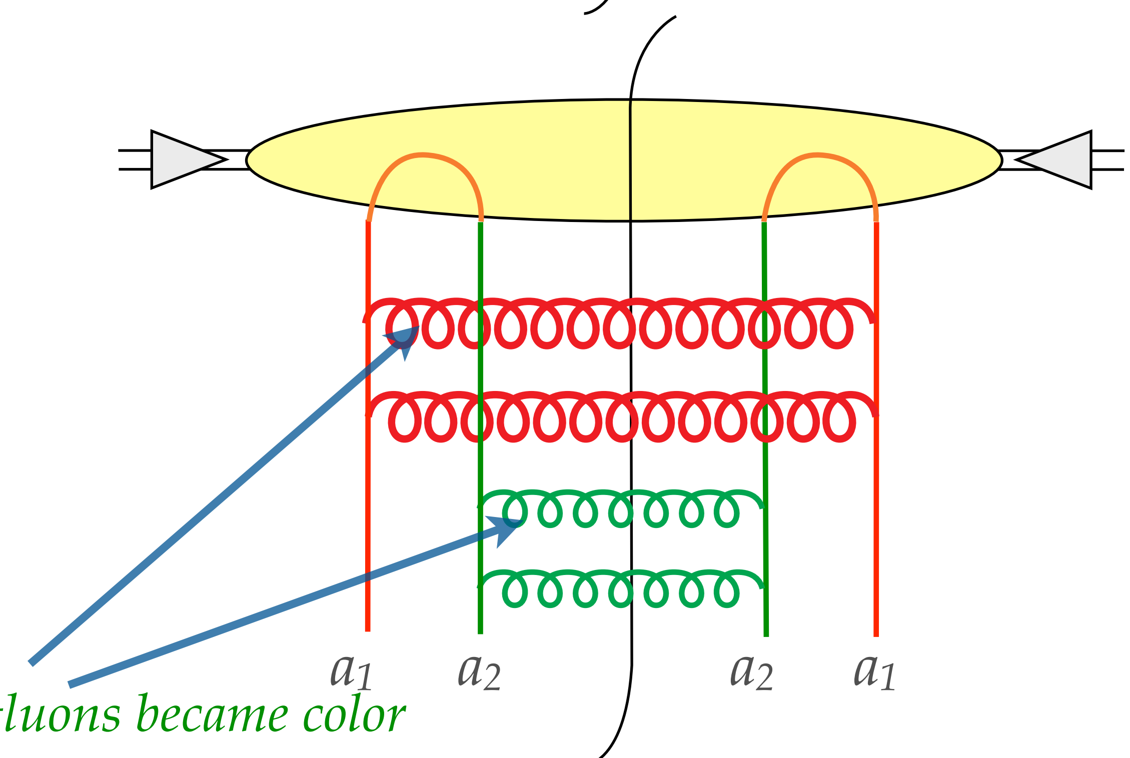
In Pythia or Herwig the effective mPDF is always color singlet state. But it can be a color octet state



$=$

Two independent interactions





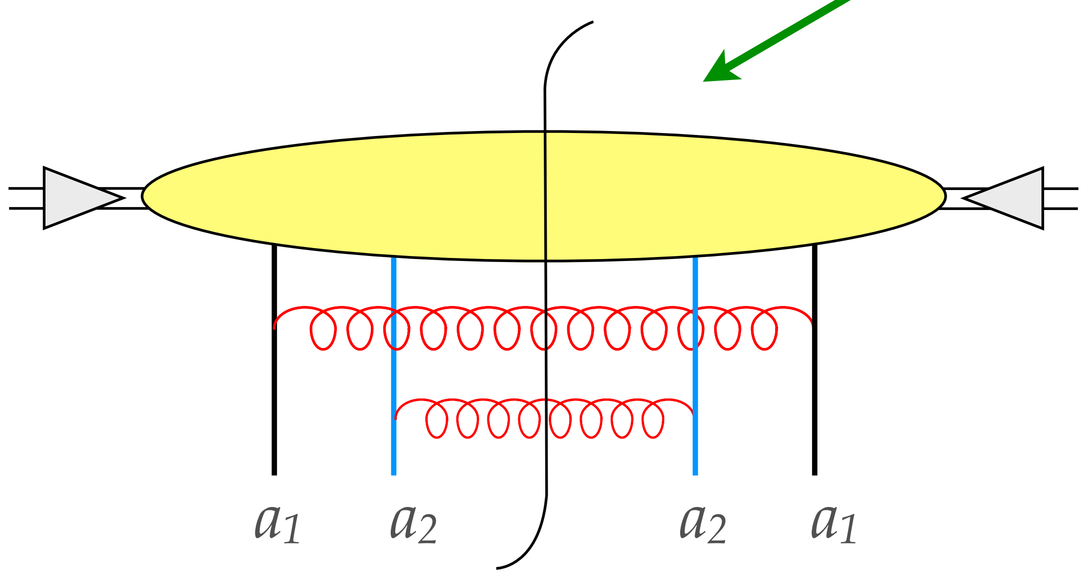
These two gluons became color connected.

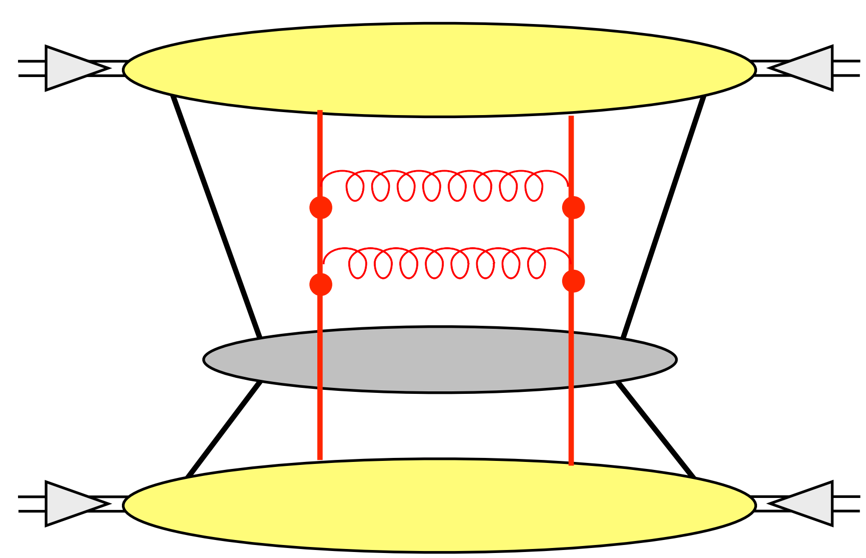
➠ *Color reconnection effect in the perturbative part.*

MPI Evolution Operator

From this approach one can find that the full evolution operator is

$$\mathcal{U}(t,t') = \mathbb{T}\exp\left\{\int_t^{t'} d\tau\,\left[\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau) + \mathcal{H}_{\rm MI}(\tau) - \mathcal{V}_{\rm MI}(\tau)\right]\right\}$$





$$\frac{f_{\{\mathrm{a}_1,\mathrm{a}_2\}/\mathrm{A}}(\hat{\eta}_{a_1},\hat{\eta}_{a_2},\mu_F^2)}{f_{\{\mathrm{a}_1,\mathrm{a}_2\}/\mathrm{A}}(\eta_{a_1},\eta_{a_2},\mu_F^2)}$$

$$\frac{f_{\{\mathrm{a}_1,\mathrm{a}_2\}/\mathrm{A}}(\hat{\eta}_{a_1},\hat{\eta}_{a_2},\mu_F^2)}{f_{\{\mathrm{a}_1\}/\mathrm{A}}(\eta_{a_1},\mu_F^2)}$$

Conclusion

- Multiple Interaction is very complicated from theory point of view.
- There are MC tool available mostly based on some tunable models (Color reconnection, simple mPDF assumption,...)
- Running of the mPDF, modeling mPDF
- Some perturbative effects are not included in our MC (Coulomb gluon,...)
- Lack of theorems (factorization,...)

MPI11@DESY

DESY DFG HELMHOLTZ ASSOCIATION PHYSICS AT THE TERA SCALE Helmholtz Alliance UH Universität Hamburg DER FORSCHUNG | DER LEHRE | DER BILDUNG

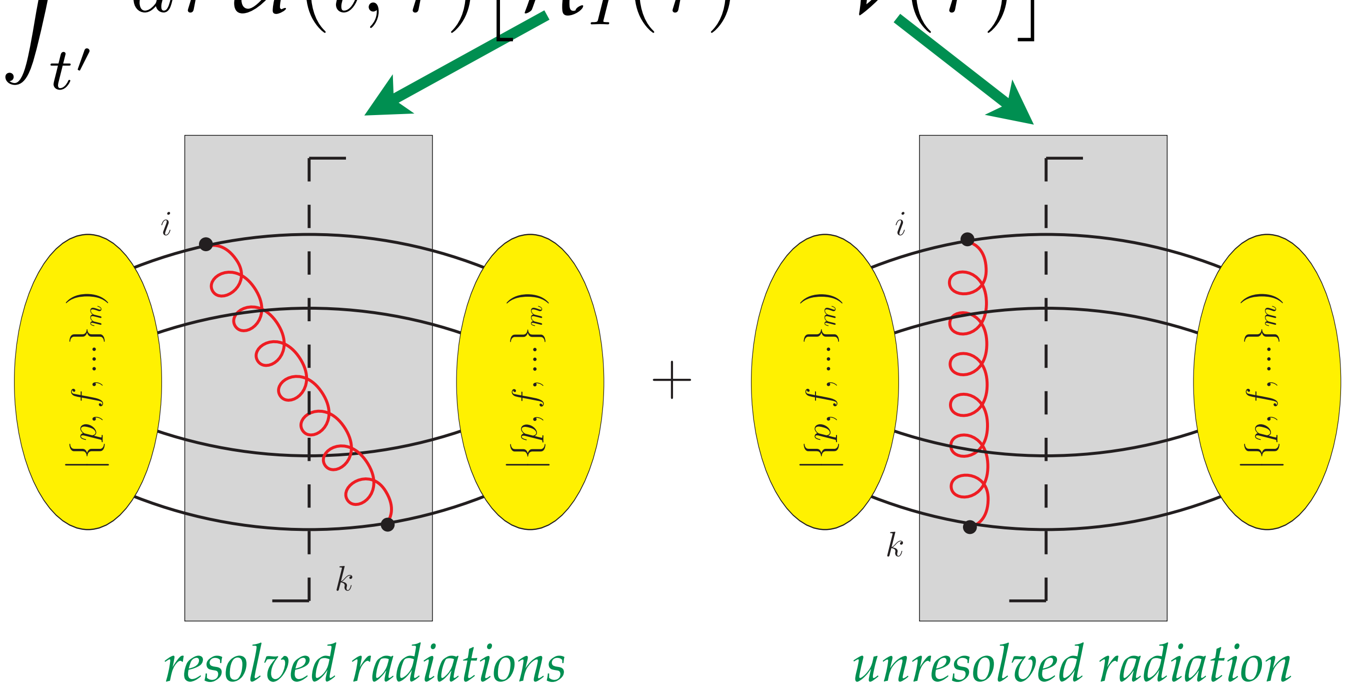
Shower Evolution

Using the factorization properties of the QCD the approximated order by order calculation can be organized according to

$$\mathcal{U}(t,t') = 1 + \int_{t'}^{t} d\tau\, \mathcal{U}(t,\tau)\big[\mathcal{H}_I(\tau) - \mathcal{V}(\tau)\big]$$

From the unitary condition:

$$\big(1\big|\mathcal{V}(t) = \big(1\big|\mathcal{H}_\mathrm{I}(t)$$



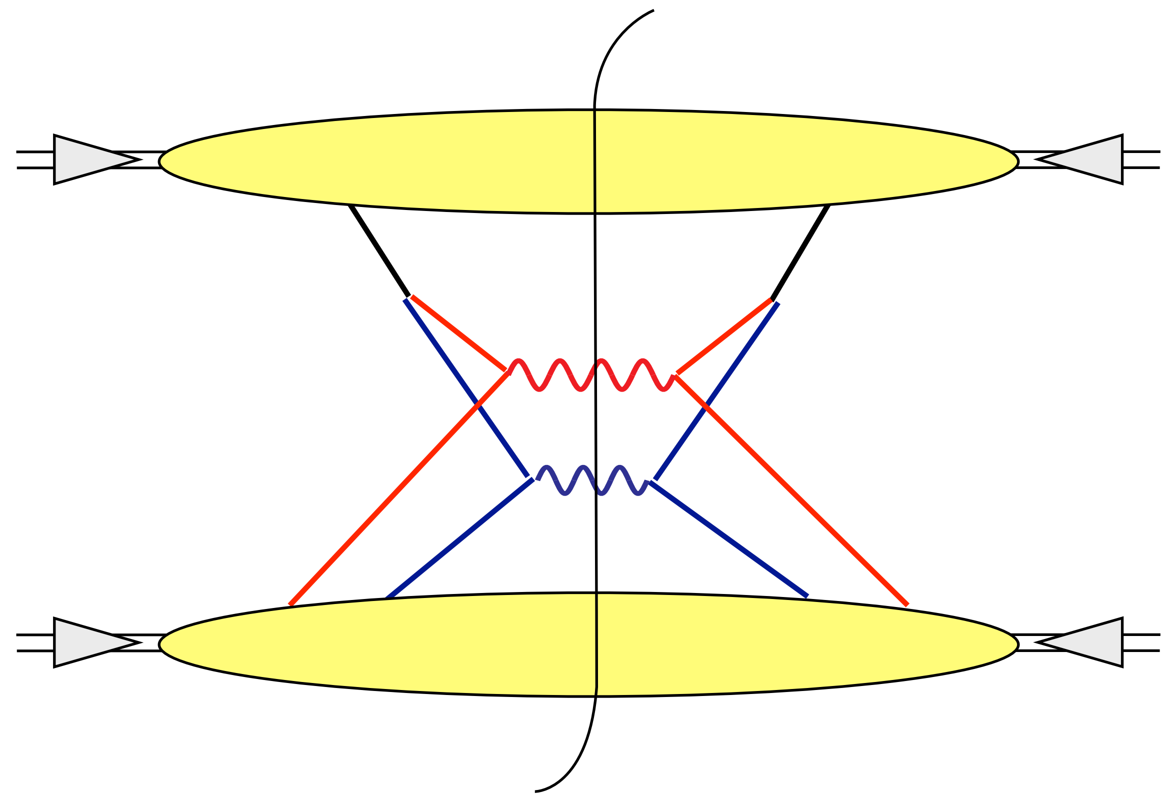
The shower form of the solution is

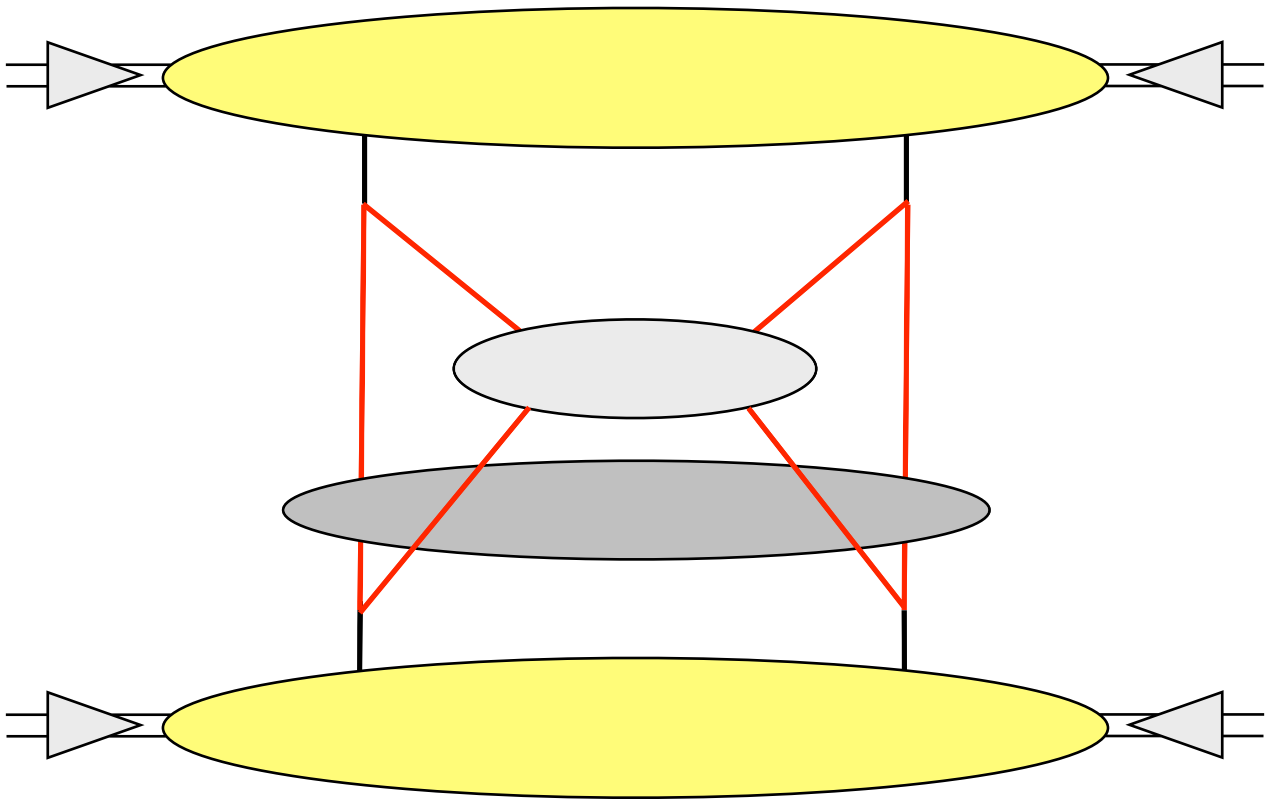
$$\mathcal{U}(t,t') = \mathcal{N}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}(t,\tau)\, \mathcal{H}_I(\tau)\, \mathcal{N}(\tau,t')$$

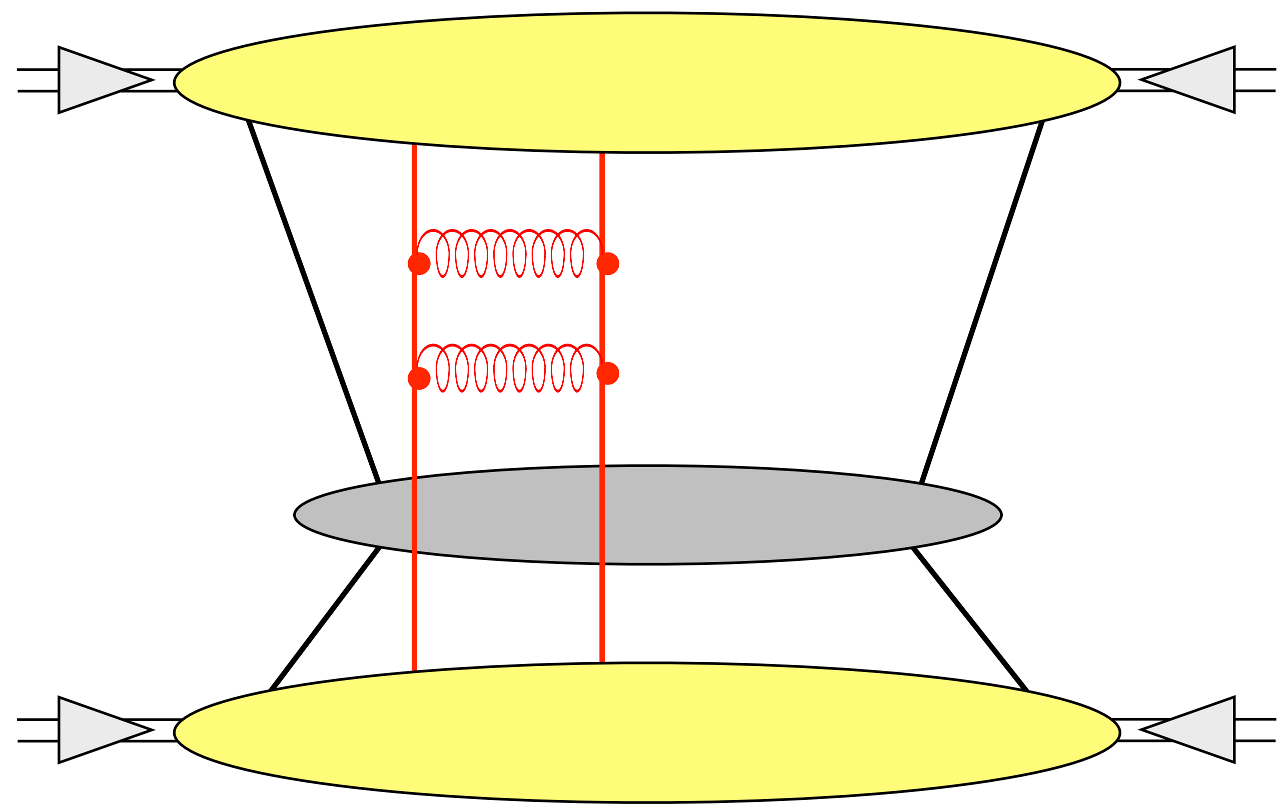
and the Sudakov operator is

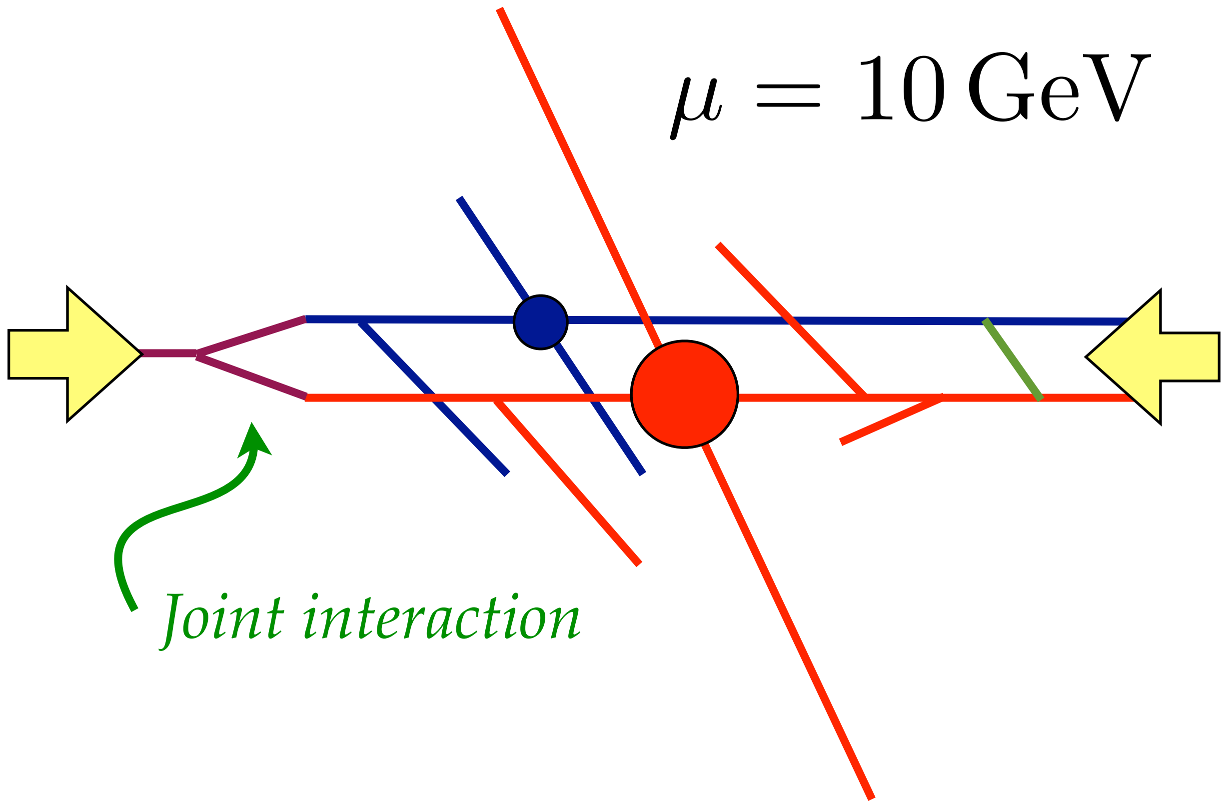
$$\mathcal{N}(t,t') = \mathbb{T}\exp\left(-\int_{t'}^{t} d\tau\, \mathcal{V}(\tau)\right)$$

This graph is singular








$\mu = 10\,\mathrm{GeV}$
Joint interaction